Joint Digital Precoding and Power Allocation for QoS-Aware Weighted Sum-Rate Maximization in mmWave Hybrid Beamforming Systems

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***Abstract*—Beam Division Multiple Access (BDMA) with hybrid precoding has recently been proposed for multiuser multiple-input multiple-output (MU-MIMO) systems by simultaneously transmitting multiple digitally precoded users' data streams via different beams. In the hybrid precoding system, the analog and digital precoder design as well as the power allocation problem are highly non-convex. In this paper, we first design the analog precoder in BDMA transmission systems. After obtaining the analog precoder, the non-convex problem can be reformulated as an iterative rank-constrained D.C. (difference of two convex functions) programming problem by setting a dummy variable. In contrast to the conventional approximation algorithms like zero-forcing or D.C. algorithms, the proposed algorithm directly maximizes the weighted sum-rate (WSR) $\tau_u \log_2(1+\mathbf{SINR}_u)$ of u-th user by jointly optimizing the digital precoder and power allocation. Furthermore, the iterative algorithm is proved to be monotonic and convergent. The simulation results confirm the effectiveness of the proposed iterative algorithm.**

***Index Terms*—BDMA, Hybrid Beamforming, D.C. Programming, Power Allocation.**

I. INTRODUCTION

To meet the ever-increasing demand for higher user data rates, it is envisioned that the next-generation cellular systems will be equipped with massive antenna arrays [1]. Capitalizing on a large number of antennas at the base station (BS), BDMA was first proposed to decompose the multiuser multiple-input multiple-output (MU-MIMO) system into multiple single-user MIMO channels by multiplexing multiple users' data onto non-overlapping beams in [2]. BDMA is particularly attractive in practice as beamforming is commonly implemented in the analog domain using low-cost phase shifters. Meanwhile, to eliminate the multiuser interference while avoiding the unaffordable hardware cost and power consumption of full digital beamforming, hybrid digital and analog beamforming has been proposed for massive MIMO transmissions by dividing the precoding process into two steps, namely analog and digital precoding [3]. More specifically, the transmitted signals are first precoded digitally using a smaller number of RF chains followed by the analog precoding implemented with a much larger number of low-cost phase shifters. As a result, the hybrid analog-digital precoding architecture requires significantly less RF chains as compared to the fully digital precoding.

Despite the advantages for hybrid beamforming structure, the analog and digital precoders are intractable to be designed in closed-form since its non-convexity and computational complexity. Besides, power allocation is also an important problem in co-channel interference management for the multiuser wireless network. To maximize the weighted sum-rate (WSR) in hybrid beamforming systems, the problem is usually highly non-convex, especially for the coupled hybrid precoding and power constraints. Therefore, most of the existing works separate the precoding and power allocation problem to two isolated problems. For instance, [4] alternatively optimizes the power allocation for sum-rate capacity maximization problem implementing water filling by assuming the analog precoders are strictly orthogonal among distinct users. For single antenna transceivers, the difference of two convex functions (D.C.) programming algorithm is presented to maximize WSR by finding the optimal power allocation in [5].

Furthermore, in many MIMO applications, it is desirable to design a system satisfying the quality of service (QoS) constraint for each user by adjusting the power on different channels [6]. Recently, as the extension of signal-to-interference ratio (SIR) balancing for multiuser code division multiple access (CDMA) system [7], signal-to-interference-and-noise ratio (SINR)- balancing is proposed to simplify the QoS-aware problem by allocating power on each data stream. This optimization problem satisfies the QoS requirement because at the optimum all the weighted SINRs are equal. However, balancing SINR approach implies the system performances are limited by the worst user causing a reduction of the overall sum-rate capacity.

Finally, it is worth noting that most of the exiting works approximate the sum-rate maximization problem as an interference minimization (zero-forcing) or SINR maximization problem.

Motivated by addressing the challenges, the main contributions of this paper are summarized as follows:

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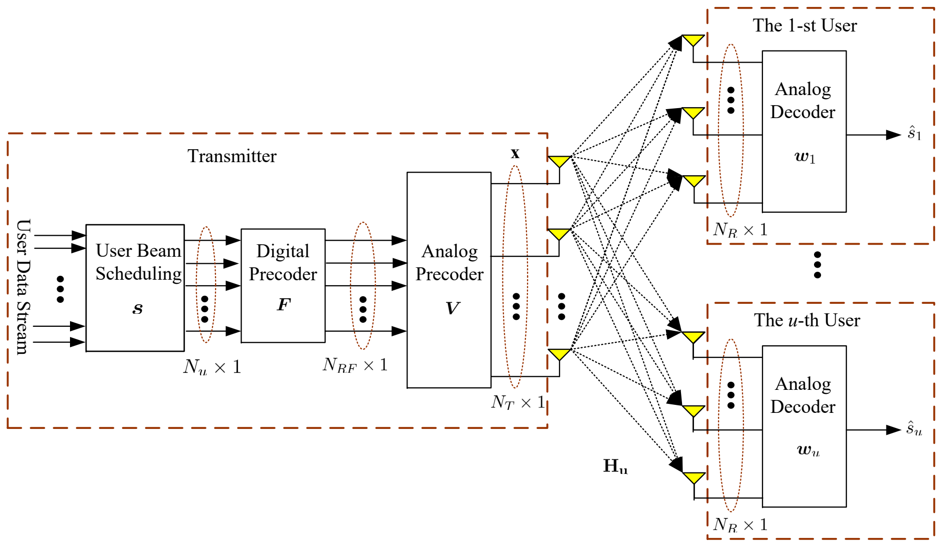

Fig. 1. Block diagram of the hybrid precoding system under consideration.

