Unified Analysis of Asynchronous-type Algorithms

KAUST Rising Stars in AI Symposium 2024

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Outline

1. The Problem Formulation
2. Theoretical Analysis
3. Numerical Experiments

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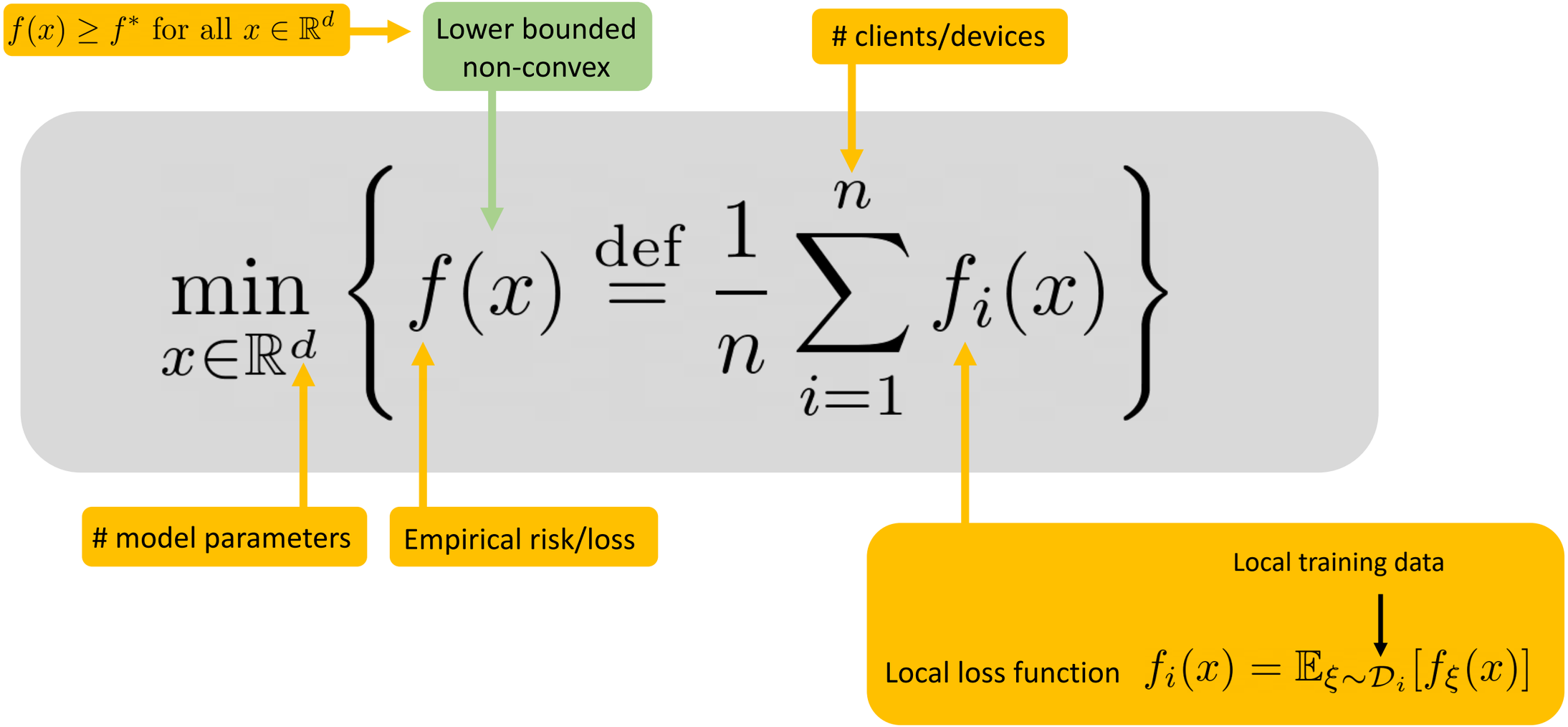
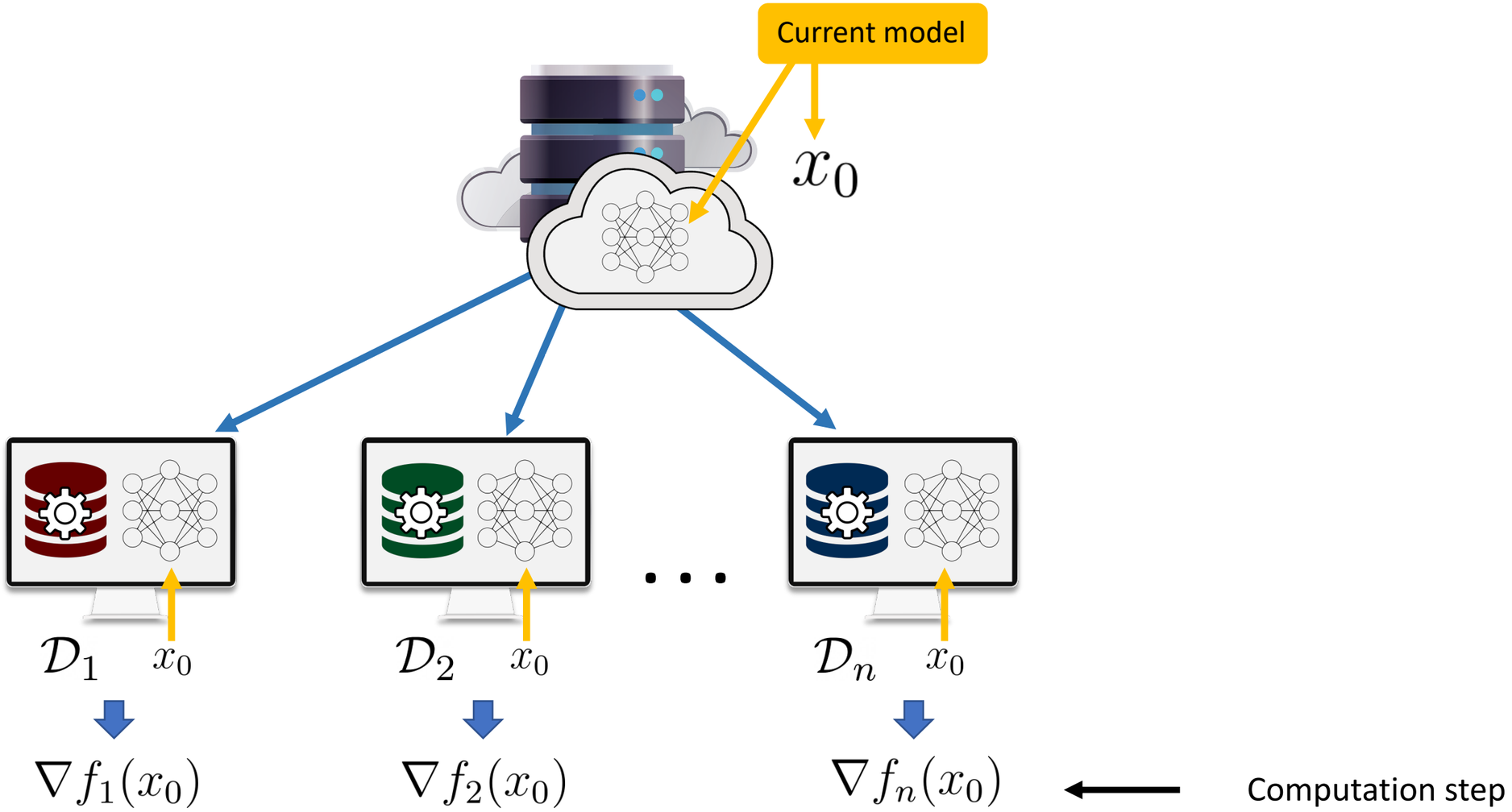
The Problem: Centralized Distributed Training

