

Session 13 Additional Exercise

Prob 1: [Binomial Distribution method - either choose faulty bulb or not]

$$P(r) = {}^nC_r p^r q^{n-r} \quad \text{where } q = (1-p)$$

Probability of Success, $p = 30/100 = 0.3$

Max. no. of trials, $n = 6$

Total no. of Successes, $r = 2$

$$(i) {}^nC_r = \frac{n!}{r!(n-r)!} = \frac{6!}{2!(6-2)!} = \frac{6 \times 5 \times 4!}{2! \times (4!)} = 15$$

$$(ii) p^r = (0.3)^2 = 0.09$$

$$(iii) q^{n-r} = (1-p)^{n-r} = (1-0.3)^{6-2} = 0.7^4 = 0.24$$

$$\Rightarrow P(r) = 15 (0.09) (0.24) = 0.324$$

Average Value of this process, $\mu = n P(r) = 6 (0.324)$

$$\Rightarrow \boxed{\mu = 1.944}$$

$$\text{Std. deviation, } \sigma = \sqrt{n P(r) [1-p]} = \sqrt{6 (0.324) (1-0.3)} = \sqrt{1.3464} = 1.16$$

$$\Rightarrow \boxed{\sigma = 1.12}$$

Prob 2: [Method - Binomial Distribution (either solve correctly or not)]

Gaurav:

Probability of Success, $p = \frac{75}{100} = 0.75$

Max. no. of trials, $n = 8$

(i) Total no. of Success, $r = 5$

$$P(r) = {}^nC_r p^r q^{n-r} = \frac{n!}{r!(n-r)!} p^r q^{n-r}$$

$$P(5) = \frac{8!}{5!(8-5)!} (0.75)^5 (1-0.75)^{8-5}$$

$$P(5)_G = 0.207$$

(ii) Total no. of Success, $r = 4$

$$P(4) = \frac{8!}{4!(8-4)!} (0.75)^4 (1-0.75)^{8-4}$$

$$P(4)_G = 0.086$$

Barakha:

Probability of Success, $p = \frac{45}{100} = 0.45$

Max. no. of trials, $n = 12$

(i) Total no. of Success, $r = 5$

$$P(5) = \frac{12!}{5!(12-5)!} (0.45)^5 (1-0.45)^{12-5}$$

$$P(5)_B = 0.222$$

(ii) Total no. of Success, $r = 4$

$$P(4) = \frac{12!}{4!(12-4)!} (0.45)^4 (1-0.45)^{12-4}$$

$$P(4)_B = 0.169$$

(iii) Total no. of Success, $r = 6$

$$P(6)_G = \frac{8!}{6!(8-6)!} (0.75)^6 (1-0.75)^{8-6}$$

$$P(6)_G = 0.266$$

$$P(6)_B = \frac{12!}{6!(12-6)!} (0.45)^6 (1-0.45)^{12-6}$$

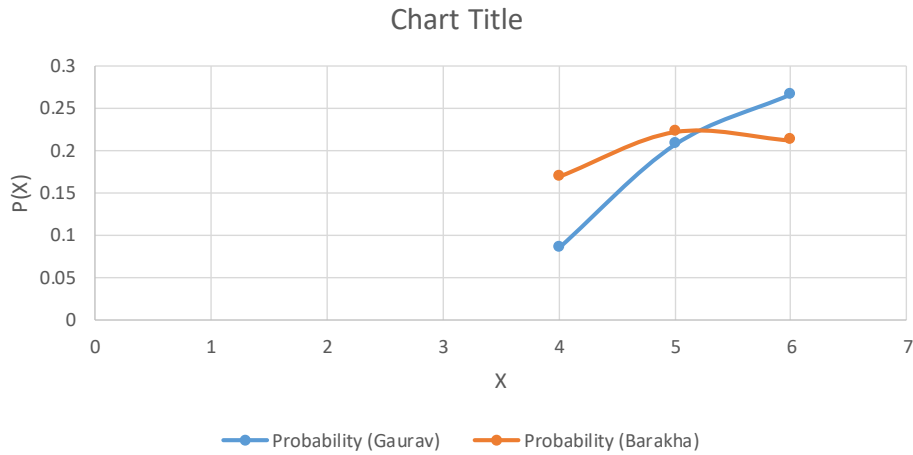
$$P(6)_B = 0.212$$

(iv) Factors affecting their ability to solve questions are n & p i.e., probability of success & max. no. of trials.

Occurrence should

Problem 2

Random variable (X)	Probability (Gaurav)	Probability (Barakha)
4	0.086	0.169
5	0.207	0.222
6	0.266	0.212



Prob 3:
 [Poisson distribution Method - Since Probability of occurrence should be found at fixed / same interval i.e., 4 mins]

$$P(x) = \frac{e^{-\mu} \cdot \mu^x}{x!}$$

Mean, $\mu = \frac{72}{60} \times 4 = 4.8$ (fixed interval for all occurrences)

(a) 5 customers :-
 No. of occurrences, $k = x = 5$
 $\therefore P(5) = \frac{e^{-4.8} (4.8)^5}{5!} = 0.18\%$

(b) Not more than 3 customers :-
 $P(x \leq 3) = P(0) + P(1) + P(2) + P(3)$
 $= e^{-4.8} \left[\frac{(4.8)^0}{0!} + \frac{(4.8)^1}{1!} + \frac{(4.8)^2}{2!} + \frac{(4.8)^3}{3!} \right]$
 $= e^{-4.8} (1 + 4.8 + 11.52 + 18.43)$
 $\Rightarrow \boxed{P(x \leq 3) = 0.303}$

(c) More than 3 customers :-
 $P(x > 3) = P(4) + P(5)$
 $= e^{-4.8} \left[\frac{(4.8)^4}{4!} + \frac{(4.8)^5}{5!} \right]$
 $\boxed{P(x > 3) = 0.368}$

Prob 4:-

Poisson distribution Method - Since interval of error occurrence is fixed/same i.e., 6 errors per hour
 $6 \text{ errors/hr.} = \frac{6}{60} \text{ errors/min.}$
 $= 0.1 \text{ errors/min.}$

Mean, $\mu_{77} = \frac{0.1}{77}$

$\mu_{455} = \frac{0.1}{77} (455) = 0.59$

$\mu_{1000} = \frac{0.1}{77} (1000) = 1.29$

$\mu_{255} = \frac{0.1}{77} (255) = 0.331$

Gn. total no. of success, $x=2$ | $P(x) = \frac{e^{-\mu} \cdot \mu^x}{x!}$

$P(2)_{455} = \frac{e^{-0.59} (0.59)^2}{2!} = 0.096$

$P(2)_{1000} = \frac{e^{-1.29} (1.29)^2}{2!} = 0.229$

$P(2)_{255} = \frac{e^{-0.331} (0.331)^2}{2!} = 0.039$

Problem 4

Mean for 77 words = 0.001

Total no. of success = 2

X	Mean(λ)	P(X)
255	0.331	0.039
256	0.332	0.040
257	0.334	0.040
258	0.335	0.040
259	0.336	0.040
260	0.338	0.041
261	0.339	0.041
262	0.340	0.041
263	0.342	0.041
264	0.343	0.042
265	0.344	0.042
266	0.345	0.042
267	0.347	0.043
268	0.348	0.043
269	0.349	0.043
270	0.351	0.043
271	0.352	0.044
272	0.353	0.044
273	0.355	0.044

