Prototyping Preference Inference Using Stan

John Mount
Win Vector LLC
https://www.win-vector.com/
August 30, 2024

Win-Vector LLC

Win Vector LLC

- Win Vector LLC is a statistics, machine learning, and data science consultancy and training organization.
- Specialize in solution design, technology evaluation, and prototyping.
- Contact: jmount@win-vector.com .

Win-Vector LLC

Outline

- The problem
  - Some history
- Bayesian solution
  - Experiments/Results
- Simpler solutions
- Observations
- Conclusions/recommendations
- Next steps

The Situation

- Users (or user personas) have ***preferences*** or valuations (either implicit or explicit and unobserved/ shared)
    - Personas are either representatives of a group of users or a label collecting together a group of users.
    - This lets us assume we can collect a lot of per-personal data.
- We observe user ***behaviors***
    - Typical observation: we presented 5 alternatives and the user purchased one.
- Can we estimate persona preferences from persona behaviors?
    - Often called “learning to rank.”
    - We can use estimated preferences to plan (a lot more than just predicting future behaviors).

Example Problems

- User is shown 5 products online and clicks on one.
- User tastes 5 wines and buys one
  - (repeat with 100 users thought to be in same persona)

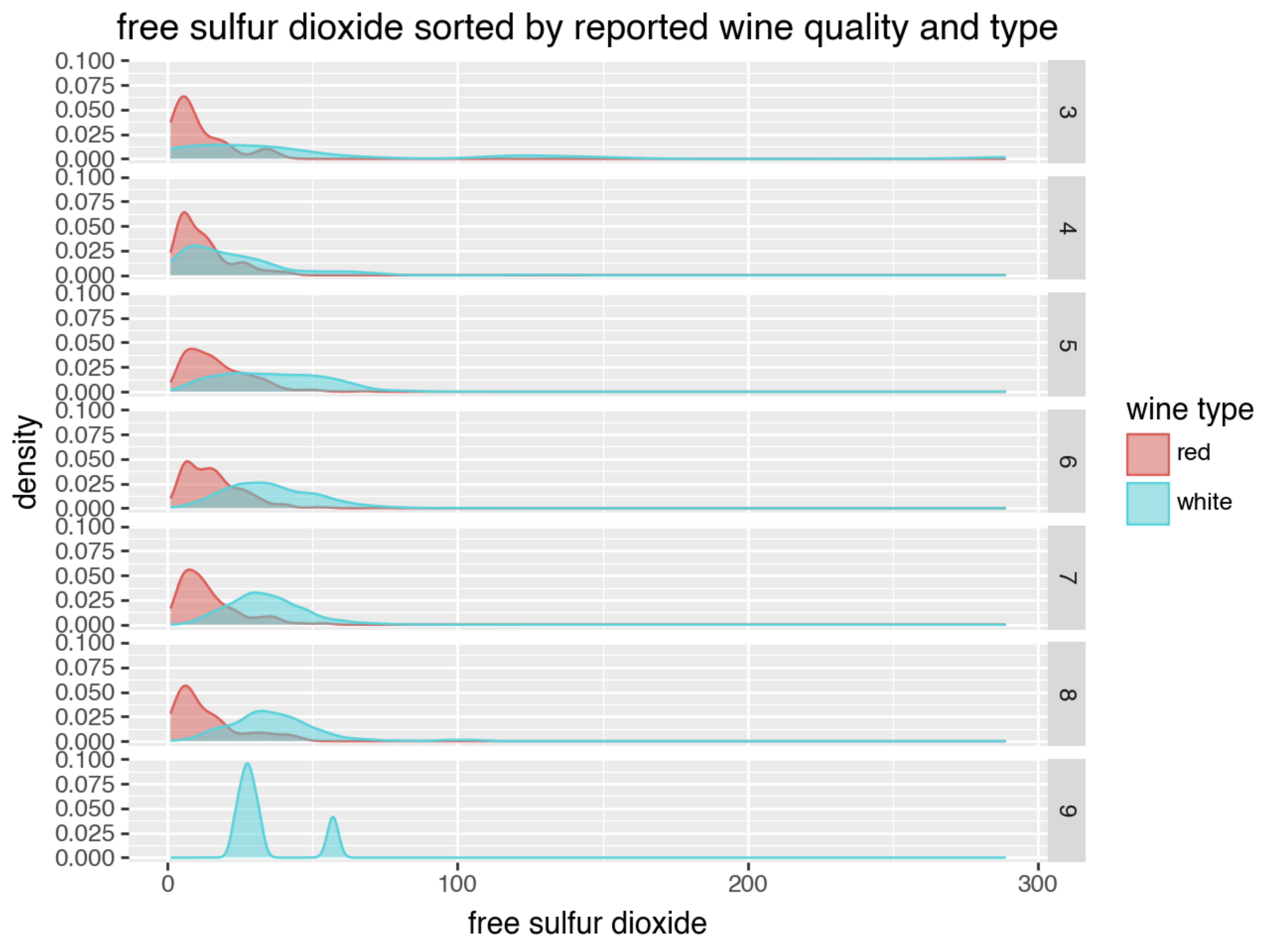
We **wish** they would tell us their numerical valuation of each wine!

Win-Vector LLC

The Wine Example

Variables:

alcohol
chlorides
citric acid
density
fixed acidity
free sulfur dioxide *(shown as example)*
is_red *(we interact this with all other variables)*
pH
residual sugar
sulphates
total sulfur dioxide
volatile acidity



Not actually a good set of variables for the task (physical chemical variables instead of domain perceptual and provenance variables).