- In sharp contrast to the conventional optimization algorithms that separate the precoding and power allocation problems independently, a rank-constrained D.C. problem is presented to jointly optimize the digital precoder and power in this paper.
- To solve the general rank-constrained D.C. problems, an iterative algorithm is proposed to formulate the objective function as a standard convex optimization problem in each iteration. The convergence of the proposed algorithm is also proved.
- To address the widely-known disadvantages of zero-forcing and SINR-balancing algorithms in WSR maximization, the proposed iterative approach can maximize the WSR with QoS constraint for each user by jointly optimizing the digital precoder and power allocation. The simulation results show the effectiveness of our proposed algorithm.

Notation: In this paper, we use uppercase boldface letters to denote matrices and lowercase boldface letters to denote vectors. $j = \sqrt{-1}$ and $\boldsymbol{I}_N$ denotes the identity matrix with size $N \times N$. $\boldsymbol{A}^T$ and $\boldsymbol{A}^H$ denote transpose and conjugate transpose of $\boldsymbol{A}$, respectively. $[\boldsymbol{a}]_i$ denotes the i-element of $\boldsymbol{a}$. $\|\boldsymbol{A}\|$ stands for the L2 norm of $\boldsymbol{A}$ while $|A|$ denotes the absolute value of A. $\mathbf{rank}(\boldsymbol{A})$ and $\mathbf{trace}(\boldsymbol{A})$ represent the rank and trace of $\boldsymbol{A}$, respectively. Finally, $\bigtriangledown f(\boldsymbol{A})$ represents the gradient of function $f(\boldsymbol{A})$.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a multiuser mmWave MIMO system shown in Fig. 1, in which a transmitter equipped with N_{RF} RF chains and N_T antennas transmits N_U data streams to N_U receivers with N_R receive antennas. Following the same assumption commonly employed in the literature [8], we assume only one data stream is designated to each scheduled receiver. We use $\boldsymbol{s}(n)$ to denote the n-th block of N_U data to be transmitted with $\mathbb{E}\left[\boldsymbol{s}\boldsymbol{s}^H\right] = \frac{1}{N_U}\boldsymbol{I}_{N_U}$. In the sequel, we concentrate on a single block and omit the temporal index n for notational simplicity.

The hybrid precoding system first multiplies $\boldsymbol{s}$ with the digital precoding matrix $\boldsymbol{F} = [\boldsymbol{f}_1, \boldsymbol{f}_2, \cdots, \boldsymbol{f}_{N_U}]$ with $\boldsymbol{f}_u$ of dimension $N_{RF} \times 1$ being the digital beamforming vector for the u-th user, $u = 1, 2, \cdots, N_U$. After that, the output signal will be multiplied by the analog precoding matrix $\boldsymbol{V} = [\boldsymbol{v}_1, \cdots, \boldsymbol{v}_u, \cdots, \boldsymbol{v}_{N_{RF}}]$ with $\boldsymbol{v}_u$ of dimension $N_T \times 1$ being the u-th analog beamforming vector for $u = 1, 2, \cdots, N_{RF}$. The resulting precoded signal $\boldsymbol{x}$ of dimension $N_T \times 1$ can be expressed as

$$\boldsymbol{x} = \boldsymbol{V} \cdot \boldsymbol{F} \cdot \boldsymbol{s} = \boldsymbol{V} \sum_{u=1}^{N_U} \boldsymbol{f}_u s_u. \tag{1}$$

The precoded signal $\boldsymbol{x}$ is then broadcast to N_U users. The signal received by the u-th user is given by

$$\begin{aligned} \boldsymbol{y}_u &= \boldsymbol{H}_u \boldsymbol{x} + \boldsymbol{n}_u & (2)\\ &= \boldsymbol{H}_u \boldsymbol{V} \boldsymbol{f}_u s_u + \boldsymbol{H}_u \boldsymbol{V} \sum_{\substack{i=1\\ i\neq u}}^{N_U} \boldsymbol{f}_i s_i + \boldsymbol{n}_u, & (3) \end{aligned}$$

where $\boldsymbol{H}_u \in \mathbb{C}^{N_R \times N_T}$ is the MIMO channel matrix between the transmitter and the u-th receiver[9]. Furthermore, $\boldsymbol{n}_u$ is complex additive white Gaussian noise with zero mean and variance equal to σ_u^2.

Assuming the receivers are all low-cost terminals that perform analog beamforming only in decoding, the decoded signal by the u-th user denoted by $\hat{s}_u$ is given by

$$\hat{s}_u = \boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{V} \boldsymbol{f}_u \boldsymbol{s} + \boldsymbol{w}_u^H \tilde{\boldsymbol{n}}_u, \tag{4}$$

where $\boldsymbol{w}_u$ of dimension $N_R \times 1$ is the analog beamforming vector employed by the u-th receiver with the power constraint of $\|\boldsymbol{w}_u\|^2 = 1$ and

$$\tilde{\boldsymbol{n}}_u = \boldsymbol{H}_u \boldsymbol{V} \sum_{\substack{i=1 \\ i \neq u}}^{N_U} \boldsymbol{f}_i s_i + \boldsymbol{n}_u. \tag{5}$$

Note that the first term in Eq. (4) stands for the desired signal while the second term is the sum of its own receiver noise and interference from other users.

