Preference Inference Using Stan

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- Win Vector LLC is a statistics, machine learning, and data science consultancy and training organization.
- Specialize in solution design, technology evaluation, and prototyping.
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Outline

- What is Stan
- The problem
    - Some history
- Solution
    - Experiments/Results
    - Methods
- Conclusions/recommendations

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What is Stan?

Stan

- A Markov Chain Monte Carlo sampler
- Can be used for complicated Bayesian inference
  - Guesses likely values of parameters, and nuisance variables conditioned on observed data.
  - Returns distributional answers, allowing us to see if there incompatible solution.
- A massive piece of software.

The Problem

The Situation

- Users (or user personas) have intrinsic unobserved ***preferences*** or valuations
    - Personas are either representatives of a group of users or a label collecting together a group of users.
    - This lets us assume we can collect a lot of per-persona data.
- We observe user ***behaviors***
    - Typical observation: we presented 5 alternatives and the user purchased one.
    - Observation contaminated both by presentation position and alternative items in the presentation.
- Can we estimate persona preferences from persona behaviors?
    - Often called “learning to rank.”
    - We can use estimated preferences to plan (a lot more than just predicting future list behaviors).

Our Goal

- Estimate the user (or user cohort) *intrinsic* preferences (independent of presentation position and other items).

  - Allows us to rate items with respect to user to retrieve, sort, estimate utilities and so on.

  - Eliminates some systems, such as **`xgb.XGBRanker`** (as it reproduces predictions over whole lists

  - From: https://xgboost.readthedocs.io/en/stable/python/examples/learning_to_rank.html#sphx-glr-python-examples-learning-to-rank-py

```
    X_test, clicks_test, y_test, qid_test = sort_ltr_samples(
        test.X,
        test.y,
        test.qid,
        test.click,
        test.pos,
    )
…
ranker.predict(X_test)
```

(notice the `qid` column, which is means data is in presentation and we `xgb.XGBRanker` is compute probability of selection with respect to the alternatives, not utility. Would have to "sum out" many proposed panels to get intrinsic utility.)

Our Thesis

- Factor overall system performance into the product of:
  - "**Model Structure**" (how well the explanatory variables can reproduce scores).
    - We used logistic regression (great at converting yes/no observations back into coefficients and probability scores).
  - "**Presentation Un-Censoring Hygiene**" (how well we account for the censoring effects of presentation on the problem). We are treating this as a *required user supplied causal structure*.
    - May involve joining in some "facts about presentation" and "user profile" interactions.
- To improve: must know where we *actually* are losing quality.
  - Stan can simulate inverting presentation censorship.

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Example Problems

- User is shown 5 products online and clicks on one.
- User tastes 5 wines and buys at most one.
    - (repeat with 100 users thought to be in same persona)

We **wish** they would tell us their numerical valuation of each wine!

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Leaning to Rank History

- Tradition: each field develops solutions ignoring all other fields
    - BIG topic in search engines
        - Joachims, T. (2002), "Optimizing Search Engines using Clickthrough Data", *Proceedings of the ACM Conference on Knowledge Discovery and Data Mining*
        - Tie-Yan Liu (2009), "Learning to Rank for Information Retrieval", *Foundations and Trends in Information Retrieval*, **3** (3): 225–331
- Huge survey: https://en.wikipedia.org/wiki/Learning_to_rank
    - Claims that logistic regression does not work
        - “Bill Cooper proposed logistic regression for the same purpose in 1992 [19] and used it with his Berkeley research group to train a successful ranking function for TREC. Manning et al. [40] suggest that these early works achieved limited results in their time due to little available training data and poor machine learning techniques.”

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Maybe Things Can Be Easy?

- If: most of the unobserved activation lists have fewer than two activations.
- Then:
  - Most of the unobserved activation lists are then identical to the observed selection lists.
  - Problem is solvable by direct item-wise logistic regression.
- Example: online shopping with low activation rates.

Maybe Things are Difficult?

