

As of now, this document is mostly for myself. Its missing a lot of prose and explanations (and may eventually be discarded).

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1 Introduction

All valid terms have a type who's type is of the form $*$.

All valid types have a type who's type is of the form of either \Box^σ . where σ is a Unification.

Variables in the type environment are annotated with their multiplicity. ie $x :^1 \dots$

Function literals are syntactically required to be top level only. Top level function types are internal.

Overlines are used to mean 0 or more.

After β reduction, terms not of type σ^π must either be a free variable or correspond to these β normal forms.

Type	Term
$\sigma \xrightarrow{\pi} \tau$	$\lambda x.e$
$\forall \alpha : \kappa. \sigma$	$\Lambda \alpha.e$
$\sigma \xrightarrow{\text{text}} \tau$	$\text{function}(x).e$

Some types use syntactic unification, others use boolean unification (denoted by their kind). Types which use boolean unification all permit the standard boolean operations(**true**, **false**, \wedge , \vee , etc).

2 Object Categories

e	(Terms)
x	(Variables)
$\sigma, \tau, \pi, \kappa, \rho$	(Types)
α, β	(Type Variables)
Γ	(Type Environment)
Φ	(Misc Environment)

2.0.1 Misc Categories

n	(Numbers)
sym	(Symbol)
c	(Erasure)
$c ::=$	
\circ	(Transparent)
\bullet	(Concrete)

3 Judgment Forms

The term rules are not syntax directed. The type of a term must be a valid type ($\Phi | \Gamma \vdash e : \sigma$, requires $\Phi \vdash \sigma : \kappa$).

$$\begin{array}{ll} \Phi | \Gamma \vdash e : \sigma & \text{(Term Validation)} \\ \Phi \vdash \sigma : \kappa & \text{(Type Validation)} \\ \Phi \Vdash^c \sigma & \text{(Erasure Entailment)} \end{array}$$

4 Synonyms

Synonyms for true / false in boolean unification.

Syntax	Synonym
1	false
ω	true
pure	false

5 Typing Environments

$$\begin{array}{l} \Gamma ::= \\ \quad \Gamma, x :^\pi \sigma \\ \quad \emptyset \\ \Phi ::= \\ \quad \Phi, \alpha :^c \kappa \\ \quad \emptyset \end{array}$$

Type environments can be multiplied by a multiplicity, following from Linear Haskell. Except due to the representation of multiplicity as booleans, multiplication gets mapped to disjunction.

$$\begin{aligned} \pi \Gamma &= \{(x :^{\pi \vee \pi'}) \mid (x :^{\pi'}) \in \Gamma\} \\ \Gamma, \Gamma' &= (\Gamma \oplus \Gamma') \cup \{(x^\omega) \mid (x :^\pi) \in \Gamma, (x :^{\pi'}) \in \Gamma'\} \end{aligned}$$

6 Meta

Multiplicity uses boolean unification where ω (**true**) is unrestricted and 1 (**false**) is linear.

$$\sigma, \tau, \pi, \kappa, \rho ::=$$

$*$ (Type)

\Box^σ (Kind)

\top (Top)

Syntactic

Propositional

Unification

Multiplicity

Label

$$\overline{\Phi \vdash * : \Box^{\text{Syntactic}}}$$

$$\frac{\Phi \vdash \sigma : \text{Unification}}{\Phi \vdash \Box^\sigma : \top}$$

$$\overline{\Phi \vdash \text{Syntactic} : \text{Unification}}$$

$$\overline{\Phi \vdash \text{Propositional} : \text{Unification}}$$

$$\overline{\Phi \vdash \text{Unification} : \top}$$

$$\overline{\Phi \vdash \text{Multiplicity} : \Box^{\text{Propositional}}}$$

$$\overline{\Phi \vdash \text{Label} : \top}$$

6.1 Variables

$e ::=$
 x (Variable)

\dots

$\sigma, \tau, \pi, \kappa, \rho ::=$
 α (Type Variable)

