A KALMAN FILTER FOR INTEGRATING DEAD RECKONING, MAP MATCHING AND GPS POSITIONING

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ABSTRACT

A Kalman filter has been developed to integrate the three positioning systems (differential odometer dead reckoning, map matching, and GPS) used in the Automatic Vehicle Location System AVL 2000™ being designed and developed in the Department of Surveying Engineering at The University of Calgary. The system is being targeted for "on road" applications and incorporates a digital map. The filter has been designed to take into account uncertainties, via covariance matrices. In wide open spaces GPS positioning will dominate while in zones where GPS signal has been obstructed, dead reckoning will be used as an automatic interpolation between GPS position fixes. Simulation studies and covariance analyses have been performed on a test route located in a sector of the city of Calgary.

INTRODUCTION

The primary objective of any land based, on-road, Automatic Vehicle Location (AVL) system is to combine navigational devices with route information to the benefit of the user by relating vehicle location to the surrounding environment. To be effective, the vehicle's position must be continuously maintained in real-time as the vehicle journeys throughout its operating region.

Currently a wide variety of AVL systems exist employing diverse positioning technology ranging from LORAN-C, to satellite methods, to dead reckoning devices, and combinations thereof [1] and [2]. One of the methods of positioning for on-road AVL systems which is gaining considerable recognition is dead reckoning augmented with map matching [3]. Unaided, dead reckoning systems (e.g. odometer and compass) require positional updates to control the significant error accumulation over time and distance. To provide positional updates to dead reckoning devices, a clever and economical technique called map matching was devised for street based applications [4]. By taking advantage of the fact that vehicles are constrained to road networks, a correlation (or match) can be made between vehicular movement patterns and digitally defined roads. For example, if relative direction changes can be detected when a vehicle turns at a known, identified intersection, the vehicle's dead reckoned position may be updated automatically by utilizing the known intersection coordinates retrieved from a digital map database. In this manner, error accumulation characteristic to dead reckoning may be controlled periodically, providing the required accuracy demanded by the system of about 20-30 metres.

PROBLEM AND SOLUTIONS

Although augmented dead reckoning components are presently being installed in some AVL systems (e.g. Bosch Blaupunkt's EVA and Etak's Navigator), one obvious positioning limitation still exists. In order for the necessary map matching updates to occur, turns at intersections or recognizable curvatures must be made. If this is not done, errors will accumulate to such a degree that a lost situation will occur. Honey et al. [5] report that Etak's Navigator requires manual re-initialization after approximately 250 miles of normal travel on average. Undoubtedly on long straight highways such as those roads forming state, provincial and federal highway networks, this problem will become more acute, due to inadequate map matched updates, and the lost situation will occur more frequently. Potentially, periodic manual re-initialization would quickly remedy such occurrences but such an effort hardly seems appropriate for an automatic vehicle location system. Also, modern day users will most likely not accept such an inconvenience. In addition to location loss due to infrequent map matching

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updates, augmented dead reckoning also requires manual re-initialization whenever undetected vehicular movement occurs (e.g. ferry crossings, towing).

A complementary approach to augmented dead reckoning is absolute positioning provided by radio location systems such as Navstar GPS and LORAN-C. Such systems do not possess the error accumulation problems inherent in dead reckoning. They do however suffer from signal loss due to shadowing effects created by man-made and natural structures such as sky scrapers, mountains, and foliage. Both Chrysler and GMC have incorporated GPS positioning into their current AVL system designs and realize the limitations due to shadowing. GMC, thus have included in their test and demonstration vehicle GPS, LORAN-C, and dead reckoning using an odometer and compass [6]. Presently, GPS receivers are relatively expensive which has persuaded many AVL system designers to decide against GPS implementation, especially those attacking present markets.

It is strongly felt, however, that when the full GPS satellite constellation is launched in the early 90's and 24 hour coverage is available, increased demands will dramatically reduce present high equipment costs. In this paper we thus present a positioning method that integrates GPS with two complimenting positioning techniques, namely: 1) dead reckoning - involving differential odometry; and 2) map matching - utilizing digital map correlation techniques.

