Spectrum and magnetic moment of hidden heavy-flavor pentaquarks

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In light of the recently observed resonance states $P_{\psi s}^{\Lambda}(4338)^0$ and $P_{cs}(4459)^0$ by LHCb Collaboration in $J/\psi \Lambda$ decay channel, we perform a systematical study of all possible hidden heavy-flavor pentaquarks with strangeness S=0,-1,-2,-3, in unified framework of MIT bag model. The color-spin wavefunctions presented in terms of Young-Yamanouchi bases and transformed into baryon-meson couplings, are utilized to calculate masses, magnetic moments and ratios of partial widths. With numerical analysis, the observed $P_{\psi s}^{\Lambda}(4338)^0$ is likely to be a $1/2^-$ compact P_{cs} pentaquark, and $P_{cs}(4459)^0$ can hold two-peak structure of $3/2^-$ and $1/2^ P_{cs}$ states. Futher predictions on hadron properties and decay channels are given to compact P_{css} , P_{csss} states and bottom sectors.

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Key Words: Multiquark, Heavy pentaquark, Mass, Magnetic moment, Quantum number

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$$\begin{split} \phi_1^P &= \frac{1}{4\sqrt{3}} \Big[(2bbgr - 2bbrg + gbrb - gbbr + bgrb - bgbr \\ &- rbgb + rbbg - brgb + brbg) \bar{b} + (2rrbg - 2rrgb \\ &+ rgrb - rgbr + grrb - grbr + rbgr - rbrg + brgr \\ &- brrg) \bar{r} + (2ggrb - 2ggbr - rggb + rgbg - grgb \\ &+ grbg + gbgr - gbrg + bggr - bgrg) \bar{g} \Big], \end{split}$$

$$\begin{split} \phi_2^P &= \frac{1}{12} \Big[(3bgbr - 3gbbr - 3brbg + 3rbbg - rbgb - 2rgbb \\ &+ 2grbb + brgb + gbrb - bgrb) \bar{b} + (3grrb - 3rgrb \\ &- 3brrg + 3rbrg - rbgr - 2gbrr + 2bgrr - grbr \\ &+ rgbr + brgr) \bar{r} + (3grgb - 3rggb + 3bggr - 3gbgr \\ &- grbg + rgbg + 2rbgg - 2brgg + gbrg - bgrg) \bar{g} \Big], \end{split}$$

$$\begin{split} \phi_3^P &= \frac{1}{3\sqrt{2}} \Big[(grbb - rgbb + rbgb - brgb + bgrb - gbrb) \bar{b} \\ &+ (grbr - rgbr + rbgr - brgr + bgrr - gbrr) \bar{r} \\ &+ (grbg - rgbg + rbgg - brgg + bgrg - gbrg) \bar{g} \Big]. \end{split}$$

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TABLE I: Predicted spectra of pentaquarks $P_{nnnb\bar{n}}$. δm is the mass calculated relative to corresponding threshold energy.

	I P	P_{nnnbi}	ī		$\delta m = N$	$I-M_{Thi}$	eshold							
I_{nnn}	J	R_0	M	μ	$\Sigma_b^*\omega$	$\Sigma_b \omega$	$\Sigma_b^*\pi$	$\Sigma_b \pi$	$\Lambda_b \omega$	$\Lambda_b\pi$	ΔB^*	ΔB	NB^*	NB
3/2	5/2-	5.94	6.735	7.33, 4,15, 0.97, -2.20, -5.38	0.119						0.178			
	3/2-	5.95	6.852	3.93, 1.33, 0.74, 0.15, -2.45	0.236	0.255	0.879				0.295	0.340	0.588	
		5.91	6.714	6.55, 0.78, 0.92, 1.06, -4.71	0.098	0.117	0.741				0.157	0.202	0.450	
		5.88	6.571	3.96, 1.36, 0.76, 0.15, -2.45	-0.045	-0.026	0.598				0.014	0.059	0.307	
	$1/2^{-}$	5.99	6.954	0.15, 0.39, 0.08, -0.22, 0.01	0.338	0.357		1.000		1.194	0.397		0.690	0.735
		5.96	6.860	2.53, 0.68, 0.48, 0.26, -1.58	0.244	0.263		0.906		1.100	0.303		0.596	0.641
		5.87	6.563	2.56, 0.66, 0.47, 0.28, -1.62	-0.053	-0.034		0.609		0.803	0.006		0.299	0.344
1/2	5/2-	5.94	6.735	4.15, 0.97, -2.20	0.119						0.178			
	3/2-	5.93	6.726	2.62, 0.88, -0.86	0.110	0.129	0.753		0.323		0.169	0.214	0.462	
		5.90	6.705	3.25, 0.60, -2.06	0.089	0.108	0.732		0.302		0.148	0.193	0.441	
		5.87	6.489	2.01, 0.09, -1.83	-0.127	-0.108	0.516		0.086		-0.068	-0.023	0.225	
		5.81	6.236	1.99, 0.75, -0.50	-0.380	-0.361	0.263		-0.167		-0.321	-0.276	-0.028	
	$1/2^{-}$	5.91	6.709	1.42, 0.44, -0.54	0.093	0.112		0.755	0.306	0.949	0.152		0.445	0.490
		5.88	6.574	-0.06, -0.19, -0.32	-0.042	-0.023		0.620	0.171	0.814	0.017		0.310	0.355
		5.85	6.476	$1.41,\ 0.15,\ -1.12$	-0.140	-0.121		0.522	0.073	0.716	-0.081		0.212	0.257
		5.80	6.274	-0.05, 0.00, 0.05	-0.342	-0.323		0.320	-0.129	0.514	-0.283		0.010	0.055
		5.74	6.195	1.36, 0.56, -0.25	-0.421	-0.402		0.241	-0.208	0.435	-0.362		-0.069	-0.024

TABLE II: Predicted spectra of pentaquarks $P_{nnnc\bar{n}}$. δm is the mass calculated relative to corresponding threshold energy.