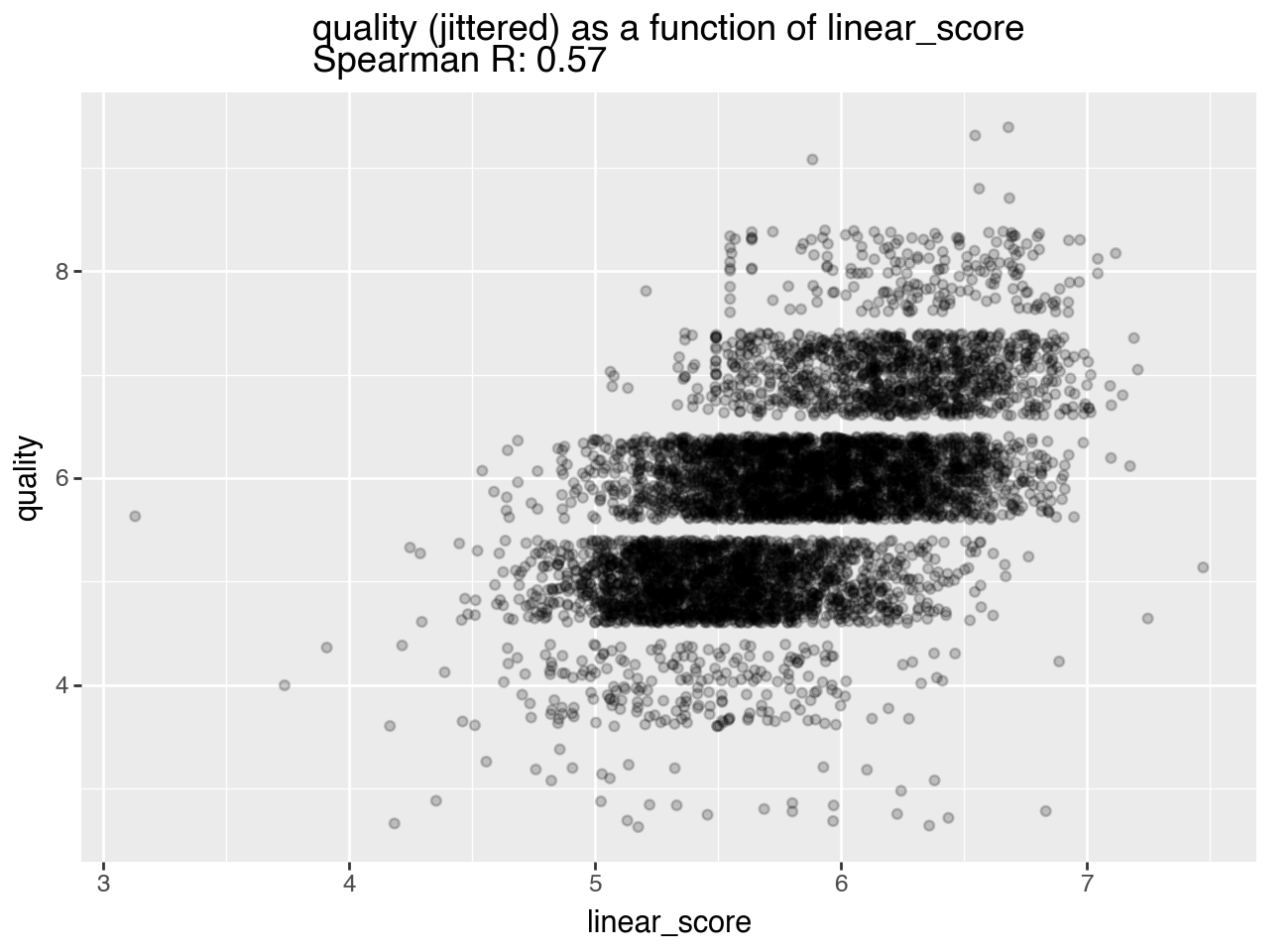
https://archive.ics.uci.edu/dataset/186/wine+quality

Win-Vector LLC

The Data we Wish For

User tells us their darn scores!

With 100 tasting lists we observe heavily noisy and censored outcomes from about 500 wines out of our 6497.



(Above fit on all 6497 wines, without noise or list structure. Notice reported quality unfortunately isn't very well modeled as a linear function of *these* variables. Please remember 0.57 as "best possible" for this model structure.)

Win-Vector LLC

Confounding Issues

- Presentation position may be a strong influence
    - Users may be biased to pick in earlier presentation positions, forget earlier positions, or even not even try/look-at later positions!
    - Item quality may correlate with position (would require additional interactions to be introduced in the model).
- Data is noisy
    - User may make different choice when re-presented
- Data is low information or censored
    - Selecting 1 out of 5 positions is only 2.3 bits of information
- Presentation is not an ideal experiment (influenced by business needs)
    - Ideal statistical procedure would be either:
        - Only present 2 alternatives and force a selection (Conjoint Analysis)
        - Bully exact scores out of the user (unlikely, they may not even be able to express numeric utilities)

My Intended Points

- The critical concern is having the right model structure.
    - Must approximate the data generation process.
    - Stan is great for prototyping inference strategies.
- The data presentation is statistical censoring
    - We can remove the censoring, but that doesn't change if we have the right or wrong model.
    - The censoring increases the required amount of data
- There are several "right ways" to undo the censoring
    - Gives us some useful trade-offs

Leaning to Rank History

- Each field develops solutions ignoring all other fields
  - BIG topic in search engines
    - Joachims, T. (2002), "Optimizing Search Engines using Clickthrough Data", *Proceedings of the ACM Conference on Knowledge Discovery and Data Mining*
    - Tie-Yan Liu (2009), "Learning to Rank for Information Retrieval", *Foundations and Trends in Information Retrieval*, **3** (3): 225–331
  - Econometrics has its own methods
  - *Tons* of ink spilled on “what is the right way to undo the pick censorship?” (delaying working on the actual problem)
  - …
  - Issues such as “point-wise” (each comparison is an event) versus “list-wise” (each presented list is an event) dominate problem design.
  - Huge emphasis on efficiency of calculation and problem as an optimization problem
    - Trade-off may be different a quarter of a century later.
- Huge survey: https://en.wikipedia.org/wiki/Learning_to_rank
  - Claims that logistic regression does not work
  - “Bill Cooper proposed logistic regression for the same purpose in 1992 [19] and used it with his Berkeley research group to train a successful ranking function for TREC. Manning et al.[40] suggest that these early works achieved limited results in their time due to little available training data and poor machine learning techniques.”

Win-Vector LLC

Our Goal

- Estimate the user (or user cohort) intrinsic preferences (independent of presentation).

- Eliminates some systems, such as **`xgb.XGBRanker`** (as it reproduces predictions over whole lists

- From: https://xgboost.readthedocs.io/en/stable/python/examples/learning_to_rank.html#sphx-glr-python-examples-learning-to-rank-py

```
    X_test, clicks_test, y_test, qid_test = sort_ltr_samples(
        test.X,
        test.y,
        test.qid,
        test.click,
        test.pos,
    )
…
ranker.predict(X_test)
```

(notice the `qid` column, which is means data is in presentation and we `xgb.XGBRanker` is compute probability of selection with respect to the alternatives, not utility.)

A Bayesian Solution

- Suppose the user has a hidden valuation parameter β vector such that their valuation of a an item with features x_i is $f_\beta(x_i)$
  - Often this is realized as $f_\beta(x_i) = \beta \cdot x_i$.
- Further assume for a list of items $x_1, \ldots, x_5$ the user values item i with a value of $f_\beta(x_i) + e_i$ (e_i being a mean-zero noise term).
- Notice I have *not* chosen a probabilistic model!
  - Rank valuation is just an abstract number, utility, value, preference, or affinity.
- Let's introduce probabilities by modeling the user as picking the i such that this expression is maximized.
  - This formulation differs from the standard logistic solution in that we are not assuming a link function and error-rate, but instead a value denominated noise process.

Bayesian Reasoning

- We apply Bayes' Theorem.

