Flexible Snow Model scientific documentation

Version 2.1.1

March 1, 2025

The Flexible Snow Model (FSM2) allows alternative process parametrizations to be combined in a complete model of the mass and energy balances of snow on the ground and in forest canopies. Parametrizations are selected by setting option numbers in a text file before the model is compiled. Parameter values and input data are read from text files when the model is run. Outputs can be written to text or netCDF files. Physical constants, meteorological driving variables, site characteristics, model state variables and parameters are listed in tables 1 to 6; refer to these tables for any variables that are not explicitly defined in the text. Calculations are described below in the order in which they are performed in the code.

1 Constants and variables

Table 1. Physical constants and quantities assumed to be constant in FSM2 (module `CONSTANTS`).

Constant	Value
Heat capacity of air c_p	1005 J K^{-1} kg^{-1}
Heat capacity of ice c_{ice}	2100 J K^{-1} kg^{-1}
Heat capacity of water c_{wat}	4180 J K^{-1} kg^{-1}
Saturation vapour pressure at T_m, e_0	611.213 Pa
Acceleration due to gravity g	9.81 m s^{-2}
Solar constant I_0	1367 W m^{-2}
Von Kármán constant k	0.4
Latent heat of fusion of water L_f	0.334×10^6 J kg^{-1}
Latent heat of sublimation of ice L_s	2.835×10^6 J kg^{-1}
Latent heat of vapourisation of water L_v	2.501×10^6 J kg^{-1}
Gas constant for air R_{air}	287 J K^{-1} kg^{-1}
Gas constant for water vapour R_{wat}	462 J K^{-1} kg^{-1}
Melting point of ice T_m	273.15 K
Ratio of molecular weights of water and dry air ε	0.622
Thermal conductivity of air λ_{air}	0.025 W m^{-1} K^{-1}
Thermal conductivity of clay λ_{clay}	1.16 W m^{-1} K^{-1}
Thermal conductivity of ice λ_{ice}	2.24 W m^{-1} K^{-1}
Thermal conductivity of sand λ_{sand}	1.57 W m^{-1} K^{-1}
Thermal conductivity of water λ_{wat}	0.56 W m^{-1} K^{-1}
Dynamic viscosity of water μ_{wat}	1.78×10^{-3} kg m^{-1}
Density of ice ρ_{ice}	917 kg m^{-3}
Density of water ρ_{wat}	1000 kg m^{-3}
Stefan-Boltzmann constant σ	5.26×10^{-8} W m^{-2} K^{-4}

Table 2. Meteorological driving variables.

Variable	Units
Incoming longwave radiation $LW_{\downarrow}$	W m^{-2}
Surface air pressure P_s	Pa
Specific humidity Q_a	kg kg^{-1}
Relative humidity RH	%
Rainfall rate R_f	kg m^{-2} s^{-1}

Snowfall rate S_f	kg m^{-2} s^{-1}
Incoming shortwave radiation $SW_\downarrow$	W m^{-2}
Air temperature T_a	K
Wind speed U_a	m s^{-1}

Table 3. State variables.

Variable	Units
Forest canopy (N_{cnpy} layers)	
Canopy air space specific humidity Q_c	kg kg^{-1}
Snow mass on canopy S_v	W m^{-2}
Canopy air space temperature T_c	K
Vegetation temperature T_v	K
Surface	
Surface skin temperature T_s	K
Snow on the ground (up to N_{smax} layers)	
Number of snow layers N_{snow}	-
Albedo of snow α_s	-
Thickness of snow layers D_{sn}	m
Radii of grains in snow layers r	m
Ice content of snow layers I	kg m^{-2}
Liquid water content of snow layers W	kg m^{-2}
Temperature of snow layers T_{sn}	K
Soil (N_{soil} layers)	
Temperature of soil layers T_{sl}	K
Volumetric moisture content of soil layers θ_{sl}	-

Table 4. Model layers.

Documentation	Namelist	Default
Fraction of vegetation in upper canopy layer f_Λ	fvg1	0.5
Maximum number of snow layers N_{smax}	`Nsmax`	3
Fixed snow layer thicknesses Δz_{sn}	`Dzsnow`	0.1, 0.2, 0.4 m
Number of soil layers N_{soil}	`Nsoil`	4
Soil layer thicknesses Δz_{sl}	`Dzsoil`	0.1, 0.2, 0.4, 0.8 m

Table 5. Site and driving data characteristics.

Documentation	Namelist	Default
Snow-free albedo α_0	`alb0`	0.2
Timestep δt	`dt`	3600 s
Latitude ϕ	`lat`	0°
Local time of solar noon h_{12}	`noon`	12:00
Vegetation area index Λ	`VAI`	0
Canopy height h_c	`vegh`	0 m
Temperature and humidity measurement height z_T	`zT`	2 m
Wind speed measurement height z_U	`zU`	10 m

Table 6. Parameters (subroutine `FSM2_PARAMS`).

Documentation	Namelist	Default
Snow-free dense canopy albedo α_{c0}	`acn0`	0.1
Snow-covered dense canopy albedo α_{cs}	`acns`	0.3
Minimum albedo for melting snow $\alpha_{\min}$	`asmn`	0.5
Maximum albedo for fresh snow $\alpha_{\max}$	`asmx`	0.85
Canopy element reflectivity $\alpha_{\Lambda 0}$	`avg0`	0.27

Canopy snow reflectivity $\alpha_{\Lambda s}$	`avgs`	0.65
Vegetation heat capacity per unit VAI C_Λ	`cvai`	3.6×10^4 J K^{-1} m^{-2}
Vegetation turbulent transfer coefficient C_{veg}	`cveg`	20
Reference snow viscosity η_0	`eta0`	3.7×10^7 Pa s
Exponential unloading timescale τ_u	`eunl`	240 hours
Soil clay fraction f_{clay}	`fcly`	0.3
Soil sand fraction f_{sand}	`fsnd`	0.6
Surface conductance for saturated soil g_{sat}	`gsat`	0.01 m s^{-1}
Surface conductance for snow-free vegetation g_{veg}	`gsnf`	0.01 m s^{-1}
Canopy base height h_b	`hbas`	2 m
Snow cover fraction depth scale h_f	`hfsn`	0.1 m
Canopy light extinction coefficient k_{ext}	`kext`	0.5
Fixed snow thermal conductivity λ_0	`kfix`	0.24 W m^{-1} K^{-1}
Leaf boundary layer resistance r_{leaf}	`leaf`	20 s$^{1/2}$ m$^{-1/2}$
Melt unloading fraction m_u	`munl`	0.4
Number of snow hydrology substeps N_{shyd}	`nhyd`	10
Maximum density for cold snow ρ_{cold}	`rcld`	300 kg m^{-3}
Fixed snow density ρ_0	`rfix`	300 kg m^{-3}
Fresh snow grain radius r_0	`rgr0`	5×10^{-5} m
Fresh snow density ρ_f	`rhof`	100 kg m^{-3}
Maximum density for melting snow ρ_{melt}	`rmlt`	500 kg m^{-3}
Snowfall to refresh albedo S_α	`Salb`	10 kg m^{-2}
Thermal metamorphism parameter c_1	`snda`	2.8×10^{-6} s^{-1}
Intercepted snow capacity per unit VAI S_Λ	`svai`	4.4 kg m^{-2}
Snow albedo decay temperature threshold T_α	`Talb`	-2°C
Cold snow albedo decay time scale τ_{cold}	`tcld`	1000 hours
Melting snow albedo decay time scale τ_{melt}	`tmlt`	100 hours
Snow compaction time scale τ_ρ	`trho`	200 hours
Temperature unloading parameter C_T	`Tunl`	1.87×10^5 K s
Wind unloading parameter C_U	`Uunl`	1.56×10^5 m
Canopy wind decay coefficient η	`wcan`	2.5
Irreducible liquid water content of snow W_{irr}	`Wirr`	0.03
Snow-free surface roughness length z_{0f}	`z0sf`	0.1 m
Snow surface roughness length z_{0s}	`z0sn`	0.001 m

2 Driving data (subroutine `FSM2_DRIVE`)

1D driving data are read from a text file with optional formats described in the FSM2 User Guide. A minimum wind speed of 0.1 m s^{-1} is imposed to avoid dividing by small numbers in aerodynamic calculations.

