In [2]: !pip install pca
!pip install fredapi

```
Collecting pca
  Downloading pca-2.0.9-py3-none-any.whl.metadata (10 kB)
Collecting datazets (from pca)
  Downloading datazets-1.1.0-py3-none-any.whl.metadata (11 kB)
Requirement already satisfied: statsmodels in /usr/local/lib/python3.11/dist-package
s (from pca) (0.14.4)
Requirement already satisfied: matplotlib in /usr/local/lib/python3.11/dist-packages
(from pca) (3.10.0)
Requirement already satisfied: numpy in /usr/local/lib/python3.11/dist-packages (fro
m pca) (2.0.2)
Requirement already satisfied: scikit-learn in /usr/local/lib/python3.11/dist-packag
es (from pca) (1.6.1)
Requirement already satisfied: scipy in /usr/local/lib/python3.11/dist-packages (fro
m pca) (1.14.1)
Collecting colourmap>=1.1.19 (from pca)
  Downloading colourmap-1.1.20-py3-none-any.whl.metadata (5.5 kB)
Requirement already satisfied: pandas in /usr/local/lib/python3.11/dist-packages (fr
om pca) (2.2.2)
Collecting scatterd>=1.3.7 (from pca)
  Downloading scatterd-1.3.7-py3-none-any.whl.metadata (5.1 kB)
Collecting adjusttext (from pca)
  Downloading adjustText-1.3.0-py3-none-any.whl.metadata (3.1 kB)
Requirement already satisfied: seaborn in /usr/local/lib/python3.11/dist-packages (f
rom scatterd>=1.3.7->pca) (0.13.2)
Requirement already satisfied: requests in /usr/local/lib/python3.11/dist-packages
(from datazets->pca) (2.32.3)
Requirement already satisfied: contourpy>=1.0.1 in /usr/local/lib/python3.11/dist-pa
ckages (from matplotlib->pca) (1.3.1)
Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.11/dist-packag
es (from matplotlib->pca) (0.12.1)
Requirement already satisfied: fonttools>=4.22.0 in /usr/local/lib/python3.11/dist-p
ackages (from matplotlib->pca) (4.57.0)
Requirement already satisfied: kiwisolver>=1.3.1 in /usr/local/lib/python3.11/dist-p
ackages (from matplotlib->pca) (1.4.8)
Requirement already satisfied: packaging>=20.0 in /usr/local/lib/python3.11/dist-pac
kages (from matplotlib->pca) (24.2)
Requirement already satisfied: pillow>=8 in /usr/local/lib/python3.11/dist-packages
(from matplotlib->pca) (11.1.0)
Requirement already satisfied: pyparsing>=2.3.1 in /usr/local/lib/python3.11/dist-pa
ckages (from matplotlib->pca) (3.2.3)
Requirement already satisfied: python-dateutil>=2.7 in /usr/local/lib/python3.11/dis
t-packages (from matplotlib->pca) (2.8.2)
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.11/dist-packag
es (from pandas->pca) (2025.2)
Requirement already satisfied: tzdata>=2022.7 in /usr/local/lib/python3.11/dist-pack
ages (from pandas->pca) (2025.2)
Requirement already satisfied: joblib>=1.2.0 in /usr/local/lib/python3.11/dist-packa
ges (from scikit-learn->pca) (1.4.2)
Requirement already satisfied: threadpoolctl>=3.1.0 in /usr/local/lib/python3.11/dis
t-packages (from scikit-learn->pca) (3.6.0)
Requirement already satisfied: patsy>=0.5.6 in /usr/local/lib/python3.11/dist-packag
es (from statsmodels->pca) (1.0.1)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.11/dist-packages
(from python-dateutil>=2.7->matplotlib->pca) (1.17.0)
Requirement already satisfied: charset-normalizer<4,>=2 in /usr/local/lib/python3.1
1/dist-packages (from requests->datazets->pca) (3.4.1)
```

```
Requirement already satisfied: idna<4,>=2.5 in /usr/local/lib/python3.11/dist-packag
es (from requests->datazets->pca) (3.10)
Requirement already satisfied: urllib3<3,>=1.21.1 in /usr/local/lib/python3.11/dist-
packages (from requests->datazets->pca) (2.3.0)
Requirement already satisfied: certifi>=2017.4.17 in /usr/local/lib/python3.11/dist-
packages (from requests->datazets->pca) (2025.1.31)
Downloading pca-2.0.9-py3-none-any.whl (36 kB)
Downloading colourmap-1.1.20-py3-none-any.whl (10 kB)
Downloading scatterd-1.3.7-py3-none-any.whl (12 kB)
Downloading adjustText-1.3.0-py3-none-any.whl (13 kB)
Downloading datazets-1.1.0-py3-none-any.whl (14 kB)
Installing collected packages: datazets, colourmap, adjusttext, scatterd, pca
Successfully installed adjusttext-1.3.0 colourmap-1.1.20 datazets-1.1.0 pca-2.0.9 sc
atterd-1.3.7
Collecting fredapi
 Downloading fredapi-0.5.2-py3-none-any.whl.metadata (5.0 kB)
Requirement already satisfied: pandas in /usr/local/lib/python3.11/dist-packages (fr
om fredapi) (2.2.2)
Requirement already satisfied: numpy>=1.23.2 in /usr/local/lib/python3.11/dist-packa
ges (from pandas->fredapi) (2.0.2)
Requirement already satisfied: python-dateutil>=2.8.2 in /usr/local/lib/python3.11/d
ist-packages (from pandas->fredapi) (2.8.2)
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.11/dist-packag
es (from pandas->fredapi) (2025.2)
Requirement already satisfied: tzdata>=2022.7 in /usr/local/lib/python3.11/dist-pack
ages (from pandas->fredapi) (2025.2)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.11/dist-packages
(from python-dateutil>=2.8.2->pandas->fredapi) (1.17.0)
Downloading fredapi-0.5.2-py3-none-any.whl (11 kB)
Installing collected packages: fredapi
Successfully installed fredapi-0.5.2
```

#### In [19]: import matplotlib.pyplot as plt import numpy as np import pandas as pd import seaborn as sns import time import yfinance as yf from sklearn import svm from sklearn.cluster import AgglomerativeClustering from sklearn.datasets import load breast cancer from sklearn.decomposition import PCA from sklearn.discriminant\_analysis import LinearDiscriminantAnalysis from sklearn.ensemble import GradientBoostingClassifier, RandomForestClassifier, St from sklearn.inspection import DecisionBoundaryDisplay from sklearn.linear\_model import Lasso, LassoCV, LogisticRegression, Ridge, LinearR from sklearn.metrics import accuracy\_score, auc, classification\_report, confusion\_m from sklearn.model\_selection import cross\_val\_score, GridSearchCV, KFold, Randomize from sklearn.neural\_network import MLPClassifier, MLPRegressor from sklearn.pipeline import Pipeline from sklearn.preprocessing import LabelEncoder, PolynomialFeatures, StandardScaler, from sklearn.tree import DecisionTreeClassifier, DecisionTreeRegressor, plot\_tree from scipy.cluster.hierarchy import dendrogram, fcluster, linkage

# **Issue 1: Optimizing Hyperparameters**

# **How to Optimize Hyperparameters**

Hyperparameters vary with different models.

- The first steps to optimizing hyperparameters is to identify the model in question: Is it a Supervised machine learning models (Classifiers, Regressors), Unsupervised learning model or Neural Network.
- Secondly, Identify all of the relevant hyperparameters associated with the model in question.
- Specify a search range for the parameters for tuning
- For a supervised model, apply optimization algorithm search (e.g GridSearch, K-fold cross validation) to determine best parameters for the models. While for an unsupervised model implement a manual search function on the search range.
- Apply the best hyperparameters in the modelling operations.
- Evaluate the model perfromance by using metrics such as Mean-Squared Error, R-squared value, Classification report depending on kind of model measures its accuracy

For this python code, we will show how to optimize hyparameters for unsupervised learning models, supervised learning models, Combination of both learning models (in an ML pipeline) and Neural Networks.

# **Unsupervised Learning Models**

## **Hierarchical Clustering**

There are 3 main hyperparamters that can be tuned in hierarchical clustering model(say, an Agglomerative Clustering problem), these parameters are namely:

- n clusters: number of clusters
- linkage method: which measures how to compute distance between clusters. Linkage methods could be ward, complete, average, single
- distance metric: euclidean, manhattan etc.

```
In [4]: # Defining the date range for daily data from 2018 to 2023
start_date = "2018-01-01"
end_date = "2023-12-31"
```

```
# Downloading historical data for SPY (S&P 500 ETF) and QQQ (Nasdaq-100 ETF)
spy_data = yf.download("SPY", start=start_date, end=end_date)
qqq_data = yf.download("QQQ", start=start_date, end=end_date)
# Debug: Print the column names to verify available columns
print("SPY Data Columns:", spy_data.columns)
print("QQQ Data Columns:", qqq_data.columns)
# Using the 'Adj Close' column if available; otherwise, fallback to 'Close'
spy_col = "Adj Close" if "Adj Close" in spy_data.columns else "Close"
qqq_col = "Adj Close" if "Adj Close" in qqq_data.columns else "Close"
# Creating DataFrames for the selected columns
spy_df = pd.DataFrame(spy_data[spy_col], columns=["SPY"])
qqq_df = pd.DataFrame(qqq_data[qqq_col], columns=["QQQ"])
# Merge the DataFrames on the date index and drop rows with missing values
indices_df = pd.merge(spy_df, qqq_df, left_index=True, right_index=True, how="inner
if indices_df.empty:
   raise ValueError("Merged DataFrame is empty. Check the date range and ticker sy
# ------
# Data Preprocessing
# -----
# Standardizing the data so both SPY and QQQ are on the same scale
scaler = StandardScaler()
scaled_data = scaler.fit_transform(indices_df)
X = pd.DataFrame(scaled_data, columns=indices_df.columns, index=indices_df.index)
# Manual Grid Search + Sillhoutte Score Analysis to determine the optimal linkage m
linkage_options = ['ward', 'complete', 'average']
n_clusters_range = range(2, 10)
best score = -1
best_params = None
for linkage method in linkage options:
   for k in n_clusters_range:
        if linkage_method == 'ward':
           metric = 'euclidean'
       else:
           metric = 'manhattan'
       model = AgglomerativeClustering(n clusters=k, linkage=linkage method, metri
       labels = model.fit_predict(X)
       score = silhouette_score(X, labels)
       if score > best_score:
           best_score = score
           best_params = (k, linkage_method, metric)
print("Best params:", best_params)
print("Best (Silhouette), Score", best_score)
```

YF.download() has changed argument auto\_adjust default to True

```
[******** 100%*********** 1 of 1 completed
[******** 100%*********** 1 of 1 completed
SPY Data Columns: MultiIndex([( 'Close', 'SPY'),
           ( 'High', 'SPY'),
              'Low', 'SPY'),
           ( 'Open', 'SPY'),
           ('Volume', 'SPY')],
          names=['Price', 'Ticker'])
QQQ Data Columns: MultiIndex([( 'Close', 'QQQ'),
             'High', 'QQQ'),
              'Low', 'QQQ'),
           ( 'Open', 'QQQ'),
           ('Volume', 'QQQ')],
         names=['Price', 'Ticker'])
Best params: (2, 'complete', 'manhattan')
Best (Silhouette), Score 0.7027569580923678
```

Silhouette score of 0.70 and above depicts an excellent hierarchical clustering

## **Principal Component Analysis**

There is only one main hyperparameter that can be tuned in Principal Coomponent Analysis problem which is the **number of components (n\_components)** 

There are 3 commons ways for optimizing number components:

- Specify Standard Explained variance: This helps tailor the model to only work with optimal number of components required for dimensionality reduction. A standard explained variance for a principal component should be greater or equal to 95%.
- Scree plot to visualise number of components with explained variance greater than a threshold value (preferaly 95%)
- Run a GridSearch or Randomized Search or a search specific to the model in question to determine the optimal number of components. Used when PCA is integrated in a pipeline with supervised learning tasks. For this case, we'll make use of LassoCV (as this was one of the models run in GWP1)

```
In [5]: from pca import pca
# Load in data from FRED Economics, drop null values
import pandas_datareader.data as web
import datetime
from datetime import datetime
import scipy as sp

# Plotting
import seaborn as sns
from matplotlib import pyplot
from sklearn.preprocessing import StandardScaler

from sklearn.decomposition import PCA
from fredapi import Fred
```