Define:

$$Z_i = 1/\mathrm{P}[select(i)\,|\,x_1,...,x_k]$$
$$select(i) = \wedge_{j,\,j \neq i}\,(\beta \cdot x_i + e_i > \beta \cdot x_j + e_j)$$

The trick in working with Stan:
make things conditionally independent.

Then:

$$\begin{aligned}
\mathrm{P}[\beta\,|\,select(i), e_1,...,e_k] &= \mathrm{P}[\beta\,|\,e_1,...,e_k]\ \mathrm{P}[select(i)\,|\,\beta, e_1,...,e_k]\ /\ \mathrm{P}[select(i)\,|\,e_1,...,e_k] && \text{//Bayes' theorem} \\
&= Z_i\ \mathrm{P}[\beta]\ \mathrm{P}[\ \wedge_{j,\,j\neq i}\,(\beta \cdot x_i + e_i > \beta \cdot x_j + e_j)\,|\,\beta, \wedge_j\, e_j] && \text{//expanding definitions, remove unused conditions} \\
&= Z_i\ \mathrm{P}[\beta] \prod_{j,\,j\neq i} \mathrm{P}[\beta \cdot x_i + e_i > \beta \cdot x_j + e_j\,|\,\beta, \wedge_j\, e_j] && \text{//conditional independence} \\
&= Z_i\ \mathrm{P}[\beta] \prod_{j,\,j\neq i} \mathrm{P}[\beta \cdot x_i + e_i > \beta \cdot x_j + e_j\,|\,\beta, e_i, e_j] && \text{//remove unused conditions}
\end{aligned}$$

- (Z_i can be ignored in finding a maximum likelihood β, as it does not depend on β.)

- The point is: each of the checks become independent (can be written as a product) once we know e_i for the picked index i. If we explicitly draw e_i and β in our simulation, we can exploit the independence. We do not worry about the e_j, as they appear only once- so can't carry conditioning information between terms. We will call this formulation the list-wise model.

- Or: I am going to type stuff into Stan, and try not to feel overly bad about it.

Win-Vector LLC

The Pick Data

Explanatory features are a combination of table lookup by item_id and presentation facts (i.e. presentation position).

	item_id_0	pick_value_0	item_id_1	pick_value_1	item_id_2	pick_value_2	item_id_3	pick_value_3	item_id_4	pick_value_4
0	1569	0	1754	0	6425	1	2780	0	2646	0
1	4390	1	2031	0	2692	0	4416	0	1913	0
2	599	1	1808	0	64	0	59	0	1671	0
3	1392	0	2324	0	5815	0	1819	1	4567	0
4	2063	0	6283	0	3610	1	2085	0	5610	0
5	2010	1	1465	0	6388	0	25	0	420	0
6	5903	1	1374	0	312	0	926	0	5467	0
7	5194	1	3651	0	1494	0	1749	0	5865	0
8	5946	1	4527	0	5988	0	3021	0	4821	0
9	6469	1	6044	0	2787	0	5786	0	3709	0

Feature Encoding

$$x(item = 6423, position = 2) =$$

	6425
fixed acidity	6.0
volatile acidity	0.34
citric acid	0.29
residual sugar	6.1
chlorides	0.046
free sulfur dioxide	29.0
total sulfur dioxide	134.0
density	0.99462
pH	3.48
sulphates	0.57
alcohol	10.7
is_red	False
posn_0	0
posn_1	0
posn_2	1
posn_3	0
posn_4	0

Feature table lookup

Encoding of presentation position. Could also encode demographics of participant Especially useful if we interact the demographic variables with the feature variables.

Outcome Encoding: List-Wise Observations

Outcome (y):

Encodes as:

[0, 0, 0, 1, 0]

Simulates a single pick from a single tasting of each wine.

Point-Wise Observations

Outcome (y):

Simulates the (odd) situation of us guessing the user's preferred wine and then giving them 4 paired tastings of it against the 4 non-preferred wines. Notice wine 3 is tasted 4 times (all other wines tasted once).
Not what anyone wants, but easiest for most modeling systems.

Encodes as (independent events):

$$score_3 > score_0$$
$$score_3 > score_1$$
$$score_3 > score_2$$
$$score_3 > score_4$$

Win-Vector LLC

Our Models

- Stan position utility mode
    - Models later positions as costing some utility or score
    - Fairly flexible, even when it doesn’t match the data generation process
    - Can also try quicker point-wise model (to bound importance of modeling dependence for a given data set).
- Stan sequential inspection model
    - Matches the data generation process
- Logistic regression model
    - Less flexible (can’t imitate as many proposed generative processes). Closest to independent or point-wise model.
    - Much faster and easier to deploy

