Joint Precoding and Power Allocation for Hybrid Precoding Systems via Rank-Constrained D.C. Programming

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***Abstract*—Hybrid analog and digital precoding has recently been proposed for massive multiple-input multiple-output (MIMO) systems. However, it is challenging to optimize the design of precoding and power allocation as the optimization problem is highly non-convex. In this work, the non-convex problem is cast into the D.C. (difference of two convex functions) programming framework. To cope with the high dimensionality of the problem, an iterative rank-constrained D.C. programming technique is developed. As a result, the proposed algorithm is capable of directly maximizing the weighted sum-rate (WSR) of each user to take in account the quality of service (QoS) requirement. Furthermore, the monotonic convergence behavior of the proposed iterative algorithm is proved. Finally, simulation results confirm the effectiveness of the proposed iterative algorithm.**

***Index Terms*—Massive MIMO, Hybrid Beamforming, D.C. Programming, Power Allocation.**

I. Introduction

To meet the ever-increasing demand for higher user data rates, it is envisioned that the next-generation cellular systems will be equipped with massive antenna arrays [1]. Capitalizing on a large number of antennas at the base station (BS), BDMA was first proposed to decompose the multiuser multiple-input multiple-output (MU-MIMO) system into multiple single-user MIMO channels by multiplexing multiple users' data onto non-overlapping beams in [2]. BDMA is particularly attractive in practice as beamforming is commonly implemented in the analog domain using low-cost phase shifters. Meanwhile, to eliminate the multiuser interference while avoiding the unaffordable hardware cost and power consumption of full digital beamforming, hybrid digital and analog beamforming has been proposed for massive MIMO transmissions by dividing the precoding process into two steps, namely analog and digital precoding [3]. More specifically, the transmitted signals are first precoded digitally using a smaller number of RF chains followed by the analog precoding implemented with a much larger number of low-cost phase shifters. As a result, the hybrid analog-digital precoding architecture requires significantly less RF chains as compared to the fully digital precoding.

Despite its many advantages, the analog and digital beamforming requires highly computational design of the analog and digital precoding vectors. In addition, power allocation design plays an important role in managing co-channel interference in multiuser networks. However, joint precoding and power allocation design is usually highly non-convex as these precoding and power constraints are coupled. To the authors' best knowledge, no closed-form solutions to the joint precoding and power allocation design have been reported in the literature. Instead, most existing works separate the precoding and power allocation problem to two isolated problems. For instance, [4] alternatively optimizes the power allocation for the sum-rate capacity maximization problem implementing water filling by assuming the analog precoders are strictly orthogonal among distinct users.

Finally, for multi-user wireless networks, it is highly desirable to design a system that satisfies the quality of service (QoS) constraint for each user. In the literature, such QoS requirements are commonly met by adjusting the power allocated to different users in the system [5]. For a simple case, *i.e.* single antenna transceivers, the difference of two convex functions (D.C.) programming algorithm is presented to maximize the weighted sum-rate (WSR) with QoS guarantee by finding the optimal power allocation in [6]. D.C programming can be regarded as an elegant extension of convex optimization. To solve the D.C. problem, the non-convex problem is approximately solved by an iterative algorithm where each iteration requires solution of a convex program. Thus, the convexity of the constraints is mandatory for the conventional D.C. programming. In [7], a rank-constrained problem is solved via a convex-concave procedure (CCP) while the rank constraint is relaxed by using singular value decomposition (SVD). The solution is approximately represented by the vectors corresponding the largest singular values. However, this algorithm is not suitable for the general cases.

Motivated by the aforementioned challenges, this work develops joint precoding and power allocation for hybrid precoding systems using the rank-constrained D.C. programming. The main contributions of this work are summarized as follows:

This work was supported, in part, by the Shenzhen Science and Technology Innovation Committee under Grant No. ZDSYS20170725140921348, the Robotic Discipline Development Fund (2016-1418) from the Shenzhen Government and the National Natural Science Foundation of China under Grant No. 61731018.

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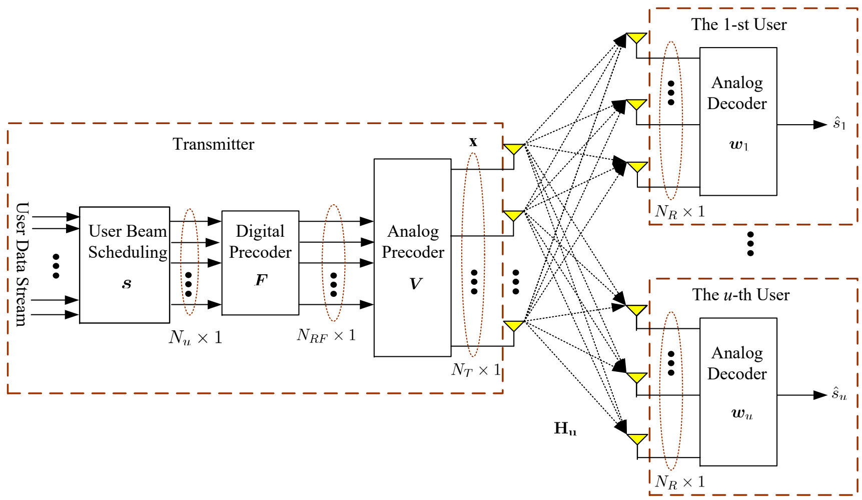

Fig. 1. Block diagram of the hybrid precoding system.

