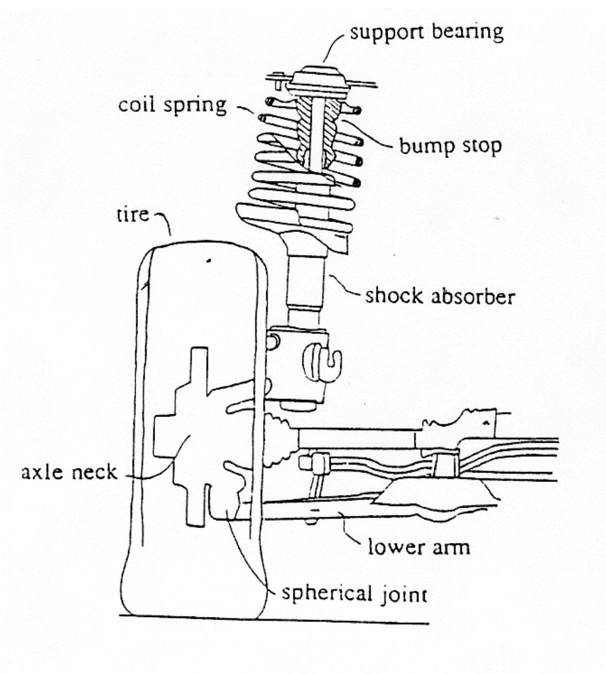
Equation of motion for vehicle ride comfort 2

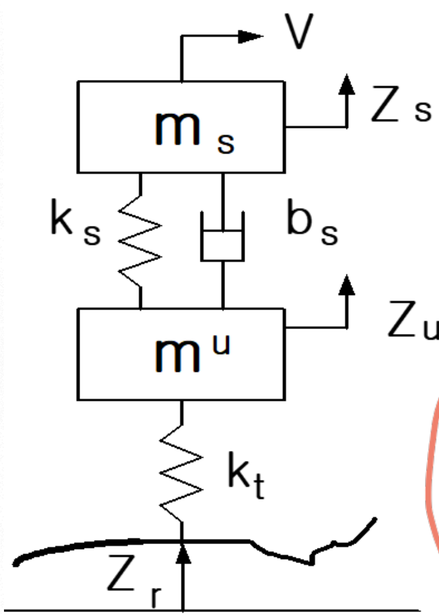
Professor Seunghoon Woo

2022-1

Last Class…

- ❖ **Suspension system components and dynamics**
- ❖ **Passive suspension effects: Spring, Damper and Tire.**





$x_1 = z_s - z_u$ → suspension deflection (rattle space)

$x_2 = \dot{z}_s$ → absolute velocity of sprung mass

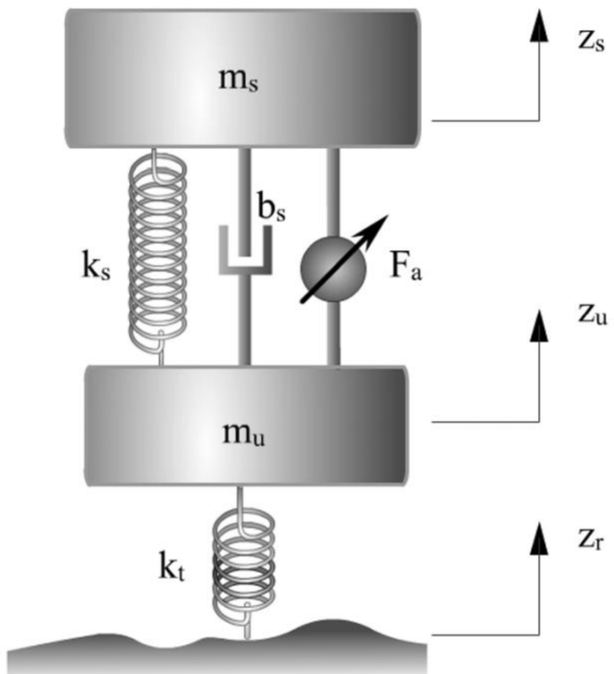
$x_3 = z_u - z_r$ → tire deflection

$x_4 = \dot{z}_u$ → absolute velocity of unsprung mass

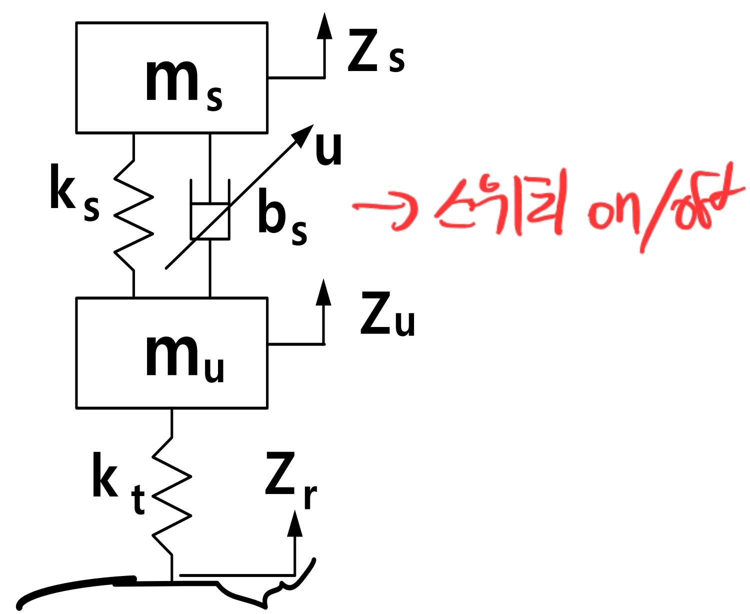
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \dfrac{-k_s}{m_s} & \dfrac{-b_s}{m_s} & 0 & \dfrac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & \dfrac{b_s}{m_u} & \dfrac{-k_t}{m_u} & \dfrac{-b_s}{m_u} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dot{z}_r$$

Today's Topic: Vehicle Suspension Control

- Active Suspension Feedback Control Model
- Semi-Active Suspension Feedback Control Model



Active



Semi-Active

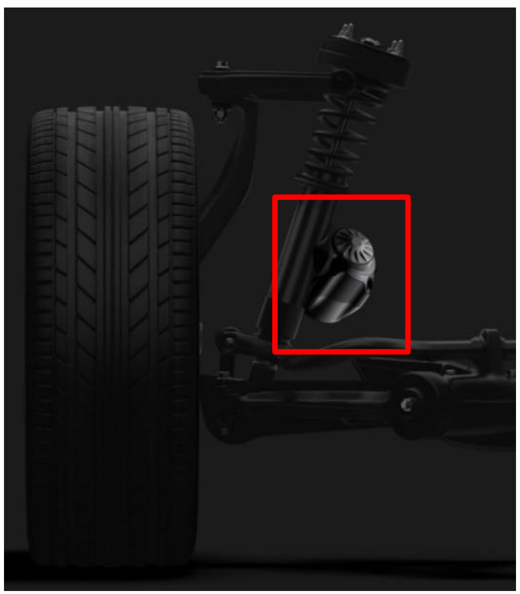
Active Suspension System (Revisited)

❖ **Ex 2: Active suspensions of BMW and Benz**

https://www.youtube.com/watch?v=akySq8g8jRA

Active Suspension System (Revisited)

❖ Ex 2: Active suspensions of BMW and Benz

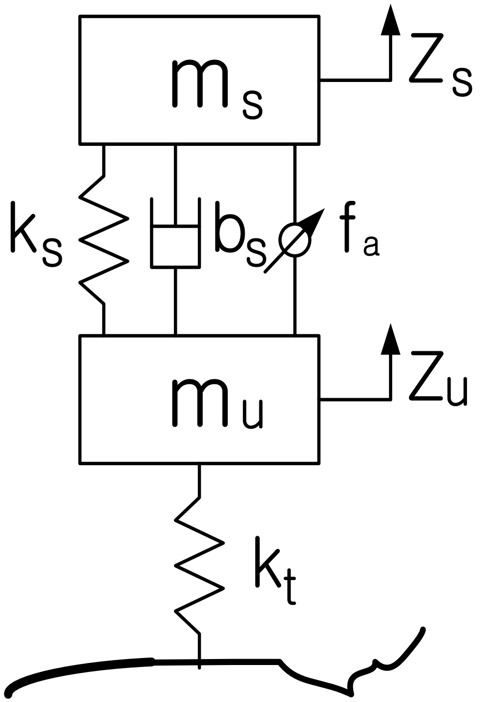


https://www.clearmotion.com/technology

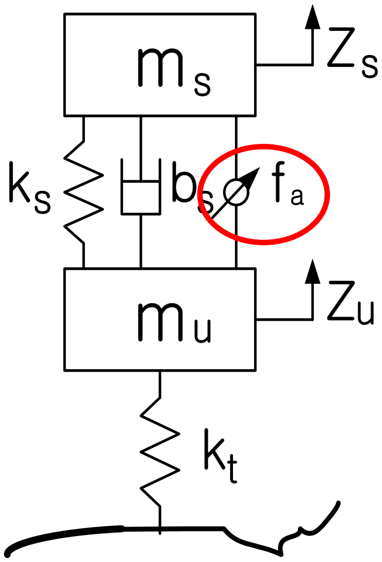
Suspension Control System

❖ PERFORMANCE INDICES FOR ADVANCED SUSPENSION DESIGN

1. Ride Quality
2. Rattle Space
3. Tire Force Variations
4. Actuator Size
5. Power Consumption
6. Component Failure
7. Road Damage