Stan List-Wise Position Utility Model

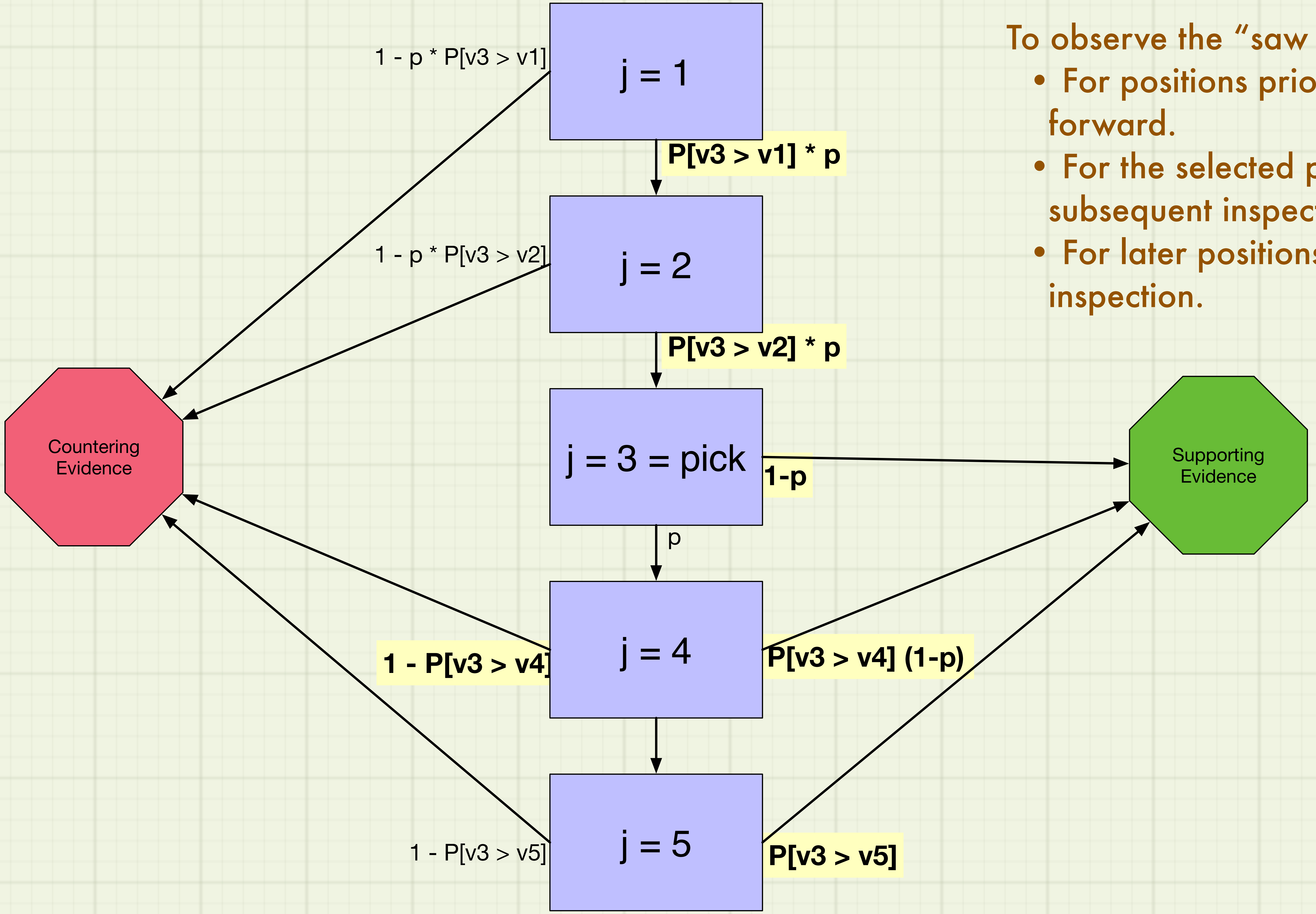
Pass one-hot encoded position indicators to the model as features. Models presentation position as a utility trade-off.

```
...
transformed parameters {
  …
  expect_picked = x_picked * beta;           // modeled expected score of picked item
  v_picked = expect_picked + error_picked;   // reified actual score of picked item
  expect_passed_1 = x_passed_1 * beta;       // modeled expected score of passed item
  expect_passed_2 = x_passed_2 * beta;       // modeled expected score of passed item
  expect_passed_3 = x_passed_3 * beta;       // modeled expected score of passed item
  expect_passed_4 = x_passed_4 * beta;       // modeled expected score of passed item
}
model {
    // basic priors
  beta ~ normal(0, 10);
  error_picked ~ normal(0, 10);
    // log probability of observed ordering as a function of parameters
    // terms are independent conditioned on knowing value of v_picked!
  target += normal_lcdf( v_picked | expect_passed_1, 10);
  target += normal_lcdf( v_picked | expect_passed_2, 10);
  target += normal_lcdf( v_picked | expect_passed_3, 10);
  target += normal_lcdf( v_picked | expect_passed_4, 10);
}
```

Point-wise model, simply uses expect_picked instead of v_picked throughout.

Win-Vector LLC

Sequential Inspection Model



To observe the "saw position 3 picked" we must:

- For positions prior to pick: not see a larger value *and* also continue forward.
- For the selected position either stop continuing forward, or pass subsequent inspections
- For later positions: not see a larger value or also stop forward inspection.

Does not use position indicator columns.

Logistic Model

- Trick 1: encode as a difference
  - Item 6425 in position 2 picked and item 1569 in position 2 not picked encoded as:

$$x_i = x(item = 6423, position = 2) - x(item = 1569, position = 0)$$
$$y_i = True$$

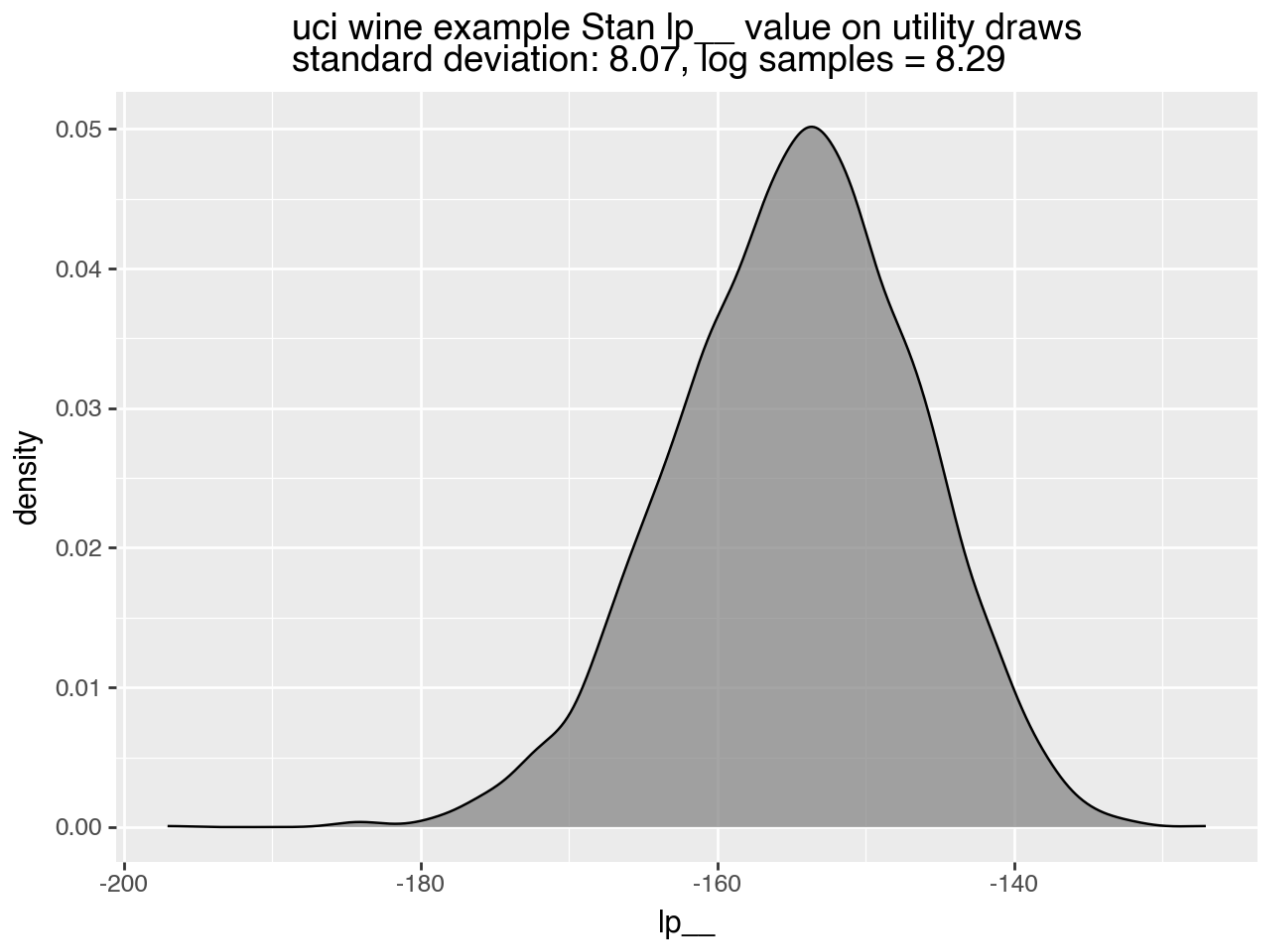
  - The subtraction enforces that features have the same interpretation in picks and non-picks. This is needed to have a model we can apply to items outside of lists and not knowing if they are picked or not. It also halves the number of model parameters, presumably making inference more statistically efficient (requiring less data). This is also why we are not using multinomial logistic regression, or multi class classification.
  - Problem: creates a data set with only “True” outcomes (can’t run fitter!).
- Trick 2: also encode reversal (in addition to seeing the winner winning, we saw losers lose)
  - Add in extra data rows of the form:

$$x_i = -\,(x(item = 6423, position = 2) - x(item = 1569, position = 0))$$
$$y_i = False$$