- In sharp contrast to the conventional optimization algorithms that separate the precoding and power allocation problems independently, a rank-constrained D.C. problem is presented to jointly optimize the digital precoder and power allocation to different data streams;
- To solve the general non-convex rank-constrained D.C. problems, an iterative algorithm is proposed to formulate the objective function as a standard convex optimization problem in each iteration. The convergence of the proposed algorithm is proved.
- Finally, we apply the newly established iterative ank-constrained D.C. programming technique to maximize the WSR with QoS constraints for each user by jointly optimizing the precoder design and power allocation. Simulation results are shown to demonstrate the effectiveness of the proposed algorithm.

Notation: In this paper, we use uppercase boldface letters to denote matrices and lowercase boldface letters to denote vectors. $j = \sqrt{-1}$ and $\boldsymbol{I}_N$ denotes the identity matrix with size $N \times N$. $\boldsymbol{A}^T$ and $\boldsymbol{A}^H$ denote transpose and conjugate transpose of $\boldsymbol{A}$, respectively. $[\boldsymbol{a}]_i$ denotes the i-element of vector $\boldsymbol{a}$. $\|\boldsymbol{A}\|$ stands for the L2 norm of $\boldsymbol{A}$ while $|A|$ denotes the absolute value of A. $\textbf{rank}(\boldsymbol{A})$ and $\textbf{trace}(\boldsymbol{A})$ represent the rank and trace of $\boldsymbol{A}$, respectively. Finally, $\bigtriangledown f(\boldsymbol{A})$ represents the gradient of function $f(\boldsymbol{A})$.

II. Review of D.C. Programming

In this section, we give a review of D.C. programming. D.C. problems arise in many applications in fields such as signal processing, machine learning, computer vision, and statistics.

In general, D.C. programming problems have the form

$$\begin{aligned} &\text{minimize} \quad f_0(\boldsymbol{x}) - g_0(\boldsymbol{x}) \qquad (1)\\ &\text{subject to} \quad f_i(\boldsymbol{x}) - g_i(\boldsymbol{x}) \le 0, i = 1, \cdots, m, \end{aligned}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the optimization variable, and the functions f_i: $\mathbb{R}^n \to \mathbb{R}$ and g_i: $\mathbb{R}^n \to \mathbb{R}$ for $i = 0, \cdots, m$ are convex.

The non-convex problem can be approximated by an iterative procedure. In $(n+1)$-th iteration, the objective function is formulated as

$$\text{minimize} \quad f_0(\boldsymbol{x}) - g_0(\boldsymbol{x}^{(n)}) - \langle \bigtriangledown g_0(\boldsymbol{x}^{(n)}), \boldsymbol{x} - \boldsymbol{x}^{(n)} \rangle, \quad (2)$$

which can be solved via available software packages [11]. $g_0(\boldsymbol{x})$ can be well approximated by its first oder approximation $g_0(\boldsymbol{x}^{(n)}) + \langle \bigtriangledown g_0(\boldsymbol{x}^{(n)}), \boldsymbol{x} - \boldsymbol{x}^{(n)} \rangle$ at a fairly large neighborhood of $\boldsymbol{x}^{(n)}$. Since the convex property, the $(n+1)$-th solution is always better than the previous one. Thus, the convergence can be straightforwardly verified.

Consequently, the solution of Equation (1) can be approximated by the iterative algorithm. An elegant application for power allocation can be found in [6].

III. System model and problem formulation

A. System Model

We consider a mmWave MU-MIMO system shown in Fig. 1, in which a transmitter equipped with N_{RF} RF chains and N_T antennas transmits N_U data streams to N_U receivers with N_R receive antennas. Following the same assumption commonly employed in the literature [8], we assume only one data stream is designated to each scheduled receiver. We use $\boldsymbol{s}(n)$ to denote the n-th block of N_U data to be transmitted with $\mathbb{E}\left[\boldsymbol{s}\boldsymbol{s}^H\right] = \frac{1}{N_U}\boldsymbol{I}_{N_U}$. In the sequel, we concentrate on a single block and omit the temporal index n for notational simplicity.

The hybrid precoding system first multiplies $\boldsymbol{s}$ with the digital precoding matrix $\boldsymbol{F} = [\boldsymbol{f}_1, \boldsymbol{f}_2, \cdots, \boldsymbol{f}_{N_U}]$ with $\boldsymbol{f}_u$ of dimension $N_{RF} \times 1$ being the digital beamforming vector for the u-th user, $u = 1, 2, \cdots, N_U$. After that, the output signal will be multiplied by the analog precoding matrix $\boldsymbol{V} = [\boldsymbol{v}_1, \cdots, \boldsymbol{v}_u, \cdots, \boldsymbol{v}_{N_{RF}}]$ with $\boldsymbol{v}_u$ of dimension $N_T \times 1$ being the u-th analog beamforming vector for $u = 1, 2, \cdots, N_{RF}$.