- Presentation position may be a strong influence
  - Users may be biased to pick earlier presentation positions, forget earlier positions, or even not even try/look-at later positions!
  - Item quality may correlate with presentation position.
  - What other items are in the list or panel having an effect (such as high priced irrelevant alternatives).
- Data is noisy
  - User may make different choice when re-presented
- Data is low fidelity or censored
  - Selecting 1 out of 5 positions is only 2.3 bits of information
- No-select lists ***may* be coming from an unknown mixture of two sources**
    - Motivated Shoppers (with intent to buy, so non-buying strong evidence against all presented wines).
    - Idle Browsers (with no intent to buy, so non-buying against any wine).
  - These are not the same persona! Need per-user variables to sort these users out.
  - Punishes highly desirable selections *more* than less desirable selections.

Our “Wine” Example

- Artificial Problem
    - Know answer
    - 3 explanatory variables: x1, x2, x3
- Visible Data
    - Panels of 5 wines tasted in one sitting
    - explanatory variables
    - selected (if any) wine
- Many repetitions over many customers thought to be in same persona cluster.

	group	position	example_index	x1	x2	x3	score	hidden_activation	selected
0	0	0	7	0.00	1.00	1.00	0.295	False	False
1	0	1	2	0.00	0.00	0.10	0.257	False	False
2	0	2	3	0.00	1.00	1.00	0.304	False	False
3	0	3	8	0.00	1.00	1.00	0.288	False	False
4	0	4	10	0.00	1.00	1.00	0.348	False	False
5	1	0	1	1.00	6.00	0.00	0.546	True	False
6	1	1	0	12.00	1.00	0.00	0.796	True	True
7	1	2	3	0.00	1.00	1.00	0.326	True	False
8	1	3	7	0.00	1.00	1.00	0.309	False	False
9	1	4	2	0.00	0.00	0.10	0.210	True	False
10	2	0	6	0.00	1.00	1.00	0.349	False	False
11	2	1	0	12.00	1.00	0.00	0.817	True	True
12	2	2	5	0.00	1.00	1.00	0.297	False	False
13	2	3	10	0.00	1.00	1.00	0.327	True	False
14	2	4	8	0.00	1.00	1.00	0.327	False	False

The (Assumed) Data Generating Process

Evidence pattern on selection

User considers 5 alternatives

(unobserved) explanatory variables converted into activation probabilities (scores)

0.349	**0.817**	0.297	0.327	0.327

wish we knew this

(unobserved) probabilities converted into “interested” activations check-marks

✗	✓	✗	✓	✗

activation list

(observed) checked item with highest activation score is picked

	✓			

selection list

experiment design

model structure

$f_\beta(x)$ (fit for β)

presentation censoring process

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Solution

Stan Model

- Guess: β (coefficient vector including intercept), ζ (hidden per chosen selection vector noise term).

- Let $x_{i,j}$ denote the vector of explanatory values for the ith example's jth alternative

  We model the hidden scores as being distributed normal$(x_{i,j} \cdot \beta, \sigma)$ (or $\zeta_i \sim$ normal$(0,\sigma)$).

  For lists with no selection model:

$$\text{P}[data_i \,|\, \beta, \zeta_i] \sim \prod_j (1 - \text{sigmoid}(x_{i,j} \cdot \beta))$$

  (No selection arrises from failing to select any of the alternatives.)

  For lists with a selection s model:

$$\text{P}[data_i \,|\, \beta, \zeta_i] \sim \text{sigmoid}(x_{i,s} \cdot \beta + \zeta_i) \prod_{j, j \neq s} (1 - \text{sigmoid}(x_{i,j} \cdot \beta)(1 - \Phi(x_{i,j} \cdot \beta \,|\, x_{i,s} \cdot \beta + \zeta_i, \sigma))$$

  (For a selection to prevail it must be activated, and none of the alternative must simultaneously activate and exceed the selection's score
  We (optionally) condition on an explicit ζ_i is to make the terms conditionally independent and justify multiplying.

- Set up Stan to sample β, ζ such that P$[\beta, \zeta]$P$[data \,|\, \beta, \zeta]$ is large.