B. Channel Model

As shown in [10], the mmWave wireless channel can be well modeled by the Saleh-Valenzuela model. Following the same approach developed in [8], we assume that each scatter only contributes one single propagation path. As a result, the u-th user's channel model can been modeled as

$$\boldsymbol{H}_u = \sqrt{\frac{N_T N_R}{L_u}} \sum_{l=1}^{L_u} \alpha_{u,l} \cdot \boldsymbol{a}_R(\phi_{u,l}^r, \theta_{u,l}^r) \cdot \boldsymbol{a}_T^H(\phi_{u,l}^t, \theta_{u,l}^t), \tag{6}$$

where L_u is the number of scatters of the u-th user's channel. Furthermore, $\alpha_{u,l}$, $\theta_{u,l}^r/\phi_{u,l}^r$ and $\theta_{u,l}^t/\phi_{u,l}^t$ are the complex path gain, azimuth/elevation angles of arrival(AoA) and azimuth/elevation angles of departure(AoD) of the l-th path of the u-th user, respectively. Finally, $\boldsymbol{a}$ is the array response vector. For an uniform planar array (UPA) of size $W \times Q$ considered in this work, the array response vector $\boldsymbol{a}$ is given by

$$\begin{aligned}\boldsymbol{a}(\phi, \theta) = &\frac{1}{\sqrt{WQ}} \Big[1, e^{j\kappa d(\sin\phi \sin\theta + \cos\theta)}, \\ &\cdots, e^{j\kappa d((P-1)\sin\phi\sin\theta + (Q-1)\cos\theta)}\Big]^T,\end{aligned} \tag{7}$$

where $\kappa = \frac{2\pi}{\lambda}$ is the wavenumber and d is the distance between two adjacent antennas.

C. Problem Formulation

For notational simplicity, we denote by $\boldsymbol{g}_u^H$ the effective array gain of the u-th user with

$$\boldsymbol{g}_u^H = \boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{V}. \tag{8}$$

Then, the channel capacity of the u-th user is given by

$$R_u(\boldsymbol{p}, \boldsymbol{W}, \boldsymbol{V}, \boldsymbol{F}) = \log\left(1 + \frac{p_u |\boldsymbol{g}_u^H \boldsymbol{f}_u|^2}{\sum_{\substack{i=1 \\ i\neq u}}^{N_U} p_i |\boldsymbol{g}_u^H \boldsymbol{f}_i|^2 + \sigma_u^2}\right). \tag{9}$$

Subsequently, the system WSR can be formulated as

$$R_{tot} = \sum_{u=1}^{N_U} \tau_u R_u(\boldsymbol{p}, \boldsymbol{W}, \boldsymbol{V}, \boldsymbol{F}), \tag{10}$$

where $\boldsymbol{\tau} = \left[\tau_1, \tau_2, \cdots, \tau_{N_U}\right]$ with $\sum_{u=1}^{N_U} \tau_u = N_U$ are the corresponding weights for users and $\boldsymbol{p} = [p_1, p_2, \cdots, p_{N_U}]$.

Finally, the optimal design of the digital and analog precoding matrices as well as power allocation can be formulated as

$$\begin{aligned} \mathcal{P}_1 : \quad & \underset{\boldsymbol{W}, \boldsymbol{V}, \boldsymbol{F}}{\text{maximize}} \quad R_{tot}(\boldsymbol{p}, \boldsymbol{W}, \boldsymbol{V}, \boldsymbol{F}) \\ \text{subject to} \quad & C_1 : |[\boldsymbol{v}_u]_i|^2 = 1/N_T, \quad 0 < i \leq N_T; \\ & C_2 : |[\boldsymbol{w}_u]_j|^2 = 1/N_R, \quad 0 < j \leq N_R; \\ & C_3 : \|\boldsymbol{V}\boldsymbol{f}_u\|^2 = 1, \quad 0 < u < N_U; \\ & C_4 : \sum_{u=1}^{N_U} p_u \leq P; \\ & C_5 : R_u \geq \lambda_u, \quad 0 < u < N_U, \end{aligned} \tag{11}$$

where C_1 and C_2 are the analog constraints for receiver and transmitter. C_3 ensures that each RF chain is of unit power. The total transmitter power is constrained by P as shown in C_4. Finally, C_5 ensures that the u-th user's capacity should be guaranteed not less than a certain threshold λ_u.

The problem $\mathcal{P}_1$ is highly non-convex and intractable to derive a closed-form solution. In this paper, we first design the analog precoder to relax the phase constraints. Then, a general rank-constrained D.C. programming algorithm is proposed to derive the optimal digital precoder and power allocation while formulating the problem in each iteration as a standard convex optimization problem.

