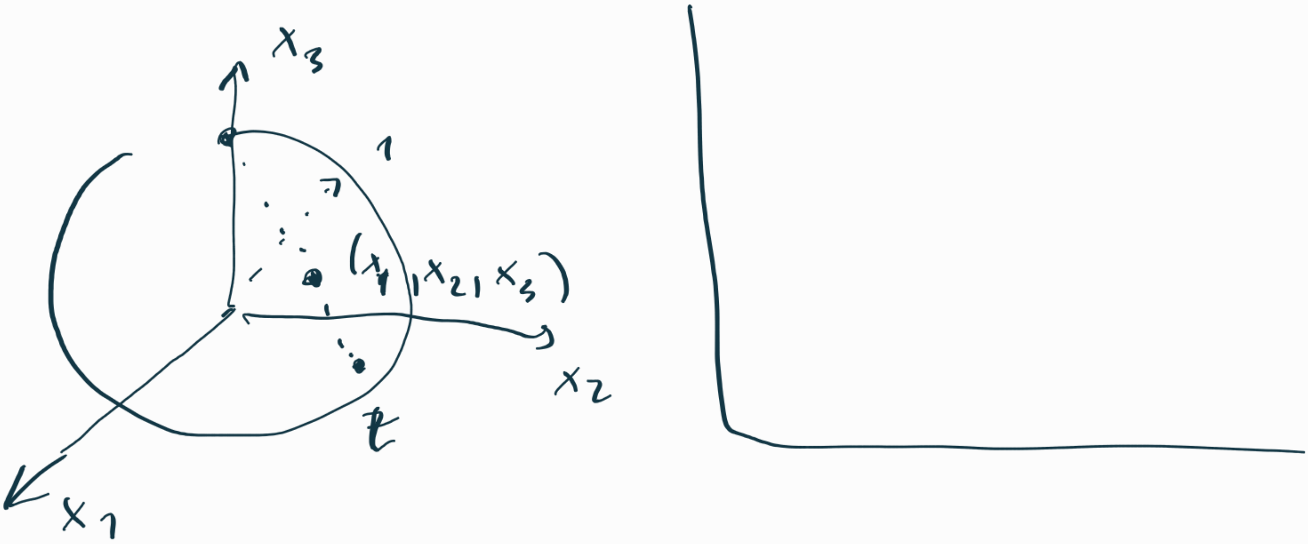


$$(x_1, x_2, x_3) \longmapsto z = \frac{x_1 + i x_2}{1 - x_3}$$

$$z_1 = x_1 + i x_2$$

$$x_1^2 + x_2^2 + x_3^2 = 1$$

$$z_2 = 1 - x_3$$

$$z = \frac{x_1 + i x_2}{1 - x_3} \frac{x_1 - i x_2}{x_1 - i x_2} =$$

$$= \frac{x_1^2 + x_2^2}{(1 - x_3)(x_1 - i x_2)} = \frac{1 - x_3^2}{(1 - x_3)(x_1 - i x_2)}$$

$$= \frac{1 + x_3}{x_1 - i x_2}$$

Rotation in the $x_1 x_2$ plane

$$(x_1\ x_2\ x_3) \longmapsto (x_1\ x_2\ x_3) \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\ldots \varphi - x_2 \sin\varphi + i(x_1 \sin\varphi + x_2 \cos\varphi)$$

$$z \longmapsto \frac{x_1 \cos\varphi - x_2 \sin\varphi + i\,(x_1 \sin\varphi + x_2 \cos\varphi)}{1-x_3} =$$

$$= \frac{x_1 e^{i\varphi} + i x_2 e^{i\varphi}}{1-x_3} = e^{i\varphi} z = \frac{e^{i\frac{\varphi}{2}} z}{e^{-i\frac{\varphi}{2}}}$$

