Flexible Snow Model (FSM2) scientific documentation

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The Flexible Snow Model allows alternative process parametrizations to be combined in a complete model of the mass and energy balances of snow on the ground and in forest canopies. Existing models from which parametrizations are taken for inclusion in FSM2 include CLASS, CLM, Crocus, ISBA and MOSES. Parametrizations are selected by setting option numbers in a text file before the model is compiled. Parameter values and input data are read from text files when the model is run. Physical constants, meteorological driving variables, site characteristics, model state variables and parameters are listed in tables 1 to 5; refer to these tables for any variables that are not explicitly defined in the text.

1   Snow-free and snow-covered ground albedos

Bare ground with albedo α_0 and snow cover fraction f_s with albedo α_s have average albedo

$$\alpha_g = (1 - f_s)\alpha_0 + f_s\alpha_s. \tag{1}$$

The snow cover is a function of snow depth h parametrized as

$$f_s(h) = \frac{h}{h + h_f} \tag{2}$$

if option `SNFRAC=0` is set in the compilation file, and

$$f_s(h) = \tanh\left(\frac{h}{h_f}\right) \tag{3}$$

if `SNFRAC=1`.

1.1   Diagnosed snow albedo (option `ALBEDO=0`)

Following a common approach in earlier generations of climate models, snow albedo is diagnosed as a function of surface temperature

$$\alpha_s(T_*) = \alpha_{\min} + (\alpha_{\max} - \alpha_{\min})\min\left(\frac{T_* - T_m}{T_\alpha}, 1\right). \tag{4}$$

1.2   Prognostic snow albedo (option `ALBEDO=1`)

Based on CLASS and ISBA, decreasing albedo as snow ages with time t and increasing albedo as fresh snow falls at rate S_f are parametrized by

$$\frac{d\alpha_s}{dt} = \frac{1}{\tau_\alpha}(\alpha_{\min} - \alpha_s) + \frac{S_f}{S_\alpha}(\alpha_{\max} - \alpha_s), \tag{5}$$

where the timescale τ_α has different values τ_{cold} and τ_{melt} for cold and melting snow. Equation (5) is implemented by integrating over a timestep of length δt to give the change in snow albedo as

$$\delta\alpha_s = (\alpha_{\lim} - \alpha_s)(1 - e^{-\gamma\delta t}), \tag{6}$$

where

$$\gamma = \frac{1}{\tau_\alpha} + \frac{S_f}{S_\alpha} \tag{7}$$

and

$$\alpha_{\lim} = \frac{1}{\gamma}\left(\frac{1}{\tau_\alpha}\alpha_{\min} + \frac{S_f}{S_\alpha}\alpha_{\max}\right). \tag{8}$$

2 Snow density

2.1 Compaction

Snow may be assumed to have constant density ρ_0 (option `DENSTY=0`) or to compact at rate

$$\frac{d\rho_s}{dt} = f_\rho(\rho_s, T_s). \tag{9}$$

Snow density ρ_s in layer k with ice mass I and liquid water mass W at the beginning of timestep n is diagnosed as

$$\rho_{s,k}^{(n)} = \frac{I_k^{(n)} + W_k^{(n)}}{D_{s,k}}. \tag{10}$$

Equation (9) is then implemented as

$$\rho_{s,k}^{(n+1)} = \rho_{s,k}^{(n)} + f_\rho\left(\rho_{s,k}^{(n)}, T_{s,k}^{(n)}\right)\delta t. \tag{11}$$

Finally, the thickness of the compacted layer at the end of the timestep is calculated by inverting Equation (10).

2.1.1 Empirical maximum densities (option `DENSTY=1`)

Based on CLASS, the compaction rate function is

$$f_\rho = \frac{1}{\tau_\rho}(\rho_{\max} - \rho_s) \tag{12}$$

with the same time constant τ_ρ but different asymptotic values ρ_{cold} and ρ_{melt} for $\rho_{\max}$ in cold and melting snow.