\dots

$$\overline{\Phi \mid \omega(\Gamma \setminus \{x :^\pi \sigma\}), x :^\pi \sigma \vdash x : \sigma}$$

$$\frac{(\alpha : \kappa) \in \Phi}{\Phi \vdash \alpha : \kappa}$$

6.2 Macro Linear Lambda Calculus

$$\begin{aligned}
e ::= & \\
& \lambda^{\uparrow} x. e && \text{(Macro Lambda)} \\
& e(\uparrow e') && \text{(Macro Application)} \\
& \dots \\
\sigma, \tau, \pi, \kappa, \rho ::= & \\
& \sigma \xrightarrow{\uparrow}_{\pi} \tau && \text{(Macro)} \\
& \dots \\
& \dots
\end{aligned}$$

$$\frac{\Phi | \Gamma, x :^{\pi} \sigma \vdash e : \tau}{\Phi | \Gamma \vdash \lambda^{\uparrow} x. e : \sigma \xrightarrow{\uparrow}_{\pi} \tau}$$

$$\frac{\Phi | \Gamma \vdash e_1 : \sigma \xrightarrow{\uparrow}_{\pi} \tau \quad \Phi | \Gamma' \vdash e_2 : \sigma}{\Phi | \Gamma, \pi \Gamma' \vdash e_1(\uparrow e_2) : \tau}$$

$$\frac{\Phi \vdash \sigma : * \quad \Phi \vdash \tau : * \quad \Phi \vdash \pi : \text{Multiplicity}}{\Phi \vdash \sigma \xrightarrow{\uparrow}_{\pi} \tau : *}$$

6.3 System-F

$$\begin{aligned}
e ::= & \\
& \Lambda \alpha. e && \text{(Type Lambda)} \\
& e(\sigma) && \text{(Type Application)} \\
& \dots \\
\sigma, \tau, \pi, \kappa, \rho ::= & \\
& \forall \alpha :^c \kappa. \sigma && \text{(Type Poly)} \\
& \dots
\end{aligned}$$

$$\frac{\Phi, \alpha :^c \kappa | \Gamma \vdash e : \sigma}{\Phi | \Gamma \vdash \Lambda \alpha. e : \forall \alpha :^c \kappa. \sigma}$$

$$\frac{\Phi|\Gamma \vdash e : \forall \alpha :^c \kappa. \sigma \quad \Phi \vdash \tau : \kappa \quad \Phi \Vdash^c \tau}{\Phi|\Gamma \vdash e(\tau) : \sigma[\tau/\alpha]}$$

$$\frac{\Phi, \alpha :^\circ \kappa \vdash \sigma : * \quad \Phi \vdash \kappa : \Box^\tau}{\Phi \vdash \forall \alpha :^c \kappa. \sigma : *}$$

7 Runtime

$$\begin{array}{ll} \sigma, \tau, \pi, \kappa, \rho ::= & \\ & +_\pi^\rho \quad \text{(Pretyping)} \\ & \text{Representation} \quad \text{(Representation)} \\ & \dots \end{array}$$

$$\frac{\Phi \vdash \rho :^\tau \text{Representation} \quad \Phi \vdash \pi : \text{Multiplicity}}{\Phi \vdash +_\pi^\rho : \Box^{\text{Syntactic}}}$$

$$\overline{\Phi \vdash \text{Representation} : \Box^{\text{Syntactic}}}$$

7.1 Regions

todo add proper patterns to rules

Regions use boolean unification where **pure** (**false**) means using no regions.

$$\begin{array}{ll} e ::= & \\ & \text{letRGN}(\Lambda \alpha. e) \quad \text{(Bind Region Type Variable (Unused))} \\ & \text{let}^\downarrow x = e; e' \quad \text{(Runtime Let)} \\ & \text{case } e \text{ of } \{x \rightarrow e'; x' \rightarrow' e''\} \quad \text{(Case)} \\ & \dots \\ \sigma, \tau, \pi, \kappa ::= & \\ & \sigma^\pi \quad \text{(Region Effect)} \\ & \text{IO} \quad \text{(IO Region)} \\ & \text{Region} \quad \text{(Region)} \\ & \dots \end{array}$$

$$\frac{\Phi, \alpha : \text{Region} \mid \Gamma \vdash e : \sigma^{\pi \vee (\alpha \wedge \rho)} \quad \alpha \notin \text{Free}(\sigma, \pi)}{\Phi \mid \Gamma \vdash \text{letRGN}(\Lambda \alpha. e) : \sigma^{\pi}}$$