Differential odometry involves averaging and differencing of odometer measurements taken from both wheels of the non-driven wheel pair. This technique provides the system with a means of self-contained positioning which is both, relatively economical and straight forward in concept. It avoids problematic characteristics inherent in other forms of dead reckoning such as magnetic compass configurations which are plagued by magnetic anomalies.

Map matching, provides the system with a valuable form of updating which controls differential odometer error accumulation while traveling. As well, map matching provides an efficient means of monitoring vehicle location with respect to the digital road network as defined in the digital map database.

Navstar GPS satellite positioning determines, in real-time, absolute position fixes by observing ranges from at least four different satellites in polar orbits 20,000 km above the earth's surface. In point positioning mode GPS can achieve accuracies at about the 20 to 30 metre level. In differential mode (i.e. relative to another station) accuracies of about 5 metres or less level are attainable [7] and [8]. The latter mode implies station - vehicle communication via a digital data link. Once in full operation (estimated early next decade) GPS will provide multi-user(s) of AVL systems positional updates anywhere in the world, any time of the day, and under any weather condition. As well GPS can conceptually aid in automatic initialization of an AVL system by link fitting [9].

Integration of the above three complimenting techniques will provide required positional information of a vehicle anywhere within the framework of an automobile transportation network. The integration strategy investigated in this paper blends together the in-coming navigational information by way of a Kalman filter, which takes into account uncertainties, via covariance matrices associated with each of the three positioning methodologies employed. Design and simulation analysis form the basis of this paper.

INTEGRATION CONCEPT

Imagine yourself traveling in an automobile. As you journey along various streets and avenues your surrounding environment is constantly changing. En route, you may encounter a multitude of events which may include stopping at street lights, passing large buildings, changing lanes, and turning at intersections, to name only a few [Figure 1].

Now consider the problem of keeping track of the vehicle's position in real-time as it manoeuvres through the road network. Upon start up, at time t_k^{INIT} (Figure 1), an initialization position is required. Coordinates and azimuth at this point may be entered manually, computed by GPS satellite observations or retrieved from computer storage (i.e. bubble memory) - carried over from the last computed vehicular position prior to shut down (e.g. parked car's position). As the vehicle advances, new positions relative to the initial coordinates are computed using differential odometry. As the vehicle proceeds further, periodic updates from GPS and map matching (MM) respectively at times t_{k+1}^{GPS} and t_{k+2}^{MM} will intermittently control the error accumulation of the on-board differential odometers. While operating in GPS shadow zones (e.g. urban centers), map matching will serve to provide these necessary positional updates (t_{k+3}^{MM} to t_{k+5}^{MM}). In contrast, during highway travel (i.e. long, open straight aways) where map matching is not possible, GPS will provide the required positional updates from