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Why That Works

- We want to maximize plausibility of parameters given data: P$[\beta, \zeta \mid data]$.
- By Bayes' Law P$[\beta, \zeta \mid data] =$ P$[\beta, \zeta]$P$[data \mid \beta, \zeta]$/P$[data]$.
    - Can ignore P$[data]$ as it is free of our parameter estimates.
    - So picking β, ζ such that P$[\beta, \zeta]$P$[data \mid \beta, \zeta]$ is large is the same as picking such that P$[\beta, \zeta \mid data]$ is large.
- Can invoke ideas such as the Bernstein-von Mises theorem to argue "large" is going to concentrate samples near the (unknown) true parameter value as we add more data.
    - With enough data and flat enough P$[\beta, \zeta]$, we can even omit the P$[\beta, \zeta]$ term.
        - Structural mis-specification more risky than "wrong priors."

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Actual Stan Code

```
data {
  int<lower=1> n_vars;  // number of variables per alternative
  int<lower=2> n_alternatives;  // number of items per presentation list
  int<lower=1> m_examples;  // number of examples
  array[m_examples] int<lower=0, upper=n_alternatives> pick_index;   // which item is picked, 0 means no pick
  array[n_alternatives] matrix[m_examples, n_vars] x;  // explanatory variables
}
parameters {
  real beta_0;  // model parameters
  vector[n_vars] beta;  // model parameters
  vector[m_examples] noise_in_pick_link;
}
transformed parameters {
  array[n_alternatives] vector[m_examples] link;  // link values
  for (sel_j in 1:n_alternatives) {
    link[sel_j] = beta_0 + x[sel_j] * beta;
  }
}
model {
    // basic priors
  beta_0 ~ normal(0, 10);
  beta ~ normal(0, 10);
  noise_in_pick_link ~ normal(0, 0.1);
    // log probability of observed situation
  for (row_i in 1:m_examples) {
    if (pick_index[row_i] <= 0) {
      for (sel_j in 1:n_alternatives) {
        target += log1m(inv_logit(link[sel_j][row_i]));  // non-activation odds
      }
    } else {
      for (sel_j in 1:n_alternatives) {
        if (sel_j == pick_index[row_i]) {
          target += log(inv_logit(link[pick_index[row_i]][row_i] + noise_in_pick_link[row_i]));  // probability selection indicates
        } else {
          target += log1m(inv_logit(link[sel_j][row_i])  // probability potential spoiler indicates
                            // probability potential spoiler outscores selection
                            * (1 - normal_cdf(link[pick_index[row_i]][row_i] + noise_in_pick_link[row_i] | link[sel_j][row_i], 0.1)));
        }
      }
    }
  }
}
```

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Calling Stan from R or Python

```r
library(rstan)
library(jsonlite)

data <- fromJSON("rank_src_censored_picks.stan")

sample <- stan(
  file = "rank_src_censored_picks.stan",  # Stan program
  data = data,                            # named list of data
  chains = 4,                             # number of Markov chains
  cores = 4,                              # number of cores (could use one per chain)
  refresh = 0,                            # no progress shown
  pars=c("lp__", "beta_0", "beta")        # parameters to bring back
)

draws <- as.data.frame(sample)
```

```python
from cmdstanpy import CmdStanModel

# quiet down Stan
logger = logging.getLogger("cmdstanpy")
logger.addHandler(logging.NullHandler())

# instantiate the model object
model_comp = CmdStanModel(stan_file="rank_src_censored_picks.stan")
# sample high probability parameter settings
sample_Stan = model_comp.sample(
    data=“rank_src_censored_picks.stan”,
    show_progress=True,
    show_console=False,
)

draws = sample_Stan.draws_pd(vars=['lp__', 'beta_0', 'beta'])
```

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What you Get From Stan

```
draws |>
  head() |>
  knitr::kable()
```

lp__	beta_0	beta[1]	beta[2]	beta[3]
-1251.130	-1.520818	0.2282623	0.2879370	0.3201965
-1250.358	-1.506511	0.2233696	0.2475490	0.1977965
-1251.936	-1.552996	0.1998336	0.2522806	0.3323120
-1249.657	-1.533951	0.2080136	0.2574112	0.3573067
-1248.750	-1.493253	0.2228508	0.2416577	0.3151417
-1248.531	-1.221375	0.2030706	0.2373745	0.0258106

```
nrow(draws)
```

```
## [1] 4000
```

Result:
Activation Probabilities

	example_index	x1	x2	x3	data generation process	model from visible selections	model from pair differences	Stan model MCMC	Stan model optimizer
0	0	12.0	1.0	0.0	0.80	0.80	0.99	0.80	0.80
1	1	1.0	6.0	0.0	0.55	0.39	0.94	0.56	0.56
2	2	0.0	0.0	0.1	0.24	0.04	0.51	0.26	0.32
3	3	0.0	1.0	1.0	0.31	0.10	0.73	0.31	0.34

Notice the Stan models dominate.