2.1 FSM format driving data (option `DRIV1D 1`)

Assuming that relative humidity is measured with respect to water at all temperatures, specific humidity is calculated as

$$Q_a = \frac{RH}{100}\frac{\varepsilon e_0}{P_s}\exp\left(\frac{17.5043T_a}{241.3 + T_a}\right) \tag{1}$$

for air temperature in °C. The humidity is limited to not exceed saturation with respect to ice.

2.2 ESM-SnowMIP format driving data (option `DRIV1D 2`)

Specific humidity is included in ESM-SnowMIP driving data files.

2.3 Shortwave radiation partitioning (option SWPART 1)

Subroutine SOLARPOS approximates solar declination δ (radians) and equation of time E_t (hours) by Fourier series

$$\begin{aligned}\delta = 0.006918 &- 0.399912\cos\Gamma + 0.070257\sin\Gamma \\ &- 0.006758\cos 2\Gamma + 0.000907\sin 2\Gamma \\ &- 0.002697\cos 3\Gamma + 0.001480\sin 3\Gamma \end{aligned} \tag{2}$$

and

$$E_t = (12/\pi)(0.000075 + 0.001868\cos\Gamma - 0.032077\sin\Gamma - 0.014615\cos 2\Gamma - 0.04089\sin 2\Gamma) \tag{3}$$

for day of year d_n and day angle $\Gamma = 2\pi(d_n - 1)/365$. From a date given by integers y, m and d for year, month and day, the day of year can be found using integer division in the magic formula

$$d_n = (7y)/4 - 7(y + (m+9)/12)/4 + (275m)/9 + d - 30. \tag{4}$$

For hour h of a day, the hour angle is defined by

$$\omega = (\pi/12)(h_{12} - h - E_t), \tag{5}$$

the sine of the solar elevation is

$$\sin\theta = \sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega \tag{6}$$

and the cosine of the solar azimuth is

$$\cos\psi = (\sin\theta\sin\phi - \sin\delta)/(\cos\theta\cos\phi). \tag{7}$$

Azimuth is measured anticlockwise from south, so ψ has the same sign as ω (positive in the morning and negative in the afternoon).

An atmospheric clearness parameter is found by dividing global radiation at the surface by incoming radiation at the top of the atmosphere, giving

$$k_t = \frac{SW_\downarrow}{I_0\sin\theta}. \tag{8}$$

The diffuse fraction of shortwave radiation is then estimated as

$$\frac{S_{\mathrm{dif}}}{SW_\downarrow} = \begin{cases} 1 - 0.09k_t & k_t \le 0.22 \\ 0.95 - 0.16k_t + 4.39k_t^2 - 16.64k_t^3 + 12.34k_t^4 & 0.22 < k_t \le 0.8 \\ 0.165 & k_t > 0.8. \end{cases} \tag{9}$$

Direct-beam shortwave radiation is the remainder $S_{\mathrm{dir}} = SW_\downarrow - S_{\mathrm{dif}}$.

3 Forest canopy properties (subroutine CANOPY)

A one-layer canopy model (option CANMOD 1) has vegetation area index Λ provided as an input. The layers in a two-layer model (CANMOD 2) have vegetation area indices $\Lambda_1 = f_\Lambda\Lambda$ (upper) and $\Lambda_2 = (1 - f_\Lambda)\Lambda$ (lower). A canopy layer with intercepted snow has heat capacity $C_v = C_\Lambda\Lambda + c_{\mathrm{ice}}S_v$, snow interception capacity $S_c = S_\Lambda\Lambda$ and snow cover fraction

$$f_{cs} = \left(\frac{S_v}{S_c}\right)^{2/3}. \tag{10}$$

Vegetation layer temperatures at the start of the timestep are saved for use in subroutine SRFEBAL.

4 Shortwave radiation (subroutine SWRAD)

4.1 Ground albedos

Bare ground with albedo α_0 and snow cover fraction f_s with albedo α_s have average albedo

$$\alpha = (1 - f_s)\alpha_0 + f_s\alpha_s. \tag{11}$$

The snow cover fraction for snow depth h_s is

$$f_s = \min\left(\frac{h_s}{h_f}, 1\right) \tag{12}$$

for option `SNFRAC 1`,

$$f_s = \tanh\left(\frac{h_s}{h_f}\right) \tag{13}$$

for `SNFRAC 2` and

$$f_s = \frac{h_s}{h_s + h_f} \tag{14}$$

for `SNFRAC 3`.

4.1.1 Diagnosed snow albedo (option ALBEDO 1)

Snow albedo is diagnosed as a function of surface temperature

$$\alpha_s = \alpha_{\min} + (\alpha_{\max} - \alpha_{\min}) \min\left(\frac{T_s - T_m}{T_\alpha}, 1\right). \tag{15}$$

4.1.2 Prognostic snow albedo (option ALBEDO 2)

Snow albedo decreases with time and increases as fresh snow falls with update

$$\alpha_s \to \alpha_s + (\alpha_{\mathrm{lim}} - \alpha_s)(1 - e^{-\gamma\delta t}) \tag{16}$$

each timestep, where

$$\gamma = \frac{1}{\tau_\alpha} + \frac{S_f}{S_\alpha} \tag{17}$$

and

$$\alpha_{\mathrm{lim}} = \frac{1}{\gamma}\left(\frac{1}{\tau_\alpha}\alpha_{\min} + \frac{S_f}{S_\alpha}\alpha_{\max}\right). \tag{18}$$

Timescale τ_α has different values τ_{cold} and τ_{melt} for cold and melting snow.

4.2 Canopy reflection and transmission (subroutine SWRAD)

A canopy layer has reflectivities R_b, R_d and transmissivities τ_b, τ_d for direct-beam and diffuse radiation, respectively, and forward-scattering fraction s_b for direct-beam radiation. Upwards and downwards shortwave radiation fluxes at the top and bottom of canopy layers are found by solving a matrix equation

$$\begin{pmatrix} 1 & -R_d & 0 \\ -\alpha & 1 & 0 \\ 0 & -\tau_d & 1 \end{pmatrix} \begin{pmatrix} S_{\downarrow 1} \\ S_{\uparrow 1} \\ S_{\uparrow 0} \end{pmatrix} = \begin{pmatrix} \tau_d \\ 0 \\ R_d \end{pmatrix} S_{\downarrow \mathrm{dif}} + \begin{pmatrix} s_b \\ \alpha\tau_b \\ R_b \end{pmatrix} S_{\downarrow \mathrm{dir}} \tag{19}$$

for a one-layer model or

$$\begin{pmatrix} 1 & 0 & 0 & -R_{d,1} & 0 \\ -\tau_{d,2} & 1 & -R_{d,2} & 0 & 0 \\ 0 & -\alpha & 1 & 0 & 0 \\ -R_{d,2} & 0 & -\tau_{d,2} & 1 & 0 \\ 0 & 0 & 0 & -\tau_{d,1} & 1 \end{pmatrix} \begin{pmatrix} S_{\downarrow 1} \\ S_{\downarrow 2} \\ S_{\uparrow 2} \\ S_{\uparrow 1} \\ S_{\uparrow 0} \end{pmatrix} = \begin{pmatrix} \tau_{d,1} \\ 0 \\ 0 \\ 0 \\ R_{d,1} \end{pmatrix} S_{\downarrow \mathrm{dif}} + \begin{pmatrix} s_{b,1} \\ s_{b,2}\tau_{b,1} \\ \alpha\tau_{b,1}\tau_{b,2} \\ R_{b,2}\tau_{b,1} \\ R_{b,1} \end{pmatrix} S_{\downarrow \mathrm{dir}} \tag{20}$$

