Preference Inference Using Stan

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Bay Area Use R Group

https://www.meetup.com/r-users/events/303488652/

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Slides, links, and code: https://github.com/WinVector/Examples/tree/main/rank

Who I am

- John Mount, General Partner at Win Vector LLC.
    - Co-author of *Practical Data Science with R*, 2nd edition, Manning, 2020.
    - Co-author of the **`vtreat`** R package for re-encoding high cardinality explanatory variables.
- Win Vector LLC is a statistics, machine learning, and data science consultancy and training organization.
    - Specialize in solution design, technology evaluation, and prototyping.
    - We help greatly speed up solution and production deployment.
    - Looking for some more engagements.
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- Please follow us on the Win Vector blog: https://win-vector.com/blog-2/

Outline

- The problem
  - Some history
- What is Stan?
- Solution
  - Experiments/Results
  - Methods
- Conclusions/recommendations

The Problem

Example Problem

- User is shown 5 products online and clicks on one.
- User tastes 5 wines and buys at most one.
    - (repeat with 100 users thought to be in same persona)

We **wish** they would tell us a numerical valuation of each wine!

Our Goal

- Estimate the user (or user cohort) *intrinsic* preferences (independent of presentation position and other items).

  - Allows us to rate items with respect to user to retrieve, sort, estimate utilities and so on.

  - Eliminates some systems, such as **`xgb.XGBRanker`** (as it reproduces predictions over whole lists).

Formal Set Up

- Users (or user personas) have intrinsic *unobserved* ***preferences*** or valuations
    - Personas are either representatives of a group of users or a label collecting together a group of users.
    - This lets us assume we can collect a lot of per-persona data.
- We (unfortunately) only observe user ***behaviors***
    - Typical observation: we presented 5 alternatives and the user selected one.
    - Observation contaminated both by presentation position and alternative items in the presentation.
- Can we estimate persona preferences from persona behaviors?
    - Often called “learning to rank.”
    - We can use estimated preferences to plan (a lot more than just predicting future list behaviors).
- Typically inferring hidden state from observations is very high value.
    - Stan can simulate inverting fairly complicated “hidden state to observations” processes.

Leaning to Rank History

- Tradition: each field develops solutions ignoring all other fields
    - BIG topic in search engines
        - Joachims, T. (2002), "Optimizing Search Engines using Clickthrough Data", *Proceedings of the ACM Conference on Knowledge Discovery and Data Mining*
        - Tie-Yan Liu (2009), "Learning to Rank for Information Retrieval", *Foundations and Trends in Information Retrieval*, **3** (3): 225–331
- Huge survey: https://en.wikipedia.org/wiki/Learning_to_rank
    - Claims that logistic regression does not work
        - “Bill Cooper proposed logistic regression for the same purpose in 1992 [19] and used it with his Berkeley research group to train a successful ranking function for TREC. Manning et al. [40] suggest that these early works achieved limited results in their time due to little available training data and poor machine learning techniques.”

Maybe Things Can Be Easy?

- If:
  - Most of the unobserved activation lists have two alternatives?
- Then:
  - This looks like the question structure from conjoint analysis
  - Most of the unobserved activation lists are then identical to the observed selection lists.
  - Problem is solvable by direct item-wise logistic regression.

Maybe Things are Difficult?

- Presentation position *may* be a strong influence
    - Users may be biased to pick earlier presentation positions, forget earlier positions, or even not even try/ look-at later positions!
    - Item quality may correlate with presentation position.
    - What other items are in the list or panel having an effect (such as high priced irrelevant alternatives).
- Data is low fidelity or censored
    - Selecting 1 out of 5 positions is only 2.3 bits of information
- No-select lists ***may* be coming from an unknown mixture of two sources**
        - Motivated Shoppers (with intent to buy, so non-buying strong evidence against all presented wines).
        - Idle Browsers (with no intent to buy, so non-buying against any wine).
    - These are not the same persona! Would need per-user variables to sort these users out.
    - Punishes highly desirable selections *more* than less desirable selections.

Our “Wine” Example

- Artificial Problem
  - Know answer
  - 3 explanatory variables: x1, x2, x3
- Visible Data
  - Panels of 5 wines tasted in one sitting
  - explanatory variables
  - selected (if any) wine
- Many repetitions over many customers thought to be in *the same persona cluster*.