III. Analog Beamforming Design

Since the inherent NP-hard of the problem $\mathcal{P}_1$, we assume the digital precoder is $\boldsymbol{f}_u \boldsymbol{f}_u^H = \boldsymbol{I}_{N_U}$ with uniform power allocation for BDMA transmissions in this section. We begin with the analog beamforming design for both transmitter and receiver. It is well-known that distinct array response vectors are asymptotically orthogonal as the number of antennas in an antenna array goes to infinity [2], *i.e.*

$$\lim_{N \to +\infty} \boldsymbol{a}^H(\phi_{k,u}^t, \theta_{k,u}^t) \cdot \boldsymbol{a}(\phi_{\ell,v}^t, \theta_{\ell,v}^t) = \delta(k-\ell)\delta(u-v). \tag{12}$$

However, since the antenna number is finite in practice, the residual interference must be considered in the analog precoding design. Recalling the channel model presented in Equation (6), we can asymptotically orthogonalize the transmitted signals by optimizing the design of $\boldsymbol{w}_u$ and $\boldsymbol{v}_u$:

$$\begin{aligned} \{\boldsymbol{w}_u^*, \boldsymbol{v}_u^*\} = &\arg\max_{\tilde{\boldsymbol{w}}_u, \tilde{\boldsymbol{v}}_u} \sum_{u=1}^{M_k} \log\left(1 + \text{SINR}\left(\tilde{\boldsymbol{w}}_u, \tilde{\boldsymbol{v}}_u\right)\right) \\ \text{subject to} \quad & \tilde{\boldsymbol{v}}_u \in \mathcal{A}_u^t; \\ & \tilde{\boldsymbol{w}}_u \in \mathcal{A}_u^r, \end{aligned} \tag{13}$$

where the array response vectors in transmitter and receiver are denoted by

$$\begin{aligned} \mathcal{A}_u^t &= \left[\boldsymbol{a}_T(\phi_{u,1}^t, \theta_{u,1}^t), \cdots, \boldsymbol{a}_T(\phi_{u,L_u}^t, \theta_{u,L_u}^t)\right], \\ \mathcal{A}_u^r &= \left[\boldsymbol{a}_R(\phi_{u,1}^r, \theta_{u,1}^r), \cdots, \boldsymbol{a}_R(\phi_{u,L_u}^r, \theta_{u,L_u}^r)\right]. \end{aligned} \tag{14}$$

Furthermore, SINR $(\tilde{\boldsymbol{w}}_u, \tilde{\boldsymbol{v}}_u)$ is given by

$$\mathrm{SINR}_u\left(\tilde{\boldsymbol{w}}_u, \tilde{\boldsymbol{v}}_u\right) = \frac{|\tilde{\boldsymbol{w}}_u^H \boldsymbol{H}_u \tilde{\boldsymbol{v}}_u|^2}{\sum\limits_{i=1, i\neq u}^{N_U} |\tilde{\boldsymbol{w}}_u^H \boldsymbol{H}_u \tilde{\boldsymbol{v}}_i|^2 + \frac{1}{\gamma}}, \tag{15}$$

where $\gamma = \frac{P}{N_U \sigma_u^2}$ is the uniform SNR for each user.

The optimal analog precoder can be straightforwardly found by exhaustively searching in the feasible sets of $\mathcal{A}_{k,u}^T$ and $\mathcal{A}_{k,u}^R$.

IV. Proposed rank-constrained D.C. problem

As the analog precoder is given, we will derive an optimization problem to jointly find the optimal digital precoder and power allocation in this section. Recalling the problem $\mathcal{P}_1$, the function of $R_{tot}(\boldsymbol{F}, \boldsymbol{p})$ is still non-convex. To settle this challenge, we form the digital precoder as

$$\bar{\boldsymbol{F}}_u = p_u \boldsymbol{f}_u \boldsymbol{f}_u^H, \tag{16}$$

with constraints $\mathbf{rank}(\bar{\boldsymbol{F}}_u) \leq 1$ and $\bar{\boldsymbol{F}}_u \succeq \mathbf{0}$. Denote by $\boldsymbol{\mathcal{F}} = [\bar{\boldsymbol{F}}_1^{(n)}, \bar{\boldsymbol{F}}_2^{(n)}, \cdots, \bar{\boldsymbol{F}}_{N_U}^{(n)}]$.

Furthermore, the constraints C_3 and C_4 in $\mathcal{P}_1$ can be transformed as

$$\sum_{u=1}^{N_U} p_u \|\boldsymbol{V}\boldsymbol{f}_u\|^2 = \sum_{u=1}^{N_U} \boldsymbol{V}\bar{\boldsymbol{F}}_u \boldsymbol{V}^H \leq P. \tag{17}$$

It is worth noting that the optimal digital precoder $\boldsymbol{f}_u^*$ is the eigenvector corresponding to the only one non-zero eigenvalue p_u^* of optimal $\bar{\boldsymbol{F}}_u^*$, where p_u^* is the optimal power allocation.

