

## Forelesning 12

Et stort skritt i retning av generell lineær optimering (såkalt lineær programering). Her ser vi på to tilsynelatende forskjellige problemer, som viser seg å være *duale* av hverandre, noe som hjelper oss med å finne en løsning.

### Pensum

- Kap. 26. Maximum flow:  
Innledning og 26.1–26.3

### Læringsmål

- [L<sub>1</sub>] Kunne definere *flytnettverk*, *flyt* og *maks-flyt-problemet*
- [L<sub>2</sub>] Kunne håndtere *antiparallele kanter* og *flere kilder og sluk*
- [L<sub>3</sub>] Kunne definere *residualnettverket* til et nettverk med en gitt flyt
- [L<sub>4</sub>] Forstå *oppheving* av flyt
- [L<sub>5</sub>] Forstå *forøkende stier*
- [L<sub>6</sub>] Forstå *snitt*, *snitt-kapasitet* og *minimalt snitt*
- [L<sub>7</sub>] Forstå *maks-flyt/min-snitt*
- [L<sub>8</sub>] Forstå FORD-FULKERSON
- [L<sub>9</sub>] Vite at FORD-FULKERSON med BFS er EDMONDS-KARP
- [L<sub>10</sub>] Kunne finne en *maksimum bipartitt matching* vha. flyt
- [L<sub>11</sub>] Forstå *heltallsteoremet*

# Forelesningen filmes



**1. Problemet**

**2. Ideer**

**3. Ford-Fulkerson**

**4. Minimalt snitt**

**5. Matching**

# Forelesning 12

Maksimal flyt

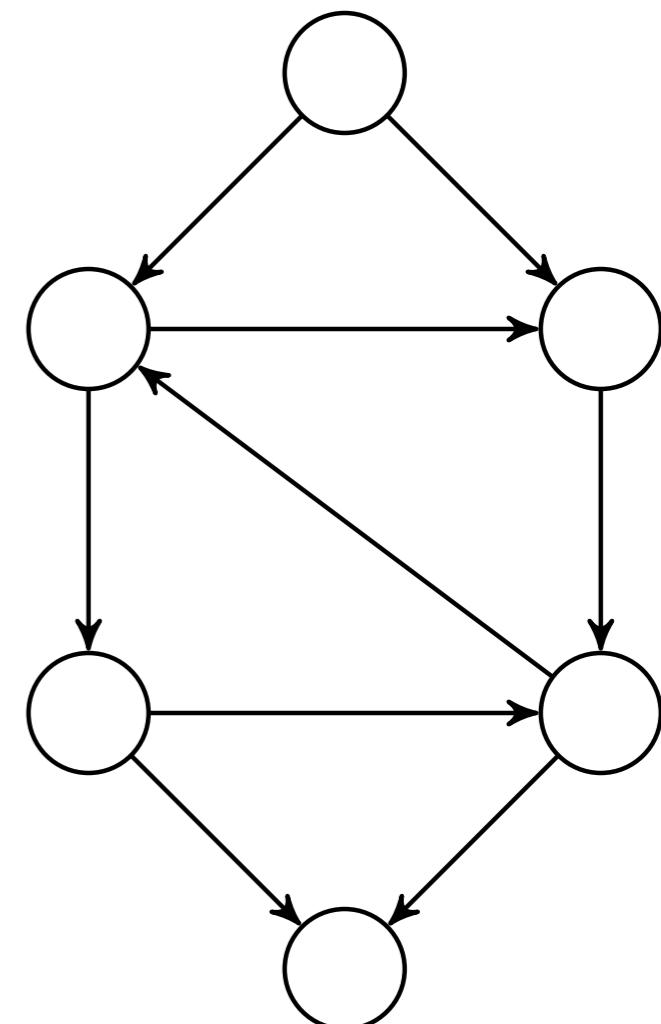


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Problemet

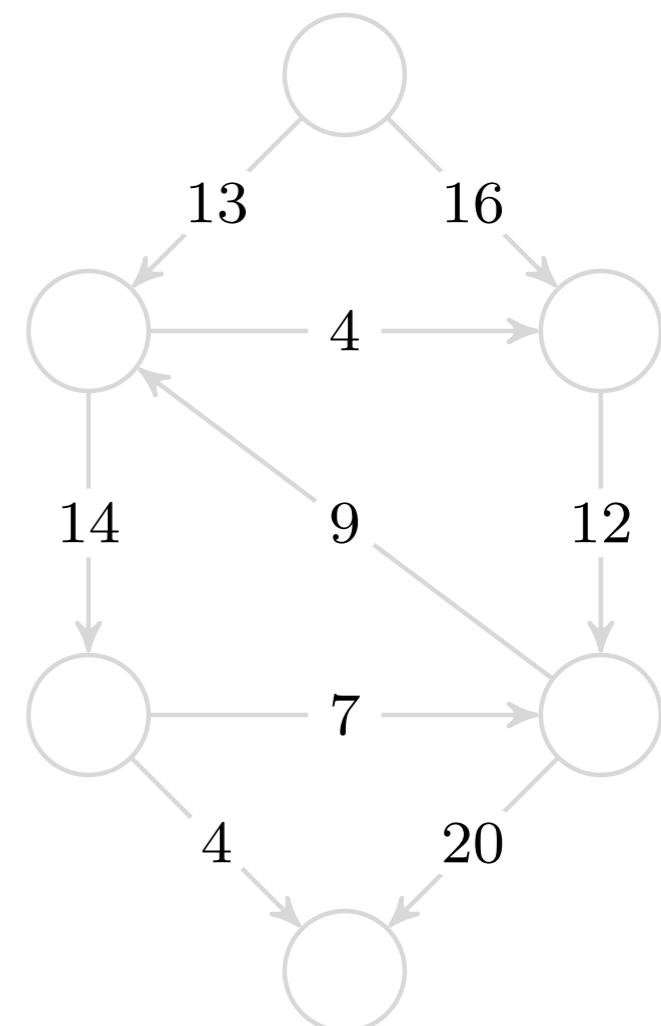
**Flytnett:** Rettet graf  $G = (V, E)$

Ofte kalt flytnettverk.