The resulting precoded signal $\boldsymbol{x}$ of dimension $N_T \times 1$ can be expressed as

$$\boldsymbol{x} = \boldsymbol{V} \cdot \boldsymbol{F} \cdot \boldsymbol{s} = \boldsymbol{V} \sum_{u=1}^{N_U} \boldsymbol{f}_u s_u. \tag{3}$$

The precoded signal $\boldsymbol{x}$ is then broadcast to N_U users. The signal received by the u-th user is given by

$$\begin{aligned} \boldsymbol{y}_u &= \boldsymbol{H}_u \boldsymbol{x} + \boldsymbol{n}_u & (4)\\ &= \boldsymbol{H}_u \boldsymbol{V} \boldsymbol{f}_u s_u + \boldsymbol{H}_u \boldsymbol{V} \sum_{\substack{i=1\\ i\neq u}}^{N_U} \boldsymbol{f}_i s_i + \boldsymbol{n}_u, & (5) \end{aligned}$$

where $\boldsymbol{H}_u \in \mathbb{C}^{N_R \times N_T}$ is the MIMO channel matrix between the transmitter and the u-th receiver[9]. Furthermore, $\boldsymbol{n}_u$ is complex additive white Gaussian noise with zero mean and variance equal to σ_u^2.

Assuming the receivers are all low-cost terminals that perform analog beamforming only in decoding, the decoded signal by the u-th user denoted by $\hat{s}_u$ is given by

$$\hat{s}_u = \boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{V} \boldsymbol{f}_u \boldsymbol{s} + \boldsymbol{w}_u^H \tilde{\boldsymbol{n}}_u, \tag{6}$$

where $\boldsymbol{w}_u$ of dimension $N_R \times 1$ is the analog beamforming vector employed by the u-th receiver with the power constraint of $\|\boldsymbol{w}_u\|^2 = 1$ and

$$\tilde{\boldsymbol{n}}_u = \boldsymbol{H}_u \boldsymbol{V} \sum_{\substack{i=1\\ i\neq u}}^{N_U} \boldsymbol{f}_i s_i + \boldsymbol{n}_u. \tag{7}$$

Note that the first term in Eq. (6) stands for the desired signal while the second term is the sum of its own receiver noise and interference from other users.

B. Channel Model

As shown in [3], the mmWave wireless channel can be well modeled by the Saleh-Valenzuela model. Following the same approach developed in [8], we assume that each scatter only contributes one single propagation path. As a result, the u-th user's channel model can been modeled as

$$\boldsymbol{H}_u = \sqrt{\frac{N_T N_R}{L_u}} \sum_{l=1}^{L_u} \alpha_{u,l} \cdot \boldsymbol{a}_R(\phi_{u,l}^r, \theta_{u,l}^r) \cdot \boldsymbol{a}_T^H(\phi_{u,l}^t, \theta_{u,l}^t), \tag{8}$$

where L_u is the number of scatters of the u-th user's channel. Furthermore, $\alpha_{u,l}$, $\theta_{u,l}^r/\phi_{u,l}^r$ and $\theta_{u,l}^t/\phi_{u,l}^t$ are the complex path gain, azimuth/elevation angles of arrival(AoA) and azimuth/elevation angles of departure(AoD) of the l-th path of the u-th user, respectively. Finally, $\boldsymbol{a}$ is the array response vector. For an uniform planar array (UPA) of size $W \times Q$ considered in this work, the array response vector $\boldsymbol{a}$ is given by

$$\begin{aligned} \boldsymbol{a}(\phi, \theta) =& \frac{1}{\sqrt{WQ}} \Big[1, e^{j\kappa d(\sin\phi \sin\theta + \cos\theta)}, \\ & \cdots, e^{j\kappa d((P-1)\sin\phi \sin\theta + (Q-1)\cos\theta)}\Big]^T, \end{aligned} \tag{9}$$

where $\kappa = \frac{2\pi}{\lambda}$ is the wavenumber and d is the distance between two adjacent antennas.

C. Analog Beamforming Design

We begin with the analog beamforming design for both transmitter and receiver. It is well-known that distinct array response vectors are asymptotically orthogonal as the number of antennas in an antenna array goes to infinity [2], *i.e.*

$$\lim_{N \to +\infty} \boldsymbol{a}^H(\phi_{k,u}^t, \theta_{k,u}^t) \cdot \boldsymbol{a}(\phi_{\ell,v}^t, \theta_{\ell,v}^t) = \delta(k-\ell)\delta(u-v). \tag{10}$$

However, since the antenna number is finite in practice, the residual interference must be considered in the analog precoding design. Recalling the channel model presented in Equation (8), we can asymptotically orthogonalize the transmitted signals by optimizing the design of $\boldsymbol{w}_u$ and $\boldsymbol{v}_u$:

$$\begin{aligned} \{\boldsymbol{w}_u^*, \boldsymbol{v}_u^*\} = & \arg\max_{\tilde{\boldsymbol{w}}_u, \tilde{\boldsymbol{v}}_u} \sum_{u=1}^{M_k} \log\left(1 + \text{SINR}\left(\tilde{\boldsymbol{w}}_u, \tilde{\boldsymbol{v}}_u\right)\right) \\ \text{subject to} \quad & \tilde{\boldsymbol{v}}_u \in \mathcal{A}_u^t; \\ & \tilde{\boldsymbol{w}}_u \in \mathcal{A}_u^r, \end{aligned} \tag{11}$$

where the array response vectors in transmitter and receiver are denoted by

$$\begin{aligned} \mathcal{A}_u^t &= \left[\boldsymbol{a}_T(\phi_{u,1}^t, \theta_{u,1}^t), \cdots, \boldsymbol{a}_T(\phi_{u,L_u}^t, \theta_{u,L_u}^t)\right], \\ \mathcal{A}_u^r &= \left[\boldsymbol{a}_R(\phi_{u,1}^r, \theta_{u,1}^r), \cdots, \boldsymbol{a}_R(\phi_{u,L_u}^r, \theta_{u,L_u}^r)\right]. \end{aligned} \tag{12}$$