Active Suspension Control System (cont'd)



❖ Equation of Motion:

actuater

$$m_s\ddot{z}_s = -k_s(z_s - z_u) - b_s(\dot{z}_s - \dot{z}_u) + f_a$$

$$m_u\ddot{z}_u = k_s(z_s - z_u) + b_s(\dot{z}_s - \dot{z}_u) + k_t(z_r - z_u) - f_a$$

❖ The state variables:

$x_1 = z_s - z_u$ ⟶ suspension deflection (rattle space)

$x_2 = \dot{z}_s$ ⟶ absolute velocity of sprung mass

$x_3 = z_u - z_r$ ⟶ tire deflection

$x_4 = \dot{z}_u$ ⟶ absolute velocity of unsprung mass

Active Suspension Control System (cont'd)

$$m_s \ddot{z}_s = -k_s(z_s - z_u) - b_s(\dot{z}_s - \dot{z}_u) + f_a$$

$$m_u \ddot{z}_u = k_s(z_s - z_u) + b_s(\dot{z}_s - \dot{z}_u) + k_t(z_r - z_u) - f_a$$

where

$$u \equiv F_a = -K \cdot x = -\begin{bmatrix} K_1 & K_2 & K_3 & K_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Active control input

Matrix of State Equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{b_s}{m_u} & -\frac{k_t}{m_u} & -\frac{b_s}{m_u} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_u} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dot{z}_r$$

Full-state feedback controller equation

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{1}{m_s}(k_s + K_1) & -\frac{1}{m_s}(b_s + K_2) & -\frac{1}{m_s}K_3 & \frac{1}{m_s}(b_s - K_4) \\ 0 & 0 & 0 & 1 \\ \frac{1}{m_u}(k_s + K_1) & \frac{1}{m_u}(b_s + K_2) & -\frac{1}{m_u}(k_t - K_3) & -\frac{1}{m_u}(b_s - K_4) \end{bmatrix}}_{A_c} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}}_{L} \dot{z}_r$$

Active Suspension Control System (cont'd)

Matrix of State Equation

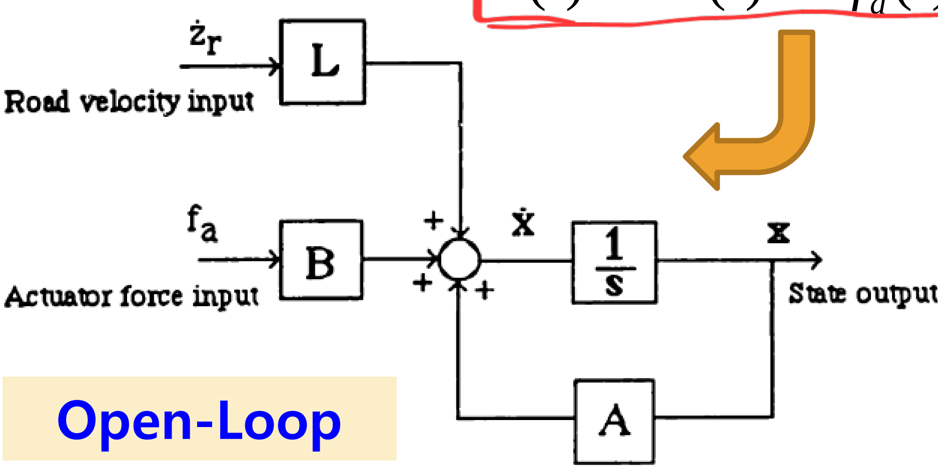
where

$$u \equiv F_a = -K \cdot x = -\begin{bmatrix} K_1 & K_2 & K_3 & K_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

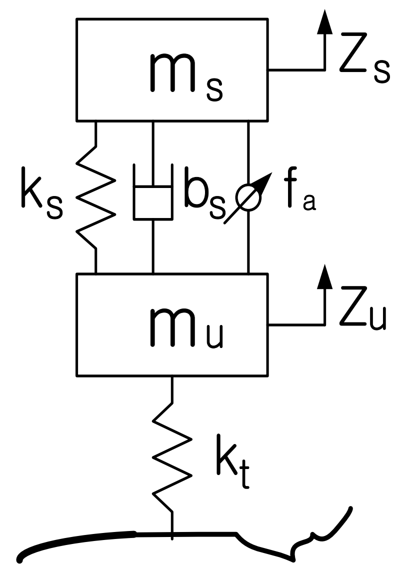
Active control input

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{b_s}{m_u} & -\frac{k_t}{m_u} & -\frac{b_s}{m_u} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_u} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dot{z}_r$$

$$\boxed{\dot{x}(t) = Ax(t) + Bf_a(t) + L\dot{z}_r(t)}$$



Open-Loop



Active Suspension Control System (cont'd)

Full-state feedback controller equation comes to

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{1}{m_s}(k_s+K_1) & -\frac{1}{m_s}(b_s+K_2) & -\frac{1}{m_s}K_3 & \frac{1}{m_s}(b_s-K_4) \\ 0 & 0 & 0 & 1 \\ \frac{1}{m_u}(k_s+K_1) & \frac{1}{m_u}(b_s+K_2) & -\frac{1}{m_u}(k_t-K_3) & -\frac{1}{m_u}(b_s-K_4) \end{bmatrix}}_{A_c} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}}_{L} \dot{z}_r$$

for exmaple,

$$y(t) = Cx(t)$$

$$= \dot{z}_s(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$$

$$\left.\begin{aligned} \dot{x}(t) &= A_c x(t) + Lu(t) \\ y(t) &= Cx(t) \end{aligned}\right\}$$

State equation (First-order matrix differential eq.)

With zero initial condition,

$$sX(s) = A_c X(s) + LU(s)$$
$$Y(s) = CX(s)$$

$$sX(s) - A_c X(s) = LU(s) \qquad \boxed{Y(s) = CX(s)}$$
$$(sI - A_c)X(s) = LU(s)$$
$$X(s) = (sI - A_c)^{-1} LU(s)$$

$$Y(s) = C \cdot (sI - A_c)^{-1} LU(s)$$

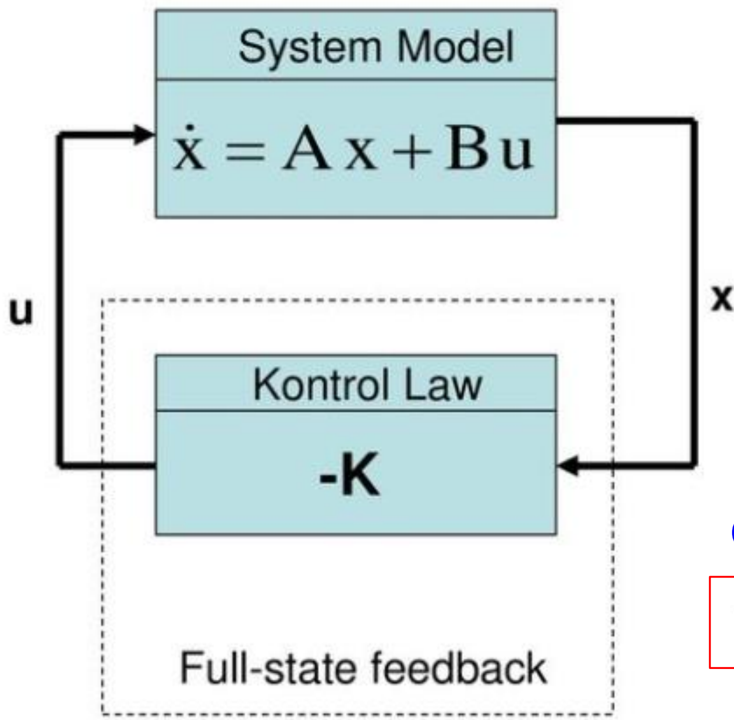
$$\boxed{\frac{Y(s)}{U(s)} = C \cdot (sI - A_c)^{-1} L}$$

❖ **Important finding:**

- **With active suspension controller, system dynamics can be adjusted to reduce disturbance (e.g., road roughness) effects!!**

Full-State Feedback Controller Design

❖ **Full-state feedback block diagram (without no reference input)**



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{b_s}{m_u} & -\frac{k_t}{m_u} & -\frac{b_s}{m_u} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_u} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dot{z}_r$$

Active control Input

Passive Input

Control Law:

$$\boxed{u = -Kx}$$

https://slideplayer.com/slide/12389328/

Full-State Feedback Controller Design

❖ Full-state feedback block diagram (with no reference input)

With the system defined by the state variable model

$$\dot{x} = Ax + Bu$$

and the control feedback given by

$$u = -Kx$$

we find the closed-loop system to be

$$\dot{x} = Ax + Bu = Ax - BKx = [A - BK]x$$

The characteristic equation associated with above equation is

$$\det(\lambda I - [A - BK]) = 0$$

If all the roots of the characteristic equation lie in the left-half plain, then the closed loop is stable. In other words, for any initial condition $x(t_0)$, it follows that

$$x(t) = e^{(A-BK)t} x(t_0) \rightarrow 0 \quad \text{as } t \rightarrow \infty$$

Given the pair (A,B), we can always determine K to place all the system closed loop poles in the left half-plane if and only if the system is completely controllable-that is, if and only if the controllability matrix P_C is full rank (for a SISO system, full rank implies that P_C is invertible).