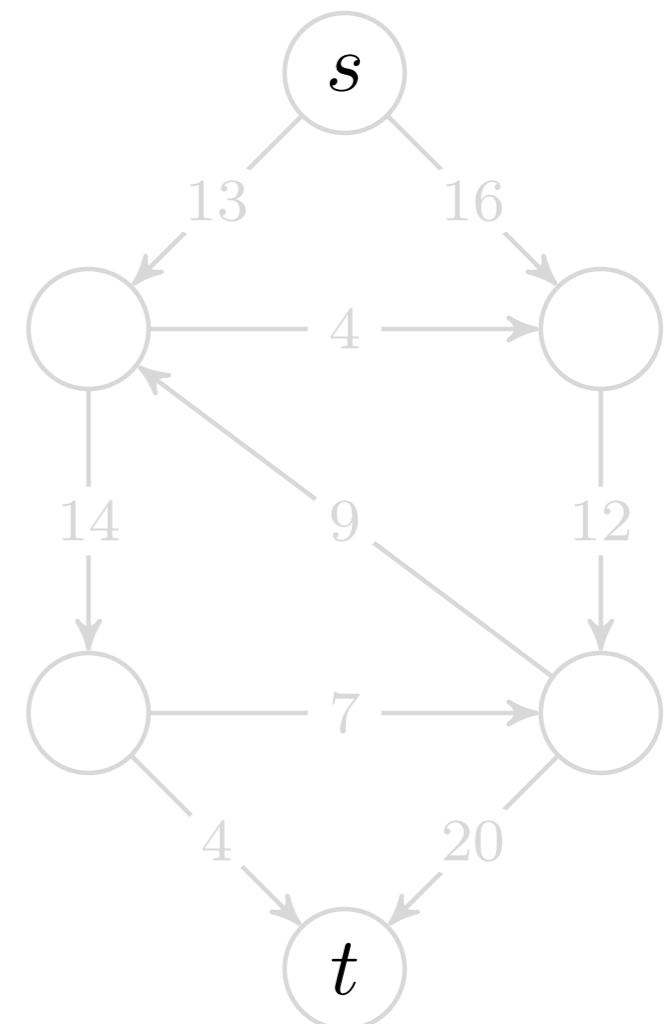
**Flytnett:** Rettet graf  $G = (V, E)$

- › Kapasiteter  $c(u, v) \geq 0$



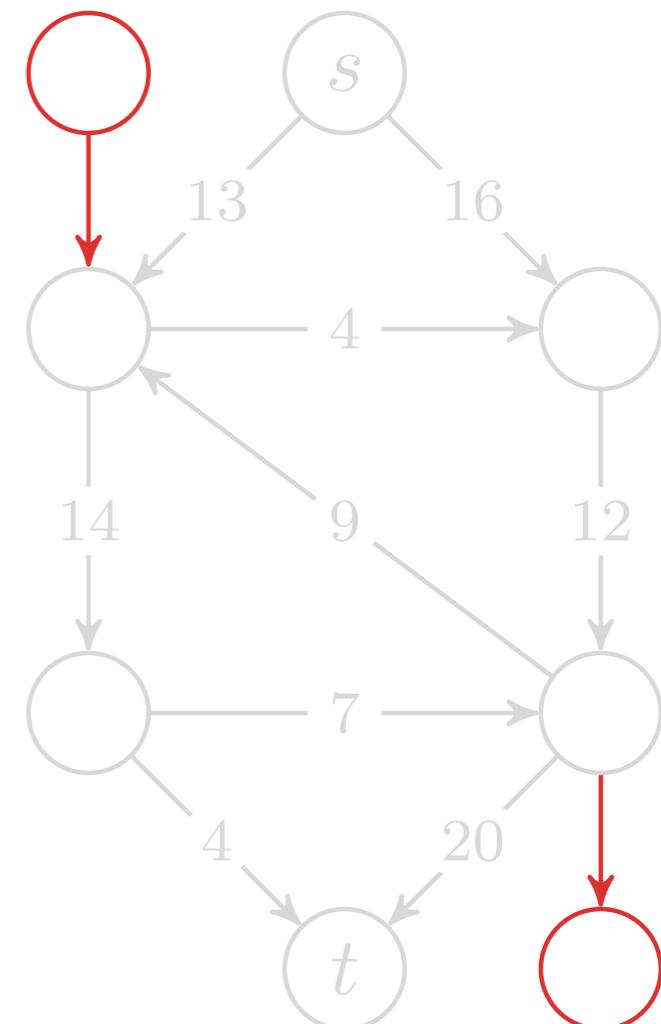
**Flytnett:** Rettet graf  $G = (V, E)$

- › Kapasiteter  $c(u, v) \geq 0$
- › Kilde og sluk  $s, t \in V$



**Flytnett:** Rettet graf  $G = (V, E)$

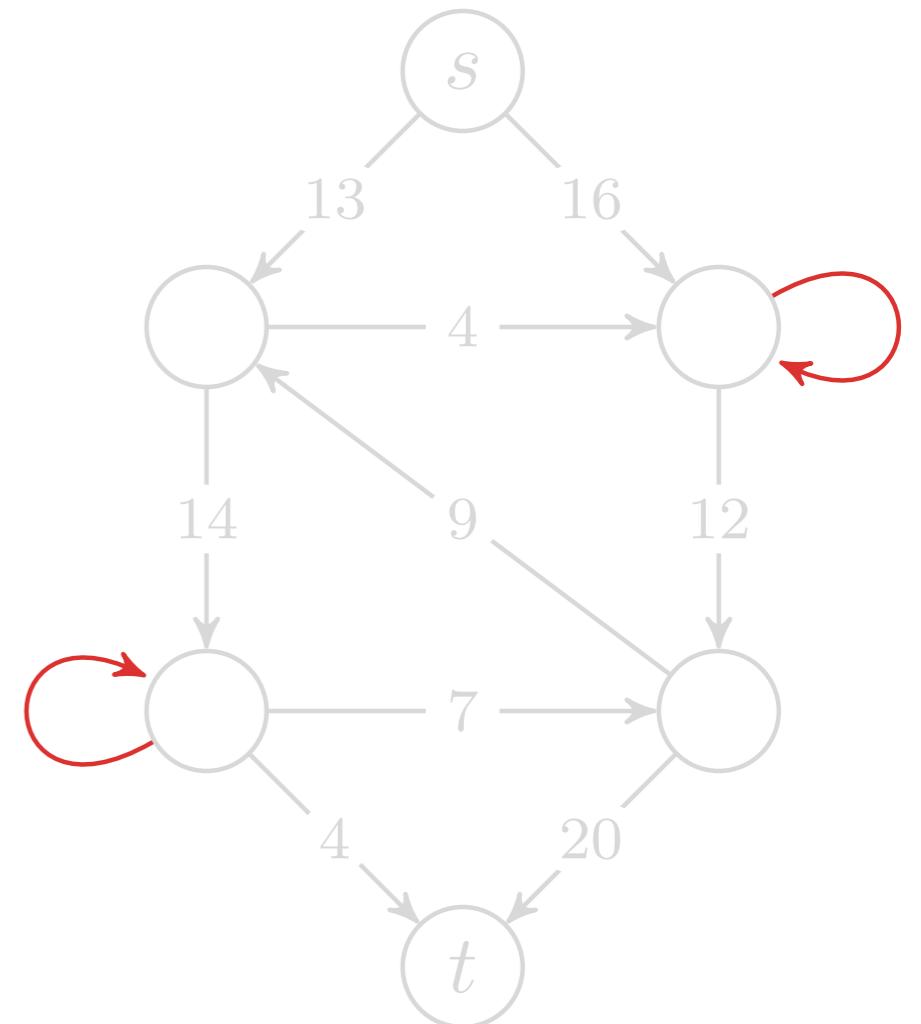
- › Kapasiteter  $c(u, v) \geq 0$
- › Kilde og sluk  $s, t \in V$
