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INCREASED ACCURACY OF MOTOR VEHICLE POSITION ESTIMATION BY UTILISING MAP DATA, VEHICLE DYNAMICS, AND OTHER INFORMATION SOURCES

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ABSTRACT

Many positioning systems employed to track motor vehicles use location techniques that were not designed solely for this purpose (GPS, terrestrial radio signals). As such they fail to utilise the restriction of motor vehicles to the road network and thus a valuable source of position information is lost. Techniques exist that make use of map information to improve the position estimate of a motor vehicle but the techniques lack a mathematical framework. The authors have addressed this problem by developing a map-aided position estimation system whereby the raw position measurements are optimally translated so that they lie on the roads. The accuracy of the map-aided estimates is derived for an arbitrary positioning system with Gaussian measurement noise demonstrating significant improvements over the raw measurements. Further performance improvements are achieved through the use of a one-dimensional kalman filter developed to utilise the fact that all of the map-aided position estimates lie along known curves. The mathematical framework utilised by the map-aided estimator readily allows other sources of position information such as road type and road rules to be quantified and optimally incorporated into the estimation process.

I. INTRODUCTION

Motor vehicles are, in general, restricted to travel on roads but the positioning systems used to estimate their position do not inherently have the ability to locate the vehicles onto the roads. The various noise sources that affect the signals and instrumentation used by the positioning system result in the measured position not necessarily lying on the road network. Apart from the beginning and end of a journey, a vehicle is highly unlikely to be in the middle of a building as possibly reported by a positioning system and therefore the position estimate needs to be refined to make use of the knowledge that the vehicle is restricted to the road network.

Map matching is well established as a means of utilising map information in positioning systems. The effectiveness of the technique being illustrated by the significant proportion of IVHS (Intelligent Vehicle/Highway System) navigation systems that utilise it [11]. Map matching techniques have been used in many different ways including the correction of drift errors in dead reckoned (DR) navigation systems, [4, 7] and road identification within ADIS (Advanced Driver Information Systems) [8]. Map information has also been incorporated directly into the estimation process of a GPS system for situations where there are insufficient visible satellites [2]. Despite the proliferation of map matching systems, there technique appears to lack a mathematical framework.

This paper presents an estimation process within a well defined mathematical framework that allows map information and other sources of position information to be optimally incorporated. The work presented represents a part of a larger research program into the general theory of positioning systems [9] and the notation used in this paper is based on this.

Since the noise in the positioning system results in measurements that do not necessarily lie on the road, the amount of noise present must be at least the distance from the measurement to the road. It should be possible to remove this noise to produce a better position estimate. Translating the measurement to the nearest point on the road network is the intuitive and most straight forward method for achieving this but it is not necessarily the optimal position estimate and there is a significant probability that the measurement will be translated onto the wrong road. A MAP (maximum *à posteriori*) estimator has been developed to optimally translate raw position measurements onto the road network with the resultant increase in position estimation accuracy.

A consequence of this process is that all of the position measurements now lie on the road network and if the road is modelled by its centreline, the vehicle's trajectory lies along a curve – the majority of which are straight lines. A motor vehicle has two degrees of freedom and normally requires a two-dimensional kalman filter to take advantage of the knowledge of the vehicle's dynamics, but since the position estimates now lie along a curve a kalman filter with reduced degrees of freedom can be developed. Such a filter is presented.

II. MAP AIDED POSITIONING

In order to utilise road network information for map aided positioning, the maps and the vehicle's domain must be modelled. The roads have finite width but the lateral position on the road is not of interest so the road is modelled by its centreline. The restriction of the vehicle's domain to the road network $\mathcal{R}$ will be represented

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by a uniform positional probability distribution $p(\boldsymbol{x})$.

$$p(\boldsymbol{x}) = \begin{cases} 0 & \boldsymbol{x} \notin R \\ k & \boldsymbol{x} \in R \end{cases} \quad (1)$$

where k is a constant defined such that $\int_{\mathcal{R}} k d\boldsymbol{x} = 1$. The uniform positional PDF implies that no knowledge of the vehicle's behaviour has been incorporated into the estimation process except for the restricted domain. The addition of supplementary information sources will be discussed later in this paper.