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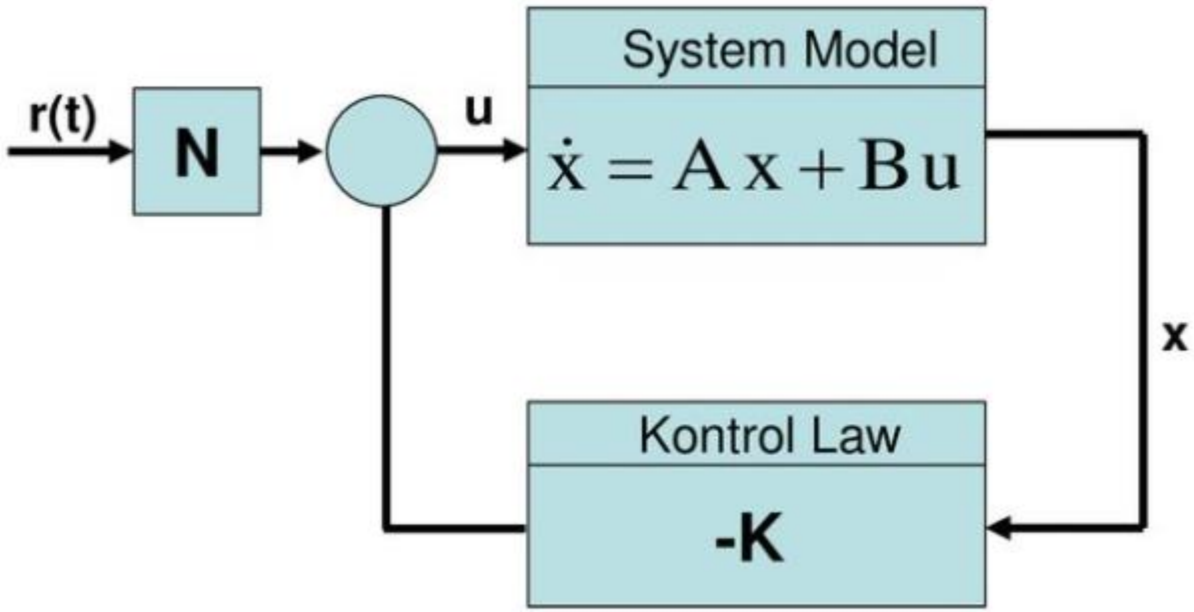
Full-State Feedback Controller Design

❖ **Full-state feedback block diagram (with reference input)**

$$u(t) = -K\,x(t) + N\,r(t)$$



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Transfer functions: Active vs. Passive

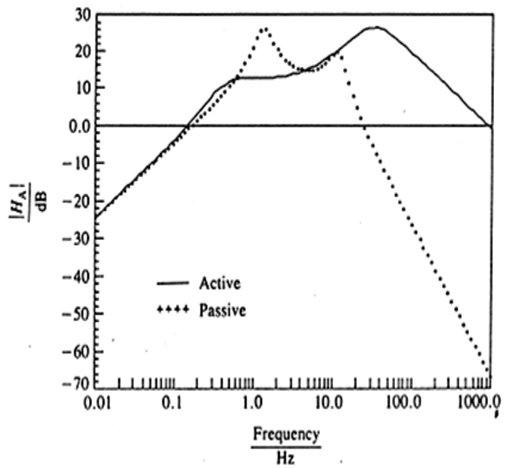

Fig. 3a Acceleration transfer functions

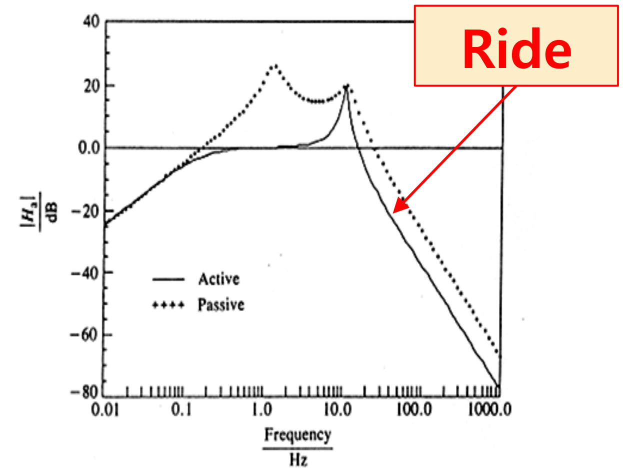

Fig. 4a Acceleration transfer functions

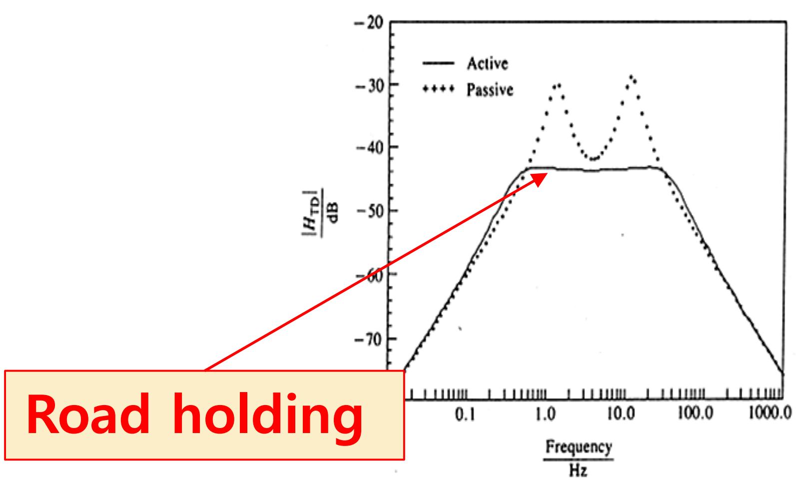

Fig. 3b Tyre deflection transfer functions

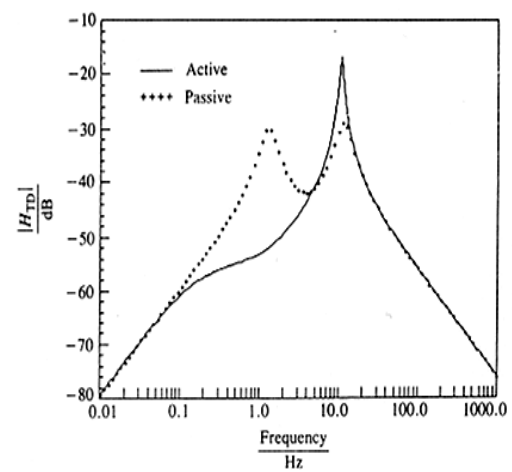

Fig. 4b Tyre deflection transfer functions

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Transfer functions: Active vs. Passive (cont'd)

Road holding

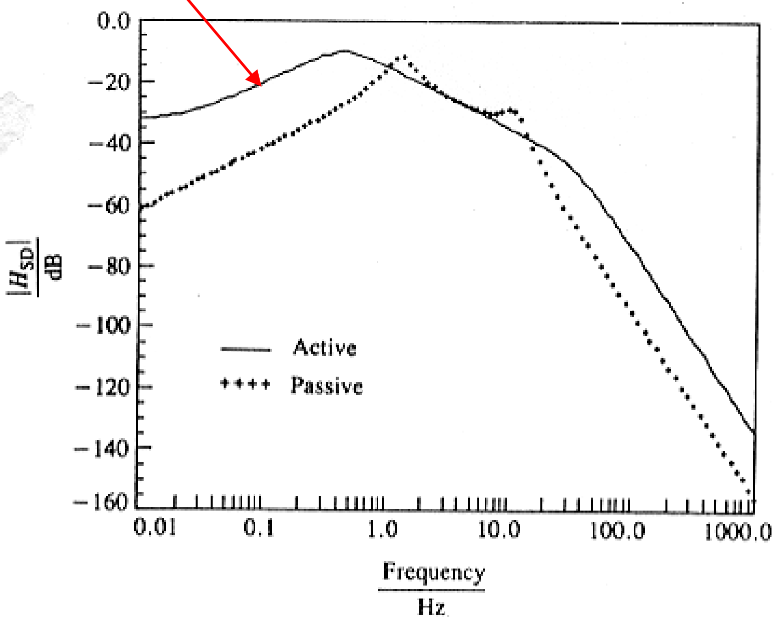

Fig. 6 Suspension deflection transfer functions

Ride

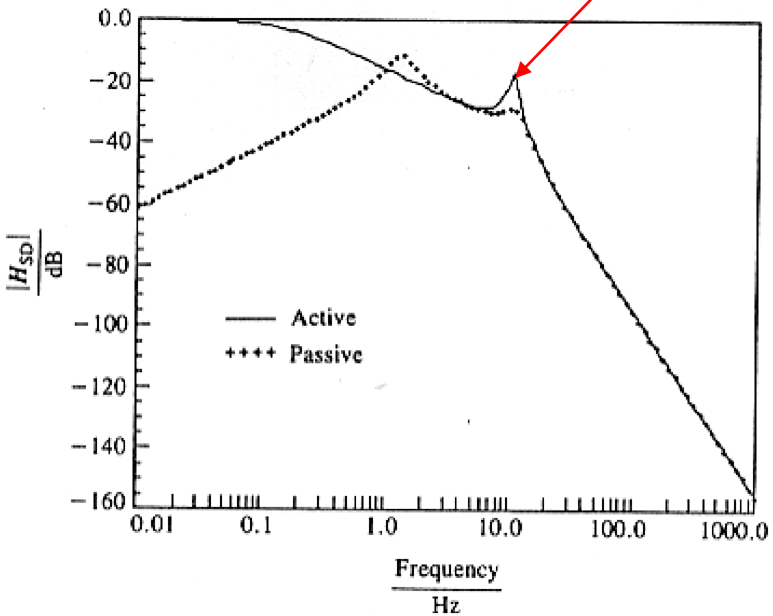

Fig. 8 Suspension deflection transfer functions

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Active Suspension: Transfer Functions