$$\frac{\Phi \mid \Gamma \vdash e : \tau^{\pi} \quad \Phi \vdash \tau : +_{\kappa}^{\rho} \quad \Phi \mid \Gamma', x :^{\kappa} \tau^{\text{pure}} \vdash e' : \sigma^{\pi'} \quad \Phi \Vdash^{\bullet} \rho}{\Phi \mid \Gamma, \Gamma' \vdash \text{let}^{\downarrow} x = e; e' : \sigma^{\pi \vee \pi'}}$$

$$\frac{\Phi \mid \Gamma \vdash e : \tau^{\pi} \quad \Phi \vdash \tau : +_{\kappa}^{\rho} \quad \overline{\Phi \mid \Gamma', x :^{\kappa} \tau^{\text{pure}} \vdash e' : \sigma^{\pi}} \quad \Phi \Vdash^{\bullet} \rho}{\Phi \mid \Gamma, \overline{\Gamma'} \vdash \text{case } e \text{ of } \{x' \rightarrow e'\} : \sigma^{\pi \vee \pi'}}$$

$$\frac{\Phi \vdash \pi : \text{Region} \quad \Phi \vdash \sigma : +_{\tau}^{\rho}}{\Phi \vdash \sigma^{\pi} : *}$$

$$\overline{\Phi \vdash \text{IO} : \text{Region}}$$

$$\overline{\Phi \vdash \text{Region} : \square^{\text{Propositional}}}$$

7.2 Boxed

$$\begin{array}{ll} \sigma, \tau, \pi ::= & \\ \text{unique } \sigma & \\ \sigma @ \pi & \\ - & \text{(Boxed)} \\ \text{Pointer} & \text{(Pointer Representation)} \\ \dots & \end{array}$$

$$\frac{\Phi \vdash \sigma : -}{\Phi \vdash \text{unique } \sigma : +_1^{\text{Pointer}}}$$

$$\frac{\Phi \vdash \sigma : - \quad \pi : \text{Region}}{\Phi \vdash \sigma @ \pi : +_{\omega}^{\text{Pointer}}}$$

$$\overline{\Phi \vdash - : \Box^{\text{Syntactic}}}$$

$$\overline{\Phi \vdash \text{Pointer} : \text{Representation}}$$

7.3 Pointers

$$\begin{array}{ll}
e ::= & \\
& * e \quad \text{(Read Pointer)} \\
& * e = e' \quad \text{(Write Pointer)} \\
& \& * \quad \text{(Array to Pointer)} \\
& \& e[e'] \quad \text{(Array Increment)} \\
& \dots \\
\sigma, \tau, \pi, \kappa, \rho ::= & \\
& \sigma * \quad \text{(Pointer)} \\
& \sigma [] \quad \text{(Array Pointer)}
\end{array}$$

$$\frac{\Phi | \Gamma \vdash e : (\sigma * @ \pi')^\pi \quad \Phi \vdash \sigma : +_\omega^\rho \quad \Phi \Vdash^\bullet \rho}{\Phi | \Gamma \vdash * e : \sigma^{\pi \vee \pi'}}$$

$$\frac{\Phi | \Gamma \vdash e : (\sigma * @ \pi')^\pi \quad \Phi | \Gamma \vdash e' : \sigma^{\pi''} \quad \Phi \vdash \sigma : +_\omega^\rho \quad \Phi \Vdash^\bullet \rho}{\Phi | \Gamma \vdash * e = e' : ()^{\pi \vee \pi' \vee \pi''}}$$

$$\frac{\Phi | \Gamma \vdash e : (\sigma [] @ \pi')^\pi}{\Phi | \Gamma \vdash \& * e : (\sigma * @ \pi')^\pi}$$

$$\frac{\Phi | \Gamma \vdash e : (\sigma [] @ \pi')^\pi \quad \Phi | \Gamma' \vdash e' : (\text{unsigned integer}(\text{native}))^\pi \quad \Phi \vdash \sigma : +_\tau^\rho \quad \Phi \Vdash^\bullet \rho}{\Phi | \Gamma, \Gamma' \vdash \& e[e'] : (\sigma [] @ \pi')^\pi}$$

$$\frac{\Phi \vdash \sigma : +_\sigma^\rho}{\Phi \vdash \sigma * : -}$$

$$\frac{\Phi \vdash \sigma : +_\sigma^\rho}{\Phi \vdash \sigma [] : -}$$

7.4 Functions

function and $\sigma \xrightarrow{\text{text}}_{\pi} \tau$ are internal.