The positioning system makes a measurement $\boldsymbol{y}$, in the presence of noise $\boldsymbol{\eta}$, of the position $\boldsymbol{x}$ of a vehicle. In general, positioning systems are affected by a number of independent noise sources so it is reasonable to assume that the noise $\boldsymbol{\eta}$ has a gaussian distribution which will have a zero mean provided that the system is properly calibrated.

$$\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{\eta} \quad (2)$$

$$p(\boldsymbol{y}|\boldsymbol{x}) = \frac{1}{|2\pi \mathrm{N}|^{\frac{1}{2}}} e^{-\frac{1}{2}\left[(\boldsymbol{y}-\boldsymbol{x})^T \mathrm{N}^{-1}(\boldsymbol{y}-\boldsymbol{x})\right]} \quad (3)$$

where

$$\mathrm{N} = E\left[(\boldsymbol{\eta} - E[\boldsymbol{\eta}])^T(\boldsymbol{\eta} - E[\boldsymbol{\eta}])\right] \quad (4)$$

In the case of motor vehicle positioning, only two dimensions are of interest and the covariance matrix is therefore defined as

$$\mathrm{N} = \begin{pmatrix} \sigma_{\eta_1}^2 & \sigma_{\eta_{12}} \\ \sigma_{\eta_{12}} & \sigma_{\eta_2}^2 \end{pmatrix} \quad (5)$$

and from [12]

$$p(\boldsymbol{y}|\boldsymbol{x}) = \frac{1}{2\pi\sigma_{\eta_1}\sigma_{\eta_2}\sqrt{1-r^2}} \cdot e^{-\frac{1}{2(1-r^2)}\left[\frac{(y_1-x_1)^2}{\sigma_{\eta_1}^2} - \frac{2r(y_1-x_1)(y_2-x_2)}{\sigma_{\eta_1}\sigma_{\eta_2}} + \frac{(y_2-x_2)^2}{\sigma_{\eta_2}^2}\right]} \quad (6)$$

where

$$r = \frac{\sigma_{\eta_{12}}}{\sigma_{\eta_1}\sigma_{\eta_2}} \quad (7)$$

is the spatial correlation coefficient and $|r| \leq 1$.

Having quantified the assumptions regarding the vehicle's domain and the measurement noise of the positioning system, the estimation process to optimally combine the position measurement with the vehicle's domain can now be defined. The MAP estimate of $\boldsymbol{x}$ at time k, denoted by $\hat{\boldsymbol{x}}(k)$, is defined by [1, 9, 10]

$$\hat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x}} [p(\boldsymbol{x})p(\boldsymbol{y}|\boldsymbol{x})] \quad (8)$$

III. Long Straight Roads

The simplest scenario for matching a measured position to a map is the case where a vehicle is known to be travelling on a very long straight single-lane road. The road $\mathcal{R}$ will be modelled by its centreline and a new coordinate frame $\mathcal{X}$ is defined by translating and rotating the global frame $\mathcal{W}$ such that the road is collinear with the x_1 axis (fig. 1). Thus the road is defined as

$$\mathcal{R} = \{(x_1, x_2) : x_2 = 0\} \quad (9)$$

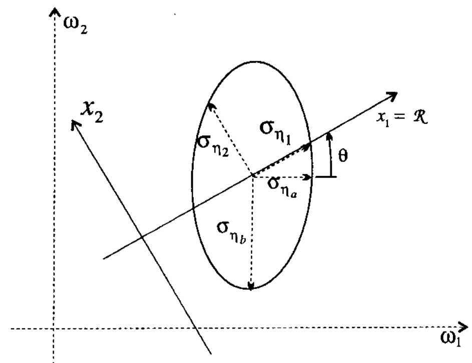

Figure 1: *Coordinate frame for map matching: The ellipse represents a contour of constant error probability. σ_{η_1} and σ_{η_2}, are the standard deviations of the measurement error as observed in the $\mathcal{X}$ frame. Similarly, σ_{η_a} and σ_{η_b} are the errors for the $\mathcal{A}$ frame.*