for a two-layer model. Net shortwave radiation absorbed by vegetation in canopy layers and the underlying snow or ground surface are

$$SW_v = S_{\downarrow \mathrm{dif}} - S_{\downarrow 1} + S_{\uparrow 1} - S_{\uparrow 0} + (1 - \tau_b) S_{\downarrow \mathrm{dir}}, \tag{21}$$
$$SW_s = (1 - \alpha)(S_{\downarrow 1} + \tau_b S_{\downarrow \mathrm{dir}}) \tag{22}$$

for a one-layer model, and

$$SW_{v,1} = S_{\downarrow \mathrm{dif}} - S_{\downarrow 1} + S_{\uparrow 1} - S_{\uparrow 0} + (1 - \tau_{b,1}) S_{\downarrow \mathrm{dir}}, \tag{23}$$
$$SW_{v,2} = S_{\downarrow 1} - S_{\downarrow 2} + S_{\uparrow 2} - S_{\uparrow 1} + \tau_{b,1}(1 - \tau_{b,2}) S_{\downarrow \mathrm{dir}}, \tag{24}$$
$$SW_s = (1 - \alpha)(S_{\downarrow 2} + \tau_{b,1}\tau_{b,2} S_{\downarrow \mathrm{dir}}) \tag{25}$$

for a two-layer model. Equations (19) and (20) are solved by LU decomposition (subroutine `LUDCMP`).

4.2.1 Beer's Law (option `CANRAD 1`)

The fractions of radiation transmitted without interception through a canopy layer are

$$\tau_b = \exp(-k_{\mathrm{ext}}\Lambda / \sin\theta) \tag{26}$$

and

$$\tau_d = \exp(-1.6 k_{\mathrm{ext}}\Lambda). \tag{27}$$

Diffuse and direct-beam canopy layer reflectivities are $R_d = (1 - \tau_d)\alpha_c$ and $R_b = (1 - \tau_b)\alpha_c$ for dense canopy albedo

$$\alpha_c = (1 - f_{cs})\alpha_{c0} + f_{cs}\alpha_{cs} \tag{28}$$

and the forward-scattering fraction is zero.

4.2.2 Two-stream approximation (option `CANRAD 2`, subroutine `TWOSTREAM`)

For flat, opaque and randomly oriented leaves,

$$\omega = (1 - f_{cs})\alpha_{\Lambda 0} + f_{cs}\alpha_{\Lambda s} \tag{29}$$

is the fraction of incident radiation that is scattered, $\beta = 2/3$ is the fraction of scattered diffuse radiation that is directed back into the upward hemisphere and

$$\beta_0 = (0.5 + \mu)\left[1 - \mu \ln\left(\frac{1 + \mu}{\mu}\right)\right] \tag{30}$$

is the upscatter fraction for direct-beam radiation with $\mu = \sin\theta$. Two-stream equations involve coefficients

$$\gamma_1 = 2[1 - (1 - \beta)\omega], \ \gamma_2 = 2\beta\omega, \ \gamma_3 = \beta_0, \ \gamma_4 = 1 - \beta_0. \tag{31}$$

The reflectivity and transmittivity of a canopy layer for diffuse radiation are

$$R_d = \frac{\gamma_2(1 - e^{-2kl})}{k + \gamma_1 + (k - \gamma_1)e^{-2kl}} \tag{32}$$

and

$$\tau_d = \frac{2ke^{-kl}}{k + \gamma_1 + (k - \gamma_1)e^{-2kl}} \tag{33}$$

for extinction coefficient $k = (\gamma_1^2 - \gamma_2^2)^{1/2}$ and optical thickness $l = k_{\mathrm{ext}}\Lambda$. The direct-beam reflectivity, forward scattering fraction and transmissivity are

$$R_b = \frac{\omega[(1 - k\mu)(\alpha_2 + k\gamma_3)e^{kl} - (1 + k\mu)(\alpha_2 - k\gamma_3)e^{-kl} - 2k(\gamma_3 - \alpha_2\mu)e^{-l/\mu}]}{(1 - k^2\mu^2)[(k + \gamma_1)e^{kl} + (k - \gamma_1)e^{-kl}]} \tag{34}$$

and

$$s_b = \frac{\omega e^{-l/\mu}[(1-k\mu)(\alpha_1 - k\gamma_4)e^{-kl} - (1+k\mu)(\alpha_1 + k\gamma_4)e^{kl}] + 2k\omega(\gamma_4 + \alpha_1\mu)}{(1-k^2\mu^2)[(k+\gamma_1)e^{kl} + (k-\gamma_1)e^{-kl}]}, \tag{35}$$

and

$$\tau_b = e^{-l/\mu} \tag{36}$$

where $\alpha_1 = \gamma_1\gamma_4 + \gamma_2\gamma_3$ and $\alpha_2 = \gamma_1\gamma_3 + \gamma_2\gamma_4$.

5 Thermal properties (subroutine THERMAL)

5.1 Snow

The thermal conductivity of a snow layer of density ρ_s is a fixed parameter for option `CONDCT 0` and

$$\lambda_{sn} = 2.224 \left(\frac{\rho_s}{\rho_{\text{wat}}}\right)^{1.885} \tag{37}$$

for `CONDCT 1`.

5.2 Soil

Clapp-Hornberger exponent

$$b = 3.1 + 15.7 f_{\text{clay}} - 0.3 f_{\text{sand}}, \tag{38}$$

dry soil heat capacity

$$c_{\text{dry}} = \frac{2.128 \times 10^6 f_{\text{clay}} + 2.385 \times 10^6 f_{\text{sand}}}{f_{\text{clay}} + f_{\text{sand}}}, \tag{39}$$

saturated soil water suction

$$\Psi_s = 10^{0.17 - 0.63 f_{\text{clay}} - 1.58 f_{\text{sand}}}, \tag{40}$$

volumetric soil moisture at saturation

$$V_{\text{sat}} = 0.505 - 0.037 f_{\text{clay}} - 0.142 f_{\text{sand}}, \tag{41}$$

volumetric soil moisture at the critical point

$$V_{\text{crit}} = V_{\text{sat}} \left(\frac{\Psi_s}{3.364}\right)^{1/b} \tag{42}$$

and dry soil thermal conductivity

$$\lambda_{\text{dry}} = \lambda_{\text{air}}^{V_{\text{sat}}} (\lambda_{\text{clay}}^{f_{\text{clay}}} \lambda_{\text{sand}}^{1-f_{\text{clay}}})^{1-V_{\text{sat}}} \tag{43}$$

are derived from soil texture fractions f_{clay} and f_{sand} at the start of the run.