	group	position	example_index	x1	x2	x3	score	hidden_activation	selected
0	0	0	7	0.00	1.00	1.00	0.295	False	False
1	0	1	2	0.00	0.00	0.10	0.257	False	False
2	0	2	3	0.00	1.00	1.00	0.304	False	False
3	0	3	8	0.00	1.00	1.00	0.288	False	False
4	0	4	10	0.00	1.00	1.00	0.348	False	False
5	1	0	1	1.00	6.00	0.00	0.546	True	False
6	1	1	0	12.00	1.00	0.00	0.796	True	True
7	1	2	3	0.00	1.00	1.00	0.326	True	False
8	1	3	7	0.00	1.00	1.00	0.309	False	False
9	1	4	2	0.00	0.00	0.10	0.210	True	False
10	2	0	6	0.00	1.00	1.00	0.349	False	False
11	2	1	0	12.00	1.00	0.00	0.817	True	True
12	2	2	5	0.00	1.00	1.00	0.297	False	False
13	2	3	10	0.00	1.00	1.00	0.327	True	False
14	2	4	8	0.00	1.00	1.00	0.327	False	False

unobserved

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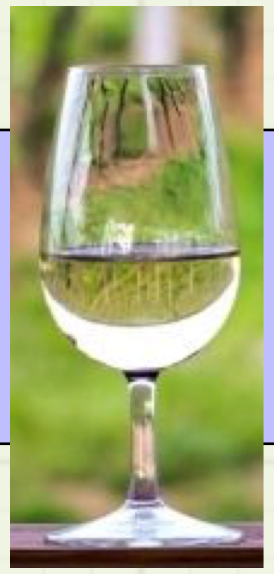
The (Assumed) Data Generating Process

Evidence pattern on selection

User considers 5 alternatives





(unobserved) explanatory variables converted into activation probabilities (scores)

0.349	**0.817**	0.297	0.327	0.327

wish we knew this

(unobserved) probabilities converted into “interested” activations check-marks

activation list

(observed) checked item with highest activation score is picked

selection list

experiment design

model structure

$f_\beta(x)$ (fit for β)

presentation censoring process

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What is Stan?

Stan

- A Markov Chain Monte Carlo sampler
- Can be used for complicated Bayesian inference
  - Guesses likely values of parameters, and nuisance variables, *and unobserved intermediate state* conditioned on observed data.
  - Returns distributional answers.
- Fairly high dependency (requires C++ compiler and linker)
  - May eventually see competition from Torch based alternatives

Solution

(Approximate) Stan Model

- Define parameters: β, β_0 (coefficient vector and intercept) *and* ζ (hidden per chosen selection vector link value).
- Let $x_{i,j}$ denote the vector of explanatory values for the ith example's jth alternative
    - We model a hidden link scores as being distributed normal$(\beta_0 + x_{i,j} \cdot \beta, \sigma)$
    - We (**evilly**) approximate this many places as $\beta_0 + x_{i,j} \cdot \beta$
- For lists with no selection model, the probability of seeing the observed non-selection is:
    - P$[data_i | \beta, \zeta_i] \sim \prod_j (1 - \text{sigmoid}(\beta_0 + x_{i,j} \cdot \beta))$
    - "all the alternatives must all fail to activate"
- For lists with a selection s model, the probability of seeing the observed selection is:
    - $\zeta_{i,s} \sim$ normal$(\beta_0 + x_{i,j} \cdot \beta, \sigma)$
    - P$[data_i | \beta, \zeta_{i,s}] \sim \text{sigmoid}(\zeta_{i,s}) \prod_{j, j \neq s} (1 - \text{sigmoid}(\beta_0 + x_{i,j} \cdot \beta))(1 - \Phi(\beta_0 + x_{i,j} \cdot \beta | \zeta_i, \sigma))$
    - "choice activates and none of the activated alternatives outscores the choice"
- Set up Stan to sample β, ζ such that P$[\beta, \zeta]$P$[data | \beta, \zeta]$ is large.

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As Stan Code

```
data {
  int<lower=1> n_vars;  // number of variables per alternative
  int<lower=2> n_alternatives;  // number of items per presentation list
  int<lower=1> m_examples;  // number of examples
  array[m_examples] int<lower=0, upper=n_alternatives> pick_index;   // which item is picked, 0 means no pick
  array[n_alternatives] matrix[m_examples, n_vars] x;  // explanatory variables
}
parameters {
  real beta_0;  // model parameters
  vector[n_vars] beta;  // model parameters
  vector[m_examples] noise_in_pick_link;
}
transformed parameters {
  array[n_alternatives] vector[m_examples] link;  // link values
  for (sel_j in 1:n_alternatives) {
    link[sel_j] = beta_0 + x[sel_j] * beta;
  }
}
model {
    // basic priors
  beta_0 ~ normal(0, 10);
  beta ~ normal(0, 10);
  noise_in_pick_link ~ normal(0, 0.1);
    // log probability of observed situation
  for (row_i in 1:m_examples) {
    if (pick_index[row_i] <= 0) {
      for (sel_j in 1:n_alternatives) {
        target += log1m(inv_logit(link[sel_j][row_i]));  // non-activation odds
      }
    } else {
      for (sel_j in 1:n_alternatives) {
        if (sel_j == pick_index[row_i]) {
          target += log(inv_logit(link[pick_index[row_i]][row_i] + noise_in_pick_link[row_i]));  // probability selection indicates
        } else {
          target += log1m(inv_logit(link[sel_j][row_i])  // probability potential spoiler indicates
                          // probability potential spoiler outscores selection
                          * (1 - normal_cdf(link[pick_index[row_i]][row_i] + noise_in_pick_link[row_i] | link[sel_j][row_i], 0.1)));
        }
      }
    }
  }
}
```