Furthermore, $\text{SINR}\left(\tilde{\boldsymbol{w}}_u, \tilde{\boldsymbol{v}}_u\right)$ is given by

$$\text{SINR}_u\left(\tilde{\boldsymbol{w}}_u, \tilde{\boldsymbol{v}}_u\right) = \frac{|\tilde{\boldsymbol{w}}_u^H \boldsymbol{H}_u \tilde{\boldsymbol{v}}_u|^2}{\sum\limits_{i=1, i\neq u}^{N_U} |\tilde{\boldsymbol{w}}_u^H \boldsymbol{H}_u \tilde{\boldsymbol{v}}_i|^2 + \dfrac{1}{\gamma}}, \tag{13}$$

where $\gamma = \frac{P}{N_U \sigma_u^2}$ is the uniform SNR for each user and we have assumed that $\boldsymbol{f}_u \boldsymbol{f}_u^H = \boldsymbol{I}_{N_U}$.

The optimal analog precoder can be straightforwardly found by exhaustively searching in the feasible sets of $\mathcal{A}_{k,u}^T$ and $\mathcal{A}_{k,u}^R$.

D. Problem Formulation

For notational simplicity, we denote by $\boldsymbol{g}_u^H$ the effective array gain of the u-th user with

$$\boldsymbol{g}_u^H = \boldsymbol{w}_u^H \boldsymbol{H}_u \boldsymbol{V}. \tag{14}$$

Then, the channel capacity of the u-th user is given by

$$R_u(\boldsymbol{p}, \boldsymbol{W}, \boldsymbol{V}, \boldsymbol{F}) = \log\left(1 + \frac{p_u |\boldsymbol{g}_u^H \boldsymbol{f}_u|^2}{\sum_{\substack{i=1\\ i\neq u}}^{N_U} p_i |\boldsymbol{g}_u^H \boldsymbol{f}_i|^2 + \sigma_u^2}\right). \tag{15}$$

Subsequently, the system WSR can be formulated as

$$R_{tot} = \sum_{u=1}^{N_U} \tau_u R_u(\boldsymbol{p}, \boldsymbol{W}, \boldsymbol{V}, \boldsymbol{F}), \tag{16}$$

where $\boldsymbol{\tau} = \left[\tau_1, \tau_2, \cdots, \tau_{N_U}\right]$ with $\sum_{u=1}^{N_U} \tau_u = N_U$ are the corresponding weights for users and $\boldsymbol{p} = [p_1, p_2, \cdots, p_{N_U}]$.

Finally, the optimal design of the digital and analog precoding matrices as well as power allocation can be formulated as

$$\begin{aligned} \mathcal{P}_1: \quad & \underset{\boldsymbol{p},\boldsymbol{F}}{\text{maximize}} \quad R_{tot}(\boldsymbol{p},\boldsymbol{F}) \\ \text{subject to} \quad & C_1: \|\boldsymbol{V}\boldsymbol{f}_u\|^2 = 1, \quad 0 < u < N_U; \\ & C_2: \sum_{u=1}^{N_U} p_u \le P; \\ & C_3: R_u \ge \lambda_u, \quad 0 < u < N_U, \end{aligned} \tag{17}$$

where C_1 ensures that each RF chain is of unit power. The total transmitter power is constrained by P as shown in C_2. Finally, C_3 ensures that the u-th user's capacity should be guaranteed not less than a certain threshold λ_u.

Since the problem $\mathcal{P}_1$ is highly non-convex, it is analytically intractable to derive a closed-form solution to $\mathcal{P}_1$. In this paper, a general rank-constrained D.C. programming technique is proposed to derive the optimal digital precoder and power allocation by transforming the problem in each iteration as a standard convex optimization problem.

IV. Proposed rank-constrained D.C. problem

For given analog precoders, we will derive the optimal digital precoder and power allocation in this section. Recalling the problem $\mathcal{P}_1$, the function of $R_{tot}(\boldsymbol{F},\boldsymbol{p})$ is still non-convex. To settle this challenge, we form the digital precoder as

$$\bar{\boldsymbol{F}}_u = p_u \boldsymbol{f}_u \boldsymbol{f}_u^H, \tag{18}$$

with constraints $\mathbf{rank}(\bar{\boldsymbol{F}}_u) \le 1$ and $\bar{\boldsymbol{F}}_u \succeq \mathbf{0}$. Denote by $\boldsymbol{\mathcal{F}} = [\bar{\boldsymbol{F}}_1^{(n)}, \bar{\boldsymbol{F}}_2^{(n)}, \cdots, \bar{\boldsymbol{F}}_{N_U}^{(n)}]$.