Models Compared

- **Model from visible selections**: treat each item selection or non-selection as a positive or negative example. Also called “item-wise” ranking. A nice logistic regression solution for comparison.

- **Model from pair differences**: encode each pair in a list where one wine is picked and one is not as an example. A common “let’s be clever trick.”

- **Stan Model MCMC**: encode an entire probability model (including guessing unseen quantities) and then use a Markov chain to sample high likelihood values. The big dog.

- **Stan Model Optimizer**: use the above encoding as a “loss”, and hope something as simple as standard optimizer can find a high likelihood solution. “Cheap Stan” (could use other numeric platforms like PyTorch for this).

Details: Feature Encoding

$$x(item = 6425, position = 2) =$$

	6425
fixed acidity	6.0
volatile acidity	0.34
citric acid	0.29
residual sugar	6.1
chlorides	0.046
free sulfur dioxide	29.0
total sulfur dioxide	134.0
density	0.99462
pH	3.48
sulphates	0.57
alcohol	10.7
is_red	False
posn_0	0
posn_1	0
posn_2	1
posn_3	0
posn_4	0

Feature table lookup

Encoding of presentation position. Could also encode demographics of participant Especially useful if we interact the demographic variables with the feature variables.

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Details: Difference Encoding

- Exploit linear structure to directly encode differences
  - Explanatory variables: encode comparison pairs (with different outcomes).
    - For example: item 6425 in position 2 **picked** and item 1569 in position 2 **not picked** encoded as:

$$x_i = x(item = 6425, position = 2) - x(item = 1569, position = 0)$$

    - The subtraction enforces that features have the same interpretation in picks and non-picks.
    - Outcomes encodes as “True”.
- Outcomes/dependent variables must vary
  - So also encode a “False” outcome for $-x_i$
- Encoding not natural or obvious.