The temperature derivative of soil water suction Ψ in the presence of ice is

$$\frac{d\Psi}{dT} = -\frac{\rho_{\text{ice}} L_f}{g \rho_{\text{wat}} T_m}. \tag{44}$$

For a layer with volumetric soil moisture content V_s, the temperature above which all soil moisture is unfrozen is

$$T_{\max} = T_m + \Psi_s \left(\frac{d\Psi}{dT}\right)^{-1} \left(\frac{V_{\text{sat}}}{V_s}\right)^b. \tag{45}$$

The apparent areal heat capacity of a soil layer of thickness $\Delta z_{\rm sl}$ and Celsius temperature $T_c = T_{\rm sl} - T_m$, including the influence of soil moisture phase change, is

$$C_{sl} = (c_{\rm dry} + \rho_{\rm ice} c_{\rm ice} \theta_f + \rho_{\rm wat} c_{\rm wat} \theta_u)\Delta z_{\rm sl} + \rho_{\rm wat} \Delta z_{\rm sl}[(c_{\rm wat} - c_{\rm ice})T_c + L_f]\frac{d\theta_u}{dT} \tag{46}$$

for

$$\frac{d\theta_u}{dT} = -\frac{d\Psi}{dT}\frac{V_{\rm sat}}{b\Psi_s}\left(\frac{T_c}{\Psi_s}\frac{d\Psi}{dT}\right)^{-1/b-1}. \tag{47}$$

The thermal conductivity of wet soil is

$$\lambda_{sl} = (\lambda_{\rm sat} - \lambda_{\rm dry})(S_f + S_u) + \lambda_{\rm dry} \tag{48}$$

with

$$\lambda_{\rm sat} = \frac{\lambda_{\rm soil}\lambda_{\rm wat}^{\theta_{\rm wat}}\lambda_{\rm ice}^{\theta_{\rm ice}}}{\lambda_{\rm air}^{V_{\rm sat}}} \tag{49}$$

for

$$\theta_{\rm ice} = \frac{S_f}{S_f + S_u}V_{\rm sat}, \quad \theta_{\rm wat} = \frac{S_u}{S_f + S_u}V_{\rm sat}. \tag{50}$$

5.3 Surface layer

Surface fluxes are calculated using the thermal properties of a surface layer of thickness $\Delta z_1 = \max(\Delta z_{sl,1}, D_{sn,1})$ that can include both snow and soil for shallow snow. The average temperature of this layer is

$$T_1 = \begin{cases} T_{sl,1} + (T_{sn,1} - T_{sl,1})\frac{D_{sn,1}}{\Delta z_{sl,1}} & h_s \le \Delta z_{sl,1} \\ T_{sn,1} & h_s > \Delta z_{sl,1} \end{cases} \tag{51}$$

and the thermal conductivity between the surface and the middle of the layer if snow free is

$$\lambda_1 = \begin{cases} \Delta z_{sl,1}\left(\frac{2D_{sn,1}}{\lambda_{sn,1}} + \frac{\Delta z_{sl,1} - 2D_{sn,1}}{\lambda_{sl,1}}\right)^{-1} & h_s \le 0.5\Delta z_{sl,1} \\ \lambda_{sn} & h_s > 0.5\Delta z_{sl,1} \end{cases} \tag{52}$$

The moisture conductance for evaporation between this layer and the atmosphere is

$$g_1 = g_{\rm sat}\left(\frac{\theta_u}{V_{\rm crit}}\right)^2. \tag{53}$$

6 Surface and canopy energy balance (subroutine `SRFEBAL`)

Quantities set before starting the iterative solution of the surface energy balance are:

- temperature and wind speed measurement heights z_T and z_U, incremented by h_c if specified above the canopy height (option `ZOFFST 1`);
- canopy layer heights $z_1 = h_b + (h_c - h_b)/2$ (one layer) or $z_1 = (1 - f_\Lambda/2)h_c$ and $z_2 = (1 - f_\Lambda)h_c/2$ (two layers);
- vegetation fraction $f_v = 1 - \exp(-k_{\rm ext}\Lambda)$;
- vegetation roughness length $z_{0v} = 0.1h_c$ and displacement height $d = 0.67h_c$;
- momentum roughness length $z_0 = z_{0s}^{f_s} z_{0f}^{1-f_s}$;
- scalar roughness length $z_{0h} = 0.1z_0$;
- air density $P_s/(R_{\rm air}T_a)$.

Saturation humidity (function `qsat`) and latent heat

$$q_{\text{sat}}(T, P_s) = \begin{cases} \frac{\varepsilon e_0}{P_s} \exp\left(\frac{22.4422 T_a}{272.186 + T_a}\right), & L = L_s \quad T \le T_m \\ \frac{\varepsilon e_0}{P_s} \exp\left(\frac{17.5043 T_a}{241.3 + T_a}\right), & L = L_v \quad T > T_m \end{cases} \tag{54}$$

and gradient

$$D = \frac{dq_{\text{sat}}}{dT} = \frac{L q_{\text{sat}}}{R_{\text{wat}} T^2} \tag{55}$$

are set wherever required below for surface or vegetation temperatures.

6.1 Open areas ($\Lambda = 0$)

The reciprocal of the Obukhov length, the friction velocity and the aerodynamic conductance are

$$\frac{1}{L_O} = -\frac{k g g_a (T_s - T_a)}{T_a u_*^3}, \tag{56}$$

$$u_* = k U_a \left[\ln\left(\frac{z_U}{z_0}\right) - \psi_m\left(\frac{z_U}{L_O}\right) + \psi_m\left(\frac{z_0}{L_O}\right)\right]^{-1} \tag{57}$$

and

$$g_a = k u_* \left[\ln\left(\frac{z_T}{z_{0h}}\right) - \psi_h\left(\frac{z_T}{L_O}\right) + \psi_h\left(\frac{z_{0h}}{L_O}\right)\right]^{-1}, \tag{58}$$

with ψ_h and ψ_m set to zero for option `EXCHNG 0` and calculated by functions `psih` and `psim` for `EXCHNG 1`. The surface miosture flux, ground heat flux, sensible heat flux and net radiation are

$$E = \rho \chi_s g_a [q_{\text{sat}}(T_s) - q_a], \tag{59}$$

$$G = \frac{2\lambda_1}{\Delta z_1}(T_s - T_1), \tag{60}$$

$$H = \rho c_p g_a (T_s - T_a), \tag{61}$$

$$R_s = SW_s + LW_\downarrow - \sigma T_s^4, \tag{62}$$

with water availability factor $\chi_s = 1$ if $q_a > q_{\text{sat}}$ and

$$\chi_s = f_s + \frac{(1 - f_s) g_1}{g_a + g_1} \tag{63}$$

otherwise. The surface temperature and flux increments are first calculated assuming no snow melt ($M = 0$) as

$$\delta T_s = \frac{R_s - G - H - LE - L_f M}{4\sigma T_s^3 + 2\lambda_1/\Delta z_1 + \rho g_a (c_p + L D \chi_s)}, \tag{64}$$

$$\delta E = \rho \chi_s g_a D \delta T_s, \tag{65}$$

$$\delta G = \frac{2\lambda_1}{\Delta z_1} \delta T_s \tag{66}$$

and

$$\delta H = \rho c_p g_a \delta T_s. \tag{67}$$

If this gives a temperature $T_s + \delta T_s > T_m$ and there is snow with ice mass I on the ground, the temperature and flux increments are recalculated with $M = I/\delta t$ (all of the snow melts). If then $T_s + \delta T_s < T_m$, not all of the snow melts and the fluxes are recalculated with $T_s = T_m$ and the melt rate is diagnosed as

$$M = (R_s - G - H - LE)/L_f. \tag{68}$$

The increments are added to the surface temperature and fluxes, and the surface energy balance residual

$$\epsilon = R_s - G - H - LE - L_f M \tag{69}$$

is calculated. At least four iterations and at most ten are made, terminating earlier if $|\epsilon| < 0.01$ W m^{-2}.