	J^P	$P_{nnnc\bar{n}}$	i		$\delta m = N$	$I-M_{Thr}$	eshold							
I_{nnn}	J	R_0	M	μ	$\Sigma_c^*\omega$	$\Sigma_c \omega$	$\Sigma_c^*\pi$	$\Sigma_c \pi$	$\Lambda_c \omega$	$\Lambda_c \pi$	ΔD^*	ΔD	ND^*	ND
3/2	5/2-	6.07	3.348	8.09, 4.84, 1.59, -1.66, -4.92	0.047						0.107			
	$3/2^{-}$	6.06	3.442	4.55, 1.79, 1.29, 0.80, -1.96	0.141	0.205	0.784				0.201	0.342	0.494	
		6.05	3.310	5.90, 0.17, 0.55, 0.93, -4.80	0.009	0.073	0.652				0.069	0.210	0.362	
		5.94	3.147	5.09, 2.39, 1.44, 0.49, -2.21	-0.154	-0.090	0.489				-0.094	0.047	0.199	
	$1/2^{-}$	6.14	3.569	1.24, 1.57, 0.83, 0.07, 0.40	0.268	0.332		0.975		1.143	0.328		0.621	0.762
		6.05	3.451	1.95, 0.12, 0.21, 0.30, -1.53	0.150	0.214		0.857		1.025	0.210		0.503	0.644
		5.97	3.144	$2.34,\ 0.27,\ 0.22,\ 0.16,\ -1.92$	-0.157	-0.093		0.550		0.718	-0.097		0.196	0.337
1/2	5/2	6.07	3.348	4.84, 1.59, -1.66	0.047						0.107			
	$3/2^{-}$	6.07	3.338	2.88, 1.25, -0.39	0.037	0.101	0.680		0.269		0.097	0.238	0.390	
		5.98	3.270	3.32, 0.54, -2.24	-0.031	0.033	0.612		0.201		0.029	0.170	0.322	
		5.99	3.092	2.66, 1.27, -1.39	-0.209	-0.145	0.434		0.023		-0.149	-0.008	0.144	
		5.96	2.857	2.63, 1.38, 0.11	-0.444	-0.380	0.199		-0.212		-0.384	-0.243	-0.091	
	$1/2^{-}$	6.04	3.304	1.20, 0.37, -0.47	0.003	0.067		0.710	0.235	0.878	0.063		0.356	0.497
		5.97	3.160	0.58, 0.20, -0.19	-0.141	-0.077		0.566	0.091	0.734	-0.081		0.212	0.353
		5.96	3.060	1.21, -0.08, -1.38	-0.241	-0.177		0.466	-0.009	0.634	-0.181		0.112	0.253
		5.92	2.871	0.58, 0.69, 0.79	-0.430	-0.366		0.277	-0.198	0.445	-0.370		-0.077	0.064
		5.79	2.738	1.16, 0.39, -0.38	-0.563	-0.499		0.144	-0.331	0.312	-0.503		-0.210	-0.069

TABLE III: Predicted spectra of pentaquarks $P_{nnsb\bar{n}}$. δm is the mass calculated relative to corresponding threshold energy.

	J^P	$P_{nnsb\bar{n}}$	$R_0 = 5.90 \mathrm{GeV^{-1}}$	$\delta m = N$	$M - M_{Th}$	reshold									
I_{nn}	J	M	μ	$\Xi_c^*\omega$	$\Xi_c \omega$	$\Xi_c^*\pi$	$\Xi_c\pi$	$\Sigma_c^* K^*$	$\Sigma_c K^*$	$\Sigma_c^* K$	$\Sigma_c K$	$\Lambda_c K^*$	$\Lambda_c K$	ΛD^*	ΛD
1	5/2-	6.889	4.40, 1.25, -1.91, -5.07	0.152				0.162							
		6.857	4.40, 1.25, -1.91, -5.07	0.120				0.130							
	$3/2^{-}$	6.975	2.09, 0.91, -0.50, -2.17	0.238	0.398	0.881		0.248	0.267	0.646		0.461		0.534	
		6.876	2.15, 0.65, -1.17, -2.67	0.139	0.299	0.782		0.149	0.168	0.547		0.362		0.435	
		6.865	3.49,0.81,-1.58,-4.25	0.128	0.288	0.771		0.138	0.157	0.536		0.351		0.424	
		6.834	3.56, 0.81, -1.63, -4.38	0.097	0.257	0.740		0.107	0.126	0.505		0.320		0.393	
		6.718	2.95, 1.34, -0.87, -2.48	-0.019	0.141	0.624		-0.009	0.010	0.389		0.204		0.277	
		6.680	4.02, 1.51, -0.70, -3.21	l	0.103				-0.028			0.166		0.239	
		6.453	2.68, 1.36, -0.88, -2.20	-0.284	-0.124	0.359		-0.274	-0.255	0.124		-0.061		0.012	
	$1/2^{-}$	7.055	0.06, 0.18, -0.09, 0.03	0.318	0.478		1.121	0.328	0.347		0.745	0.541	0.939	0.614	0.659
		6.983	1.33, 0.49, -0.57, -1.40	0.246	0.406		1.049	0.256	0.275		0.673	0.469	0.867	0.542	0.587
		6.862	0.91, 0.13, -0.67, -1.45		0.285			0.135			0.552	0.348	0.746	0.421	0.466
		6.768	$\hbox{-}0.31,\hbox{-}0.41,\hbox{-}0.16,\hbox{-}0.26$	0.031	0.191		0.834	0.041	0.060		0.458	0.254	0.652	0.327	0.372
		6.709	2.00, 0.85, -0.49, -1.65	-0.028	0.132		0.775	-0.018	0.001		0.399	0.195	0.593	0.268	0.313
		6.665	, , ,	-0.072			0.731	-0.062	-0.043		0.355	0.151		0.224	0.269
		6.444	$\hbox{-}0.55,\hbox{-}0.37,\hbox{-}0.24,\hbox{-}0.05$	ı			0.510	-0.283	-0.264			-0.070		0.003	0.048
			2.42, 1.39, -0.33, -1.37	-0.321	-0.161		0.482	-0.311	-0.292		0.106	-0.098	0.300	-0.025	0.020
0	5/2	6.857	1.91, -1.25	0.120				0.130							
	$3/2^{-}$	6.843	0.66, -0.87	l	0.266				0.135			0.329		0.402	
		6.827	1.68, -1.18	I	0.250				0.119			0.313		0.386	
		6.653	0.13, -2.61		0.076				-0.055			0.139		0.212	
		6.621	0.96, -2.14	-0.116					-0.087			0.107		0.180	
		6.360	-0.24, -0.76	-0.377		0.266			-0.348	0.031		-0.154		-0.081	
	$1/2^{-}$	6.824	0.71, -0.19		0.247			0.097			0.514	0.310		0.383	0.428
		6.705	0.16, -0.26	-0.032				-0.022			0.395	0.191	0.589	0.264	0.309
		6.637	0.16, -1.62	-0.100				-0.090			0.327	0.123	0.521	0.196	0.241
		6.621	0.69, -1.33	-0.116				-0.106			0.311	0.107	0.505	0.180	0.225
		6.475	-0.15, 0.10	-0.262				-0.252				-0.039		0.034	
		6.325	-0.10, -0.47		-0.252			-0.402						-0.116	
		6.107	-0.09, -0.12	-0.630	-0.470		0.173	-0.620	-0.601		-0.203	-0.407	-0.009	-0.334	-0.289