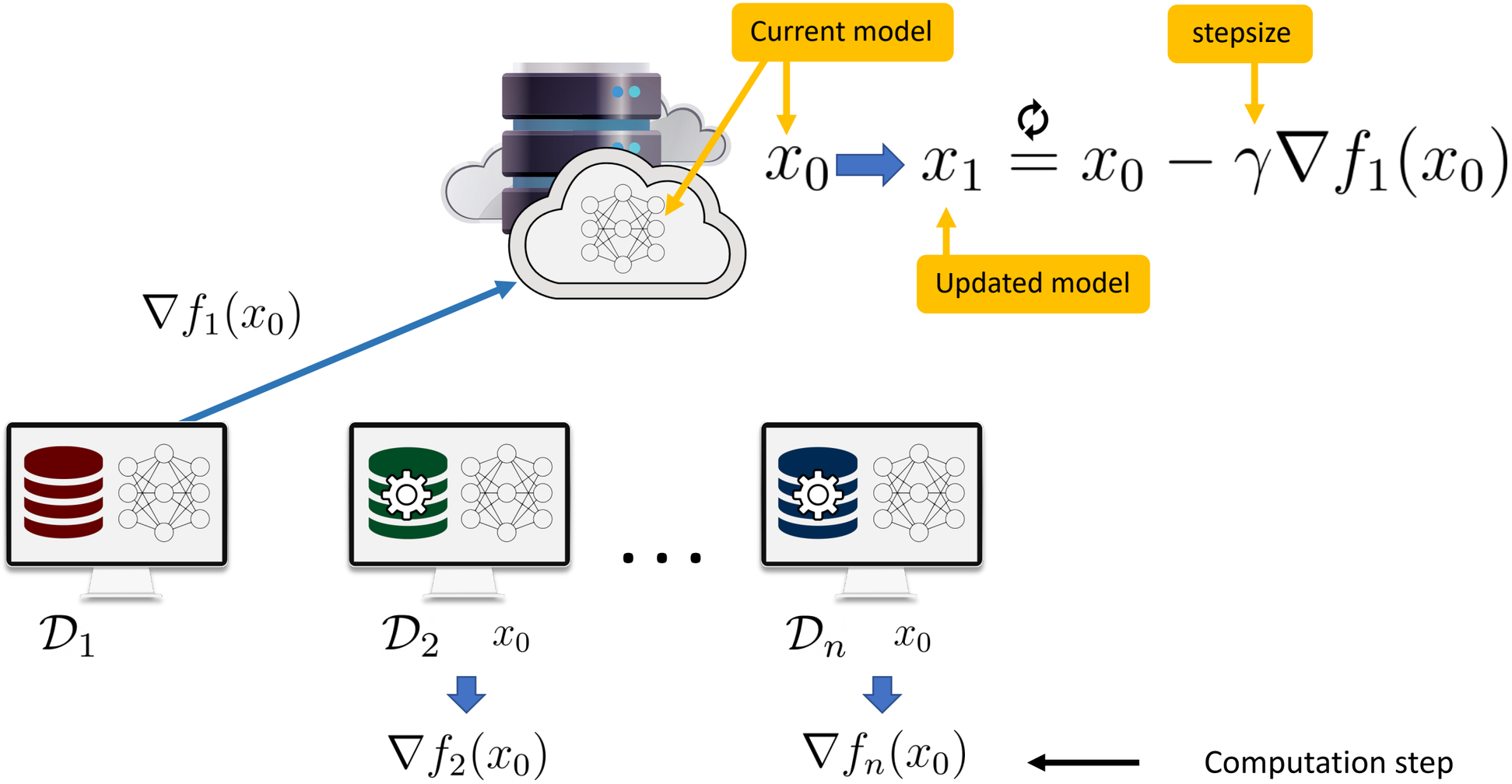


Asynchronous
Distributed Gradient Descent

