1.0 intro

In[∘]:=
```
Clear[H, Hs]
H = RandomVariate[NormalDistribution[], {6, 6}];
MatrixForm[H]
Eigenvalues[H]

Hs = (H + Transpose[H]) / 2;
MatrixForm[Hs]
Eigenvalues[Hs]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0.116971 & -1.00938 & -0.460445 & -0.648554 & -0.279209 & 0.805004 \\ -1.27886 & 1.29483 & 0.289786 & -0.7103 & -0.634491 & 0.215866 \\ -1.10051 & -1.33792 & -0.324435 & -0.25535 & -0.893412 & -0.505374 \\ 1.14786 & 0.968773 & -1.11693 & -0.831789 & 0.939036 & -0.0300643 \\ 0.0634135 & 0.447728 & 0.615986 & -1.1057 & 1.1296 & 0.232138 \\ 1.11879 & 0.942648 & 0.682628 & 1.07657 & 0.751377 & 0.26319 \end{pmatrix}$$

Out[∘]=
```
{1.7444 + 0. i, 0.505801 + 0.857755 i, 0.505801 - 0.857755 i,
 -0.420849 + 0.25378 i, -0.420849 - 0.25378 i, -0.265942 + 0. i}
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0.116971 & -1.14412 & -0.780477 & 0.249653 & -0.107898 & 0.961898 \\ -1.14412 & 1.29483 & -0.524068 & 0.129237 & -0.0933818 & 0.579257 \\ -0.780477 & -0.524068 & -0.324435 & -0.686138 & -0.138713 & 0.0886271 \\ 0.249653 & 0.129237 & -0.686138 & -0.831789 & -0.0833303 & 0.523255 \\ -0.107898 & -0.0933818 & -0.138713 & -0.0833303 & 1.1296 & 0.491758 \\ 0.961898 & 0.579257 & 0.0886271 & 0.523255 & 0.491758 & 0.26319 \end{pmatrix}$$

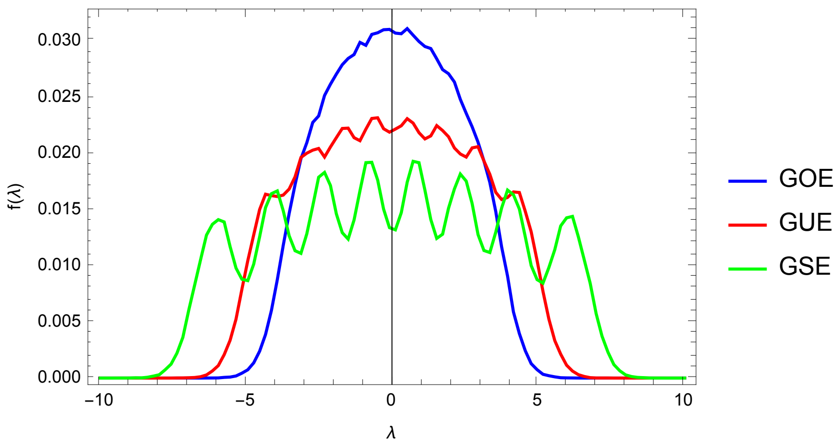
Out[∘]=
```
{-2.02114, 1.99428, 1.64009, 1.07123, -1.03916, 0.00305268}
```