Rotation in the $x_2 x_3$ plane

$$(x_1\, x_2\, x_3) \longmapsto (x_1\, x_2\, x_3) \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta & \sin\vartheta \\ 0 & -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$w = \frac{x_2 + i x_3}{1-x_1} \longmapsto w e^{i\vartheta}$$

$$\boxed{\frac{w+i}{w-i}} = \frac{\frac{x_2+ix_3}{1-x_1} + i}{\frac{x_2+ix_3}{1-x_1} - i} \cdot \frac{i(1-x_1)}{i(1-x_1)}$$

$$= \frac{ix_2 - x_3 - 1 + x_1}{ix_2 - x_3 + 1 - x_1} = \frac{x_1 + ix_2 - 1 - x_3}{-(x_1 - ix_2) + 1 - x_3}$$

$$= \frac{\frac{x_1+ix_2}{1-x_3}(1-x_3) - (1+x_3)}{-\frac{x_1-ix_2}{1+x_3}(1+x_3) + (1-x_3)} =$$

$$= \frac{z(1-x_3) - (1+x_3)}{\ldots}$$

$$= \frac{z}{z} \cdot \frac{\quad \ldots \quad}{-\frac{1}{z}(1+x_3) + (1-x_3)} =$$

$$= z \cdot \frac{z\cancel{(1-x_3) - (1+x_3)}}{z(1-x_3) \cancel{- (1+x_3)}} = \boxed{z}$$

$$\frac{w+i}{w-i} = z \implies w+i = zw - iz$$

$$w(1-z) = -i(1+z)$$

$$w = i\,\frac{(z+1)}{z-1}$$

$$\frac{z'+1}{z'-1} = e^{i\vartheta}\,\frac{z+1}{z-1} = u$$

$$z'+1 = uz' - u$$

$$z'(1-u) = -1-u$$

$$z' = \frac{u+1}{u-1}$$

$$z' = \frac{e^{i\vartheta}\,\frac{z+1}{z-1} + 1}{e^{i\vartheta}\,\frac{z+1}{z-1} - 1} = \frac{e^{i\vartheta}(z+1) + z - 1}{e^{i\vartheta}(z+1) - z + 1}$$

$$= \frac{z\left(e^{i\vartheta}+1\right) + e^{i\vartheta} - 1}{z\left(e^{i\vartheta}-1\right) + e^{i\vartheta} + 1} \quad \frac{1/2\, e^{i\vartheta/2}}{1/2\, e^{i\vartheta/2}} =$$

$$= z\left(\frac{e^{\frac{i\vartheta}{2}} + e^{-\frac{i\vartheta}{2}}}{2}\right) + i\,\frac{e^{\frac{i\vartheta}{2}} - e^{-\frac{i\vartheta}{2}}}{2i}$$

$$iz\left(\frac{e^{\frac{i\vartheta}{2}}-e^{-\frac{i\vartheta}{2}}}{2i}\right)+\frac{e^{\frac{i\vartheta}{2}}+e^{-\frac{i\vartheta}{2}}}{2}$$

$$=\frac{z\cos\frac{\vartheta}{2}+i\sin\frac{\vartheta}{2}}{z\,i\sin\frac{\vartheta}{2}+\cos\frac{\vartheta}{2}}$$

Complex projective plane

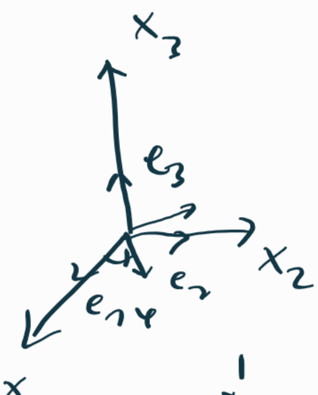
$$(z_1, z_2) \longmapsto (z_1\ z_2)\begin{pmatrix} e^{i\varphi/2} & 0 \\ 0 & e^{-i\varphi/2}\end{pmatrix}$$

$$(z_1, z_2) \sim \Big(\underbrace{\frac{z_1}{z_2}}_{z}, 1\Big) \quad \text{for } z_2 \neq 0$$

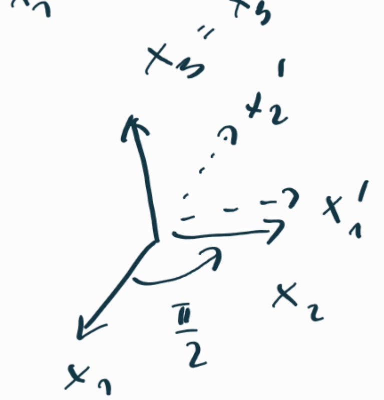
$$(z_1, z_2) \longmapsto (z_1\ z_2)\begin{pmatrix} \cos\frac{\vartheta}{2} & i\sin\frac{\vartheta}{2} \\ i\sin\frac{\vartheta}{2} & \cos\frac{\vartheta}{2}\end{pmatrix}$$