$$\begin{aligned}
 e &::= \\
 &\quad \text{extern sym} && (\text{Extern Function}) \\
 &\quad e^{\downarrow}(e') && (\text{Function Pointer Application}) \\
 &\quad \text{function}(x).e && (\text{Function Literal}) \\
 &\quad \dots \\
 \sigma, \tau, \pi, \kappa, \rho &::= \\
 &\quad \tau \xrightarrow{\downarrow}_{\pi} \sigma && (\text{Function Pointer}) \\
 &\quad \tau \xrightarrow{\text{text}}_{\pi} \sigma && (\text{Function Literal Type}) \\
 &\quad \dots
 \end{aligned}$$

$$\frac{\Phi \vdash \tau : +_{\tau'}^{\rho} \quad \Phi \Vdash^{\bullet} \rho \quad \Phi \vdash \sigma : +_{\tau''}^{\rho'} \quad \Phi \Vdash^{\bullet} \rho'}{\Phi | \Gamma \vdash \text{extern sym} : (\tau \xrightarrow{\downarrow}_{\pi} \sigma)^{\text{pure}}}$$

$$\frac{\Phi | \Gamma \vdash e : (\sigma \xrightarrow{\downarrow}_{\pi'} \tau)^{\pi} \quad \Phi | \Gamma' \vdash e' : \sigma^{\pi''} \quad \Phi \vdash \tau : +_{\tau'}^{\rho} \quad \Phi \Vdash^{\bullet} \rho}{\Phi | \Gamma, \Gamma' \vdash e^{\downarrow}(e') : \tau^{\pi \vee \pi' \vee \pi''}}$$

$$\frac{\Phi | \Gamma, x : {}^1\sigma^{\text{pure}} \vdash e : \tau^{\pi} \quad \Phi \vdash \sigma : +_{\tau'}^{\rho} \quad \Phi \Vdash^{\bullet} \rho}{\Phi | \Gamma \vdash \text{function}(x).e : \sigma \xrightarrow{\text{text}}_{\pi} \tau}$$

$$\frac{\Phi \vdash \sigma : +_{\kappa}^{\rho} \quad \Phi \vdash \tau : +_{\kappa'}^{\rho'} \quad \Phi \vdash \pi : \text{Region}}{\Phi \vdash \sigma \xrightarrow{\downarrow}_{\pi} \tau : +_{\omega}^{\text{Pointer}}}$$

$$\frac{\Phi \vdash \sigma : +_{\kappa}^{\rho} \quad \Phi \vdash \tau : +_{\kappa'}^{\rho'} \quad \Phi \vdash \pi : \text{Region}}{\Phi \vdash \sigma \xrightarrow{\text{text}}_{\pi} \tau : *}$$

7.5 Tuples

$$\begin{aligned}
 e &::= \\
 &\quad (\bar{e},) && (\text{Tuple Introduction})
 \end{aligned}$$

$$\begin{array}{ll}
\text{let}^\downarrow(\bar{x}) = e; e' & \text{(Tuple Elimination)} \\
\dots & \\
\sigma, \tau, \pi, \kappa, \rho ::= & \\
(\bar{\sigma},) & \text{(Tuple)} \\
\text{Struct}\{\bar{\rho}\} & \text{(Struct Representation)} \\
\dots &
\end{array}$$

$$\begin{array}{c}
\frac{\Phi|\bar{\Gamma} \vdash e : \sigma^\pi}{\Phi|\bar{\Gamma} \vdash (\bar{e},) : (\bar{\sigma},)^{\vee\pi}} \\
\\
\frac{\Phi|\Gamma \vdash e : (\bar{\tau},)^\pi \quad \Phi|\Gamma, x : \tau^{\text{pure}} \vdash e : \sigma^{\pi'} \quad \Phi \vdash \tau : +_\kappa^\rho \quad \Phi \Vdash^\bullet \rho}{\Phi|\Gamma, \Gamma' \vdash \text{let}^\downarrow(\bar{x}) = e; e' : \sigma^{\pi \vee \pi'}} \\
\\
\frac{\Phi \vdash \bar{\sigma} : +_\pi^\kappa}{\Phi \vdash (\bar{\sigma},) : +_{\bigwedge \pi}^{\text{Struct}\{\bar{\kappa}\}}} \\
\\
\frac{\Phi \vdash \bar{\rho} : \tau \text{ Representation}}{\Phi \vdash \text{Struct}\{\bar{\rho}\} : \bigwedge \tau \text{ Representation}}
\end{array}$$