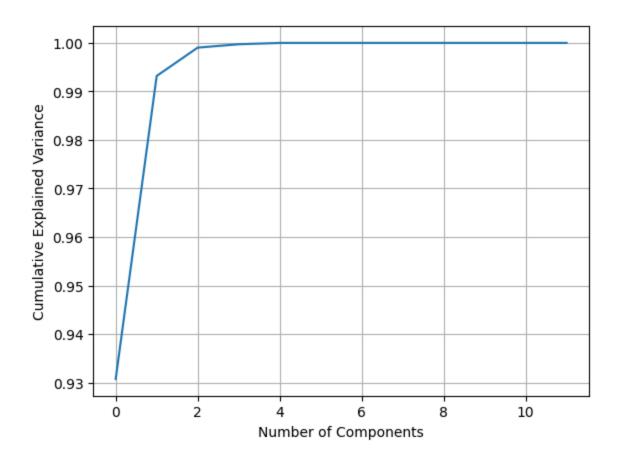
```
fred = Fred(api_key="4a686e78f0f4f1b2a194e90961e4c4f9")
```

```
In [6]: #Load 20-year spot rate data
        start = datetime(1992, 12, 31)
        end = datetime(2022, 12, 31)
        data = [
            "HQMCB1YR",
            "HQMCB2YR",
            "HOMCB3YR",
             "HOMCB5YR",
            "HQMCB7YR",
             "HQMCB10YR",
             "HQMCB15YR",
             "HQMCB20YR",
             "HQMCB25YR",
            "HOMCB30YR",
             "HQMCB40YR",
             "HQMCB50YR",
        data = web.DataReader(data, "fred", start, end).dropna(how="all").ffill()
        data.rename(
            columns={
                 "HQMCB1YR": "1y",
                 "HQMCB2YR": "2y",
                 "HQMCB3YR": "3y",
                 "HQMCB5YR": "5y",
                 "HQMCB7YR": "7y",
                 "HQMCB10YR": "10y",
                 "HQMCB15YR": "15y",
                 "HQMCB20YR": "20y",
                 "HQMCB25YR": "25y",
                 "HQMCB30YR": "30y",
                 "HQMCB40YR": "40y",
                 "HQMCB50YR": "50y",
            inplace=True,
        df = data.copy()
        # Rescale data
        scaler = StandardScaler().fit(df)
        rescaleddf = pd.DataFrame(scaler.fit_transform(df), columns=df.columns, index=df.in
        # summarize transformed data
        df.dropna(how="any", inplace=True)
        rescaleddf.dropna(how="any", inplace=True)
        pca = PCA(n_components=0.95) # keep 95% of variance
        X_pca = pca.fit_transform(rescaleddf)
        # 1. Specifying Standard Explained Variance (of 95%)
        print(f" Optimal number of components selected: {pca.n_components_}")
        # 2. Scree plot
```

```
pca = PCA().fit(rescaleddf)
plt.plot(np.cumsum(pca.explained_variance_ratio_))
plt.xlabel('Number of Components')
plt.ylabel('Cumulative Explained Variance')
plt.suptitle('Scree Plot')
plt.grid(True)
plt.show()
```

Optimal number of components selected: 2

#### Scree Plot



Scree plot also shows optimal number of components.

```
In [7]: # 3. Pipeline Implementation (PCA + LassoCV)

# Download data
tickers = ['SPY', '^VIX', '^TNX', 'GLD', 'USO', 'TIP', 'XLY', '^IXIC', 'EURUSD=X',
data = yf.download(tickers, start='2010-01-01', end='2025-01-01')['Close']

# Compute returns (using 1-day returns)
returns = data.pct_change().shift(-1).dropna()

# Feature matrix X (including the new macroeconomic variables)
X = returns[['^VIX', '^TNX', 'GLD', 'USO', 'TIP', 'XLY', '^IXIC', 'EURUSD=X', 'VNQ'
# Target variable Y (future returns of SPY)
y = returns['SPY']
# Drop NaN values
X = X.dropna()
y = y.loc[X.index]
```

```
# Train-test split (ensure data is in chronological order)
 X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, shuffle=Fa
 # Pipeline: StandardScaler → PCA → LassoCV
 pipeline = Pipeline([
     ('scaler', StandardScaler()),
     ('pca', PCA(n_components=0.95)), # Keep 95% variance
     ('lasso', LassoCV(alphas=np.logspace(-6, 6, 13), cv=5, random_state=42)) #cv is
 ])
 # Fit pipeline
 pipeline.fit(X_train, y_train)
 # Predict
 y_pred = pipeline.predict(X_test)
 # Evaluate model accuracy
 rmse = np.sqrt(mean_squared_error(y_test, y_pred))
 r2 = r2_score(y_test, y_pred)
 print(f"R^2: {r2:.3f}")
 print(f"Test RMSE: {rmse:.3f}")
 print(f"Best Alpha (λ): {pipeline.named_steps['lasso'].alpha_}")
 print(f"Number of PCA components used: {pipeline.named_steps['pca'].n_components_}'
[********* 13 of 13 completed
<ipython-input-7-f76a9c97d7fd>:8: FutureWarning: The default fill_method='pad' in Da
taFrame.pct_change is deprecated and will be removed in a future version. Either fil
l in any non-leading NA values prior to calling pct_change or specify 'fill_method=N
one' to not fill NA values.
 returns = data.pct_change().shift(-1).dropna()
R^2: 0.930
Test RMSE: 0.003
Best Alpha (λ): 1e-06
Number of PCA components used: 9
```

Compared to results obtained in GWP1 where we ran Lasso cross validation on the same dataset, we obtained almost similar results for the Root mean Sqaure Error (RMSE), R-squared value and Optimal Alpha when we integrated PCA in the same pipeline with LassoCv. However, runnning a *cross validation* on a supervised model (LASSO Regression) is enough to optimize hyperparameters of supervised learning model.

# **Supervised Models**

## **Linear Discriminant Analysis**

```
In [8]: # Define the tickers and FRED series
tickers = ['^GSPC', '^NDX', '^VIX', '^TNX', 'GC=F', 'BTC-USD']
fred_series = {'FFR': 'FEDFUNDS', 'Baa_Yield': 'BAA', 'Aaa_Yield': 'AAA'}
# Define the date range
```

```
start_date = '2010-01-01'
end_date = '2025-01-01'
# Function to fetch data from Yahoo Finance
def fetch_yahoo_data(tickers, start, end):
   data = yf.download(tickers, start=start, end=end)['Close']
   return data
# Function to fetch data from FRED
def fetch_fred_data(series_dict, start, end):
   data = pd.DataFrame()
   for name, series_id in series_dict.items():
        series_data = fred.get_series(series_id, start, end)
        data[name] = series_data
   return data
# Fetch macroeconomic data
macro_data = fetch_fred_data(fred_series, start_date, end_date)
# Fetch market-based data
market_data = fetch_yahoo_data(tickers, start_date, end_date)
# Data Prepreocessing-----
# Calculate daily returns for market data
returns = market_data.pct_change().dropna()
# Calculate credit spread (Baa - Aaa Yield)
macro_data['Credit_Spread'] = macro_data['Baa_Yield'] - macro_data['Aaa_Yield']
# Merge datasets
data = returns.join(macro_data[['FFR', 'Credit_Spread']], how='inner')
# Drop rows with NaN values
data.dropna(inplace=True)
data
# Define the target variable
data['Target'] = (data['^GSPC'] > 0).astype(int)
# Define feature set and labels
X = data.drop(columns=['Target', '^GSPC'])
y = data['Target']
# Standardize the features
scaler = StandardScaler()
X_scaled = scaler.fit_transform(X)
# Model Implementation-----
# Split the data into training and testing sets
split_date = '2021-01-01'
X_train = X_scaled[data.index < split_date]</pre>
X_test = X_scaled[data.index >= split_date]
y_train = y[data.index < split_date]</pre>
```

```
y_test = y[data.index >= split_date]
        # Run a GridSearch validation
        param_grid = {
            'solver': ['lsqr', 'eigen'],
            'shrinkage': [None, 'auto', 0.1, 0.5, 0.9] # will be ignored if solver='svd'
        # fit the grid search
        lda = LinearDiscriminantAnalysis()
        grid = GridSearchCV(lda, param_grid, cv=5, scoring='accuracy', error_score='raise')
        grid.fit(X_train, y_train)
        # Print best parameters, accururacy of grid algorithm & fit the lda model using bes
        print("Best parameters:", grid.best_params_)
        print("Best accuracy:", grid.best_score_.round(4))
        lda = LinearDiscriminantAnalysis(solver='eigen', shrinkage='auto')
        lda.fit(X_train, y_train)
       [******** 6 of 6 completed
       <ipython-input-8-7a31ec88726b>:31: FutureWarning: The default fill_method='pad' in D
      ataFrame.pct_change is deprecated and will be removed in a future version. Either fi
      11 in any non-leading NA values prior to calling pct_change or specify 'fill_method=
      None' to not fill NA values.
        returns = market data.pct change().dropna()
      Best parameters: {'shrinkage': 'auto', 'solver': 'lsqr'}
      Best accuracy: 0.88
Out[8]:
                        LinearDiscriminantAnalysis
        LinearDiscriminantAnalysis(shrinkage='auto', solver='eigen')
        y_pred = lda.predict(X_test)
```

```
In [9]: # Make predictions
y_pred = lda.predict(X_test)
y_pred_proba = lda.predict_proba(X_test)[:, 1]

# Evaluate the model
conf_matrix = confusion_matrix(y_test, y_pred)
class_report = classification_report(y_test, y_pred)
accuracy = accuracy_score(y_test, y_pred)

# Print Confusion matrix and Class Report
print("Confusion Matrix:\n", conf_matrix)
print("\nClassification Report:\n", class_report)
print("\nAccuracy Score:", accuracy)
```

```
Confusion Matrix:
[[28 0]
 [ 3 17]]
Classification Report:
             precision recall f1-score support
                0.90 1.00
                                 0.95
                                            28
         1
               1.00
                         0.85
                                 0.92
                                            20
                                 0.94
                                            48
   accuracy
                                 0.93
  macro avg
              0.95
                       0.93
                                            48
                0.94
weighted avg
                       0.94
                                 0.94
```

Accuracy Score: 0.9375

## **Support Vector Classifiers**

There are 4 main hyperparameters that can be tuned in support vector machines modelling:

- C: Regularization parameter
- kernel type: 'linear" "poly", "rbf", "sigmoid" etc
- gamma: kernel coefficients for non-linear kernel functions
- epsilon: measures margin tolerance of the decision boundary and support vectors
- degree: controls the "poly" kernel function degree.

```
In [10]: from google.colab import files
         uploaded = files.upload()
         # Load the dataset
         data_df = pd.read_csv("loan_predictor.csv")
         # Encode categorical variables
         label_encoders = {}
         for col in data_df.select_dtypes(include=['object']).columns:
             label encoders[col] = LabelEncoder()
             data_df[col] = label_encoders[col].fit_transform(data_df[col])
         # Data Preprocessing
         X = data_df.drop(columns=['Loan_ID','Loan_Status', "Gender", "Married", "Dependents
         X.dropna()
         y = data_df['Loan_Status']
         y.dropna()
         # from sklearn.cross_validation import train_test_split (training_size = 80, test_s
         X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_sta
         # 1. GridSearch with Cross validation
         # defining parameter (search) range
         param_grid = {
```

```
"C": [0.01, 0.1, 1, 10, 100],
    # "gamma": [1, 0.1, 0.01, 0.001, 0.0001],
    "kernel": ["rbf", "linear", "poly", "sigmoid"],
}
grid = GridSearchCV(svm.SVC(), param_grid, refit=True, verbose=3, cv=5)
# fitting the model for grid search
grid.fit(X_train, y_train)
# print best parameter after tuning
print(grid.best_params_)
# print how our model looks after hyper-parameter tuning
print(grid.best_estimator_)
```

Choose Files No file chosen Upload widget is only available when the cell has been executed in the current browser session. Please rerun this cell to enable.

```
Saving loan_predictor.csv to loan_predictor.csv
Fitting 5 folds for each of 20 candidates, totalling 100 fits
[CV 1/5] END .................C=0.01, kernel=rbf;, score=0.697 total time=
                                                                     0.1s
[CV 2/5] END .................C=0.01, kernel=rbf;, score=0.694 total time=
                                                                     0.0s
[CV 3/5] END ......C=0.01, kernel=rbf;, score=0.694 total time=
                                                                     0.1s
[CV 4/5] END .................C=0.01, kernel=rbf;, score=0.694 total time=
                                                                     0.0s
[CV 5/5] END .................C=0.01, kernel=rbf;, score=0.704 total time=
                                                                     0.1s
[CV 1/5] END ...........C=0.01, kernel=linear;, score=0.697 total time=
                                                                    36.0s
[CV 2/5] END ...........C=0.01, kernel=linear;, score=0.694 total time=
[CV 3/5] END ......C=0.01, kernel=linear;, score=0.684 total time= 1.1min
[CV 4/5] END ......C=0.01, kernel=linear;, score=0.704 total time=
[CV 5/5] END ..........C=0.01, kernel=linear;, score=0.704 total time=
[CV 1/5] END ............C=0.01, kernel=poly;, score=0.707 total time=
                                                                     0.0s
[CV 2/5] END .............C=0.01, kernel=poly;, score=0.694 total time=
                                                                     0.0s
[CV 3/5] END .............C=0.01, kernel=poly;, score=0.684 total time=
                                                                     0.0s
[CV 4/5] END ................C=0.01, kernel=poly;, score=0.704 total time=
                                                                     0.0s
[CV 5/5] END .................C=0.01, kernel=poly;, score=0.704 total time=
                                                                     0.0s
[CV 1/5] END ...........C=0.01, kernel=sigmoid;, score=0.697 total time=
                                                                     0.0s
[CV 2/5] END ...........C=0.01, kernel=sigmoid;, score=0.694 total time=
                                                                     0.0s
[CV 3/5] END ..........C=0.01, kernel=sigmoid;, score=0.694 total time=
                                                                     0.0s
[CV 4/5] END ...........C=0.01, kernel=sigmoid;, score=0.694 total time=
                                                                     0.0s
[CV 5/5] END ...........C=0.01, kernel=sigmoid;, score=0.704 total time=
                                                                     0.0s
[CV 1/5] END ......C=0.1, kernel=rbf;, score=0.697 total time=
                                                                     0.0s
0.0s
0.0s
0.0s
[CV 5/5] END ..................C=0.1, kernel=rbf;, score=0.704 total time=
                                                                     0.0s
[CV 1/5] END ............C=0.1, kernel=linear;, score=0.697 total time=
                                                                    20.8s
[CV 2/5] END ............C=0.1, kernel=linear;, score=0.704 total time= 1.8min
[CV 3/5] END ...........C=0.1, kernel=linear;, score=0.663 total time= 1.1min
[CV 4/5] END ............C=0.1, kernel=linear;, score=0.704 total time=
[CV 5/5] END ............C=0.1, kernel=linear;, score=0.704 total time=
[CV 1/5] END ..................C=0.1, kernel=poly;, score=0.707 total time=
[CV 2/5] END ..................C=0.1, kernel=poly;, score=0.694 total time=
                                                                     0.0s
[CV 3/5] END ................C=0.1, kernel=poly;, score=0.684 total time=
                                                                     0.0s
[CV 4/5] END .................C=0.1, kernel=poly;, score=0.704 total time=
                                                                     0.0s
[CV 5/5] END .................C=0.1, kernel=poly;, score=0.704 total time=
                                                                     0.0s
[CV 1/5] END ............C=0.1, kernel=sigmoid;, score=0.697 total time=
                                                                     0.0s
[CV 2/5] END ..........C=0.1, kernel=sigmoid;, score=0.694 total time=
                                                                     0.0s
[CV 3/5] END ..........C=0.1, kernel=sigmoid;, score=0.694 total time=
                                                                     0.0s
[CV 4/5] END ..........C=0.1, kernel=sigmoid;, score=0.694 total time=
                                                                     0.0s
[CV 5/5] END ...........C=0.1, kernel=sigmoid;, score=0.704 total time=
                                                                     0.0s
[CV 1/5] END ......C=1, kernel=rbf;, score=0.687 total time=
                                                                     0.0s
[CV 2/5] END ..................C=1, kernel=rbf;, score=0.684 total time=
                                                                     0.0s
[CV 3/5] END ......C=1, kernel=rbf;, score=0.684 total time=
                                                                     0.0s
[CV 4/5] END ......C=1, kernel=rbf;, score=0.694 total time=
                                                                     0.0s
[CV 5/5] END ......C=1, kernel=rbf;, score=0.704 total time=
                                                                     0.0s
[CV 1/5] END ......C=1, kernel=linear;, score=0.707 total time=
                                                                    43.9s
[CV 2/5] END ......C=1, kernel=linear;, score=0.704 total time=
[CV 3/5] END ......C=1, kernel=linear;, score=0.653 total time= 1.9min
[CV 4/5] END ......C=1, kernel=linear;, score=0.694 total time= 1.2min
[CV 5/5] END ......C=1, kernel=linear;, score=0.704 total time=
                                                                    46.7s
[CV 1/5] END ..................C=1, kernel=poly;, score=0.707 total time=
                                                                     0.5s
[CV 2/5] END ......C=1, kernel=poly;, score=0.694 total time=
                                                                     0.3s
[CV 3/5] END ......C=1, kernel=poly;, score=0.684 total time=
                                                                     0.3s
[CV 4/5] END ..................C=1, kernel=poly;, score=0.704 total time=
                                                                     0.2s
```