2.1.2 Viscous compaction by overburden and thermal metamorphism (option `DENSTY=2`)

Following ISBA, the compaction rate function is

$$f_\rho = \rho_s\left\{\frac{gm}{\eta} + c_1\exp\left[\frac{(T_s - T_m)}{23.8} - \max\left(\frac{\rho_s - 150}{21.7}, 0\right)\right]\right\} \tag{13}$$

where

$$\eta = \eta_0\exp\left[-\frac{(T_s - T_m)}{12.4} + \frac{\rho_s}{55.6}\right]. \tag{14}$$

and the snow mass overlying the middle of a layer is

$$m_k = \sum_{j=1}^{k-1}\left[I_j^{(n)} + W_j^{(n)}\right] + 0.5\left[I_k^{(n)} + W_k^{(n)}\right]. \tag{15}$$

2.2 Fresh snow density

Fresh snow deposited with density ρ_f over a timestep increases the snow depth before compaction by $\rho_f^{-1}S_f\delta t$. Unless snow density is fixed, the density of fresh snow is taken from

$$\rho_f = \max[\rho_0 + b_\rho(T_a - T_m) + c_\rho U_a^{1/2}, 50]. \tag{16}$$

Parameter values can be selected to match CLM ($\rho_0 = 100$ kg m^{-3}) or Crocus ($\rho_0 = 109$ kg m^{-3}, $a_\rho = 6$ kg m^{-3} K^{-1} and $b_\rho = 26$ kg m$^{-7/2}$ s$^{1/2}$). Snow unloading from a forest canopy is added to snow on the ground with the bulk density of the existing snow.
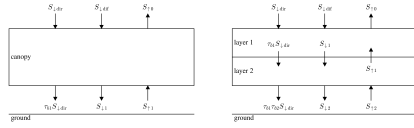
Figure 1: Reflection and transmission of diffuse and direct-beam shortwave radiation by a forest canopy and diffuse reflection by the ground surface.

3 Canopy radiative transfer

Forest structure is defined by canopy height h, vegetation area index Λ (including leaves and stems) and vegetation fraction f_v, which is either parametrized as $1 - \exp(-k_{\text{veg}}\Lambda)$ or specified as an input. The fraction of radiation incident from above at elevation angle θ that is transmitted through the canopy without interception is parametrized as

$$\tau_{\text{dir}} = \exp[-(G_1 + G_2 \sin\theta)\Lambda/\sin\theta]. \tag{17}$$

The canopy transmissivity for diffuse radiation is parametrized as

$$\tau_{\text{dif}} = \exp(-k_{\text{dif}}\Lambda) \tag{18}$$

by default but can also be specified as a sky view fraction f_{sky} for sites which are not directly under trees ($\Lambda = 0$) but which are shaded by surrounding vegetation or topography ($f_{\text{sky}} < 1$). In principle, the parameters G_1, G_2 and k_{dif} are not independent.

3.1 Shortwave radiation

A forest canopy with intercepted snow mass S_v and interception capacity S_{cap} is assumed to have snow cover fraction

$$f_{cs} = \frac{S_v}{S_{\text{cap}}} \tag{19}$$

and albedo

$$\alpha_c = f_v[(1 - f_{cs})\alpha_{c0} + f_{cs}\alpha_{cs}]. \tag{20}$$

As illustrated in Figure 1, the canopy is assumed to reflect fraction α_c of incoming shortwave radiation, but radiation reflected from the ground is transmitted and absorbed by the canopy without further reflections. For incoming shortwave radiation made up of a diffuse component S_{dif} and a direct-beam component S_{dir}, the net absorption is

$$SW_v = (1 - \alpha_c)[1 - (1 - \alpha_g)\tau_{\text{dif}} - \alpha_g \tau_{\text{dif}}\tau_{\text{dif}}]S_{\text{dif}} + \tag{21}$$
$$(1 - \alpha_c)[1 - (1 - \alpha_g)\tau_{\text{dir}} - \alpha_g \tau_{\text{dif}}\tau_{\text{dir}}]S_{\text{dir}} \tag{22}$$

by the forest canopy and

$$SW_g = (1 - \alpha_c)(1 - \alpha_g)(\tau_{\text{dif}}S_{\text{dif}} + \tau_{\text{dir}}S_{\text{dir}}). \tag{23}$$

by the ground or snow surface. The effective above-canopy albedo, including shortwave radiation reflected from the canopy and the ground, is

$$\alpha = \alpha_c + (1 - \alpha_c)\alpha_g\tau_{\text{dif}}\left(\frac{\tau_{\text{dif}}S_{\text{dif}} + \tau_{\text{dir}}S_{\text{dir}}}{S_{\text{dif}} + S_{\text{dir}}}\right). \tag{24}$$

Usually, only measurements of global shortwave radiation are available. Option `SWPART=0` treats all of the global radiation as diffuse and option `SWPART=1` partitions global shortwave radiation into direct and diffuse components as described in Appendix A. If separate measurements of direct and diffuse components are available, they can be read as inputs using option `SWPART=2`. *Two-stream radiative transfer to be added as an option*

