Strategy in Conquest: A Formal Knowledge Approach to Multi-lateral War Games

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1 Introduction

For our project in LAMAS, we chose to analyze the logic of a very simplified version of the games in the Sid Meier's Civilization (Civ) series. Civ is a game where each player controls a nation that has access to many resources, among which military units (army) and scouts. Nations/players are commonly referred to by the name of a specific historical leader, such as queen Elizabeth, Cleopatra, Julius Caesar, Ghandi, etc. The game has complex and complete army strength mechanics and vision mechanics, as well as politics and many other mechanics outside the scope of this course. This makes Civ a very good candidate to apply the contents of the LAMAS course, with some simplifications and adaptations.

Instead of having a dynamic army strength, with an army having many units, we will consider each nation as having either a "weak", "medium" or "powerful" military force. We are going to be analysing the logic of the equivalent of one turn of combat in Civ, so, the strength of the nations armies don't change for the entire logic analysis. That is done so we do not have changing atom values - which would fall outside the scope of this project and the LAMAS course.

To incorporate multi-agent systems logic into this project, we will focus on scouting. Instead of having a complex vision system, we will simplify it and say that each player has access to only one scout. Each scouting round the player can send the scout to any nation. When a scout goes to a nation, the scout's controlling player gains knowledge of the scouted player's military force. All the scouts that go to the same nation at the same time also see each other - meaning that if players A and B send their scouts to player C, players A and B know player C's military, and players A also knows that player B knows C's military strength, and vice-versa.

2 Formalizing the rules of the game

Our war game is played among n players, where n can be any number from 2 to 6.

The game is structured into 2 scouting rounds followed by 1 combat round.

2.1 Setup

Each player is arbitrarily (and potentially randomly) assigned a military strength - “weak”, “medium” or “powerful”.

2.2 Scouting Round

During a scouting round, all the players, at the same time, choose a target to scout. This means there is no “order” to the scouting of different players and, by extension, no ”first player”. We can refer to the players by their numbers/indexes, but that has no bearing on the order of play. After all players chose a target for their scouts, each player learns their target’s military strength. In addition to that, the scouts can ”see each other” - meaning that each player also knows all other players who scouted the same target.

2.3 Combat Round

For the combat round, we attribute numeric values to the strengths of the players military force. A “weak” military is considered to have 1 strength, a “medium” military is considered to have 2 strength, and a “powerful” military is considered to have 3 strength.

(Note that, from this state of the game onwards, the logic analysis of the system is over, and we will compute some strength values that might potentially end up resulting in a player having less than 1 or more than 3 strength. That is fine, because, since the logic analysis is over, we don’t need any propositional atoms for strengths not in 1, 2, 3. The numbers only define the final standings.)

Players then, simultaneously, either declare war against some other player, or they declare they are defending.

- Defending players temporarily gain +1 strength, for the purposes of calculating the combat results only. This bonus is taken away after combat itself is resolved, but before final score is computed.
- Players that get attacked by a total military power bigger than their own military are defeated in that combat power lose 1 strength. All players that attacked a defeated player are said to have defeated someone.
- Players that defeated someone gain 1 strength.
- If a player gets attacked by a total strength equal to their strength, it is a stalemate and none of the involved players gain or lose any strength.

Note that a player can defeat someone and be defeated at the same time (+1 from defeating someone and -1 for being defeated, resulting in a net +0 strength).

After all of this is computed, the bonus defensive points that were applied to the defending players are removed.

And in the end, the players are ranked by strength, resulting in the final standings. The player with most strength wins first place, the player with the second most strength is second place, and so on, until the player with the least strength, in last place.

3 Logic modelling

The epistemic logic part of this implementation is primarily present in the scouting rounds. As explained earlier, each player has the option to place their scout anywhere on the "map" to gain knowledge. For instance, when in an example game player 'Elizabeth' scouts player 'Gandhi', Elizabeth will learn the strength of Gandhi, which is represented in the following way: $K_E m_G$ where E stands for Elizabeth, G stands for Gandhi and m indicates that Gandhi has medium strength. To correctly model the knowledge gain, our implementation starts with an initial model in which players only know their own strength. The accessibility relations are defined in such a way that for each state a player will have a relation between two states if their strength is the same in both states, i.e. a player cannot access a world that is contradictory with their own strength, since a player always knows his strength.