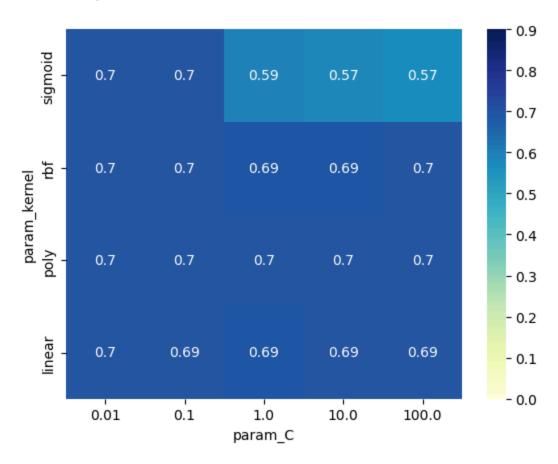
```
[CV 5/5] END ......C=1, kernel=poly;, score=0.704 total time=
                                                                      0.3s
[CV 1/5] END ......C=1, kernel=sigmoid;, score=0.596 total time=
                                                                      0.0s
[CV 2/5] END ......C=1, kernel=sigmoid;, score=0.612 total time=
                                                                      0.0s
[CV 3/5] END ......C=1, kernel=sigmoid;, score=0.561 total time=
                                                                      0.0s
[CV 4/5] END ......C=1, kernel=sigmoid;, score=0.571 total time=
                                                                      0.0s
[CV 5/5] END ......C=1, kernel=sigmoid;, score=0.622 total time=
                                                                      0.0s
[CV 1/5] END ..................C=10, kernel=rbf;, score=0.687 total time=
                                                                      0.0s
[CV 2/5] END .................C=10, kernel=rbf;, score=0.684 total time=
                                                                      0.0s
[CV 3/5] END ..................C=10, kernel=rbf;, score=0.684 total time=
                                                                      0.0s
[CV 4/5] END .................C=10, kernel=rbf;, score=0.704 total time=
                                                                      0.0s
[CV 5/5] END ......C=10, kernel=rbf;, score=0.704 total time=
                                                                      0.0s
[CV 1/5] END .................C=10, kernel=linear;, score=0.707 total time= 1.1min
[CV 2/5] END .................C=10, kernel=linear;, score=0.704 total time= 1.2min
[CV 3/5] END .................C=10, kernel=linear;, score=0.663 total time= 1.0min
[CV 4/5] END ............C=10, kernel=linear;, score=0.694 total time=
[CV 5/5] END ......C=10, kernel=linear;, score=0.704 total time=
[CV 1/5] END ......C=10, kernel=poly;, score=0.707 total time=
                                                                      3.0s
[CV 2/5] END ............C=10, kernel=poly;, score=0.694 total time=
                                                                      1.6s
[CV 3/5] END ............C=10, kernel=poly;, score=0.684 total time=
                                                                      1.4s
[CV 4/5] END ......C=10, kernel=poly;, score=0.704 total time=
                                                                      1.0s
[CV 5/5] END ......C=10, kernel=poly;, score=0.704 total time=
                                                                      1.9s
[CV 1/5] END .....C=10, kernel=sigmoid;, score=0.606 total time=
                                                                      0.0s
[CV 2/5] END ......C=10, kernel=sigmoid;, score=0.582 total time=
                                                                      0.0s
[CV 3/5] END ......C=10, kernel=sigmoid;, score=0.500 total time=
                                                                      0.0s
[CV 4/5] END ......C=10, kernel=sigmoid;, score=0.561 total time=
                                                                      0.0s
[CV 5/5] END .....C=10, kernel=sigmoid;, score=0.622 total time=
[CV 1/5] END ......C=100, kernel=rbf;, score=0.697 total time=
                                                                      0.1s
[CV 2/5] END .............C=100, kernel=rbf;, score=0.684 total time=
                                                                      0.1s
[CV 3/5] END ............C=100, kernel=rbf;, score=0.694 total time=
                                                                      0.1s
[CV 4/5] END ......C=100, kernel=rbf;, score=0.704 total time=
                                                                      0.1s
[CV 5/5] END ..............C=100, kernel=rbf;, score=0.704 total time=
[CV 1/5] END ......C=100, kernel=linear;, score=0.707 total time= 1.4min
[CV 2/5] END ......C=100, kernel=linear;, score=0.704 total time= 1.7min
[CV 3/5] END ......C=100, kernel=linear;, score=0.663 total time= 1.1min
[CV 4/5] END ......C=100, kernel=linear;, score=0.694 total time= 1.4min
[CV 5/5] END ......C=100, kernel=linear;, score=0.704 total time= 1.6min
[CV 1/5] END ......C=100, kernel=poly;, score=0.707 total time= 12.4s
[CV 2/5] END ......C=100, kernel=poly;, score=0.694 total time=
[CV 3/5] END ......C=100, kernel=poly;, score=0.684 total time=
[CV 4/5] END ......C=100, kernel=poly;, score=0.704 total time=
                                                                    11.0s
[CV 5/5] END ......C=100, kernel=poly;, score=0.704 total time=
[CV 1/5] END ......C=100, kernel=sigmoid;, score=0.606 total time=
                                                                      0.0s
[CV 2/5] END ......C=100, kernel=sigmoid;, score=0.592 total time=
                                                                      0.0s
[CV 3/5] END ......C=100, kernel=sigmoid;, score=0.490 total time=
                                                                      0.0s
[CV 4/5] END ..........C=100, kernel=sigmoid;, score=0.561 total time=
                                                                      0.0s
[CV 5/5] END ......C=100, kernel=sigmoid;, score=0.602 total time=
                                                                      0.0s
{'C': 0.01, 'kernel': 'poly'}
SVC(C=0.01, kernel='poly')
 accuracy = grid.score(X_test, y_test)
```

The test accuracy score of the grid-searched crossvalidation is: 0.650

out[11]:									
	mean_f	it_time	std_fit_tim	e mean_score_	time std_	score_time	param_C	param_kernel	
	<b>0</b> 0.	.032074	0.01172	9 0.01	5020	0.004541	0.01	rbf	
	<b>1</b> 39.	.650884	16.85334	0 0.00	3102	0.000309	0.01	linear	
	<b>2</b> 0.	.009946	0.00099	7 0.00	2605	0.000078	0.01	poly	
	<b>3</b> 0.	.008894	0.00013	7 0.00	3205	0.000091	0.01	sigmoid	
	<b>4</b> 0.	.006608	0.00071	4 0.00	3132	0.000117	0.10	rbf	
In [12]:	column_recv_result	sults = sults + s = cv_	[f"param_ = ["mean_t results[co	<pre>{name}" for na est_score", ": lumn_results]</pre>				score"]	
	cv_result  # Move re pivoted_cv	_" in p eturn p n param s = cv_  Levant v_resul s="mean	<pre>raram_name: raram_name. results.re cv results ts = cv_re _test_scor</pre>		param, ax: rt able(		nns=["para	am_C"]	
ut[12]:	return  cv_result  # Move re pivoted_cv value )	_"in p eturn p n param  s = cv_  Levant v_resul s="mean  v_resul	<pre>raram_name: raram_name. results.re cv results ts = cv_re _test_scor</pre>	rsplit("", : name(shorten_)  to pivot char sults.pivot_t	param, ax: rt able(	el"], colur	mns=["para	am_C"]	
ut[12]:	return  cv_result  # Move re pivoted_cv value )  pivoted_cv	_" in p eturn p n param s = cv_  Levant v_resul s="mean v_resul n_C	results.re  cv results  ts = cv_re  _test_scor	rsplit("", : name(shorten_   to pivot chan sults.pivot_t; e", index=["pa	param, ax: rt able( aram_kerne	el"], colur	nns=["para	am_C"]	
ut[12]:	return  cv_result  # Move re pivoted_cv value )  pivoted_cv  param_ker	_" in p eturn p n param  s = cv_  Levant v_resul s="mean  v_resul n_C	results.re  cv results ts = cv_re test_scor	rsplit("", : name(shorten_   to pivot chan sults.pivot_t; e", index=["pa	param, ax:  nt able( aram_kerne	el"], colur 100.00	mns=["para	am_C"]	
Out[12]:	return  cv_result  # Move re pivoted_cv value )  pivoted_cv  paran  param_ker	"in peturn pn params s = cv_ Levant v_resul s="mean v_resul n_C rnel ear 0.6	results.re  cv results ts = cv_re  test_scor  ts  0.01	rsplit("", iname(shorten_)  to pivot chan sults.pivot_ta e", index=["pa	param, ax:  nt able( aram_kerne	el"], colur 100.00 0.694475	nns=["para	am_C"]	
ut[12]:	return  cv_result  # Move re pivoted_cv value )  pivoted_cv  param param_ker	_"in peturn pn params s = cv_ Levant v_resul s="mean v_resul n_C resul ear 0.6	results.re cv results ts = cv_re test_scor  ts  0.01  96537 0.69	rsplit("", iname(shorten_)  to pivot chan sults.pivot_ta e", index=["pa	param, ax:  rt able( aram_kerne  10.00  0.694475  0.698557	100.00 0.694475 0.698557	mns=["para	am_C"]	
ut[12]:	return  cv_result  # Move re pivoted_cv value )  pivoted_cv  param  param_ker  line	"in peturn pn params s = cv_ Levant v_resul s="mean v_resul n_C resul ear 0.6 ref 0.6	results.re cv results ts = cv_re ts  0.01  96537 0.69 98557 0.69	rsplit("", :: name(shorten_   to pivot chan sults.pivot_t: e", index=["pi  0.10	param, ax:  rt able( aram_kerne  10.00  0.694475 0.698557 0.692476	100.00 0.694475 0.698557 0.696537	mns=["para	am_C"]	

```
In [13]: # Plot heatmap
    ax = sns.heatmap(pivoted_cv_results, annot=True, cmap="YlGnBu", vmin=0.0, vmax=0.9)
    ax.invert_yaxis()
    plt.suptitle(
        " Heatmap of the Model Results",
        fontweight="bold",
        horizontalalignment="right",
    )
    plt.show()
```

## Heatmap of the Model Results



```
# Classification Report
print(classification_report(y_test, y_pred))
```

```
Accuracy score of the test data: 0.6504
              precision
                           recall f1-score
                                               support
           0
                   0.00
                             0.00
                                                    43
                                       0.00
                   0.65
                             1.00
                                       0.79
                                                    80
    accuracy
                                       0.65
                                                   123
                             0.50
   macro avg
                   0.33
                                       0.39
                                                   123
                                                   123
weighted avg
                   0.42
                             0.65
                                       0.51
```

```
/usr/local/lib/python3.11/dist-packages/sklearn/metrics/_classification.py:1565: Und efinedMetricWarning: Precision is ill-defined and being set to 0.0 in labels with no predicted samples. Use `zero_division` parameter to control this behavior.

_warn_prf(average, modifier, f"{metric.capitalize()} is", len(result))
/usr/local/lib/python3.11/dist-packages/sklearn/metrics/_classification.py:1565: Und efinedMetricWarning: Precision is ill-defined and being set to 0.0 in labels with no predicted samples. Use `zero_division` parameter to control this behavior.

_warn_prf(average, modifier, f"{metric.capitalize()} is", len(result))
/usr/local/lib/python3.11/dist-packages/sklearn/metrics/_classification.py:1565: Und efinedMetricWarning: Precision is ill-defined and being set to 0.0 in labels with no predicted samples. Use `zero_division` parameter to control this behavior.

_warn_prf(average, modifier, f"{metric.capitalize()} is", len(result))
```

