IMPRO3

Logistic Regression

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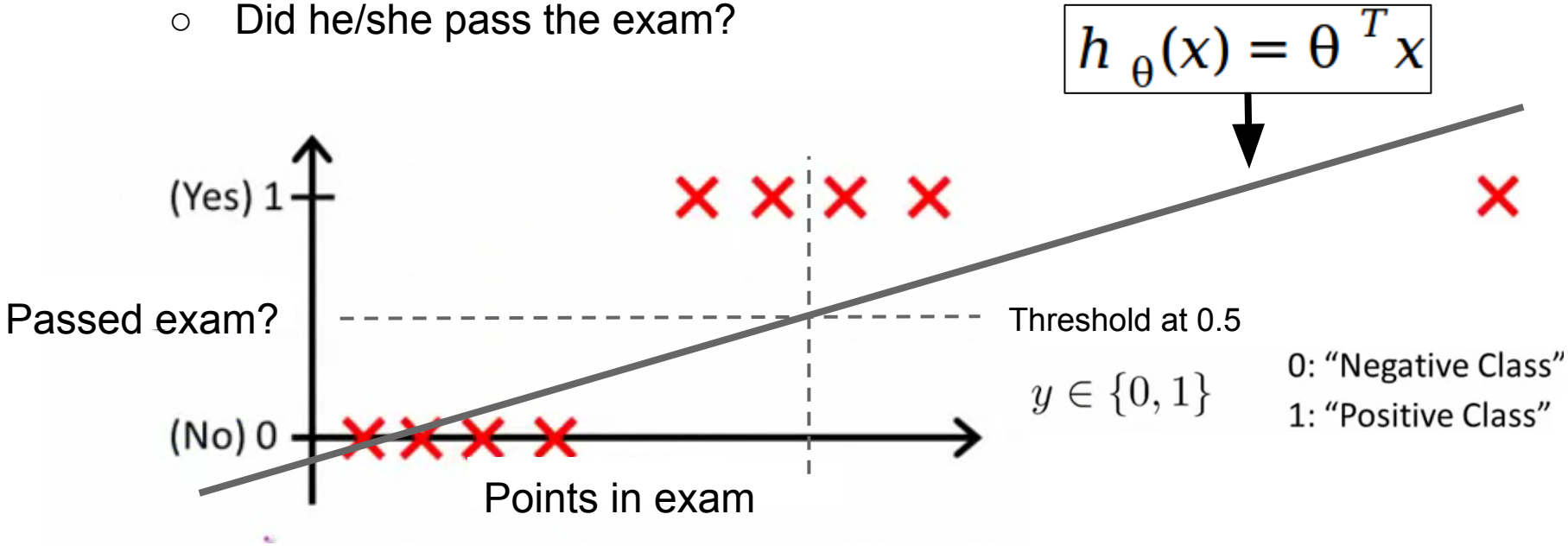
Motivation

- Logistic Regression is for **Classification**
- Typically binary classification
  - Is this mail spam?
  - Did he/she pass the exam?



- In example: $h_\theta(x) < 0$ *and* $h_\theta(x) > 1$ *are possible*
- With Logistic Regression: $0 \leq h_\theta(x) \leq 1$

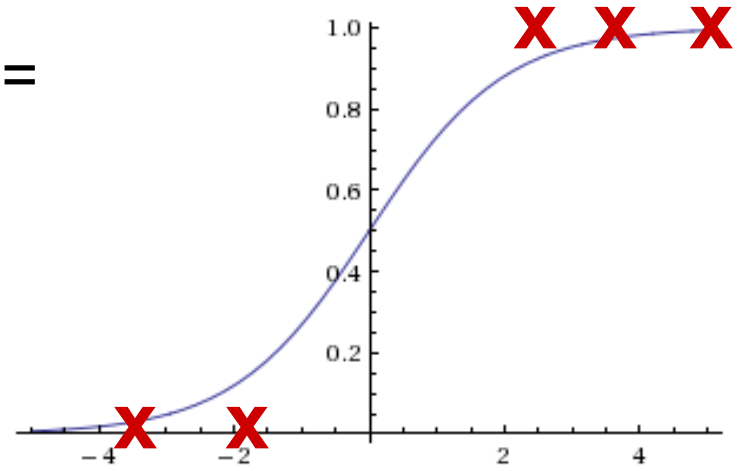
source: coursera/Stanford Machine Learning by Andrew Ng

Hypothesis and Cost Representation

Sigmoid Function = Logistic Function =

$$g(z) = \frac{1}{1+e^{-z}} \quad with\ h_\theta(x) = g(\theta^T x)$$

$$\Rightarrow h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$



wolframalpha.com: plot g(z)=(1/(1+e^(-z))) from z = -5 to +5

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost(h_\theta(x^{(i)}), y^{(i)}) \qquad Cost(h_\theta(x), y) = \begin{cases} -log(h_\theta(x)) & if\ y = 1 \\ -log(1-h_\theta(x)) & if\ y = 0 \end{cases}$$

=> We want to minimize cost J

=> Gradient Descent, repeat:

$$\theta_j = \theta_j - \alpha \frac{\Delta J(\theta)}{\Delta \theta_j} with\ \frac{\Delta J(\theta)}{\Delta \theta_j} = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})\, x_j^{(i)}$$

Pseudocode

```
X = [m, n]      // training set of features
y = [m]         // vector of classification
alpha = 1       // learning rate
theta = [n] -> all 0

Gradient descent:
for 1:number_iterations
   for i = 1:n
      grad(i) = 0;
      for j = 1:m
         grad(i) += (sigmoid(X(j,:)*theta)-y(j))*X(j,i));
      end
      grad(i) = grad(i)/m;
   end
   theta = theta - alpha * grad;
end
```

derivative of cost function

h(x)

Very naive way,
can be vectorized