3.2 Longwave radiation

The net absorption of longwave radiation is

$$LW_v = (1 - f_{\text{sky}})(LW_\downarrow + \sigma T_*^4 - 2\sigma T_v^4) \tag{25}$$

by the forest canopy and

$$LW_g = f_{\text{sky}}LW_\downarrow - \sigma T_*^4 + (1 - f_{\text{sky}})\sigma T_v^4 \tag{26}$$

by the ground or snow surface. Upwelling longwave radiation above a forest canopy is

$$LW_\uparrow = f_{\text{sky}}\sigma T_*^4 + (1 - f_{\text{sky}})\sigma T_v^4. \tag{27}$$

4 Turbulent fluxes

4.1 Roughness lengths and aerodynamic resistances

Roughness lengths for snow-free ground and snow on fraction f_s of the ground are combined to give a ground roughness length

$$z_{0g} = z_{0f}^{1-f_s} z_{0s}^{f_s}. \tag{28}$$

For vegetation of height h covering fraction f_v of the ground, the roughness length and displacement height are $z_{0v} = R_{z0}h$ and $d = f_v R_d h$. The combined surface roughness length is

$$z_0 = z_{0g}^{1-f_v} z_{0v}^{f_v}. \tag{29}$$

The aerodynamic resistance network for turbulent exchanges of heat between the ground, vegetation and the atmosphere is shown in Figure 2. The resistance between the canopy air space and the atmosphere is

$$r_{aa} = \frac{1}{f_H k u_*} \ln\left(\frac{z_T - d}{z_0}\right) \tag{30}$$

with atmospheric stability factor f_H and friction velocity

$$u_* = k U_a \left[\ln\left(\frac{z_U - d}{z_0}\right)\right]^{-1}. \tag{31}$$

Vegetated and unvegetated fractions of the ground share a common surface temperature but have separate aerodynamic resistances that combine in parallel to give resistance

$$r_{ag} = \frac{1}{k u_*}\left[\frac{(1 - f_v)f_h}{\ln(z_{0g}/z_{0h})} + f_v f_c C_{\text{dense}}\right]^{-1} \tag{32}$$

between the ground and the canopy air space, with sub-canopy stability factor f_c. The roughness length for heat z_{0h} is assumed to be equal to $0.1z_{0g}$. Between vegetation and the canopy air space, the aerodynamic resistance is

$$r_{av} = \frac{C_{\text{veg}}}{\Lambda u_*^{1/2}}. \tag{33}$$

C_{dense}, C_{veg} are constant parameters. In the absence of vegetation ($\Lambda = f_v = 0$), resistances for heat transfer combine to give

$$r_h \equiv r_{aa} + r_{ag} = \frac{1}{f_h k^2 U_a}\ln\left(\frac{z_U}{z_{0g}}\right)\ln\left(\frac{z_T}{z_{0h}}\right). \tag{34}$$

4.1.1 Neutral stability (option EXCHNG=0)
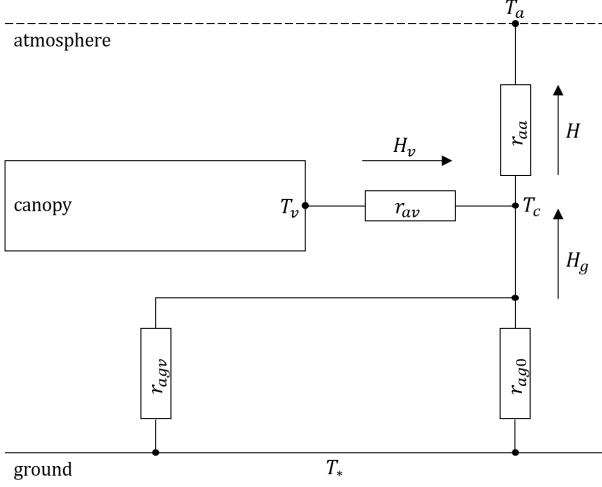
Atmospheric stability is neglected by setting $f_H = 1$.

Figure 2: Aerodynamic resistance network for heat exchanges between the ground, vegetation and the atmosphere.

4.1.2 Richardson number stability functions (option `EXCHNG=1`)

Atmospheric stability is characterized by a bulk Richardson number

$$\text{Ri}_\text{B} = \frac{g(z_U - d)^2[T_a - f_v T_v - (1 - f_v)T_*]}{(z_T - d)T_a U_a^2}. \tag{35}$$

The aerodynamic resistances in Equation (30) and (32) are adjusted by factor

$$f_H = \begin{cases} [1 + 3b_h \text{Ri}_\text{B}(1 + b_h \text{Ri}_\text{B})^{1/2}]^{-1} & \text{Ri}_\text{B} \geq 0 \\ 1 - 3b_h \text{Ri}_\text{B}[1 + c(-\text{Ri}_\text{B})^{1/2}]^{-1} & \text{Ri}_\text{B} < 0 \end{cases} \tag{36}$$

with

$$c = 3b_h^2 k^2 \left(\frac{z_U}{z_0}\right)^{1/2} \left[\ln\left(\frac{z_U}{z_0}\right)\right]^{-2}. \tag{37}$$

The sub-canopy stability factor is

$$f_c = \frac{1}{1 + \text{Ri}_c/2} \tag{38}$$

with Richardson number

$$\text{Ri}_c = \frac{gh(T_c - T_*)}{T_c u_*^2} \tag{39}$$

limited to values in the range 0 - 10.