### **Neural Networks**

## **Multi-Layer Perceptron**

# Issue 2: Optimizing the Bias-Variance Tradeoff

Reference Equation

```
In [24]: # Demonstrating the Bias-Variance Tradeoff
# using the reference equation:
# MSE = Var(f^(x)) + [Bias(f^(x))]^2 + Var(ε)

# 1. Creating a true function (non-linear) and generating noisy observations
np.random.seed(42)

def true_function(x):
    # A cubic polynomial for demonstration
    return 1.5 * x**3 - 4.0 * x**2 + 2.0 * x - 1.0

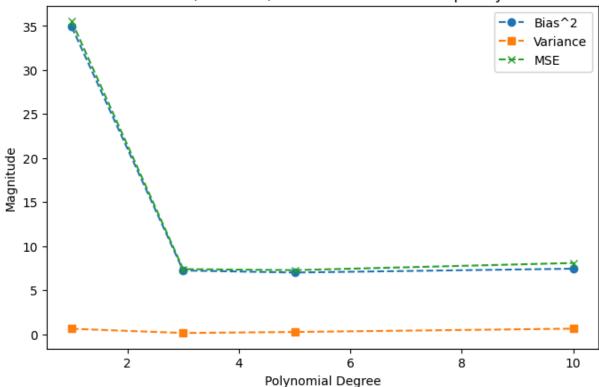
# Generating X values and corresponding y_true + noise
X_all = np.linspace(-2.0, 2.0, 200).reshape(-1, 1)
y_true_all = true_function(X_all).ravel()
noise = np.random.normal(loc=0.0, scale=3.0, size=len(X_all))
y_noisy_all = y_true_all + noise
```

```
# 2. Training/testing split
X_train, X_test, y_train, y_test = train_test_split(
   X_all, y_noisy_all, test_size=0.3, random_state=42
# 3. Function to approximate bias^2, variance, and MSE
     We do multiple runs with random re-sampling to estimate these values.
def estimate bias variance(degree, runs=50):
   For a specified polynomial 'degree', we:
     1. Randomly re-sample training data multiple times
     2. Train a model on each sample
     3. Record predictions on a fixed test set
     4. Compute average predictions, then derive bias^2, variance, and MSE
   predictions_matrix = []
   # repeated sampling for better approximation
   for _ in range(runs):
       # re-sample the training set
        idx = np.random.randint(0, len(X_train), len(X_train))
       X_sample, y_sample = X_train[idx], y_train[idx]
        # polynomial transform
        poly = PolynomialFeatures(degree=degree)
        X_sample_poly = poly.fit_transform(X_sample)
       X_test_poly = poly.transform(X_test)
       # fit the model
       model = LinearRegression()
       model.fit(X_sample_poly, y_sample)
        # predict on test set
        y pred = model.predict(X test poly)
        predictions_matrix.append(y_pred)
   # Converting to np array for easy aggregation
   predictions_matrix = np.array(predictions_matrix)
   # mean prediction across all runs for each test sample
   mean_predictions = np.mean(predictions_matrix, axis=0)
   # computing bias^2: average[(mean_predictions - y_true)^2]
   # here we use y_test as the "true" labels in the test set
   bias_sq = np.mean((mean_predictions - y_test)**2)
   # computing variance: average of the variance of predictions for each test samp
   # for each test sample i, we find var_i among runs, then average over all i
   var_each_test_point = np.var(predictions_matrix, axis=0)
   variance = np.mean(var_each_test_point)
   # total MSE: mean of all predictions vs. y_test
   # (averaged across runs, but we do it by referencing each run's difference)
   # simpler route: average over all runs, for each run's MSE
   mse_runs = []
   for run_i in range(runs):
```

```
mse_i = np.mean((predictions_matrix[run_i] - y_test)**2)
       mse_runs.append(mse_i)
   mse = np.mean(mse runs)
   return bias_sq, variance, mse
# 4. Evaluating for various polynomial degrees
degrees = [1, 3, 5, 10]
biases, variances, mses = [], [], []
for deg in degrees:
   b_sq, var, mse_val = estimate_bias_variance(deg, runs=50)
   biases.append(b_sq)
   variances.append(var)
   mses.append(mse val)
   print(f"Degree {deg}: Bias^2={b_sq:.3f}, Variance={var:.3f}, MSE={mse_val:.3f}"
# 5. Plot results showing how bias^2, variance, and MSE evolve with degree
plt.figure(figsize=(8,5))
plt.plot(degrees, biases, 'o--', label='Bias^2')
plt.plot(degrees, variances, 's--', label='Variance')
plt.plot(degrees, mses, 'x--', label='MSE')
plt.xlabel("Polynomial Degree")
plt.ylabel("Magnitude")
plt.title("Bias^2, Variance, and MSE vs. Model Complexity")
plt.legend()
plt.show()
```

Degree 1: Bias^2=34.891, Variance=0.634, MSE=35.525 Degree 3: Bias^2=7.246, Variance=0.156, MSE=7.403 Degree 5: Bias^2=7.016, Variance=0.271, MSE=7.288 Degree 10: Bias^2=7.452, Variance=0.646, MSE=8.098

Bias^2, Variance, and MSE vs. Model Complexity



Approaches to Controlling Bias and Variance

Regularization and Cross-validation

```
In [25]: # 1. Fetching data from FRED
         fred = Fred(api_key='3085603a10d918d77f7a6789fd1a57ef')
         def fetch_fred_data(series_list, start_date, end_date):
             data_holder = {}
             for s in series_list:
                 data_holder[s] = fred.get_series(
                     s, observation_start=start_date, observation_end=end_date
             df = pd.DataFrame(data_holder)
             return df.dropna()
         fred_series = ['FEDFUNDS', 'BAA', 'AAA']
         macro_data = fetch_fred_data(fred_series, '2017-11-01', '2025-01-01')
         # 2. Set up feature matrix (X) and target vector (y)
         X_raw = macro_data[['FEDFUNDS', 'AAA']].values
         y = macro_data['BAA'].values
         # 3. Polynomial degrees to investigate
         degrees = [1, 4, 15]
         # 4. Dictionaries for storing predictions and average MSE
         predictions_dict = {}
         mse_dict = {}
```

```
# 5. Looping over polynomial degrees
 for deg in degrees:
     # Transform input features
     poly = PolynomialFeatures(degree=deg, include_bias=False)
     X_poly = poly.fit_transform(X_raw)
     # Initialize Ridge
     ridge_model = Ridge(alpha=1.0)
     # Cross-validation with MSE
     cv_scores = cross_val_score(
         ridge_model,
         X_poly,
         cv=KFold(n splits=5, shuffle=True, random state=42),
         scoring='neg_mean_squared_error'
     mse_values = -cv_scores
     avg_mse = mse_values.mean()
     mse_dict[deg] = avg_mse
     # Train on full data
     ridge_model.fit(X_poly, y)
     preds = ridge_model.predict(X_poly)
     predictions_dict[deg] = preds
     print(f"Degree={deg}: MSE per fold -> {mse_values}, Mean MSE -> {avg_mse:.3f}")
 # 6. Plotting lines of predictions vs. actual values
 plt.figure(figsize=(8, 5))
 # Ideal fit line (diagonal)
 plt.plot([y.min(), y.max()], [y.min(), y.max()], 'k--', label='Ideal Fit')
 colors = ['blue', 'orange', 'green']
 # For each degree, sort by actual y so we can draw a line
 for i, deg in enumerate(degrees):
     combined = sorted(zip(y, predictions_dict[deg]), key=lambda z: z[0])
     y_sorted, preds_sorted = zip(*combined)
     plt.plot(y_sorted, preds_sorted, color=colors[i],
              label=f'Ridge deg={deg} (MSE={mse_dict[deg]:.3f})')
 plt.xlabel("Actual BAA")
 plt.ylabel("Predicted BAA")
 plt.title("Ridge Regression on FRED Data: Degrees 1, 4, and 15")
 plt.legend()
 plt.show()
Degree=1: MSE per fold -> [0.02486246 0.04775349 0.03180452 0.0487431 0.07010729],
Mean MSE -> 0.045
Degree=4: MSE per fold -> [0.02940225 0.04999149 0.02273009 0.04925371 0.08405904],
Mean MSE -> 0.047
```

```
/usr/local/lib/python3.11/dist-packages/sklearn/linear_model/_ridge.py:254: UserWarn
ing: Singular matrix in solving dual problem. Using least-squares solution instead.
 warnings.warn(
/usr/local/lib/python3.11/dist-packages/sklearn/linear_model/_ridge.py:254: UserWarn
ing: Singular matrix in solving dual problem. Using least-squares solution instead.
  warnings.warn(
/usr/local/lib/python3.11/dist-packages/sklearn/linear_model/_ridge.py:254: UserWarn
ing: Singular matrix in solving dual problem. Using least-squares solution instead.
  warnings.warn(
/usr/local/lib/python3.11/dist-packages/sklearn/linear model/ ridge.py:254: UserWarn
ing: Singular matrix in solving dual problem. Using least-squares solution instead.
  warnings.warn(
/usr/local/lib/python3.11/dist-packages/sklearn/linear_model/_ridge.py:254: UserWarn
ing: Singular matrix in solving dual problem. Using least-squares solution instead.
 warnings.warn(
/usr/local/lib/python3.11/dist-packages/sklearn/linear_model/_ridge.py:254: UserWarn
```

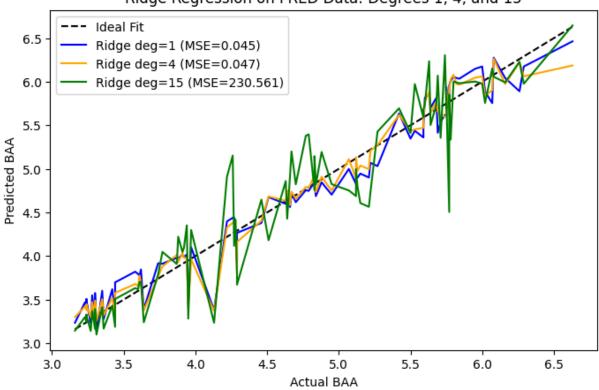
Degree=15: MSE per fold -> [6.54751776e+00 3.78886243e+00 5.60067984e-01 9.85918224e +02

ing: Singular matrix in solving dual problem. Using least-squares solution instead.

1.55990874e+02], Mean MSE -> 230.561

warnings.warn(

#### Ridge Regression on FRED Data: Degrees 1, 4, and 15



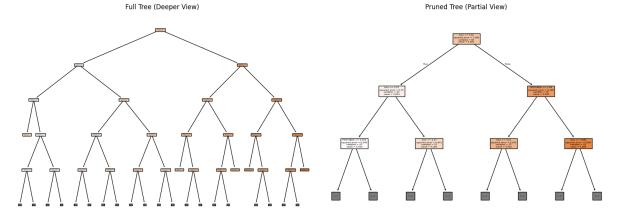
#### **Pruning Decision Trees**

```
In [26]: # 1. Fetching and preparing data
fred = Fred(api_key='3085603a10d918d77f7a6789fd1a57ef')
fred_series = ['FEDFUNDS', 'BAA', 'AAA']

def fetch_fred_data(series, start_date, end_date):
    data_dict = {}
    for s in series:
```

```
data_dict[s] = fred.get_series(s, observation_start=start_date, observation
    df = pd.DataFrame(data_dict)
   return df.dropna()
macro_data = fetch_fred_data(fred_series, '2017-11-01', '2025-01-01')
X = macro_data[['FEDFUNDS','AAA']].values
y = macro_data['BAA'].values
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3, random_sta
# 2. Fully grown tree
full_tree = DecisionTreeRegressor(max_depth=None, random_state=42)
full_tree.fit(X_train, y_train)
# 3. Pruned tree
pruned_tree = DecisionTreeRegressor(max_depth=3, random_state=42)
pruned_tree.fit(X_train, y_train)
# 4. Evaluate models
print("Full Tree MSE:", np.mean((full_tree.predict(X_test) - y_test)**2))
print("Pruned Tree MSE:", np.mean((pruned_tree.predict(X_test) - y_test)**2))
# 5. Visualizing deeper tree structure
plt.figure(figsize=(16, 6))
plt.subplot(1, 2, 1)
plot_tree(full_tree, max_depth=4, feature_names=['FEDFUNDS','AAA'], filled=True)
plt.title("Full Tree (Deeper View)")
plt.subplot(1, 2, 2)
plot_tree(pruned_tree, max_depth=2, feature_names=['FEDFUNDS','AAA'], filled=True)
plt.title("Pruned Tree (Partial View)")
plt.tight_layout()
plt.show()
```

Full Tree MSE: 0.049270370370370346 Pruned Tree MSE: 0.044310910675459766



#### Feature Selection

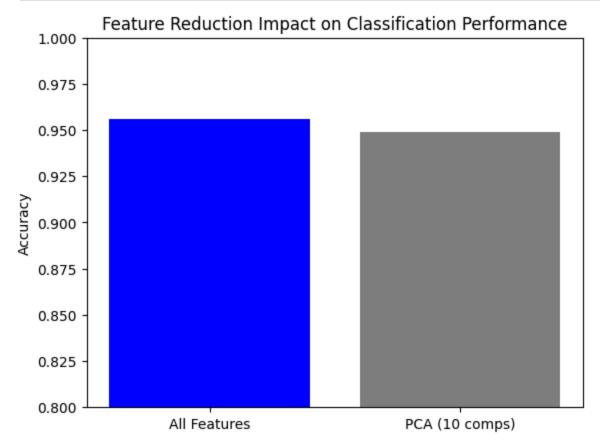
```
In [27]: # Load data
data = load_breast_cancer()
```

```
X, y = data.data, data.target

# Baseline with all features
rf = RandomForestClassifier(random_state=42)
base_scores = cross_val_score(rf, X, y, cv=5).mean()

# PCA with top 10 components
pca = PCA(n_components=10)
X_pca = pca.fit_transform(X)
pca_scores = cross_val_score(rf, X_pca, y, cv=5).mean()

plt.bar(['All Features','PCA (10 comps)'], [base_scores, pca_scores], color=['blue' plt.title("Feature Reduction Impact on Classification Performance")
plt.ylabel("Accuracy")
plt.ylabel("Accuracy")
plt.ylim([0.8,1.0])
plt.show()
```



