The Emergence of Patterns in Nature and Chaos Theory

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1.1Hausdorff Dimension Ryan

1.1.1 Topological Equivalence

Sources for this section on topology are primarily.

Topology is an area of mathematics concerned with ideas of continuity through the study of figures that are preserved under homeomorphic transformations. cite:gilmoreTopologyChaosAlice200

Two figures are said to be homeomorphic if there is a continuous bijective mapping between the two shapes .

So for example deforming a cube into a sphere would be homeomorphic, but deforming a sphere into a torus would not, because the the surface of the shape would have to be compromised to acheive that.

Historically the concept of dimension was a difficult problem with a tenuous definition, while an inutitive definition related the dimension of a shape to the number of parameters needed to describe that shape, this definition is not sufficient to be preserved under a homeomorphic transform however.

Consider the koch fractal in figure 1 (see also figure 2), at each iteration the perimeter is given by $p_n = p_{n-1}\left(\frac{4}{3}\right)$, this means if the shape is scaled by some factor s the the following relationship holds.

The number of edges in the koch fractal is given by:

$$
\begin{aligned}
N_n &= N_{n-1} \cdot 4 && (1)\\
&= 3 \cdot 4^n && (2)
\end{aligned}
$$

If the length of any individual side was given by l and scaled by some value s then the length of each individual edge would be given by:

$$l = \frac{s \cdot l_0}{3^n} \quad (3)$$

The total perimeter would be given by:

$$
\begin{aligned}
p_n &= N_n \times l && (4)\\
&= 3 \cdot 4^n \times \frac{s \cdot l_o}{3^n} && (5)\\
&= 3 \cdot s \cdot l_0 \left(\frac{4}{3}\right)^n && (6)
\end{aligned}
$$

The koch snowflake, is defined such that there are no edges, every point on the curve is the vertex of an equilateral triangle. Every time the koch curve is iterated, one edge is reduced in length by a scale of 3 and the overall length increases by a factor of 4, this means if the overall shape was scaled by a factor of s the number of segments.

Briggs and Tyree provide a great introduction.

the scale of resolution increases 3 fold THIS IS NOT CORRECT, I MUST SHOW THAT THE DIMENSION IS $\frac{ln(4)}{\ln(3)}$ BUT I'M SIMPLY OUT OF TIME.

$$
\begin{aligned}
s \cdot p_n &= (4/3)^n \cdot s \cdot P_0 && (7)\\
&\propto \left(\frac{4}{3}\right)^n && (8)\\
\implies n &= \frac{\ln(4)}{\ln(3)} && (9)
\end{aligned}
$$

In ordinary geometric shapes this value n will be the dimension of the shape,

See for working.

The idea is we start with the similarity dimension which should be equal to the hausdorff and box counting for most fractals, but for fractals that aren't so obviously self similar it won't be feasible but for the julia set we'll need to expand the concept to box counting, we don't know whether or not the dimension of the julia set is constant across scales so we use linear regression to check, this is more important for things like coastlines.

with respect to that shapes *measure*. For example consider measure similar to mass, a piece of wire when scaled in length, will increase in mass by a factor of that scale, whereas a sheet of material would increase in mass by a factor proportional to the square of that scaling.

In the case of the koch snowflake, the measure of the shape, when scaled, will increase by a factor of

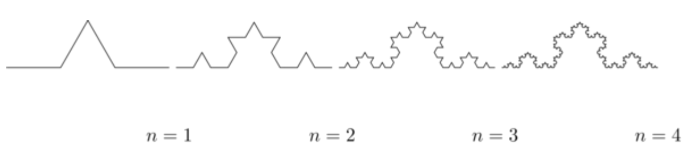

Figure1: Progression of the Koch Snowflake

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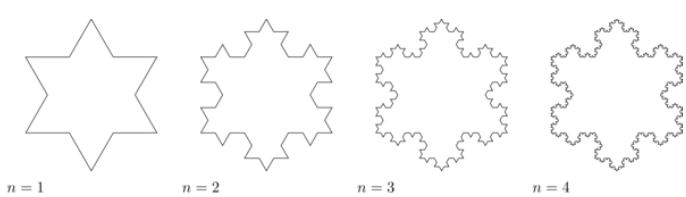

Figure2: Progression of the Koch Snowflake

In the development of topology