incoming data

values to guess at

notational convenience

the criticism

Stan Encoding

- Stan code is to propose a desirability of the simultaneous settings of data and parameters.
    - This desirability is encoded in a variable called “target”
    - Stan tries to sample parameter values proportional to exp(target).
- We add our criticisms to target:
    - Directly as in $target$ += $\log(1 - \text{inv\_logit}(link))$
        - That is encoding: “our data has a non-select here, so we want the modeled probability of selection to be low.”
    - Distributionally as in $beta \sim normal(0,10)$
        - Equivalent to $target$ += $\text{normal\_lpdf}(beta \,|\, 0,10)$

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Stan Code Again

```
data {
  int<lower=1> n_vars;  // number of variables per alternative
  int<lower=2> n_alternatives;  // number of items per presentation list
  int<lower=1> m_examples;  // number of examples
  array[m_examples] int<lower=0, upper=n_alternatives> pick_index;   // which item is picked, 0 means no pick
  array[n_alternatives] matrix[m_examples, n_vars] x;  // explanatory variables
}
parameters {
  real beta_0;  // model parameters
  vector[n_vars] beta;  // model parameters
  vector[m_examples] noise_in_pick_link;
}
transformed parameters {
  array[n_alternatives] vector[m_examples] link;  // link values
  for (sel_j in 1:n_alternatives) {
    link[sel_j] = beta_0 + x[sel_j] * beta;
  }
}
model {
    // basic priors
  beta_0 ~ normal(0, 10);
  beta ~ normal(0, 10);
  noise_in_pick_link ~ normal(0, 0.1);
    // log probability of observed situation
  for (row_i in 1:m_examples) {
    if (pick_index[row_i] <= 0) {
      for (sel_j in 1:n_alternatives) {
        target += log1m(inv_logit(link[sel_j][row_i]));  // non-activation odds
      }
    } else {
      for (sel_j in 1:n_alternatives) {
        if (sel_j == pick_index[row_i]) {
          target += log(inv_logit(link[pick_index[row_i]][row_i] + noise_in_pick_link[row_i]));  // probability selection indicates
        } else {
          target += log1m(inv_logit(link[sel_j][row_i])  // probability potential spoiler indicates
                        // probability potential spoiler outscores selection
                        * (1 - normal_cdf(link[pick_index[row_i]][row_i] + noise_in_pick_link[row_i] | link[sel_j][row_i], 0.1)));
        }
      }
    }
  }
}
```

the math

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Calling Stan from R or Python

```r
library(rstan)
library(jsonlite)

data <- fromJSON("rank_src_censored_picks.stan")

sample <- stan(
  file = "rank_src_censored_picks.stan",  # Stan program
  data = data,                            # named list of data
  chains = 4,                             # number of Markov chains
  cores = 4,                              # number of cores (could use one per chain)
  refresh = 0,                            # no progress shown
  pars=c("lp__", "beta_0", "beta")        # parameters to bring back
)

draws <- as.data.frame(sample)
```

```python
from cmdstanpy import CmdStanModel

# quiet down Stan
logger = logging.getLogger("cmdstanpy")
logger.addHandler(logging.NullHandler())

# instantiate the model object
model_comp = CmdStanModel(stan_file="rank_src_censored_picks.stan")
# sample high probability parameter settings
sample_Stan = model_comp.sample(
    data="rank_src_censored_picks.stan",
    show_progress=True,
    show_console=False,
)

draws = sample_Stan.draws_pd(vars=['lp__', 'beta_0', 'beta'])
```

Why That Works

- We want to maximize plausibility of parameters given data: P$[\beta_0, \beta, \zeta \,|\, data]$.
- By Bayes' Law P$[\beta_0, \beta, \zeta \,|\, data] =$ P$[\beta_0, \beta, \zeta]$P$[data \,|\, \beta_0, \beta, \zeta]$/P$[data]$.
  - Can ignore P$[data]$ as it is free of our parameter estimates.
  - So picking β, ζ such that P$[\beta_0, \beta, \zeta]$P$[data \,|\, \beta_0, \beta, \zeta]$ is large is the same as picking such that P$[\beta_0, \beta, \zeta \,|\, data]$ is large.
- Can invoke ideas such as the Bernstein-von Mises theorem to argue "large" is going to concentrate samples near the (unknown) true parameter value as we add more data.
  - (Technically better to argue that for a model with no data size dependent variables ζ.)
  - With enough data and flat enough P$[\beta_0, \beta, \zeta]$, we can even omit the P$[\beta_0, \beta, \zeta]$ term.
    - Structural mis-specification *more* risky than "wrong priors."