4.1.3 Obukhov length stability functions (*option to be added*)

4.2 Snow on short vegetation and bare ground

Sensible heat and moisture fluxes from the ground or snow surface without exposed vegetation to the atmosphere are

$$H_g = \frac{\rho c_p}{r_h}(T_* - T_a) \tag{40}$$

and

$$E_g = \frac{\rho \psi_g}{r_h}[Q_\text{sat}(T_*, P_s) - Q_a] \tag{41}$$

where Q_sat is the specific humidity at saturation over water or ice (Appendix B) and $\rho = P_s/(R_\text{air}T_a)$ is the air density. The moisture availability factor ψ_g is set to 1 if there is snow on the ground or if $Q_\text{sat}(T_*, P_s) < Q_a$ (moisture flux onto the surface) and

$$\psi_g = \frac{r_h}{r_h + r_{sg}} \tag{42}$$

otherwise, where r_{sg} is a resistance for evaporation from soil moisture. Without exposed vegetation, moisture and sensible heat fluxes to the atmosphere are $E = E_g$ and $H = H_g$. The latent heat flux is LE, with L taken to be the latent heat of vapourisation if $T_* > T_m$ and the latent heat of sublimation otherwise.

4.3 Forest canopies and underlying ground

Sensible heat fluxes are

$$H = \frac{\rho c_p}{r_{aa}}(T_c - T_a) \tag{43}$$

upwards from the canopy air space,

$$H_g = \frac{\rho c_p}{r_{ag}}(T_* - T_c) \tag{44}$$

from the ground to the canopy air space, and

$$H_v = \frac{\rho c_p}{r_{av}}(T_v - T_c) \tag{45}$$

from the vegetation to the canopy air space. Similarly, moisture fluxes are

$$E = \frac{\rho}{r_{aa}}(Q_c - Q_a) \tag{46}$$

upwards from the canopy air space,

$$E_g = \frac{\rho \psi_g}{r_{ag}}[Q_{\text{sat}}(T_*, P_s) - Q_c] \tag{47}$$

from the ground to the canopy air space, and

$$E_v = \frac{\rho \psi_v}{r_{av}}[Q_{\text{sat}}(T_v, P_s) - Q_c] \tag{48}$$

from the vegetation to the canopy air space, with moisture availability factor $\psi_v = 1$ if there is snow on the canopy or if $Q_{\text{sat}}(T_v) < Q_c$ (moisture flux onto the canopy) and

$$\psi_v = \frac{r_{av}}{r_{av} + r_{sg}} \tag{49}$$

otherwise.

5 Energy balance

5.1 Snow on short vegetation and bare ground

The surface energy balance

$$f(T_*) = LW_g + SW_g - G - H_g - LE_g - L_f M = 0 \tag{50}$$

with flux parametrizations substituted from Equations (26), (40) and (41) is a nonlinear equation for the unknown surface temperature and snowmelt rate M. From an initial guess of temperature T_{*0} and no melt, a linear estimate of T_* is given by

$$T_* = T_{*0} - f(T_{*0}) \left(\frac{df}{dT_*} \right)^{-1}. \tag{51}$$

A single application of Equation (51) gives an approximate solution, and repeated applications with T_* calculated in each iteration being used as T_{*0} in the next is the Newton-Raphson method for solving Equation (50). Neglecting the complicated temperature dependence of r_{aa}, this gives

$$T_* = T_{*0} + \frac{LW_g + SW_g - G - H_g - LE_g - L_f M}{4\sigma T_{*0}^3 + 2\lambda_1/\Delta h_1 + \rho(c_p + LD_g\psi_g)/r_h}. \tag{52}$$

where the fluxes in the numerator are calculated using $T_* = T_{*0}$, and

$$D_g = \left. \frac{dQ_{\text{sat}}}{dT} \right|_{T=T_{*0}} = \frac{LQ_{\text{sat}}(T_{*0})}{R_{\text{wat}} T_{*0}^2}. \tag{53}$$

Equation (52) is first evaluated with $M = 0$. If this gives $T_* > T_m$ and there is snow with ice mass I on the ground, Equation (52) is re-evaluated assuming that all of the snow melts and $M = I/\delta t$. If this gives $T_* < T_m$, the snow does not all melt and $T_* = T_m$ is known; Equation (50) is solved instead for the unknown melt rate by substitution of $T_* = T_m$ in the equations for the other fluxes.

