

## Step 1: Import packages and classes

```
In [2]: ▶ import numpy as np
        from sklearn.linear_model import LinearRegression
```

The fundamental data type of NumPy is the array type called `numpy.ndarray`. The term array to refer to instances of the type `numpy.ndarray`.

The class `sklearn.linear_model.LinearRegression` will be used to perform linear and polynomial regression and make predictions accordingly.

## Step 2: Provide data

```
In [3]: ▶ x = np.array([5, 10, 25, 35, 45, 55]).reshape((-1, 1))
        y = np.array([5, 20, 14, 32, 22, 38])
```

`.reshape()` on `x` because this array is required to be two-dimensional, or to be more precise, to have one column and as many rows as necessary. That's exactly what the argument `(-1, 1)` of `.reshape()` specifies.

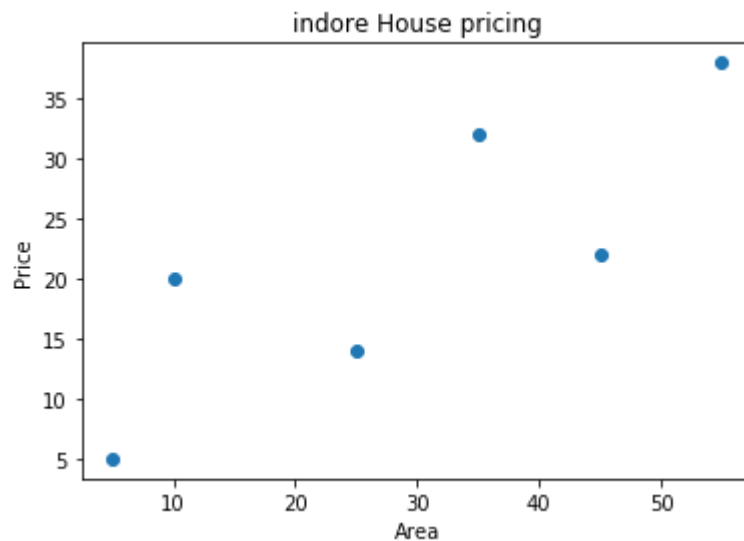
```
In [94]: ▶ print(x)
```

```
[[ 5]
 [10]
 [25]
 [35]
 [45]
 [55]]
```

```
In [95]: ▶ print(y)
```

```
[ 5 20 14 32 22 38]
```

```
In [97]: ▶ import matplotlib.pyplot as plt
plt.scatter(x,y)
plt.title('indore House pricing')
plt.xlabel('Area')
plt.ylabel('Price')
plt.show()
```



```
In [3]: ▶ print(x)
```

```
[[ 5]
 [15]
 [25]
 [35]
 [45]
 [55]]
```

```
In [4]: ▶ print(y)
```

```
[ 5 20 14 32 22 38]
```

## Step 3: Create a model and fit it

```
In [5]: ▶ model = LinearRegression()
```

```
In [6]: ▶ model.fit(x, y)
```

```
Out[6]: LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=False)
```

This statement creates the variable model as the instance of LinearRegression .

parameters to LinearRegression :

fit\_intercept is a Boolean ( True by default) that decides whether to calculate the intercept b0 ( True ) or consider it equal to zero ( False ).

normalize is a Boolean ( False by default) that decides whether to normalize the input variables ( True ) or not ( False ).

copy\_X is a Boolean ( True by default) that decides whether to copy ( True ) or overwrite the input variables ( False ).

n\_jobs is an integer or None (default) and represents the number of jobs used in parallel computation. None usually means one job and -1 to use all processors

```
In [7]: ▶ model = LinearRegression().fit(x, y)
```

## Step 4: Get results

```
In [8]: ▶ r_sq = model.score(x, y)
```

.score() , the arguments are also the predictor x and regressor y , and the return value is R2.