The positional PDF $p(\boldsymbol{x})$ is given by equation (1) where for this example, k is a small positive constant. The MAP position estimate can now be determined by solving equation (8) [13].

$$\hat{\boldsymbol{x}} = \left(y_1 - \frac{\sigma_{\eta_1}}{\sigma_{\eta_2}} r y_2, 0\right) \quad (10)$$

Analysis of this estimator proves it to be unbiased and the variance is given by

$$\sigma_{\hat{x}_1}^2 = (1 - r^2)\sigma_{\eta_1}^2 \quad (11)$$

As mentioned previously, the minimum amount of measurement noise present can be determined by the distance between the measured position nearest road. Thus the nearest point (NP) estimator is defined by translating a given position measurement to the nearest point on the road network. By again using the local coordinate frame $\mathcal{X}$, the NP estimator is described by

$$\tilde{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \left[(\boldsymbol{y} - \boldsymbol{x})^T(\boldsymbol{y} - \boldsymbol{x})\right] \quad (12)$$

$$= (y_1, 0) \quad (13)$$

$$(14)$$

This is also an unbiased estimator but the variance of the estimate is larger than the optimal estimate from the MAP estimator.

$$\sigma_{\tilde{x}_1}^2 = \sigma_{\eta_1}^2 \quad (15)$$

The difference between the optimal MAP estimate and the more intuitive NP estimator arises from the utilisation of the spatial correlation of the measurement errors by the MAP estimator. As the correlation increases, the MAP estimator variance decreases eventually becoming the ideal estimator when the errors are fully correlated ($r = \pm 1$), but to evaluate estimator's true performance, the source and effect the error correlation needs to be investigated.

The spatial correlation arises from the angle between

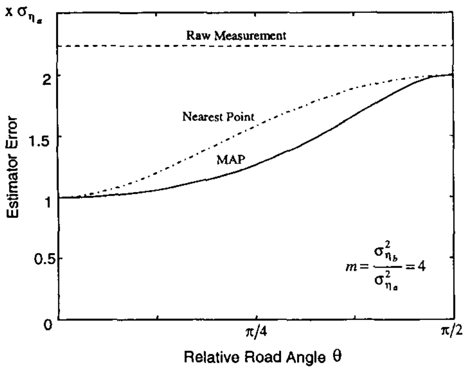

Figure 2: *Performance of MAP & NP estimators and the raw measurment as a function of road heading (or more specifically, the angle between the $\mathcal{A}$ and $\mathcal{X}$ frames. The symmetry of the equations meant that estimator errors only needed to be plotted for $0 \le \theta \le \frac{\pi}{2}$*

the measurement error ellipse minor axis and the local coordinate frame $\mathcal{X}$. That is, the measurement error parameters and consequently, the correlation coefficient r, are a function of the angle θ between the coordinate frames $\mathcal{A}$ and $\mathcal{X}$ where the coordinate frame $\mathcal{A}$ is defined by the rotation of the $\mathcal{W}$ frame such that the covariance observed in the $\mathcal{A}$ frame is zero (see figure 1). By using standard coordinate transformation techniques it can be shown that these measurement error parameters are given by [13]

$$\begin{aligned}
{\sigma_{\eta_1}}^2 &= \sigma^2_{\eta_a}\cos^2\theta + \sigma^2_{\eta_b}\sin^2\theta & (16)\\
{\sigma_{\eta_2}}^2 &= \sigma^2_{\eta_a}\sin^2\theta + \sigma^2_{\eta_b}\cos^2\theta & (17)\\
\sigma_{\eta_{12}} &= \sin\theta\cos\theta(\sigma^2_{\eta_b} - \sigma^2_{\eta_a}) & (18)\\
r &= \left(1 + \frac{4m}{(m-1)^2}\mathrm{cosec}^2 2\theta\right)^{-\frac{1}{2}} & (19)
\end{aligned}$$

where $m = \sigma^2_{\eta_b}/\sigma^2_{\eta_a}$ and $\sigma^2_{\eta_a}$,$\sigma^2_{\eta_b}$ are the error variances as measured from the $\mathcal{A}$ reference frame. From these parameters, the performance of the various estimators can be expressed as a function of the road heading.