To better comprehend the logical statements given in the examples in the following sections, each player's name is abbreviated to their first letter, i.e. E for Elizabeth, G for Gandhi, etc. Formally our model consists out of the following sets:

Set of propositional atoms $P = \{w_0, m_0, p_0, w_1, m_1, p_1, ..., w_n, m_n, p_n\}$

Given the set P we can formally define our Kripke model $\mathbb{M}$ where $\mathbb{M} = \langle S, \pi, R_1, ..., R_n \rangle$:

- $S = \{s = \langle x_0, x_1, .., x_n \rangle | x_i \in \{p, m, w\}\}$
- $\pi(\langle x_0, x_1, ..., x_n \rangle)(p_i) = \mathbb{t} \Leftrightarrow x_i = p$
- $\pi(\langle x_0, x_1, ..., x_n \rangle)(m_i) = \mathbb{t} \Leftrightarrow x_i = m$
- $\pi(\langle x_0, x_1, ..., x_n \rangle)(w_i) = \mathbb{t} \Leftrightarrow x_i = w$
- $R_i = \{\langle s, t \rangle \, | \pi(s)(a_i) = \pi(t)(a_i)$ for all a_i in $\{p_i, m_i, w_i\}\}$

After each turn of scouting, the new knowledge is checked for every player in all states for validity. This determines which worlds in our model currently invalidate our model and are therefore excluded when we construct the new model. This is not a complete and comprehensive way of updating Kripke models to fully reflect changes that may happen, and we acknowledge that -

that is a pretty simplified view on the situation, it is important to note that. This will be further demonstrated in the upcoming examples of runs.

Notice how we defined the set of propositional atoms using number subscripts instead of the player's initials. If we have a clear mapping from player name/letter to number, we can use numbers or letters interchangeably in our model. We will often use the subscript as letters to make all the formulas a bit more human-readable, but it is still consistent with the definition with the number subscripts.

3.1 Knowledge gain

In this implementation the scouting actions create knowledge for a player. Each player has their own "Knowledge Base" (KB) that contains all their knowledge about the other players that has been accumulated over all turns. Each player knows their own strength so their KB always contains the formula $K_i a_i$ where $i \in \{0, 1, ..., n\}$ is the player and $a_i \in \{p, m, w\}$ indicates their strength. As the scouting rounds proceed and the players take their actions, their knowledge increases, and this can happen in several ways. In this section we will make use of an example scenario where we have players A, B and C and player A is powerful. The simplest case is that some player scouts another player. Take for example player B scouts player A. Player B learns the strength of player A, so the formula $K_B p_A$ is added.

When two players scout the same location, they also gain knowledge that the other player has scouted that location. If for instance player A scouts their own country and player B also scouts their country then the following formula is added; $K_A K_B p_A$. While this statement can in fact be extended to be of the form $K_A K_B K_A K_B K_A ...$ we will be limiting all formulas to the one stated earlier. This is based on the finding of de Weerd et al., 2018 that states that adults tend to use a maximum of second-order higher knowledge, therefore we also stop at the second-order knowledge.

When every player scouts the same location, e.g. player A, B and C all scout A, then we establish that the strength of player A is common knowledge as we have that $K_A K_B K_C p_A \wedge K_A K_C K_B p_A$..etc. Therefore, we have that every player knows that everyone knows that p_A since all scouts were present at the same location in the same turn. Therefore the formula $C p_A$ is added to every player's KB in this example. If for instance only players B and C scout player A then we do not establish common knowledge because while everyone knows that p_A, not everyone knows that everyone knows that p_A.

3.2 Updating the model

After each scouting round the current KB of each player is checked against the most recent model. That means that for each formula in a player's KB, all states are checked for validity. We then use the knowledge of which states are valid to construct the new model for the next round. By doing so we create a new model in which we have made sure that each formula in every player's KB is valid. To demonstrate how this is applied in the implementation we take an example with two players A and B where player A is 'powerful'. In this example in round 1 player A scouts player B and therefore gains $K_B p_A$, however player A has scouted himself and therefore gains $K_A K_B p_A$.

After this turn we have a look at the newly gained knowledge and establish a new model in which these formulas are valid. This is done by breaking down the formula into a series of equivalences. The initial model of this example is shown in figure 1(a).

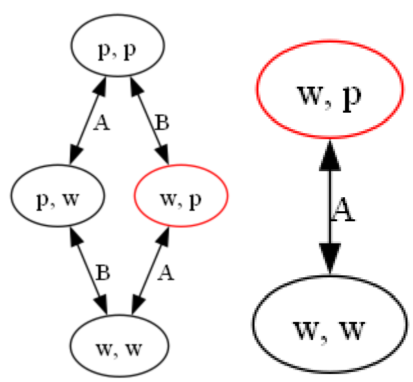
Now we can take the formula $K_A K_B w_A$ for any state s in our model. In this case we will take the state (w, p) to check whether $(\mathbb{M}, (w, p)) \models K_A K_B w_A$:

$(\mathbb{M}, (w, p) \models K_A K_B w_A \Leftrightarrow$ Truth definition of 'K_A'

For all t with $((w, p), t) \in R_A, (\mathbb{M}, t) \models K_B w_A \Leftrightarrow$ Truth definition of 'K_B'

For all t with $((w, p), t) \in R_A$ and for all u with $(t, u) \in R_B, (\mathbb{M}, u) \models w_A$

In the initial model displayed in 1(a) this statement is false as we can take (w, w) for t to have $((w, p), (w, w)) \in R_A$ and we can then take (p, w) for u to have $((w, w), (p, w)) \in R_B$. We then have that $(\mathbb{M}, (p, w) \not\models w_A$. To then make a new model we omit the world (p, w) in our new model such that the formula $K_A K_B w_A$ is valid. Additionally, we have that we can take for u to be (p, p) since $((w, p), (p, p)) \in R_B$. We then have $(\mathbb{M}, (p, p)) \not\models w_A$ and we thus also have to omit world (p, p) in the creation of the new model. This give us the new graph displayed in figure 1(b).



(a) Initial model (b) After round 1

Figure 1: Example scenario of two players A and B where we have p_A and w_B. Reflexive relations are present but not displayed in this graph

It is important to note that this is a simplification that does not account for the addition of states in the model. We are aware of this limitation, and that this does not model the full knowledge all players should have access to. By not adding new states, we are effectively not modelling the possibility of players not knowing something. So, if $K_A \neg K_B p_A$ should be valid in a model at any time, this is not taken into account.

3.3 Simplified model

To make the analysis of the Kripke models have a reasonable size to work with, we will be analyzing the case with only 2 possible strengths - "weak" and "powerful". That implicates that as opposed to our model proposed above, the model used in this example is basically the same except for the exclusion of all m_i from the set P and its implications. Formalizing that, the model we will be analysing is defined as such:

- $P = \{w_0, p_0, w_1, p_1, ..., w_n, p_n\}$
- $\mathbb{M} = \langle S, \pi, R_1, ..., R_n \rangle$ where:
- $S = \{s = \langle x_0, x_1, .., x_n \rangle | x_i \in \{p, w\}\}$
- $\pi(\langle x_0, x_1, ..., x_n \rangle)(p_i) = \mathbb{t} \Leftrightarrow x_i = p$
- $\pi(\langle x_0, x_1, ..., x_n \rangle)(w_i) = \mathbb{t} \Leftrightarrow x_i = w$
- $R_i = \{\langle s, t \rangle \ | \pi(s)(a_i) = \pi(t)(a_i)$ for all a_i in $\{p_i, w_i\}\}$

3.4 Higher-order knowledge

To demonstrate how this model is applied and uses higher order knowledge in our implementation, we will show an example of a game using three players. For simplicity we will use the players Alexander (A), Boudica (B) and Cleopatra (C), where we have $P = w_A, p_A, w_B, p_B, w_C, p_C$. Player A is weak, and players B and C are powerful. So our real world is the state (w, p, p)

Before any scouting takes place, we have the model described in Figure 2. You can notice, for player A, that all the relationship arrows that come out of any state lead directly only to states where the strength of A doesn't change, following the definition of our relationship functions.

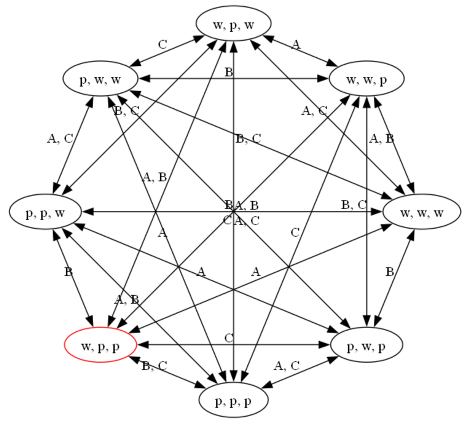


Figure 2: Initial model for 3 players and 2 values of strength

Note that all the models in this document are reflexive and symmetrical. The reflexive arrows are not drawn for clarity's sake, but for all the Kripke models shown in this document, assume they are reflexive.