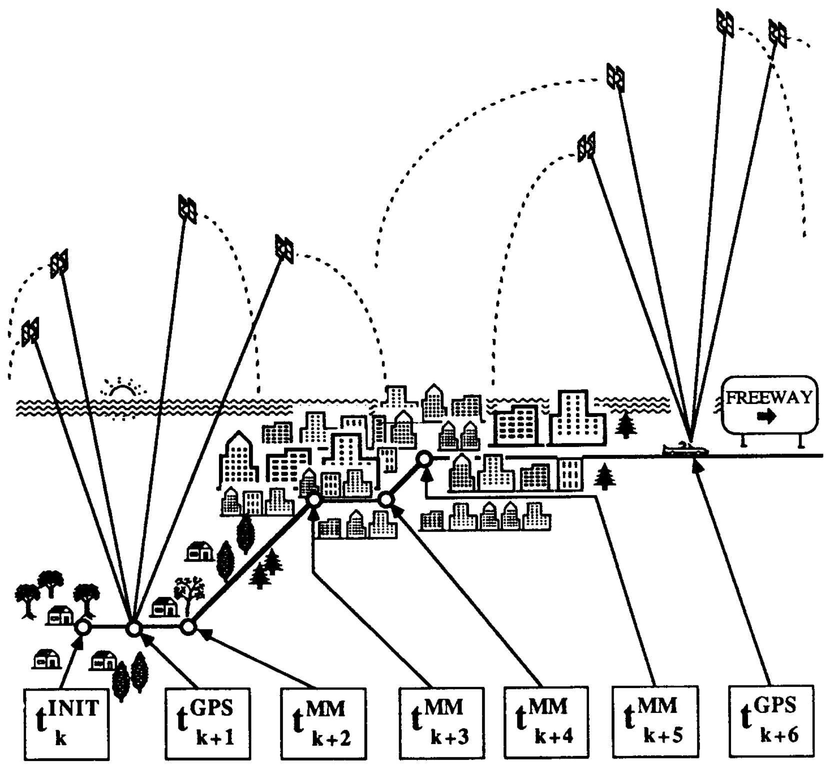

Figure 1 - Intergration Concept

time to time, e.g. t^{GPS}_{k+6} (Figure 1).

MATHEMATICAL FORMULATION

The above concept of integrating dead reckoning, map matching, and GPS positioning can be represented mathematically as follows. Three mathematical models f are needed to capture the basic unit, namely f_k and f_{k+1} at the two time epochs t_k and t_{k+1}, and $g_{k,k+1}$ for the interval between them. Map matching and GPS corresponds to f_k and f_{k+1}, while dead reckoning corresponds to $g_{k,k+1}$. This basic unit is then successively applied to pairs of epochs beginning with t_k and t_{k+1}, until all epochs (e.g. up to t_{k+6} in Figure 1) have been covered.

Considering the state vector of unknowns (vehicle coordinates) to be denoted by the vector $\mathbf{x}$ and GPS observed ranges by the vector $\mathbf{l}$, along with an a priori covariance matrix $\mathbf{C}_l$, we can write the models as follows:

$$f_k(\mathbf{x}_k, \mathbf{l}_k) = \mathbf{0}, \quad \mathbf{C}_{l_k} \qquad (1)$$

$$f_{k+1}(\mathbf{x}_{k+1}, \mathbf{l}_{k+1}) = \mathbf{0}, \quad \mathbf{C}_{l_{k+1}} \qquad (2)$$

$$g_{k,k+1}(\mathbf{x}_k, \mathbf{x}_{k+1}, \mathbf{y}_{k,k+1}, t) = \mathbf{0}, \quad \mathbf{C}_y \qquad (3)$$

where $\mathbf{y}_{k,k+1}$ denotes the vector of model errors corresponding to dead reckoning uncertainties with a priori covariance matrix $\mathbf{C}_y$. Map matching is considered to be the direct measurement $\mathbf{l}_k$ or $\mathbf{l}_{k+1}$ of the coordinates in $\mathbf{x}_k$ or $\mathbf{x}_{k+1}$, thus f_k and f_{k+1} become the simplest of functions in this case.

The solution for the state vector $\mathbf{x}_{k+1}$ and its covariance matrix is achieved by the simultaneous solution of the above three equations. Given the fact that they are in general non-linear, the first step is to linearize them [10], namely:

$$\mathbf{r}_k = \mathbf{A}_k\delta_k + \mathbf{w}_k, \quad \mathbf{C}_{l_k} \tag{4}$$

$$\mathbf{r}_{k+1} = \mathbf{A}_{k+1}\,\delta_{k+1} + \mathbf{w}_{k+1}, \quad \mathbf{C}_{l_{k+1}} \tag{5}$$

$$\varepsilon_{k,k+1} = \delta_{k+1} - \Phi_{k,k+1}\,\delta_k, \quad \mathbf{C}_y \tag{6}$$

where: $\mathbf{A}_k$ and $\mathbf{A}_{k+1}$ are the well known design matrices for GPS positioning [11] and are unit matrices for map matching [12]; $\mathbf{w}_k$ and $\mathbf{w}_{k+1}$ represent the models evaluated at approximate values of the state vectors $\mathbf{x}^o_k$ and $\mathbf{x}^o_{k+1}$ and observed values of the observables $\mathbf{l}_k$ and $\mathbf{l}_{k+1}$; $\Phi_{k,k+1}$ is the transition matrix and for dead reckoning it is a function of changes in odometer distances Δd and azimuths $\Delta\alpha$ [12]; δ_k and δ_{k+1} are the unknown corrections to the state vectors; $\mathbf{r}_k$ and $\mathbf{r}_{k+1}$ are unknown residuals while $\varepsilon_{k,k+1}$ are unknown model errors. All unknowns can be solved for, but it is usually only δ_k and δ_{k+1} that are of prime interest.