- ›  $v \in V \implies s \rightsquigarrow v \rightsquigarrow t$



Forenkrende (harmløs) antagelse: Alle noder er på en sti fra  $s$  til  $t$

**Flytnett:** Rettet graf  $G = (V, E)$

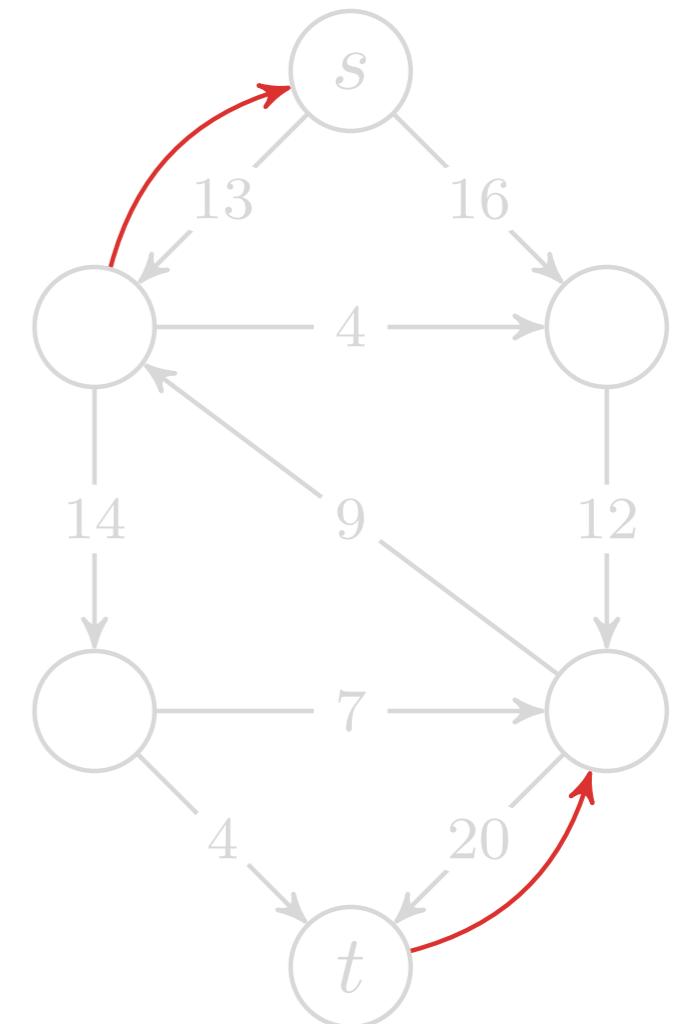
- › Kapasiteter  $c(u, v) \geq 0$
- › Kilde og sluk  $s, t \in V$
- ›  $v \in V \implies s \rightsquigarrow v \rightsquigarrow t$
- › Ingen løkker (*self-loops*)



Merk: Vi kan ha sykler! (Se  $1 \rightarrow 3 \rightarrow 2 \rightarrow 1$ , f.eks.)