Furthermore, the constraints C_3 and C_4 in $\mathcal{P}_1$ can be transformed as

$$\sum_{u=1}^{N_U} p_u \|\boldsymbol{V}\boldsymbol{f}_u\|^2 = \sum_{u=1}^{N_U} \boldsymbol{V}\bar{\boldsymbol{F}}_u\boldsymbol{V}^H \le P. \tag{19}$$

It is worth noting that the optimal digital precoder $\boldsymbol{f}_u^*$ is the eigenvector corresponding to the only one non-zero eigenvalue p_u^* of optimal $\bar{\boldsymbol{F}}_u^*$, where p_u^* is the optimal power allocation.

Then the Equation (15) can be rewritten as

$$\begin{aligned} R_u(\bar{\boldsymbol{F}}_u) =& \log_2\left(1 + \frac{\boldsymbol{g}_u^H \bar{\boldsymbol{F}}_u \boldsymbol{g}_u}{\sum_{\substack{i=1\\i\neq u}}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_u^2}\right) \\ =& \log_2\left(\sum_{i=1}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_u^2\right) \\ & - \log_2\left(\sum_{\substack{i=1\\i\neq u}}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_u^2\right). \end{aligned} \tag{20}$$

Thereby the problem $\mathcal{P}_1$ can be reformulated as a D.C. problem with a rank constraint:

$$\begin{aligned} \mathcal{P}_2: \quad & \underset{\bar{\boldsymbol{F}}}{\text{maximize}} \quad f(\boldsymbol{\mathcal{F}}) - g(\boldsymbol{\mathcal{F}}) \\ \text{subject to} \quad & C_1: \sum_{u=1}^{N_U} \boldsymbol{V}^H \bar{\boldsymbol{F}}_u \boldsymbol{V} \le P; \\ & C_2: R_u \ge \lambda_u, u = 1, 2, \cdots, N_U; \\ & C_3: \bar{\boldsymbol{F}}_u \succeq \mathbf{0}, u = 1, 2, \cdots, N_U; \\ & C_4: \mathbf{rank}(\bar{\boldsymbol{F}}_u) \le 1, u = 1, 2, \cdots, N_U, \end{aligned} \tag{21}$$

where

$$f(\boldsymbol{\mathcal{F}}) = \sum_{u=1}^{N_U} \tau_u \log_2\left(\sum_{i=1}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_u^2\right), \tag{22}$$

$$g(\boldsymbol{\mathcal{F}}) = \sum_{u=1}^{N_U} \tau_u \log_2\left(\sum_{\substack{i=1\\i\neq u}}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_u^2\right). \tag{23}$$

It is worth noting that the QoS constraint C_2 can be straightforwardly reformulated as a convex constraint by the following transformation:

$$\boldsymbol{g}_u^H \bar{\boldsymbol{F}}_u \boldsymbol{g}_u + (1 - 2^{\lambda_u})\left(\sum_{\substack{i=1\\i\neq u}}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_u^2\right) \ge 0. \tag{24}$$

Finally, a standard D.C programming problem is derived after dropping C_4. Next, the iterative algorithm will be given to solve the problem $\mathcal{P}_2$ by ignoring C_4 constraint.

Since the problem $\mathcal{P}_2$ still has high computational complexity, the optimal $\boldsymbol{\mathcal{F}}^*$ can be obtained by iteratively solving $\bar{\boldsymbol{F}}_u$ for $u = 1, 2, \cdots, N_U$ until no improvement for next iteration as detailed in next section.

Similar to the procedures in [6], the iterative algorithm gives the optimal $\bar{\boldsymbol{F}}_u^{(n+1)}$ at n-th iteration by solving the following convex problem:

$$\begin{aligned} & \underset{\bar{\boldsymbol{F}}_u}{\text{maximize}} \quad f(\bar{\boldsymbol{F}}_u) - g(\bar{\boldsymbol{F}}_u^{(n)}) - \langle \triangledown g(\bar{\boldsymbol{F}}_u^{(n)}), \bar{\boldsymbol{F}}_u - \bar{\boldsymbol{F}}_u^{(n)} \rangle \\ & \text{subject to} \quad C_1, C_2, C_3 \quad \text{in} \quad \mathcal{P}_2, \end{aligned} \tag{25}$$

where $\langle \cdot \rangle$ denotes the inner product of two matrices, *i.e.* $\langle \boldsymbol{A}, \boldsymbol{B} \rangle = \mathbf{trace}(\boldsymbol{A}^T\boldsymbol{B})$. The gradient of $g(\bar{\boldsymbol{F}}^{(n)})$ is straightforwardly given by

$$\triangledown g(\bar{\boldsymbol{F}}_u^{(n)}) = \sum_{\substack{j=1\\j\neq u}}^{N_U} \frac{w_j / \ln 2}{\sum_{\substack{i=1\\i\neq j}}^{N_U} \boldsymbol{g}_u^H \bar{\boldsymbol{F}}_i \boldsymbol{g}_u + \sigma_j^2} \boldsymbol{g}_j \boldsymbol{g}_j^H. \tag{26}$$

It is worth noting that $\langle \triangledown g(\bar{\boldsymbol{F}}_u^{(n)}), \bar{\boldsymbol{F}}_u - \bar{\boldsymbol{F}}_u^{(n)} \rangle$ is a real value since $\triangledown g(\bar{\boldsymbol{F}}_u^{(n)})$ and $\bar{\boldsymbol{F}}_u - \bar{\boldsymbol{F}}_u^{(n)}$ are both hermitian.