KONVENCE Z-X-Z $(\varphi, \vartheta, \psi)$ intrinsic $1.\ \psi,\ 2.\ \vartheta,\ 3.\ \varphi$

$$\begin{pmatrix} e^{i\varphi/2} & 0 \\ 0 & e^{-i\varphi/2}\end{pmatrix}\begin{pmatrix} \cos\vartheta/2 & i\sin\vartheta/2 \\ i\sin\vartheta/2 & \cos\vartheta/2\end{pmatrix}\begin{pmatrix} e^{i\psi/2} & 0 \\ 0 & e^{-i\psi/2}\end{pmatrix}$$



$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \longmapsto \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$



$$W_z(t) = \begin{pmatrix} e^{it/2} & 0 \\ 0 & e^{-it/2} \end{pmatrix}$$

$$W_x(t) = \begin{pmatrix} \cos t/2 & i \sin t/2 \\ i \sin t/2 & \cos t/2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix} \longleftarrow$$ vekt. prostor M komplexní dimenze 2 (reálné dimenze 4)

uvažujme prostor $C^\infty(M, \mathbb{C})$ a prostory $\Omega_r^\infty(M) \otimes_{\mathbb{R}} \mathbb{C}$ forem na M.

$$i : S^3 \hookrightarrow M$$

$$\frac{d}{dt}\left[f\left(\underbrace{\begin{pmatrix} e^{-it/2} & \\ & e^{it/2} \end{pmatrix} \begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix} \begin{pmatrix} e^{it/2} & \\ & e^{-it/2} \end{pmatrix}}_{} \right) \right]_{t=0}$$

$$\frac{d}{dt} f\left(\begin{pmatrix} a & b e^{-it} \\ -e^{it} \bar{b} & \bar{a} \end{pmatrix} \right) \Big|_{t=0} =$$

$$\frac{\partial f}{\ } \, 0 + \frac{\partial f}{\ } e^{-it} (-i) +$$

$$= \frac{\partial f}{\partial a}\cdot 0 + \frac{\partial f}{\partial b} e \ldots$$

$$+ \frac{\partial f}{\partial \bar b} e^{it} i \Big|_{t=0} = i\left(\frac{\partial}{\partial \bar b} - \frac{\partial}{\partial b}\right) f$$

$$\bar X_z = i\left(\frac{\partial}{\partial \bar b} - \frac{\partial}{\partial b}\right)$$

$$\bar X_x f = \frac{\partial f}{\partial a} i \cdot \frac{b+\bar b}{2} + \frac{\partial f}{\partial b} i \cdot \frac{a-\bar a}{2} + \frac{\partial f}{\partial \bar b} i \frac{a-\bar a}{2} + \frac{\partial f}{\partial \bar a}(-i)\frac{b+\bar b}{2}$$

$$\bar X_y f = \frac{\partial f}{\partial a}\frac{\bar b - b}{2} + \frac{\partial f}{\partial b}\frac{a-\bar a}{2} + \frac{\partial f}{\partial \bar b}\frac{\bar a - a}{2} + \frac{\partial f}{\partial \bar a}\frac{b-\bar b}{2}$$

$$\bar X_x = i\left(\frac{b+\bar b}{2}\frac{\partial}{\partial a} + \frac{a-\bar a}{2}\frac{\partial}{\partial b} - \frac{b+\bar b}{2}\frac{\partial}{\partial \bar a} + \frac{a-\bar a}{2}\frac{\partial}{\partial \bar b}\right) = i\left[\frac{b+\bar b}{2}\left(\frac{\partial}{\partial a} - \frac{\partial}{\partial \bar a}\right) + \frac{a-\bar a}{2}\left(\frac{\partial}{\partial b} + \frac{\partial}{\partial \bar b}\right)\right]$$

$$\bar X_y = i\left[\left(\frac{b-\bar b}{2}\right)\left(\frac{\partial}{\partial \bar a} - \frac{\partial}{\partial a}\right) + \frac{a-\bar a}{2}\left(\frac{\partial}{\partial b} - \frac{\partial}{\partial \bar b}\right)\right]$$