Minimizing the quadratic forms of $\mathbf{r}_k$, $\mathbf{r}_{k+1}$ and $\varepsilon_{k,k+1}$ is the least squares criterion which yields the well known Kalman filter equations [13]; [10]; [14]. We will now describe the Kalman filter equations within the context of the problem at hand, and note that the cap "^" signifies that the above mentioned least squares estimate is being made. The prediction equations are, first, for the predicted state vector

$$\hat{\delta}^{(-)}_{k+1} = \Phi_{k,k+1}\,\hat{\delta}_k; \tag{7}$$

and for the predicted covariance matrix

$$\mathbf{C}^{(-)}_{k+1} = \Phi_{k,k+1}\,\mathbf{C}^x_k\,\Phi^T_{k,k+1}, \tag{8}$$

where $\mathbf{C}^x_k$ is a covariance matrix related to the solution involving information principally in f_k [10]. The f_k solution for position is based on either GPS or MM or both. The (-) designation signified that the predicted solution consists of GPS and/or MM at t_k, and dead reckoning up to t_{k+1}, but not the GPS or MM at t_{k+1}. The filtered state vector is

$$\hat{\delta}^{(+)}_{k+1} = \hat{\delta}^{(-)}_{k+1} - \mathbf{K}(\mathbf{w}_{k+1} + \mathbf{A}_{k+1}\,\delta^{(-)}_{k+1}) \tag{9}$$

and the corresponding covariance matrix is

$$\mathbf{C}^{(+)}_{k+1} = \mathbf{C}^{(-)}_{k+1} - \mathbf{K}\,\mathbf{A}_{k+1}\,\mathbf{C}^{(-)}_{k+1}\,. \tag{10}$$

where $\mathbf{K}$ is the Kalman gain matrix and is a function of principally $\mathbf{C}^{(-)}_{k+1}$, $\mathbf{C}_{l_{k+1}}$, and $\mathbf{A}_{k+1}$. By way of interpretation, the above filtered equations (denoted by "+") account for the addition of the GPS and/or MM coordinates at t_{k+1} to the accumulated solution for the position of the vehicle. Equations (9) and (10) form the basis for the covariance analysis in the next section.

COVARIANCE ANALYSIS

The basis for the covariance analysis is equation 9 for the state vector , and equation 10 for the covariance matrix. The input variances to these equations were as follows:

1. positioning by map matching, 225 m^2;
2. positioning by GPS, 400 m^2
3. dead reckoning system noise - odometer, 4.0×10^{-2} m^2 (for a 20 m interval); dead reckoning change in azimuth from differential odometer, 1.0×10^{-10} rad^2 (for a 20 m interval), and a Gauss Markov process for the average scale error of the odometer [12]. The above variable value for the change in azimuth is somewhat optimistic; a more realistic value will be used once more experience is gained from field testing.

The covariance analysis comprises of a simulation of a route as shown in Figure 2. An actual route of 13.5 km through a sector of the City of Calgary was chosen for the simulation. It has a total of eleven Kalman update locations. Updates numbered 0 through 4 (t_0 through t_4) are in a residential subdivision: t_0 corresponds to the start time update, while times t_1, t_2 and t_3 correspond to turns at which positioning updates are performed by map matching. Updates at t_5 and t_6 are on a straight road segment where map matching cannot be performed, thus positioning is presumed to be made by GPS. At time t_7 map matching is again possible. The selected route ends in another residential subdivision containing three turns. Updates at t_8, t_9 and t_{10} correspond to intersections where positioning is made by map matching; t_{11} is the end.