$$\psi'_{4} = \frac{1}{\sqrt{3}}\phi_{1}^{P}\chi_{5}^{P} - \frac{1}{\sqrt{6}}\phi_{2}^{P}\chi_{3}^{P} + \frac{1}{\sqrt{3}}\phi_{2}^{P}\chi_{4}^{P} + \frac{1}{\sqrt{6}}\phi_{3}^{P}\chi_{4}^{P}, \quad (A6)$$

$$\psi'_{5} = -\frac{1}{\sqrt{3}}\phi_{1}^{P}\chi_{3}^{P} + \frac{1}{\sqrt{6}}\phi_{1}^{P}\chi_{4}^{P} - \frac{1}{\sqrt{6}}\phi_{2}^{P}\chi_{5}^{P} - \frac{1}{\sqrt{3}}\phi_{3}^{P}\chi_{5}^{P}, \quad (A7)$$

$$\psi'_{6} = -\frac{1}{\sqrt{6}}\phi_{1}^{P}\chi_{5}^{P} - \frac{1}{\sqrt{3}}\phi_{2}^{P}\chi_{3}^{P} - \frac{1}{\sqrt{6}}\phi_{2}^{P}\chi_{4}^{P} + \frac{1}{\sqrt{3}}\phi_{3}^{P}\chi_{4}^{P}, \quad (A8)$$

$$\psi'_{7} = -\frac{1}{\sqrt{2}}\phi_{2}^{P}\chi_{3}^{P} - \frac{1}{\sqrt{2}}\phi_{3}^{P}\chi_{4}^{P}, \quad (A9)$$

$$\psi'_{8} = -\frac{1}{\sqrt{2}}\phi_{1}^{P}\chi_{3}^{P} + \frac{1}{\sqrt{2}}\phi_{3}^{P}\chi_{5}^{P}, \quad (A10)$$

$$\psi_9' = \frac{1}{\sqrt{2}} \phi_1^P \chi_4^P + \frac{1}{\sqrt{2}} \phi_2^P \chi_5^P, \tag{A11}$$

$$\psi_{10}' = \phi_1^P \chi_2^P, \tag{A12}$$

$$\psi'_{11} = \phi_3^P \chi_2^P, \tag{A13}$$

$$\psi'_{12} = \phi_2^P \chi_2^P. \tag{A14}$$

$$\psi_1 = \frac{1}{\sqrt{3}} \phi_1^P \chi_8^P - \frac{1}{\sqrt{3}} \phi_2^P \chi_7^P + \frac{1}{\sqrt{3}} \phi_3^P \chi_6^P, \tag{A15}$$

$$\psi_2 = -\frac{1}{\sqrt{6}}\phi_1^P \chi_6^P - \frac{1}{\sqrt{3}}\phi_1^P \chi_7^P + \frac{1}{\sqrt{3}}\phi_2^P \chi_8^P - \frac{1}{\sqrt{6}}\phi_3^P \chi_8^P, \tag{A16}$$

$$\psi_3 = -\frac{1}{\sqrt{6}}\phi_1^P \chi_8^P + \frac{1}{\sqrt{6}}\phi_2^P \chi_7^P + \sqrt{\frac{2}{3}}\phi_3^P \chi_6^P, \tag{A17}$$

$$\psi_4 = \frac{1}{\sqrt{3}} \phi_1^P \chi_8^P - \frac{1}{\sqrt{6}} \phi_2^P \chi_6^P + \frac{1}{\sqrt{3}} \phi_2^P \chi_7^P + \frac{1}{\sqrt{6}} \phi_3^P \chi_7^P, \quad (A18)$$

$$\psi_5 = -\frac{1}{\sqrt{3}}\phi_1^P \chi_6^P + \frac{1}{\sqrt{6}}\phi_1^P \chi_7^P - \frac{1}{\sqrt{6}}\phi_2^P \chi_8^P - \frac{1}{\sqrt{3}}\phi_3^P \chi_8^P, \tag{A19}$$

$$\psi_6 = -\frac{1}{\sqrt{6}}\phi_1^P \chi_8^P - \frac{1}{\sqrt{3}}\phi_2^P \chi_6^P - \frac{1}{\sqrt{6}}\phi_2^P \chi_7^P + \frac{1}{\sqrt{3}}\phi_3^P \chi_7^P,$$
(A20)

$$\psi_7 = -\frac{1}{\sqrt{2}}\phi_2^P\chi_6^P - \frac{1}{\sqrt{2}}\phi_3^P\chi_7^P, \tag{A21}$$

$$\psi_8 = -\frac{1}{\sqrt{2}}\phi_1^P \chi_6^P + \frac{1}{\sqrt{2}}\phi_3^P \chi_8^P, \tag{A22}$$

TABLE IV: Predicted spectra of pentaquarks $P_{mnsc\bar{n}}$. ΣD channels are neglected for no evidences of experiments. δm is the mass calculated relative to corresponding threshold energy.