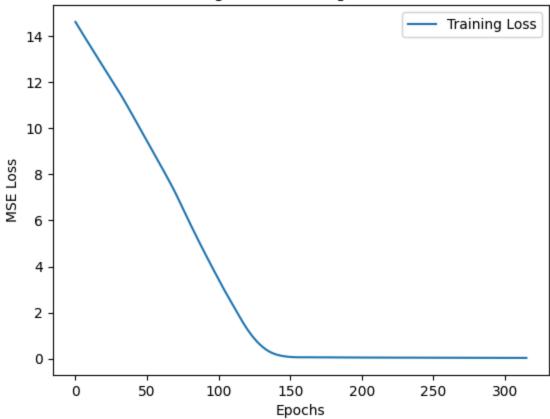
Early Stopping in Neural Networks

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from fredapi import Fred
from sklearn.model_selection import train_test_split
from sklearn.neural_network import MLPRegressor
from sklearn.metrics import mean_squared_error

#Fetch FRED data
def fetch_fred_data(series, start_date, end_date):
```

```
fred = Fred(api_key='3085603a10d918d77f7a6789fd1a57ef')
   data = \{\}
   for s in series:
        data[s] = fred.get_series(s, observation_start=start_date, observation_end=
   return pd.DataFrame(data).dropna()
# 1. Prepare data
macro_data = fetch_fred_data(['FEDFUNDS','BAA','AAA'], '2017-11-01', '2025-01-01')
X = macro_data[['FEDFUNDS', 'AAA']].values
y = macro_data['BAA'].values
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3, random_sta
# 2. Build neural net using MLPRegressor
model = MLPRegressor(hidden_layer_sizes=(16, 8), activation='relu', solver='adam',
                     max_iter=500, early_stopping=True, validation_fraction=0.2, ra
model.fit(X_train, y_train)
# 3. Plot training and validation loss
plt.plot(model.loss_curve_, label='Training Loss')
plt.title("MLPRegressor Training Loss Curve")
plt.xlabel("Epochs")
plt.ylabel("MSE Loss")
plt.legend()
plt.show()
# 4. Evaluate
y_pred = model.predict(X_test)
mse_test = mean_squared_error(y_test, y_pred)
print(f"Test MSE: {mse_test:.3f}")
```

#### MLPRegressor Training Loss Curve



Test MSE: 0.093

# Issue 3.

# **Data Acquisition and Preprocessing**

```
In [29]:
         # Define the tickers and FRED series
         tickers = ['^GSPC', '^NDX', '^VIX', '^TNX', 'DX-Y.NYB', '^FTSE', '^STOXX50E']
         # Define the date range
         start_date = '2010-01-01'
         end_date = '2025-01-01'
In [30]:
         def get_financial_data(ticker, start_date, end_date):
             data = yf.download(ticker, start=start_date, end=end_date)['Close']
             return data
         def preprocess_data(data):
             # Create multi index
             multi_index_data = pd.DataFrame()
             for col in data.columns:
                 temp_df = pd.DataFrame()
                 temp_df[('Close',col)] = data[col]
                 temp_df[('Return', col)] = data[col].pct_change()
                 temp_df[('Volatility', col)] = data[col].pct_change().rolling(window=20).st
```

```
multi_index_data = pd.concat([multi_index_data,temp_df],axis = 1)
# Drop rows with NaN from rolling calculations
multi_index_data.dropna(inplace=True)
return multi_index_data
```

```
In [31]: data = get_financial_data(tickers, start_date, end_date)
processed_data = preprocess_data(data)
```

```
[********* 7 of 7 completed
<ipython-input-30-0bdc32b8fbd8>:11: FutureWarning: The default fill_method='pad' in
Series.pct_change is deprecated and will be removed in a future version. Either fill
in any non-leading NA values prior to calling pct_change or specify 'fill_method=Non
e' to not fill NA values.
  temp_df[('Return', col)] = data[col].pct_change()
<ipython-input-30-0bdc32b8fbd8>:12: FutureWarning: The default fill_method='pad' in
Series.pct_change is deprecated and will be removed in a future version. Either fill
in any non-leading NA values prior to calling pct_change or specify 'fill_method=Non
e' to not fill NA values.
  temp_df[('Volatility', col)] = data[col].pct_change().rolling(window=20).std()
<ipython-input-30-0bdc32b8fbd8>:11: FutureWarning: The default fill_method='pad' in
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 temp_df[('Return', col)] = data[col].pct_change()
```

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Series.pct\_change is deprecated and will be removed in a future version. Either fill
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temp\_df[('Return', col)] = data[col].pct\_change()

<ipython-input-30-0bdc32b8fbd8>:12: FutureWarning: The default fill\_method='pad' in
Series.pct\_change is deprecated and will be removed in a future version. Either fill
in any non-leading NA values prior to calling pct\_change or specify 'fill\_method=Non
e' to not fill NA values.

temp\_df[('Volatility', col)] = data[col].pct\_change().rolling(window=20).std()

In	[32]	:	data
			GG CG

Out[32]:	Ticker	DX-Y.NYB	^FTSE	^GSPC	^NDX	^STOXX50E	^TNX	^VIX
	Date							
	2010- 01-04	77.529999	5500.299805	1132.989990	1886.699951	3017.800049	3.841	20.040001
	2010- 01-05	77.620003	5522.500000	1136.520020	1888.430054	3012.360107	3.755	19.350000
	2010- 01-06	77.489998	5530.000000	1137.140015	1878.420044	3009.659912	3.808	19.160000
	2010- 01-07	77.910004	5526.700195	1141.689941	1876.719971	3007.340088	3.822	19.059999
	2010- 01-08	77.470001	5534.200195	1144.979980	1892.589966	3017.850098	3.808	18.129999
	•••	•••						
	2024- 12-24	108.260002	8137.000000	6040.040039	21797.650391	NaN	4.591	14.270000
	2024- 12-26	108.129997	NaN	6037.589844	21768.310547	NaN	4.579	14.730000
	2024- 12-27	108.000000	8149.799805	5970.839844	21473.019531	4898.879883	4.619	15.950000
	2024- 12-30	108.129997	8121.000000	5906.939941	21197.089844	4869.279785	4.545	17.400000
	2024- 12-31	108.489998	8173.000000	5881.629883	21012.169922	NaN	4.573	17.350000

3868 rows × 7 columns

In [33]: processed\_data

$\cap$	+Г	20	> 7 ∘
υu	니	2	)   •

		(Close, DX-Y.NYB)	(Return, DX- Y.NYB)	(Volatility, DX- Y.NYB)	(Close, ^FTSE)	(Return, ^FTSE)	(Volatility, ^FTSE)	(Close, ^GSPC)
D	ate							
20° 02-	10- -01	79.239998	-0.002769	0.004302	5247.399902	0.011352	0.008220	1089.189941
	10- -02	79.010002	-0.002903	0.004394	5283.299805	0.006841	0.008357	1103.319946
	10- -03	79.370003	0.004556	0.004423	5253.200195	-0.005697	0.008349	1097.280029
	10- -04	79.919998	0.006930	0.004511	5139.299805	-0.021682	0.009362	1063.109985
20° 02-	10- -05	80.440002	0.006507	0.004343	5060.899902	-0.015255	0.009633	1066.189941
	•••		•••					
202 12-	24- -19	108.410004	0.003518	0.004393	8105.299805	-0.011440	0.005751	5867.080078
202 12-	24- -20	107.620003	-0.007287	0.004619	8084.600098	-0.002554	0.004710	5930.850098
202 12-	24- -23	108.040001	0.003903	0.004443	8102.700195	0.002239	0.004649	5974.069824
202 12-	24- -27	108.000000	-0.001202	0.003932	8149.799805	0.001573	0.004726	5970.839844
202 12-	24- -30	108.129997	0.001204	0.003811	8121.000000	-0.003534	0.004750	5906.939941

3606 rows × 21 columns

```
In [34]: # Feature and Target Selection
def feature_target_selection(data):
    # Drop 'Close' columns
    close_cols = [col for col in data.columns if col[0] == 'Close']
    features = data.drop(columns=close_cols)

# Set target
    target = features[('Return', '^GSPC')]

# Drop target from features
    features = features.drop(columns=[('Return', '^GSPC')])

# Shift features by one day
    features = features.shift(1)
```

```
# Drop NaN created by shift
features.dropna(inplace=True)
target = target.loc[features.index]
return features, target
```

```
In [35]: features, target = feature_target_selection(processed_data)
```

```
In [36]: # Discretize the target variable (Based on percentiles)
bins = [-float('inf'), -0.01, 0.01, float('inf')] # Define bins
labels = [0, 1, 2] # Labels for the bins (negative, neutral, positive)
target_discrete = pd.cut(target, bins=bins, labels=labels, right=False)
```

#### Classes:

Assign a numerical value to each bin.

- Class 0 represents negative returns.
- Class 1 represents neutral returns [-1% +1%]
- Class 2 represents positive returns.

```
In [37]: print("Features:")
features
```

Features:

Out[37]:		(Return, DX- Y.NYB)	(Volatility, DX- Y.NYB)	(Return, ^FTSE)	(Volatility, ^FTSE)	(Volatility, ^GSPC)	(Return, ^NDX)	(Volatility, ^NDX)	( ^STO
	Date								
	2010- 02-02	-0.002769	0.004302	0.011352	0.008220	0.009905	0.011304	0.012639	0
	2010- 02-03	-0.002903	0.004394	0.006841	0.008357	0.010401	0.009201	0.012920	0
	2010- 02-04	0.004556	0.004423	-0.005697	0.008349	0.010428	0.004378	0.013008	-0
	2010- 02-05	0.006930	0.004511	-0.021682	0.009362	0.012218	-0.028974	0.014282	-0
	2010- 02-08	0.006507	0.004343	-0.015255	0.009633	0.012219	0.007577	0.014243	-0
	•••								
	2024- 12-19	0.010004	0.004370	0.000476	0.005388	0.008014	-0.035987	0.011704	0
	2024- 12-20	0.003518	0.004393	-0.011440	0.005751	0.007901	-0.004659	0.011767	-0
	2024- 12-23	-0.007287	0.004619	-0.002554	0.004710	0.008269	0.008462	0.011886	-0
	2024- 12-27	0.003903	0.004443	0.002239	0.004649	0.008413	0.010053	0.012047	-0
	2024- 12-30	-0.001202	0.003932	0.001573	0.004726	0.009007	-0.013565	0.012608	0

3605 rows × 13 columns

In [38]: print("\nTarget Discrete:")
 target\_discrete

Target Discrete:

Out[38]: Return

^GSPC

Date	
2010-02-02	2
2010-02-03	1
2010-02-04	0
2010-02-05	1
2010-02-08	1
•••	
2024-12-19	1
2024-12-19	1 2
	•
2024-12-20	2
2024-12-20	2

3605 rows × 1 columns

#### dtype: category

```
In [39]: # Split Data
X_train, X_test, y_train, y_test = train_test_split(features, target_discrete, test
# Scale Data
scaler = StandardScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)
```

# **Bagging (Random Forest)**

```
In [40]: def train_and_evaluate_base_bagmodel(X_train, y_train, X_test, y_test):
    """Trains and evaluates a base Random Forest Classifier."""
    bagmodel = RandomForestClassifier(n_estimators=10, random_state=42)
    bagmodel.fit(X_train, y_train)

    print("Base Bagging (Random Forest) Results:")
    print("Accuracy on train set: %0.4f" % (bagmodel.score(X_train, y_train)))
    print("Accuracy on test set: %0.4f" % (bagmodel.score(X_test, y_test)))
    return bagmodel

In [41]: # Main Execution - Base
```

```
base_bagmodel = train_and_evaluate_base_bagmodel(X_train_scaled, y_train, X_test_sc

Base Bagging (Random Forest) Results:
Accuracy on train set: 0.9834
Accuracy on test set: 0.6533

def tune hyperparameters rf(X train, y train):
```

```
In [42]:

def tune_hyperparameters_rf(X_train, y_train):
    """Tunes hyperparameters for RandomForest using GridSearchCV."""
    param_grid = {
        "n_estimators": [10],
        "max_depth": [2, 3, 4],
        "min_samples_split": [2, 4, 8],
    }

    grid = GridSearchCV(
        RandomForestClassifier(random_state=42), param_grid, refit=True, verbose=3,
    )

    grid.fit(X_train, y_train)

# Get the best model from the grid search
    rf_best_model = grid.best_estimator_

    print("\n---")
    print("Best Random Forest Parameters:")
    print(grid.best_params_)

    return rf_best_model
```

```
In [43]: # Main Execution - Best
best_bagmodel = tune_hyperparameters_rf(X_train_scaled, y_train)
```

