## Tutorial 3: Dimensionality Reduction

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#### Outline

- 1 Generalized Rayleigh Quotient
- Lagrange Optimization
- 3 Exercises

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- 2 Lagrange Optimization
- 3 Exercises

# 1. Generalized Rayleigh Quotient

#### Fisher Criterion

$$J(\mathbf{w}) = \frac{\mathbf{w}^t \mathbf{S}_B \mathbf{w}}{\mathbf{w}^t \mathbf{S}_W \mathbf{w}} \tag{1}$$

 $J(\mathbf{w})$  is the generalized Rayleigh quotient. A vector  $\mathbf{w}$  that maximizes  $J(\cdot)$  must satisfy

$$\mathbf{S}_B \mathbf{w} = \lambda \mathbf{S}_W \mathbf{w} \tag{2}$$

for some constant  $\lambda$ .

### 1. Generalized Rayleigh Quotient

Maximizing  $J(\mathbf{w})$  is equivalent to

$$\max_{\mathbf{w}} \ \mathbf{w}^t \mathbf{S}_B \mathbf{w} \tag{3}$$

$$s.t. \quad \mathbf{w}^t \mathbf{S}_W \mathbf{w} = K \tag{4}$$

which can be solved using Lagrange multipliers.

## 1. Generalized Rayleigh Quotient