	J^P	$P_{nnsc\bar{n}}$	$R_0 = 5.90 \mathrm{GeV^{-1}}$	$\delta m = M$	$I-M_{Thi}$	reshold									
I_{nn}	J.	M	μ	$\Xi_c^*\omega$	$\Xi_c \omega$	$\Xi_c^*\pi$	$\Xi_c\pi$	$\Sigma_c^* K^*$	$\Sigma_c K^*$	$\Sigma_c^* K$	$\Sigma_c K$	$\Lambda_c K^*$	$\Lambda_c K$	ΛD^*	ΛD
1	5/2-	3.500	5.09, 1.87, -1.35, -4.57	0.071				0.088							
		3.476	5.09, 1.87, -1.35, -4.57	0.047				0.064							
	$3/2^{-}$	3.567	2.76, 1.46, -0.43, -1.73	0.138	0.315	0.781		0.155	0.219	0.553		0.387		0.442	
		3.493	0.44, 1.51, -0.04, -2.13	0.064	0.241	0.707		0.081	0.145	0.479		0.313		0.368	
		3.453	3.90, 0.99, -1.66, -4.58	0.024	0.201	0.667		0.041	0.105	0.439		0.273		0.328	
		3.410	$2.93,\ 0.28,\ -1.82,\ -4.47$	-0.019	0.158	0.624		-0.002	0.062	0.396		0.230		0.285	
		3.301	$3.92,\ 2.14,\ -0.49,\ -2.27$	1	0.049			-0.111	-0.047	0.287		0.121		0.176	
		3.286	4.43, 1.98, -0.24, -2.68	-0.143	0.034	0.500		-0.126	-0.062	0.272		0.106		0.161	
		3.076	3.33, 1.98, -0.30, -1.65	-0.353	-0.176	0.290		-0.336	-0.272	0.062		-0.104		-0.049	
	$1/2^{-}$	3.677	0.78, 0.91, 0.23, 0.36	l	0.425		1.068	0.265	0.329		0.727	0.497	0.895	0.552	0.693
		3.576	1.04, 0.23, -0.49, -1.31	0.147	0.324		0.967	0.164	0.228		0.626	0.396	0.794	0.451	0.592
		3.466	0.92, 0.25, -0.66, -1.33	0.037	0.214		0.857	0.054	0.118		0.516	0.286	0.684	0.341	0.482
		3.354	$\hbox{-}0.02,\hbox{-}0.34,0.16,\hbox{-}0.16$	-0.075	0.102		0.745	-0.058	0.006		0.404	0.174	0.572	0.229	0.370
		3.293	1.82, 0.59, -0.70, -1.93	-0.136				-0.119			0.343	0.113	0.511	0.168	0.309
		3.251	2.65, 1.00, -0.50, -2.14	-0.178	-0.001		0.642	-0.161	-0.097		0.301	0.071	0.469	0.126	0.267
		3.049	0.00, 0.25, 0.23, 0.48	-0.380	-0.203		0.440	-0.363	-0.299			-0.131			
		l	2.47, 1.39, -0.42, -1.50	-0.460	-0.283		0.360	-0.443	-0.379		0.019	-0.211	0.187	-0.156	-0.015
0	,	3.476	1.87, -1.35	0.047				0.064							
	$3/2^{-}$	3.459	0.97, -0.14	!	0.207				0.111			0.279		0.334	
		3.407	1.68, -1.53	-0.022				-0.005		0.393		0.227		0.282	
		3.255	0.75, -2.19	-0.174					-0.093			0.075		0.130	
		3.225	1.54, -1.64	-0.204					-0.123			0.045		0.100	
		2.985	0.35, -0.15	-0.444		0.199			-0.363	-0.029		-0.195		-0.140	
	$1/2^{-}$	3.426	0.40, -0.16	-0.003				0.014			0.476	0.246	0.644	0.301	0.442
		3.299	1.02, -0.04	-0.130				-0.113			0.349	0.119	0.517	0.174	
		3.227	0.35, -1.60	-0.202				-0.185			0.277	0.047	0.445	0.102	0.243
		3.219	0.24, -1.79	-0.210				-0.193			0.269	0.039	0.437	0.094	
		3.069	0.24, 0.91	-0.360				-0.343				-0.111			
		2.884	-0.32, -0.69	-0.545				-0.528				-0.296			
		2.713	0.48, 0.40	-0.716	-0.539		0.104	-0.699	-0.635		-0.237	-0.467	-0.069	-0.412	-0.271

$$\psi_9 = \frac{1}{\sqrt{2}} \phi_1^P \chi_7^P + \frac{1}{\sqrt{2}} \phi_2^P \chi_8^P, \tag{A23}$$

$$\psi_{13} = -\frac{1}{2} \phi_1^P \chi_{10}^P - \frac{1}{2} \phi_2^P \chi_9^P - \frac{1}{\sqrt{2}} \phi_3^P \chi_9^P, \tag{A27}$$

$$\psi_{10} = -\frac{1}{2}\phi_1^P \chi_9^P + \frac{1}{2}\phi_2^P \chi_{10}^P + \frac{1}{\sqrt{2}}\phi_3^P \chi_{10}^P, \tag{A24}$$

$$\psi_{14} = \frac{1}{2}\phi_1^P \chi_9^P - \frac{1}{2}\phi_2^P \chi_{10}^P + \frac{1}{\sqrt{2}}\phi_3^P \chi_{10}^P, \tag{A28}$$

$$\psi_{11} = \frac{1}{\sqrt{2}} \phi_1^P \chi_{10}^P - \frac{1}{\sqrt{2}} \phi_2^P \chi_9^P, \tag{A25}$$

$$\psi_{15} = -\frac{1}{\sqrt{2}} \phi_1^P \chi_9^P - \frac{1}{\sqrt{2}} \phi_2^P \chi_{10}^P.$$

$$\psi_{12} = \frac{1}{2}\phi_1^P \chi_{10}^P + \frac{1}{2}\phi_2^P \chi_9^P - \frac{1}{\sqrt{2}}\phi_3^P \chi_9^P, \tag{A26}$$

TABLE V: Predicted spectra of pentaquarks $P_{ssnb\bar{n}}$. $\Xi_b'K$ channels are neglected for simplification. δm is the mass calculated relative to corresponding threshold energy.

