CIS 4930/6930-002
DATA VISUALIZATION

Histograms & Correlation

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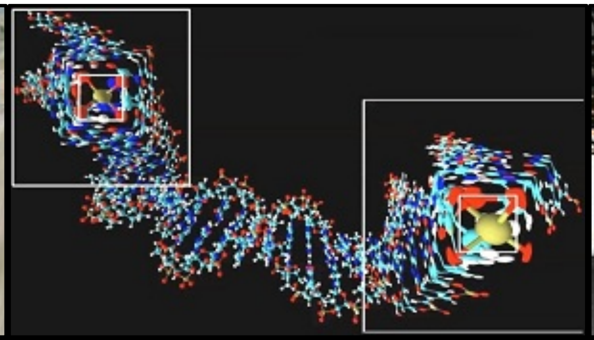
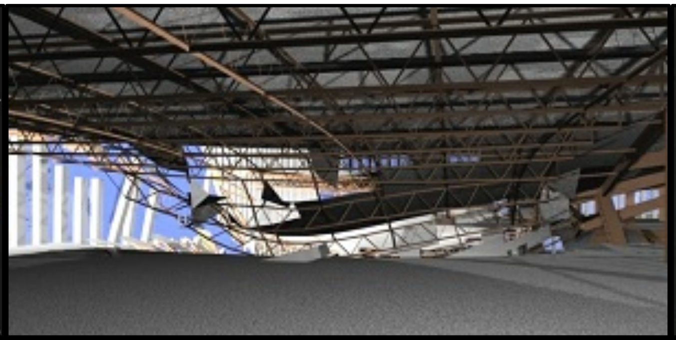
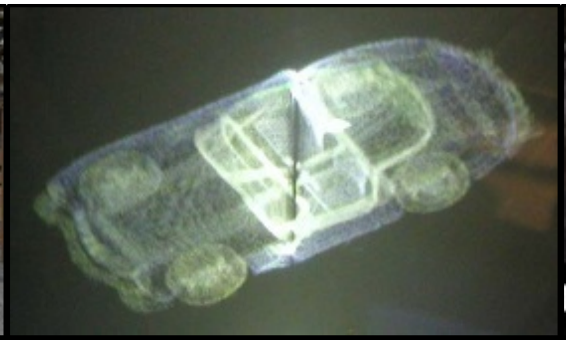





HISTOGRAMS

Bar chart-based visualization that allows evaluating distribution of values.