V. Proposed iterative algorithm for Rank-constrained D.C. problem

To solve the problem $\mathcal{P}_2$, we will first introduce a rank constrained optimization problem (RCOP). Inspired by the RCOP, we propose an iterative algorithm to solve $\mathcal{P}_2$ by formulating a convex optimization problem in each iteration.

A. The General RCOP

We now consider the rank constraint C_4 in $\mathcal{P}_2$. A general RCOP to optimize a convex objective subject to a set of convex and rank constraints can be formulated as follows:

$$\begin{aligned} \mathcal{P}_3: \quad & \underset{\boldsymbol{X}}{\text{minimize}} \quad f(\boldsymbol{X}) \\ \text{subject to} \quad & \boldsymbol{X} \succeq 0; \\ & \boldsymbol{X} \in \mathcal{C}; \\ & \textbf{rank}(\boldsymbol{X}) \leq r, \end{aligned} \tag{27}$$

where $f(\boldsymbol{X})$ is a convex function, $\mathcal{C}$ is the set of given convex constraints and $\boldsymbol{X} \in \mathbb{C}^{m\times m}$ is a general positive semidefinite matrix.

The RCOP can be solved by an iterative method by gradually approaching the constrained rank [10]. At each iteration n, we will solve the following semidefinite programming (SDP) problem:

$$\begin{aligned} \underset{\boldsymbol{X}^{(n+1)}, e^{(n)}}{\text{minimize}} \quad & f(\boldsymbol{X}^{(n+1)}) + we^{(n+1)} \\ \text{subject to} \quad & \boldsymbol{X}^{(n+1)} \succeq \boldsymbol{0}; \\ & \boldsymbol{X}^{(n+1)} \in \mathcal{C}; \\ & e^{(n+1)}\boldsymbol{I}_{m-r} - \boldsymbol{U}^{(n)H}\boldsymbol{X}^{(n+1)}\boldsymbol{U}^{(n)} \succeq \boldsymbol{0}; \\ & e^{(n+1)} \leq e^{(n)}, \end{aligned} \tag{28}$$

where $w > 0$ is the weighting factor. Using eigenvalue decomposition (EVD), $\boldsymbol{U}^{(n)} \in \mathbb{C}^{m\times(m-r)}$ is the orthonormal eigenvectors corresponding to the $n-r$ smallest eigenvalues of $\boldsymbol{X}^{(n)}$ solved at previous n-th iteration. At the first iteration where $n = 0$, $e^{(0)}$ is the $(m-r)$-th smallest eigenvalue of $\boldsymbol{X}^{(0)}$. $\boldsymbol{X}^{(0)}$ is obtained via

$$\begin{aligned} \underset{\boldsymbol{X}^{(0)}}{\text{minimize}} \quad & f(\boldsymbol{X}^{(0)}) \\ \text{subject to} \quad & \boldsymbol{X}^{(0)} \succeq 0; \\ & \boldsymbol{X}^{(0)} \in \mathcal{C}, \end{aligned} \tag{29}$$

and $\boldsymbol{U}^{(0)}$ is the eigenvectors corresponding to $n-r$ smallest eigenvalues of $\boldsymbol{X}^{(0)}$.

B. Iterative Algorithm for $\mathcal{P}_2$

As shown in Section IV, Equation (25) is obviously a concave optimization problem. By combining the $\mathcal{P}_2$ and $\mathcal{P}_3$, an iterative algorithm for the rank-constrained D.C programming problem is derived. The optimal $g(\bar{\boldsymbol{F}}^{(n+1)})$ in the n-th iteration is given by solving the following convex problem:

$$\begin{aligned} \underset{\bar{\boldsymbol{F}}_u, e^{(n+1)}}{\text{minimize}} \quad & t(\bar{\boldsymbol{F}}_u, e^{(n+1)}) \\ \text{subject to} \quad & \sum_{u=1}^{N_U} \boldsymbol{V}^H \bar{\boldsymbol{F}}_u \boldsymbol{V} \leq N_U; \\ & R_u \geq \lambda_u, u = 1, 2, \cdots, N_U; \\ & \bar{\boldsymbol{F}}_u \succeq \boldsymbol{0}; \\ & e^{(n+1)}\boldsymbol{I}_{N_U-1} - \boldsymbol{U}^{(n)H}\bar{\boldsymbol{F}}_u\boldsymbol{U}^{(n)} \succeq \boldsymbol{0}; \\ & e^{(n+1)} \leq e^{(n)}, \end{aligned} \tag{30}$$

where

$$\begin{aligned} & t(\bar{\boldsymbol{F}}_u, e^{(n+1)}) \\ =& g(\bar{\boldsymbol{F}}_u^{(n)}) + \langle \bigtriangledown g(\bar{\boldsymbol{F}}_u^{(n)}), \bar{\boldsymbol{F}}_u - \bar{\boldsymbol{F}}_u^{(n)} \rangle - f(\bar{\boldsymbol{F}}_u) + we^{(n+1)}, \end{aligned}$$

and $\boldsymbol{U}^{(n)}$ is the eigenvectors corresponding to $N_U - 1$ smallest eigenvalues of $\bar{\boldsymbol{F}}_u^{(n)}$.