a) Acceleration transfer function

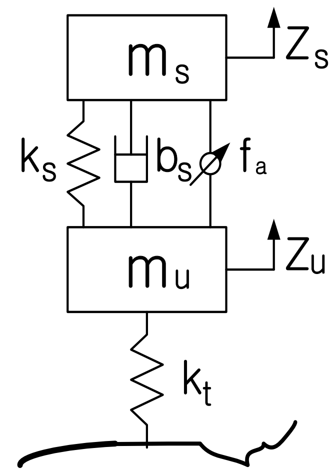
$$H_A(s) = \frac{\ddot{z}_s(s)}{\dot{z}_r(s)}$$

b) Rattle Space transfer function

$$H_{RS}(s) = \frac{z_s(s) - z_u(s)}{\dot{z}_r(s)}$$

c) Tire deflection transfer function

$$H_{TD}(s) = \frac{z_u(s) - z_r(s)}{\dot{z}_r(s)}$$



$x_1 = z_s - z_u$ → suspension deflection (rattle space)

$x_2 = \dot{z}_s$ → absolute velocity of sprung mass

$x_3 = z_u - z_r$ → tire deflection

$x_4 = \dot{z}_u$ → absolute velocity of unsprung mass

Active Suspension: LQR control design

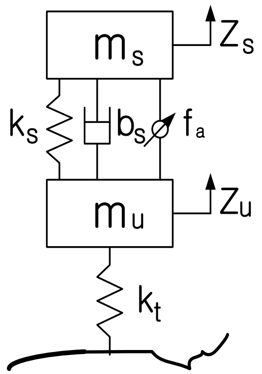
Linear Quadratic Regulator (LQR)

- Consider the following plant

$$\dot{x} = Ax + B_1 d + B_2 u \quad A \in R^{n \times n}, B_1 \in R^n, B_2 \in R^n$$

$$z = C_1 x + D_{12} u \quad C_1 \in R^{m \times n}, D_{12} \in R^{m \times 1}$$

where $d \in R$ is a disturbance input,



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$x_4 = \dot{z}_u$ → absolute velocity of unsprung mass

- In the LQR problem, the controller is to be designed so that the **following performance index (variable) is minimized**:

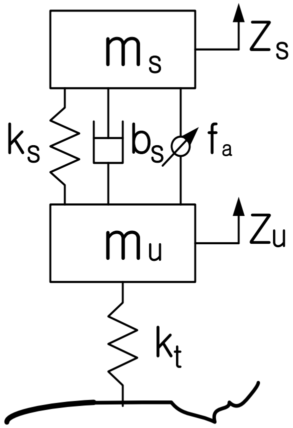
$$J = \int_0^\infty z^T z dt = \int_0^\infty \left[x^T C_1^T C_1 x + 2x^T C_1^T D_{12} u + u^T D_{12}^T D_{12} u \right] dt$$

for all initial conditions $x_0 = x(0)$.

Active Suspension: LQR control (cont'd)

$x_1 = z_s - z_u$ → suspension deflection (rattle space)
$x_2 = \dot{z}_s$ → absolute velocity of sprung mass
$x_3 = z_u - z_r$ → tire deflection
$x_4 = \dot{z}_u$ → absolute velocity of unsprung mass

❖ LQR formulation for active suspension design



- Define the following quadratic (squared) performance index.

$$J = \left[\int_0^\infty \ddot{z}_s^{\,2} + \rho_1 (z_s - z_u)^2 + \rho_2 \dot{z}_s^{\,2} + \rho_3 (z_u - z_r)^2 + \rho_4 \dot{z}_u^{\,2} dt \right]$$

where the weighting factors ρ_1, ρ_2, ρ_3 and ρ_4

$$J = \int_0^\infty z^T z dt = \int_0^\infty \left[x^T C_1^T C_1 x + 2x^T C_1^T D_{12} u + u^T D_{12}^T D_{12} u \right] dt$$

Then, we interest in the acceleration of sprung mass,

$$\ddot{z}_s^{\,2} = \frac{1}{m_s^{\,2}} [k_s^{\,2} x_1^{\,2} + b_s^{\,2} x_2^{\,2} + b_s^{\,2} x_4^{\,2} + F_a^{\,2} + 2k_s b_s x_1 x_2 - 2k_s b_s x_1 x_4$$
$$- 2b_s^{\,2} x_2 x_4 - 2k_s x_1 F_a - 2b_s x_2 F_a + 2b_s x_4 F_a]$$

Active Suspension: LQR control (cont'd)

$x_1 = z_s - z_u \longrightarrow$ suspension deflection (rattle space)
$x_2 = \dot{z}_s \longrightarrow$ absolute velocity of sprung mass
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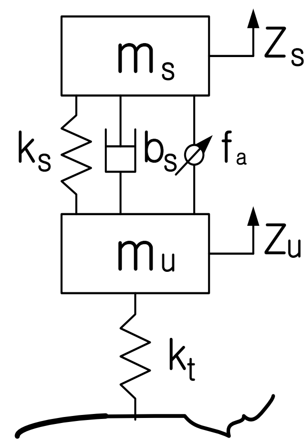
❖ LQR formulation for active suspension design

Then, we interest in the acceleration of sprung mass,



$$\ddot{z}_s^{\,2} = \frac{1}{m_s^{\,2}}[k_s^{\,2}x_1^{\,2} + b_s^{\,2}x_2^{\,2} + b_s^{\,2}x_4^{\,2} + F_a^{\,2} + 2k_s b_s x_1 x_2 - 2k_s b_s x_1 x_4 - 2b_s^{\,2}x_2 x_4 - 2k_s x_1 F_a - 2b_s x_2 F_a + 2b_s x_4 F_a]$$

$$\ddot{z}_s^{\,2} + \rho_1 (z_s - z_u)^2 + \rho_2 \dot{z}_s^{\,2} + \rho_3 (z_u - z_r)^2 + \rho_4 \dot{z}_u^{\,2} = x^T Q x + 2x^T N F_a + F_a^{\,T} R F_a$$

where

$$Q = \begin{bmatrix} \frac{k_s^{\,2}}{m_s^{\,2}} + \rho_1 & \frac{b_s k_s}{m_s^{\,2}} & 0 & -\frac{b_s k_s}{m_s^{\,2}} \\ \frac{b_s k_s}{m_s^{\,2}} & \frac{b_s^{\,2}}{m_s^{\,2}} + \rho_2 & 0 & -\frac{b_s^{\,2}}{m_s^{\,2}} \\ 0 & 0 & \rho_3 & 0 \\ -\frac{b_s k_s}{m_s^{\,2}} & -\frac{b_s^{\,2}}{m_s^{\,2}} & 0 & \frac{b_s^{\,2}}{m_s^{\,2}} + \rho_4 \end{bmatrix}, \quad N = \begin{Bmatrix} -\frac{k_s}{m_s^{\,2}} \\ -\frac{b_s}{m_s^{\,2}} \\ 0 \\ \frac{b_s}{m_s^{\,2}} \end{Bmatrix} \quad \text{and}$$

$$R = \frac{1}{m_s^{\,2}}.$$

Active Suspension: LQR control (cont'd)

$x_1 = z_s - z_u$ → suspension deflection (rattle space)

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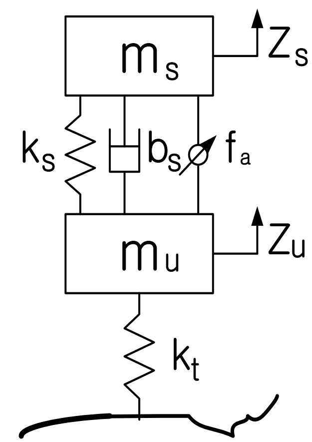
$x_4 = \dot{z}_u$ → absolute velocity of unsprung mass

❖ LQR formulation for active suspension design

The performance index is then written as

$$J = \left[\int_0^\infty \left(x^T Q x + 2x^T N u + u^T R u \right) dt \right]$$