The performance of a positioning system is often quantified by the circular error probable (CEP) [16] but in this instance, the measurements are not distributed in two dimensions so the RMS error between the estimate and the true position $\bar{\boldsymbol{x}}$ is more suitable. The MAP and NP estimators are unbiased and therefore, the RMS errors are equal to the estimator standard deviations.

$$\begin{aligned}
e(\hat{\boldsymbol{x}}) &= \sqrt{E\left[(\bar{\boldsymbol{x}} - \hat{\boldsymbol{x}})^T(\bar{\boldsymbol{x}} - \hat{\boldsymbol{x}})\right]} & (20)\\
&= \sigma_{\eta_1}\sqrt{1 - r^2} & (21)\\
e(\tilde{\boldsymbol{x}}) &= \sigma_{\eta_1} & (22)\\
e(\boldsymbol{y}) &= \sqrt{\sigma^2_{\eta_1} + \sigma^2_{\eta_2}} & (23)
\end{aligned}$$

Using Eqs. (16)–(19), these RMS errors are now plotted as a function of the road heading (fig. 2). The MAP estimator is clearly a better estimator than the NP estimator, equivalence only occurring when the errors are uncorrelated ($\theta = 0, \frac{\pi}{2}$).

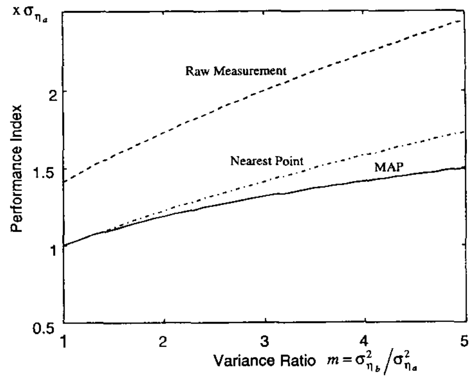

Figure 3: *Estimator performance indicies (Eq. 26–28) are plotted as a function of the measurement noise ratio m.*

The RMS estimator errors quantify the performance of the estimators for a given road heading and measurement noise but they do not provide a means of evaluating the performance of the estimators across an entire network. Now since the road headings across an entire network can be assumed to be distributed uniformly an average performance can be determined for each estimator given the raw measurement errors. This allows a performance index i to be defined as follows and these indicies are plotted as a function of the measurement error variance ratio m (fig. 3).

$$\begin{aligned}
i^2(x) &= E\left[e^2(\boldsymbol{x})\right] & (24)\\
&= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} e^2(\boldsymbol{x})d\theta & (25)\\
i^2(\hat{x}) &= \sigma^2_{\eta_a}\sqrt{m} & (26)\\
i^2(\tilde{x}) &= \sigma^2_{\eta_a}\frac{m+1}{2} & (27)\\
i^2(y) &= \sigma^2_{\eta_a}(m+1) & (28)
\end{aligned}$$

IV. Finite Straight Roads

The above analysis was for very long straight roads but in an urban area, the roads cannot necessarily be considered long with respect to the positioning system errors. A map-aided position estimator for a road of finite length is required. A straight road $\mathcal{R}$ of length $2l$ is defined by

$$\mathcal{R} = \{(x_1, x_2) : -l \le x_1 \le l, x_2 = 0\} \qquad (29)$$

The vehicle's existence on this road is represented by a uniform PDF (Eq. 1).

$$p(\boldsymbol{x}) = \frac{1}{2l}\left[u(x_1 + l) - u(x_1 - l)\right]\delta(x_2) \qquad (30)$$

where $u(x)$ is the unit step function. Using the same measurement noise distribution as for the very long

