ERRATUM TO "SLOPE BOUNDEDNESS AND EQUIDISTRIBUTION THEOREM"

by

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Introduction

Let E be an elliptic curve over a number field or a function field K, with a non-torsion line bundle $N \in \mathrm{Pic}^0(E)$.

We may and do assume that $N = T_y^*L - L$ for some symmetric ample line bundle L on E and $y \in E(K) \setminus E_{tors}(K)$. Let $\overline{L}$ be a canonical adelic line bundle whose underlying line bundle is L. Set $\overline{N} = T_y^*\overline{L} - \overline{L}$.

Now consider $X = \mathbb{P}(T_y^*L \oplus L^\vee)$, and $M = O_X(1)$. Note that M is big on X since the maximal slope of the vecctor bundle $T_y^*L \oplus L^\vee$ is $\deg_L(E) > 0$. Let

$$V_n = H^0(X, nM) = \underset{n_1+n_2=n}{\oplus} H^0(E, n_1T_y^*L - n_2L).$$

We equip V_n with the direct sum norms of the supnorms on $H^0(E, n_1T_y^*L - n_2L)$, and denote the adelically normed vector space by $\overline{V}_n$. Then

$$\widehat{\mu}_{\min}(\overline{V}_n) = \min_{n_1+n_2=n}\{\widehat{\mu}_{\min}(H^0(E, n_1T_y^*\overline{L} - n_2\overline{L})\}.$$

If $\liminf_{n\to\infty} \frac{\widehat{\mu}_{\min}(\overline{V}_n)}{n} > -\infty$, then $\frac{1}{n_1+n_2}\widehat{\mu}_{\min}(H^0(E, n_1T_y^*\overline{L} - n_2\overline{L})$ is bounded from below. In particular

$$\liminf_{n\to\infty} \frac{1}{(2m-1)n}\widehat{\mu}_{\min}(H^0(E, n(m(T_y^*\overline{L} - \overline{L}) + \overline{L}))$$

is bounded from below uniformly for $m \in \mathbb{N}_+$.

On the other hand, since $m(T_y^*L - L) + L = mN + L$ is ample,

$$\liminf_{n\to\infty} \frac{1}{n}\widehat{\mu}_{\min}(H^0(E, nmT_y^*\overline{L} - (nm-n)\overline{L})) = \inf_{x\in E(\overline{K})} h_{mT_y^*\overline{L} - m\overline{L} + \overline{L}}(x).$$

Notice that

$$\begin{aligned} h_{mT_y^*\overline{L}-m\overline{L}+\overline{L}}(x) &= mh_{\overline{L}}(x+y)-(m-1)h_{\overline{L}}(x) \\ &= m(h_{\overline{L}}(x)+h_{\overline{L}}(y)+2\langle x,y\rangle)-(m-1)h_{\overline{L}}(x) \\ &= h_{\overline{L}}(x)+2m\langle x,y\rangle+mh_{\overline{L}}(y) \\ &= h_{\overline{L}}(x+my)-(m^2-m)h_{\overline{L}}(y). \end{aligned}$$

We have $h_{mT_y^*\overline{L}-m\overline{L}+\overline{L}}(-my) = -(m^2+m)h_{\overline{L}}(y)$, which gives a contradiction as $\frac{-m^2+m}{2m-1}$ is unbounded from below.

So the asymptotic minimal slope in the sense of [**1, 3, 4**] is unbounded. Moreover, $\overline{N}$ can be taken to be relative semipositive, so the asymptotic minimal slope in the sense of [**2**] is also unbounded.

References

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