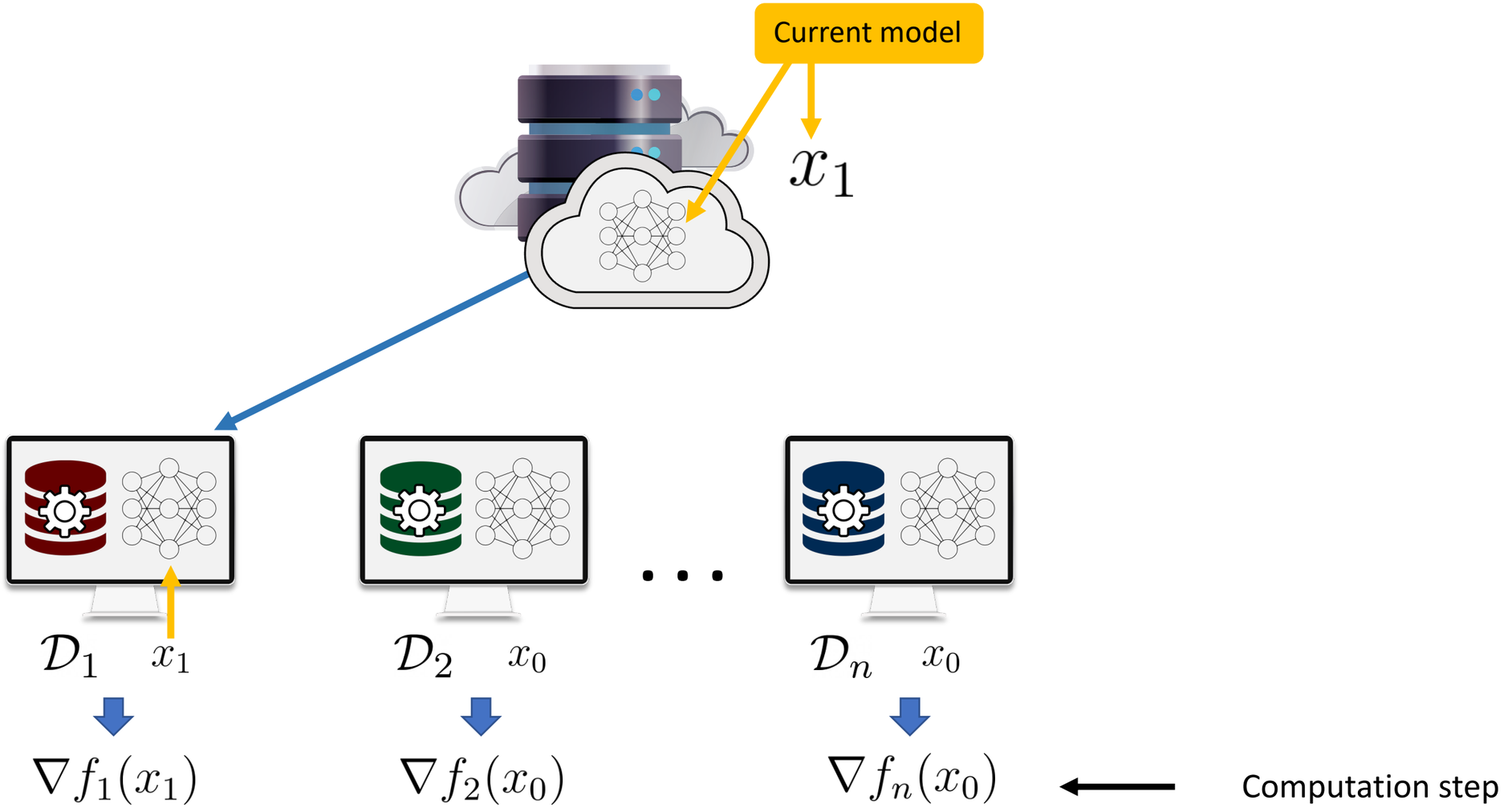
Asynchronous Distributed Gradient Descent



