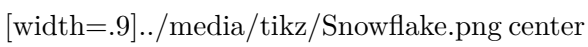
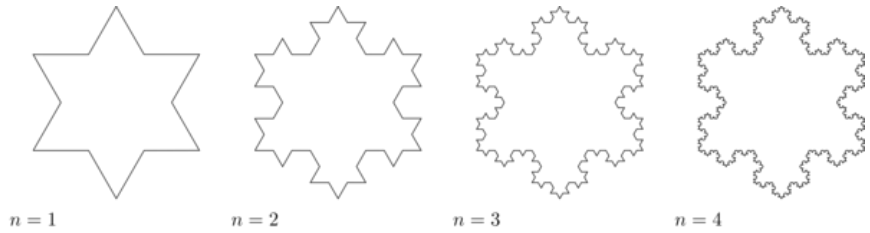


The Emergence of Order



Ryan & James

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1 Introduction

- Looked at the emergence of patterns from natural and iterative processes.
- This lead to an investigation of fractals mostly

2 Definition of a Fractal

- Shapes with a complex structure
- Tend to exhibit self-similarity
  - Although they may not!

2.1 Examples of Fractals

To motivate the concept, here are some fractals we generated in our investigation:

- Self-Similar Fractals

$$\mathbf{B} \leftarrow \begin{bmatrix} \mathbf{B} & \mathbf{Z} & \mathbf{B} \\ \mathbf{Z} & \mathbf{B} & \mathbf{Z} \\ \mathbf{B} & \mathbf{Z} & \mathbf{B} \end{bmatrix}$$

where:

- $\mathbf{B} = [1]$
- $\mathbf{Z} = [0]$

`media/vicsek_fractal.gif`

$$\mathbf{B} \leftarrow \begin{bmatrix} \mathbf{B} & \mathbf{B} & \mathbf{B} \\ \mathbf{B} & \mathbf{Z} & \mathbf{B} \\ \mathbf{B} & \mathbf{B} & \mathbf{B} \end{bmatrix}$$

`media/sierpinski_carpet.gif`

- Can also use the Chaos Game

[width=.9]media/chaos$_game$/1

[width=.9]media/chaos$_game$/2

[width=.9]media/chaos$_game$/3

[width=.9]media/chaos$_game$/4

[width=.9]media/chaos$_game$/5

[width=.9]media/chaos$_game$/6

[width=.9]media/chaos$_game$/7

[width=.9]media/chaos$_game$/8 `media/sierpinsky_triangle_chaos.gif`

- and sometimes thay just fall out of otherwise simple math:

$$z \leftarrow z^2 + c$$

- What follows is an illustration of all the points that converge to zero for values on the circle:

$$z \leftarrow z^2 + e^{i\frac{9k}{2}}$$

`media/julia_sets.gif`

2.2 Generating Fractals

- Matrices and Iteration
- Drawing them
- Chaos Game
- Recurrence

2.3 Mandelbrots Definition

- Defined it as:
- Later went back on this
- Efforts have been made to more clearly define it.

2.4 The Fractal Dimension

3 Defining Dimension

3.1 Hausdorff Measure

3.2 Hausdorff Dimension

4 My fractal if I have time

5 Measuring the Dimension of a non-self-similar Fractal