The state vector of positions was perturbed by adding random errors to differential dead reckoning, MM and GPS commensurate with their input variances as stated above. This perturbed state vector (containing X and Y coordinates) is plotted along with its corresponding standard deviation (square root of the corresponding element of the covariance matrix). In Figures 3 and 4 results are presented for only dead reckoning (DR) and map matching

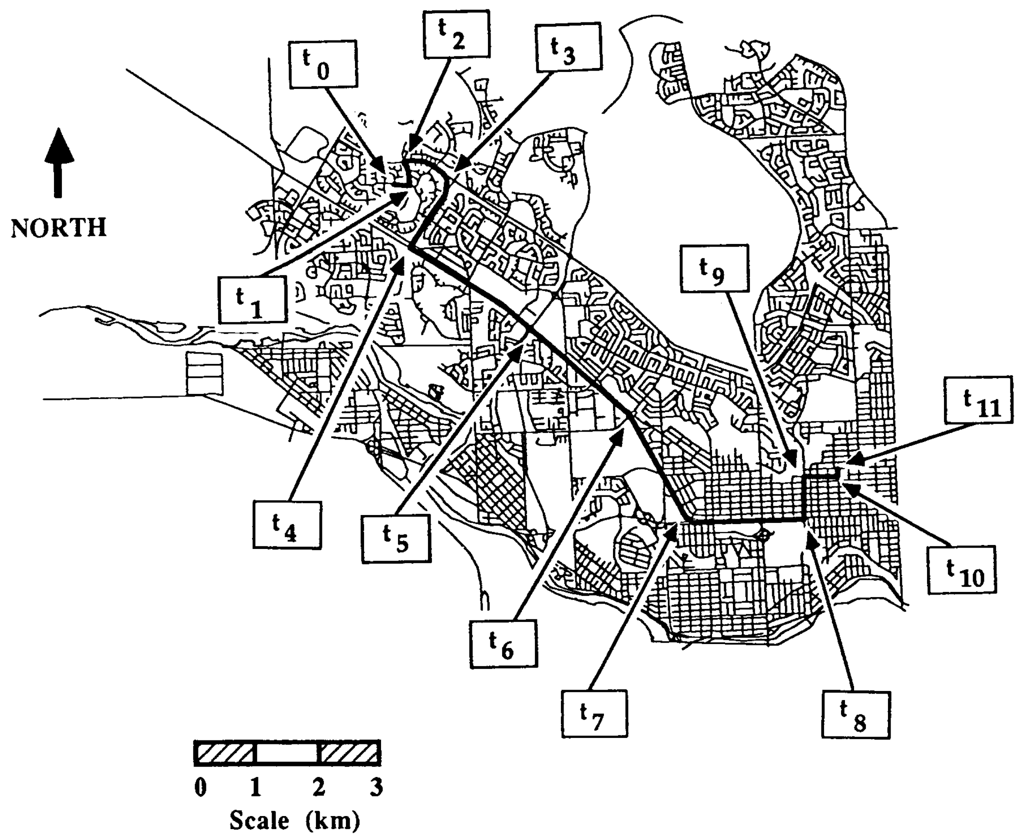

Figure 2 - N.W. Calgary Test Route for Simulation Analysis

(MM), while in Figures 5 and 6 results are shown for the case where GPS is added.

The DR and MM results (Figures 3 and 4) clearly indicate that MM can control the accumulated error (less than 20 m - updates t_1, t_2, and t_3) in an area of several turns, while in the open stretch of road (updates t_5 and t_6) where MM cannot be performed the error grows to nearly 60 m. Note the perturbed state vector stays within the 1σ error envelope.

Consider the case of DR, MM and GPS (Figures 5 and 6). GPS positioning is performed at updates t_5 and t_6 and clearly the accumulated error is brought down and contained to about 20 m. Note that GPS fixes would be needed only where the errors begin to grow over the 20 m level.

CONCLUSIONS

Dead reckoning (DR), map matching (MM), and GPS positioning have been shown to be complementary positioning technologies worthy of integration in the urban environment. The Kalman filter has been shown to be an easy and automatic way to combine these three positioning technologies.

In areas masking GPS signals, such as amidst tall buildings, the filter is dominated by the dead reckoning and if turns are made the MM component takes over. In open spaces of long straight stretches of road, MM cannot be performed, thus the filter is dominated by DR and GPS fixes. The GPS fixes act to control the accumulation of DR errors.