  - Now we have both True and False outcomes, and can try a logistic regression.
- Collect all the above rows as a logistic regression training set (one row for each pair with a different outcome per list).

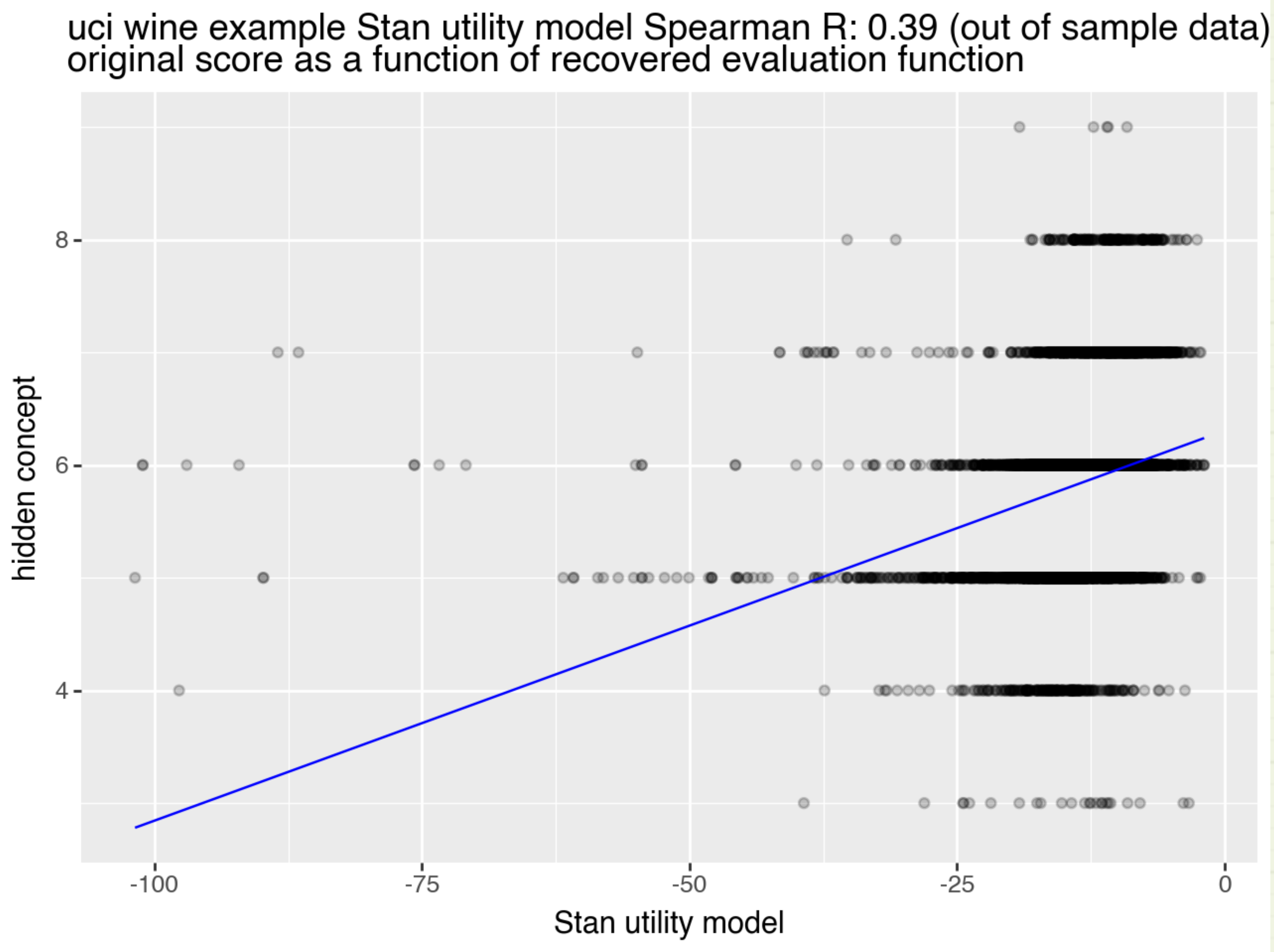
Win-Vector LLC

Stan Double Check



At least during development, you want to check lp__ is unimodal with a standard deviation not *too much* wider than +- log(n-samples) = log(4000) ~ 8.29. Though I have seen good runs that violate this.

Quality of Reproduced Preference Score



Recovered estimates of the *underlying* preference or utility scores. This may not look great, but remember we established 0.57 as best possible Spearman R for a linear model with this set of features. So Spearman 0.39 isn't *that* bad.