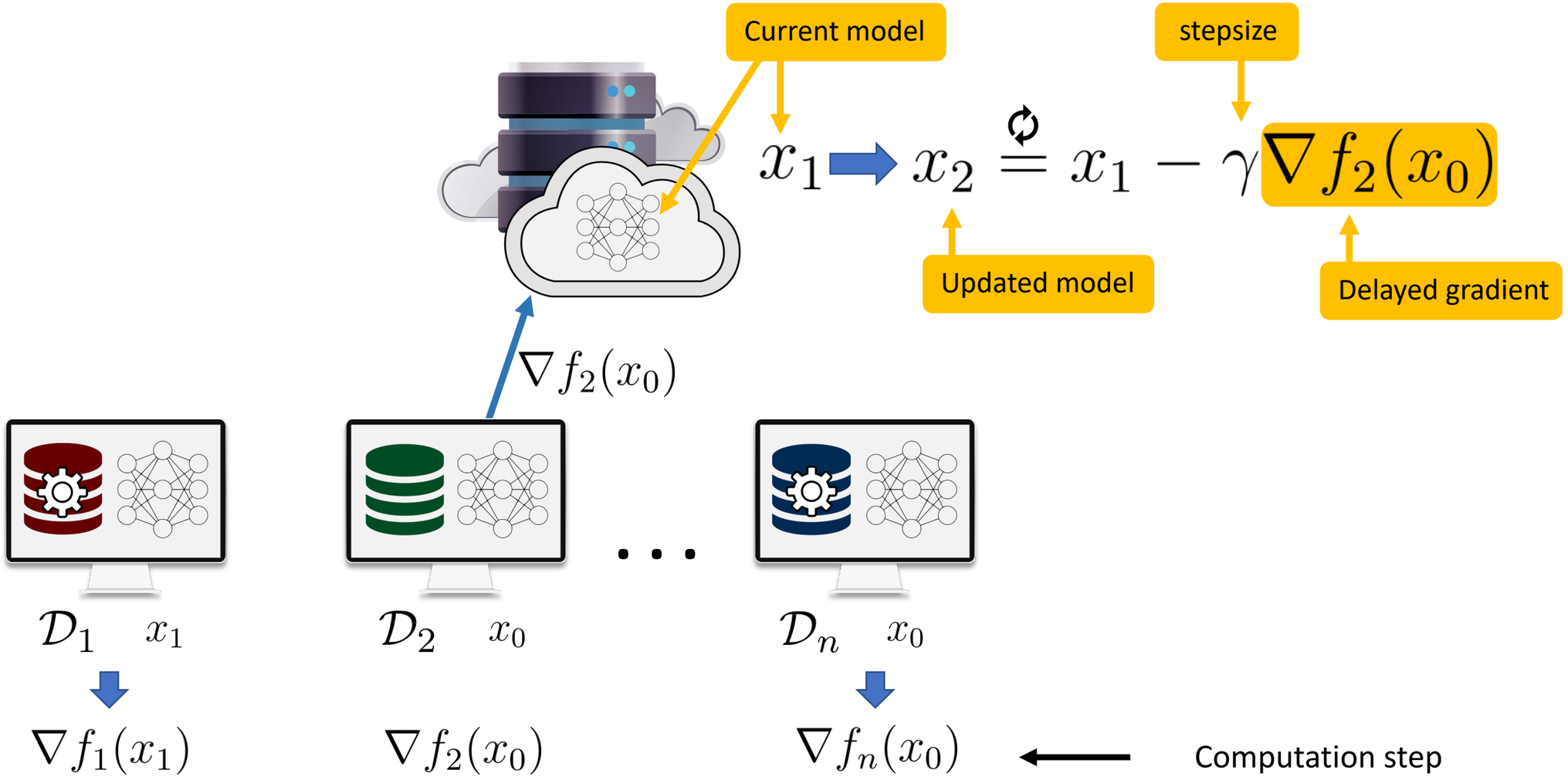
Asynchronous Distributed Gradient Descent



Asynchronous Distributed Gradient Descent



Asynchronous Distributed Gradient Descent



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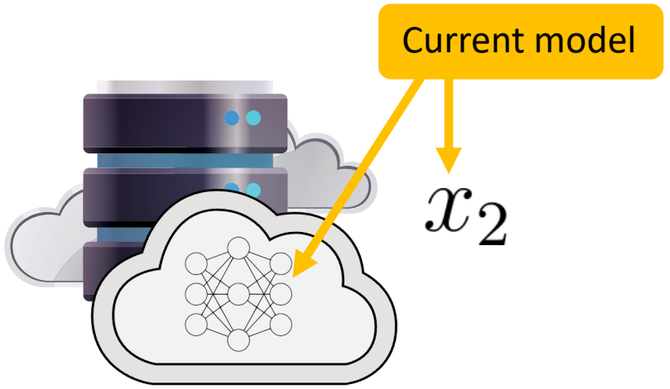

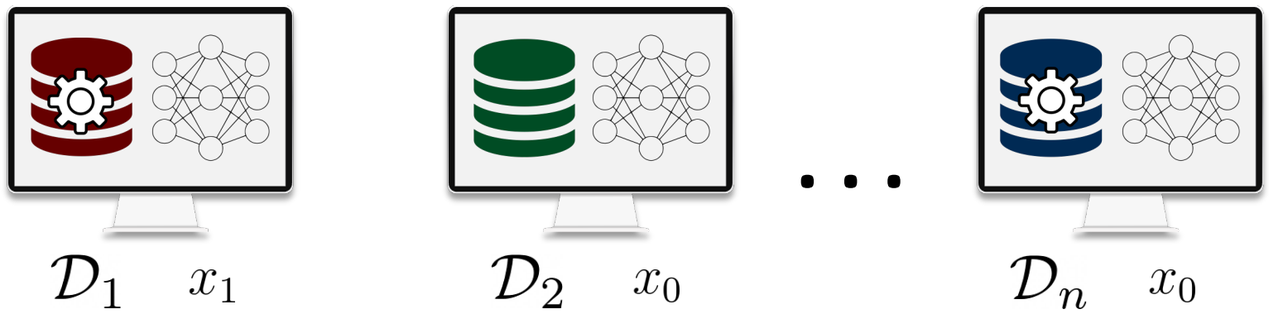

We save time by **asychronizing** the communication, but it comes with a cost of applying **delayed gradients**