5.2 Forest canopies and underlying ground

5.2.1 Zero-layer canopy model (option CANMOD=0)

The zero-layer canopy model does not attempt to calculate the canopy energy balance, instead assuming that the canopy temperature is equal to the air temperature. Substituting $T_v = T_a$ in Equations (43) to (48) and rearranging gives

$$H_g = \frac{\rho c_p}{r_h}(T_* - T_a) \tag{54}$$

with

$$r_h = r_{ag} + \left(\frac{1}{r_{aa}} + \frac{1}{r_{av}}\right)^{-1}. \tag{55}$$

and

$$E_g = \frac{\rho \psi_g}{r_h}(T_* - T_a) \tag{56}$$

with

$$\frac{r_h}{\psi_g} = r_{ag} + r_{sg} + \left(\frac{1}{r_{aa}} + \frac{\psi_v}{r_{av}}\right)^{-1}. \tag{57}$$

Surface temperature and melt rate are found by solving Equations (52) and (50) with the modified aerodynamic resistance and moisture factor from Equations (55) and (57) instead of Equations (34) and (42). The rate of moisture transfer from or to the canopy is

$$E_v = -\left(\frac{\psi_v r_{aa}}{\psi_v r_{aa} + r_{av}}\right)E_g. \tag{58}$$

5.2.2 One-layer canopy model (option CANMOD=1)

Energy and mass conservation equations

$$f_1 = (H - H_g - H_v)/(\rho c_p) = 0, \tag{59}$$

$$f_2 = (E - E_g - E_v)/\rho = 0, \tag{60}$$

$$f_3 = LW_g + SW_g - G - H_g - LE_g = 0, \tag{61}$$

and

$$f_4 = LW_v + SW_v - H_v - LE_v - C_{can}\frac{dT_v}{dt} = 0, \tag{62}$$

form a set of four nonlinear equations with four unknowns: Q_c, T_c, T_* and T_v. Writing vectors $\mathbf{f} = (f_1, f_2, f_3, f_4)^T$ and $\mathbf{x} = (Q_c, T_c, T_*, T_v)^T$, a solution is found by iterating

$$\mathbf{x} = \mathbf{x}_0 - \mathbf{J}^{-1}\mathbf{f}(\mathbf{x}_0) \tag{63}$$

where J is the Jacobian matrix of $\mathbf{f}$ with elements

$$\mathrm{J}_{ij} = \frac{\partial f_i}{\partial x_j} \tag{64}$$

or

$$\mathrm{J} = \begin{bmatrix} 0 & -\left(r_{aa}^{-1} + r_{ag}^{-1} + r_{av}^{-1}\right) & r_{ag}^{-1} & r_{av}^{-1} \\ -\left(r_{aa}^{-1} + \psi_g r_{ag}^{-1} + \psi_v r_{av}^{-1}\right) & 0 & \psi_g D_g r_{ag}^{-1} & \psi_v D_v r_{av}^{-1} \\ -L\rho\psi_g r_{ag}^{-1} & -\rho c_p r_{ag}^{-1} & J_{33} & -4f_v\sigma T_v^3 \\ -L\rho\psi_v r_{av}^{-1} & -\rho c_p r_{av}^{-1} & -4f_v\sigma T_*^3 & J_{44} \end{bmatrix} \tag{65}$$

with

$$J_{33} = (c_p + L\psi_g D_g)\frac{\rho}{r_{ag}} + 4\sigma T_*^3 + 2\frac{\lambda_1}{\Delta h_1} \tag{66}$$

and

$$J_{44} = \frac{C_{can}}{\delta t} + (c_p + L\psi_v D_v)\frac{\rho}{r_{av}} + 8f_v\sigma T_v^3 \tag{67}$$

Equation (63) is implemented by solving

$$J(\mathbf{x} - \mathbf{x}_0) = \mathbf{f}(\mathbf{x}_0) \tag{68}$$

for $\mathbf{x}$ by LU decomposition. If this gives $T_* > T_m$ and there is snow with ice mass I on the ground, Equation (61) is replaced by

$$f_3 = LW_g + SW_g - G - H_g - LE_g - L_f M. \tag{69}$$

It is first assumed that all of the snow melts and Equation (68) is solved again with $M = I/\delta t$. If this gives $T_* < T_m$, the snow does not all melt and the surface temperature is known but the melt rate is unknown. Fluxes in f are recalculated with $T_* = T_m$, elements in the third column of J are replaced by $J_{13} = J_{23} = J_{43} = 0$ and $J_{33} = 1$, and $M = x_3/L_f$ is found from the solution of Equation (68).