**Flytnett:** Rettet graf  $G = (V, E)$

- › Kapasiteter  $c(u, v) \geq 0$
- › Kilde og sluk  $s, t \in V$
- ›  $v \in V \implies s \rightsquigarrow v \rightsquigarrow t$
- › Ingen løkker (*self-loops*)
- ›  $(u, v) \in E \implies (v, u) \notin E$

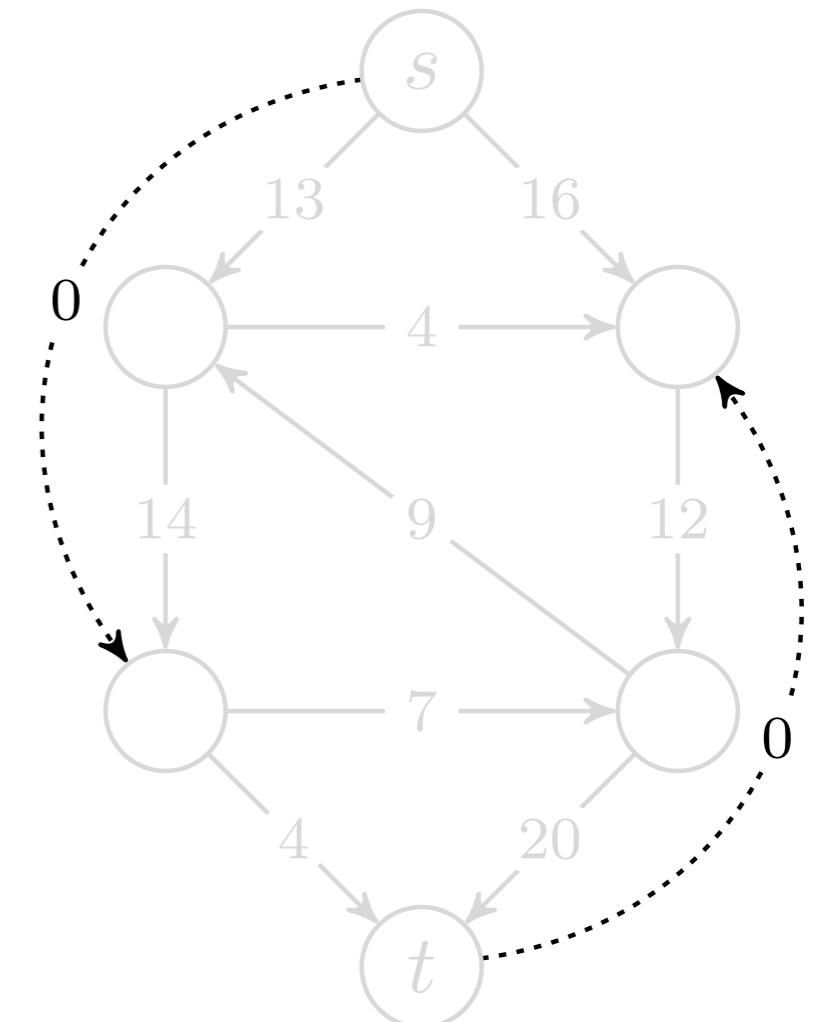


Nok en forenkling: Tillater ikke antiparallelle kanter

## Flytnett:

Rettet graf  $G = (V, E)$

- › Kapasiteter  $c(u, v) \geq 0$
- › Kilde og sluk  $s, t \in V$
- ›  $v \in V \implies s \rightsquigarrow v \rightsquigarrow t$
- › Ingen løkker (*self-loops*)
- ›  $(u, v) \in E \implies (v, u) \notin E$
- ›  $(u, v) \notin E \implies c(u, v) = 0$