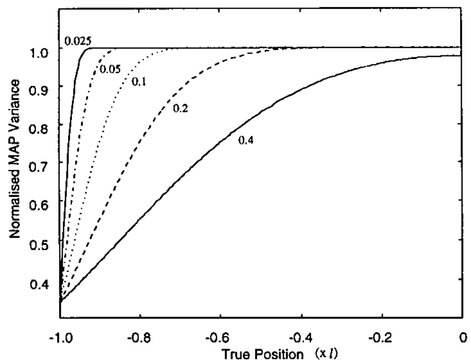

Figure 4: *MAP estimator variance on a finite road normalised to the that of a very long road. Each trace represents a different measurement error standard deviation to length ratio.*

road, the MAP and NP estimates can be shown to be [13]

$$\hat{\boldsymbol{x}} = \begin{cases} (-l,0) & \text{for } y_1 - \frac{\sigma_{\eta_1}}{\sigma_{\eta_2}} r y_2 \geq l \\ (y_1 - \frac{\sigma_{\eta_1}}{\sigma_{\eta_2}} r y_2, 0) & \text{for } \left|y_1 - \frac{\sigma_{\eta_1}}{\sigma_{\eta_2}} r y_2\right| < l \\ (l,0) & \text{for } y_1 - \frac{\sigma_{\eta_1}}{\sigma_{\eta_2}} r y_2 \leq l \end{cases} \tag{31}$$

$$\tilde{\boldsymbol{x}} = \begin{cases} (-l,0) & \text{for } y_1 \leq -l \\ (y_1,0) & \text{for } |y_1| < l \\ (l,0) & \text{for } y_1 \leq l \end{cases} \tag{32}$$

The normalised variance of the MAP estimators is plotted in figure 4 as a function of position. Each trace represents a different measurement-error to road-length ratio which is effectively a measure of the relative length of the road; the greater the ratio the shorter the relative length of the road. Position measurements that lie a significant distance away from the endpoints result in MAP and NP estimates that are no different to those presented earlier for very long roads. When the vehicle is closer to the endpoints, there is an increased possibility that the measurements will lie beyond the road endpoints resulting in some measurements being translated to the nearest endpoint. The resulting error is smaller than would have occurred had the vehicle been on a long road. The finite length of the road is a source of position information for both estimators with the MAP estimator again being the better of the two through the utilisation of the measurement error correlation.

V. Curved Roads

In an urban area the majority of roads are straight but the curved roads must still be included in a map-aided position estimation system. The process to determine the MAP estimate for the curved road $\mathcal{R}$ is similar to that for a straight road. The curved road is defined by:

$$\mathcal{R} = \{(x_1, x_2) : x_2 = \rho(x_1)\} \tag{33}$$

where $\rho(x_1)$ is a function that models the road's centreline to a desired accuracy.

The knowledge that the vehicle lies on the road is again represented by a uniform probability distribution (Eq. 1) where k is the inverse of the length of the road.

$$k = \left[\int_{x_{i_1}}^{x_{f_1}} \sqrt{1 + [\rho'(x_1)]^2}\,\mathrm{d}x_1\right]^{-1} \tag{34}$$

A number of possible functions suitable for modelling roads were investigated (see [13]), the most flexible and consistent model being piecewise continuous curves. The road Ξ is modelled by a sequence of spline functions $\rho_i(x_1)$ between the positions ξ_{i1} and $\xi_{(i+1)1}$.

$$\rho(x_1) = \begin{cases} \rho_1(x_1) & \text{for } \xi_{11} \leq x_1 < \xi_{21} \\ \rho_2(x_1) & \text{for } \xi_{21} \leq x_1 < \xi_{31} \\ \vdots & \\ \rho_n(x_1) & \text{for } \xi_{n1} \leq x_1 < \xi_{(n+1)1} \end{cases} \tag{35}$$