Despite the formidable appearance of Equation (30), it is indeed a standard convex optimization problem that can be solved via available convex software packages, such as CVX [11].

The proposed iterative algorithm can be summarized as **Algorithm 1**. In each iteration, we solve a D.C programming problem and update the optimal $\bar{\boldsymbol{F}}_u^{*(n)}$. If the WSR $s^{(n)}$ can be no longer improved, the digital precoder $\bar{\boldsymbol{F}}_{u+1}^{(n)}$ of $(u+1)$-th user is then optimized successively. The iteration will stop if no any improvement for $t(\bar{\boldsymbol{F}}_u^{(n)})$.

Algorithm 1 Proposed Iterative Algorithm for Rank-constrained D.C. Problem

Input:

Effective channel: $\boldsymbol{g}_1, \boldsymbol{g}_2, \cdots, \boldsymbol{g}_{N_U}$;

Random initialization matrices of $\bar{\boldsymbol{F}}_u^{(0)}$, $u = 1, 2, \cdots, N_U$;

The initialization information: s, s', $t^{(0)}$, $t^{(1)}$, ϵ;

The initialization value by solving Equation (29): $\boldsymbol{U}^{(0)}, e^{(0)}$

Procedures:

while $|(s'-s)/s| \leq \epsilon$ **do**
  Update s: $s = s'$
  for $0 \leq u \leq N_U$ **do**
    $n = 0$
    while $|(t(\bar{\boldsymbol{F}}_u^{(n+1)}) - t(\bar{\boldsymbol{F}}_u^{(n)})| \leq \epsilon$ **do**
      Obtain the optimal value $t^{(n+1)}$ of objective function and $\bar{\boldsymbol{F}}_u^{(n+1)}, e^{(n+1)}$ in Equation (30)
      Update $\boldsymbol{U}^{(n)}$ from $\bar{\boldsymbol{F}}_u^{(n+1)}$ via EVD
      Update $n = n + 1$
    end while
  end for
  Update s': $s' = t^{(n+1)}$
end while
Outputs: $\boldsymbol{\mathcal{F}}^* = [\bar{\boldsymbol{F}}_1^{*(n)}, \bar{\boldsymbol{F}}_2^{*(n)}, \cdots, \bar{\boldsymbol{F}}_{N_U}^{*(n)}]$.

C. Convergence Analysis

In the following, we provide the convergence analysis of the proposed iterative algorithm for solving the rank-constrained D.C. programming problem.

As the function $g(\bar{\boldsymbol{F}}_u)$ is concave, its gradient $\bigtriangledown g(\bar{\boldsymbol{F}}_u)$ is also its super-gradient, therefore

$$g(\bar{\boldsymbol{F}}_u) \leq g(\bar{\boldsymbol{F}}_u^{(n)}) + \langle \bigtriangledown g(\bar{\boldsymbol{F}}_u^{(n)}), \bar{\boldsymbol{F}}_u - \bar{\boldsymbol{F}}_u^{(n)} \rangle. \tag{31}$$

Since $e^{(n+1)} \leq e^{(n)}$ and $\bar{\boldsymbol{F}}_u^{(n)}$ is feasible to Equation (30), it follows

$$\begin{aligned} & g(\bar{\boldsymbol{F}}_u^{(n+1)}) - f(\bar{\boldsymbol{F}}_u^{(n+1)}) + e^{(n+1)} \\ \leq & g(\bar{\boldsymbol{F}}_u^{(n)}) + \langle \bigtriangledown g(\bar{\boldsymbol{F}}_u^{(n)}), \bar{\boldsymbol{F}}_u - \bar{\boldsymbol{F}}_u^{(n)} \rangle - f(\bar{\boldsymbol{F}}_u) + e^{(n)}. \\ \leq & g(\bar{\boldsymbol{F}}_u^{(n)}) - f(\bar{\boldsymbol{F}}_u^{(n)}) + e^{(n)}, \end{aligned} \tag{32}$$

which shows that the solution $\bar{\boldsymbol{F}}_u^{(n+1)}$ is always better than or equal to the previous solution $\bar{\boldsymbol{F}}_u^{(n)}$. Thus, the algorithm must converge.