Then the Equation (9) can be rewritten as

$$\begin{aligned} R_u(\bar{\boldsymbol{F}}_u) =& \log_2 \left(1 + \frac{\boldsymbol{g}_u^H \bar{\boldsymbol{F}}_u \boldsymbol{g}_u}{\sum_{\substack{i=1\\ i\neq u}}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_u^2}\right) \\ =& \log_2 \left(\sum_{i=1}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_u^2\right) \\ &- \log_2 \left(\sum_{\substack{i=1\\ i\neq u}}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_u^2\right). \end{aligned} \tag{18}$$

Thereby the problem $\mathcal{P}_1$ can be reformulated as a D.C. problem with a rank constraint:

$$\begin{aligned} \mathcal{P}_2: \quad & \underset{\bar{\boldsymbol{F}}}{\text{maximize}} \quad f(\boldsymbol{\mathcal{F}}) - g(\boldsymbol{\mathcal{F}}) \\ \text{subject to} \quad & C_1: \sum_{u=1}^{N_U} \boldsymbol{V}^H \bar{\boldsymbol{F}}_u \boldsymbol{V} \leq P; \\ & C_2: R_u \geq \lambda_u, u = 1, 2, \cdots, N_U; \\ & C_3: \bar{\boldsymbol{F}}_u \succeq \mathbf{0}, u = 1, 2, \cdots, N_U; \\ & C_4: \mathbf{rank}(\bar{\boldsymbol{F}}_u) \leq 1, u = 1, 2, \cdots, N_U, \end{aligned} \tag{19}$$

where

$$f(\boldsymbol{\mathcal{F}}) = \sum_{u=1}^{N_U} \tau_u \log_2 \left(\sum_{i=1}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_u^2\right), \tag{20}$$

$$g(\boldsymbol{\mathcal{F}}) = \sum_{u=1}^{N_U} \tau_u \log_2 \left(\sum_{\substack{i=1\\ i\neq u}}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_u^2\right). \tag{21}$$

It is worth noting that the QoS constraint C_2 can be straightforwardly reformulated as a convex constraint by the following transformation:

$$\boldsymbol{g}_u^H \bar{\boldsymbol{F}}_u \boldsymbol{g}_u + (1 - 2^{\lambda_u}) \left(\sum_{\substack{i=1\\ i\neq u}}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_u^2\right) \geq 0. \tag{22}$$

Finally, a standard D.C programming problem is derived after dropping C_4. Next, the iterative algorithm will be given to solve the problem $\mathcal{P}_2$ by ignoring C_4 constraint.

Since the problem $\mathcal{P}_2$ still has high computational complexity, the optimal $\boldsymbol{\mathcal{F}}^*$ can be obtained by iteratively solving $\bar{\boldsymbol{F}}_u$ for $u = 1, 2, \cdots, N_U$ until no improvement for next iteration as detailed in next section.

Similar to the procedures in [5], the iterative algorithm gives the optimal $\bar{\boldsymbol{F}}_u^{(n+1)}$ at n-th iteration by solving the following convex problem:

$$\begin{aligned} & \underset{\bar{\boldsymbol{F}}_u}{\text{maximize}} \quad f(\bar{\boldsymbol{F}}_u) - g(\bar{\boldsymbol{F}}_u^{(n)}) - \langle \bigtriangledown g(\bar{\boldsymbol{F}}_u^{(n)}), \bar{\boldsymbol{F}}_u - \bar{\boldsymbol{F}}_u^{(n)} \rangle \\ & \text{subject to} \quad C_1, C_2, C_3 \quad \text{in} \quad \mathcal{P}_2, \end{aligned} \tag{23}$$

where $\langle \cdot \rangle$ denotes the inner product of two matrices, *i.e.* $\langle \boldsymbol{A}, \boldsymbol{B} \rangle = \mathbf{trace}(\boldsymbol{A}^T \boldsymbol{B})$. The gradient of $g(\bar{\boldsymbol{F}}^{(n)})$ is easily given by

$$\bigtriangledown g(\bar{\boldsymbol{F}}_u^{(n)}) = \sum_{\substack{j=1\\ j\neq u}}^{N_U} \frac{w_j / \ln 2}{\sum_{\substack{i=1\\ i\neq j}}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_j^2} \boldsymbol{g}_j \boldsymbol{g}_j^H. \tag{24}$$

It is worth noting that $\langle \bigtriangledown g(\bar{\boldsymbol{F}}_u^{(n)}), \bar{\boldsymbol{F}}_u - \bar{\boldsymbol{F}}_u^{(n)} \rangle$ is a real value since $\bigtriangledown g(\bar{\boldsymbol{F}}_u^{(n)})$ and $\bar{\boldsymbol{F}}_u - \bar{\boldsymbol{F}}_u^{(n)}$ are both hermitian.

V. Proposed iterative algorithm for Rank-constrained D.C. problem

To solve the problem $\mathcal{P}_2$, we will first introduce a rank constrained optimization problem (RCOP). Inspired by the RCOP, we propose an iterative algorithm to solve $\mathcal{P}_2$ by formulating a convex optimization problem in each iteration.

A. The General RCOP

We now consider the rank constraint C_4 in $\mathcal{P}_2$. A general RCOP to optimize a convex objective subject to a set of convex and rank constraints can be formulated as follows:

$$\begin{aligned} \mathcal{P}_3: \quad & \underset{\boldsymbol{X}}{\text{minimize}} \quad f(\boldsymbol{X}) \\ \text{subject to} \quad & \boldsymbol{X} \succeq 0; \\ & \boldsymbol{X} \in \mathcal{C}; \\ & \mathbf{rank}(\boldsymbol{X}) \leq r, \end{aligned} \tag{25}$$

where $f(\boldsymbol{X})$ is a convex function, $\mathcal{C}$ is the set of given convex constraints and $\boldsymbol{X} \in \mathbb{C}^{m\times m}$ is a general positive semidefinite matrix.