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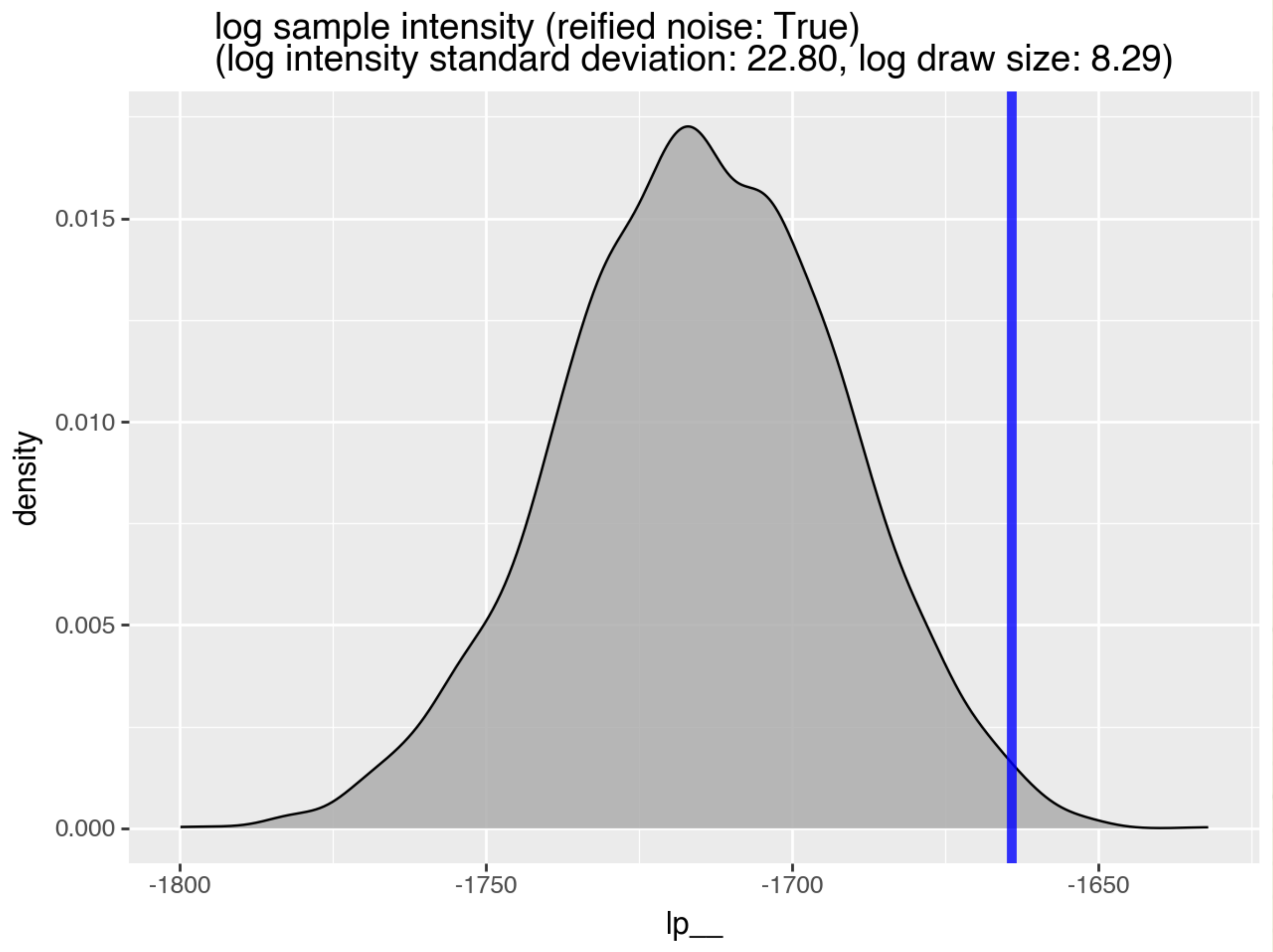
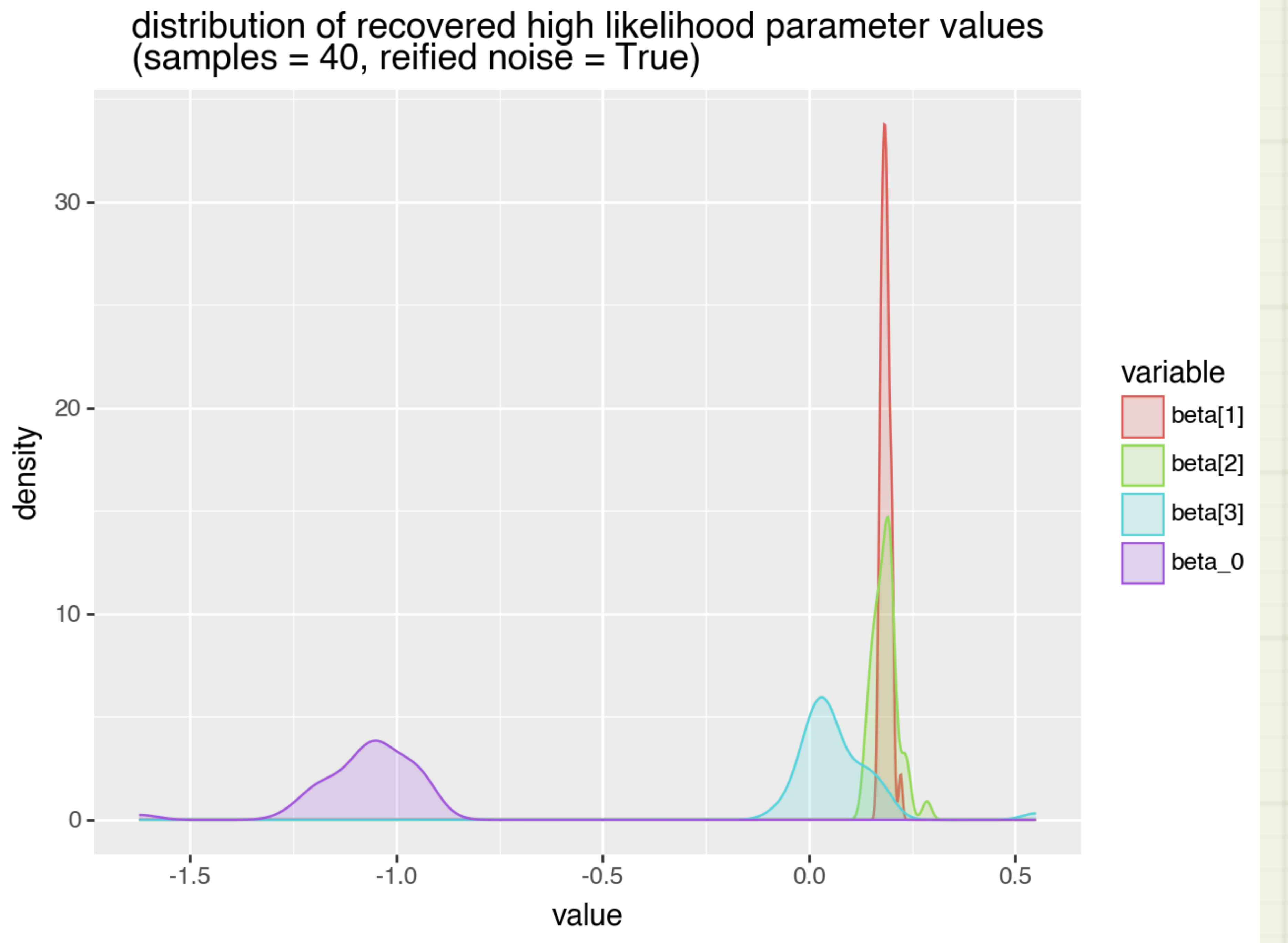
Stan Diagnostics