5.2.3 Two-layer canopy model (*option to be added*)

6 Mass balance

A forest canopy can intercept a fraction of falling snow up to a maximum capacity which is either parametrized as $S_{\mathrm{cap}} = c_{\mathrm{vai}}\Lambda$ or read from a file. If the canopy holds snow mass S_v at the beginning of a timestep, the amount of snow intercepted in the timestep is

$$\delta S_v = (S_{\mathrm{cap}} - S_v)\left[1 - \exp\left(-\frac{f_v S_f \delta t}{S_{\mathrm{cap}}}\right)\right] \tag{70}$$

and the rate of snowfall reaching the ground is reduced to

$$T_f = S_f - \frac{\delta S_v}{\delta t}. \tag{71}$$

Snow sublimates from the canopy at rate E_v and unloads at rate $U_c = \tau_{\mathrm{can}}^{-1} S_v$ with different values of the time constant τ_{can} for cold and melting snow; if $\tau_{\mathrm{can}} \leq \delta t$, all of the snow will unload immediately from the canopy. Interception and storage of liquid water in the canopy are neglected. The combined mass balance equation for canopy snow is

$$\frac{dS_v}{dt} = S_f - T_f - U_c - E_v. \tag{72}$$

If there is no forest canopy, the throughfall and unloading rates are obviously $T_f = S_f$ and $U_c = 0$. The mass balance equations for ice and liquid water in snow on the ground are

$$\frac{dI}{dt} = T_f + U_c - E_g - M + F \tag{73}$$

and

$$\frac{dW}{dt} = R_f + M - F - R_o. \tag{74}$$

If storage of liquid water in snow is neglected (option `HYDROL=0`), internal refreezing rate in the snow $F = 0$ and runoff rate at the base of the snow $R_o = R_f + M$. If bucket storage is selected (option `HYDROL=1`), snow layer k with porosity

$$\phi_k = 1 - \frac{I_k}{\rho_{\mathrm{ice}} D_{s,k}} \tag{75}$$

can hold a maximum mass $\rho_{\mathrm{wat}}\phi_k W_{\mathrm{irr}}$ of liquid water.

Tables

Documentation	Code	Value
Heat capacity of air c_p	`cp`	1005 J K^{-1} kg^{-1}
Heat capacity of ice c_{ice}	`hcap_ice`	2100 J K^{-1} kg^{-1}
Heat capacity of water c_{wat}	`hcap_wat`	4180 J K^{-1} kg^{-1}
Acceleration due to gravity g	`g`	9.81 m s^{-2}
Solar constant I_0	`I0`	1367 W m^{-2}
Von Kármán constant k	`vkman`	0.4
Latent heat of fusion of water L_f	`Lf`	0.334×10^6 J kg^{-1}
Latent heat of sublimation of ice L_s	`Ls`	2.835×10^6 J kg^{-1}
Latent heat of vapourisation of water L_v	`Lv`	2.501×10^6 J kg^{-1}
Gas constant for air R_{air}	`Rair`	287 J K^{-1} kg^{-1}
Gas constant for water vapour R_{wat}	`Rwat`	462 J K^{-1} kg^{-1}
Melting point of ice T_m	`Tm`	273.15 K
Thermal conductivity of air λ_{air}	`hcon_air`	0.025 W m^{-1} K^{-1}
Thermal conductivity of clay λ_{clay}	`hcon_clay`	1.16 W m^{-1} K^{-1}
Thermal conductivity of ice λ_{ice}	`hcon_ice`	2.24 W m^{-1} K^{-1}
Thermal conductivity of sand λ_{sand}	`hcon_sand`	1.57 W m^{-1} K^{-1}
Thermal conductivity of water λ_{wat}	`hcon_wat`	0.56 W m^{-1} K^{-1}
Density of ice ρ_{ice}	`rho_ice`	917 kg m^{-3}
Density of water ρ_{wat}	`rho_wat`	1000 kg m^{-3}
Stefan-Boltzmann constant σ	`sb`	5.26×10^{-8} W m^{-2} K^{-4}

Table 1. Physical constants and quantities assumed to be constant in the code and documentation.