Then, the solution to the optimal control problem that minimizes this performance index is

a state feedback law $F_a = -Gx$

where the feedback gain G is determined by solving the following Riccati equation

$$\left(A - BR^{-1}N\right)^T P + P\left(A - BR^{-1}N\right) + \left(Q - N^T R^{-1} N\right) - PBR^{-1}B^T P = 0 \tag{11.16}$$

$$G = R^{-1}\left(B^T P + N\right) \tag{11.17}$$

https://www.mathworks.com/help/control/ref/lqr.html

lqr

Linear-Quadratic Regulator (LQR) design

Syntax

```
[K,S,e] = lqr(SYS,Q,R,N)
[K,S,e] = LQR(A,B,Q,R,N)
```

MathWorks®

Active Suspension: LQR control (cont'd)

$x_1 = z_s - z_u$ → suspension deflection (rattle space)
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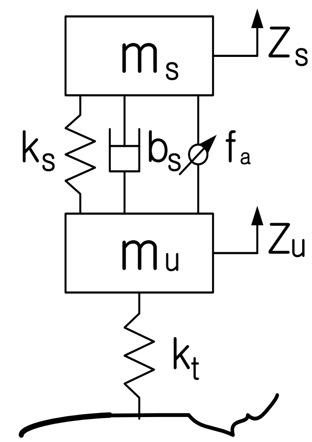

❖ **LQR formulation for active suspension design**

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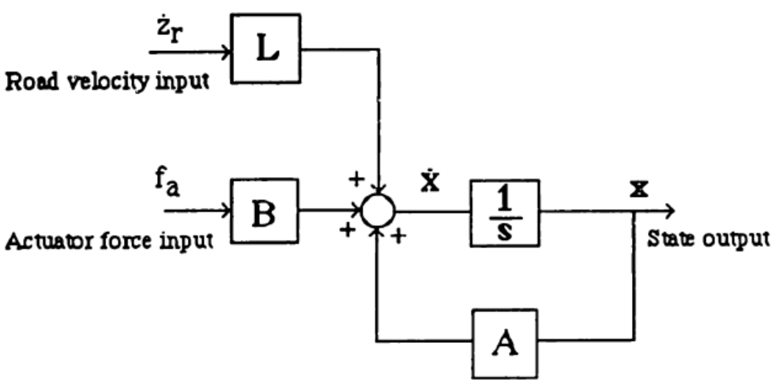
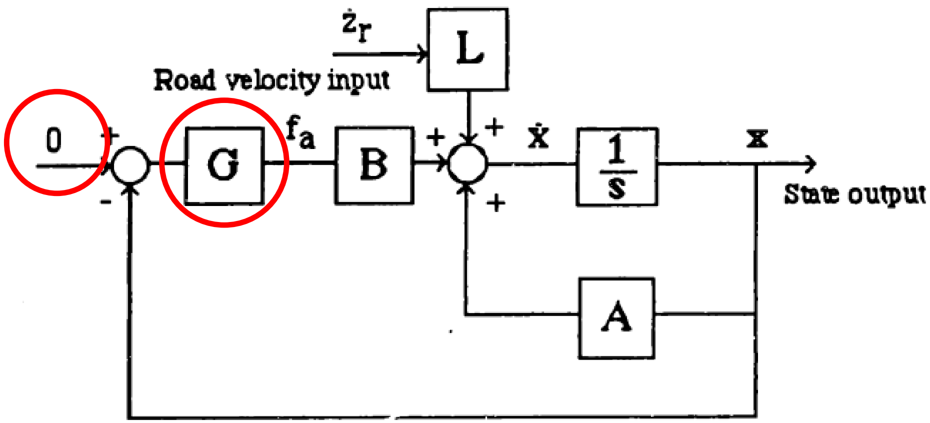
$$z = C_1 x + D_{12} u \quad C_1 \in R^{m \times n}, D_{12} \in R^{m \times 1}$$

$$\left(A - BR^{-1}N\right)^T P + P\left(A - BR^{-1}N\right) + \left(Q - N^T R^{-1} N\right) - PBR^{-1}B^T P = 0 \tag{11.16}$$

$$G = R^{-1}\left(B^T P + N\right) \tag{11.17}$$



Open-Loop



Closed-Loop

Active Suspension: LQR control (cont'd)

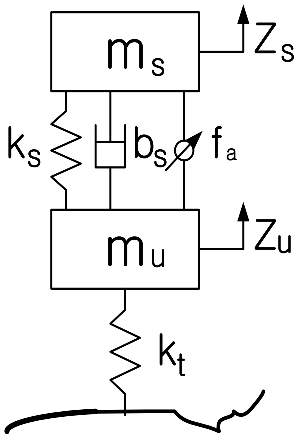
❖ **Simulation Parameters: LQR formulation for active suspension design**

- **Numerical values**

$m_s = 240kg,\quad k_s = 16{,}000 N/m,\quad b_s = 980 N\sec/m$

$m_u = 36kg,\quad k_t = 1{,}600{,}000 N/m$

Road roughness : slope variance = 22×10^{-6}



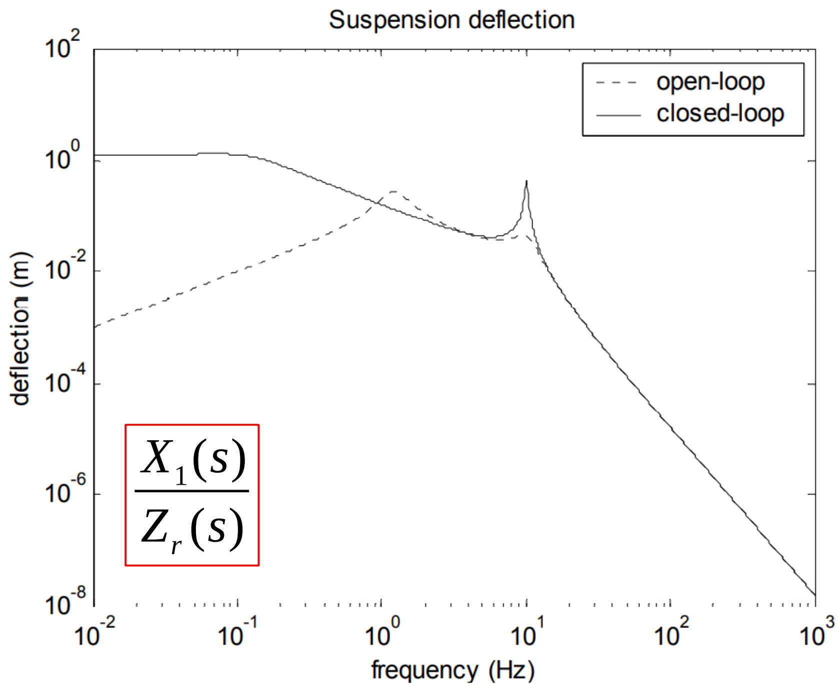
- **Original passive system has following eigenvalues**

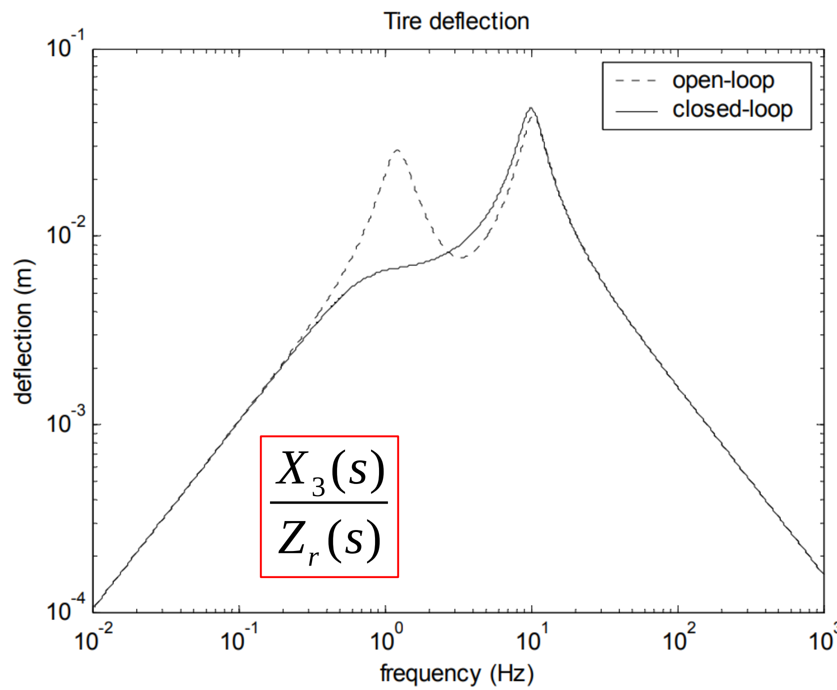
sprung mass mode: $\omega_n = 1.255$ Hz $\zeta = 0.22$

unsprung mass mode: $\omega_n = 11.0$ Hz $\zeta = 0.20$

Active Suspension: LQR control (cont'd)

❖ Simulation Result: LQR formulation for active suspension design





$$J = \left[\int_0^\infty \ddot{z}_s^{\,2} + \rho_1 (z_s - z_u)^2 + \rho_2 \dot{z}_s^{\,2} + \rho_3 (z_u - z_r)^2 + \rho_4 \dot{z}_u^{\,2} dt \right]$$

$\rho_1 = 0.4$, $\rho_2 = 0.16$, $\rho_3 = 0.4$ and $\rho_4 = 0.16$.

