1 Single Hop

To start, we define $D(c)$, which calculates the amount of debt received when issuing with an underlying amount of collateral c. This equation just uses a scalar R, which can be easily calculated with the getRequiredComponentIssuanceUnits function of DebtIssuanceModule.

$$D(c) = R \cdot c$$

Next, we define $U(c)$, which calculates the amount of the debt asset required to produce an output amount c on Uniswap V2.

$$U(c) = \frac{r_d \cdot c}{r_c - c} \cdot \frac{1000}{997}$$

Where:
- r_d is the Uniswap debt token reserve amount
- r_c is the Uniswap collateral token reserve amount
- c is the collateral output amount

In order to calculate the amount that must be flash loaned for the issuance operation, we must balance the proceeds of selling the debt token refund with the initial amount flash loaned. This situation can be described with the following equation.

$$D(c_i + c_f) = U(a \cdot c_f)$$

Where:
- c_i is the initially supplied collateral amount by the issuer
- c_f is the flash loaned collateral
- a is the Aave fee scalar. This value currently 1.0009 (a 0.9% fee)

Rewriting the equations without functions we get:

$$R \cdot (c_i + c_f) = \frac{r_d \cdot c_f \cdot a}{r_c - c_f \cdot a} \cdot \frac{1000}{997}$$

Solving for c_f, we get:

$$c_f = \frac{(R \cdot (r_c - c_i \cdot a) - \frac{1000}{997} \cdot r_d \cdot a) \pm \sqrt{(R \cdot (r_c - c_i \cdot a) - \frac{1000}{997} \cdot r_d \cdot a)^2 + 4 \cdot R^2 \cdot a \cdot c_i \cdot r_c}}{2 \cdot R \cdot a}$$

2 Two Hop

For a two hop trade, the logic for calculating the flash loan amount is mostly the same. The main difference is in how $U(c)$ is calculated. This equation must be changes to account for the intermediary asset in the trade path. The new equation is:

$$U(c) = \frac{r_{id} \cdot \frac{r_{ic} \cdot c}{r_{ci} - c}}{r_{di} - \frac{r_{ic} \cdot c}{r_{ci} - c} \cdot \frac{1000}{997}} \cdot \frac{1000}{997}$$

Where:

r_{id} is the intermediary token reserves in the intermediary-debt pool
r_{ic} is the intermediary token reserves in the intermediary-collateral pool
r_{di} is the debt token reserves in the intermediary-debt pool
r_{ci} is the collateral token reserves in the intermediary-collateral pool
c is the collateral output amount

This equation can be simplified to:

$$\frac{r_{id} \cdot r_{ic} \cdot c \cdot \frac{1000}{997}}{r_{di} \cdot (r_{ci} - c) - r_{ic} \cdot c \cdot \frac{1000}{997}}$$

Like last time, we must solve:

$$D(c_i + c_f) = U(a \cdot c_f)$$

Which we can rewrite without the functions as:

$$R \cdot (c_i + c_f) = \frac{r_{id} \cdot r_{ic} \cdot c_f \cdot a \cdot \frac{1000}{997}}{r_{di} \cdot (r_{ci} - c_f \cdot a) - r_{ic} \cdot c_f \cdot a \cdot \frac{1000}{997}}$$

We can then solve for c_f like before. We will split this equation into multiple parts for clarity.

$$A = -R \cdot r_{di} \cdot a - \frac{1000}{997} \cdot R \cdot r_{ic} \cdot a$$

$$B = R \cdot r_{di} \cdot r_{ci} - R \cdot c_i \cdot r_{di} \cdot a - \frac{1000}{997}(R \cdot c_i \cdot r_{ic} \cdot a + r_{id} \cdot r_{ic} \cdot a)$$

$$C = R \cdot c_i \cdot r_{di} \cdot r_{ci}$$

$$c_f = \frac{-B \pm \sqrt{-B^2 - 4AC}}{2A}$$