J^P	$P_{ssnb\bar{n}}$	$R_0 = 5.92 \mathrm{GeV^{-1}}$	$\delta m = N$	$M - M_{Thi}$	reshold									
	M	μ	$\Omega_b^*\omega$	$\Omega_b \omega$	$\Omega_b^*\pi$	$\Omega_b\pi$	Ξ^*B^*	Ξ^*B	ΞB^*	ΞB	$\Xi_b^* K^*$	$\Xi_b K^*$	Ξ_b^*K	$\Xi_b K$
5/2-	7.030	1.53, -1.64, -4.80	0.135				0.172				0.182			
	6.982	1.53, -1.64, -4.80	0.087				0.124				0.134			
$3/2^{-}$	7.102	0.23, -0.85, -1.93	0.207	0.273	0.850		0.244	0.289	0.459		0.254	0.414	0.652	
	7.008	1.63, -1.29, -4.19	0.113	0.179	0.756		0.150	0.195	0.365		0.160	0.320	0.558	
	6.976	0.54, -1.07, -2.69	0.081	0.147	0.724		0.118	0.163	0.333		0.128	0.288	0.526	
	6.959	2.24, -0.88, -3.99	0.064	0.130	0.707		0.101	0.146	0.316		0.111	0.271	0.509	
	6.858	1.27, -0.58, -2.43	-0.037	0.029	0.606		0.000	0.045	0.215		0.010	0.170	0.408	
	6.833	-0.56, -1.67, -2.77	-0.062	0.004	0.581		-0.025	0.020	0.190		-0.015	0.145	0.383	
	6.530	-1.07, -1.15, -1.82	-0.365	-0.299	0.278		-0.328	-0.283	-0.113		-0.318	-0.158	0.080	
$1/2^{-}$	7.159	-0.17, -0.06, 0.06	0.264	0.330		0.973	0.301		0.516	0.561	0.311	0.471		0.869
	7.110	0.26, -0.49, -1.24	0.215	0.281		0.924	0.252		0.467	0.512	0.262	0.422		0.820
	6.963	0.81, -0.34, -1.48	0.068	0.134		0.777	0.105		0.320	0.365	0.115	0.275		0.673
	6.872	0.44, 0.03, -0.38	-0.023	0.043		0.686	0.014		0.229	0.274	0.024	0.184		0.582
	6.849	1.02, -0.30, -1.62	-0.046	0.020		0.663	-0.009		0.206	0.251	0.001	0.161		0.559
	6.816	-0.48, -1.06, -1.65	-0.079	-0.013		0.630	-0.042		0.173	0.218	-0.032	0.128		0.526
	6.657	-0.17, -0.08, 0.00	-0.238	-0.172		0.471	-0.201		0.014	0.059	-0.191	-0.031		0.367
	6.500	-0.70, -0.92, -1.13	-0.395	-0.329		0.314	-0.358		-0.143	-0.098	-0.348	-0.188		0.210

TABLE VI: Predicted spectra of pentaquarks $P_{ssnc\bar{n}}$. δm is the mass calculated relative to corresponding threshold energy.

J^P	$P_{ssnc\bar{n}}$	$R_0 = 5.92 \mathrm{GeV^{-1}}$	$\delta m = N$	$\overline{I-M_{Thr}}$	eshold									
J	M	μ	$\Omega_c^*\omega$	$\Omega_c \omega$	$\Omega_c^*\pi$	$\Omega_c\pi$	Ξ^*D^*	Ξ^*D	ΞD^*	ΞD	$\Xi_c^* K^*$	$\Xi_c K^*$	$\Xi_c^* K$	$\Xi_c K$
5/2-	3.643	2.17, -1.06, -4.29	0.094				0.101				0.103			
	3.607	2.17, -1.06, -4.29	0.058				0.065				0.067			
$3/2^{-}$	3.697	0.93, -0.31, -1.54	0.148	0.219	0.791		0.155	0.296	0.370		0.157	0.334	0.555	
	3.600	0.71, -0.87 -2.45	0.051	0.122	0.694		0.058	0.199	0.273		0.060	0.237	0.458	
	3.595	1.81, -1.15, -4.10	0.046	0.117	0.689		0.053	0.194	0.268		0.055	0.232	0.453	
	3.547	2.16, -0.99, -4.15	-0.002	0.069	0.641		0.005	0.146	0.220		0.007	0.184	0.405	
	3.447	1.49, -0.31, -2.10	-0.102	-0.031	0.541		-0.095	0.046	0.120		-0.093	0.084	0.305	
	3.436	0.04, -1.13, -2.28	-0.113	-0.042	0.530		-0.106	0.035	0.109		-0.104	0.073	0.294	
	3.160	-0.48, -0.87, -1.25	-0.389	-0.318	0.254		-0.382	-0.241	-0.167		-0.38	-0.203	0.018	
$1/2^{-}$	3.789	0.11, 0.21, 0.30	0.240	0.311		0.954	0.247		0.462	0.603	0.249	0.426		0.824
	3.703	0.36, -0.35, -1.06	0.154	0.225		0.868	0.161		0.376	0.517	0.163	0.340		0.738
	3.572	0.47, -0.43, -1.31	0.023	0.094		0.737	0.030		0.245	0.386	0.032	0.209		0.607
	3.470	1.35, 0.47, -0.41	-0.079	-0.008		0.635	-0.072		0.143	0.284	-0.070	0.107		0.505
	3.438	0.94, -0.47, -1.87	-0.111	-0.040		0.603	-0.104		0.111	0.252	-0.102	0.075		0.473
	3.399	-0.55, -1.14, -1.73	-0.150	-0.079		0.564	-0.143		0.072	0.213	-0.141	0.036		0.434
	3.255	0.16, 0.39, 0.62	-0.294	-0.223		0.420	-0.287		-0.072	0.069	-0.285	-0.108		0.290
	3.070	-1.01, -1.16, -1.30	-0.479	-0.408		0.235	-0.472		-0.257	-0.116	-0.470	-0.293		0.105