7.6 Choices

$$\begin{array}{ll}
\sigma, \tau, \pi, \kappa, \rho ::= & \\
\text{Union}\{\bar{\rho}\} & \text{(Union Representation)} \\
\dots &
\end{array}$$

$$\frac{\Phi \vdash \bar{\rho} : \tau \text{ Representation}}{\Phi \vdash \text{Union}\{\bar{\rho}\} : \bigwedge \tau \text{ Representation}}$$

7.7 Integer Arithmetic

$$\begin{array}{ll}
e ::= & \\
n & \text{(Numeric Literal)} \\
e + e' & \text{(Addition)}
\end{array}$$

$e - e'$	(Subtraction)
$e * e'$	(Multiplication)
e / e'	(Division)
$e = e'$	(Equality)
$e \neq e'$	(Inequality)
$e < e'$	(Less Than)
$e \leq e'$	(Less Than Equal)
$e > e'$	(Greater Than)
$e \geq e'$	(Greater Than Equal)
...	
$\sigma, \tau, \pi ::=$	
$\rho \text{ integer}(\rho)$	(Number)
...	
$\sigma, \tau, \pi, \kappa, \rho ::=$	
$\text{Word}(\rho)$	(Word Representation)
8	(Byte Size)
16	(Short Size)
32	(Int Size)
64	(Long Size)
native	(Native Size)
signed	(Signed)
unsigned	(Unsigned)
Size	(Size Sort)
Signedness	(Signedness Sort)
...	

$$\frac{\Phi \Vdash^\bullet \rho'}{\Phi | \Gamma \vdash n : (\rho \text{ integer}(\rho'))^{\text{pure}}}$$

$$\frac{\Phi | \Gamma \vdash e : (\rho \text{ integer}(\rho'))^\pi \quad \Phi | \Gamma, \Gamma' \vdash e' : (\rho \text{ integer}(\rho'))^{\pi'} \quad \Phi \Vdash^\bullet \rho \quad \Phi \Vdash^\bullet \rho'}{\Phi | \Gamma, \Gamma' \vdash e + e' : (\rho \text{ integer}(\rho'))^{\pi \vee \pi'}}$$

...

$$\frac{\Phi | \Gamma \vdash e : (\rho \text{ integer}(\rho'))^\pi \quad \Phi | \Gamma, \Gamma' \vdash e' : (\rho \text{ integer}(\rho'))^{\pi'} \quad \Phi \Vdash^\bullet \rho \quad \Phi \Vdash^\bullet \rho'}{\Phi | \Gamma, \Gamma' \vdash e < e' : \text{Boolean}^{\pi \vee \pi'}}$$

...

$$\frac{\Phi \vdash \rho :^\tau \text{Signedness} \quad \Phi \vdash \rho' :^{\tau'} \text{Size}}{\Phi \vdash \rho \text{integer}(\rho') : +_\omega^{\text{Word}(\rho')}$$

$$\frac{\Phi \vdash \rho :^\tau \text{Size}}{\Phi \vdash \text{Word}(\rho) :^\tau \text{Representation}}$$

$$\overline{\Phi \vdash 8 : \text{Size}}$$

$$\overline{\Phi \vdash 16 : \text{Size}}$$

$$\overline{\Phi \vdash 32 : \text{Size}}$$

$$\overline{\Phi \vdash 64 : \text{Size}}$$

$$\overline{\Phi \vdash \text{native} : \text{Size}}$$

$$\overline{\Phi \vdash \text{signed} : \text{Signedness}}$$

$$\overline{\Phi \vdash \text{unsigned} : \text{Signedness}}$$

$$\overline{\Phi \vdash \text{Size} : \Box^{\text{Syntactic}}}$$

$$\overline{\Phi \vdash \text{Signedness} : \Box^{\text{Syntactic}}}$$

7.8 Boolean Logic

Note this has nothing to do with boolean unification.