VI. Simulation Results

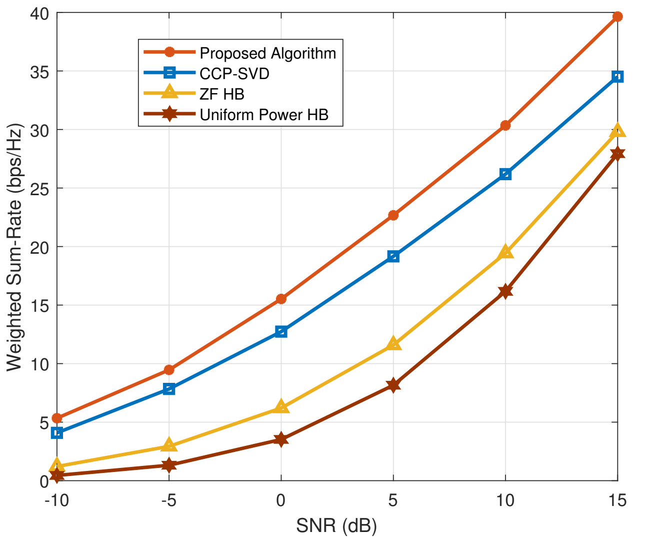

Fig. 2. Performance comparison for different algorithms.

In this section, we will use computer simulation to show the effectiveness of the proposed rank-constrained D.C. programming algorithm. In the simulation schemes, we consider a transmitter equipped with a 4×4 UPA (*i.e.* $N_T = 16$) and $N_U = 12$ users each equipped with a 2×2 UPA (*i.e.* $N_R = 4$). The number of paths is set as $L_u = 4$. We consider the azimuth AoAs/AoDs uniformly distributed over $[0, 2\pi]$ while the elevation AoAs/AoDs uniformly distributed over $[-\pi/2, \pi/2]$, respectively. For each computer experiment, we compute the average over 100 realizations.

In Fig. 2, the wights are set to be $\tau_u = 1.1$, $u = 1, \cdots, 6$ and $\tau_u = 0.9$, $u = 7, \cdots, 12$. we can observe that the proposed algorithm has the best performance compared to the conventional algorithms. The line named "CCP-SVD" is realized for the algorithm reported in [7]. The lines "ZF HB" and "Uniform Power HB" represent the zero-forcing scheme with power allocation proposed in [6] and zero-forcing with uniform power allocation scheme in [3] respectively.

Fig. 3 depicts the CDF of the user's data rate. It is evident that all users served by the QoS-aware power allocation algorithms satisfy the minimum QoS requirement (*i.e.* 1 bps/Hz). In contrast to the proposed scheme in [3], the conventional zero-forcing algorithm with uniform power allocation suffers from an outage rate of about 20% where the outage is defined as the user data rate being below the minimum required data rate. Finally, Fig. 4 validates the convergence of the proposed algorithm as detailed in V-C.

VII. Conclusion

In this paper, we present an algorithm to jointly optimize the digital precoder and power allocation for QoS-aware WSR

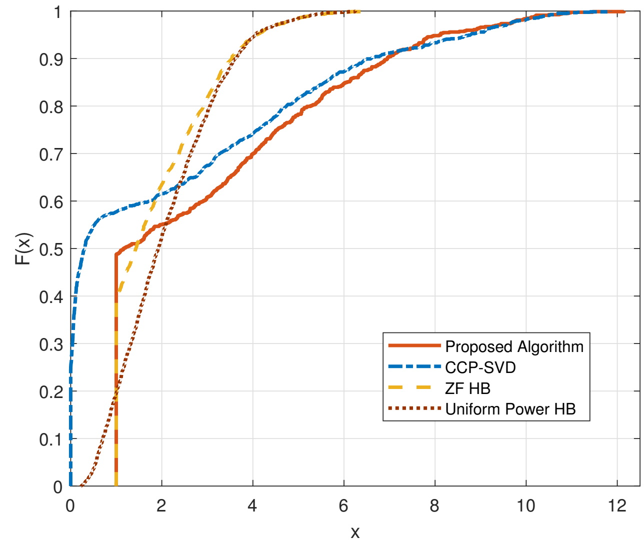

Fig. 3. CDF of users for different algorithms with SNR = 10 dB.

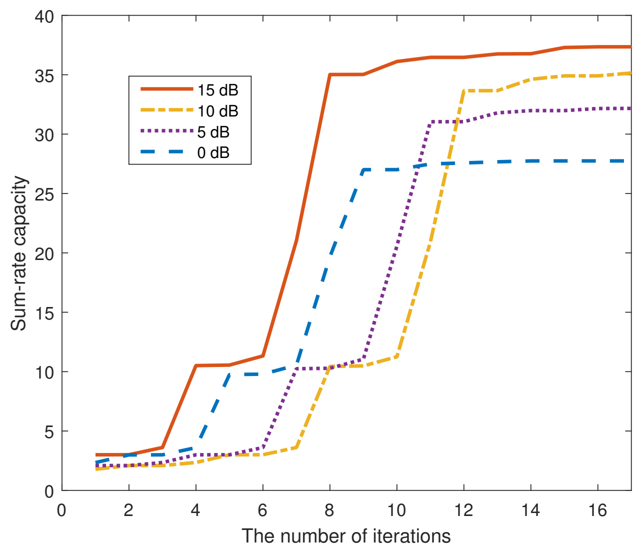

Fig. 4. Convergence proof for realizations of the proposed algorithm.

maximization in the hybrid BDMA transmission systems. To solve the coupled non-convex problem, we first derive the analog precoder by exhaustive searching. Then a rank-constrained D.C. problem is developed. To address the challenge of rank constraint, we propose an iterative algorithm by combining the conventional D.C. problem with RCOP. Finally, the simulation results are given to confirm the effectiveness of the proposed algorithm.

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