NATIONAL UNIVERSITY OF SINGAPORE

ME5402/EE5106 – ADVANCED ROBOTICS

(Semester II: AY2020-2021)

Time Allowed: ONE (1) Hour

INSTRUCTIONS TO CANDIDATES:

1. This examination paper contains **TWO (2)** questions and comprises **FIVE (5)** printed pages.
2. Answer **all TWO (2)** questions.
3. All questions carry **equal** marks.
4. This is an **OPEN BOOK** examination.

<u>Question 1</u> (50 marks)

(a) Figure 1 shows the kinematic structure of a manipulator with the link coordinate systems defined according to the standard Denavit-Hartenberg notation.

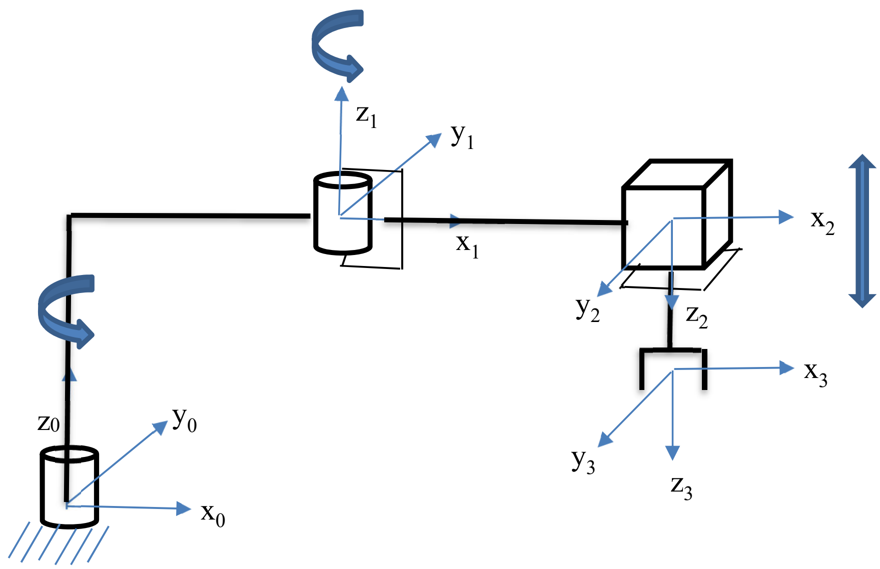

Figure 1

(i) Identify the kinematic parameters of the robotic manipulator and find the missing Denavit-Hartenberg parameters as shown in the table below.

Link number	Θ_i	d_i	α_i	a_i
1				a_1
2				a_2
3				

(ii) Derive the forward kinematic equations of the manipulator.

(15 marks)

(b) Given a purely Cartesian manipulator with 3 orthogonal prismatic joints and $\dot{\theta} = [\dot{d}_1 \quad \dot{d}_2 \quad \dot{d}_3]^T$. What is the linear velocity Jacobian of the manipulator?

Will singularity occurs with this manipulator? Briefly explain your answer.

(10 marks)

(c) Figure 2 shows the kinematic structure of a simple manipulator. Suppose that the location of the end effector is $(x, y, z)^T$ and d_3 is greater or equal to 0, solve the inverse kinematics problem by determining all the expressions of θ_1 and θ_2.

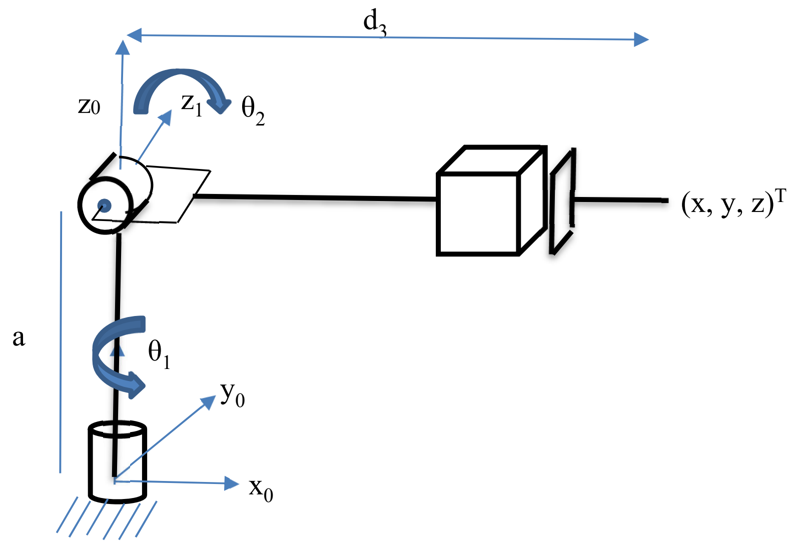

Figure 2

(10 marks)

(d) A two-link manipulator has the following Jacobian:

$$\mathbf{J} = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}$$

Assume that we can ignore the gravity. What are the joint torques if the manipulator applies a static force $\mathbf{F} = [5\ 0]^T$?