$$e ::= \begin{array}{l} \text{true} \\ \text{false} \end{array}$$

$$\begin{array}{c} \text{if}(e)\{e'\}\text{else}\{e''\} \\ \sigma, \tau, \pi, \kappa, \rho ::= \\ \text{Boolean} \end{array}$$

$$\overline{\Phi|\Gamma \vdash \text{true} : \text{Boolean}^{\text{pure}}}$$

$$\overline{\Phi|\Gamma \vdash \text{false} : \text{Boolean}^{\text{pure}}}$$

$$\frac{\Phi|\Gamma \vdash e : \text{Boolean}^\pi \quad \Phi|\Gamma' \vdash e' : \sigma^{\pi'} \quad \Phi|\Gamma' \vdash e'' : \sigma^{\pi''}}{\Phi|\Gamma, \Gamma' \vdash \text{if}(e)\{e'\}\text{else}\{e''\} : \sigma^\pi \vee \pi' \vee \pi''}$$

$$\overline{\Phi \vdash \text{Boolean} : +_\omega^{\text{Word}(8)}}$$

7.9 Loops

$$\begin{array}{c} e ::= \\ \text{continue } e \quad \quad \quad (\text{continue}) \\ \text{break } e \quad \quad \quad (\text{break}) \\ \text{loop}(\text{let } x = e)\{e'\} \quad \quad \quad (\text{loop}) \\ \sigma, \tau, \pi, \kappa, \rho ::= \\ \text{Step } \sigma \tau \quad \quad \quad (\text{Loop Instruction}) \end{array}$$

$$\frac{\Phi|\Gamma \vdash e : \sigma^\pi}{\Phi|\Gamma \vdash \text{continue } e : (\text{Step } \tau \sigma)^\pi}$$

$$\frac{\Phi|\Gamma \vdash e : \tau^\pi \quad \Phi \vdash \sigma : +_\kappa^\rho \quad \Phi \Vdash^\bullet \rho}{\Phi|\Gamma \vdash \text{break } e : (\text{Step } \tau \sigma)^\pi}$$

$$\frac{\Phi|\Gamma \vdash e : \sigma^\pi \quad \Phi|\Gamma', x : {}^1\sigma^{\text{pure}} \vdash e' : (\text{Step } \tau \sigma)^{\pi'}}{\Phi|\Gamma, \omega\Gamma' \vdash \text{loop}(\text{let } x = e)\{e'\} : \tau^\pi \vee \pi'}$$

$$\frac{\Phi \vdash \sigma : +_\pi^\rho \quad \Phi \vdash \tau : +_{\pi'}^\rho}{\Phi \vdash \text{Step } \sigma \tau : +_1^{\text{Struct}(\text{Word}(8), \text{Union}(\rho, \rho'))}}$$

8 Erasure Entailment

$$\overline{\Phi \Vdash^\circ \sigma}$$

$$\frac{(x :^\bullet \kappa) \in \Phi}{\Phi \Vdash^\bullet x}$$

$$\overline{\Phi \Vdash^\bullet \text{Pointer}}$$

$$\frac{\overline{\Phi \Vdash^\bullet \sigma}}{\Phi \Vdash^\bullet \text{Struct}\{\overline{\sigma}\}}$$

$$\frac{\overline{\Phi \Vdash^\bullet \sigma}}{\Phi \Vdash^\bullet \text{Union}\{\overline{\sigma}\}}$$

$$\frac{\Phi \Vdash^\bullet \sigma}{\Phi \Vdash^\bullet \text{Word}(\sigma)}$$

$$\overline{\Phi \Vdash^\bullet 8}$$

$$\overline{\Phi \Vdash^\bullet 16}$$

$$\overline{\Phi \Vdash^\bullet 32}$$

$$\overline{\Phi \Vdash^\bullet 64}$$

$$\overline{\Phi \Vdash^\bullet \text{native}}$$

$$\overline{\Phi \Vdash^\bullet \text{unsigned}}$$

$$\overline{\Phi \Vdash^\bullet \text{signed}}$$