To find the MAP position estimate (Eq. 10), each spline of the curve is treated as a separate road and a locally optimum position estimate $\hat{\boldsymbol{x}}_i$ is determined for that spline. The overall position estimate is then determined by evaluating each local estimate to determine the optimal estimate for the entire curve.

$$\hat{\boldsymbol{x}}_i = \arg\max_{\boldsymbol{x}} p_i(\boldsymbol{x}) p(\boldsymbol{y}|\boldsymbol{x}), i = 1, \ldots, n \tag{36}$$

where

$$p_i(\boldsymbol{x}) = \begin{cases} k_i & \xi_{i1} \leq x_1 < \xi_{(i+1)1}, x_2 = \rho_i(x_1) \\ 0 & \text{elsewhere} \end{cases} \tag{37}$$

$$k_i = \left[\int_{\xi_{i1}}^{\xi_{(i+1)1}} \sqrt{1 + [\rho_i'(x_1)]^2}\right]^{-1} \tag{38}$$

$$\hat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x}} p(\boldsymbol{x}) p(\boldsymbol{y}|\boldsymbol{x}) \tag{39}$$

$$\hat{\boldsymbol{x}} = \arg\max_{\hat{\boldsymbol{x}}_i} p(\hat{\boldsymbol{x}}_i) p(\boldsymbol{y}|\hat{\boldsymbol{x}}_i) \tag{40}$$

A number of interpolation functions were considered including linear splines, cubic splines, Bezier curves, and B-splines but all of these, with the exception of the linear splines, resulted expressions describing $\hat{\boldsymbol{x}}_i$ that could only be solved numerically. Also, the literature on digital maps [6, 15] imply that there is insufficient map detail to allow the higher order splines to be more accurate than the linear splines. As a result of these two factors, the linear spline model was chosen with the added advantage that an entire road network could be modelled by a single component, the finite straight road. The piecewise linear model of a road is illustrated in figure 5 and described by

$$\rho_i(x_1) = m_i x_1 + b_i \text{ for } \xi_{i1} \leq x_1 < \xi_{(i+1)1} \tag{41}$$

where

$$m_i = \frac{\xi_{(i+1)2} - \xi_{i2}}{\xi_{(i+1)1} - \xi_{i1}} = \tan\theta_i \tag{42}$$

$$b_i = \xi_{i2} - m_i \xi_{i1} \tag{43}$$

The MAP and NP position estimates for a given spline are given by finite road estimators (31) and (32) respec-

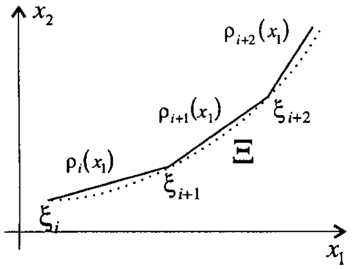

Figure 5: *Piecewise road model $\rho(x_1)$ is shown in contrast to the true road path Ξ.*

tively. The performance of the MAP and NP estimators on a piecewise linear road will be comparable to that of the finite straight road except that the endpoint effects will only be noticeable at the curve endpoints and not at every node of the piecewise curve.

The nature of curves, irrespective of the model used, causes a new problem to arise. It is possible to construct a situation where two, and possibly more, local position estimates are equally likely and consequently, the MAP estimate is not unique. This type of situation is not very likely but it is the manifestation of a bigger problem arising from the likelihood function $p(y|x)$ having multiple local maxima. For a given measurement the MAP estimate may be unique but another significantly different position may be almost as good. The resolution of this problem, which also affects a road network but to a greater degree, requires other sources of position information to be identified and incorporated into the estimation process.

VI. Reduced-Order Kalman Filter

Kalman filters provide an optimal means of filtering position measurements [3] by utilising knowledge about the dynamics of the vehicle and the measurement errors. Normally, two decoupled filters would be required in order to cope with a motor vehicle and measurement noise which have two degrees of freedom [5] but the map-aided positioning estimator results in all measurements lying on a known curve and thus the vehicle is reduced to a single degree of freedom. By choosing an appropriate one-dimensional coordinate system, a single kalman filter can be used with resultant savings in terms of computation but more importantly this filter will be more accurate than its two-dimensional counterpart as it will operate with less measurement noise and the dynamic model will be able to better match the capabilities of the vehicle.