$x_1 = z_s - z_u$ → suspension deflection (rattle space)

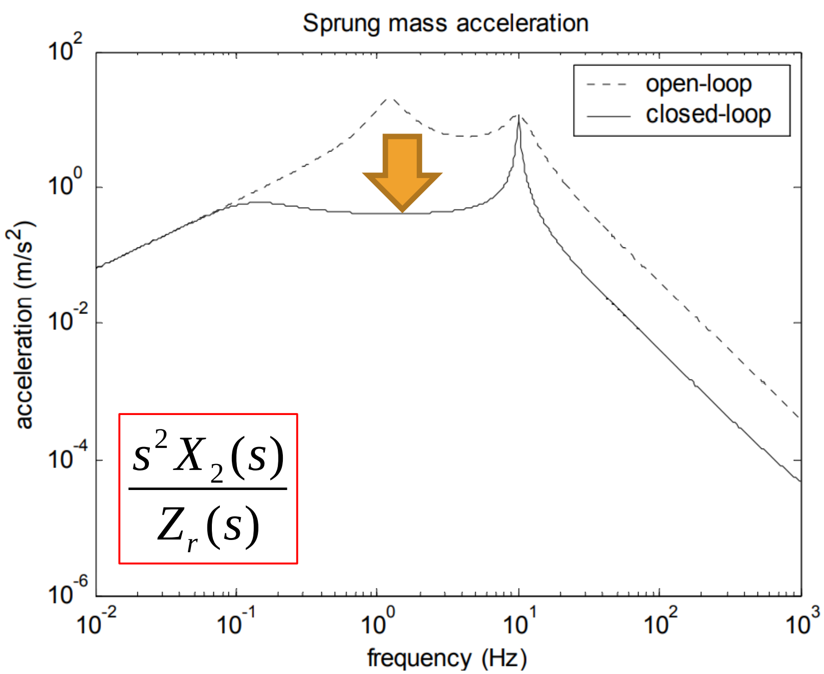
$x_2 = \dot{z}_s$ → absolute velocity of sprung mass

$x_3 = z_u - z_r$ → tire deflection

$x_4 = \dot{z}_u$ → absolute velocity of unsprung mass

Active Suspension: LQR control (cont'd)

❖ Simulation Result: LQR formulation for active suspension design



❖ Important Findings:

- **With active suspension, sprung mass acceleration dramatically becomes smaller than original one.**
- **But, resonance peak at the eigenvalues of suspension system.**
- **Then, how to reduce this one?**

$$J = \left[\int_0^\infty \ddot{z}_s^{\,2} + \rho_1 (z_s - z_u)^2 + \rho_2 \dot{z}_s^{\,2} + \rho_3 (z_u - z_r)^2 + \rho_4 \dot{z}_u^{\,2} dt \right]$$

$\rho_1 = 0.4$, $\rho_2 = 0.16$, $\rho_3 = 0.4$ and $\rho_4 = 0.16$.

$x_1 = z_s - z_u \longrightarrow$ suspension deflection (rattle space)

$x_2 = \dot{z}_s \longrightarrow$ absolute velocity of sprung mass

$x_3 = z_u - z_r \longrightarrow$ tire deflection

$x_4 = \dot{z}_u \longrightarrow$ absolute velocity of unsprung mass

Active Suspension Control System

Mercedes-Benz MAGIC BODY CONTROL | S-Class

https://www.youtube.com/watch?v=ScpgI1w5F6A

Design of Semi-active Suspensions

1. Damping Control

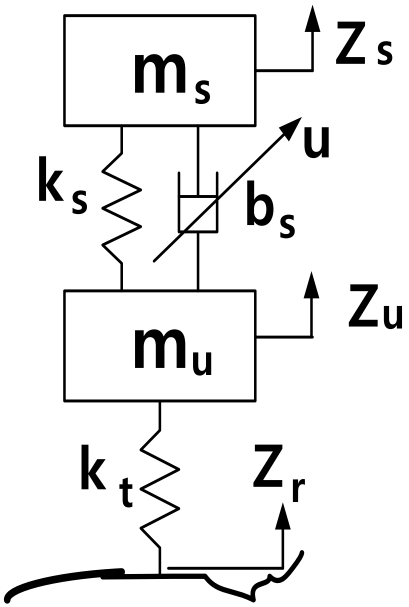
- Control Laws?

2. Semi-active Dampers

- Min. & Max. Damping Rate?
- Continuous or Multi-state?
- How many Steps?
- How fast? (actuator dynamics)

3. Performance Potential

- How much improvement?



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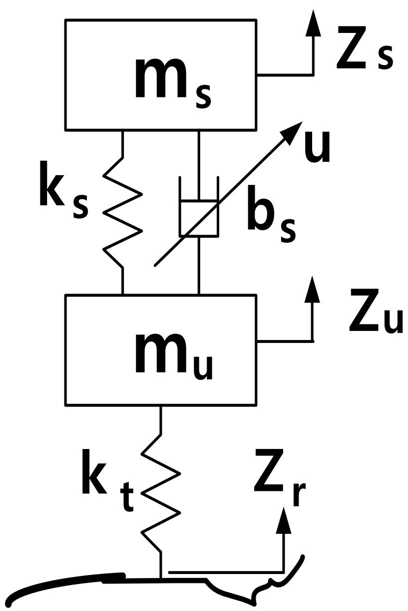
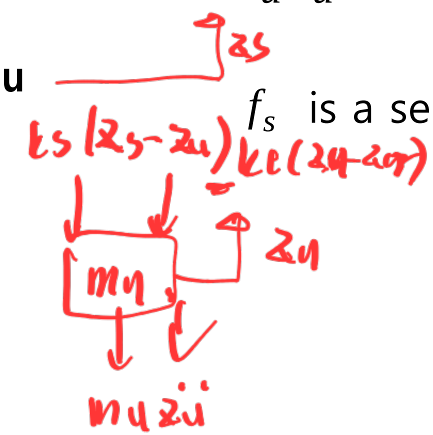
Semi-active Suspension Control Model