One-form on M ... function on S^3

Two-form on M ... one-form on S^3

$$\iota : S^3 \hookrightarrow M$$

$$da = d(a_R + i a_I) = da_R + i\, da_I$$

$$d\bar a = da_R - i\, da_I$$

$$a\bar a + b\bar b = 1$$

$$d = \boxed{\bar a\, da + a\, d\bar a + \bar b\, db + b\, d\bar b = 0}$$

$$da\left(\frac{\partial}{\partial a}\right) = 1 \qquad d\bar a\left(\frac{\partial}{\partial a}\right) = 0$$

$$(da_R + i\, da_I)\left(\frac{\frac{\partial}{\partial a_R} - i\frac{\partial}{\partial a_I}}{2}\right) = \frac{1}{2} + \frac{1}{2} = 1$$

$$(da_R - i\, da_I)\left(\frac{\frac{\partial}{\partial a_R} - i\frac{\partial}{\partial a_I}}{2}\right) = \frac{1}{2} - \frac{1}{2} = 0$$

$$X_z = i\left(\frac{\partial}{\partial \bar b} - \frac{\partial}{\partial b}\right) =$$

$$= i\left(\frac{\frac{\partial}{\partial b_R} + i\frac{\partial}{\partial b_I}}{2} - \frac{\frac{\partial}{\partial b_R} - i\frac{\partial}{\partial b_I}}{2}\right) =$$

$$= -\frac{\partial}{\partial b_I}$$

$?\ 0 = [X_z, X_x]$ ... divné?

znaménka u jednotli-vých členů!

$$X_x = -\left(b_R\frac{\partial}{\partial a_I} + a_I\frac{\partial}{\partial b_R}\right)$$

$$X_y = -\left(b_I \frac{\partial}{\partial a_R} + a_I \frac{\partial}{\partial b_R} \right)$$

$SU(2) \hookrightarrow M(2,\mathbb{C})$ ... a vector space

$SU(2)$ is isomorphic to S^3 as a manifold.

$M(2,\mathbb{C}) \cong \mathbb{C}^2$ as a vector space

funkce 4 proměnných

$(a, \bar{a}, b, \bar{b})$

$\begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix} \longleftrightarrow (a, b)$

$\begin{pmatrix} a & b \end{pmatrix}$

$\begin{pmatrix} \bar{a} & \bar{b} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$\det \begin{vmatrix} a & b \\ -\bar{b} & \bar{a} \end{vmatrix} = a\bar{a} + b\bar{b}$

Start with $V \cong \mathbb{C}^2$, construct the conjugate vector space $\bar{V}$ with scalar multiplication by complex conjugates.

$(1+i) \cdot (1, 0) = (1-i, 0)$

$(1-i) \cdot (0, 1) = (0, 1+i)$

Start with a complex vector space $V \cong \mathbb{C}^2$ with a (complex) symplectic form ω.

$\omega(u,v) = -\omega(v,u)$, ($\omega$ is nondegenerate).

In a symplectic basis we have

$\omega(e_1, e_2) = 1$

$(\omega(e_2, e_1) = -1)$

$$\omega(u^1 e_1 + u^2 e_2, v^1 e_1 + v^2 e_2) = u^1 v^2 - u^2 v^1$$

$$= \begin{Vmatrix} u^1 & u^2 \\ v^1 & v^2 \end{Vmatrix}.$$

$M \mapsto U M V^{-1}$ $\quad U, V \in SU(2)$

$\det(U M V^{-1}) = \det M$

$$\begin{pmatrix} 1 & & \cancel{0} & \\ & 1 & & \\ & & \cos t & -\sin t \\ \cancel{0} & & \sin t & \cos t \end{pmatrix}$$

$$\frac{d}{dt} f(W_z(-t)\cdot m)\Big|_{t=0}$$

$$= -\frac{i}{2}\frac{\partial f}{\partial z_1} z_1 - \frac{i}{2}\frac{\partial f}{\partial z_2} z_2 \qquad m = \begin{pmatrix} z_1 & z_2 \\ -\bar z_2 & \bar z_1 \end{pmatrix}$$

$$+ \frac{i}{2}\frac{\partial f}{\partial \bar z_1}\bar z_1 + \frac{i}{2}\frac{\partial f}{\partial \bar z_2}\bar z_2$$

$$= -\frac{i}{2}\left(\frac{\partial f}{\partial z_1} z_1 + \frac{\partial f}{\partial z_2} z_2 - \frac{\partial f}{\partial \bar z_1}\bar z_1 - \frac{\partial f}{\partial \bar z_2}\bar z_2\right)$$

