Simulating bee movement

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Introduction

Foraging by colony building pollinators, such as bees, is characterised by the fact that the pollen is not consumed directly but is brought to a central location. This strategy is defined as Central Place Foraging [5]. Here we will look at the first phase of foraging, the exploration phase, in which workers explore a previously unknown area in their search for food. For modelling purposes, it is important to note that each foraging trip starts and ends at the nest site [2]. There are two different approaches that are well suited to modelling the exploratory trajectories of animals:

- **Active Brownian Motion**
  Active Brownian motion describes trajectories through a stochastic process that governs the direction of movement by a particle. A central component for foraging can be implemented using, for example, stochastic resetting or harmonic potentials [1].
- **Persistent Random/Turning Walker**
  A persistent random walk can be regarded as a multidimensional Markov process [4], where a trajectory is described as a sequence of straight movements interrupted by sudden changes in direction. A special form of this is the **Persistent Turning Walker**, in which the trajectory is a sequence of movements with constant curvature. Here, the curvature changes instantaneously. This allows a continuously defined velocity [3].

Model description

To model bee movement, we use an agent-based modelling approach in which each bee is represented by an agent with a velocity $\vec{v}$ with constant magnitude and changing direction angle θ. We assume the bee to behave like a Persistent Turning Walker in two dimensions. Hence, the angular speed (change of flight angle) $w(t)$ over time is given by this differential equation

$$dw(t) = -\gamma[w(t) - w^*(t)]dt + \sigma dW(t), \tag{1}$$

where $w^*(t)$ is the target direction of the flight and $\sigma dW(t)$ describes Gaussian noise. The auto-correlation coefficient γ controls the "determination" of the movement and σ is the amplitude of the noise. For $w^* = 0$, there is no preferred direction, which would lead to a random movement without coming back to the hive.
Since the computer can only calculate this process numerically, we need a discretised version of this equation to approximate $w(t)$ at time step t.

$$w(t + \Delta t) = w(t)e^{-\gamma\Delta t} + w^*(1 - e^{-\gamma\Delta t}) + \epsilon \tag{2}$$

where ϵ is a random number given by a Gaussian process with mean 0 and variance

$$s^2 = \Omega \cdot (1 + e^{-2\gamma\Delta t}). \tag{3}$$

Now, since we have the direction of the bee/agent, it is possible to calculate the position $\vec{x}$ at the given time point.

$$\vec{x}(t + \Delta t) = \vec{x}(t) + \vec{v}(t)\Delta t, \tag{4}$$

where the velocity $\vec{v}$ is calculated with

$$\vec{v}(t + \Delta t) = v \begin{pmatrix} \cos(\theta(t + \Delta t)) \\ \sin(\theta(t + \Delta t)) \end{pmatrix} \text{ and} \tag{5}$$

$$\theta(t + \Delta t) = \theta(t) + w(t)\Delta(t). \tag{6}$$

Here, v is a constant providing the magnitude of the velocity.
To simulate a whole flight, we need to follow steps (2)-(6) at each time point, with given initial conditions (for $t = 0$):

condition	value
$\vec{x}(t)$	(0,0)
$w(t)$	0
$\theta(t)$	0

and standard parameter values:

parameter	value
γ	1
Ω	0.07
Δt	0.01

Tasks Part 1

First, we consider $\gamma = 0$ to investigate the role of the term $\sigma dW(t)$.

1. Implement the model given in equations (2)-(6) for $\gamma = 0$ and simulate until t=10 and save the results.
2. To control your results, find a way to plot the positions over time from Task 1.
3. Since this is a stochastic process, your results should change every time the simulation is run. Run the simulation several times and put all the paths in one plot.
4. Calculate the mean squared displacement (MSD) as an ensemble average for more than 100 runs.

Tasks Part 2

1. Implement the model given in equations (2)-(6) for $w^* = 0$ and simulate until $t = 10$ and save the results.
2. To control your results, find a way to plot the positions over time from Task 1.
3. Since this is a stochastic process, your results should change every time the simulation is run. Run the simulation several times and put all the paths in one plot.
4. Calculate the MSD as an ensemble average for more than 100 runs.

Part 3

To model central place foraging, we need to introduce an attraction component to the simulation. For this, we assume that a bee always knows its position in relation to the hive. Then this attraction w^* is defined as

$$w^*(t) = \eta(t)\angle(\vec{v}(t), \vec{H}(t)), \tag{7}$$

where $\eta(t)$ is the attraction strength and $\angle(\vec{v}(t), \vec{H}(t))$ is the angle between the bee's velocity $\vec{v}(t)$ and the homing vector $\vec{H}(t)$. For the first phase of the flight, the exploration phase $\eta(t)$ is 0. After some time τ, $\eta(t)$ is set to some value η^* or in form of an equation $\eta(t)$ is given by

$$\eta(t) = \begin{Bmatrix} 0 \text{ if } t < \tau \\ \eta^* \text{ if } t \geq \tau \end{Bmatrix}. \tag{8}$$

The values for the additional parameters are the following.

parameter	value
η	0.2
τ	$T/2$
T	100

Tasks Part 3

1. Adapt the model for w^* given in equations (7) and (8).
2. Repeat your analysis from Part 1 and 2, do you notice any difference?
3. Find a way to calculate the distance of a bee to it's hive at any given time point and plot the result
4. Let's consider a bee to be back at the hive if it's closer than 3 meters and end its trip. From there it will start again. How many trips does a bee do on average for T = 300?

References

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