```
Fitting 3 folds for each of 9 candidates, totalling 27 fits
[CV 1/3] END max_depth=2, min_samples_split=2, n_estimators=10;, score=0.770 total t
      0.0s
[CV 2/3] END max_depth=2, min_samples_split=2, n_estimators=10;, score=0.772 total t
ime= 0.0s
[CV 3/3] END max_depth=2, min_samples_split=2, n_estimators=10;, score=0.706 total t
ime= 0.0s
[CV 1/3] END max_depth=2, min_samples_split=4, n_estimators=10;, score=0.770 total t
ime= 0.0s
[CV 2/3] END max_depth=2, min_samples_split=4, n_estimators=10;, score=0.772 total t
ime= 0.0s
[CV 3/3] END max_depth=2, min_samples_split=4, n_estimators=10;, score=0.706 total t
      0.0s
[CV 1/3] END max_depth=2, min_samples_split=8, n_estimators=10;, score=0.770 total t
      0.0s
[CV 2/3] END max_depth=2, min_samples_split=8, n_estimators=10;, score=0.772 total t
ime= 0.0s
[CV 3/3] END max_depth=2, min_samples_split=8, n_estimators=10;, score=0.706 total t
ime= 0.0s
[CV 1/3] END max_depth=3, min_samples_split=2, n_estimators=10;, score=0.770 total t
ime= 0.0s
[CV 2/3] END max_depth=3, min_samples_split=2, n_estimators=10;, score=0.772 total t
      0.0s
[CV 3/3] END max_depth=3, min_samples_split=2, n_estimators=10;, score=0.695 total t
      0.0s
[CV 1/3] END max_depth=3, min_samples_split=4, n_estimators=10;, score=0.770 total t
     0.0s
[CV 2/3] END max_depth=3, min_samples_split=4, n_estimators=10;, score=0.772 total t
ime= 0.0s
[CV 3/3] END max_depth=3, min_samples_split=4, n_estimators=10;, score=0.693 total t
ime= 0.0s
[CV 1/3] END max_depth=3, min_samples_split=8, n_estimators=10;, score=0.770 total t
ime= 0.0s
[CV 2/3] END max_depth=3, min_samples_split=8, n_estimators=10;, score=0.772 total t
ime= 0.1s
[CV 3/3] END max_depth=3, min_samples_split=8, n_estimators=10;, score=0.689 total t
      0.0s
[CV 1/3] END max_depth=4, min_samples_split=2, n_estimators=10;, score=0.769 total t
ime= 0.0s
[CV 2/3] END max_depth=4, min_samples_split=2, n_estimators=10;, score=0.772 total t
ime= 0.0s
[CV 3/3] END max_depth=4, min_samples_split=2, n_estimators=10;, score=0.690 total t
ime= 0.0s
[CV 1/3] END max_depth=4, min_samples_split=4, n_estimators=10;, score=0.770 total t
      0.0s
[CV 2/3] END max_depth=4, min_samples_split=4, n_estimators=10;, score=0.772 total t
      0.0s
[CV 3/3] END max_depth=4, min_samples_split=4, n_estimators=10;, score=0.704 total t
ime= 0.1s
[CV 1/3] END max depth=4, min samples split=8, n estimators=10;, score=0.767 total t
      0.1s
[CV 2/3] END max_depth=4, min_samples_split=8, n_estimators=10;, score=0.772 total t
ime= 0.1s
[CV 3/3] END max_depth=4, min_samples_split=8, n_estimators=10;, score=0.706 total t
ime= 0.1s
```

```
Best Random Forest Parameters:
        {'max_depth': 2, 'min_samples_split': 2, 'n_estimators': 10}
In [44]: def evaluate_model(model, X_test, y_test, X_train, y_train):
             """Evaluates the model (classification metrics)."""
             start time = time.time()
             y_pred = model.predict(X_test)
             end_time = time.time()
             execution_time = end_time - start_time
             # Classification Metrics
             accuracy = accuracy_score(y_test, y_pred)
             precision = precision_score(y_test, y_pred, average='weighted') #Weighted avera
             recall = recall_score(y_test, y_pred, average='weighted')
             f1 = f1_score(y_test, y_pred, average='weighted')
             print(f"Accuracy: {accuracy:.4f}")
             print(f"Precision: {precision:.4f}")
             print(f"Recall: {recall:.4f}")
             print(f"F1 Score: {f1:.4f}")
             print(f"Execution Time: {execution_time:.4f} seconds")
             return y_pred
In [45]: print("\nBase Bagging (Random Forest) Model Results:")
         y_pred_base_bag = evaluate_model(base_bagmodel, X_test_scaled, y_test, X_train_scal
         print("\n---")
         print("\nBest Bagging (Random Forest) Model Results:")
         y_pred_best_bag = evaluate_model(best_bagmodel, X_test_scaled, y_test, X_train_scal
        Base Bagging (Random Forest) Model Results:
        Accuracy: 0.6533
        Precision: 0.5466
        Recall: 0.6533
        F1 Score: 0.5757
        Execution Time: 0.0088 seconds
        _ _ _
        Best Bagging (Random Forest) Model Results:
        Accuracy: 0.6824
        Precision: 0.4657
        Recall: 0.6824
        F1 Score: 0.5536
        Execution Time: 0.0023 seconds
        /usr/local/lib/python3.11/dist-packages/sklearn/metrics/ classification.py:1565: Und
        efinedMetricWarning: Precision is ill-defined and being set to 0.0 in labels with no
        predicted samples. Use `zero_division` parameter to control this behavior.
         _warn_prf(average, modifier, f"{metric.capitalize()} is", len(result))
```

### Illustration:

```
In [46]: def plot_multiclass_roc(y_test, y_pred_proba, title_prefix=""):
             """Plots both individual and macro-averaged ROC curves."""
             y test bin = label binarize(y test, classes=np.unique(y test))
             n_classes = y_test_bin.shape[1]
             fpr = dict()
             tpr = dict()
             roc_auc = dict()
             for i in range(n_classes):
                 fpr[i], tpr[i], _ = roc_curve(y_test_bin[:, i], y_pred_proba[:, i])
                 roc_auc[i] = auc(fpr[i], tpr[i])
             # Macro-average ROC curve
             all_fpr = np.unique(np.concatenate([fpr[i] for i in range(n_classes)]))
             mean_tpr = np.zeros_like(all_fpr)
             for i in range(n_classes):
                 mean_tpr += np.interp(all_fpr, fpr[i], tpr[i])
             mean_tpr /= n_classes
             fpr macro = all fpr
             tpr macro = mean tpr
             roc_auc_macro = auc(fpr_macro, tpr_macro)
             # Create subplots
             fig, axes = plt.subplots(1, 2, figsize=(16, 6))
             # Plot individual class ROC curves
             colors = ['red', 'blue', 'green'] # Adjust colors as needed
             for i, color in zip(range(n_classes), colors):
                 axes[0].plot(fpr[i], tpr[i], color=color, lw=1,
                               label='ROC curve of class {0} (AUC = {1:0.2f})'.format(i, roc_
             # Random guess model
             axes[0].plot([0, 1], [0, 1], 'k--', lw=1, label="Random") # Added Label here
             axes[0].set_xlim([0.0, 1.0])
             axes[0].set_ylim([0.0, 1.05])
             axes[0].set_xlabel('False Positive Rate')
             axes[0].set_ylabel('True Positive Rate')
             axes[0].set_title(f'{title_prefix}Multiclass ROC Curve')
             axes[0].legend(loc="lower right")
             # Plot macro-average ROC curve
             axes[1].plot(fpr_macro, tpr_macro,
                              label='Macro-average ROC curve (AUC = {0:0.2f})'.format(roc_au
                               color='navy', linewidth=2)
             # Random quess model
             axes[1].plot([0, 1], [0, 1], 'k--', lw=1, label="Random") # Added Label here
             axes[1].set_xlim([0.0, 1.0])
             axes[1].set_ylim([0.0, 1.05])
             axes[1].set_xlabel('False Positive Rate')
             axes[1].set_ylabel('True Positive Rate')
             axes[1].set_title(f'{title_prefix}Macro-average ROC Curve')
             axes[1].legend(loc="lower right")
             plt.show()
```

---- Random

```
In [47]: y_pred_proba_bag = best_bagmodel.predict_proba(X_test_scaled)
               plot_multiclass_roc(y_test, y_pred_proba_bag, "Bagging (RF) Model - ")
                           Bagging (RF) Model - Multiclass ROC Curve
                                                                                             Bagging (RF) Model - Macro-average ROC Curve
              1.0
                                                                                  1.0
             0.8
                                                                                  0.8
                                                                                Rate
             0.6
                                                                                  0.6
            Positive
                                                                                Positive
            를 0.4
                                                                                 0.4
                                                                                  0.2
                                                ROC curve of class 0 (AUC = 0.63)
                                                ROC curve of class 1 (AUC = 0.67)
                                                ROC curve of class 2 (AUC = 0.64)
                                                                                                                Macro-average ROC curve (AUC = 0.65)
```

## **Boosting (Gradient Boosting)**

False Positive Rate

Random

```
def train_and_evaluate_base_boostmodel(X_train, y_train, X_test, y_test):
In [48]:
             """Trains and evaluates a base Gradient Boosting Classifier."""
             boostmodel = GradientBoostingClassifier(n_estimators=100, learning_rate=0.1, ra
             boostmodel.fit(X_train, y_train)
             print("Base Boosting (Gradient Boosting) Results:")
             print("Accuracy on train set: %0.4f" % (boostmodel.score(X_train, y_train)))
             print("Accuracy on test set: %0.4f" % (boostmodel.score(X_test, y_test)))
             return boostmodel
In [49]: # Main Execution - Base
         base_boostmodel = train_and_evaluate_base_boostmodel(X_train_scaled, y_train, X_tes
        Base Boosting (Gradient Boosting) Results:
        Accuracy on train set: 0.8412
        Accuracy on test set: 0.6644
In [50]: def tune_hyperparameters_boost(X_train, y_train):
             """Tunes hyperparameters for Gradient Boosting using GridSearchCV."""
             param_grid = {
                 "n estimators": [10],
                 "learning_rate": [0.01, 0.05, 0.1],
                 "max_depth": [3, 4, 5],
             }
             grid = GridSearchCV(
                 GradientBoostingClassifier(random_state=42), param_grid, refit=True, verbos
             grid.fit(X_train, y_train)
             # Get the best model from the grid search
```

```
boostmodel = grid.best_estimator_

print("\n---")
print("Best Gradient Boosting Parameters:")
print(grid.best_params_)

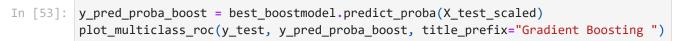
return boostmodel
```

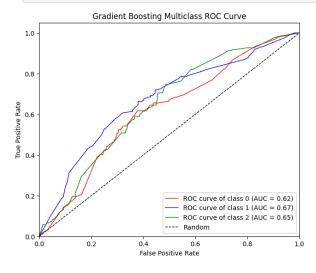
```
In [51]: # Main Execution - Best
best_boostmodel = tune_hyperparameters_boost(X_train_scaled, y_train)
```

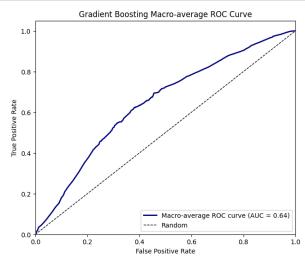
```
Fitting 3 folds for each of 9 candidates, totalling 27 fits
[CV 1/3] END learning_rate=0.01, max_depth=3, n_estimators=10;, score=0.770 total ti
     0.4s
[CV 2/3] END learning_rate=0.01, max_depth=3, n_estimators=10;, score=0.772 total ti
[CV 3/3] END learning_rate=0.01, max_depth=3, n_estimators=10;, score=0.772 total ti
    0.8s
[CV 1/3] END learning_rate=0.01, max_depth=4, n_estimators=10;, score=0.770 total ti
     0.5s
[CV 2/3] END learning_rate=0.01, max_depth=4, n_estimators=10;, score=0.772 total ti
    0.5s
[CV 3/3] END learning_rate=0.01, max_depth=4, n_estimators=10;, score=0.772 total ti
     0.5s
[CV 1/3] END learning_rate=0.01, max_depth=5, n_estimators=10;, score=0.770 total ti
[CV 2/3] END learning_rate=0.01, max_depth=5, n_estimators=10;, score=0.772 total ti
    0.7s
[CV 3/3] END learning_rate=0.01, max_depth=5, n_estimators=10;, score=0.772 total ti
    0.7s
[CV 1/3] END learning_rate=0.05, max_depth=3, n_estimators=10;, score=0.770 total ti
     0.5s
[CV 2/3] END learning_rate=0.05, max_depth=3, n_estimators=10;, score=0.772 total ti
     0.5s
[CV 3/3] END learning_rate=0.05, max_depth=3, n_estimators=10;, score=0.755 total ti
    0.5s
[CV 1/3] END learning_rate=0.05, max_depth=4, n_estimators=10;, score=0.769 total ti
[CV 2/3] END learning_rate=0.05, max_depth=4, n_estimators=10;, score=0.772 total ti
    0.7s
[CV 3/3] END learning_rate=0.05, max_depth=4, n_estimators=10;, score=0.749 total ti
    0.5s
[CV 1/3] END learning_rate=0.05, max_depth=5, n_estimators=10;, score=0.770 total ti
     0.7s
[CV 2/3] END learning_rate=0.05, max_depth=5, n_estimators=10;, score=0.772 total ti
    0.6s
[CV 3/3] END learning_rate=0.05, max_depth=5, n_estimators=10;, score=0.746 total ti
     1.0s
[CV 1/3] END learning_rate=0.1, max_depth=3, n_estimators=10;, score=0.772 total tim
   0.8s
[CV 2/3] END learning_rate=0.1, max_depth=3, n_estimators=10;, score=0.772 total tim
   0.4s
[CV 3/3] END learning_rate=0.1, max_depth=3, n_estimators=10;, score=0.718 total tim
   0.4s
[CV 1/3] END learning_rate=0.1, max_depth=4, n_estimators=10;, score=0.768 total tim
   0.5s
[CV 2/3] END learning_rate=0.1, max_depth=4, n_estimators=10;, score=0.771 total tim
    0.5s
[CV 3/3] END learning_rate=0.1, max_depth=4, n_estimators=10;, score=0.719 total tim
   0.5s
[CV 1/3] END learning rate=0.1, max depth=5, n estimators=10;, score=0.771 total tim
    0.7s
[CV 2/3] END learning_rate=0.1, max_depth=5, n_estimators=10;, score=0.767 total tim
   0.6s
[CV 3/3] END learning_rate=0.1, max_depth=5, n_estimators=10;, score=0.701 total tim
e= 0.6s
```

```
Best Gradient Boosting Parameters:
        {'learning rate': 0.01, 'max depth': 3, 'n estimators': 10}
In [52]: print("\nBase Boosting (Gradient Boosting) Model Results:")
         y_pred_base = evaluate_model(base_boostmodel, X_test_scaled, y_test, X_train_scaled
         print("\n---")
         print("\nBest Boosting (Gradient Boosting) Model Results:")
         y_pred_best = evaluate_model(best_boostmodel, X_test_scaled, y_test, X_train_scaled
        Base Boosting (Gradient Boosting) Model Results:
        Accuracy: 0.6644
        Precision: 0.5538
        Recall: 0.6644
        F1 Score: 0.5711
        Execution Time: 0.0066 seconds
        Best Boosting (Gradient Boosting) Model Results:
        Accuracy: 0.6824
        Precision: 0.4657
        Recall: 0.6824
        F1 Score: 0.5536
        Execution Time: 0.0019 seconds
        /usr/local/lib/python3.11/dist-packages/sklearn/metrics/_classification.py:1565: Und
        efinedMetricWarning: Precision is ill-defined and being set to 0.0 in labels with no
        predicted samples. Use `zero_division` parameter to control this behavior.
          _warn_prf(average, modifier, f"{metric.capitalize()} is", len(result))
```

### Illustration:







# Stacking.