6.2 Forest ($\Lambda > 0$)

Aerodynamic quantities for forests are calculated as weighted averages between open and dense-canopy values, according to the vegetation fraction. The friction velocity, aerodynamic conductance between the highest canopy layer and the atmosphere, and aerodynamic conductance between the lowest canopy layer and the ground for a dense canopy are

$$u_{*d} = kU_a \left[\ln\left(\frac{z_U - d}{z_{0v}}\right) - \psi_m\left(\frac{z_U - d}{L_O}\right) + \psi_m\left(\frac{z_{0v}}{L_O}\right) \right]^{-1}, \tag{70}$$

$$g_{ad} = \left\{ \frac{1}{ku_*} \left[\ln\left(\frac{z_T - d}{h_c - d}\right) - \psi_H\left(\frac{z_T - d}{L_O}\right) + \psi_H\left(\frac{h_c - d}{L_O}\right) \right] + \frac{h_c \left[e^{\eta(1 - z_1/h_c)} - 1\right]}{\eta K_H} \right\}^{-1} \tag{71}$$

and

$$g_{sd} = \left[\frac{1}{k^2 U_b} \ln\left(\frac{h_b}{z_0}\right) \ln\left(\frac{h_b}{z_{0h}}\right) + \frac{e^\eta h_c}{\eta K_H} \left(e^{-\eta h_b/h_c} - e^{-\eta z_N/h_c} \right) \right]^{-1} \tag{72}$$

with

$$K_H = \begin{cases} ku_*(h_c - d)/[1 + 5(h_c - d)/L_O] & L_O > 0 \\ ku_*(h_c - d)[1 - 16(h_c - d)/L_O]^{1/2} & L_O < 0. \end{cases} \tag{73}$$

Conductance between a vegetation layer and its corresponding canopy air space is

$$g_v = \frac{U_c^{1/2} \Lambda_n}{r_{\text{leaf}}} \tag{74}$$

for canopy wind speed

$$U_c = f_v \exp\left[\eta\left(\frac{z_n}{h_c} - 1\right)\right] U_h + (1 - f_v)\frac{u_*}{k} \left[\ln\left(\frac{z_n}{z_0}\right) - \psi_m\left(\frac{z_n}{L_O}\right) + \psi_m\left(\frac{z_0}{L_O}\right) \right] \tag{75}$$

and canopy-top wind speed

$$U_h = \frac{u_*}{k} \left[\ln\left(\frac{h_c - d}{z_{0v}}\right) - \psi_m\left(\frac{h_c - d}{L_O}\right) + \psi_m\left(\frac{z_{0v}}{L_O}\right) \right]. \tag{76}$$

If there are two canopy layers, the conductance between the air spaces is

$$g_c = \frac{f_v \eta K_H}{e^\eta h_c} \left(e^{-\eta z_2/h_c} - e^{-\eta z_1/h_c} \right)^{-1} + (1 - f_v) k u_* \left[\ln\left(\frac{z_1}{z_2}\right) - \psi_m\left(\frac{z_1}{L_O}\right) + \psi_m\left(\frac{z_2}{L_O}\right) \right]^{-1}. \tag{77}$$

The water availability factor for a canopy layer with snow cover fraction f_{cs} is $\chi_v = 1$ if $q_c > q_{\text{sat}}(T_v)$ and

$$\chi_v = f_{cs} + \frac{(1 - f_{cs}) g_{\text{veg}}}{g_v + g_{\text{veg}}} \tag{78}$$

otherwise.

6.2.1 One-layer canopy (option `CANMOD 1`)

Sensible heat fluxes are

$$H = \rho c_p g_a (T_c - T_a) \tag{79}$$

from the canopy air space to the atmosphere,

$$H_v = \rho c_p g_v (T_v - T_c) \tag{80}$$

from vegetation to the canopy air space, and

$$H_s = \rho c_p g_s (T_s - T_c) \tag{81}$$

from the snow or ground surface to the canopy air space. The corresponding moisture fluxes are

$$E = \rho g_a(q_c - q_a), \tag{82}$$
$$E_v = \chi_v \rho g_v[q_{\rm sat}(T_v) - q_c], \tag{83}$$

and

$$E_s = \chi_s \rho g_s[q_{\rm sat}(T_s) - q_c]. \tag{84}$$

Net radiation absorbed by the surface and the vegetation are

$$R_s = SW_s + \tau_d LW_\downarrow - \sigma T_s^4 + (1-\tau_d)\sigma T_v^4 \tag{85}$$

and

$$R_v = SW_v + (1-\tau_d)(LW_\downarrow + 4\sigma T_s^4 - 2\sigma T_v^4). \tag{86}$$

State variable increments in a vector

$$\delta\mathbf{x} = (\delta T_s, \delta Q_c, \delta T_c, \delta T_v) \tag{87}$$

are found by using LU decomposition to solve matrix equation $\mathrm{J}\delta\mathbf{x} = \mathbf{f}$ with Jacobian matrix elements

$$\begin{aligned}
&J_{11} = -\rho g_s(c_p + LD\chi_s) - 4\sigma T_s^3 - 2k_1/\Delta z_1, \quad J_{12} = L\rho\chi_s g_s, \quad J_{13} = \rho c_p g_s, \quad J_{14} = 4(1-\tau_d)\sigma T_v^3,\\
&J_{21} = 4(1-\tau_d)\sigma T_s^3, \quad J_{22} = L\rho\chi_v g_v, \quad J_{23} = \rho c_p g_v, \quad J_{24} = -\rho g_v(c_p + LD\chi_v) - 8(1-\tau_d)\sigma T_v^3 - C_v\delta t,\\
&J_{31} = -g_s, \quad J_{32} = 0, \quad J_{33} = g_a + g_s + g_v, \quad J_{34} = -g_v,\\
&J_{41} = -D\chi_s g_s, \quad J_{42} = g_a + \chi_s g_s + \chi_v g_v, \quad J_{43} = 0, \quad J_{44} = -D\chi_v g_v
\end{aligned} \tag{88}$$

and flux conservation vector elements

$$\begin{aligned}
&f_1 = -(R_s - G_s - H_s - LE_s),\\
&f_2 = -[R_v - H_s - LE_s - C_v(T_v - T_v^{(0)})/\delta t],\\
&f_3 = -(H - H_s - H_v)/(\rho c_p),\\
&f_4 = -(E - E_s - E_v)/\rho.
\end{aligned} \tag{89}$$

The flux increments are then

$$\begin{aligned}
&\delta E_s = \rho\chi_s g_s(D\delta T_s - \delta Q_c),\\
&\delta E_v = \rho\chi_v g_v(D\delta T_v - \delta Q_c),\\
&\delta G_s = 2k_1\delta T_s/\Delta z_1,\\
&\delta H_s = \rho c_p g_s(\delta T_s - \delta T_c),\\
&\delta H_v = \rho c_p g_v(\delta T_v - \delta T_c).
\end{aligned} \tag{90}$$

6.2.2 Two-layer canopy (option `CANMOD 2`)

Sensible heat fluxes are

$$H = \rho c_p g_a(T_{c,1} - T_a) \tag{91}$$

from the canopy air space to the atmosphere,

$$H_{v,n} = \rho c_p g_{v,n}(T_{v,n} - T_{c,n}) \tag{92}$$

from vegetation layer n to canopy air space layer n, and

$$H_s = \rho c_p r_s(T_s - T_{c,2}) \tag{93}$$

from the ground to the lower canopy air space. The flux between the canopy air space layers is

$$H_c = \rho c_p g_c(T_{c,2} - T_{c,1}). \tag{94}$$

The corresponding moisture fluxes are

$$E = \rho g_a (q_{c,1} - q_a), \tag{95}$$
$$E_{v,n} = \chi_{v,n} \rho g_{v,n} [q_{\mathrm{sat}}(T_{v,n}) - q_{c,n}], \tag{96}$$
$$E_s = \chi_s \rho g_s [q_{\mathrm{sat}}(T_s) - q_{c,2}] \tag{97}$$

and

$$E_c = \rho g_c (q_{c,2} - q_{c,1}). \tag{98}$$

Net radiation absorbed by the surface and the vegetation layers are

$$R_s = SW_s + \tau_{d,1}\tau_{d,2} LW_\downarrow + (1-\tau_{d,1})\tau_{d,2}\sigma T_{v,1}^4 + (1-\tau_{d,2})\sigma T_{v,2}^4 - \sigma T_s^4, \tag{99}$$
$$R_{v,1} = SW_{v,1} + (1-\tau_{d,1})[LW_\downarrow - 2\sigma T_{v,1}^4 + (1-\tau_{d,2})\sigma T_{v,2}^4 + \tau_{d,2}\sigma T_s^4] \tag{100}$$

and

$$R_{v,2} = SW_{v,2} + (1-\tau_{d,2})[\tau_{d,1} LW_\downarrow + (1-\tau_{d,1})\sigma T_{v,1}^4 - 2\sigma T_{v,2}^4 + \sigma T_s^4]. \tag{101}$$

