

Chemically Homogeneous Evolution in COMPAS

This implementation of Chemically Homogeneous Evolution in COMPAS is a fairly naïve treatment of Chemically Homogeneous stars. Changes to COMPAS to support chemically homogeneous stars are:

- A new stellar type and corresponding class for chemically homogeneous stars. The new stellar type is `CHEMICALLY_HOMOGENEOUS`, and the class `CHE`. The `CHE` class inherits from the `MS_GT_07` class.
- A new Program Option, “chemically-homogeneous-evolution”, of type `CHE_OPTION` (declared in `constants.h`). Program option “chemically-homogeneous-evolution” can take the values:
 - `NONE` indicating that the chemically homogeneous functionality is disabled - no check will be made at birth for chemical homogeneity, and no stars will be assigned the stellar type `CHEMICALLY_HOMOGENEOUS`.
 - `PESSIMISTIC` indicating that the chemically homogeneous functionality is enabled - stars will be checked at birth against the criterion for chemical homogeneity, and assigned the stellar type `CHEMICALLY_HOMOGENEOUS` if the criterion is satisfied. While the star remains on the `CHEMICALLY_HOMOGENEOUS` phase, the criterion for chemical homogeneity will be checked at every timestep and, if the criterion is no longer satisfied the star will evolve immediately to a main sequence star (`MS_GT_07`) and continue to evolve on the main sequence.
 - `OPTIMISTIC` indicating that the chemically homogeneous functionality is enabled - stars will be checked at birth against the criterion for chemical homogeneity, and assigned the stellar type `CHEMICALLY_HOMOGENEOUS` if the criterion is satisfied. The criterion for chemical homogeneity will not be checked at every timestep on the `CHEMICALLY_HOMOGENEOUS` phase – the star is assumed to continue to be chemically homogeneous until the phase ends (at `tMS` – the main sequence timescale).
- Changes to the evolution algorithms for both Single Star Evolution and Binary Star Evolution:
 - Per Marchant et al., 2016, a constituent star in a very close binary needs to expand up to 1.32 times its RL radius before it reaches L2 – so survives for some time through an over-contact phase. As an approximation to this, if `CHE` is enabled (`CHE_OPTION` is `OPTIMISTIC` or `PESSIMISTIC`), for a binary where at least one of the constituent stars is overflowing its Roche Lobe at birth, the masses of the stars are made equal, the orbit made circular, and the separation recalculated with angular momentum conserved. If the stars are not then touching, evolution continues. See the description of the algorithm below.
 - Changes to the determination of the stellar class at the birth of a star – whether the star should be a main sequence star (stellar types `MS_LTE_07` and `MS_GT_07`) or a chemically homogeneous star (stellar type `CHEMICALLY_HOMOGENEOUS`). See the description of the algorithm below.
 - Other changes in the evolution algorithm are handled by the new stellar class – calculation of the star’s radius, whether the star should continue to evolve as a chemically homogeneous star, and what stellar type it evolves to when it finishes evolving as a chemically homogeneous star. See the description of the algorithm below.

The sole criterion for determining if a star is chemically homogeneous is whether its rotational frequency Ω is at least as large as the minimum rotational frequency for chemical homogeneity to occur (Ω_{che}), according to the fit developed by Ilya from the plots in Butler, 2018.

For a star of mass M (M_{\odot}), and metallicity Z , the minimum rotational frequency Ω_{che} ($rad \cdot s^{-1}$) for which chemical homogeneity will occur is given by:

$$\Omega_{che}(M, Z) = \frac{\Omega_{che}(M, Z = 0.004)}{0.09 \ln(\frac{Z}{0.004}) + 1}$$

where

$$\Omega_{che}(M, Z = 0.004) = \sum_{i=0}^5 \begin{cases} a_i M^{i-0.4}, & M \leq 100 \\ \frac{a_i 100^i}{M^{0.4}}, & M > 100 \end{cases}$$

and

$$\begin{aligned} a_0 &= 5.7914\text{e-}04 \\ a_1 &= -1.9196\text{e-}06 \\ a_2 &= -4.0602\text{e-}07 \\ a_3 &= 1.0150\text{e-}08 \\ a_4 &= -9.1792\text{e-}11 \\ a_5 &= 2.9051\text{e-}13 \end{aligned}$$

The general algorithm implemented in COMPAS to evolve chemically homogeneous stars is:

1. At birth, calculate Ω and Ω_{che} for the star
2. At birth, if $\Omega \geq \Omega_{che}$ assign the stellar type CHEMICALLY_HOMOGENEOUS to the star
3. At each timestep, for a CHEMICALLY_HOMOGENEOUS star:
 - (a) the radius of the star is kept constant at the value calculated at birth
 - (b) if the relative age of the star < tMS AND
if ($\Omega \geq \Omega_{che}$ OR CHE_OPTION == OPTIMISTIC)
continue to evolve as a CHEMICALLY_HOMOGENEOUS star
 - (c) if the relative age of the star < tMS AND
if ($\Omega < \Omega_{che}$ AND CHE_OPTION == PESSIMISTIC)
switch stellar type to MS_GT_07 and continue to evolve as a MS_GT_07 star
 - (d) if the relative age of the star \geq tMS
switch stellar type to NAKED_HELIUM_STAR_MS
set He Core Mass = remaining mass of star
set relative age = 0, and
continue to evolve as a NAKED_HELIUM_STAR_MS star

There are some differences in the details of the algorithm between Single Star Evolution and Binary Star Evolution. They are:

- For Single Star Evolution, the rotational frequency (Ω) of a star is currently only calculated at the birth of the star, and is not changed throughout the life of the star. Effectively, for Single Star Evolution, chemically homogeneous evolution functionality always operates as though the OPTIMISTIC program option has been set (this could change in the future if the rotational frequency (Ω) of the star is calculated at every timestep).
- For Binary Star Evolution, for any binary that has at least one of its constituent stars overflowing its Roche Lobe at birth:
 - the masses of the stars are made equal,
 - the orbit made circular, and
 - the separation recalculated with angular momentum conserved

If the stars are not then touching, evolution continues.

- For Binary Star Evolution, tidal locking is assumed, and the rotational frequency (Ω) of both of the constituent stars is set at birth to the orbital frequency of the binary, and updated throughout the life of the binary as the orbital frequency of the binary changes. The PESSIMISTIC program option is therefore honoured for Binary Star Evolution. The determination of the stellar type of the constituent stars is done after the rotational frequencies are set at birth.

References

- [Butler, 2018] Butler, E., 2018, Evolution of Chemically Homogeneous Stars using MESA, School of Physics and Astronomy, University of Birmingham.
- [Marchant et al., 2016] Marchant, P., Langer, N., Podsiadlowski, P., Tauris, T. M., and Moriya, T. J., 2016, A new route towards merging massive black holes, *Astronomy & Astrophysics*, 588, A50 (2016).