The RCOP can be solved by an iterative method by gradually approaching the constrained rank [11]. At each iteration n, we will solve the following semidefinite programming (SDP) problem:

$$\begin{aligned} \underset{\boldsymbol{X}^{(n+1)}, e^{(n)}}{\text{minimize}} \quad & f(\boldsymbol{X}^{(n+1)}) + we^{(n+1)} \\ \text{subject to} \quad & \boldsymbol{X}^{(n+1)} \succeq \boldsymbol{0}; \\ & \boldsymbol{X}^{(n+1)} \in \mathcal{C}; \\ & e^{(n+1)}\boldsymbol{I}_{m-r} - \boldsymbol{U}^{(n)H}\boldsymbol{X}^{(n+1)}\boldsymbol{U}^{(n)} \succeq \boldsymbol{0}; \\ & e^{(n+1)} \leq e^{(n)}, \end{aligned} \tag{26}$$

where $w > 0$ is the weighting factor. Using eigenvalue decomposition (EVD), $\boldsymbol{U}^n \in \mathbb{C}^{m\times(m-r)}$ is the orthonormal eigenvectors corresponding to the $n-r$ smallest eigenvalues of $\boldsymbol{X}^{(n)}$ solved at previous n-th iteration. At the first iteration where $n=0$, $e^{(0)}$ is the $(m-r)$-th smallest eigenvalue of $\boldsymbol{X}^{(0)}$. $\boldsymbol{X}^{(0)}$ is obtained via

$$\begin{aligned} \underset{\boldsymbol{X}^{(0)}}{\text{minimize}} \quad & f(\boldsymbol{X}^{(0)}) \\ \text{subject to} \quad & \boldsymbol{X}^{(0)} \succeq 0; \\ & \boldsymbol{X}^{(0)} \in \mathcal{C}, \end{aligned} \tag{27}$$

and $\boldsymbol{U}^{(0)}$ is the eigenvectors corresponding to $n-r$ smallest eigenvalues of $\boldsymbol{X}^{(0)}$.

B. Iterative Algorithm for $\mathcal{P}_2$

As shown in Section IV, Equation (23) is obviously a concave optimization problem. By combining the $\mathcal{P}_2$ and $\mathcal{P}_3$, an iterative algorithm for the rank-constrained D.C programming problem is derived. The optimal $g(\bar{\boldsymbol{F}}^{(n+1)})$ in the n-th iteration is given by solving the following convex problem:

$$\begin{aligned} \underset{\bar{\boldsymbol{F}}_u, e^{(n+1)}}{\text{minimize}} \quad & t(\bar{\boldsymbol{F}}_u, e^{(n+1)}) \\ \text{subject to} \quad & \sum_{u=1}^{N_U} \boldsymbol{V}^H \bar{\boldsymbol{F}}_u \boldsymbol{V} \leq N_U; \\ & R_u \geq \lambda_u, u = 1, 2, \cdots, N_U; \\ & \bar{\boldsymbol{F}}_u \succeq \boldsymbol{0}; \\ & e^{(n+1)}\boldsymbol{I}_{N_U-1} - \boldsymbol{U}^{(n)H}\bar{\boldsymbol{F}}_u\boldsymbol{U}^{(n)} \succeq \boldsymbol{0}; \\ & e^{(n+1)} \leq e^{(n)}, \end{aligned} \tag{28}$$

where

$$\begin{aligned} & t(\bar{\boldsymbol{F}}_u, e^{(n+1)}) \\ =& g(\bar{\boldsymbol{F}}_u^{(n)}) + \langle \bigtriangledown g(\bar{\boldsymbol{F}}_u^{(n)}), \bar{\boldsymbol{F}}_u - \bar{\boldsymbol{F}}_u^{(n)} \rangle - f(\bar{\boldsymbol{F}}_u) + we^{(n+1)}, \end{aligned}$$

and $\boldsymbol{U}^{(n)}$ is the eigenvectors corresponding to $N_U - 1$ smallest eigenvalues of $\bar{\boldsymbol{F}}_u^{(n)}$.

Despite the formidable appearance of Equation (28), it is indeed a standard convex optimization problem that can be solved via available convex software packages, such as CVX [12].

The proposed iterative algorithm can be summarized as **Algorithm 1**. In each iteration, we solve a D.C programming problem and update the optimal $\bar{\boldsymbol{F}}_u^{*(n)}$. If the WSR $s^{(n)}$ can be no longer improved, the digital precoder $\bar{\boldsymbol{F}}_{u+1}^{(n)}$ of $(u+1)$-th user is then optimized successively. The iteration will stop if no any improvement for $t(\bar{\boldsymbol{F}}_u^{(n)})$.