The next step in this research is to

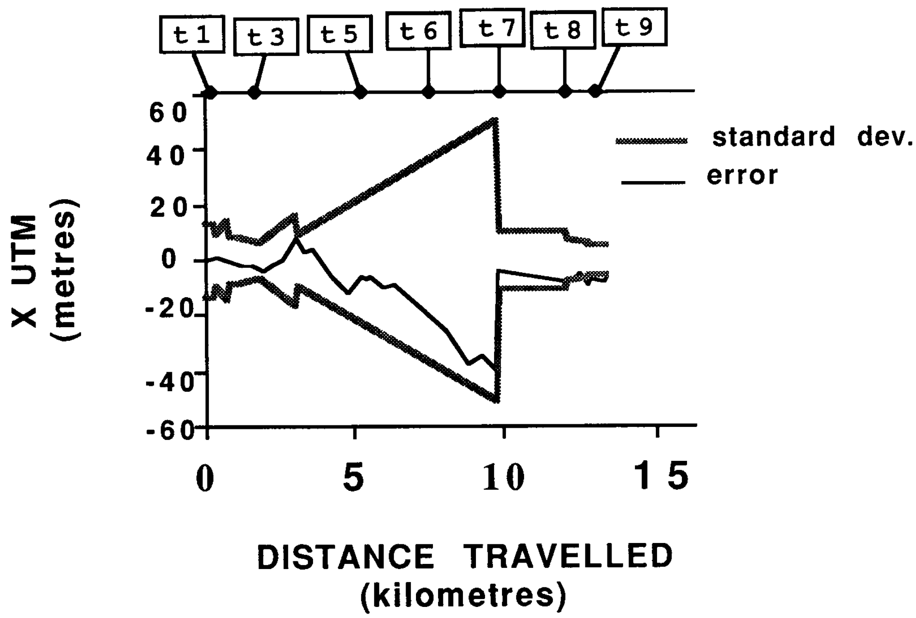

Figure 3 - X (UTM) Standard Deviation and Error
[DR and MM only]

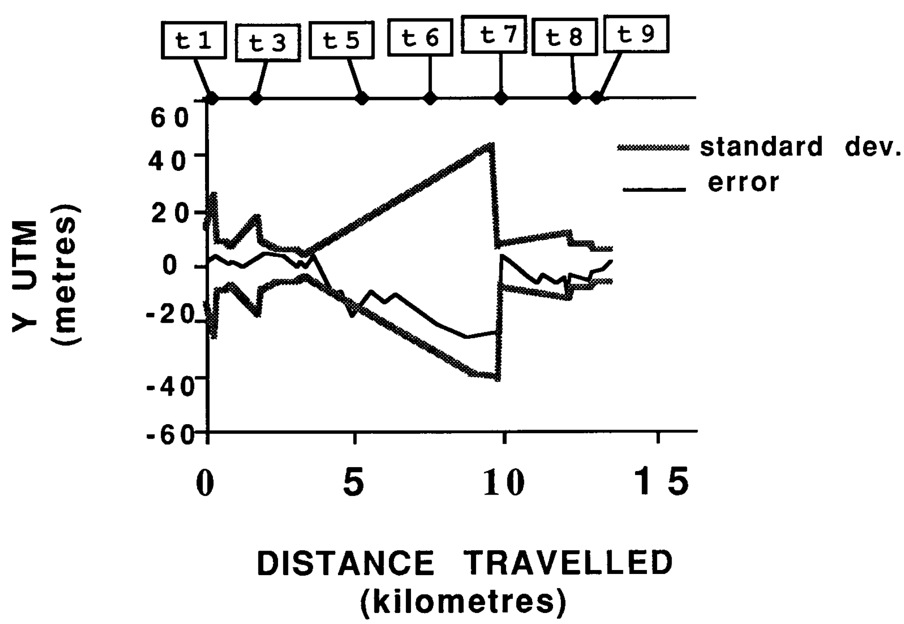

Figure 4 - Y (UTM) Standard Deviation and Error
[DR and MM only]