Computation step

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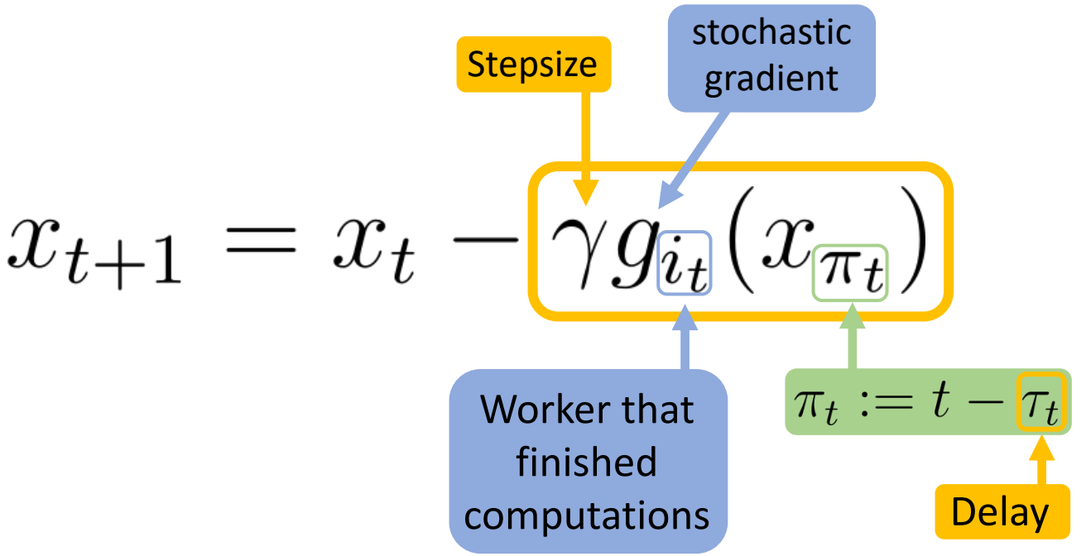
General Asynchronous Algorithm



1. The characteristics of a particular asynchronous method depend on how sequences $\{i_t\}$ and $\{\pi_t\}$ are defined.

2. In order to analyze a particular asynchronous method, we only need to understand how sequences $\{i_t\}$ and $\{\pi_t\}$ affect the method.

3. Knowing 2, we can design more efficient asynchronous SGD-type algorithms.

Having the above stated, we propose a **unified framework, called AsGrad,** to analyze asynchronous SGD-type algorithms.

Theoretical Analysis: Main Result

The analysis is performed utilizing the **restarting sequence**

$$\widetilde{x}_0 = x_0, \quad \widetilde{x}_{t+1} = \begin{cases} \widetilde{x}_t - \gamma \nabla f(x_t) & \text{if } t+1 \neq \tau k \text{ for any } k \geq 1, \\ x_{t+1} & \text{if } t+1 = \tau k \text{ for some } k \geq 1. \end{cases}$$

Choice of τ : $\Theta(1/\gamma)$

Under standard assumptions we can guarantee to find a stationary point such that

$$\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\left[\|\nabla f(x_t)\|^2\right] \leq \mathcal{O}\left(\frac{1}{\gamma T} + \gamma\sigma^2 + \gamma^2\left[\frac{1}{\lfloor T/\tau \rfloor}\sum_{k=0}^{\lfloor T/\tau \rfloor}\sigma_{k,\tau}^2 + \frac{1}{T}\nu^2\right]\right)$$

Noise variance

Delay variance of $\{i_t\}$

Hides numerical constants

Sequence correlation of $\{i_t\}$

Stepsize restrictions:
$20L\gamma\sqrt{\tau_{\max}\tau_C} \leq 1$
$6L\gamma \leq 1$

1. The closer the sequence $\{i_t\}$ to uniform sampling, the better convergence is.
2. The closer the sequence $\{\pi_t\}$ to $\{t\}$, the better convergence is.