TABLE VII: Predicted spectra of pentaquarks $P_{nnnn\bar{b}}$. δm is the mass calculated relative to corresponding threshold energy.

	J^P	$P_{nnnar{b}}$			$\delta m = M$	- M _{Threshold}		
1	J	R_0	M	μ	ΔB^*	ΔB	NB^*	NB
2	3/2-	5.96	6.916	4.22, 2.67, 1.11, -0.44, -1.99	0.359	0.404		
	$1/2^{-}$	6.00	6.945	2.83, 1.76, 0.69, -0.39, -1.46	0.388			
1	5/2-	5.94	6.735	5.38, 2.20, -0.97	0.178			
	$3/2^{-}$	5.92	6.717	1.61, 0.64, -0.34	0.160	0.205	0.453	
		5.88	6.617	2.26, 0.94, -0.37	0.060	0.105	0.353	
	$1/2^{-}$	5.91	6.633	1.46, 0.57, -0.31	0.076		0.369	0.414
		5.87	6.571	0.17,0.11,0.06	0.014		0.307	0.352
0	$3/2^{-}$	5.88	6.486	1.13			0.222	
	$1/2^{-}$	5.82	6.450	0.66			0.186	0.231

TABLE VIII: Predicted spectra of pentaquarks $P_{nnnn\bar{c}}$. δm is the mass calculated relative to corresponding threshold energy.

	J^P	$P_{nnnnar{c}}$			$\delta m = M$	- M _{Threshold}		
1	J	R_0	M	μ	ΔD^*	ΔD	ND^*	ND
2	3/2-	6.05	3.495	3.81, 2.19, 0.57, -1.05, -2.67	0.254	0.395		
	$1/2^{-}$	6.15	3.574	3.10, 2.00, 0.90, -0.20, -1.29	0.333			
1	$5/2^{-}$	6.07	3.348	4.91, 1.66, -1.59	0.107			
	$3/2^{-}$	6.06	3.320	4.38, 1.80, -0.77	0.079	0.220	0.372	
		5.92	3.176	2.93, 1.00, -0.93	-0.065	0.076	0.228	
	$1/2^{-}$	6.07	3.255	1.50, 0.63, -0.25	0.014		0.307	0.448
		5.96	3.152	-0.04, -0.24, -0.45	-0.089		0.204	0.345
0	$3/2^{-}$	6.02	3.105	0.57			0.157	
	1/2-	5.88	3.003	0.87			0.055	0.196

TABLE IX: Predicted spectra of pentaquarks $P_{nnns\bar{b}}$. δm is the mass calculated relative to corresponding threshold energy.

	I^P	$P_{nnns\bar{b}}$			$\delta m = N$	$I-M_{Thr}$	eshold							
I	J	R_0	M	μ	Σ^*B^*	Σ^*B	ΣB^*	ΣB	ΛB^*	ΛB	ΔB_s^*	ΔB_s	NB_s^*	NB_s
3/2	5/2-	5.95	6.836	5.68, 2.49, -0.69, -3.88	0.126						0.189			
	$3/2^{-}$	6.00	7.033	2.36, 0.96, -0.45, -1.86	0.323	0.368	0.515				0.386	0.434		
		5.94	6.823	4.33,1.76,-0.81,-3.38	0.113	0.158	0.305				0.176	0.224		
		5.88	6.763	5.45, 2.87, 0.29, -2.30	0.053	0.098	0.245				0.116	0.164		
	$1/2^{-}$	6.04	7.060	1.48,0.54,-0.39,-1.33	0.350		0.542	0.587			0.413			
		5.90	6.800	0.21,0.06, -0.09, -0.24	0.090		0.282	0.327			0.153			
		5.88	6.767	3.02, 1.63, 0.25, -1.14	0.057		0.249	0.294			0.120			
1/2	5/2	5.97	6.884	2.50, -0.69	0.174									
	$3/2^{-}$	5.95	6.865	2.18, -0.68	0.155	0.200	0.347		0.424				0.511	
		5.91	6.764	0.77, -0.56	0.054	0.099	0.246		0.323				0.410	
		5.87	6.662	2.12, 0.06	-0.048	-0.003	0.144		0.221				0.308	
		5.84	6.496	0.86, -0.47	-0.214	-0.169	-0.022		0.055				0.142	
	$1/2^{-}$	5.94	6.778	0.69, -0.30	0.068		0.260	0.305	0.337	0.382				0.472
		5.90	6.700	-0.12, -0.02	-0.010		0.182	0.227	0.259	0.304				0.394
		5.82	6.627	1.31, -0.05	-0.083		0.109	0.154	0.186	0.231				0.321
		5.83	6.479	-0.34, -0.55	-0.231		-0.039	0.006	0.038	0.083				0.173
		5.76	6.433	0.91, 0.23	-0.277		-0.085	-0.040	-0.008	0.037				0.127

TABLE X: Predicted spectra of pentaquarks $P_{nnns\bar{c}}$. δm is the mass calculated relative to corresponding threshold energy.