$$\boxed{L_z = -\frac{i}{2}\left(z_1\frac{\partial}{\partial z_1} + z_2\frac{\partial}{\partial z_2} - \bar z_1\frac{\partial}{\partial \bar z_1} - \bar z_2\frac{\partial}{\partial \bar z_2}\right)}$$

$$\frac{d}{dt} f(m\cdot W_z(t))\Big|_{t=0}$$

$$= \frac{i}{2}\left(\frac{\partial f}{\partial z_1} z_1 - \frac{\partial f}{\partial z_2} z_2 - \frac{\partial f}{\partial \bar z_1}\bar z_1 + \frac{\partial f}{\partial \bar z_2}\bar z_2\right)$$

$$\boxed{R_z = -\frac{i}{2}\left(-z_1\frac{\partial}{\partial z_1} + z_2\frac{\partial}{\partial z_2} + \bar z_1\frac{\partial}{\partial \bar z_1} - \bar z_2\frac{\partial}{\partial \bar z_2}\right)}$$

$$L_x = -\frac{i}{2}\left(-\bar z_2\frac{\partial}{\partial z_1} + \bar z_1\frac{\partial}{\partial z_2} + z_2\frac{\partial}{\partial \bar z_1} - z_1\frac{\partial}{\partial \bar z_2}\right)$$

$$R_x = -\frac{i}{2}\left(-z_2\frac{\partial}{\partial z_1} - z_1\frac{\partial}{\partial z_2} + \bar z_2\frac{\partial}{\partial \bar z_1} + \bar z_1\frac{\partial}{\partial \bar z_2}\right)$$

Varianta 1:

$$-\frac{i\cdot i_x + i_y}{\sqrt2} = \begin{pmatrix} 0 & 0 \\ 1/\sqrt2 & 0 \end{pmatrix} = \bar f$$

$$-\frac{i\cdot i_x - i_y}{\sqrt2} = \begin{pmatrix} 0 & 1/\sqrt2 \\ 0 & 0 \end{pmatrix} = \bar e$$

$$-i\cdot i_z = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} = \bar h$$

$$[\bar e, \bar f] = \bar h \qquad [\bar h, \bar e] = \bar e \qquad [\bar h, \bar f] = -\bar f$$

$$\boxed{\begin{aligned} [i_x, i_y] &= i_z \\ [i_z, i_x] &= i_y \\ [i_y, i_z] &= i_x \end{aligned}}$$

$$i_x = \begin{pmatrix} 0 & i/2 \\ i/2 & 0 \end{pmatrix} \qquad i_y = \begin{pmatrix} 0 & -1/2 \\ 1/2 & 0 \end{pmatrix} \qquad i_z = \begin{pmatrix} 0 & i/2 \\ i/2 & 0 \end{pmatrix}$$

Varianta 2: (better, convention with math)

$$-i\cdot i_x + i_y = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = f$$

$$-i\cdot i_x - i_y = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = e$$

$$V = \langle x, y \rangle$$

$$e(x) = 0$$

$-2i \cdot i_2 \qquad = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = h$

$[e,f] = h, \quad [h,e] = 2e, \quad [h,f] = -2f$

$\boxed{e = x\frac{\partial}{\partial y} \quad f = y\frac{\partial}{\partial x} \quad h = x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y}}$

$f(x) = y$

$h(x) = x$

$e(y) = x$

$f(y) = 0$

$h(y) = -y$

One can define other rep. spaces homogeneous polynomials of degree m.

$m=4 \qquad W = \langle x^4, x^3y, x^2y^2, xy^3, y^4 \rangle$

$(e \cdot x^4 = 0)$

$$h \cdot (x^j y^{m-j}) = [j - (m-j)] x^j y^{m-j}$$

$$= (2j-m) x^j y^{m-j}$$

$\lambda \in \{-m, -m+2, \ldots, m-2, m\} \qquad 2j - m = \lambda$

$$e \cdot V_\lambda = x^{\frac{m+\lambda+2}{2}} \frac{m-\lambda}{2} y^{\frac{m-\lambda-2}{2}} \qquad V_\lambda = \langle x^{\frac{m+\lambda}{2}} y^{\frac{m-\lambda}{2}} \rangle$$

$$\in V_{\lambda+2} \qquad W_\lambda^m = x^{\frac{m+\lambda}{2}} y^{\frac{m-\lambda}{2}}$$