❖ Dynamic Model with Semi-active Dampers



$$m_s \ddot{z}_s = -k_s(z_s - z_u) + f_s$$

$$m_u \ddot{z}_u = k_s(z_s - z_u) - k_t(z_u - z_r) - f_s$$



f_s is a semi-active force

Semi-active Suspension Control Model

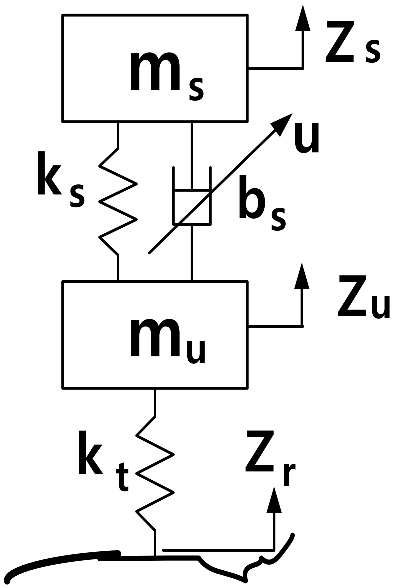
- **State Variables**

$x_1 = z_s - z_u$: Suspension Deflection

$x_2 = \dot{z}_s$: Sprung Mass Velocity

$x_3 = z_u - z_r$: Tire Deflection

$x_4 = \dot{z}_u$: Unsprung Mass Velocity



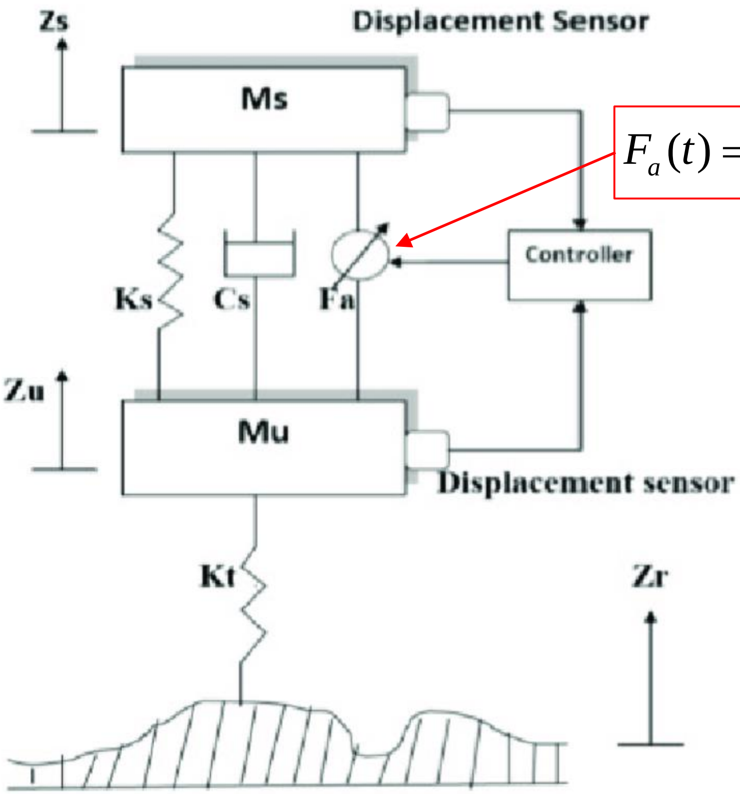
- **State Equations**

$$\dot{x} = Ax + Bf_s + F\dot{z}_r$$

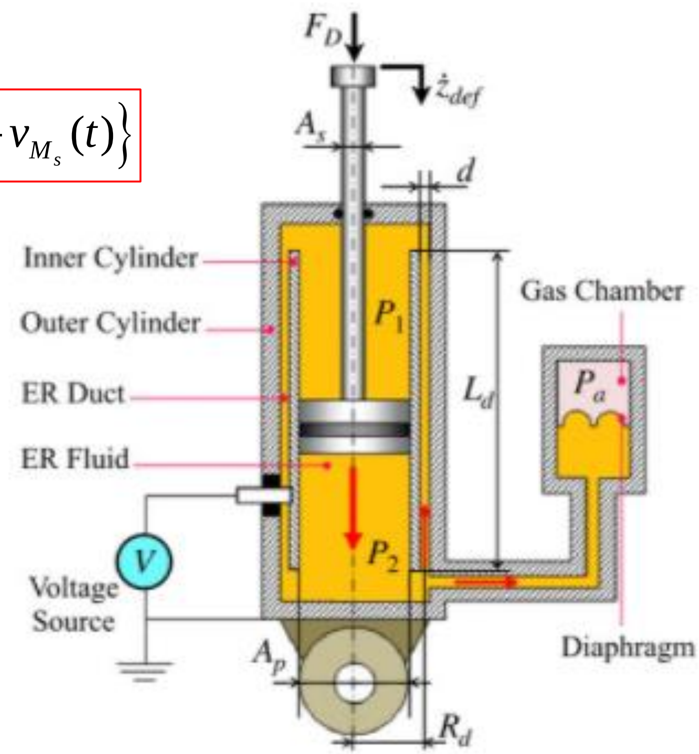
$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & 0 & -\frac{k_t}{m_u} & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_u} \end{bmatrix} \quad F = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

Semi-active Suspension Control Model

❖ Semi-active Suspension Control System Structure



$$F_a(t) = c\left\{v_{M_u}(t) - v_{M_s}(t)\right\}$$



https://www.mdpi.com/2073-8994/12/8/1286

https://www.researchgate.net/figure/Quarter-Semi-active-suspension-system-4-From-fig-1-following-equation-has-concluded_fig1_327949153

Semi-active Suspension Control Model

❖ Bilinear Model Approach

The semi-active force:
Based on unspring mass velocity

$$f_s = f_s(x_2 - x_4, \upsilon) = \upsilon(x_2 - x_4)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & 0 & -\frac{k_t}{m_u} & 0 \end{bmatrix} \qquad F = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

Bilinear state equation by sensing velocity:

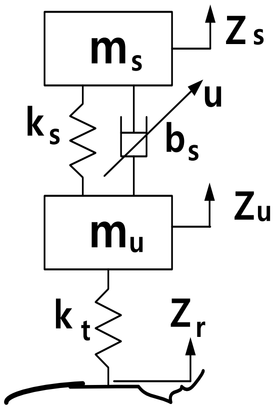
$$\dot{x} = Ax + Bf_s + F\dot{z}_r \qquad \dot{x} = Ax + Dx\upsilon + F\dot{z}_r$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{m_s} & 0 & \frac{1}{m_s} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_u} & 0 & -\frac{1}{m_u} \end{bmatrix}$$

A reasonable and very effective control law:

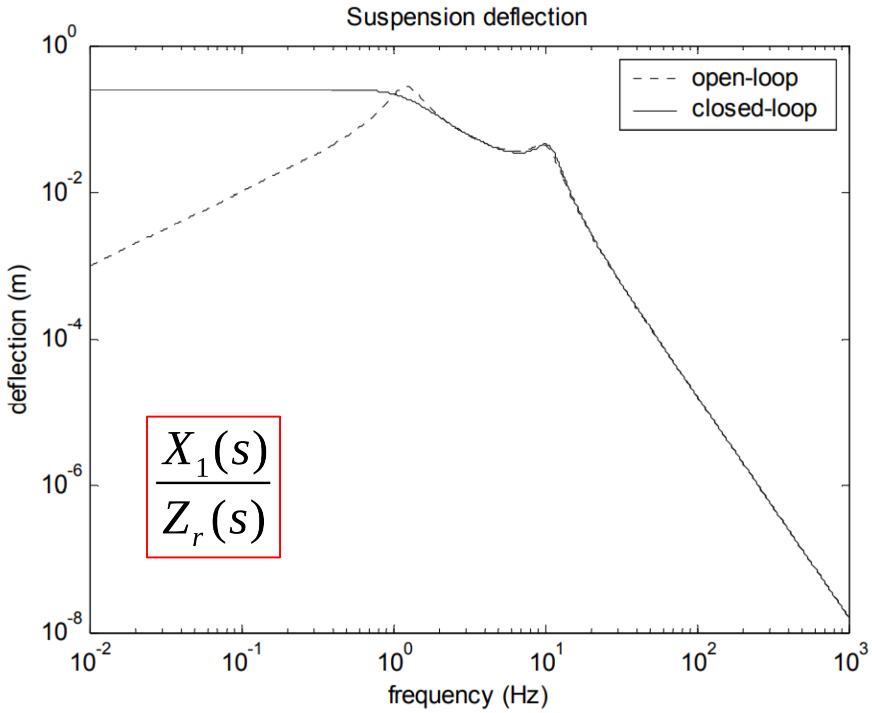
$$\upsilon(t) = \begin{cases} \upsilon_{\min} & if \quad \upsilon^*(t) \le \upsilon_{\min} \\ \upsilon^*(t) & if \quad \upsilon_{\min} < \upsilon^*(t) < \upsilon_{\max} \\ \upsilon_{\max} & if \quad \upsilon_{\max} \le \upsilon^*(t) \end{cases}$$

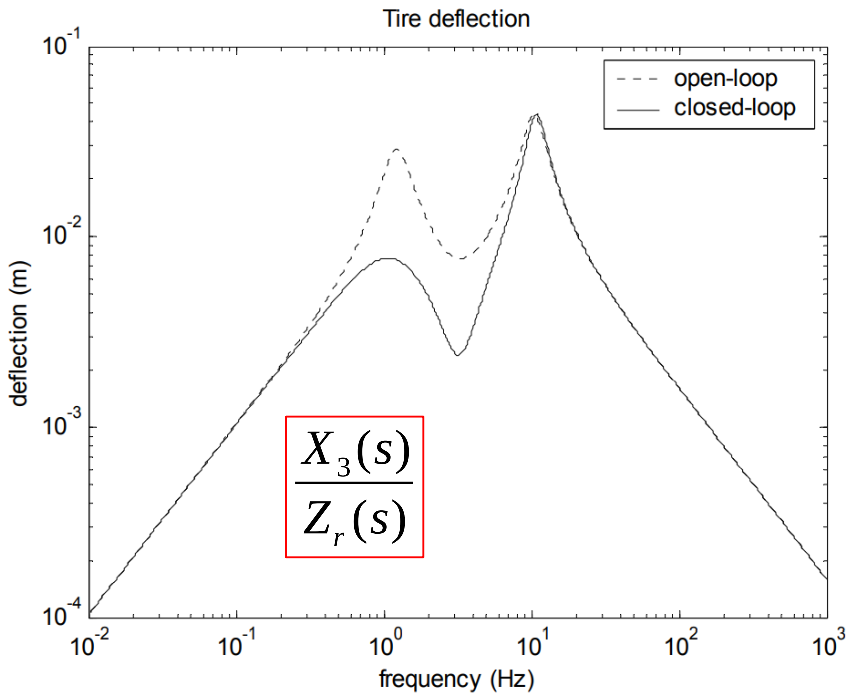
$$\upsilon^*(t) = \frac{f_{s,des}}{(\dot{z}_s - \dot{z}_u)}$$



Semi-active Suspension Control Model

❖ Simulation Result: LQR formulation for active suspension design





$x_1 = z_s - z_u \longrightarrow$ suspension deflection (rattle space)

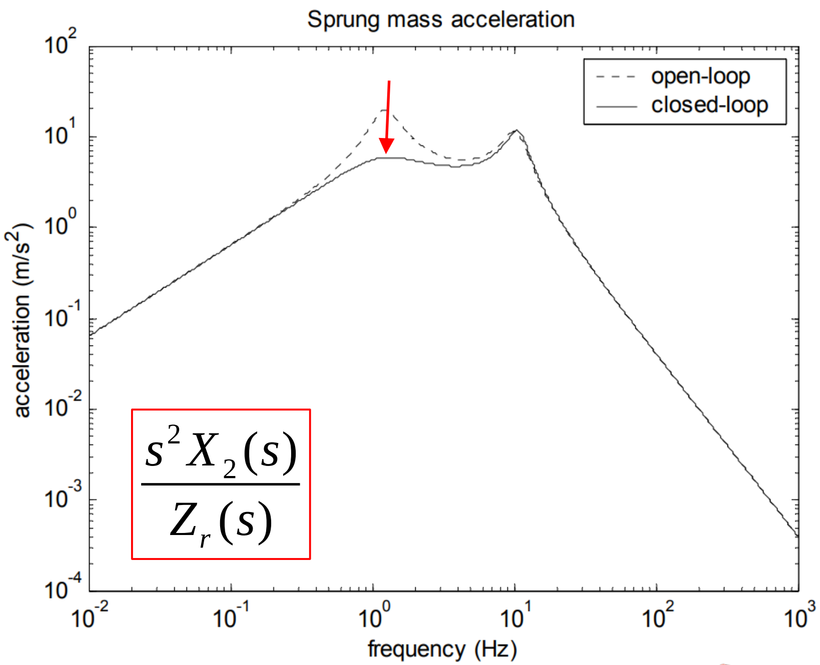
$x_2 = \dot{z}_s \longrightarrow$ absolute velocity of sprung mass