	ı P	$P_{nnns\bar{c}}$:		$\delta m = N$	$I-M_{Thi}$	eshold							
Ι	J	R_0	M	μ	Σ^*D^*	Σ^*D	ΣD^*	ΣD	ΛD^*	ΛD	ΔD_s^*	ΔD_s	ND_s^*	ND_s
3/2	5/2-	6.09	3.461	5.22, 1.96, -1.30, -4.56	0.067						0.117			
	$3/2^{-}$	6.09	3.615	1.90, 0.43, -1.03, -2.49	0.221	0.362	0.413				0.271	0.415		
		6.08	3.447	4.15, 1.76, -0.63, -3.03	0.053	0.194	0.245				0.103	0.247		
		5.93	3.334	5.45, 2.64, -0.16, -2.96	-0.060	0.081	0.132				-0.010	0.134		
	$1/2^{-}$	6.19	3.689	1.75, 0.78, -0.19, -1.16	0.295		0.487	0.628			0.345			
		6.06	3.430	0.68, 0.17, -0.33, -0.83	0.036		0.228	0.369			0.086			
		5.96	3.347	2.16, 1.12, 0.08, -0.96	-0.047		0.145	0.286			0.003			
1/2	5/2-	6.10	3.496	1.97, -1.30	0.102									
	$3/2^{-}$	6.09	3.465	2.20, -0.48	0.071	0.212	0.263		0.340				0.414	
		5.96	3.324	0.53, -1.03	-0.070	0.071	0.122		0.199				0.273	
		6.02	3.282	1.59, -0.53	-0.112	0.029	0.080		0.157				0.231	
		5.99	3.124	0.33, -1.06	-0.270	-0.129	-0.078		-0.001				0.073	
	$1/2^{-}$	6.09	3.396	0.94, -0.08	0.002		0.194	0.335	0.271	0.412				0.489
		6.00	3.288	-0.71, -0.62	-0.106		0.086	0.227	0.163	0.304				0.381
		5.90	3.186	1.43, 0.12	-0.208		-0.016	0.125	0.061	0.202				0.279
		5.97	3.105	-0.75, -0.71	-0.289		-0.097	0.044	-0.020	0.121				0.198
		5.78	2.980	0.96, 0.00	-0.414		-0.222	-0.081	-0.145	-0.004				0.073

TABLE XI: Predicted spectra of pentaquarks $P_{nnss\bar{b}}$. δm is the mass calculated relative to corresponding threshold energy.

	J^P	$P_{nnssar{b}}$			$\delta m = M$	$I-M_{Thre}$	shold							
1	J	R_0	M	μ	Ξ^*B^*	Ξ^*B	ΞB^*	ΞB	$\Sigma^* B_s^*$	$\Sigma^* B_s$	ΣB_s^*	ΣB_s	ΛB_s^*	ΛB_s
1	5/2	5.99	7.004	2.80, -0.41, -3.61	0.146				0.204					
	$3/2^{-}$	6.04	7.151	0.62, -0.54, -1.70	0.293	0.338	0.508		0.351	0.399	0.543			
		5.97	6.988	2.16, -0.54, -3.25	0.130	0.175	0.345		0.188	0.236	0.380			
		5.92	6.912	1.56, -0.15, -1.86	0.054	0.099	0.269		0.112	0.160	0.304			
		5.90	6.802	2.31, 0.24, -1.83	-0.056	-0.011	0.159		0.002	0.050	0.194			
	$1/2^{-}$	6.08	7.175	0.30, -0.46, -1.22	0.317		0.532	0.577	0.375		0.567	0.615		
		5.95	6.924	$1.29,\ 0.04,\ -1.20$	0.066		0.281	0.326	0.124		0.316	0.364		
		5.91	6.875	-0.22, -0.10, 0.02	0.017		0.232	0.277	0.075		0.267	0.315		
		5.85	6.773	1.30, 0.01, -1.28	-0.085		0.130	0.175	-0.027		0.165	0.213		
0	$5/2^{-}$	6.00	7.036	-0.41	0.178									
	$3/2^{-}$	5.98	7.016	-0.32	0.158	0.203	0.373						0.485	
		5.94	6.910	-0.78	0.052	0.097	0.267						0.379	
		5.86	6.681	-1.02	-0.177	-0.132	0.038						0.150	
	$1/2^{-}$	5.96	6.923	-0.30	0.065		0.280						0.392	0.440
		5.92	6.829	-0.08	-0.029		0.186						0.298	0.346
		5.84	6.661	-0.08	-0.197		0.018						0.130	0.178
		5.79	6.611	-0.62	-0.247		-0.032						0.080	0.128

TABLE XII: Predicted spectra of pentaquarks $P_{nnss\bar{c}}$. δm is the mass calculated relative to corresponding threshold energy.

		$P_{nnss\bar{c}}$			$\delta m = N$	$I-M_{Thre}$	-ll J							
Ι	J^P	R_0	M	μ	Ξ^*D^*	Ξ^*D	ΞD^*	ΞD	$\Sigma^*D_s^*$	Σ^*D_s	ΣD_s^*	ΣD_s	ΛD_s^*	ΛD_s
1	5/2-	6.12	3.624	2.27, -1.00, -4.28	0.082				0.127					
	$3/2^{-}$	6.13	3.737	0.20, -1.07, -2.33	0.195	0.336	0.410		0.240	0.384	0.432			
		6.11	3.599	2.07, -0.39, -2.85	0.057	0.198	0.272		0.102	0.246	0.294			
		5.97	3.485	1.24, -0.65, -2.53	-0.057	0.084	0.158		-0.012	0.132	0.180			
		6.03	3.426	1.86, -0.31, -2.48	-0.116	0.025	0.099		-0.071	0.073	0.121			
	$1/2^{-}$	6.22	3.806	0.60, -0.23, -1.05	0.264		0.479	0.620	0.309		0.501	0.645		
		6.10	3.552	1.32, 0.15, -1.01	0.010		0.225	0.366	0.055		0.247	0.391		
		6.00	3.464	-0.37, -0.49, -0.61	-0.078		0.137	0.278	-0.033		0.159	0.303		
		5.91	3.343	1.19, 0.07, -1.06	-0.199		0.016	0.157	-0.154		0.038	0.182		
0	5/2-	6.13	3.648	-1.00	0.106									
	$3/2^{-}$	6.12	3.613	-0.02	0.071	0.212	0.286						0.385	
		5.98	3.471	-1.31	-0.071	0.070	0.144						0.243	
		6.00	3.304	-1.62	-0.238	-0.097	-0.023						0.076	
	$1/2^{-}$	6.11	3.540	0.06	-0.002		0.213						0.312	0.456
		6.02	3.425	-0.83	-0.117		0.098						0.197	0.341
		6.00	3.284	0.01	-0.258		-0.043						0.056	0.200
		5.80	3.152	-1.09	-0.390		-0.175						-0.076	0.068