Documentation	Code	Units
Incoming longwave radiation $LW_\downarrow$	`LW`	W m^{-2}
Surface air pressure P_s	`Ps`	Pa
Specific humidity Q_a	`Qa`	kg kg^{-1}
Rainfall rate R_f	`Rf`	kg m^{-2} s^{-1}
Snowfall rate S_f	`Sf`	kg m^{-2} s^{-1}
Diffuse shortwave radiation S_{dif}	`Sdif`	W m^{-2}
Direct-beam shortwave radiation S_{dir}	`Sdir`	W m^{-2}
Incoming shortwave radiation $SW_\downarrow$	`SW`	W m^{-2}
Air temperature T_a	`Ta`	K
Wind speed U_a	`Ua`	m s^{-1}
Wind direction (not yet used)	`Udir`	degrees clockwise of N

Table 2. Meteorological driving variables in the code and documentation

Documentation	Code	Units
Forest canopy		
Canopy air space specific humidity Q_c	`Qcan`	kg kg^{-1}
Snow mass on canopy S_v	`Sveg`	W m^{-2}
Canopy air space T_c	`Tveg`	K
Vegetation temperature T_v	`Tveg`	K
Surface		
Surface skin temperature T_*	`Tsrf`	K
Snow on the ground (up to `Nsmax` layers)		
Number of snow layers N_{snow}	`Nsnow`	-
Albedo of snow α_s	`albs`	-
Thickness of snow layers D_s	`Ds`	m
Radii of grains in snow layers r	`rgrn`	m
Ice content of snow layers I	`Sice`	kg m^{-2}
Liquid water content of snow layers W	`Sliq`	kg m^{-2}
Temperature of snow layers T_s	`Tsnow`	K
Soil (`Nsoil` layers)		
Temperature of soil layers T_g	`Tsoil`	K
Volumetric moisture content of soil layers θ_g	`theta`	-

Table 3. Model state variables in the code and documentation.

Documentation	Code	Default
Snow-free albedo α_0	`alb0`	0.2
Canopy heat capacity C_{can}	`canh`	2500Λ (J K^{-1} m^{-2})
Soil clay fraction f_{clay}	`fcly`	0.3
Soil sand fraction f_{sand}	`fsnd`	0.6
Sky view fraction f_{sky}	`fsky`	$\exp(-k_{\mathrm{dif}}\Lambda)$
Canopy cover fraction f_{veg}	`fveg`	$1 - \exp(-\Lambda)$
Canopy height h_{can}	`hcan`	0 m
Latitude ϕ	`lat`	0°
Vegetation area index Λ	`VAI`	0
Snow-free ground roughness length z_{0g}	`z0sf`	0.1 m
Timestep δt	`dt`	3600 s
Temperature and humidity measurement height z_T	`zT`	2 m
Wind speed measurement height z_U	`zU`	10 m

Table 4. Site and driving data characteristics in the code and documentation.

Documentation	Code	Default
Maximum albedo for fresh snow $\alpha_{\max}$	`asmx`	0.8
Minimum albedo for melting snow $\alpha_{\min}$	`asmn`	0.5
Snow-free vegetation albedo α_{c0}	`avg0`	0.1
Snow-covered vegetation albedo α_{cs}	`avgs`	0.4
Atmospheric stability adjustment parameter b_h	`bstb`	5
Snow thermal conductivity exponent b_λ	`bthr`	2
Dense canopy turbulent transfer coefficient C_{dense}	`cden`	0.004
Canopy snow capacity per unit VAI c_{vai}	`cvai`	4.4 kg m^{-2}
Vegetation turbulent transfer coefficient C_{veg}	`cveg`	20
Snow cover fraction depth scale h_f	`hfsn`	0.1 m
Reference snow viscosity η_0	`eta0`	3.7×10^7 Pa s
Surface conductance for saturated soil g_{sat}	`gsat`	0.01 m s^{-1}
Leaf angle distribution parameter G_1	`Gcn1`	0.5
Leaf angle distribution parameter G_2	`Gcn2`	0
Diffuse radiation extinction coefficient k_{dif}	`kdif`	0.5
Fixed snow thermal conductivity λ_0	`kfix`	0.24 W m^{-1} K^{-1}
Canopy cover coefficient k_{veg}	`kveg`	1
Displacement height to canopy height ratio R_d	`rchd`	0.67
Roughness length to canopy height ratio R_{z0}	`rchz`	0.1
Fresh snow grain radius r_0	`rgr0`	5×10^{-5} m
Fixed snow density ρ_0	`rho0`	300 kg m^{-3}
Temperature factor in fresh snow density b_ρ	`rhob`	0 kg m^{-3} K^{-1}
Wind factor in fresh snow density c_ρ	`rhoc`	0 kg m$^{-7/2}$ s$^{1/2}$
Fresh snow density ρ_f	`rhof`	100 kg m^{-3}
Maximum density for cold snow ρ_{cold}	`rcld`	300 kg m^{-3}
Maximum density for melting snow ρ_{melt}	`rmlt`	500 kg m^{-3}
Snowfall to refresh albedo S_α	`Salb`	10 kg m^{-2}
Thermal metamorphism parameter c_1	`snda`	2.8×10^{-6} s^{-1}
Snow albedo decay temperature threshold T_α	`Talb`	-2°C
Canopy unloading time scale τ_{can} for cold snow	`tcnc`	240 h
Canopy unloading time scale τ_{can} for melting snow	`tcnm`	2.4 h
Cold snow albedo decay time scale τ_{cold}	`tcld`	1000 h
Melting snow albedo decay time scale τ_{melt}	`tmlt`	100 h
Snow compaction time scale τ_ρ	`trho`	200 h
Irreducible liquid water content of snow W_{irr}	`Wirr`	0.03
Ratio of roughness lengths for momentum and heat z_0/z_{0h}	`z0zh`	10
Snow surface roughness length z_{0s}	`z0sn`	0.01 m