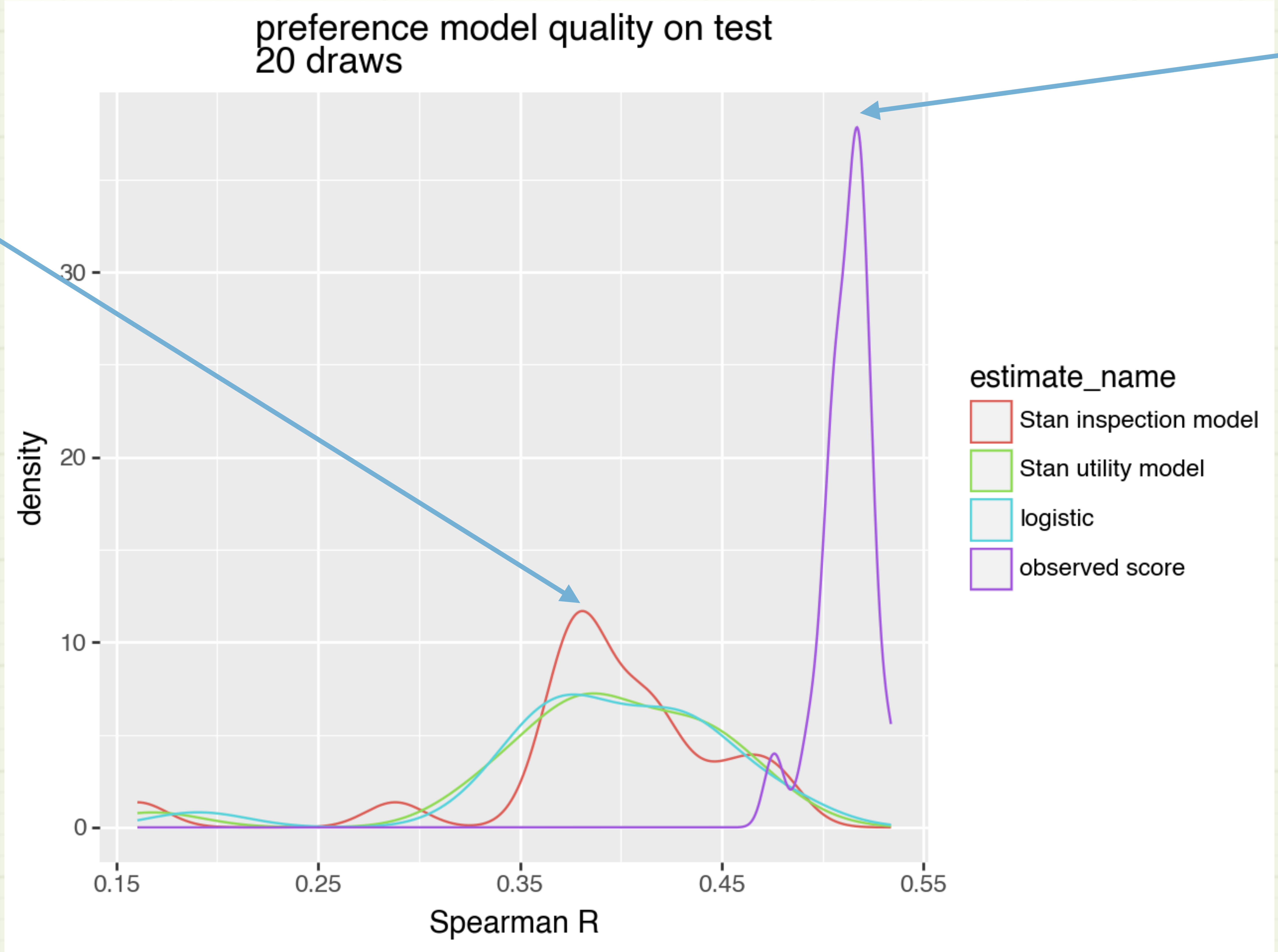
Our formulation works only on order data. So it is shift invariant (no reason to match absolute scores), and estimated scale (or differences) comes from the modeled signal to noise ratio.

Win-Vector LLC

Comparison of Methods

(100 lists, redrawn 20 times)

Various "try to infer from picks" methods.
What we can do in practice.
Has been implemented from just observed picks.



The "we wish users would tell us their scores" situation.
An upper bound on what is possible with the given model structure.
Can't be implemented from pick data.

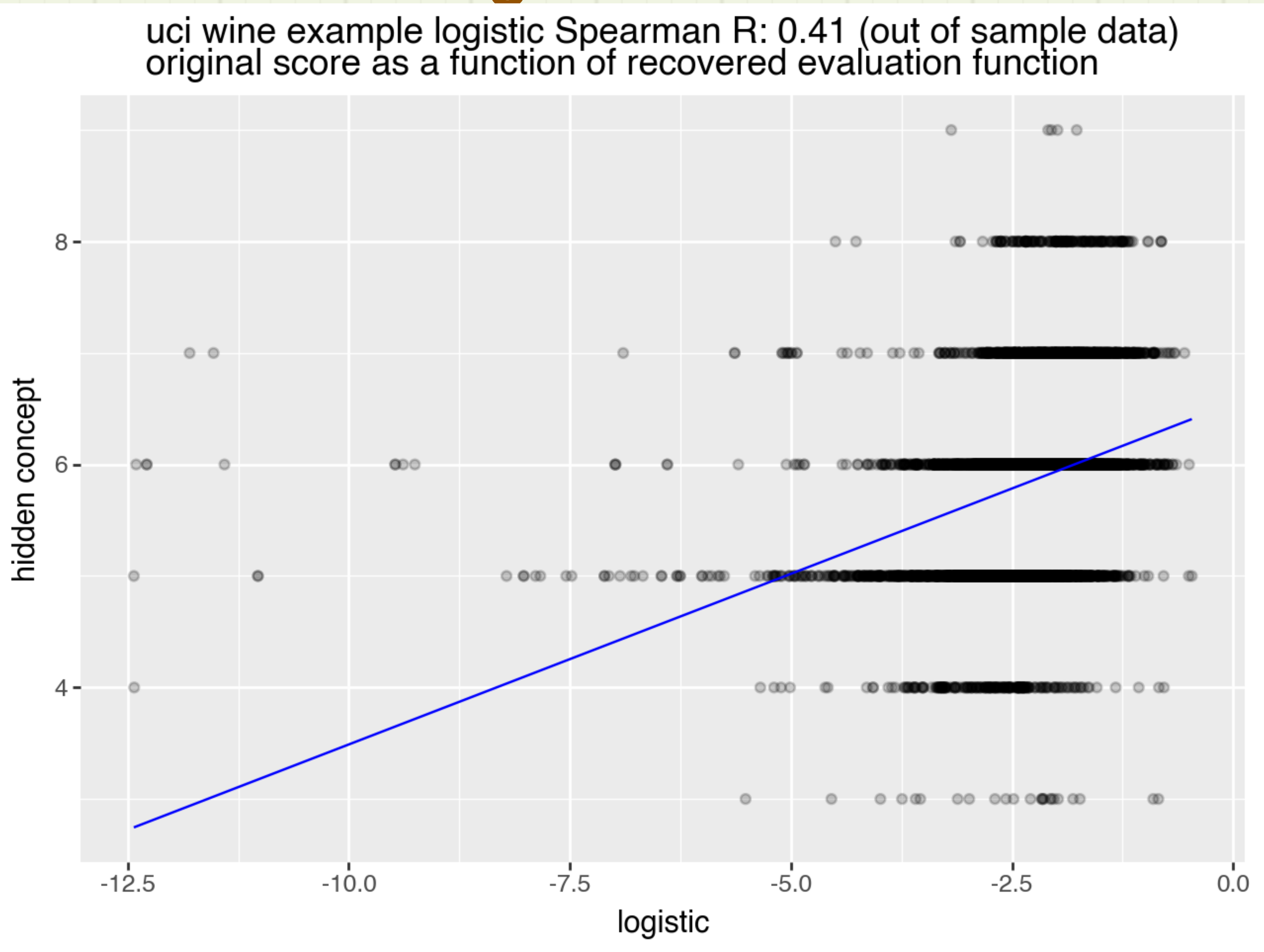
Gap goes away if we have more data!