TABLE XIII: Color-spin wave functions of pentaquarks $q_1q_2q_3q_4\bar{q}_5$ with J^P quantum number and $I_{q_1q_2q_3q_4}$ the isospin of configuration $q_1q_2q_3q_4$. Symbol q denotes any other flavors in $q_1q_2q_3q_4$.

$q_1q_2q_3q_4$	$I_{q_1q_2q_3q_4}$	J^P	Color-spin wave functions
nnnn	2	3/2-	ψ_1'
		1/2-	ψ_1
nnnn	1	5/2-	$rac{1}{\sqrt{3}} \psi_1^* - rac{1}{\sqrt{3}} \psi_2^* + rac{1}{\sqrt{3}} \psi_3^*$
		3/2-	$\frac{1}{\sqrt{3}}\psi_2' - \frac{1}{\sqrt{3}}\psi_4' + \frac{1}{\sqrt{3}}\psi_3', \ \frac{1}{\sqrt{3}}\psi_{10}' - \frac{1}{\sqrt{3}}\psi_{12}' + \frac{1}{\sqrt{3}}\psi_{11}'$
		$1/2^{-}$	$\frac{1}{\sqrt{3}}\psi_2 - \frac{1}{\sqrt{3}}\psi_4 + \frac{1}{\sqrt{3}}\psi_3, \ \frac{1}{\sqrt{3}}\psi_{10} - \frac{1}{\sqrt{3}}\psi_{12} + \frac{1}{\sqrt{3}}\psi_{11}$
nnnn	0	3/2-	$\frac{1}{\sqrt{2}}{\psi'_6} - \frac{1}{\sqrt{2}}{\psi'_5}$
		$1/2^{-}$	$rac{1}{\sqrt{2}}\psi_6-rac{1}{\sqrt{2}}\psi_5$
nnnq	3/2	5/2-	ψ_3^*
		3/2-	$\psi_1',\psi_3',\psi_{11}'$
		$1/2^{-}$	ψ_1,ψ_3,ψ_{11}
nnnq	1/2	5/2-	$rac{1}{\sqrt{2}} \psi_2^* - rac{1}{\sqrt{2}} \psi_1^*$
		3/2-	$\tfrac{1}{\sqrt{2}}\psi_4' - \tfrac{1}{\sqrt{2}}\psi_2', \ \tfrac{1}{\sqrt{2}}\psi_6' - \tfrac{1}{\sqrt{2}}\psi_5', \ \tfrac{1}{\sqrt{2}}\psi_7' - \tfrac{1}{\sqrt{2}}\psi_8', \ \tfrac{1}{\sqrt{2}}\psi_{12}' - \tfrac{1}{\sqrt{2}}\psi_{10}'$
		$1/2^{-}$	$\frac{1}{\sqrt{2}}\psi_4 - \frac{1}{\sqrt{2}}\psi_2, \ \frac{1}{\sqrt{2}}\psi_6 - \frac{1}{\sqrt{2}}\psi_5, \ \frac{1}{\sqrt{2}}\psi_7 - \frac{1}{\sqrt{2}}\psi_8, \ \frac{1}{\sqrt{2}}\psi_{12} - \frac{1}{\sqrt{2}}\psi_{10}, \ \frac{1}{\sqrt{2}}\psi_{13} - \frac{1}{\sqrt{2}}\psi_{14}$
nnss	1	5/2-	$\sqrt{rac{2}{3}}\psi_2^* - \sqrt{rac{1}{3}}\psi_3^*$
		3/2-	$\psi_1', \ \sqrt{\frac{2}{3}}\psi_4' - \sqrt{\frac{1}{3}}\psi_3', \ \psi_6', \ \sqrt{\frac{2}{3}}\psi_{12}' - \sqrt{\frac{1}{3}}\psi_{11}'$
		1/2-	$\psi_1, \ \sqrt{\frac{2}{3}}\psi_4 - \sqrt{\frac{1}{3}}\psi_3, \ \psi_6, \ \sqrt{\frac{2}{3}}\psi_{12} - \sqrt{\frac{1}{3}}\psi_{11}$
nnss	0	5/2-	ψ_1^*
		3/2-	$\psi_2',\; \sqrt{rac{2}{3}}\psi_9' - \sqrt{rac{1}{3}}\psi_8', \psi_{10}'$
		$1/2^{-}$	$\psi_2, \ \sqrt{\frac{2}{3}}\psi_9 - \sqrt{\frac{1}{3}}\psi_8, \ \psi_{10}, \ \sqrt{\frac{2}{3}}\psi_{15} - \sqrt{\frac{1}{3}}\psi_{14}$
nnsq	1	$5/2^{-}$	ψ_2^*,ψ_3^*
		3/2-	$\psi_1',\psi_3',\psi_4',\psi_6',\psi_7',\psi_{11}',\psi_{12}'$
		$1/2^{-}$	$\psi_1, \psi_3, \psi_4, \psi_6, \psi_7, \psi_{11}, \psi_{12}, \psi_{13}$
nnsq	0	5/2-	ψ_1^*
		$3/2^{-}$	$\psi_2',\psi_5',\psi_8',\psi_9',\psi_{10}'$
		1/2-	$\psi_2,\psi_5,\psi_8,\psi_9,\psi_{10},\psi_{14},\psi_{15}$