For a straight road the coordinate frame for the proposed kalman filter is self evident. Curved roads must exist in two-dimensions so the mapping of a curve onto a line is not as straight forward. The assumption that the radius of the curves are large enough that the dynamic performance of the vehicle around the corner is approximately equal to the straight line performance allows the curve to be straightened. The curve $\rho(x_1)$ can now be mapped onto a line z by choosing an appropriate point on the curve as the origin ξ_0 and measuring the distance along the curve from this origin to each point

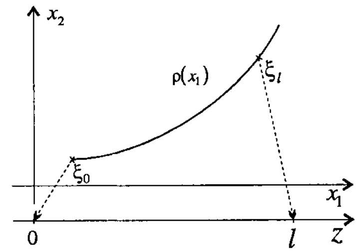

Figure 6: *Curve straightening to permit a one-dimensional kalman filter to be used.*

on the curve ξ_l (fig. 6). Once the filtering has been performed, the mapping is reversed to place the position estimate in the original cartesian frame.

$$l_z \rightleftharpoons \xi_{l_z} \tag{44}$$

$$l_z = \int_{\xi_{01}}^{\xi_{l1}} \sqrt{1 + [\rho'(x_1)]^2} dx_1 \tag{45}$$

This mapping procedure and the subsequent kalman filter will be implemented in the next phase of the research. The piecewise linear curve model making the implementation straight forward. Since the curves are a sequence of straight lines, the transformation to and from the one-dimensional kalman frame be expressed in closed form [13].

VII. Road Networks

A road network is a set of roads along with a set of nodes which join the various roads together. As this is very similar to the piecewise linear curve model, the same MAP technique can be used (Eqs.36–40) where $\rho_i(x_1)$ represents the ith road of the network. The resultant MAP position estimate is optimal but there is the problem of ambiguous estimates. The optimal estimate may actually be on the wrong road resulting in a position estimate further from the true position than the raw measurement. The local estimate on the correct road is a better estimate but it was slightly less likely and hence not chosen. The problem lies in determining which road the vehicle is on, the solution to which lies in the use of other sources of information.

The trajectory formed by the most recent raw position measurements should be able to provide an assist in determining which road the vehicle is on. Map matching attempts to determine which road a vehicle is travelling on but it is better suited to relative positioning systems and it is not an optimal solution. The information contained in the vehicle's trajectory needs to be quantified so that a probability can be associated with each road to represent the likelihood that the vehicle is on that road. To achieve this, the authors propose to filter the sequence of estimates $\hat{x}(n)$ for each road. The error covariance matrix of the kalman filter estimate will provide a measure of the probability that a vehicle is travelling on a given road; the larger the error, the smaller the probability. This probability will be used to appropriately adjust the positional PDF prior to calculating the MAP estimate. In this manner, there is a much less chance of ambiguous estimates and the plotted position of the vehicle will not be seen "jumping" from road to road.

VIII. Supplementary Information Sources

The performance of the MAP map-aided position estimator is easily improved through the incorporation of additional information sources such as:

Road Type: Each road type is assigned a probability according to it's traffic flow capability – a vehicle is more likely to be on a major road than the lane running parallel to the major road.

Road Rules: Any rule restricting the vehicle's freedom, (e.g. No Right Turn, one way streets, etc) can be modelled by setting appropriate probabilities to zero.

Route Knowledge: Direct route knowledge or route preferences as determined from past journeys [14] (driver behaviour) can be used to bias the positional PDFs in a manner similar to the road type above.

The incorporation of this type of information is straight forward as the information applies uniformly to each road. That is, the positional PDF for a given road is still uniform and as a result the local position estimate (Eq. 36) is unaffected by the additional information. The information, incorporated by modifying the positional PDF of each road, is only used when determining the globally optimal estimate (Eq. 40).