Algorithm 1 Proposed Iterative Algorithm for Rank-constrained D.C. Problem

Input:
Effective channel: $\boldsymbol{g}_1, \boldsymbol{g}_2, \cdots, \boldsymbol{g}_{N_U}$;
Random initialization matrices of $\bar{\boldsymbol{F}}_u^{(0)}$, $u = 1, 2, \cdots, N_U$;
The initialization information: s, s', $t^{(0)}$, $t^{(1)}$, ϵ;
The initialization value by solving Equation (27): $\boldsymbol{U}^{(0)}, e^{(0)}$

Procedures:
while $|(s'-s)/s| \leq \epsilon$ **do**
  Update s: $s = s'$
  for $0 \leq u \leq N_U$ **do**
    $n = 0$
    while $|(t(\bar{\boldsymbol{F}}_u^{(n+1)}) - t(\bar{\boldsymbol{F}}_u^{(n)})| \leq \epsilon$ **do**
      Obtain the optimal value $t^{(n+1)}$ of objective function and $\bar{\boldsymbol{F}}_u^{(n+1)}, e^{(n+1)}$ in Equation (28)
      Update $\boldsymbol{U}^{(n)}$ from $\bar{\boldsymbol{F}}_u^{(n+1)}$ via EVD
      Update $n = n + 1$
    end while
  end for
  Update s': $s' = t^{(n+1)}$
end while
Outputs: $\boldsymbol{\mathcal{F}}^* = [\bar{\boldsymbol{F}}_1^{*(n)}, \bar{\boldsymbol{F}}_2^{*(n)}, \cdots, \bar{\boldsymbol{F}}_{N_U}^{*(n)}]$.

C. Convergence Analysis

In the following, we provide the convergence analysis of the proposed iterative algorithm for solving the rank-constrained D.C. programming problem.

As the function $g(\bar{\boldsymbol{F}}_u)$ is concave, its gradient $\bigtriangledown g(\bar{\boldsymbol{F}}_u)$ is also its super-gradient, therefore

$$g(\bar{\boldsymbol{F}}_u) \leq g(\bar{\boldsymbol{F}}_u^{(n)}) + \langle \bigtriangledown g(\bar{\boldsymbol{F}}_u^{(n)}), \bar{\boldsymbol{F}}_u - \bar{\boldsymbol{F}}_u^{(n)} \rangle. \tag{29}$$

Since $e^{(n+1)} \leq e^{(n)}$ and $\bar{\boldsymbol{F}}_u^{(n)}$ is feasible to Equation (28), it follows

$$\begin{aligned} & g(\bar{\boldsymbol{F}}_u^{(n+1)}) - f(\bar{\boldsymbol{F}}_u^{(n+1)}) + e^{(n+1)} \\ \leq & g(\bar{\boldsymbol{F}}_u^{(n)}) + \langle \bigtriangledown g(\bar{\boldsymbol{F}}_u^{(n)}), \bar{\boldsymbol{F}}_u - \bar{\boldsymbol{F}}_u^{(n)} \rangle - f(\bar{\boldsymbol{F}}_u) + e^{(n)}. \\ \leq & g(\bar{\boldsymbol{F}}_u^{(n)}) - f(\bar{\boldsymbol{F}}_u^{(n)}) + e^{(n)}, \end{aligned} \tag{30}$$

which shows that the solution $\bar{\boldsymbol{F}}_u^{(n+1)}$ is always better than or equal to the previous solution $\bar{\boldsymbol{F}}_u^{(n)}$. Thus, the algorithm must converge.

VI. Simulation Results

In this section, we will use computer simulation to show the effectiveness of the proposed rank-constrained D.C. programming algorithm. In the simulation schemes, we consider a transmitter equipped with a 4×4 UPA (*i.e.* $N_T = 16$) and $N_U = 12$ users each equipped with a 2×2 UPA (*i.e.* $N_R = 4$). The number of paths is set as $L_u = 4$. We consider the azimuth AoAs/AoDs uniformly distributed over $[0, 2\pi]$ while the elevation AoAs/AoDs uniformly distributed over $[-\pi/2, \pi/2]$, respectively. For each computer experiment, we compute the average over 100 realizations.

In Fig. 2, the wights are set to be $\tau_u = 1.1$, $u = 1, \cdots, 6$ and $\tau_u = 0.9$, $u = 7, \cdots, 12$. we can observe that the proposed algorithm has the best performance compared to the conventional algorithms. The line named "CCP-SVD" is realized for the algorithm reported in [13], where a similar rank-constrained problem is solved via a convex-concave procedure (CCP) while the rank constraint is relaxed using singular value decomposition (SVD). The lines "ZF HB" and "Uniform Power HB" represent the zero-forcing scheme with power allocation proposed in [5] and zero-forcing with uniform power allocation scheme in [3] respectively.

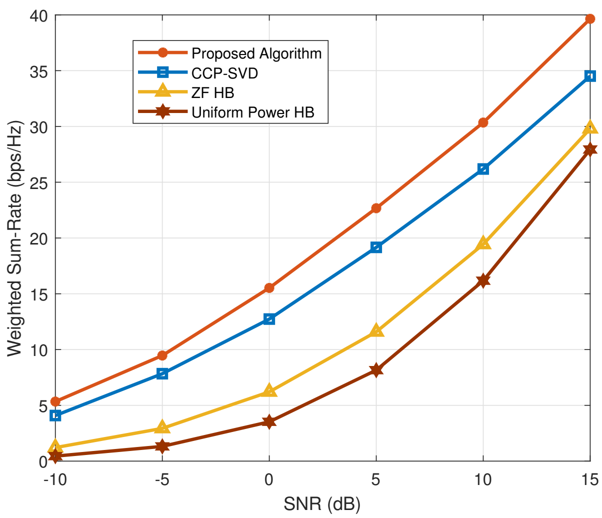

Fig. 2. Performance comparison for different algorithms.