State variable increments in a vector

$$\delta \mathbf{x} = (\delta T_s, \delta Q_{c,1}, \delta T_{c,1}, \delta T_{v,1}, \delta Q_{c,2}, \delta T_{c,2}, \delta T_{v,2}) \tag{102}$$

are found by using LU decomposition to solve matrix equation $\mathrm{J}\delta\mathbf{x} = \mathbf{f}$ with non-zero Jacobian matrix elements

$$\begin{aligned}
&J_{11} = -(c_p + LD\psi_g)\rho g_s - 4\sigma T_s^3 - 2\lambda_1/\Delta z_1, \quad J_{14} = 4(1-\tau_{v,1})\tau_{v,2}\sigma T_{v,1}^3, \\
&J_{15} = L\rho\psi_g g_g, \quad J_{16} = \rho c_p g_g, \quad J_{17} = 4(1-\tau_{v,2})\sigma T_{v,2}^3, \\
&J_{21} = 4(1-\tau_{v,1})\tau_{v,2}\sigma T_{v,2}^3, \quad J_{22} = L\rho\chi_{v,1} g_{v,1}, \quad J_{23} = \rho c_p g_{v,1}, \\
&J_{24} = -\rho g_{v,1}(c_p + LD\chi_{v,1}) - 8(1-\tau_{v,1})\sigma T_{v,1}^3 - C_{v,1}/\delta t, \quad J_{27} = 4(1-\tau_{v,1})(1-\tau_{v,2})\sigma T_{v,2}^3, \\
&J_{31} = 4(1-\tau_{v,2})\sigma T_s^3, \quad J_{34} = 4(1-\tau_{v,1})(1-\tau_{v,2})\sigma T_s^3, \\
&J_{35} = L\rho\chi_{v,2} g_{v,2}, \quad J_{36} = \rho c_p g_{v,2}, \quad J_{37} = -\rho g_{v,2}(c_p + LD\chi_{v,2}) - 8(1-\tau_{v,2})\sigma T_{v,2}^3 - C_{v,2}/\delta t, \\
&J_{43} = g_a + g_c + g_{v,1}, \quad J_{44} = -g_{v,1}, \quad J_{46} = -g_c, \\
&J_{51} = -g_s, \quad J_{53} = -g_c, \quad J_{56} = g_c + g_s + g_{v,2}, \quad J_{57} = -g_{v,2}, \\
&J_{62} = g_a + g_c + \chi_{v,1} g_{v,1}, \quad J_{64} = -D\chi_{v,1} g_{v,1}, \quad J_{65} = -g_c, \\
&J_{71} = -D\chi_s g_s, \quad J_{72} = -g_c, \quad J_{75} = g_c + \chi_s g_s + \chi_{v,2} g_{v,2}, \quad J_{77} = -D\chi_{v,2} g_{v,2},
\end{aligned} \tag{103}$$

the flux conservation vector elements are

$$\begin{aligned}
&f_1 = -(R_s - G_s - H_s - LE_s), \\
&f_2 = -[R_{v,1} - H_{v,1} - LE_{v,1} - C_{v,1}(T_{v,1} - T_{v,1}^{(0)})/\delta t], \\
&f_3 = -[R_{v,2} - H_{v,2} - LE_{v,2} - C_{v,2}(T_{v,2} - T_{v,2}^{(0)})/\delta t], \\
&f_4 = -(H - H_c - H_{v,1})/(\rho c_p), \\
&f_5 = -(H_c - H_s - H_{v,2})/(\rho c_p), \\
&f_6 = -(E - E_c - E_{v,1})/\rho, \\
&f_7 = -(E_c - E_s - E_{v,2})/\rho
\end{aligned} \tag{104}$$

and the flux increments are

$$\begin{aligned}
&\delta Es = \rho\chi_s g_s (D\delta Ts - \delta Q_{c,2}), \\
&\delta E_{v,1} = \rho\chi_{v,1} g_{v,1}(D\delta T_{v,1} - \delta Q_{c,1}), \\
&\delta E_{v,2} = \rho\chi_{v,2} g_{v,2}(D\delta T_{v,2} - \delta Q_{c,2}), \\
&\delta G_s = 2k_1 \delta T_s/\Delta z_1, \\
&\delta H_s = \rho c_p g_s (\delta Ts - \delta T_{c,2}), \\
&\delta H_{v,1} = \rho c_p g_{v,1}(\delta T_{v,1} - \delta T_{c,1}), \\
&\delta H_{v,2} = \rho c_p g_{v,2}(\delta T_{v,2} - \delta T_{c,2}).
\end{aligned} \tag{105}$$

6.2.3 Sub-canopy snow melt

If the updated surface temperature exceeds T_m and there is snow with ice mass I on the ground, it is assumed to melt at rate $M = I/\delta t$, melt energy $L_f M$ is added to f_1 and the increments are recalculated. If the updated surface temperature is then less than T_m, the snow does not all melt in the timestep. The surface fluxes and $\mathbf{f}$ are recalculated with $T_s = T_m$, elements in the first row of the Jacobian matrix are changed to 0 except $J_{11} = -1$ and the increments are recalculated. The melt rate is then given by $M = \delta x_1/L_f$.

6.2.4 Sub-canopy diagnostics

Sub-canopy windspeed at height z is diagnosed as

$$U = f_v U_b \frac{\ln(z/z_{0g})}{\ln(h_b/z_{0g})} + (1-f_v)U_a \left[\frac{\ln(z/z_{0g}) - \psi_m(z/L_O) + \psi_m(z_{0g}/L_O)}{\ln(z_U/z_{0g}) - \psi_m(z_U/L_O) + \psi_m(z_{0g}/L_O)}\right] \tag{106}$$

for canopy-base wind speed

$$U_b = \exp\left[\eta\left(\frac{h_b}{h_c} - 1\right)\right]. \tag{107}$$

The sub-canopy air temperature is

$$T = T_s - \frac{H_s}{\rho c_p g_s} \tag{108}$$

for surface resistance

$$g_s = \frac{f_v k^2 U_b}{\ln(z/z_{0g})\ln(z/z_{0h})} + \frac{(1-f_v)ku_*}{\ln(z/z_{0h}) - \psi_h(z/L_O) + \psi_h(z_{0h}/L_O)}. \tag{109}$$

6.3 Stability functions (functions `psim`, `psih`)

Arguments z and L_O define $\zeta = z/L_O$. The momentum and scalar stability functions are

$$\psi_m(\zeta) = \begin{cases} 2\ln\left(\frac{1+x}{2}\right) + \ln\left(\frac{1+x^2}{2}\right) - 2\arctan x + \frac{\pi}{2} & \zeta < 0 \\ -5\zeta & 0 \le \zeta \end{cases} \tag{110}$$

and

$$\psi_H(\zeta) = \begin{cases} 2\ln\left(\frac{1+x^2}{2}\right) & \zeta < 0 \\ -5\zeta & 0 \le \zeta, \end{cases} \tag{111}$$

with $x = (1-16\zeta)^{1/4}$ and $-2 \le \zeta \le 1$.