Other sources and forms of information are available, the most notable being the use of traffic flow rates. For example, a vehicle on a given road is more likely to be found at the intersections. This knowledge, and other knowledge of a similar form, can be represented by a non-uniform positional PDF implying that the MAP estimate will need to be derived from first principles. This is not quite the case as it can be shown that a MAP estimate based on a uniform PDF does not lose any position information [13] and consequently the estimate can be refined if and when other information becomes available such as that represented by a non-uniform PDF.

IX. Conclusion

The proposed map-aided position estimation system based on maximum *à posteriori* principles greatly increases the accuracy of any positioning system used to track objects with a restricted operating domain; most particularly motor vehicles, but also trains and trams. The position estimation process results in all measurements being translated onto a map such that all of the position estimates now lie on known curves allowing a one-dimensional kalman filter to be used to incorporate the dynamics of the vehicle being tracked. This further increases the accuracy of the positioning system and allows for a map matching function to be performed as well.

The mathematical framework of the MAP estimator used also readily allows further sources of information regarding the vehicle's position and possible movements to be optimally incorporated. At the same time, the estimator is also very flexible. These additional information sources can be added and removed as required without affecting the rest of the estimation process.

References

[1] Y. Bar-Shalom and T. E. Fortman, *Tracking and Data Association*. Academic Press Inc., 1988.

[2] A. Brown, "A low-cost vehicle location and tracking system," in *Position Location and Navigation Symposium (PLANS '92)*, March, 1992.

[3] R. G. Brown and P. Y. Hwang, *Introduction to Random Signals and Applied Kalman Filtering*. John Wiley & Sons, Inc., 2nd ed., 1992.

[4] J. L. Buxton, S. K. Honey, W. E. Suchowerskyi, and A. Tempelhof, "The Travelpilot: A second-generation automotive navigation system," *IEEE Transaction on Vehicular Technology*, vol. 40, no. 1, pp. 41–44, February, 1991.

[5] C. K. Chui and G. Chen, *Kalman Filtering with Real-Time Applications*, vol. 17 of *Springer Series in Information Sciences*. Springer-Verlag, 2nd ed., 1991.

[6] H. Claussen and W. Lichtner, "GDF - A proposed standard for digital road maps to be used in car navigation," in *Vehicle Navigation Information Systems Conference (VNIS '89)*, pp. 324–330, 1989.

[7] W. C. Collier, "In-vehicle route guidance systems using map matched dead reckoning," in *Position Location and Navigation Symposium (PLANS '90)*, pp. 359–363, 1990.

[8] W. C. Collier and R. J. Weiland, "Smart cars, smart highways," *IEEE Spectrum*, vol. 31, no. 4, pp. 27–33, April, 1994.

[9] C. R. Drane, *Positioning Systems – A Unified Approach*, vol. 181 of *Springer Series in Control and Information Sciences*. Springer-Verlag, 1992.

[10] N. Johnson, W. Tang, and G. Howell, "Terrain aided navigation using maximum a posteriori estimation," in *Position Location and Navigation Symposium (PLANS '90)*, pp. 464–469, March, 1990.

[11] E. J. Krakiwsky, "Tracking the worldwide development of IVHS navigation systems," *GPS World*, pp. 40–47, October, 1993.

[12] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*. McGraw–Hill, 1965.

[13] C. A. Scott, *Improved Positioning of Motor Vehicles Through Secondary Information Sources*. PhD thesis, University of Technology, Sydney, 1994. In preparation.

[14] C. A. Scott and C. R. Drane, "The chaotic streets of Sydney - the application of chaotic filtering to vehicle monitoring," in *The Third International Symposium on Signal Processing and its Applications (ISSPA 92)*, pp. 650–653, 1992.

[15] M. Shibata and Y. Fujita, "Current status and future plans for digital map databases in Japan," in *Vehicle Navigation Information Systems Conference (VNIS '93)*, pp. 29–33, October, 1993.

[16] D. J. Torrieri, "Statistical theory of passive location systems," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-20, no. 2, pp. 183–198, March, 1984.