The attributes of model are .intercept\_ , which represents the coefficient, b0 and .coef\_ , which represents b1:

```
In [9]: ▶ print('coefficient of determination:', r_sq)
```

```
coefficient of determination: 0.715875613747954
```

```
In [10]: ▶ print('intercept:', model.intercept_)
```

```
intercept: 5.633333333333329
```

```
In [11]: ▶ print('slope:', model.coef_)
```

```
slope: [0.54]
```

```
In [12]: ▶ new_model = LinearRegression().fit(x, y.reshape((-1, 1)))
```

```
In [13]: ▶ print('intercept:', new_model.intercept_)
```

```
intercept: [5.63333333]
```

```
In [14]: ▶ print('slope:', new_model.coef_)
```

```
slope: [[0.54]]
```

## Step 5: Predict response

```
In [15]: ▶ y_pred = model.predict(x)
```

When applying `.predict()` , you pass the regressor as the argument and get the corresponding predicted response.

```
In [16]: ▶ print('predicted response:', y_pred, sep='\n')
```

```
...
```

```
In [17]: ▶ y_pred = model.intercept_ + model.coef_ * x
```

```
In [18]: ▶ print('predicted response:', y_pred, sep='\n')
```

```
predicted response:
[[ 8.33333333]
 [13.73333333]
 [19.13333333]
 [24.53333333]
 [29.93333333]
 [35.33333333]]
```

In practice, regression models are often applied for forecasts. This means that you can use fitted models to calculate the outputs based on some other, new inputs:

```
In [19]: ▶ x_new = np.arange(5).reshape((-1, 1))
```

```
In [20]: ▶ print(x_new)
```

```
[[0]
 [1]
 [2]
 [3]
 [4]]
```

```
In [21]: ▶ y_new = model.predict(x_new)
```

```
In [22]: ▶ print(y_new)
```

```
[5.63333333 6.17333333 6.71333333 7.25333333 7.79333333]
```

## Multiple Linear Regression With scikit-learn

Steps 1 and 2: Import packages and classes, and provide data

```
In [23]: ▶ import numpy as np
from sklearn.linear_model import LinearRegression
x = [[0, 1], [5, 1], [15, 2], [25, 5], [35, 11], [45, 15], [55, 34],
y = [4, 5, 20, 14, 32, 22, 38, 43]
x, y = np.array(x), np.array(y)
```

```
In [24]: ▶ print(x)
```

```
[[ 0  1]
 [ 5  1]
 [15  2]
 [25  5]
 [35 11]
 [45 15]
 [55 34]
 [60 35]]
```

In multiple linear regression, x is a two-dimensional array with at least two columns, while y is usually a one-dimensional array. This is a simple example of multiple linear regression, and x has exactly two columns.

```
In [4]: ▶ print(y)
```

```
[ 5 20 14 32 22 38]
```

## Step 3: Create a model and fit it

```
In [5]: ▶ model = LinearRegression().fit(x, y)
```

## Step 4: Get results

```
In [27]: > r_sq = model.score(x, y)
```

```
In [28]: > print('coefficient of determination:', r_sq)
```

```
coefficient of determination: 0.8615939258756776
```

```
In [29]: > print('intercept:', model.intercept_)
```

```
intercept: 5.52257927519819
```

```
In [30]: > print('slope:', model.coef_)
```

```
slope: [0.44706965 0.25502548]
```

the value of R<sup>2</sup> using `.score()` and the values of the estimators of regression coefficients with `.intercept_` and `.coef_`. Again, `.intercept_` holds the bias  $b_0$ , while now `.coef_` is an array containing  $b_1$  and  $b_2$  respectively. In this example, the intercept is approximately 5.52, and this is the value of the predicted response when  $x_1 = x_2 = 0$ . The increase of  $x_1$  by 1 yields the rise of the predicted response by 0.45. Similarly, when  $x_2$  grows by 1, the response rises by 0.26