7 Canopy snow mass balance (subroutine `INTERCEPT`)

The mass of snow added to a canopy layer by interception of snowfall in a timestep is either

$$\delta S_v = f_v S_f \delta t \tag{112}$$

for linear interception option `CANINT 1` or

$$\delta S_v = (S_c - S_v)\left[1 - \exp\left(-\frac{f_v S_f \delta t}{S_c}\right)\right] \tag{113}$$

for non-linear interception option `CANINT 2`, with the limitation $\delta S_v \le S_c - S_v$. The rate of snowfall reaching the next layer or the ground is reduced to $S_f - \delta S_v/\delta t$. Snow sublimates from a canopy layer at rate E_v if $E_v > 0$ or is added to the canopy layer snow mass if $E_v < 0$ and $T_v < T_m$. If the vegetation layer temperature is greater than

T_m, the amount of melt is $M = L_f^{-1} C_v (T_v - T_m)$, T_v is reset to T_m, and the melt water drips from the canopy to the ground. The mass of snow removed by unloading in a timestep is

$$\delta S_v = \frac{\delta t}{\tau_u} S_v + m_u M \tag{114}$$

for time/melt-dependent unloading (`CANUNL 1`) or

$$\delta S_v = \left[\frac{1}{c_T} \max(T_v - 270.15, 0) + \frac{U_a}{c_U} \right] S_v, \tag{115}$$

for temperature/wind-dependent unloading (`CANUNL 2`).

8 Snow on the ground (subroutine `SNOW`)

8.1 Heat conduction

The heat capacity of a snow layer with ice content I and liquid water content W is

$$C_{sn} = c_{\text{ice}} I + c_{\text{wat}} W. \tag{116}$$

If there is one snow layer, the increment to the layer temperature due to heat conduction over a timestep is

$$\delta T_{sn,1} = \frac{[G_s - U_1 (T_{sn,1} - T_{sl,1})] \delta t}{C_{sn,1} + U_1 \delta t} \tag{117}$$

for thermal transmittance

$$U_1 = 2 \left(\frac{D_{sn,1}}{\lambda_{sn,1}} + \frac{\Delta z_{sl,1}}{\lambda_{sl,1}} \right)^{-1}. \tag{118}$$

If there is more than one snow layer, snow layer temperature increments are found by solving a tridiagonal matrix equation (subroutine `TRIDIAG`) with thermal transmittance between layers (U), below-diagonal (a), diagonal (b) and above-diagonal (c) matrix elements, and right-hand side vector elements (r)

$$U_1 = 2 \left(\frac{D_{sn,1}}{\lambda_{sn,1}} + \frac{D_{sn,2}}{\lambda_{sn,2}} \right)^{-1} \tag{119}$$

$$b_1 = C_{sn,1} + U_1 \delta t \tag{120}$$

$$c_1 = -U_1 \delta t \tag{121}$$

$$r_1 = [G_s - U_1 (T_{sn,1} - T_{sn,2})] \delta t \tag{122}$$

for $n = 1$,

$$U_n = 2 \left(\frac{D_{sn,n}}{\lambda_{sn,n}} + \frac{D_{sn,n+1}}{\lambda_{sn,n+1}} \right)^{-1} \tag{123}$$

$$a_n = c_{n-1} \tag{124}$$

$$b_n = C_{sn,n} + (U_{n-1} + U_n) \delta t \tag{125}$$

$$c_n = -U_n \delta t \tag{126}$$

$$r_n = U_{n-1} (T_{sn,n-1} - T_{sn,n}) \delta t + U_n (T_{sn,n+1} - T_{sn,n}) \delta t \tag{127}$$

for $n = 2, ..., N_{\text{snow}} - 1$, and

$$U_n = 2 \left(\frac{D_{sn,n}}{\lambda_{sn,n}} + \frac{\Delta z_{sl,1}}{\lambda_{sl,1}} \right)^{-1} \tag{128}$$

$$a_n = c_{n-1} \tag{129}$$

$$b_n = C_{sn,n} + (U_{n-1} + U_n) \delta t \tag{130}$$

$$r_n = U_{n-1} (T_{sn,n-1} - T_{sn,n}) \delta t + U_n (T_{sl,1} - T_{sn,n}) \delta t \tag{131}$$

for $n = N_{\text{snow}}$. The heat flux at the base of the snow into the soil is $G_{\text{soil}} = U_{N_{\text{snow}}} (T_{sn,N_{\text{snow}}} - T_{sl,1})$.

8.2 Melt

The total amount of ice removed by surface melting is $\delta I = M\delta t$. If heat conduction without phase change results in a snow layer temperature $T_{sn} > T_m$, layer melt $L_f^{-1}C_{sn}(T_{sn} - T_m)$ is added to δI and the temperature is reset to T_m. Starting from the top layer and working downwards, a layer melts entirely if δI exceeds the layer ice mass; the ice in the layer is then entirely turned to liquid water, the layer is removed by setting its thickness to zero, and the ice mass is subtracted from δI for the next layer. If the layer only partially melts, the melt is added to the liquid content of the layer, the layer thickness is reduced to $(1 - \delta I/I)D_{sn}$ and the loop through layers terminates.

8.3 Sublimation

The total amount of ice removed by surface sublimation is $\delta I = E\delta t$. Starting from the top layer and working downwards, a layer sublimates entirely if δI exceeds the layer ice mass, the layer is removed by setting its thickness to zero, and the ice mass is subtracted from δI for the next layer. If the layer only partially sublimates, the layer thickness is reduced to $(1 - \delta I/I)D_{sn}$ and the loop through layers terminates.

8.4 Snow density

8.4.1 Fixed density (option `DENSTY 0`)

Snow has constant density ρ_0.

8.4.2 Compaction with age (option `DENSTY 1`)

The density of a snow layer with ice content I, liquid water content W and thickness D_{sn} at the beginning of a timestep is diagnosed as

$$\rho_s = \frac{I + W}{D_{sn}}. \tag{132}$$

This is then incremented by

$$\delta\rho_s = (\rho_{\max} - \rho_s)(1 - e^{-\delta t/\tau_\rho}). \tag{133}$$

Maximum density $\rho_{\max}$ has different values ρ_{cold} and ρ_{melt} for cold and melting snow. The thickness of the compacted layer at the end of the timestep is retrieved as

$$D_{sn} = \frac{I + W}{\rho_s}. \tag{134}$$

8.4.3 Compaction by overburden and thermal metamorphism (option `DENSTY 2`)

Layer density is incremented by

$$\delta\rho_s = \rho_s \left\{ \frac{gm}{\eta} + c_1 \exp\left[\frac{(T_{sn} - T_m)}{23.8} - \max\left(\frac{\rho_s - 150}{21.7}, 0 \right) \right] \right\} \delta t \tag{135}$$

where

$$\eta = \eta_0 \exp\left[-\frac{(T_{sn} - T_m)}{12.4} + \frac{\rho_s}{55.6} \right] \tag{136}$$

and the snow mass overlying the middle of layer n is

$$m_n = \sum_{j=1}^{n-1} (I_j + W_j) + 0.5(I_n + W_n). \tag{137}$$

8.5 Grain growth

8.5.1 Temperature metamorphism from JULES (option `SGRAIN 1`)

The increment in grain radius in a snow layer over a timestep is $\delta r = g_r r^{-1} \delta t$ for grain area growth rate

$$g_r = \begin{cases} 2 \times 10^{-13} \text{ m}^2 \text{ s}^{-1} & T_{sn} = T_m \\ 2 \times 10^{-14} \text{ m}^2 \text{ s}^{-1} & T_{sn} < T_m, r < 1.5 \times 10^{-4}\text{m} \\ 7.3 \times 10^{-8} \exp(-4600/T_{sn}) & T_{sn} < T_m, r \geq 1.5 \times 10^{-4}\text{m} \end{cases} \tag{138}$$