Table 5. Model parameters in the code and documentation.

Appendix: Solar radiation

Following Iqbal (1983), solar declination δ (in radians) and equation of time E_t (in hours) are approximated by Fourier series

$$\delta = 0.006918 - 0.399912 \cos \Gamma + 0.070257 \sin \Gamma$$
$$- 0.006758 \cos 2\Gamma + 0.000907 \sin 2\Gamma$$
$$- 0.002697 \cos 3\Gamma + 0.001480 \sin 3\Gamma \tag{76}$$

and

$$E_t = (12/\pi)(0.000075 + 0.001868 \cos \Gamma - 0.032077 \sin \Gamma$$
$$- 0.014615 \cos 2\Gamma - 0.04089 \sin 2\Gamma) \tag{77}$$

for day of year d_n and day angle $\Gamma = 2\pi(d_n - 1)/365$. From a date given by intergers y, m and d for year, month and day, the day of year can be found using integer division in the magic formula

$$d_n = (7y)/4 - 7(y + (m + 9)/12)/4 + (275m)/9 + d - 30. \tag{78}$$

For hour h of a day at latitude ϕ, the hour angle is defined by

$$\omega = (\pi/12)(12 - h - E_t), \tag{79}$$

the sine of the solar elevation is

$$\sin\theta = \sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega \tag{80}$$

and the cosine of the solar azimuth is

$$\cos\psi = (\sin\theta\sin\phi - \sin\delta)/(\cos\theta\cos\phi). \tag{81}$$

Azimuth is measured anticlockwise from south, so ψ has the same sign as ω (positive in the morning and negative in the afternoon).

Separate direct and diffuse components of shortwave radiation are estimated by the empirical method of Erbs et al. (1982) if they are required but have not been measured. First, an atmospheric clearness parameter is found by dividing the global radiation by the radiation at the top of the atmosphere, giving

$$k_t = \frac{SW_\downarrow}{I_o\sin\theta}, \tag{82}$$

where $I_0 = 1367$ Wm^{-2} is the solar constant. The diffuse fraction is then estimated as

$$\frac{S_{\mathrm{dif}}}{S_0} = \begin{cases} 1 - 0.09k_t & k_t < 0.22 \\ 0.95 - 0.16k_t + 4.39k_t^2 - 16.64k_t^3 + 12.34k_t^4 & 0.22 < k_t < 0.8 \\ 0.165 & k_t > 0.8 \end{cases} \tag{83}$$

Direct radiation is the remainder $S_{\mathrm{dir}} = S_0 - S_{\mathrm{dif}}$.

If a digital elevation model is provided as an input, the array of elevations is assumed to follow the convention that the first row is at the north of the model domain and the first column is at the west.

$$\tan^{-1}\left[\sqrt{\left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}^2\right)}\right] \tag{84}$$

$$-\tan^{-1}\left(\frac{dz}{dy}\bigg/\frac{dz}{dx}\right) - \frac{\pi}{2} \tag{85}$$

Aspect is calculated to follow the same convention as solar azimuth and the Fortran `atan2` function is used to obtain the correct quadrant. Grid cells are indexed by row number i from north to south and column number j from west to east. The surface gradient is calculated by centred differences

$$\left(\frac{dz}{dx}, \frac{dz}{dy}\right)_{ij} = \left(\frac{z_{i,j+1} - z_{i,j-1}}{2\Delta x}, \frac{z_{i+1,j} - z_{i-1,j}}{2\Delta x}\right) \tag{86}$$

for grid spacing Δx.