$x_3 = z_u - z_r \longrightarrow$ tire deflection

$x_4 = \dot{z}_u \longrightarrow$ absolute velocity of unsprung mass

Semi-active Suspension Control Model

❖ Simulation Result: LQR formulation for active suspension design



❖ Important Findings:

- **With semi-active suspension, sprung mass acceleration becomes smaller than original one.**
- **But, not that much better than active suspension. Why?**

cost function

$$J = \left[\int_0^\infty \ddot{z}_s^{\,2} + \rho_1 (z_s - z_u)^2 + \rho_2 \dot{z}_s^{\,2} + \rho_3 (z_u - z_r)^2 + \rho_4 \dot{z}_u^{\,2} dt \right]$$

$\rho_1 = 0.4$, $\rho_2 = 0.16$, $\rho_3 = 0.4$ and $\rho_4 = 0.16$.

$x_1 = z_s - z_u \rightarrow$ suspension deflection (rattle space)

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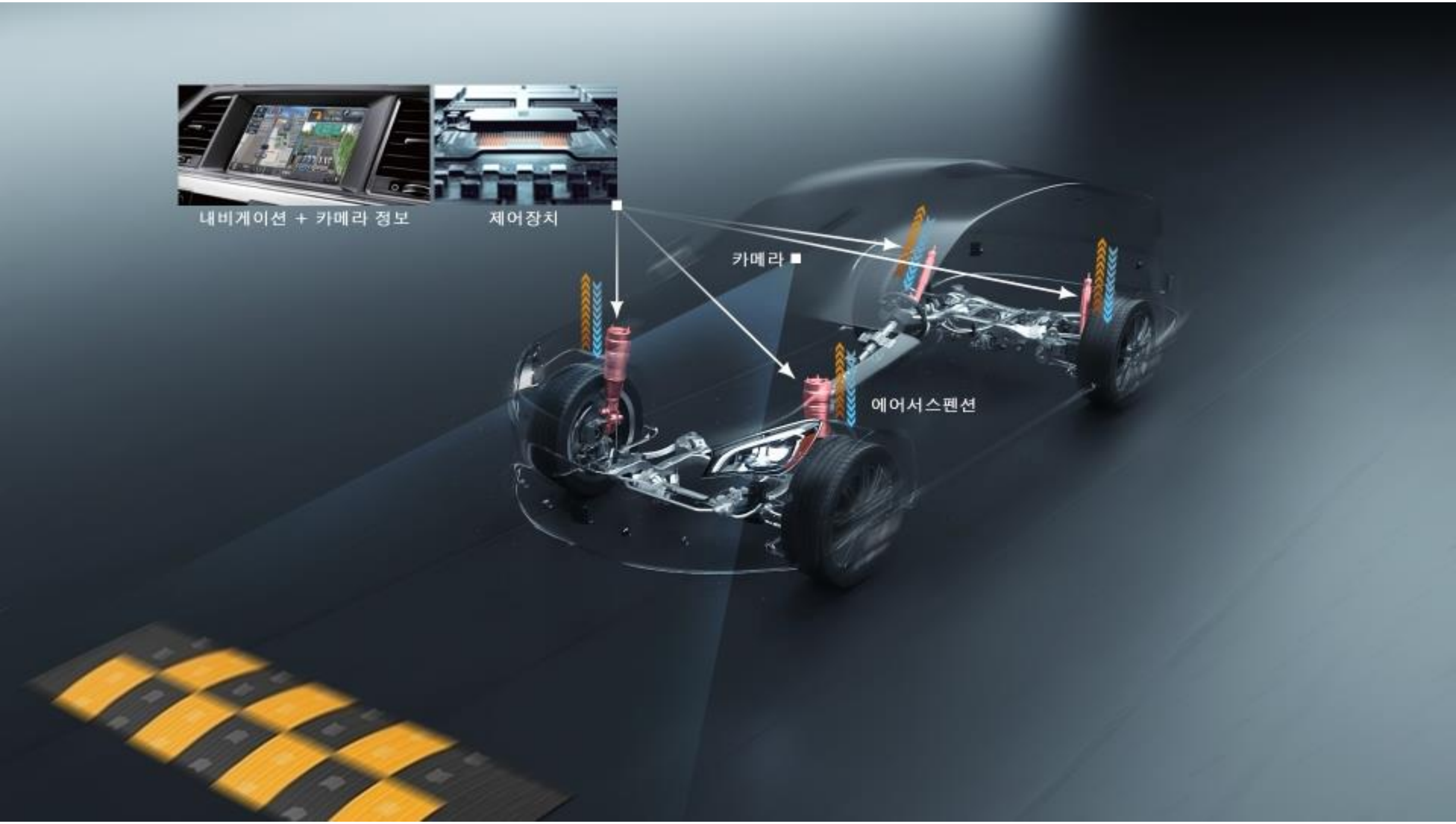
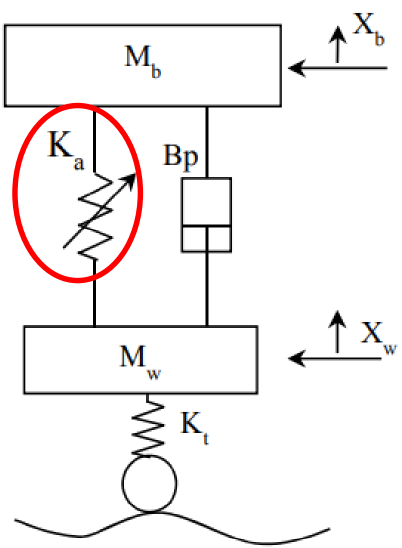
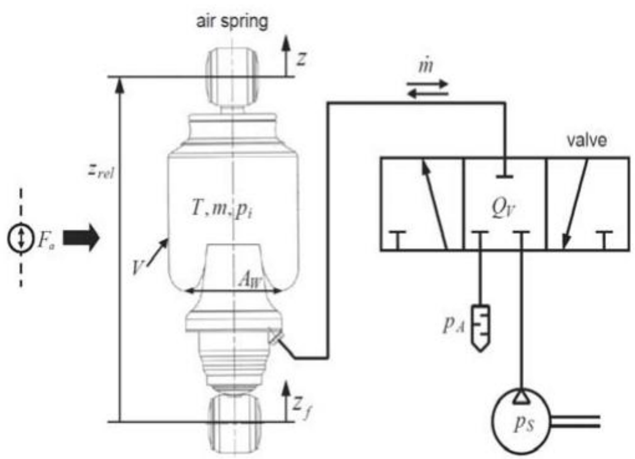
Air Suspension System (Semi-Active)



https://www.youtube.com/watch?v=C-T8I-xiM0U

Air Suspension System (Semi-Active)





- Mathematical model with linear force spring

$$M_b\ddot{x}_b = -C(\dot{x}_b - \dot{x}_w) - K(x_b - x_w)$$

$$M_w\ddot{x}_w = -C(\dot{x}_w - \dot{x}_b) - K(x_w - x_b)$$

$$-K_t(x_w - r)$$

- Mathematical model with air-spring

$$M_b\ddot{x}_b = -C(\dot{x}_b - \dot{x}_w) - K((x_b \boxed{- h_b}) - (x_w \boxed{- h_w}))$$

$$-M_b g + u_c$$

$$M_w\ddot{x}_w = -K(\dot{x}_w - \dot{x}_b) - K((x_w \boxed{- h_w}) - (x_b \boxed{- h_b}))$$

$$-K_t((x_w - h_w) - r) - u_c$$

Ref: A. Kazemini et al., IMPROVING CONTROL MECHANISM OF AN ACTIVE AIR-SUSPENSION SYSTEM, May 2014, Conference: Automotive Technologies Congress

Air Suspension System (Semi-Active)

제네시스 G90
에어 스프링 승차감 제어 단계

주행 모드		승차감 제어	
		일반 주행	고속 주행 120km/h 이상
컴포트	전륜	소프트	소프트
	후륜	소프트	소프트
쇼퍼 드리븐	전륜	노멀	노멀
	후륜	소프트	소프트
스포츠	전륜	노멀	하드
	후륜	노멀	하드

- Mathematical model with linear force spring

$$M_b\ddot{x}_b = -C(\dot{x}_b - \dot{x}_w) - \boxed{K}(x_b - x_w)$$

$$M_w\ddot{x}_w = -C(\dot{x}_w - \dot{x}_b) - \boxed{K}(x_w - x_b)$$

$$-K_t(x_w - r)$$

Air Suspension System (Semi-Active)