Given: $X=\{x_0,\ldots,x_n\}$

Select: k bins

$$bin_i = k * (x_i - \min X) / (\max X - \min X)$$

$X=\{1,2.5,3,4\}$

$k = 3$

$$X=\{1, 2.5, 3, 4\}$$

$$k = 3$$

$X=\{1,2.5,3,4\}$

$k = 3$

$$bin_i = floor(\ k * (x_i - \min X) / (\max X - \min X)\ )$$

$$X=\{1,2.5,3,4\}$$

$$k = 3$$

$$bin_i = floor(3 * (x_i - 1) / (4 - 1))$$

$X=\{1,2.5,3,4\}$

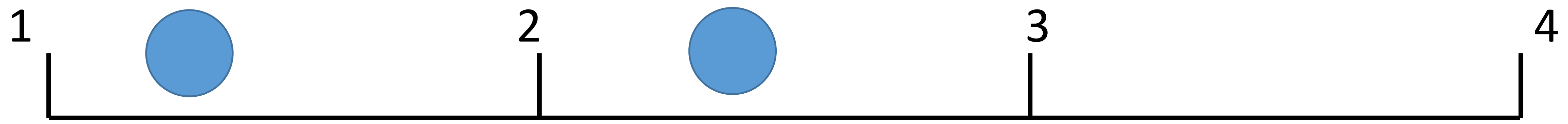
$k = 3$

1 -> floor(3 * (1 – 1) / (4 – 1)) = Bin 0

$X=\{1,2.5,3,4\}$

$k = 3$

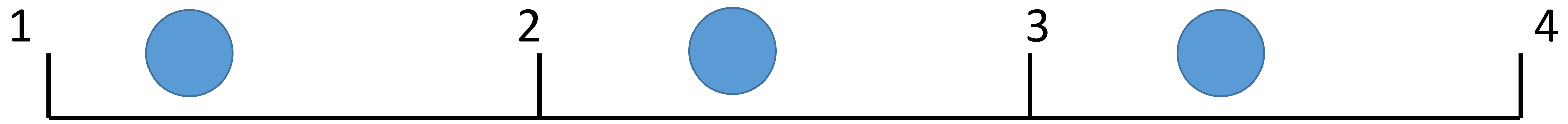
2.5 -> floor(3 * (2.5 – 1) / (4 – 1)) = Bin 1



$X=\{1,2.5,3,4\}$

$k = 3$

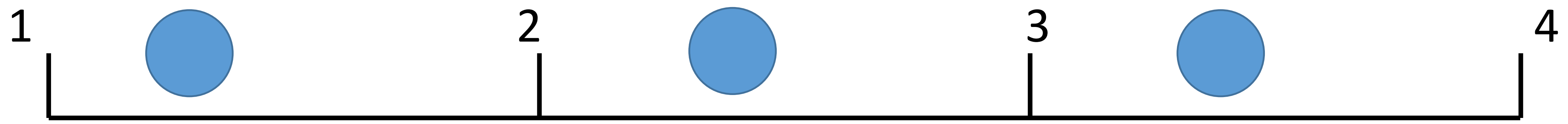
3 -> floor(3 * (3 – 1) / (4 – 1)) = Bin 2



$X=\{1,2.5,3,4\}$

$k = 3$

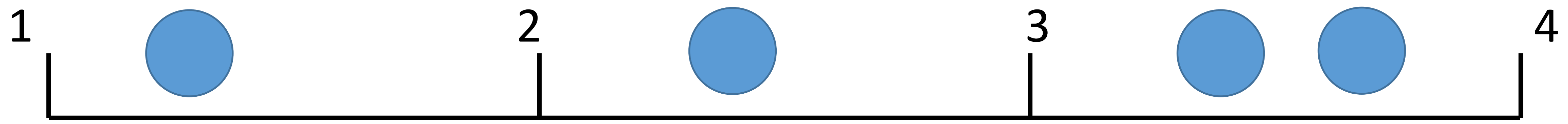
4 -> floor(3 * (4 – 1) / (4 – 1)) = Bin 3?