What you Get From Stan

```
draws |>
  head() |>
  knitr::kable()
```

lp__	beta_0	beta[1]	beta[2]	beta[3]
-1251.130	-1.520818	0.2282623	0.2879370	0.3201965
-1250.358	-1.506511	0.2233696	0.2475490	0.1977965
-1251.936	-1.552996	0.1998336	0.2522806	0.3323120
-1249.657	-1.533951	0.2080136	0.2574112	0.3573067
-1248.750	-1.493253	0.2228508	0.2416577	0.3151417
-1248.531	-1.221375	0.2030706	0.2373745	0.0258106

```
nrow(draws)
```

```
## [1] 4000
```

Result: Activation Probabilities

	example_index	x1	x2	x3	data generation process	model from visible selections	model from pair differences	Stan model MCMC	Stan model optimizer
0	0	12.0	1.0	0.0	0.80	0.80	0.99	0.80	0.80
1	1	1.0	6.0	0.0	0.55	0.39	0.94	0.56	0.56
2	2	0.0	0.0	0.1	0.24	0.04	0.51	0.26	0.32
3	3	0.0	1.0	1.0	0.31	0.10	0.73	0.31	0.34

Notice the Stan models dominate.

Models Compared

- **Model from visible selections**: treat each item selection or non-selection as a positive or negative example. Also called “item-wise” ranking. A nice logistic regression solution for comparison.

- **Model from pair differences**: encode each pair in a list where one wine is picked and one is not as an example. A common “let’s be clever trick.”

- **Stan Model MCMC**: encode an entire probability model (including guessing unseen quantities) and then use a Markov chain to sample high likelihood values. The big dog.

- **Stan Model Optimizer**: use the above encoding as a “loss”, and hope something as simple as standard optimizer can find a high likelihood solution. “Cheap Stan” (could use other numeric platforms like PyTorch for this).

Our Results: Model Coefficients

	model	beta_0	x1	x2	x3
0	data generation process	-1.20	0.20	0.20	0.20
1	model from visible selections	-3.13	0.34	0.39	0.50
2	model from pair differences	0.00	0.32	0.41	0.57
3	Stan model MCMC	-1.05	0.19	0.18	0.05
4	Stan model optimizer	-0.73	0.16	0.13	-0.06

	example_index	x1	x2	x3	data generation process	model from visible selections	model from pair differences	Stan model MCMC	Stan model optimizer
0	0	12.0	1.0	0.0	0.80	0.80	0.99	0.80	0.80
1	1	1.0	6.0	0.0	0.55	0.39	0.94	0.56	0.56
2	2	0.0	0.0	0.1	0.24	0.04	0.51	0.26	0.32
3	3	0.0	1.0	1.0	0.31	0.10	0.73	0.31	0.34

Not Just a Prevalence Issue

	model	beta_0	x1	x2	x3	recovered prevalence
0	model from visible selections	-1.78	0.34	0.39	0.50	0.38
1	model from pair differences	-1.85	0.32	0.41	0.57	0.38

	example_index	x1	x2	x3	data generation process	model from visible selections	model from pair differences
0	0	12.0	1.0	0.0	0.80	0.94	0.92
1	1	1.0	6.0	0.0	0.55	0.71	0.72
2	2	0.0	0.0	0.1	0.24	0.15	0.14
3	3	0.0	1.0	1.0	0.31	0.29	0.29

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Conclusions

- There are examples where easy inference of preference doesn’t work.
- Stan was powerful to let us pick our assumed presentation censoring process.
    - The other systems impose a presentation behavior.
- Stan is great for prototyping solution methods and experimenting with how much model structure and presentation hygiene you wish to capture.
    - Can export inferred parameters and use them elsewhere.

Tools Used

- Stan, R, and Python
- All code and data shared here:

    https://github.com/WinVector/Examples/tree/main/rank

    rstan.md

    rstan.Rmd

    rank_src_censored_picks_reified_noise.stan

    rank_src_censored_picks.stan

    generate_example.ipynb

    rank_data_censored_picks.json

    LearningToRank.pdf

Our Using Stan to Solve Problems Training Offering

- The series currently includes:
  - Dealing with range censored data, or tobit style regression.
  - Learning rank preferences from observed actions.
  - Time series with external explanatory variables.
- Contact jmount@win-vector.com for custom training and consulting.

Thank you

Slides, links, and code: https://github.com/WinVector/Examples/tree/main/rank