Win-Vector LLC

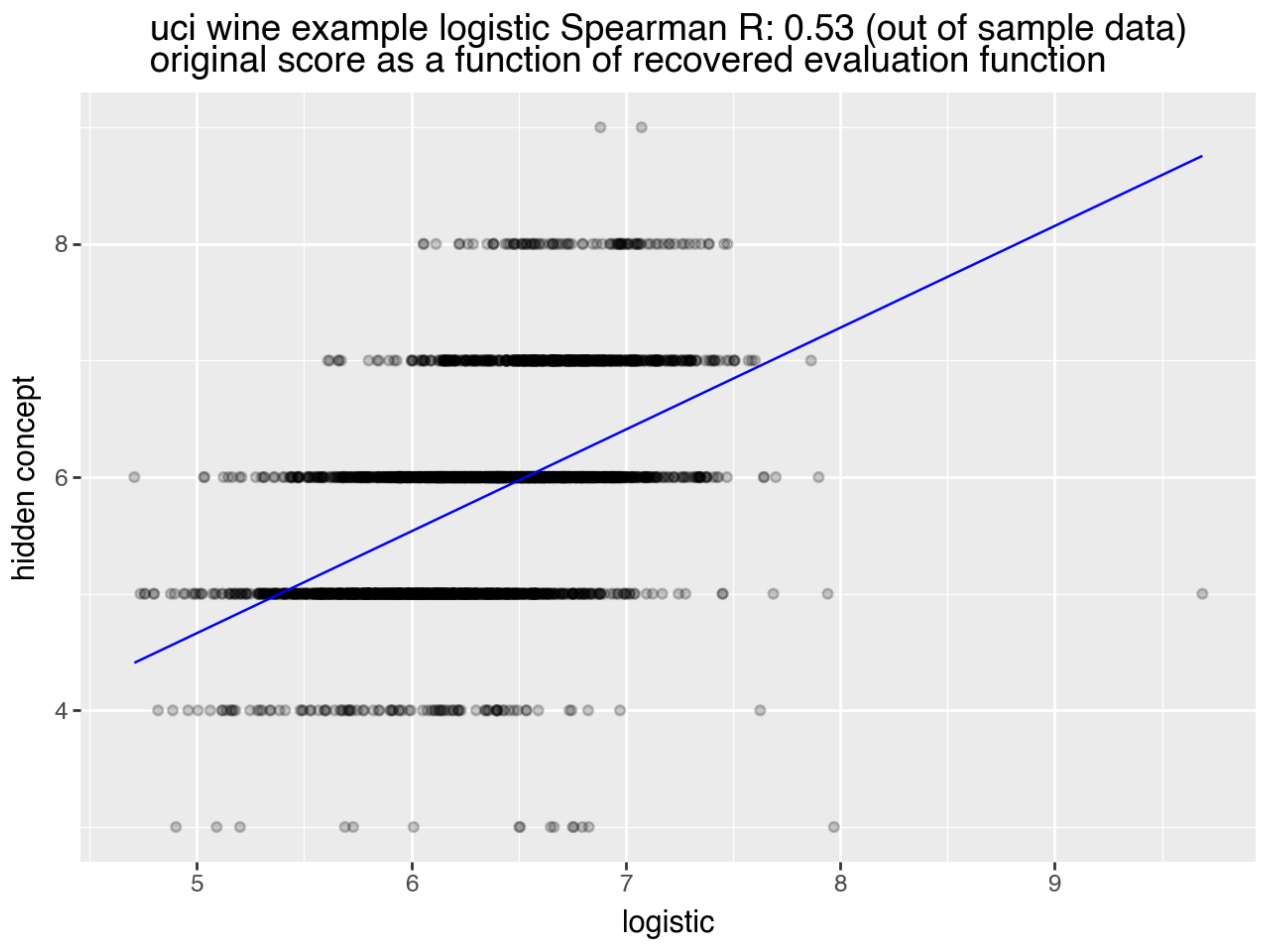
The Effect of More Data

Close to perfect (0.57)!

100 training lists



1000 training lists



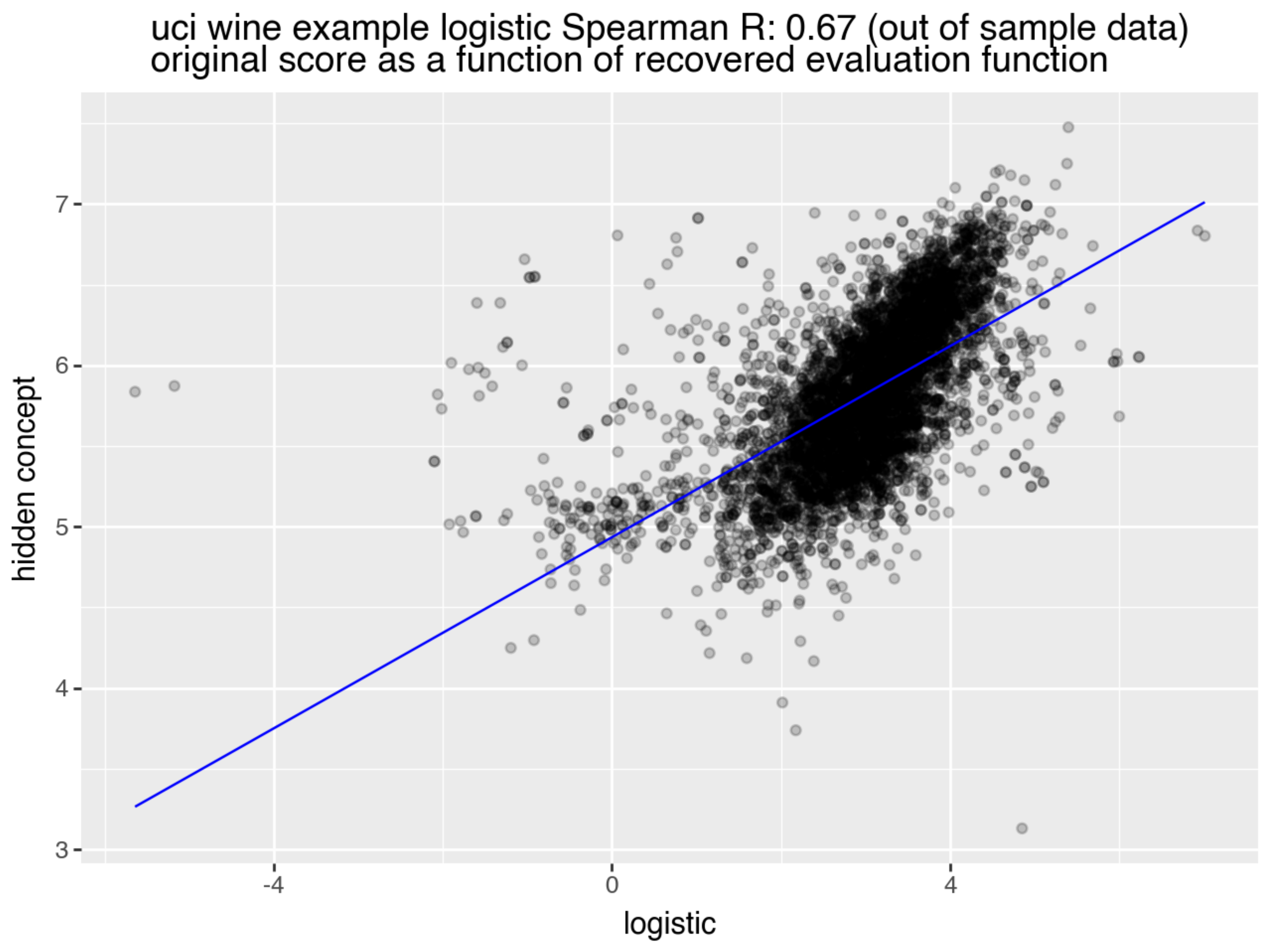
Note: next slide shows we are not hiding behind a noisy evaluation.

