

3. **Altitude of a satellite:** A satellite is to be launched into a circular orbit around the Earth so that it orbits the planet once every T seconds.

(a) Show that the altitude h above the Earth's surface that the satellite must have is

$$h = \left(\frac{GMT^2}{4\pi^2} \right)^{1/3} - R,$$

where $G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ is Newton's gravitational constant, $M = 5.97 \times 10^{24} \text{ kg}$ is the mass of the Earth, and $R = 6371 \text{ km}$ is its radius.

(b) Write a program that asks the user to enter the desired value of T and then calculates and prints out the correct altitude in meters.

(c) Use your program to calculate the altitudes of satellites that orbit the Earth once a day (so-called "geosynchronous" orbit), once every 90 minutes, and once every 45 minutes. What do you conclude from the last of these calculations?

Problem 3

(A) Show that altitude h above earth orbit is

$$h = \left(\frac{GMT^2}{4\pi^2} \right)^{1/3} - R$$

Start with: $F_g = \frac{GMm}{R^2}$, $F_c = \frac{mv^2}{R}$

set equal for circular orbit

$$\frac{GMm}{R^2} = \frac{mv^2}{R} \quad \left| \quad \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T} \right)^2}{R} \right.$$

$$\frac{GM}{R^2} = \frac{4\pi^2 R^2}{T^2 R}$$

$$\frac{GMT^2}{4\pi^2} = R^3$$

$$R = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$h = \left(\frac{GMT^2}{4\pi^2} \right)^{1/3} \quad * \text{ subtract earth's radius for } h \text{ (height above earth).}$$

$$h = \left(\frac{GMT^2}{4\pi^2} \right)^{1/3} - r_{\text{earth}}$$

```
Git CMD - python -i Prob3.py
C:\Users\lab\Dropbox\Computational Physics\Jinesh_HW1>python -i Prob3.py
>>> orbit_height()
Enter desired period value in seconds: 60*90
('The altitude for the satellite to be in a circular orbit for a period of ', 5400.0, ' seconds is ', 279321.62537285965, 'm.')
>>> _
```

- 1) Asked for user input on the period.
- 2) Calculated the value for the first result to be about 279 km.
- 3) When calculating the second part of problem c, the program returned a negative value displayed below. This means that the orbit is “inside” the earth. These calculations assume that the earth is a point mass and therefore would have this sort of result. Hence there are threshold values for T that must be reached for a realistic orbit.

```
Git CMD - python -i Prob3.py
('The altitude for the satellite to be in a circular orbit for a period of ', 5400.0, ' seconds is ', 279321.62537285965, 'm.')
>>> orbit_height()
Enter desired period value in seconds: 60*45
('The altitude for the satellite to be in a circular orbit for a period of ', 2700.0, ' seconds is ', -2181559.8978108233, 'm.')
>>>
```