(5 marks)

(e) The following single cubic trajectory is used over the interval from $t = 0$ to $t = 1$.

$$\theta(t) = 30 + 90t^2 - 60t^3$$

Sketch the position, velocity, and acceleration of the joint as a function of time. Indicate their respective starting and final values clearly on your sketches.

(10 marks)

Question 2 (50 marks)

(a) Define $\eta_j = \sum_{i=1}^{k} m_i \ddot{r}_i^T \frac{\partial r_i}{\partial q_j}$. Show that $\eta = \frac{d}{dt}\frac{\partial K}{\partial \dot{q}} - \frac{\partial K}{\partial q}$, where K is the kinetic energy, $r_i = r_i(q)$, $i = 1, \cdots, k$, $q = [q_1, \cdots, q_n]^T$ with $q_1, \cdots, q_n$ being the generalized coordinates which are independent.

(10 marks)

(b) A three-wheel differential drive mobile robot is depicted in Figure 3. The robot is powered by two rear wheels. The global frame which is fixed in the environment or plane in which the robot moves is denoted as $\{X_I, Y_I\}$. The local frame attached to the robot is denoted as $\{X_r, Y_r\}$. The two defined frames are shown in Figure 3. The origin of the local frame is defined to be the mid-point A on the axis between the wheels. The center of mass (x_c, y_c) of the robot is assumed to be on the axis of symmetry, at a distance d from the origin A. θ represents the orientation of the robot. φ_L, φ_R are the rotation angle of left and right wheel respectively. m_c is the mass of the robot without the driving wheels and actuators, m_w is the mass of each driving wheel (with actuator), I_c is the moment of inertia of the robot about the vertical axis through the center of mass, I_w is the moment of inertia of each driving wheel with a motor along the wheel axis, and I_m is the moment of inertia of each driving wheel.

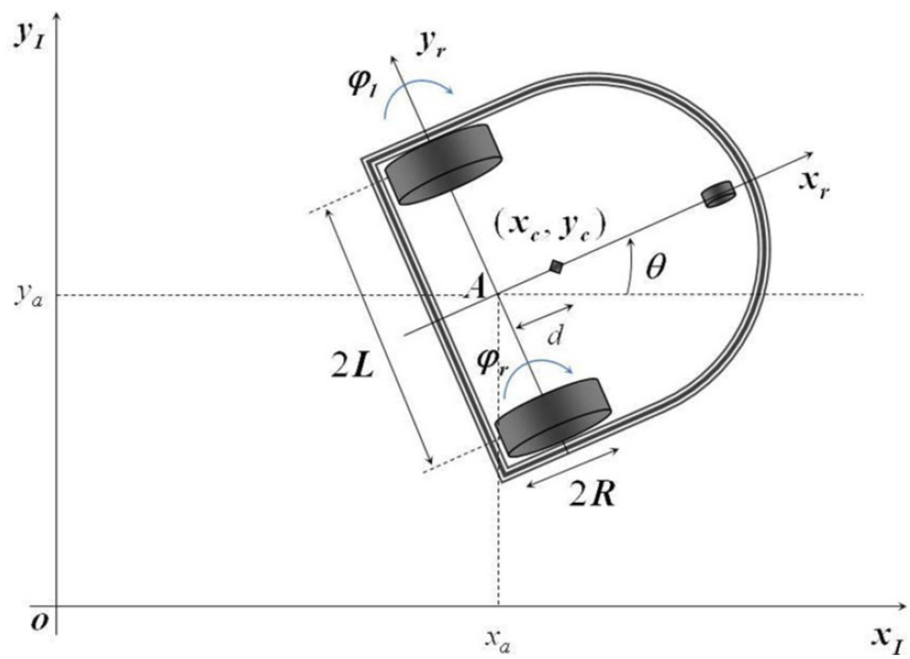

Figure 3. Schematic drawing of a differential drive mobile robot

(i) Ignore the effect the non-holonomic constraints, derive the dynamics of the robot in the following form

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = B(q)\tau$$

where $q = [x_a, y_a, \theta, \varphi_R, \varphi_L]^T$.

(20 marks)

(ii) Suppose that only q and $\dot{q}$ are measurable, design adaptive tracking control in the presence of parameter uncertainty with desired trajectory $q_d, \dot{q}_d, \ddot{q}_d$ and give a detailed stability analysis.

(20 marks)

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