## Step 5: Predict response

```
In [31]: > y_pred = model.predict(x)
```

```
In [32]: > print('predicted response:', y_pred, sep='\n')
```

```
predicted response:
[ 5.77760476  8.012953  12.73867497 17.9744479 23.97529728 29.466
 0957
 38.78227633 41.27265006]
```

```
In [33]: > y_pred = model.intercept_ + np.sum(model.coef_ * x, axis=1)
```

```
In [34]: ▶ print('predicted response:', y_pred, sep='\n')
```

```
predicted response:
[ 5.77760476  8.012953  12.73867497 17.9744479 23.97529728 29.466
0957
38.78227633 41.27265006]
```

```
In [35]: ▶ x_new = np.arange(10).reshape((-1, 2))
```

```
In [36]: ▶ print(x_new)
```

```
[[0 1]
 [2 3]
 [4 5]
 [6 7]
 [8 9]]
```

```
In [37]: ▶ y_new = model.predict(x_new)
```

```
In [38]: ▶ print(y_new)
```

```
[ 5.77760476  7.18179502  8.58598528  9.99017554 11.3943658 ]
```

## Polynomial Regression With scikit-learn

Implementing polynomial regression with scikit-learn is very similar to linear regression. There is only one extra step: just need to transform the array of inputs to include non-linear terms such as  $x^2$ .

### Step 1: Import packages and classes

In addition to `numpy` and `sklearn.linear_model.LinearRegression`, you should also import the class `PolynomialFeatures` from `sklearn.preprocessing`:

```
In [39]: ▶ import numpy as np
          ▶ from sklearn.linear_model import LinearRegression
          ▶ from sklearn.preprocessing import PolynomialFeatures
```

### Step 2a: Provide data

```
In [40]: x = np.array([5, 15, 25, 35, 45, 55]).reshape((-1, 1))
y = np.array([15, 11, 2, 8, 25, 32])
```

## Step 2b: Transform input data

This is the new step you need to implement for polynomial regression! As you've seen earlier, you need to include  $x^2$  (and perhaps other terms) as additional features when implementing polynomial regression. For that reason, you should transform the input array  $x$  to contain the additional column(s) with the values of  $x^2$  (and eventually more features).

```
In [41]: transformer = PolynomialFeatures(degree=2, include_bias=False)
```

```
In [42]: transformer.fit(x)
```

```
Out[42]: PolynomialFeatures(degree=2, include_bias=False, interaction_only=False,
                             order='C')
```

parameters to PolynomialFeatures :

- degree is an integer ( 2 by default) that represents the degree of the polynomial regression function.
- interaction\_only is a Boolean ( False by default) that decides whether to include only interaction features ( True ) or all features ( False ).
- include\_bias is a Boolean ( True by default) that decides whether to include the bias (intercept) column of ones ( True ) or not ( False ).

```
In [43]: x_ = transformer.transform(x)
```

```
In [44]: x_ = PolynomialFeatures(degree=2, include_bias=False).fit_transform(x)
```

That's fitting and transforming the input array in one statement with `.fit_transform()` . It also takes the input array and effectively does the same thing as `.fit()` and `.transform()` called in that order. It also returns the modified array



In [45]: `print(x_)`

```
[[ 5.  25.]  
 [15. 225.]  
 [25. 625.]  
 [35. 1225.]  
 [45. 2025.]  
 [55. 3025.]]
```

## Step 3: Create a model and fit it

In [46]: `model = LinearRegression().fit(x_, y)`

## Step 4: Get results

In [47]: `r_sq = model.score(x_, y)`

In [48]: `print('coefficient of determination:', r_sq)`

coefficient of determination: 0.8908516262498564

In [49]: `print('intercept:', model.intercept_)`

intercept: 21.372321428571425

In [50]: `print('coefficients:', model.coef_)`

coefficients: [-1.32357143 0.02839286]

Again, `.score()` returns R2. Its first argument is also the modified input `x_`, not `x`. The values of the weights are associated to `.intercept_` and `.coef_`: `.intercept_` represents  $b_0$ , while `.coef_` references the array that contains  $b_1$  and  $b_2$  respectively.