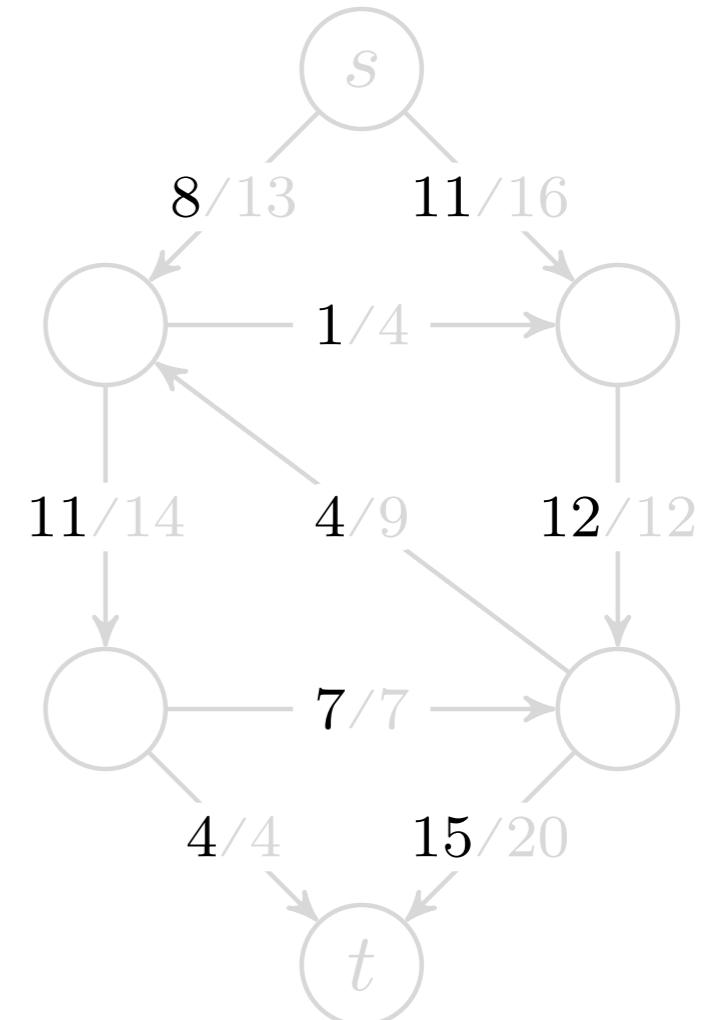


Ingen kapasitet uten kant

**Flytnett:** Rettet graf  $G = (V, E)$

- › Kapasiteter  $c(u, v) \geq 0$
- › Kilde og sluk  $s, t \in V$
- ›  $v \in V \implies s \rightsquigarrow v \rightsquigarrow t$
- › Ingen løkker (*self-loops*)
- ›  $(u, v) \in E \implies (v, u) \notin E$
- ›  $(u, v) \notin E \implies c(u, v) = 0$

**Flyt:** En funksjon  $f : V \times V \rightarrow \mathbb{R}$



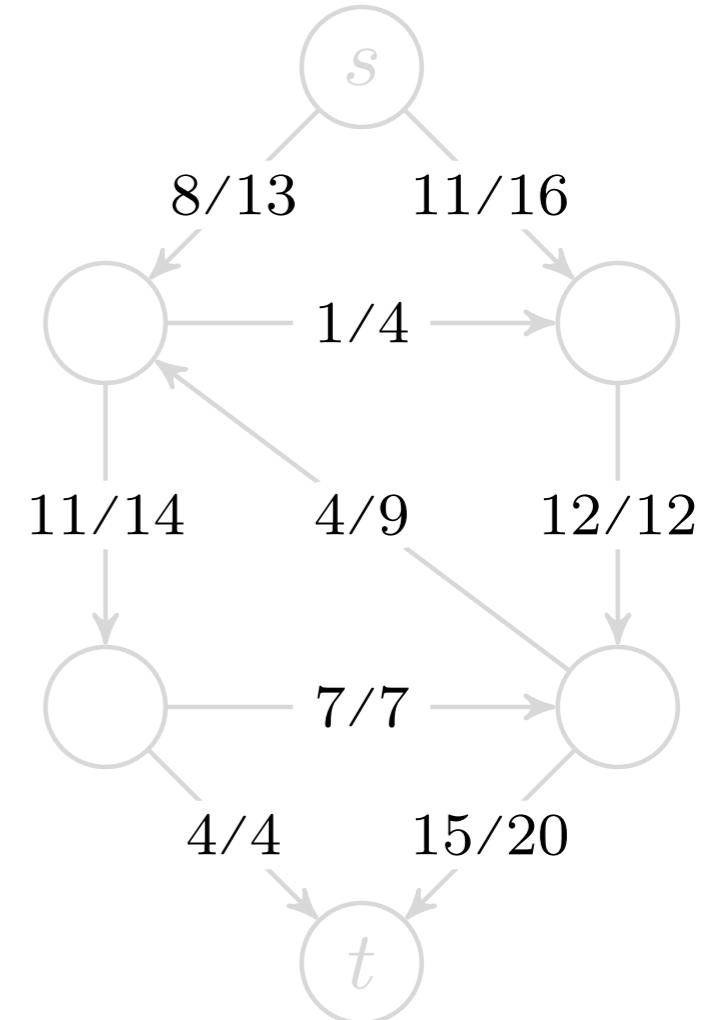
Vi har flyt fra enhver node til enhver annen

**Flytnett:** Rettet graf  $G = (V, E)$

- › Kapasiteter  $c(u, v) \geq 0$
- › Kilde og sluk  $s, t \in V$
- ›  $v \in V \implies s \rightsquigarrow v \rightsquigarrow t$
- › Ingen løkker (*self-loops*)
- ›  $(u, v) \in E \implies (v, u) \notin E$
- ›  $(u, v) \notin E \implies c(u, v) = 0$