For scouting round 1, players A and B scout player A, and player C scouts player B. We have that $K_A w_A, K_A K_B w_A, K_B w_a, K_B K_A w_A$. For completeness sake, technically, since this is the first round and players A and B know that player C did not scout player A, we should also have $K_A \left(\neg K_C w_A \wedge \neg K_C p_A\right)$, but as part of the simplifications that have to be made to a model so that it doesn't explode, we chose to not consider that knowledge. After this round of scouting, we have our model as described in Figure 3

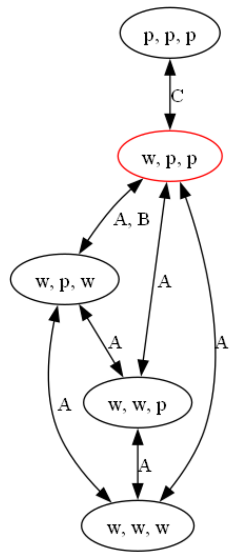


Figure 3: Game 1, after first scouting round

Let us analyze Figure 3. We will verify that $K_A K_B w_A$. Since proving that for all states would be quite a lengthy process, we will verify that for (w, p, p). This means that:

$(\mathbb{M}, (w, p, p)) \models K_A K_B w_A \Leftrightarrow$ Truth definition of 'K_A'
For all t with $((w, p, p), t) \in R_A, (\mathbb{M}, t) \models K_B w_A \Leftrightarrow$ Truth definition of 'K_B'
For all t with $((w, p, p), t) \in R_A$ and for all u with $(t, u) \in R_B, (\mathbb{M}, u) \models w_A$

We can observe that the states reachable from (w,p,p) for A are (w,p,p), (w,p,w), (w,w,p) and (w,w,w). We can verify that, for each of those worlds, B can only reach worlds where w_A is valid. Therefore, $K_A K_B w_A$ holds for state (w,p,p)

Now that the first round of scouting is done, the second round of scouting happens. We say that A scouts B, B scouts C, and C scouts A. After this round of scouting, we have $K_A p_B, K_B p_C$ and $K_C w_A$. The updated model after this round of scouting can be found in Figure 4

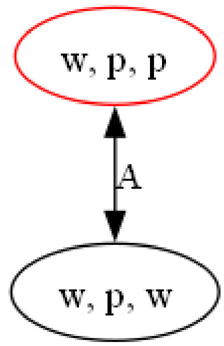


Figure 4: Game 1, after second scouting round

This is a pretty simple model, and we can see that $K_A p_B, K_B p_C$ and $K_C w_A$. Due to those being all very simple, proving and verifying those 3 formulas is left as an exercise to the reader.

3.5 Common knowledge

When all players scout the same location in one round, there is common knowledge of the player's strength at the scouted location for all players. To show this in an example, we modify the scenario from the previous section to have one round where everyone scouts the same player. The initial model is the same as in the previous example (see Figure 2), with players Alexander (A), Boudica (B) and Cleopatra (C), where the true world is $\langle w, p, p \rangle$. In the first scouting round, every player scouts Alexander. After the scouting round, Cw_A is valid in $\mathbb{M}$. We follow the truth definition of common knowledge as stated on page 46 of Meyer and Hoek, 1995:

$(\mathbb{M}, s) \models Cw_A \Leftrightarrow$ Truth definition of C

$(\mathbb{M}, t) \models w_A$ for all t with $s \twoheadrightarrow t$

As the model is reflexive, it is clear that any world s where $(\mathbb{M}, s) \not\models w_A$ is automatically no longer possible. Removing all states s with $(\mathbb{M}, s) \models p_A$ we get a model that satisfies validity of Cw_A, as shown in Figure 5.

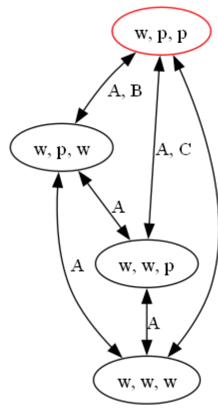


Figure 5: Model after first scouting round where every player scouted Alexander

If we add an additional scouting round where players A and C scout B, and B scouts C, we end up with a similar situation to the example from the previous section, i.e., Alexander has scouted himself and B, B has scouted A and C, and C has scouted A and B. It is clear that B and C both know every other player's strength, but A does not know C's strength. The resulting model is shown in Figure 6.

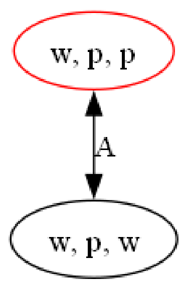


Figure 6: Game 2, after second scouting round

4 Conclusion and Discussion

This implementation shows the basic case of how higher-order knowledge and common knowledge would play into the scenario of a war game. Throughout this project, we formally analyzed many situations in which the players would have to process some logic to find out information.

The implemented model has many limitations, the greatest one being its inability to add new states to the models and, therefore, its inability to make agents aware of any "negative knowledge" (i.e., someone not knowing something). We propose for future works on the topic that this limitation should be explored and either removed or made less impactful in some way.

Other than that, the knowledge that is gained by the agents is not processed at all, and no decisions come out of this as it is. It would be interesting if a strategy could be implemented based on the knowledge of an agent.

References

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