$$f \cdot V_\lambda = x^{\frac{m+\lambda-2}{2}} \left(\frac{m+\lambda}{2}\right) y^{\frac{m-\lambda+2}{2}} \in V_{\lambda-2}$$

$$L_x = -\frac{i}{2}\left(-\bar z_2 \frac{\partial}{\partial z_1} + \bar z_1 \frac{\partial}{\partial z_2} + z_2 \frac{\partial}{\partial \bar z_1} - z_1 \frac{\partial}{\partial \bar z_2}\right)$$

$$R_x = -\frac{i}{2}\left(-z_2 \frac{\partial}{\partial z_1} - z_1 \frac{\partial}{\partial z_2} + \bar z_2 \frac{\partial}{\partial \bar z_1} + \bar z_1 \frac{\partial}{\partial \bar z_2}\right)$$

$f(x) \mapsto$

$\mapsto f(g^{-1}x)$

$f(xg)$

$$L_y = -\frac{1}{2} \bar z_2 \frac{\partial}{\partial z_1} + \frac{1}{2} \bar z_1 \frac{\partial}{\partial z_2} - \frac{1}{2} z_2 \frac{\partial}{\partial \bar z_1} + \frac{1}{2} z_1 \frac{\partial}{\partial \bar z_2}$$

$$R_y = \frac{1}{2} z_2 \frac{\partial}{\partial z_1} - \frac{1}{2} z_1 \frac{\partial}{\partial z_2} + \frac{1}{2} \bar z_2 \frac{\partial}{\partial \bar z_1} - \frac{1}{2} \bar z_1 \frac{\partial}{\partial \bar z_2}$$

$$-iL_x + L_y = -z_2 \frac{\partial}{\partial \bar z_1} + z_1 \frac{\partial}{\partial \bar z_2} = L_f$$

$$-iL_x - L_y = \bar z_2 \frac{\partial}{\partial z_1} - \bar z_1 \frac{\partial}{\partial z_2} = L_e$$

f lowers eigenvalues
e raises eigenvalues

$$-iR_x + R_y = z_2 \frac{\partial}{\partial z_1} - \bar{z}_1 \frac{\partial}{\partial \bar{z}_2} = R_f$$

$$-iR_x - R_y = z_1 \frac{\partial}{\partial z_2} - \bar{z}_2 \frac{\partial}{\partial \bar{z}_1} = R_e$$

$$-2iL_z = -z_1 \frac{\partial}{\partial z_1} - z_2 \frac{\partial}{\partial z_2} + \bar{z}_1 \frac{\partial}{\partial \bar{z}_1} + \bar{z}_2 \frac{\partial}{\partial \bar{z}_2} = L_H$$

$$-2iR_z = z_1 \frac{\partial}{\partial z_1} - z_2 \frac{\partial}{\partial z_2} - \bar{z}_1 \frac{\partial}{\partial \bar{z}_1} + \bar{z}_2 \frac{\partial}{\partial \bar{z}_2} = R_H$$

$$V \cong \{(z_1, z_2)\} = \mathbb{C}^2$$

$$\bar{V} \cong \{(\bar{z}_1, \bar{z}_2)^T\} = \bar{\mathbb{C}}^2$$

$$V \otimes (\bar{V})^{\omega} \cong \left\{ \begin{pmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{pmatrix} \right\}$$

$$\omega = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\left[\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{z}_1 \\ \bar{z}_2 \end{pmatrix} \right]^T$$

$$(-\bar{z}_2, \bar{z}_1)$$

$$D_f = z_2 \left(\frac{\partial}{\partial z_1} - \frac{\partial}{\partial \bar{z}_1} \right) + (z_1 - \bar{z}_1) \frac{\partial}{\partial \bar{z}_2}$$

$$D_e = \bar{z}_2 \left(\frac{\partial}{\partial z_1} - \frac{\partial}{\partial \bar{z}_1} \right) + (z_1 - \bar{z}_1) \frac{\partial}{\partial z_2}$$

$$D_H = -2 z_2 \frac{\partial}{\partial z_2} + 2 \bar{z}_2 \frac{\partial}{\partial \bar{z}_2}$$

$$W^{m_1 m_2}_{\bar{m}_1 \bar{m}_2} = z_1^{m_1} z_2^{m_2} \bar{z}_1^{\bar{m}_1} \bar{z}_2^{\bar{m}_2}$$