**Flyt:** En funksjon  $f : V \times V \rightarrow \mathbb{R}$

- ›  $0 \leq f(u, v) \leq c(u, v)$

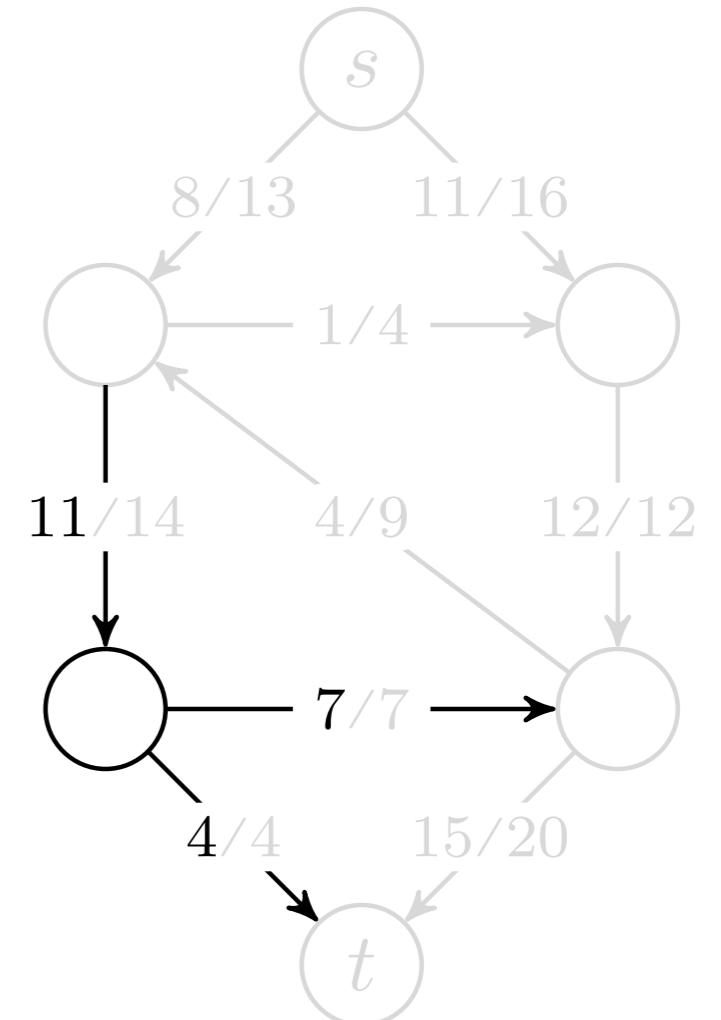


**Flytnett:** Rettet graf  $G = (V, E)$

- › Kapasiteter  $c(u, v) \geq 0$
- › Kilde og sluk  $s, t \in V$
- ›  $v \in V \implies s \rightsquigarrow v \rightsquigarrow t$
- › Ingen løkker (*self-loops*)
- ›  $(u, v) \in E \implies (v, u) \notin E$
- ›  $(u, v) \notin E \implies c(u, v) = 0$

**Flyt:** En funksjon  $f : V \times V \rightarrow \mathbb{R}$

- ›  $0 \leq f(u, v) \leq c(u, v)$
- ›  $u \neq s, t \implies \sum_v f(v, u) = \sum_v f(u, v)$



Flyt inn = flyt ut (tilsv. Kirchhoffs første lov)

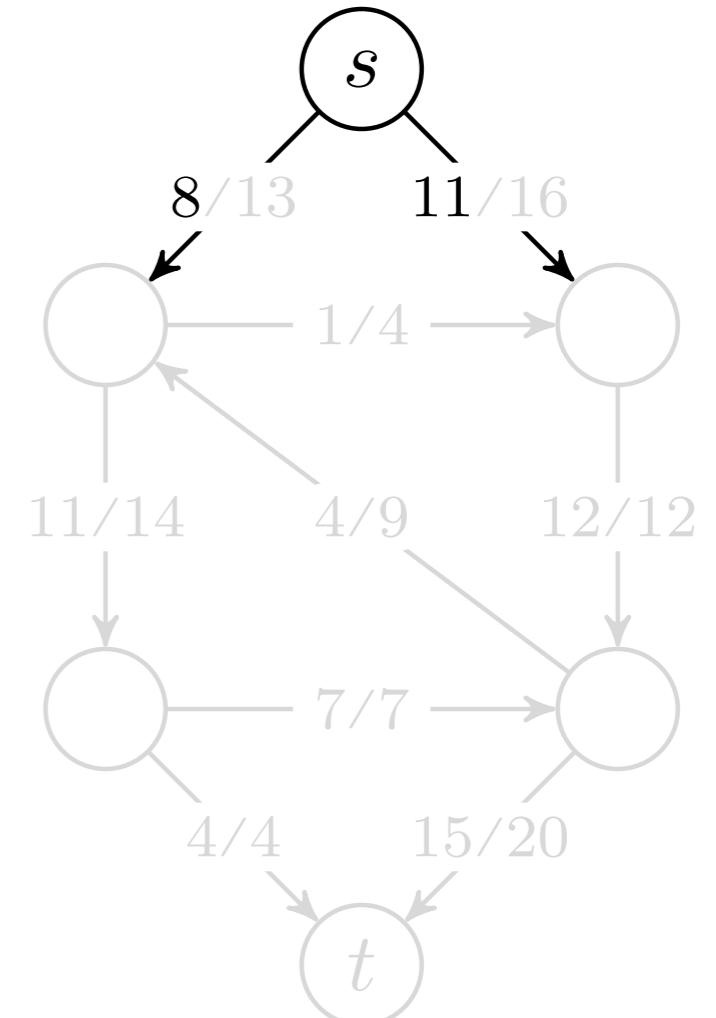
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- › Kapasiteter  $c(u, v) \geq 0$
- › Kilde og sluk  $s, t \in V$
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**Flyt:** En funksjon  $f : V \times V \rightarrow \mathbb{R}$

- ›  $0 \leq f(u, v) \leq c(u, v)$
- ›  $u \neq s, t \implies \sum_v f(v, u) = \sum_v f(u, v)$

**Flytverdi:**  $|f| = \sum_v f(s, v) - \sum_v f(v, s)$



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**Input:** Et flytnett  $G$ .

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**Input:** Et flytnett  $G$ .

