Tutorial 3: Dimensionality Reduction

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Outline

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1. Generalized Rayleigh Quotient

Fisher Criterion

$$J(\mathbf{w}) = \frac{\mathbf{w}^t \mathbf{S}_B \mathbf{w}}{\mathbf{w}^t \mathbf{S}_W \mathbf{w}} \tag{1}$$

$J(\mathbf{w})$ is the generalized Rayleigh quotient. A vector $\mathbf{w}$ that maximizes $J(\cdot)$ must satisfy

$$\mathbf{S}_B \mathbf{w} = \lambda \mathbf{S}_W \mathbf{w} \tag{2}$$

for some constant λ.

1. Generalized Rayleigh Quotient

Maximizing $J(\mathbf{w})$ is equivalent to

$$\max_{\mathbf{w}} \quad \mathbf{w}^t \mathbf{S}_B \mathbf{w} \tag{3}$$

$$s.t. \quad \mathbf{w}^t \mathbf{S}_W \mathbf{w} = K \tag{4}$$

which can be solved using Lagrange multipliers.

1. Generalized Rayleigh Quotient

Define the Lagrangian:

$$L = \mathbf{w}^t \mathbf{S}_B \mathbf{w} - \lambda(\mathbf{w}^t \mathbf{S}_W \mathbf{w} - K) \tag{5}$$

Maximize with respect to $\mathbf{w}$:

$$\nabla_{\mathbf{w}} L = 2(\mathbf{S}_B - \lambda \mathbf{S}_W)\mathbf{w} = 0 \tag{6}$$

To obtain the solution:

$$\mathbf{S}_B \mathbf{w} = \lambda \mathbf{S}_W \mathbf{w} \tag{7}$$

2. Lagrange Optimization