- Check distribution of returned `__lp`.
  - Confirms Markov chain converged and model is identifiable.
  - The standard deviation should not be too much larger than the log of the number of samples.
    - This isn't always practical.
- Check distribution of parameter estimates with high `__lp`.
  - Confirms model is identifiable.
  - Confirms we are looking at a dominant estimate and not a mixture of incompatible estimates.





Our Results: Model Coefficients

	model	beta_0	x1	x2	x3
0	data generation process	-1.20	0.20	0.20	0.20
1	model from visible selections	-3.13	0.34	0.39	0.50
2	model from pair differences	0.00	0.32	0.41	0.57
3	Stan model MCMC	-1.05	0.19	0.18	0.05
4	Stan model optimizer	-0.73	0.16	0.13	-0.06

	example_index	x1	x2	x3	data generation process	model from visible selections	model from pair differences	Stan model MCMC	Stan model optimizer
0	0	12.0	1.0	0.0	0.80	0.80	0.99	0.80	0.80
1	1	1.0	6.0	0.0	0.55	0.39	0.94	0.56	0.56
2	2	0.0	0.0	0.1	0.24	0.04	0.51	0.26	0.32
3	3	0.0	1.0	1.0	0.31	0.10	0.73	0.31	0.34

Not Just a Prevalence Issue

	model	beta_0	x1	x2	x3	recovered prevalence
0	model from visible selections	-1.78	0.34	0.39	0.50	0.38
1	model from pair differences	-1.85	0.32	0.41	0.57	0.38

	example_index	x1	x2	x3	data generation process	model from visible selections	model from pair differences
0	0	12.0	1.0	0.0	0.80	0.94	0.92
1	1	1.0	6.0	0.0	0.55	0.71	0.72
2	2	0.0	0.0	0.1	0.24	0.15	0.14
3	3	0.0	1.0	1.0	0.31	0.29	0.29

Conclusions

- There are examples where easy inference of preference doesn’t work.
- Only Stan was powerful to let us pick our assumed presentation censoring process.
    - The other systems impose a presentation behavior.
- Stan is great for prototyping solution methods and experimenting with how much model structure and presentation hygiene you wish to capture.
    - Dropping the modeled errors made for a smaller model that was nearly as effective.
    - Can export inferred parameters and use them elsewhere.
    - Doesn’t Stan in production.

Tools Used

- Stan, R, and Python
- All code and data shared here:

  https://github.com/WinVector/Examples/tree/main/rank

  rstan.md

  rstan.Rmd

  rank_src_censored_picks_reified_noise.stan

  rank_src_censored_picks.stan

  generate_example.ipynb

  rank_data_censored_picks.json

  LearningToRank.pdf

Thank you