8.5.2 Temperature gradient metamorphism from SNTHERM (option `SGRAIN 1`)

The temperature gradient in a snow layer is calculated as the temperature difference between the top and bottom of the layer divided by the layer thickness. The temperature at the top of a snow layer is T_s for the surface layer and

$$T_{sn,n-1/2} = \frac{D_{sn,n-1} T_{sn,n} + D_{sn,n} T_{sn,n-1}}{D_{sn,n} + D_{sn,n-1}} \tag{139}$$

for $n > 1$. The temperature at the base of the snow is

$$T_{sn,n+1/2} = \frac{D_{sn,n} T_{sn,n} + \Delta z_{sl,1} T_{sl,1}}{D_{sn,n} + \Delta z_{sl,1}} \tag{140}$$

for $n = N_{\text{snow}}$. The vertical vapour flux in a layer is

$$q_v = 9.2 \times 10^{-5} \left(\frac{T_{sn,n}}{T_m} \right)^6 \frac{\partial \rho_{\text{sat}}}{\partial T} \left(\frac{T_{sn,n-1/2} - T_{sn,n+1/2}}{D_{sn,n}} \right) \tag{141}$$

with

$$\frac{\partial \rho_{\text{sat}}}{\partial T} = \frac{e_0}{R_{\text{wat}} T^2} \left(\frac{L_s}{R_{\text{wat}} T} - 1 \right) \exp \left[\frac{L_s}{R_{\text{wat}}} \left(\frac{1}{T_m} - \frac{1}{T} \right) \right]. \tag{142}$$

The grain area growth rate is

$$g_r = \begin{cases} 1.25 \times 10^{-7} \min(|q_v|, 10^{-6}) & \theta_w < 10^{-4} \\ 10^{-12} \min(\theta_w + 0.05, 0.14) & \theta_w \geq 10^{-4} \end{cases} \tag{143}$$

where the volumetric liquid water content is

$$\theta_w = \frac{W}{\rho_{\text{wat}} D_{sn}}. \tag{144}$$

8.6 Adding and removing snow

Snowfall and frost ($E < 0$ for $T_s < T_m$) are added to layer 1 with fresh snow density ρ_f and grain size r_0. Snow unloading from a forest canopy is added to snow on the ground with the existing bulk density and grain size.

All snow layers except the bottom layer have fixed (user specified or default) thicknesses. As snow depth changes, the thickness of the lowest layer changes until it falls below the fixed thickness or increases to more than twice the fixed thickness. The lowest layer is then combined with the layer above (decreasing) or split into two equal parts (increasing) if the maximum number of snow layers N_{smax} has not been reached. Ice mass, liquid water mass and internal energy contents of snow layers are stored before recalculating layer thicknesses and then redistributed according to conservation.

8.7 Liquid water movement in snow

8.7.1 Free drainage (option `HYDROL 0`)

Liquid water drains from snow immediately and is added to runoff.

8.7.2 Bucket storage (option `HYDROL 1`)

A snow layer with porosity

$$\phi = 1 - \frac{I}{\rho_{\mathrm{ice}} D_{sn}} \tag{145}$$

can hold a maximum mass $W_{\max} = \rho_{\mathrm{wat}} \phi D_{sn} W_{\mathrm{irr}}$ of liquid water. If the liquid mass of a layer exceeds the maximum, the excess is passed downwards to the next layer.

8.7.3 Gravitational drainage (option `HYDROL 2`)

Liquid water in excess of layer porosities ($\theta_w > \phi$) is first added to runoff. Saturated hydraulic conductivity in a layer and downward water flux due to gravitational drainage at the bottom of the layer are then parametrized as

$$k_{\mathrm{sat},n} = 0.31 \frac{\rho_{\mathrm{wat}} g r_n^2}{\mu_{\mathrm{wat}}} \exp\left(-7.8 \frac{\rho_{s,n}}{\rho_{\mathrm{wat}}}\right) \tag{146}$$

and

$$Q_{w,n+1/2} = k_{\mathrm{sat},n} \left(\frac{\theta_{w,n} - \theta_{r,n}}{\phi_n - \theta_{r,n}}\right)^3 \tag{147}$$

for irreducible water content $\theta_{r,n} = W_{\mathrm{irr}} \phi_n$. The conservation equation for changes in liquid water content is discretized by an implicit upwind scheme

$$\frac{\theta_{w,n} - \theta_{w,n}^{(0)}}{\delta t} = \frac{Q_{w,n-1/2} - Q_{w,n+1/2}}{D_{sn,n}} \tag{148}$$

with upper boundary condition $Q_{w,-1/2} = R_f / \rho_{\mathrm{wat}}$.

Each driving data timestep is divided into $N_{\mathtt{shyd}}$ substeps of length $\delta t_s = \delta t / N_{\mathtt{shyd}}$ to improve the numerical stability of solving equation (148). Layer liquid water contents at the end of each substep are found by the Newton-Raphson method, with increments in each iteration found by forward substitution. Zeros in the liquid mass balance residuals

$$r_n = \frac{\theta_{w,n} - \theta_{w,n}^{(0)}}{\delta t_s} + \frac{Q_{w,n-1/2} - Q_{w,n+1/2}}{D_{sn,n}} \tag{149}$$

are sought by adding increments

$$\delta\theta_{w,n} = \begin{cases} -r_n / b_n & n = 1 \\ -(a_n \delta\theta_{w,n-1} + r_n)/b_n & 1 < n \leq N_{\mathrm{snow}} \end{cases} \tag{150}$$

and recalculating the water fluxes, with

$$a_n = -3 \frac{k_{\mathrm{sat},n-1}}{D_{sn,n-1}} \frac{(\theta_{w,n-1} - \theta_{r,n-1})^2}{(\phi_{n-1} - \theta_{r,n-1})^3} \tag{151}$$

and

$$b_n = \frac{1}{\delta t_s} + 3 \frac{k_{\mathrm{sat},n}}{D_{sn,n}} \frac{(\theta_{w,n} - \theta_{r,n})^2}{(\phi_n - \theta_{r,n})^3} \tag{152}$$

8.7.4 Freezing of liquid water in snow

The cold content of a snow layer is $C_c = C_{sn}(T_m - T_{sn})$. If this is greater than zero and there is liquid water in the layer, an amount $\delta I = \min(W, L_f^{-1} C_c)$ freezes and is added to the layer ice mass. An increment $L_f \delta I / C_{sn}$ is added to the layer temperature.

9 Soil temperatures (subroutine SOIL)

Heat flux at the soil surface is $G_{\text{soil}} = G_s$ if there is no snow and $G_{\text{soil}} = G_{N_{\text{snow}}}$ at the base of the snow if there is snow. Timestep increments in soil layer temperatures are found by solving a tridiagonal matrix equation with thermal transmittance between soil layers (U), below-diagonal (a), diagonal (b) and above-diagonal (c) matrix elements, and right-hand side vector elements (r)

$$U_n = 2\left(\frac{\Delta z_{sl,n}}{\lambda_{sl,n}} + \frac{\Delta z_{sl,n+1}}{\lambda_{sl,n+1}}\right)^{-1} \tag{153}$$

$$b_n = C_{sl,n} + U_n \delta t \tag{154}$$

$$c_n = -U_n \delta t \tag{155}$$

$$r_n = [G_{\text{soil}} + U_n(T_{sl,n+1} - T_{sl,n})]\delta t \tag{156}$$

for $n = 1$,

$$U_n = 2\left(\frac{\Delta z_{sl,n}}{\lambda_{sl,n}} + \frac{\Delta z_{sl,n+1}}{\lambda_{sl,n+1}}\right)^{-1} \tag{157}$$

$$a_n = -c_{n-1} \tag{158}$$

$$b_n = C_{sl,n} + (U_{n-1} + U_n)\delta t \tag{159}$$

$$c_n = -U_n \delta t \tag{160}$$

$$r_n = U_{n-1}(T_{sl,n-1} - T_{sl,n})\delta t + U_n(T_{sl,n+1} - T_{sl,n})\delta t \tag{161}$$

for $n = 2, ..., N_{\text{soil}} - 1$, and

$$U_n = \frac{\lambda_{sl,n}}{\Delta z_{sl,n}} \tag{162}$$

$$a_n = -c_{n-1} \tag{163}$$

$$b_n = C_{sl,n} + (U_{n-1} + U_n)\delta t \tag{164}$$

$$r_n = U_{n-1}(T_{sl,n-1} - T_{sl,n})\delta t \tag{165}$$

for $n = N_{\text{soil}}$. This assumes that there is no heat flux at the base of the soil model.