Theoretical Analysis: Special Cases

1. **Pure Asynchronous SGD** (new job is assigned to the *same worker* who finished computations)

$$\mathcal{O}\left(\frac{\tau_C}{T} + \left(\frac{\sigma^2}{T}\right)^{1/2} + \left(\frac{\tau_C G}{T}\right)^{2/3} + \zeta^2\right)$$

τ_C maximum number of active workers

ζ^2 gradient similarity

G gradient bound

2. **Random Asynchronous SGD** (new job is assigned to randomly selected worker)

$$\mathcal{O}\left(\frac{\tau_C}{T} + \left(\frac{\sigma^2}{T}\right)^{1/2} + \left(\frac{\zeta^2}{T}\right)^{1/2} + \left(\frac{\tau_C G}{T}\right)^{2/3}\right)$$

3. **Shuffle Asynchronous SGD [ours]** (new job is assigned according to reshuffled order of workers)

$$\mathcal{O}\left(\frac{n}{T} + \left(\frac{\sigma^2}{T}\right)^{1/2} + \left(\frac{\sqrt{n}\zeta}{T}\right)^{2/3} + \left(\frac{\tau_C G}{T}\right)^{2/3}\right)$$

Our results improve over the results of previous works or match them in terms of the dependencies of the rate on maximum delay.

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Experiments

We consider non-convex Logistic Regression

$$f_i(x) = \frac{1}{m}\sum_{j=1}^{m} \log\left(1 + e^{-b_{ij}a_{ij}^\top x}\right) + \lambda r(x), \quad r(x) = \sum_{j=1}^{d} \frac{x_j^2}{1+x_j^2}$$

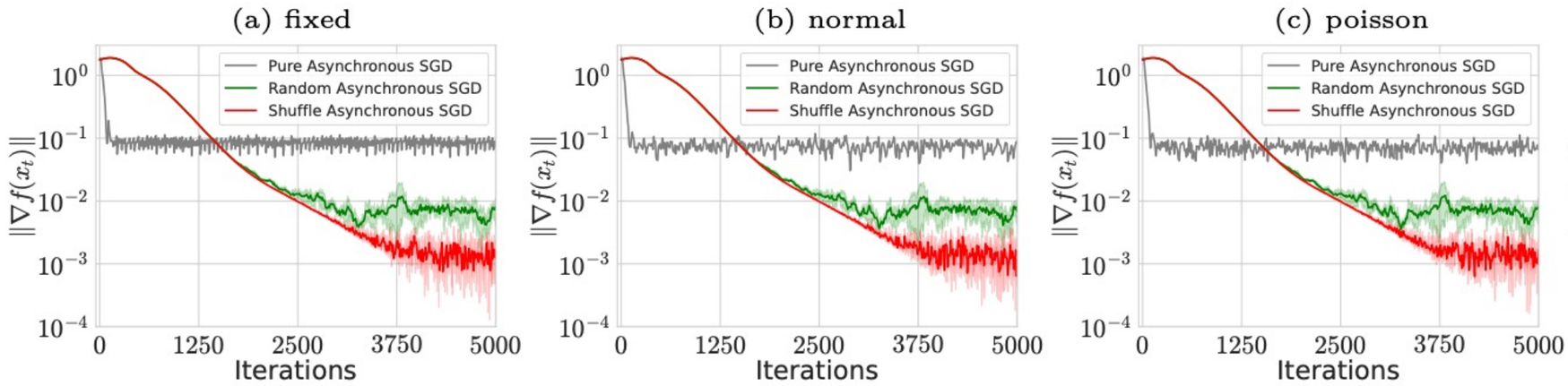
Training data points

Regularization parameter

Gradient computation time simulations:

- **Fixed:** worker *i* spends s_i seconds to compute one gradient;
- **Poisson:** worker *i spends* s_i *seconds to compute one gradient where* $s \sim \mathrm{Po}(s_i)$*;*
- **Normal:** worker *i* spends $|s| + 1$ seconds to compute one gradient where $s \sim \mathcal{N}(s_i, s_i)$.

Experiments



We observe that

- Random and Shuffled Asynchronous SGD is always faster than Pure Asynchronous SGD, and both of them find a stationary point with smaller error;
- Random and Shuffled Asynchronous SGD converge with similar speed, but Shuffled Asynchronous SGD finds a stationary point with smaller gradient norm;

Thank you

Rustem Islamov, Mher Safaryan, Dan Alistarh, AsGrad: A Sharp Unified Analysis of Asynchronous-SGD algorithms, International Conference on Artificial Intelligence and Statistics (AISTATS 2024)