Fig. 3 depicts the CDF of the user's data rate. It is evident that all users served by the QoS-aware power allocation algorithms satisfy the minimum QoS requirement (*i.e.* 1 bps/Hz). In contrast, the conventional zero-forcing algorithm with uniform power allocation suffers from an outage rate of about 20% where the outage is defined as the user data rate being below the minimum required data rate.

VII. Conclusion

In this paper, we present an algorithm to jointly optimize the digital precoder and power allocation for QoS-aware WSR maximization in the hybrid BDMA transmission systems. To solve the coupled non-convex problem, we first derive the analog precoder by exhaustive searching. Then a rank-constrained D.C. problem is developed. To address the challenge of rank constraint, we propose an iterative algorithm by combining the conventional D.C. problem with

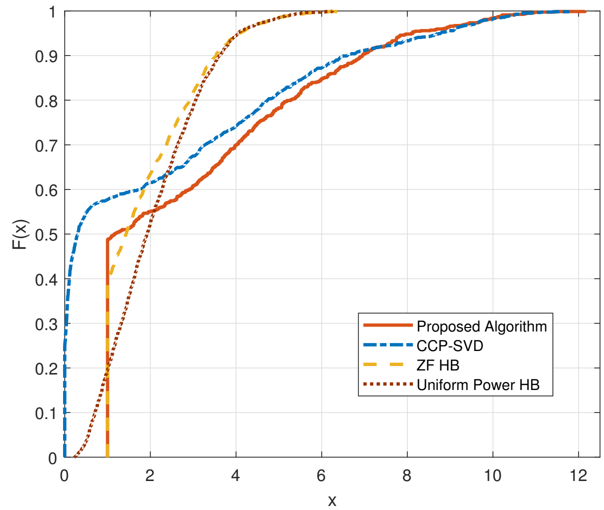

Fig. 3. CDF of users for different algorithms with SNR = 10 dB.

RCOP. Finally, the simulation results are given to confirm the effectiveness of the proposed algorithm.

References

[1] A. F. Molisch, M. Z. Win, Y.-S. Choi, and J. H. Winters, "Capacity of MIMO systems with antenna selection," *IEEE Transactions on Wireless Communications*, vol. 4, no. 4, pp. 1759–1772, 2005.

[2] C. Sun, X. Gao, S. Jin, M. Matthaiou, Z. Ding, and C. Xiao, "Beam Division Multiple Access Transmission for massive MIMO communications," *IEEE Transactions on Communications*, vol. 63, no. 6, pp. 2170–2184, June 2015.

[3] A. Alkhateeb, O. El Ayach, G. Leus, and R. W. Heath, "Channel estimation and hybrid precoding for millimeter wave cellular systems," *IEEE Journal of Selected Topics in Signal Processing*, vol. 8, no. 5, pp. 831–846, 2014.

[4] F. Sohrabi and W. Yu, "Hybrid digital and analog beamforming design for large-scale antenna arrays," *IEEE Journal of Selected Topics in Signal Processing*, vol. 10, no. 3, pp. 501–513, 2016.

[5] H. H. Kha, H. D. Tuan, and H. H. Nguyen, "Fast global optimal power allocation in wireless networks by local D.C. programming," *IEEE Transactions on Wireless Communications*, vol. 11, no. 2, pp. 510–515, 2012.

[6] D. P. Palomar, M. A. Lagunas, and J. M. Cioffi, "Optimum linear joint transmit-receive processing for MIMO channels with QoS constraints," *IEEE Transactions on Signal Processing*, vol. 52, no. 5, pp. 1179–1197, 2004.

[7] M. Xie and T.-M. Lok, "SINR balancing via base station association, beamforming, and power control in downlink multicell MISO systems," *IEEE Transactions on Wireless Communications*, vol. 17, no. 3, pp. 1811–1821, 2017.

[8] A. Alkhateeb, G. Leus, and R. W. Heath, "Limited feedback hybrid precoding for multi-user millimeter wave systems," *IEEE Transactions on Wireless Communications*, vol. 14, no. 11, pp. 6481–6494, 2015.

[9] O. El Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, "Spatially sparse precoding in millimeter wave MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 13, no. 3, pp. 1499–1513, 2014.

[10] T. S. Rappaport, R. W. Heath Jr, R. C. Daniels, and J. N. Murdock, *Millimeter wave wireless communications*. Pearson Education, 2014.

[11] C. Sun and R. Dai, "Rank-constrained optimization and its applications," *Automatica*, vol. 82, pp. 128–136, 2017.

[12] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," http://cvxr.com/cvx, Mar. 2014.

[13] B. Hu, C. Hua, C. Chen, and X. Guan, "Joint beamformer design for wireless fronthaul and access links in C-RANs," *IEEE Transactions on Wireless Communications*, vol. 17, no. 5, pp. 2869–2881, 2018.