$$L_h \Big(\underbrace{z_1^{m_1} \bar{z}_1^{\bar{m}_1} z_2^{m_2} \bar{z}_2^{\bar{m}_2}}_{W^{m_1 m_2}_{\bar{m}_1 \bar{m}_2}} \Big) = (\bar{m}_1 + \bar{m}_2 - m_1 - m_2)\, W^{m_1 m_2}_{\bar{m}_1 \bar{m}_2}$$

$$R_h \left(W^{m_1 m_2}_{\bar{m}_1 \bar{m}_2} \right) = (m_1 + \bar{m}_2 - \bar{m}_1 - m_2)\, W^{m_1 m_2}_{\bar{m}_1 \bar{m}_2}$$

$$D_h \left(W^{m_1 m_2}_{\bar{m}_1 \bar{m}_2} \right) = -2 (m_2 - \bar{m}_2)\, W^{m_1 m_2}_{\bar{m}_1 \bar{m}_2}$$

$$L_e \left(W^{m_1 m_2}_{\bar{m}_1 \bar{m}_2} \right) = -m_2\, W^{m_1 (m_2 - 1)}_{(\bar{m}_1 + 1)\, \bar{m}_2} + m_1\, W^{(m_1 - 1)\, m_2}_{\bar{m}_1 (\bar{m}_2 + 1)}$$

$$R_e \left(W^{m_1 m_2}_{\bar{m}_1 \bar{m}_2} \right) = m_2\, W^{(m_1 + 1)(m_2 - 1)}_{\bar{m}_1\, \bar{m}_2} - \bar{m}_1\, W^{m_1 m_2}_{(\bar{m}_1 - 1)(\bar{m}_2 + 1)}$$

$$D_e\left(w^{m_1\ m_2}_{\bar m_1\ \bar m_2}\right) = m_2\left(w^{(m_1+1)(m_2-1)}_{\bar m_1\quad \bar m_2} - w^{m_1(m_2-1)}_{(\bar m_1+1)\ \bar m_2}\right) - \bar m_1\, w^{m_1\ m_2}_{(\bar m_1-1)(\bar m_2+1)} + m_1\, w^{(m_1-1)\ m_2}_{\bar m_1\ (\bar m_2+1)}$$

$g \in G$

Definující akce $SU(2)$ na $\underbrace{\mathbb{C}^2}_{M}$

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \mapsto \underbrace{\begin{pmatrix} a & b \\ -\bar b & \bar a \end{pmatrix}}_{M \in SU(2)} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$m \mapsto \varphi(g, m) = g \cdot m$$

$$\underset{M}{\hat{m}} \mapsto \underset{M}{\overset{\wedge}{\varphi(g,m)}} \longmapsto \varphi(g', \varphi(g,m)) \overset{?}{=} \varphi(g'g, m)$$

$=$

$$G \times \underbrace{G}_{M} \to \underbrace{G}_{M}$$

$$(g, g') \longmapsto g g'$$

$$G \times G \longrightarrow G$$

$$\varphi: (g, h) \longmapsto g h^{-1}$$

$=$

$$\varphi(\varphi(g,h), k) = \varphi(g, kh)$$

$$\varphi(gh^{-1}, k)$$

$$\|$$

$$gh^{-1}$$

$$g^{h^{-1}k^{-1}}$$
$$\shortparallel$$
$$g(kh)^{-1}$$

$=$

$$((h,k),g) \longmapsto \underline{\underline{hgk^{-1}}}$$

$$((G\times G),G)$$

$$(h,g) \longmapsto \underline{\underline{hgh^{-1}}}$$ – konjugační akce

---

$$f \in C^\infty(G,\mathbb{C})$$

$$(g,f) \longmapsto (x \longmapsto f(g^{-1}x)) \qquad (g,f) \longmapsto (x \longmapsto f(xg))$$

$$((g_1,g_2),f) \longmapsto (x \longmapsto f(g_1^{-1}xg_2))$$

$$\left.\begin{aligned} \alpha &: V \longrightarrow \mathbb{C} \\ \alpha(av) &= \bar{a}\,\alpha(v) \end{aligned}\right\} \bar{V}$$

$$\omega : V \otimes V \to \mathbb{C}$$

$$\omega : V \to V^*$$

$$u \mapsto (v \mapsto \omega(u,v))$$

$$\omega : \bar{V} \to \underline{\underline{V}}^*$$