```
In [54]: def train_and_evaluate_base_stackmodel(X_train, y_train, X_test, y_test):
             """Trains and evaluates a base Stacking Classifier."""
             estimators = [
                 ('rf', RandomForestClassifier(n estimators=10, random state=42)),
                 ('gb', GradientBoostingClassifier(n_estimators=10, learning_rate=0.1, rando
             stackmodel = StackingClassifier(estimators=estimators)
             stackmodel.fit(X_train, y_train)
             print("Base Stacking Model Results:")
             print("Accuracy on train set: %0.4f" % (stackmodel.score(X_train, y_train)))
             print("Accuracy on test set: %0.4f" % (stackmodel.score(X_test, y_test)))
             return stackmodel
In [55]: # Main Execution - Base
         base_stackmodel = train_and_evaluate_base_stackmodel(X_train_scaled, y_train, X_tes
        Base Stacking Model Results:
        Accuracy on train set: 0.7718
        Accuracy on test set: 0.6824
In [56]: # Get best parameters
         best rf params = tune hyperparameters rf(X train scaled, y train)
         best_gb_params = tune_hyperparameters_boost(X_train_scaled, y_train)
```

```
Fitting 3 folds for each of 9 candidates, totalling 27 fits
[CV 1/3] END max_depth=2, min_samples_split=2, n_estimators=10;, score=0.770 total t
      0.0s
[CV 2/3] END max_depth=2, min_samples_split=2, n_estimators=10;, score=0.772 total t
ime= 0.0s
[CV 3/3] END max_depth=2, min_samples_split=2, n_estimators=10;, score=0.706 total t
ime= 0.0s
[CV 1/3] END max_depth=2, min_samples_split=4, n_estimators=10;, score=0.770 total t
ime= 0.0s
[CV 2/3] END max_depth=2, min_samples_split=4, n_estimators=10;, score=0.772 total t
ime= 0.0s
[CV 3/3] END max_depth=2, min_samples_split=4, n_estimators=10;, score=0.706 total t
      0.0s
[CV 1/3] END max_depth=2, min_samples_split=8, n_estimators=10;, score=0.770 total t
      0.0s
[CV 2/3] END max_depth=2, min_samples_split=8, n_estimators=10;, score=0.772 total t
ime= 0.0s
[CV 3/3] END max_depth=2, min_samples_split=8, n_estimators=10;, score=0.706 total t
ime= 0.0s
[CV 1/3] END max_depth=3, min_samples_split=2, n_estimators=10;, score=0.770 total t
ime= 0.0s
[CV 2/3] END max_depth=3, min_samples_split=2, n_estimators=10;, score=0.772 total t
      0.0s
[CV 3/3] END max_depth=3, min_samples_split=2, n_estimators=10;, score=0.695 total t
      0.0s
[CV 1/3] END max_depth=3, min_samples_split=4, n_estimators=10;, score=0.770 total t
     0.0s
[CV 2/3] END max_depth=3, min_samples_split=4, n_estimators=10;, score=0.772 total t
ime= 0.0s
[CV 3/3] END max_depth=3, min_samples_split=4, n_estimators=10;, score=0.693 total t
ime= 0.0s
[CV 1/3] END max_depth=3, min_samples_split=8, n_estimators=10;, score=0.770 total t
      0.0s
[CV 2/3] END max_depth=3, min_samples_split=8, n_estimators=10;, score=0.772 total t
ime= 0.0s
[CV 3/3] END max_depth=3, min_samples_split=8, n_estimators=10;, score=0.689 total t
      0.0s
[CV 1/3] END max_depth=4, min_samples_split=2, n_estimators=10;, score=0.769 total t
ime= 0.0s
[CV 2/3] END max_depth=4, min_samples_split=2, n_estimators=10;, score=0.772 total t
ime= 0.0s
[CV 3/3] END max_depth=4, min_samples_split=2, n_estimators=10;, score=0.690 total t
ime= 0.0s
[CV 1/3] END max_depth=4, min_samples_split=4, n_estimators=10;, score=0.770 total t
      0.0s
[CV 2/3] END max_depth=4, min_samples_split=4, n_estimators=10;, score=0.772 total t
      0.0s
[CV 3/3] END max_depth=4, min_samples_split=4, n_estimators=10;, score=0.704 total t
ime= 0.0s
[CV 1/3] END max depth=4, min samples split=8, n estimators=10;, score=0.767 total t
      0.0s
[CV 2/3] END max_depth=4, min_samples_split=8, n_estimators=10;, score=0.772 total t
ime= 0.1s
[CV 3/3] END max_depth=4, min_samples_split=8, n_estimators=10;, score=0.706 total t
ime=0.0s
```

\_ \_ \_

```
Best Random Forest Parameters:
{'max_depth': 2, 'min_samples_split': 2, 'n_estimators': 10}
Fitting 3 folds for each of 9 candidates, totalling 27 fits
[CV 1/3] END learning_rate=0.01, max_depth=3, n_estimators=10;, score=0.770 total ti
     0.4s
[CV 2/3] END learning_rate=0.01, max_depth=3, n_estimators=10;, score=0.772 total ti
     0.4s
[CV 3/3] END learning rate=0.01, max depth=3, n estimators=10;, score=0.772 total ti
[CV 1/3] END learning_rate=0.01, max_depth=4, n_estimators=10;, score=0.770 total ti
    0.5s
[CV 2/3] END learning_rate=0.01, max_depth=4, n_estimators=10;, score=0.772 total ti
     0.5s
[CV 3/3] END learning rate=0.01, max depth=4, n estimators=10;, score=0.772 total ti
     0.5s
[CV 1/3] END learning_rate=0.01, max_depth=5, n_estimators=10;, score=0.770 total ti
     0.7s
[CV 2/3] END learning_rate=0.01, max_depth=5, n_estimators=10;, score=0.772 total ti
     0.6s
[CV 3/3] END learning_rate=0.01, max_depth=5, n_estimators=10;, score=0.772 total ti
     0.6s
[CV 1/3] END learning_rate=0.05, max_depth=3, n_estimators=10;, score=0.770 total ti
    0.4s
[CV 2/3] END learning_rate=0.05, max_depth=3, n_estimators=10;, score=0.772 total ti
    0.4s
[CV 3/3] END learning_rate=0.05, max_depth=3, n_estimators=10;, score=0.755 total ti
     0.4s
[CV 1/3] END learning_rate=0.05, max_depth=4, n_estimators=10;, score=0.769 total ti
     0.5s
[CV 2/3] END learning_rate=0.05, max_depth=4, n_estimators=10;, score=0.772 total ti
[CV 3/3] END learning_rate=0.05, max_depth=4, n_estimators=10;, score=0.749 total ti
    0.5s
[CV 1/3] END learning_rate=0.05, max_depth=5, n_estimators=10;, score=0.770 total ti
     0.7s
[CV 2/3] END learning_rate=0.05, max_depth=5, n_estimators=10;, score=0.772 total ti
[CV 3/3] END learning_rate=0.05, max_depth=5, n_estimators=10;, score=0.746 total ti
    0.8s
[CV 1/3] END learning_rate=0.1, max_depth=3, n_estimators=10;, score=0.772 total tim
    0.5s
[CV 2/3] END learning_rate=0.1, max_depth=3, n_estimators=10;, score=0.772 total tim
    0.4s
[CV 3/3] END learning_rate=0.1, max_depth=3, n_estimators=10;, score=0.718 total tim
   0.4s
[CV 1/3] END learning_rate=0.1, max_depth=4, n_estimators=10;, score=0.768 total tim
   0.5s
[CV 2/3] END learning_rate=0.1, max_depth=4, n_estimators=10;, score=0.771 total tim
e = 0.5s
[CV 3/3] END learning_rate=0.1, max_depth=4, n_estimators=10;, score=0.719 total tim
   0.5s
[CV 1/3] END learning_rate=0.1, max_depth=5, n_estimators=10;, score=0.771 total tim
   0.7s
[CV 2/3] END learning_rate=0.1, max_depth=5, n_estimators=10;, score=0.767 total tim
e= 0.6s
```

```
[CV 3/3] END learning_rate=0.1, max_depth=5, n_estimators=10;, score=0.701 total tim
        e= 0.6s
        Best Gradient Boosting Parameters:
        {'learning_rate': 0.01, 'max_depth': 3, 'n_estimators': 10}
In [57]: def tune_hyperparameters_stack(X_train, y_train):
             """Tunes hyperparameters for Stacking Classifier using GridSearchCV, using pre-
             estimators = [
                 ('rf', best_rf_params),
                 ('gb', best_gb_params)
             param_grid = {
                 'final_estimator__C': [0.1, 1.0, 10.0], # Regularization parameter for Log
                 'final_estimator__solver': ['lbfgs', 'liblinear', 'mylr'],
                 'stack_method': ['auto', 'predict_proba']
             stackmodel = StackingClassifier(estimators=estimators, final estimator=Logistic
             grid = GridSearchCV(stackmodel, param_grid, refit=True, verbose=3, cv=3)
             grid.fit(X_train, y_train)
             # Get the best model from the grid search
             stackmodel = grid.best_estimator_
             print("\n---")
             print("Best Stacking Parameters:")
             print(grid.best_params_)
             return stackmodel
```

```
In [58]: # Main Execution - Best
best_stackmodel = tune_hyperparameters_stack(X_train_scaled, y_train)
```