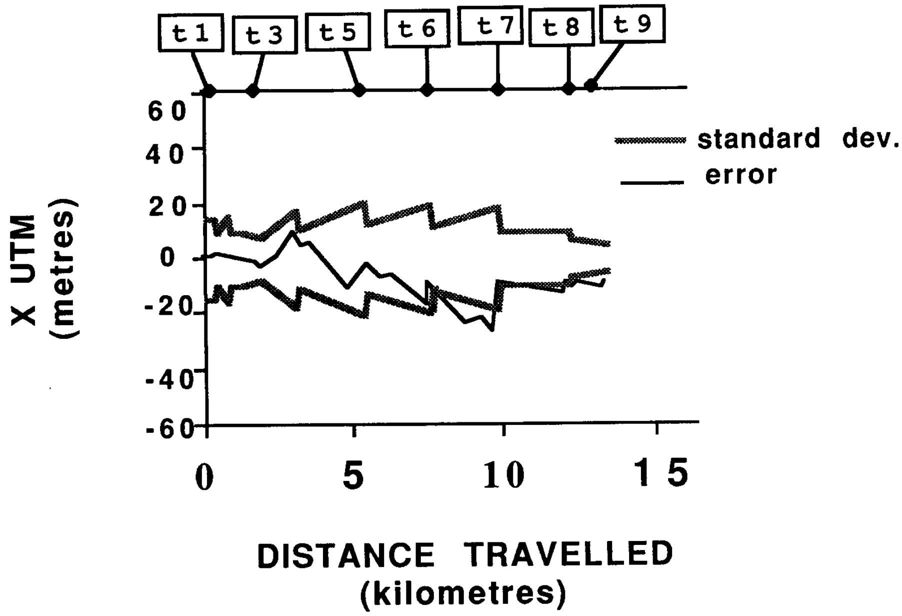

Figure 5 - X (UTM) Standard Deviation and Error
[DR, MM and GPS]

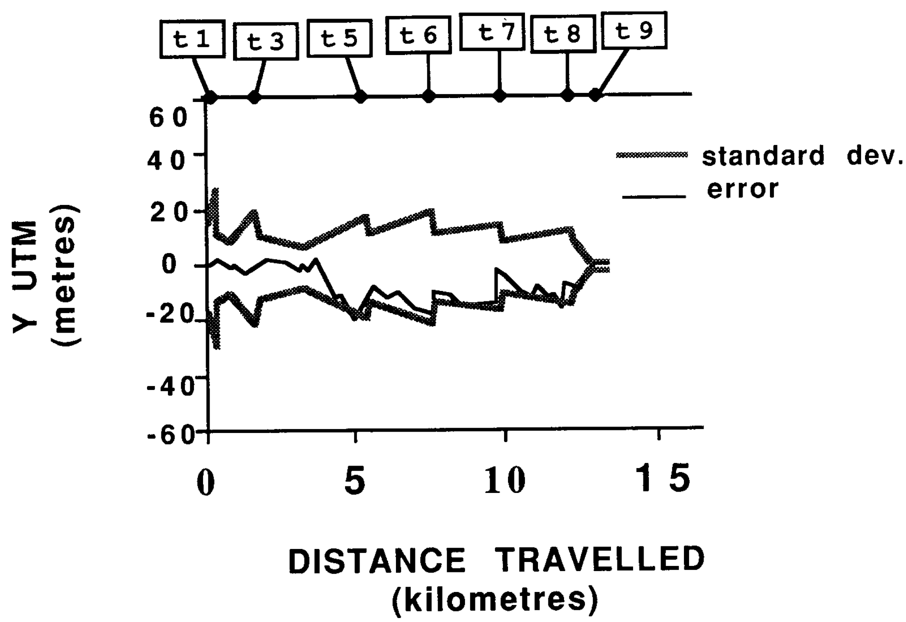

Figure 6 - Y (UTM) Standard Deviation and Error
[DR, MM and GPS]

implement the filter in the AVL 2000™ prototype, which now operates with separate DR and separate GPS positioning. It will also be necessary to implement the map matching scheme. It is envisaged that this implementation will be achieved over the winter and results will be reported in the spring of 1989.

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