$X=\{1,2.5,3,4\}$

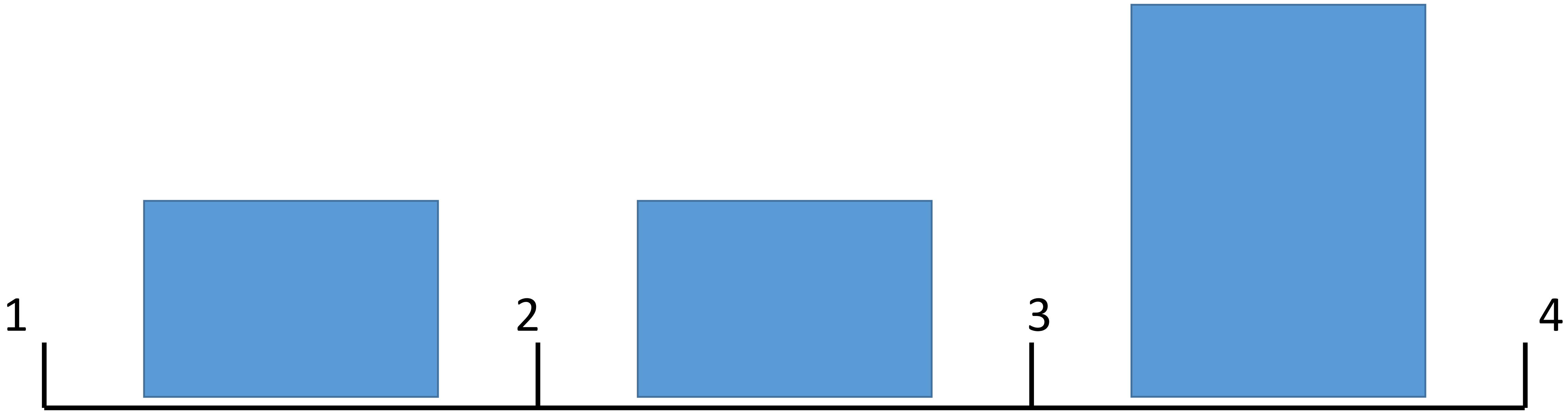
$k = 3$

$4 \rightarrow \text{floor}(\ 3 * (4 - 1) / (4 - 1)\ ) = \text{Bin } 2$



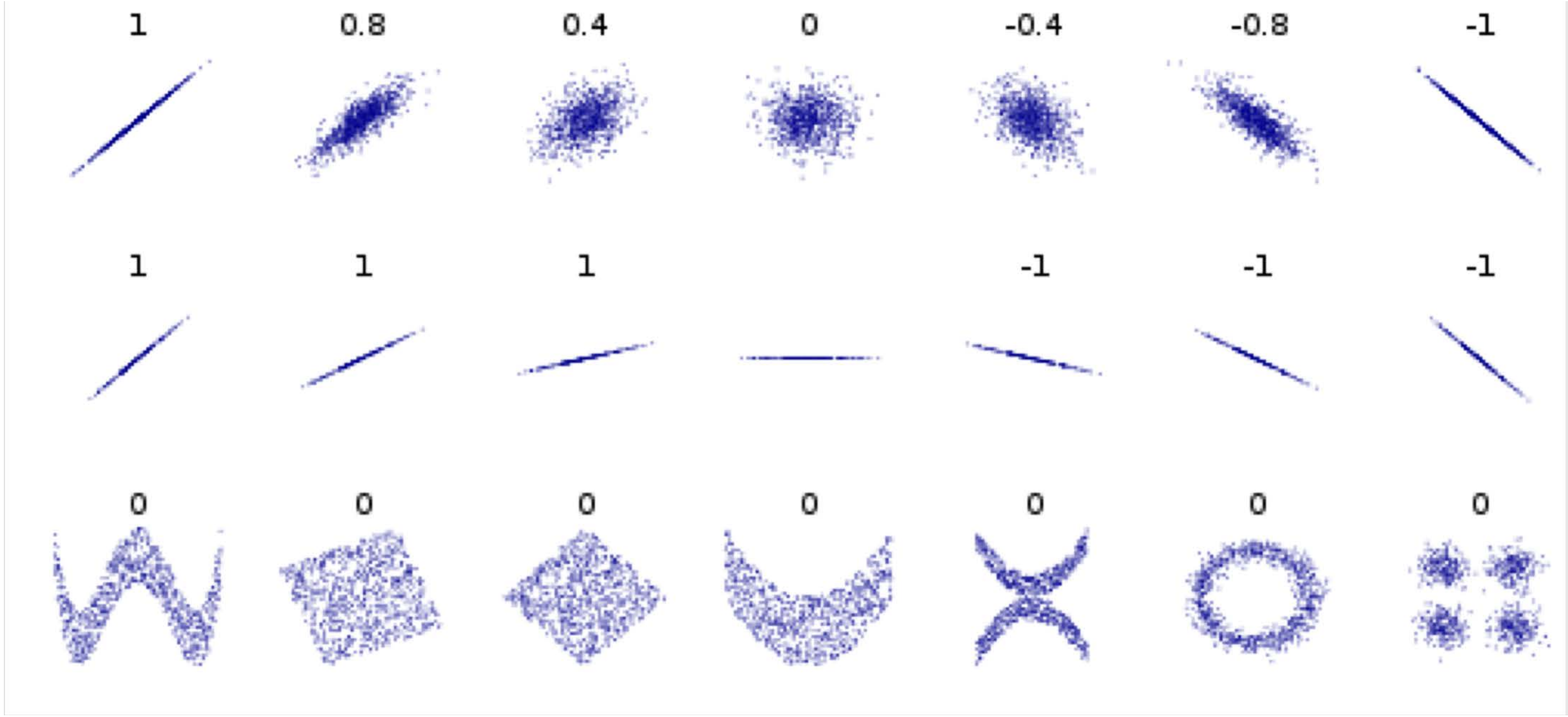
$X=\{1,2.5,3,4\}$

$k = 3$



PEARSON CORRELATION COEFFICIENT

A measure of the linearity between 2 sets


1
0.8
0.4
0
-0.4
-0.8
-1
1
1
1
-1
-1
-1
0
0
0
0
0
0
0

$$\rho_{X,Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y}$$

where:

- cov is the covariance
- σ_X is the standard deviation of X
- σ_Y is the standard deviation of Y

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

where:

- n, x_i, y_i are defined as above
- $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ (the sample mean); and analogously for $\bar{y}$

Given: $X=\{x_0,\ldots,x_n\}, Y=\{y_0,\ldots,y_n\}$

Calculate mean(X), mean(Y), stdev(X), stdev(Y)

mean(X) = $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

stdev(X) = $\sigma_X = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2}$

$$\boxed{r = \frac{1}{n}\frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sigma_X \sigma_Y}}$$

$X=\{1,2.5,3,4.5\}$
$Y=\{2,2.5,3.5,4\}$