Figure 1.1

In[768]:=
```
Clear[N, H, Hs, T, getGOE, getGUE, getGSE, eigGOE,
 eigGUE, eigGSE, histGOE, histGUE, histGSE, λdim, λlst]

(* Gaussian Orthogonal Ensemble *)
getGOE[N_] := Module[{H, Hs},
  H = RandomVariate[NormalDistribution[], {N, N}];
  Hs = (H + ConjugateTranspose[H]) / 2;
  Return[Eigenvalues[Hs]]
 ]

(* Gaussian Unitary Ensemble *)
getGUE[N_] := Module[{H, Hs, Has, M},
  M = RandomVariate[NormalDistribution[], {N, N}] +
    I RandomVariate[NormalDistribution[], {N, N}];
  Hs = (M + ConjugateTranspose[M]) / 2;
```

```
  Return[Eigenvalues[Hs]]
 ]

(* Gaussian Symplectic Ensemble *)
getGSE[N_] := Module[{H, X, Y, A},
  X = RandomVariate[NormalDistribution[], {N, N}] +
    I RandomVariate[NormalDistribution[], {N, N}];
  Y = RandomVariate[NormalDistribution[], {N, N}] +
    I RandomVariate[NormalDistribution[], {N, N}];
  A = ArrayFlatten[{{X, Y}, {-Conjugate[Y], Conjugate[X]}}];
  H = (A + ConjugateTranspose[A]) / 2;
  Return[DeleteDuplicates[Eigenvalues[H]]]
 ]

N = 8;
T = 50 000; (*50000;*)
λdim = 100; (*120*)
λlst = Table[x, {x, -10, +10, 20/(λdim - 1)}];
eigGOE = Flatten[Table[getGOE[N], {i, 1, T}]];
eigGUE = Flatten[Table[getGUE[N], {i, 1, T}]];
eigGSE = Flatten[Table[getGSE[N], {i, 1, T}]];

histGOE = Transpose[{λlst, BinCounts[eigGOE, {-10, +10, 20/λdim}] / (N T)}];
histGUE = Transpose[{λlst, BinCounts[eigGUE, {-10, +10, 20/λdim}] / (N T)}];
histGSE = Transpose[{λlst, BinCounts[eigGSE, {-10, +10, 20/λdim}] / (2 N T)}];
ListPlot[{histGOE, histGUE, histGSE},
 Joined → True, Frame → True, FrameLabel → {"λ", "f(λ)"},
 PlotStyle → {Blue, Red, Green}, PlotLegends → {"GOE", "GUE", "GSE"}]

Sqrt[Variance[eigGSE] / Variance[eigGOE]]
Sqrt[Variance[eigGUE] / Variance[eigGOE]]
Variance[eigGOE]^0.5
```

Out[781]=



Out[782]= 1.81189

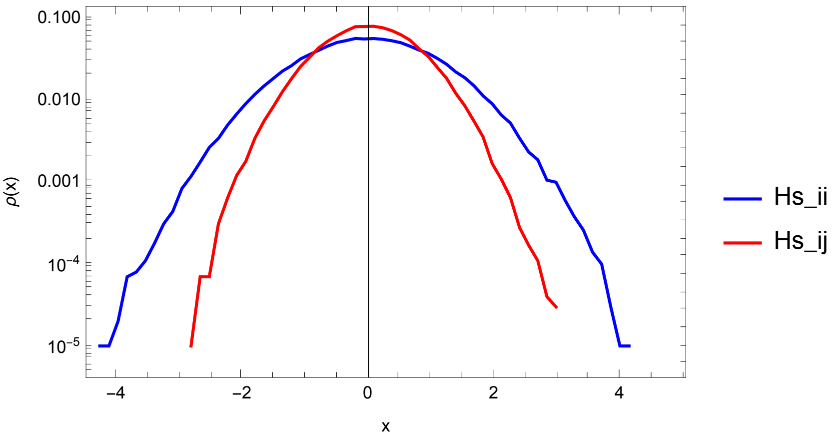
Out[783]= 1.33343

Out[784]= 2.12138

Question 1.3 jpdf of the N(N-1)/2 entries of symmetric matrix Hs

In[475]:=
```
Clear[N, H, Hs, T, getGOE, getGUE, eigGOE, eigGUE,
 histGOE, histGUE, λdim, λlst, xii, xij, μii, σii, μij, σij]

(* real symmetric matrix *)
getHs[N_] := Module[{H, Hs},
   H = RandomVariate[NormalDistribution[], {N, N}];
   Hs = (H + Transpose[H]) / 2;
   Return[Hs]
  ]
T = 100 000;
tabii = Table[getHs[2][[1, 1]], {i, 1, T}];
tabij = Table[getHs[2][[1, 2]], {i, 1, T}];

λdim = 70;
λlst = Table[x // N, {x, -5, +5, 10/(λdim - 1)}];
xii = BinCounts[tabii // N, {-5, +5, 10/λdim}] / (T);
histii = Transpose[{λlst, xii}];
xij = BinCounts[tabij // N, {-5, +5, 10/λdim}] / (T);
histij = Transpose[{λlst, xij}];
ListLogPlot[{histii, histij},
 Joined → True, Frame → True, FrameLabel → {"x", "ρ(x)"},
 PlotStyle → {Blue, Red}, PlotLegends → {"Hs_ii", "Hs_ij"}]

(* average and spread for diagonal entries *)
μii = Total[λlst xii] / Total[ xii];
σii = Sqrt[Total[(λlst - μii)^2 xii] / Total[ xii]];

(* average and spread for off-diagonal entries *)
μij = Total[λlst xij] / Total[ xij];
σij = Sqrt[Total[(λlst - μij)^2 xij] / Total[ xij]];

(* ratio of variances *)
(σii / σij)^2
```

Out[486]=



Out[491]= 2.01735