In [51]: `x_ = PolynomialFeatures(degree=2, include_bias=True).fit_transform(x_)`

In [52]: `print(x_)`

```
[[1.000e+00 5.000e+00 2.500e+01]
 [1.000e+00 1.500e+01 2.250e+02]
 [1.000e+00 2.500e+01 6.250e+02]
 [1.000e+00 3.500e+01 1.225e+03]
 [1.000e+00 4.500e+01 2.025e+03]
 [1.000e+00 5.500e+01 3.025e+03]]
```

In [53]: `model = LinearRegression(fit_intercept=False).fit(x_, y)`

In [54]: `r_sq = model.score(x_, y)`

In [55]: `print('coefficient of determination:', r_sq)`

coefficient of determination: 0.8908516262498565

In [56]: `print('intercept:', model.intercept_)`

intercept: 0.0

In [57]: `print('coefficients:', model.coef_)`

coefficients: [21.37232143 -1.32357143 0.02839286]

## Step 5: Predict response

In [58]: `y_pred = model.predict(x_)`

In [59]: `print('predicted response:', y_pred, sep='\n')`

```
predicted response:
[15.46428571  7.90714286  6.02857143  9.82857143 19.30714286 34.464
28571]
```

the prediction works almost the same way as in the case of linear regression. It just requires the modified input instead of the original.

```

In [60]: ▶ # Step 1: Import packages
import numpy as np
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import PolynomialFeatures
# Step 2a: Provide data
x = [[0, 1], [5, 1], [15, 2], [25, 5], [35, 11], [45, 15], [55, 34],
y = [4, 5, 20, 14, 32, 22, 38, 43]
x, y = np.array(x), np.array(y)
# Step 2b: Transform input data
x_ = PolynomialFeatures(degree=2, include_bias=False).fit_transform(x)
# Step 3: Create a model and fit it
model = LinearRegression().fit(x_, y)
# Step 4: Get results
r_sq = model.score(x_, y)
intercept, coefficients = model.intercept_, model.coef_
# Step 5: Predict
y_pred = model.predict(x_)
print('coefficient of determination:', r_sq)

```

coefficient of determination: 0.9453701449127822

```

In [61]: ▶ print('intercept:', intercept)

```

intercept: 0.8430556452395734

```

In [62]: ▶ print('coefficients:', coefficients, sep='\n')

```

coefficients:  
[ 2.44828275 0.16160353 -0.15259677 0.47928683 -0.4641851 ]

```

In [63]: ▶ print('predicted response:', y_pred, sep='\n')

```

predicted response:  
[ 0.54047408 11.36340283 16.07809622 15.79139 29.73858619 23.508  
34636  
39.05631386 41.92339046]

In this case, there are six regression coefficients (including the intercept), as shown in the estimated regression function  $f(x_1, x_2) = b_0 + b_1x_1 + b_2x_2 + b_3x_1^2 + b_4x_1x_2 + b_5x_2^2$ . It can also be noticed that polynomial regression yielded a higher coefficient of determination than multiple linear regression for the same problem. At first, it could be thought that obtaining such a large  $R^2$  is an excellent result. It might be. However, in real-world situations, having a complex model and  $R^2$  very close to 1 might also be a sign of overfitting. To check the performance of a model, you should test it with new data, that is with observations not used to fit (train) the model.

# Advanced Linear Regression With statsmodels

## Step 1: Import packages

```
In [64]: ▶ import numpy as np
import statsmodels.api as sm
```

## Step 2: Provide data and transform inputs

```
In [65]: ▶ x = [[0, 1], [5, 1], [15, 2], [25, 5], [35, 11], [45, 15], [55, 34],
y = [4, 5, 20, 14, 32, 22, 38, 43]
x, y = np.array(x), np.array(y)
```

```
In [66]: ▶ x = sm.add_constant(x)
```

```
In [67]: ▶ print(x)
```

```
[[ 1.  0.  1.]
 [ 1.  5.  1.]
 [ 1. 15.  2.]
 [ 1. 25.  5.]
 [ 1. 35. 11.]
 [ 1. 45. 15.]
 [ 1. 55. 34.]
 [ 1. 60. 35.]]
```

```
In [68]: ▶ print(y)
```

```
[ 4  5 20 14 32 22 38 43]
```