**Output:** En flyt  $f$  for  $G$  med maks.  $|f|$ .

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- › Cormen 1 & 2 har andre definisjoner
- › Antiparallele kanter:  
Splitt den ene med en node
- › Flere kilder og sluk:  
Legg til super-kilde og super-sluk

2.5

Ideer

Ofte kalt residualnettverk.

# Restnett

- Engelsk: Residual network
- Fremoverkant ved ledig kapasitet
- Bakoverkant ved flyt

# Forøkende sti

- Engelsk: Augmenting path
- En sti fra kilde til sluk i restnettet
- Langs fremoverkanter: Flyten kan økes
- Langs bakoverkanter: Flyten kan omdirigeres
- Altså: En sti der den totale flyten kan økes

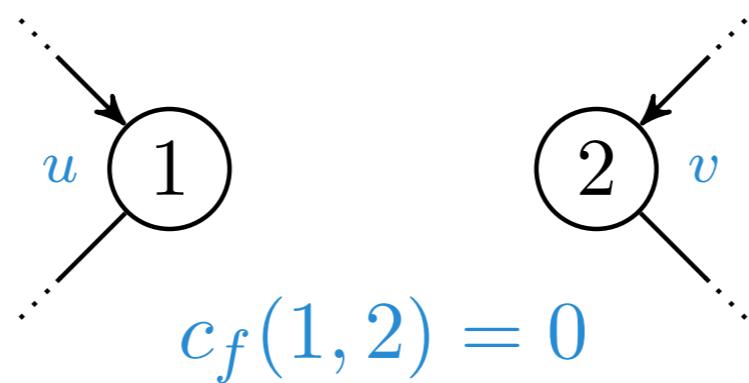
# Flytopphøving

- Vi kan «sende» flyt baklengs langs kanter der det allerede går flyt
- Vi opphever da flyten, så den kan omdirigeres til et annet sted
- Det er dette bakoverkantene i restnettet representerer

$$c_f(u, v) =$$

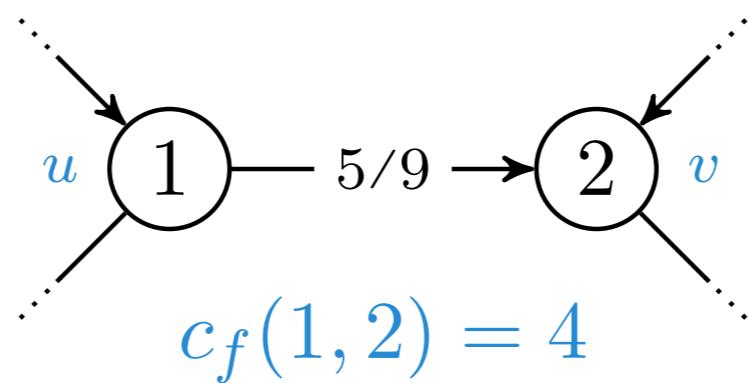
Restkapasitet: Hvor mye kan vi øke flyten fra  $u$  til  $v$ ?

$$c_f(u, v) = \begin{cases} \infty & \text{if } (u, v) \in E, \\ \infty & \text{if } (v, u) \in E, \\ 0 & \text{otherwise.} \end{cases}$$



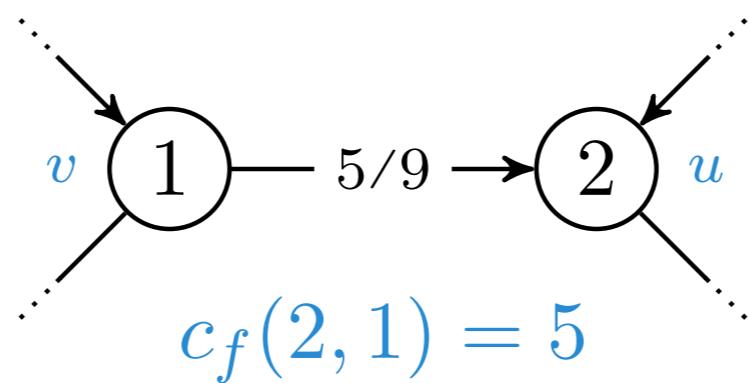
Ingen kant: Har ingen flyt og kan ikke få det

$$c_f(u, v) = \begin{cases} c(u, v) - f(u, v) & \text{if } (u, v) \in E, \\ & \text{if } (v, u) \in E, \\ 0 & \text{otherwise.} \end{cases}$$



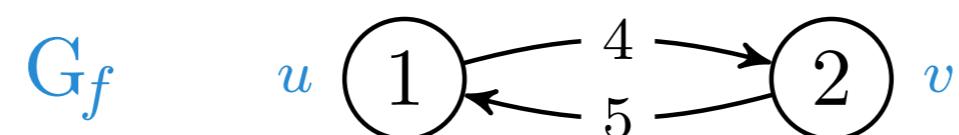
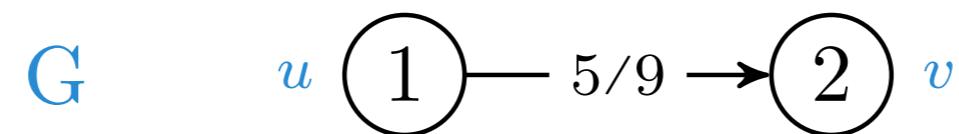
Fremover: Kan øke med det som gjenstår av  $c(u, v)$

$$c_f(u, v) = \begin{cases} c(u, v) - f(u, v) & \text{if } (u, v) \in E, \\ f(v, u) & \text{if } (v, u) \in E, \\ 0 & \text{otherwise.} \end{cases}$$