When We Have Correct Model Structure

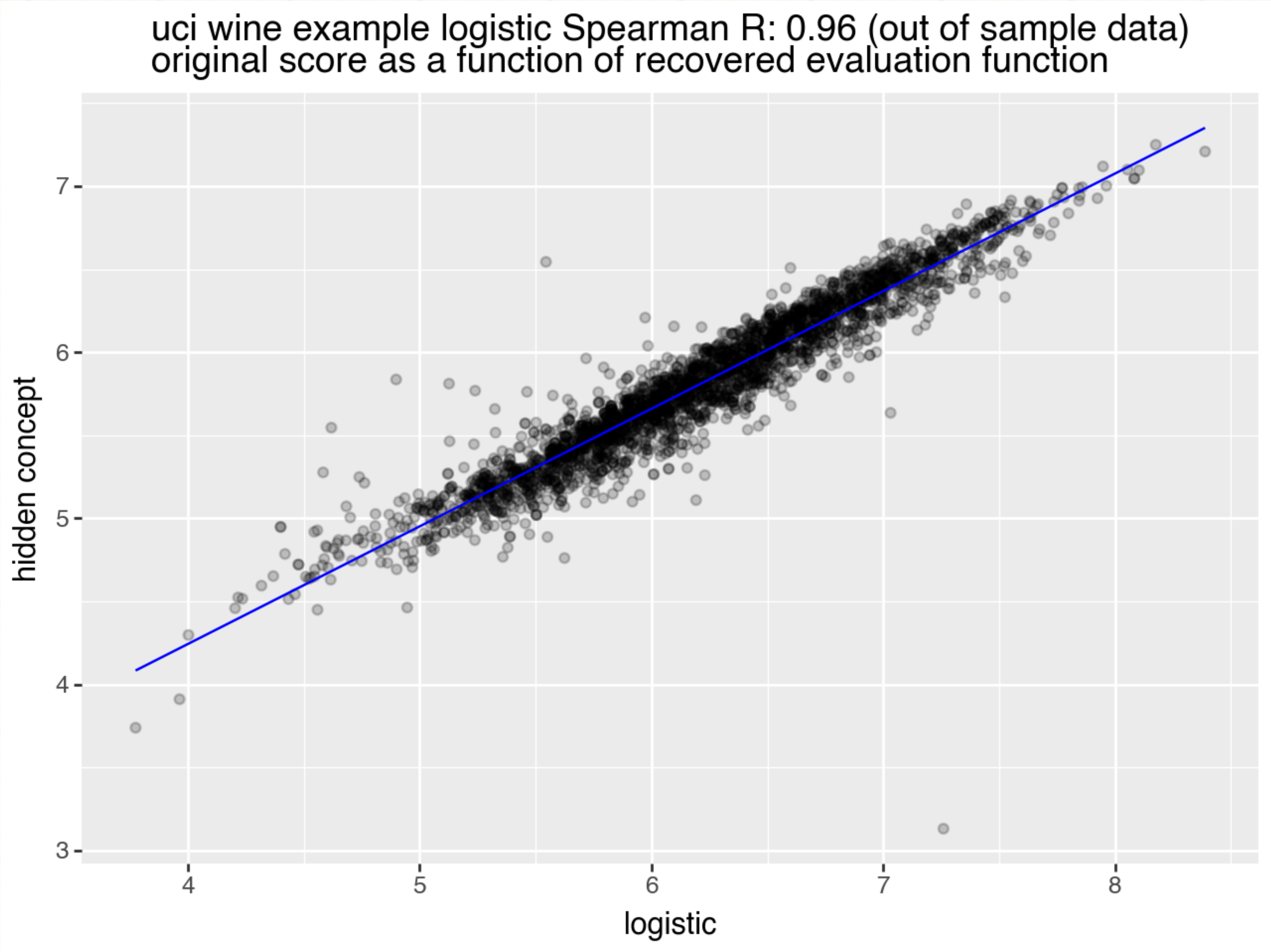
Replace training data with a linear model of user score (an easier problem).
Now (modulo noise) the concept is in our modeling space.

Nearly perfect (1.0)!

100 training lists



1000 training lists



Observations

- All 3 methods perform about as well as each other
  - With correct or incorrect model structure
  - Think of list selection observations as an issue of statistical inefficiency (or censoring), and not directly as an optimization problem.
  - The trick to encoding problems into Stan: add explicit probability conditioning until things become independent.
- The path to a high quality fit is
  - Correct model structure
  - More data
  - Though, some clever ad-hoc ideas can still be incorporated (such as training no examples past the winner).
- Previous criticism may have been mis-attributing poor fit due to bad model structure as poor fit due to “pick data” censoring.
  - Makes sense to try arbitrary classification models that return probabilities (with a logit transform after prediction).
  - For general classifiers must sum-out position encoding during evaluation to get an estimate of intrinsic score (as they may be interacting position with other variables).

Conclusions

- Problem structure likely more important than the issues of pick list presentation.
- Stan is great for prototyping solution methods and experimenting with how much model structure you wish to capture.
- Logistic regression may take some steps to justify, but often works well and is fast.

Tools Used

- Stan, Python, and R
  - All code and data shared here:

    https://github.com/WinVector/Examples/tree/main/rank
- Example code can work from just selection data
  - Though it would have no base score to compare to.
  - Can compare to proxy goal: ability to reproduce picks.

Next Steps

- See if the 3 solutions (Stan list-wise, Stan point-wise, logistic regression) behave similarly on data from chosen domains (where we may not know ground truth).
    - Code works from picks alone
        - hidden preferences used only to generate picks and check inference quality
        - can do different length lists by encoding a “never picked” extra indicator variable
- Try other classification models! (no reason to limit to linear structure)
- Implement “signal attenuation due to pick data” estimator tool.

Thank you