Recursive Relation and ODEs

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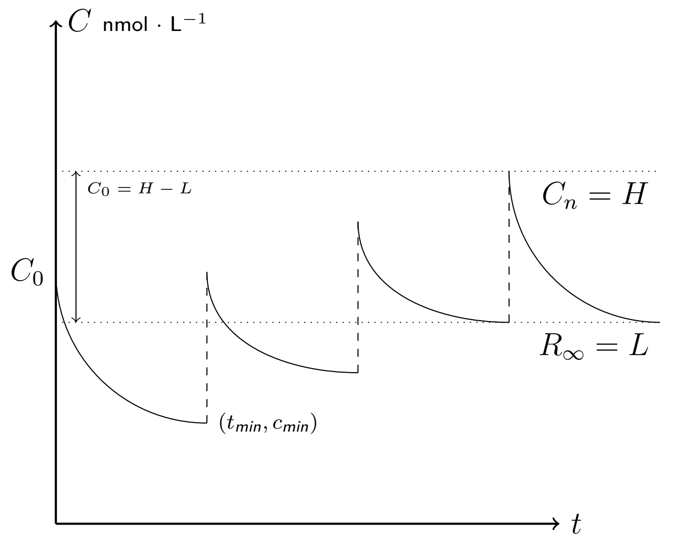

Figure 1: Diagram of Blood Levels over time

Consider a Recursive relation with constant coefficients:

$$\sum_{n=0}^{\infty} [c_i \cdot a_n] = 0, \quad \exists c \in \mathbb{R}, \ \forall i < k \in \mathbb{Z}^+$$

This can be expressed in terms of the exponential generating function $f(x)$ from (**??**):

$$\sum_{i=0}^{k} [c_i \cdot a_{n+i}] = 0$$

$$\implies \sum_{i=0}^{k} \left[\sum_{n=0}^{\infty} \left[c_i \cdot a_{n+i} \frac{x_n}{n!} \right] \right] = 0$$

From (**??**) it is known that $\frac{\mathrm{d}}{\mathrm{d}x}\left(f\left(x\right)\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\sum_{n=0}^{\infty}\left[a_n\frac{x^n}{n!}\right]\right)=\sum_{n=0}^{\infty}\left[a_{n+1}\frac{x_n}{n!}\right]$ and hence:

$$\begin{aligned}\sum_{i=0}^{k}\left[\sum_{n=0}^{\infty}\left[c_i\cdot a_{n+i}\frac{x_n}{n!}\right]\right]&=0\\ \implies\sum_{n=0}^{k}\left[c_i\cdot f^{(k)}\left(x\right)\right]&=0\end{aligned}$$

This is a homogenous k^{th} order linear ODE with constant coefficients, assume that all solutions exist, there will be k solutions either of the form e^{mx} or $x^k\cdot e^x$ will be such that