## Step 3: Create a model and fit it

The regression model based on ordinary least squares is an instance of the class

statsmodels.regression.linear\_model.OLS

```
In [69]: ▶ model = sm.OLS(y, x)
```

```
In [70]: ▶ results = model.fit()
```

## Step 4: Get results

In [71]: `print(results.summary())`

```

                                OLS Regression Results
=====
Dep. Variable:                  y    R-squared:
0.862
Model:                          OLS    Adj. R-squared:
0.806
Method:                          Least Squares    F-statistic:
15.56
Date:                            Fri, 20 Dec 2019    Prob (F-statistic):
0.00713
Time:                            20:47:32    Log-Likelihood:
-24.316
No. Observations:                8    AIC:
54.63
Df Residuals:                    5    BIC:
54.87
Df Model:                        2
Covariance Type:                nonrobust
=====
=====
                                coef    std err          t      P>|t|      [0.025
0.975]
-----
const          5.5226         4.431        1.246      0.268      -5.867
16.912
x1              0.4471         0.285        1.567      0.178      -0.286
1.180
x2              0.2550         0.453        0.563      0.598      -0.910
1.420
=====
=====
Omnibus:                0.561    Durbin-Watson:
3.268
Prob(Omnibus):          0.755    Jarque-Bera (JB):
0.534
Skew:                   0.380    Prob(JB):
0.766
Kurtosis:               1.987    Cond. No.
80.1
=====
=====

```

#### Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```

/home/vikas/anaconda3/lib/python3.7/site-packages/scipy/stats/stat
s.py:1450: UserWarning: kurtosistest only valid for n>=20 ... conti
nuing anyway, n=8
"anyway, n=%i" % int(n))

```

This table is very comprehensive. You can find many statistical values associated with linear regression including R2, b0, b1, and b2. In this particular case, you might obtain the warning related to kurtosistest . This is due to the small number of observations provided.

```
In [72]: ▶ print('coefficient of determination:', results.rsquared)
```

```
coefficient of determination: 0.8615939258756777
```

```
In [73]: ▶ print('adjusted coefficient of determination:', results.rsquared_adj)
```

```
adjusted coefficient of determination: 0.8062314962259488
```

```
In [74]: ▶ print('regression coefficients:', results.params)
```

```
regression coefficients: [5.52257928 0.44706965 0.25502548]
```

```
In [75]: ▶ print('predicted response:', results.fittedvalues, sep='\n')
```

```
predicted response:
[ 5.77760476  8.012953  12.73867497 17.9744479  23.97529728 29.466
 0.957
 38.78227633 41.27265006]
```

```
In [76]: ▶ print('predicted response:', results.predict(x), sep='\n')
```

```
predicted response:
[ 5.77760476  8.012953  12.73867497 17.9744479  23.97529728 29.466
 0.957
 38.78227633 41.27265006]
```

The results of linear regression:

1. .rsquared holds R2.
2. .rsquared\_adj represents adjusted R2 (R2 corrected according to the number of input features).
3. .params refers the array with b0, b1, and b2 respectively.

```
In [77]: ▶ x_new = sm.add_constant(np.arange(10).reshape((-1, 2)))
```

## Step 5: Predict response

In [78]: `print(x_new)`

```
[[1. 0. 1.]  
 [1. 2. 3.]  
 [1. 4. 5.]  
 [1. 6. 7.]  
 [1. 8. 9.]]
```

In [79]: `y_new = results.predict(x_new)`

In [80]: `print(y_new)`

```
[ 5.77760476  7.18179502  8.58598528  9.99017554 11.3943658 ]
```

In [ ]:

1. Import the packages and classes you need
2. Provide data to work with and eventually do appropriate transformations
3. Create a regression model and fit it with existing data
4. Check the results of model fitting to know whether the model is satisfactory
5. Apply the model for predictions

In [ ]: