Homework 3 # Solution

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Problem 1

1. The sampling density goes as $n^{1/d}$, and thus if we need n_1 samples in $d = 1$ dimensions, an "equivalent" number samples in d dimensions is n_1^d. Thus if we needed 100 points in a line (i.e., $n_1 = 100$), then for $d = 20$, we would need $n_{20} = (100)^{20} = 10^{40}$ points - roughly the number of atoms in the universe.

2. Consider points uniformly distributed in the unit interval $0 \leq x \leq 1$. The length containing fraction p of all the points is of course p. In d dimensions, the width of a hypercube containing fraction p of points is $l_d(p) = p^{1/d}$. Thus we have

$$l_5(0.01) = (0.01)^{1/5} = 0.398$$
$$l_5(0.1) = (0.1)^{1/5} = 0.631$$
$$l_{20}(0.01) = (0.01)^{1/20} = 0.794$$
$$l_{20}(0.1) = (0.1)^{1/20} = 0.891$$

3. As the dimensionality increases, to keep the same sampling density, we must increase the number of samples exponentially; on the other hand, to include the same fraction of samples in the hypercube, the length of the hypercube edge should be increased, which implies that the resolution of the estimated density will be reduced.