mean(X) = 2.75, mean(Y) = 3

stdev(X)= sqrt($(1-2.75)^2 + (2.5-2.75)^2 + (3-2.75)^2 + (4.5-2.75)^2$ / 4) = 1.25
stdev(Y)= sqrt($(2-3)^2 + (2.5-3)^2 + (3.5-3)^2 + (4-3)^2$ / 4) = 0.79

X={1,2.5,3,4.5}
Y={2,2.5,3.5,4}

mean(X) = 2.75, mean(Y) = 3
stdev(X)= 1.25, stdev(Y)= 0.79

$$\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

Covariance(X,Y) =
= 1/4 * (1-2.75)(2-3) + (2.5-2.75)(2.5-3) + (3-2.75)(3.5-3) + (4.5-2.75)(4-3)
= 3.75 / 4 = 0.94

$X=\{1,2.5,3,4.5\}$

$Y=\{2,2.5,3.5,4\}$

mean(X) = 2.75, mean(Y) = 3

stdev(X)= 1.25, stdev(Y)= 0.79

Cov(X,Y)= 0.94

$$\boxed{r = 0.94 / (1.25 * 0.79) = 0.95}$$

Spearman Rank Correlation

X={1, 2.5, 3, 4.5}
Y={2, 3.5, 2.5, 4}

X' = rank(X)
Y' = rank(Y)

SRC = PCC(X', Y')

$X=\{1, 2.5, 3, 4.5\}$

X Sorted $\{1, 2.5, 3, 4.5\}$

$X' = \text{rank}(X)$

$X' = \{ \text{rank}(1), \text{rank}(2.5), \text{rank}(3), \text{rank}(4.5) \}$

$X' = \{ 1, 2, 3, 4 \}$

$$Y=\{2, 3.5, 2.5, 4\}$$

$$Y \text{ Sorted } \{2, 2.5, 3.5, 4\}$$

$$Y' = \mathrm{rank}(Y)$$

$$Y' = \{ \mathrm{rank}(2), \mathrm{rank}(3.5), \mathrm{rank}(2.5), \mathrm{rank}(4) \}$$

$$Y' = \{ 1, 3, 2, 4 \}$$