```
Fitting 3 folds for each of 18 candidates, totalling 54 fits
[CV 1/3] END final_estimator__C=0.1, final_estimator__solver=lbfgs, stack_method=aut
o;, score=0.770 total time=
                             3.1s
[CV 2/3] END final_estimator__C=0.1, final_estimator__solver=lbfgs, stack_method=aut
o;, score=0.772 total time= 5.1s
[CV 3/3] END final_estimator__C=0.1, final_estimator__solver=lbfgs, stack_method=aut
o;, score=0.772 total time= 4.3s
[CV 1/3] END final_estimator__C=0.1, final_estimator__solver=lbfgs, stack_method=pre
dict proba;, score=0.770 total time= 3.6s
[CV 2/3] END final_estimator__C=0.1, final_estimator__solver=lbfgs, stack_method=pre
dict_proba;, score=0.772 total time= 2.3s
[CV 3/3] END final_estimator__C=0.1, final_estimator__solver=lbfgs, stack_method=pre
dict_proba;, score=0.772 total time= 2.8s
[CV 1/3] END final_estimator__C=0.1, final_estimator__solver=liblinear, stack_method
=auto;, score=0.770 total time= 2.2s
[CV 2/3] END final_estimator__C=0.1, final_estimator__solver=liblinear, stack_method
=auto;, score=0.772 total time= 2.2s
[CV 3/3] END final_estimator__C=0.1, final_estimator__solver=liblinear, stack_method
=auto;, score=0.772 total time= 2.2s
[CV 1/3] END final_estimator__C=0.1, final_estimator__solver=liblinear, stack_method
=predict_proba;, score=0.770 total time= 2.2s
[CV 2/3] END final_estimator__C=0.1, final_estimator__solver=liblinear, stack_method
=predict_proba;, score=0.772 total time= 2.7s
[CV 3/3] END final_estimator__C=0.1, final_estimator__solver=liblinear, stack_method
=predict_proba;, score=0.772 total time= 2.3s
[CV 1/3] END final_estimator__C=0.1, final_estimator__solver=mylr, stack_method=aut
o;, score=nan total time=
                           2.2s
[CV 2/3] END final_estimator__C=0.1, final_estimator__solver=mylr, stack_method=aut
o;, score=nan total time= 2.2s
[CV 3/3] END final_estimator__C=0.1, final_estimator__solver=mylr, stack_method=aut
o;, score=nan total time= 2.2s
[CV 1/3] END final_estimator__C=0.1, final_estimator__solver=mylr, stack_method=pred
ict_proba;, score=nan total time= 2.4s
[CV 2/3] END final_estimator__C=0.1, final_estimator__solver=mylr, stack_method=pred
ict_proba;, score=nan total time= 2.6s
[CV 3/3] END final_estimator__C=0.1, final_estimator__solver=mylr, stack_method=pred
ict proba;, score=nan total time=
                                  2.2s
[CV 1/3] END final_estimator__C=1.0, final_estimator__solver=lbfgs, stack_method=aut
o;, score=0.770 total time= 2.2s
[CV 2/3] END final_estimator__C=1.0, final_estimator__solver=lbfgs, stack_method=aut
o;, score=0.772 total time= 2.2s
[CV 3/3] END final_estimator__C=1.0, final_estimator__solver=lbfgs, stack_method=aut
o;, score=0.755 total time= 2.2s
[{\it CV~1/3}] \ {\it END~final\_estimator\_C=1.0,~final\_estimator\_\_solver=lbfgs,~stack\_method=pre}
dict_proba;, score=0.770 total time= 2.8s
[CV 2/3] END final_estimator__C=1.0, final_estimator__solver=lbfgs, stack_method=pre
dict_proba;, score=0.772 total time= 2.2s
[CV 3/3] END final_estimator__C=1.0, final_estimator__solver=lbfgs, stack_method=pre
dict_proba;, score=0.755 total time= 2.2s
[CV 1/3] END final_estimator__C=1.0, final_estimator__solver=liblinear, stack_method
=auto;, score=0.770 total time= 2.2s
[CV 2/3] END final_estimator__C=1.0, final_estimator__solver=liblinear, stack_method
=auto;, score=0.772 total time= 2.2s
[CV 3/3] END final_estimator__C=1.0, final_estimator__solver=liblinear, stack_method
=auto;, score=0.769 total time= 2.6s
[CV 1/3] END final_estimator__C=1.0, final_estimator__solver=liblinear, stack_method
```

- =predict\_proba;, score=0.770 total time= 2.5s
- [CV 2/3] END final\_estimator\_\_C=1.0, final\_estimator\_\_solver=liblinear, stack\_method =predict proba;, score=0.772 total time= 2.2s
- [CV 3/3] END final\_estimator\_\_C=1.0, final\_estimator\_\_solver=liblinear, stack\_method =predict\_proba;, score=0.769 total time= 2.2s
- [CV 1/3] END final\_estimator\_\_C=1.0, final\_estimator\_\_solver=mylr, stack\_method=aut o;, score=nan total time= 2.2s
- [CV 2/3] END final\_estimator\_\_C=1.0, final\_estimator\_\_solver=mylr, stack\_method=aut o;, score=nan total time= 2.3s
- [CV 3/3] END final\_estimator\_\_C=1.0, final\_estimator\_\_solver=mylr, stack\_method=aut o;, score=nan total time= 2.7s
- [CV 1/3] END final\_estimator\_\_C=1.0, final\_estimator\_\_solver=mylr, stack\_method=pred ict\_proba;, score=nan total time= 2.2s
- [CV 2/3] END final\_estimator\_\_C=1.0, final\_estimator\_\_solver=mylr, stack\_method=pred ict\_proba;, score=nan total time= 2.2s
- [CV 3/3] END final\_estimator\_\_C=1.0, final\_estimator\_\_solver=mylr, stack\_method=pred ict\_proba;, score=nan total time= 2.2s
- [CV 1/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=lbfgs, stack\_method=au to;, score=0.770 total time= 2.2s
- [CV 2/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=lbfgs, stack\_method=au to;, score=0.772 total time= 2.7s
- [CV 3/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=lbfgs, stack\_method=au to;, score=0.745 total time= 2.3s
- [CV 1/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=lbfgs, stack\_method=pr edict\_proba;, score=0.770 total time= 2.2s
- [CV 2/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=lbfgs, stack\_method=pr edict\_proba;, score=0.772 total time= 2.2s
- [CV 3/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=lbfgs, stack\_method=pr edict\_proba;, score=0.745 total time= 2.2s
- [CV 1/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=liblinear, stack\_metho d=auto;, score=0.771 total time= 2.4s
- [CV 2/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=liblinear, stack\_metho d=auto;, score=0.772 total time= 2.6s
- [CV 3/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=liblinear, stack\_metho d=auto;, score=0.741 total time= 2.2s
- [CV 1/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=liblinear, stack\_metho d=predict\_proba;, score=0.771 total time= 2.7s
- [CV 2/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=liblinear, stack\_metho d=predict\_proba;, score=0.772 total time= 2.2s
- [CV 3/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=liblinear, stack\_metho d=predict\_proba;, score=0.741 total time= 2.3s
- [CV 1/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=mylr, stack\_method=aut o;, score=nan total time= 2.7s
- [CV 2/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=mylr, stack\_method=aut o;, score=nan total time= 2.2s
- [CV 3/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=mylr, stack\_method=aut o;, score=nan total time= 2.2s
- [CV 1/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=mylr, stack\_method=pre dict\_proba;, score=nan total time= 2.2s
- [CV 2/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=mylr, stack\_method=pre dict\_proba;, score=nan total time= 2.2s
- [CV 3/3] END final\_estimator\_\_C=10.0, final\_estimator\_\_solver=mylr, stack\_method=pre dict\_proba;, score=nan total time= 2.7s

```
/usr/local/lib/python3.11/dist-packages/sklearn/model_selection/_validation.py:528:
FitFailedWarning:
18 fits failed out of a total of 54.
The score on these train-test partitions for these parameters will be set to nan.
If these failures are not expected, you can try to debug them by setting error score
='raise'.
Below are more details about the failures:
18 fits failed with the following error:
Traceback (most recent call last):
  File "/usr/local/lib/python3.11/dist-packages/sklearn/model_selection/_validation.
py", line 866, in _fit_and score
    estimator.fit(X_train, y_train, **fit_params)
  File "/usr/local/lib/python3.11/dist-packages/sklearn/utils/validation.py", line 6
3, in inner_f
    return f(*args, **kwargs)
          ^^^^^^
  File "/usr/local/lib/python3.11/dist-packages/sklearn/ensemble/_stacking.py", line
717, in fit
    return super().fit(X, y_encoded, **fit_params)
           ^^^^^^
  File "/usr/local/lib/python3.11/dist-packages/sklearn/base.py", line 1389, in wrap
per
    return fit_method(estimator, *args, **kwargs)
           ^^^^^^
  File "/usr/local/lib/python3.11/dist-packages/sklearn/ensemble/_stacking.py", line
278, in fit
    _fit_single_estimator(self.final_estimator_, X_meta, y, fit_params=fit_params)
  File "/usr/local/lib/python3.11/dist-packages/sklearn/ensemble/_base.py", line 39,
in _fit_single_estimator
   estimator.fit(X, y, **fit_params)
  File "/usr/local/lib/python3.11/dist-packages/sklearn/base.py", line 1382, in wrap
per
    estimator. validate params()
  File "/usr/local/lib/python3.11/dist-packages/sklearn/base.py", line 436, in _vali
date_params
    validate_parameter_constraints(
  File "/usr/local/lib/python3.11/dist-packages/sklearn/utils/_param_validation.py",
line 98, in validate_parameter_constraints
    raise InvalidParameterError(
sklearn.utils._param_validation.InvalidParameterError: The 'solver' parameter of Log
isticRegression must be a str among {'sag', 'lbfgs', 'liblinear', 'newton-cholesky',
'newton-cg', 'saga'}. Got 'mylr' instead.
  warnings.warn(some_fits_failed_message, FitFailedWarning)
/usr/local/lib/python3.11/dist-packages/sklearn/model_selection/_search.py:1108: Use
rWarning: One or more of the test scores are non-finite: [0.77149835 0.77149835 0.77
149835 0.77149835
                        nan
 0.76594857 0.76594857 0.77045776 0.77045776
                                                   nan
                                                              nan
0.76247996 0.76247996 0.76143902 0.76143902
                                                   nan
                                                              nan]
warnings.warn(
Best Stacking Parameters:
{'final_estimator__C': 0.1, 'final_estimator__solver': 'lbfgs', 'stack_method': 'aut
0'}
```

```
In [59]: print("\nBase Stacking Model Results:")
    y_pred_base = evaluate_model(base_stackmodel, X_test_scaled, y_test, X_train_scaled
    print("\n---")

    print("\nBest Stacking Model Results:")
    y_pred_best = evaluate_model(best_stackmodel, X_test_scaled, y_test, X_train_scaled)

Base Stacking Model Results:
    Accuracy: 0.6824
    Precision: 0.4663
    Recall: 0.6824
    F1 Score: 0.5540
```

Execution Time: 0.0067 seconds

- - -

Best Stacking Model Results:

Accuracy: 0.6824 Precision: 0.4657 Recall: 0.6824 F1 Score: 0.5536

Execution Time: 0.0041 seconds

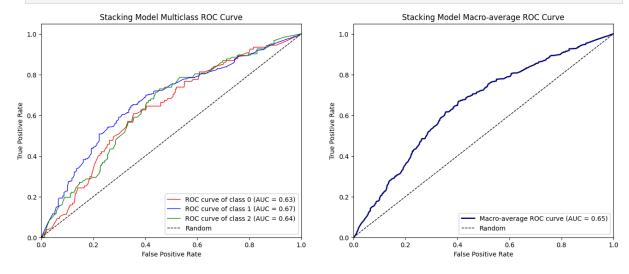
/usr/local/lib/python3.11/dist-packages/sklearn/metrics/\_classification.py:1565: Und efinedMetricWarning: Precision is ill-defined and being set to 0.0 in labels with no predicted samples. Use `zero\_division` parameter to control this behavior.

\_warn\_prf(average, modifier, f"{metric.capitalize()} is", len(result))
/usr/local/lib/python3.11/dist-packages/sklearn/metrics/\_classification.py:1565: Und efinedMetricWarning: Precision is ill-defined and being set to 0.0 in labels with no predicted samples. Use `zero\_division` parameter to control this behavior.

\_warn\_prf(average, modifier, f"{metric.capitalize()} is", len(result))

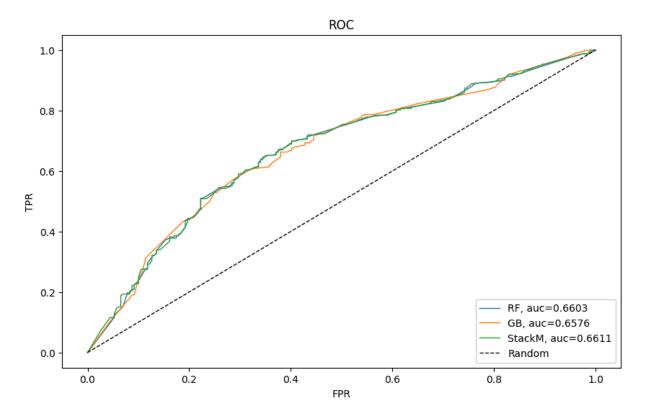
#### Illustration:

In [60]: y\_pred\_proba\_stack = best\_stackmodel.predict\_proba(X\_test\_scaled)
 plot\_multiclass\_roc(y\_test, y\_pred\_proba\_stack, title\_prefix="Stacking Model ")



```
# Get the parameters from the best_rf_params object
rf params = best rf params.get params()
# Remove 'random_state' from rf_params if it exists to avoid conflict
rf_params.pop('random_state', None)
best bagmodel = RandomForestClassifier(random state=42, **rf params).fit(X trai
# Similarly for GradientBoostingClassifier
gb params = best gb params.get params()
# Remove 'random_state' from gb_params if it exists to avoid conflict
gb_params.pop('random_state', None)
best boostmodel = GradientBoostingClassifier(random state=42, **gb params).fit(
# predicted probabilities generated by tuned classifier
y pred probaStack = best stackmodel.predict proba(X test scaled)
y_pred_proba_rf = best_bagmodel.predict_proba(X_test_scaled)
y_pred_proba_gb = best_boostmodel.predict_proba(X_test_scaled)
# Stacking Model ROC dependencies
fpr, tpr, _ = roc_curve(y_test, y_pred_probaStack[:, 1], pos_label=1)
auc = round(roc_auc_score(y_test, y_pred_probaStack, multi_class='ovr', average
# Bagging (Random Forest) Model ROC dependencies
fpr_RF, tpr_RF, _ = roc_curve(y_test, y_pred_proba_rf[:, 1], pos_label=1)
auc_RF = round(roc_auc_score(y_test, y_pred_proba_rf, multi_class='ovr', averag
# Boosting (Gradient Boosting) Model ROC dependencies
fpr_GB, tpr_GB, _ = roc_curve(y_test, y_pred_proba_gb[:, 1], pos_label=1)
auc_GB = round(roc_auc_score(y_test, y_pred_proba_gb, multi_class='ovr', averag
plt.figure(figsize=figsize) # Set the figure size here
# Bagging (Random Forest) Model
plt.plot(fpr_RF, tpr_RF, label="RF, auc=" + str(auc_RF), lw=1)
# Boosting (Gradient Boosting) Model
plt.plot(fpr_GB, tpr_GB, label="GB, auc=" + str(auc_GB), lw=1)
# Stacking Model
plt.plot(fpr, tpr, label="StackM, auc=" + str(auc), lw=1)
# Random guess model
plt.plot([0, 1], [0, 1], 'k--', lw=1, label="Random")
plt.title("ROC")
plt.ylabel("TPR")
plt.xlabel("FPR")
plt.legend(loc=4)
plt.show()
```

```
In [62]: # Plot ROC curves
plot_stacking_rf_gb_roc(best_stackmodel, best_rf_params, best_gb_params, X_test_sca
```



### Save to PDF

```
In []: from google.colab import files
    import subprocess

# Upload the notebook file
    uploaded = files.upload()

# Specify the notebook name (from the uploaded file)
    notebook_name = list(uploaded.keys())[0]
    html_name = notebook_name.replace('.ipynb', '.html')

# Convert the notebook to HTML
    subprocess.run(["jupyter", "nbconvert", "--to", "html", notebook_name])

# Verify the HTML file is created
    !ls /content

# Download the HTML file
files.download(html_name)
```

Choose Files No file chosen

Upload widget is only available when the cell has

been executed in the current browser session. Please rerun this cell to enable.

```
Saving MLiF_GWP3_g8507.ipynb to MLiF_GWP3_g8507.ipynb MLiF_GWP3_g8507.html MLiF_GWP3_g8507.ipynb sample_data
```