Bakover: Kan sende tilbake og omdirigere  $f(v, u)$  enheter

$$c_f(u, v) = \begin{cases} c(u, v) - f(u, v) & \text{if } (u, v) \in E, \\ f(v, u) & \text{if } (v, u) \in E, \\ 0 & \text{otherwise.} \end{cases}$$



$$(u, v) \in E_f \iff c_f(u, v) > 0$$

# 3:5

## Ford-Fulkerson

### MAXIMAL FLOW THROUGH A NETWORK

L. R. FORD, JR. AND D. R. FULKERSON

**Introduction.** The problem discussed in this paper was formulated by T. Harris as follows:

"Consider a rail network connecting two cities by way of a <sup>29</sup> number of intermediate cities, where each link of the network has a number representing its capacity. Assuming a steady flow from one given

Fra 1956

- › Finn forøkende stier så lenge det går
- › Deretter er flyten maksimal
- › Generell metode, ikke en algoritme
- › Om vi bruker BFS: «Edmonds-Karp»

- › Normal implementasjon:
  - › Finn forøkende sti først
  - › Finn så flaskehalsen i stien
  - › Oppdater flyt langs stien med denne verdien

FORD-FULKERSON-METHOD( $G, s, t$ )

G flytnett  
s kilde  
t sluk

Finn maksimal flyt fra  $s$  til  $t$  i  $G$

FORD-FULKERSON-METHOD( $G, s, t$ )

1 initialize flow  $f$  to 0

G flytnett  
s kilde  
t sluk  
 $f$  flyt

Lovlig, men neppe optimal løsning. Forbedres gradvis

FORD-FULKERSON-METHOD( $G, s, t$ )

- 1 initialize flow  $f$  to 0
- 2 **while** there is an augm. path  $p$  in  $G_f$

$G$	flytnett
$s$	kilde
$t$	sluk
$f$	flyt
$G_f$	restnett
$p$	$s \rightsquigarrow t$ i $G_f$

Ikke nødvendigvis en sti i  $G$ ; kanter kan gå baklengs i  $p$  der

FORD-FULKERSON-METHOD( $G, s, t$ )

- 1 initialize flow  $f$  to 0
- 2 **while** there is an augm. path  $p$  in  $G_f$
- 3     augment flow  $f$  along  $p$

$G$	flytnett
$s$	kilde
$t$	sluk
$f$	flyt
$G_f$	restnett
$p$	$s \rightsquigarrow t$ i $G_f$

Flyten i hver kant kan økes ( $\rightarrow$ ) eller oppheves og omdirigeres ( $\leftarrow$ )

FORD-FULKERSON-METHOD( $G, s, t$ )

- 1 initialize flow  $f$  to 0
- 2 **while** there is an augm. path  $p$  in  $G_f$
- 3     augment flow  $f$  along  $p$
- 4 **return**  $f$

$G$	flytnett
$s$	kilde
$t$	sluk
$f$	flyt
$G_f$	restnett
$p$	$s \rightsquigarrow t$ i $G_f$

FORD-FULKERSON( $G, s, t$ )

$G$  flyttnett  
 $s$  kilde  
 $t$  sluk

En litt mer detaljert beskrivelse av flyt-oppdateringen

FORD-FULKERSON( $G, s, t$ )  
1 **for** each edge  $(u, v) \in G.E$

$G$  flyttnett  
 $s$  kilde  
 $t$  sluk  
 $u$  node  
 $v$  node

FORD-FULKERSON( $G, s, t$ )

- 1 **for** each edge  $(u, v) \in G.E$
- 2        $(u, v).f = 0$

G flyttnett  
s kilde  
t sluk  
u node  
v node  
f flyt

FORD-FULKERSON( $G, s, t$ )

- 1 **for** each edge  $(u, v) \in G.E$
- 2      $(u, v).f = 0$
- 3 **while** there is a path  $p$  from  $s$  to  $t$  in  $G_f$

$G$	flyttnett
$s$	kilde
$t$	sluk
$u$	node
$v$	node
$f$	flyt
$G_f$	restnett
$p$	$s \rightsquigarrow t$ i $G_f$

Alle kanter i  $G_f$  har restkapasitet. En sti  $s \rightsquigarrow t$  er dermed forøkende

FORD-FULKERSON( $G, s, t$ )

- 1 **for** each edge  $(u, v) \in G.E$
- 2      $(u, v).f = 0$
- 3 **while** there is a path  $p$  from  $s$  to  $t$  in  $G_f$
- 4      $c_f(p) = \min \{c_f(u, v) : (u, v) \text{ is in } p\}$

$G$	flyttnett
$s$	kilde
$t$	sluk
$u$	node
$v$	node
$f$	flyt
$G_f$	restnett
$p$	$s \rightsquigarrow t$ i $G_f$
$c_f$	restkap.

Dette er «flaskehalsen» langs  $p$ : Minste restkapasitet

FORD-FULKERSON( $G, s, t$ )

- 1 **for** each edge  $(u, v) \in G.E$
- 2      $(u, v).f = 0$
- 3 **while** there is a path  $p$  from  $s$  to  $t$  in  $G_f$
- 4      $c_f(p) = \min \{c_f(u, v) : (u, v) \text{ is in } p\}$
- 5     **for** each edge  $(u, v)$  in  $p$

$G$	flyttnett
$s$	kilde
$t$	sluk
$u$	node
$v$	node
$f$	flyt
$G_f$	restnett
$p$	$s \rightsquigarrow t$ i $G_f$
$c_f$	restkap.

Vi vil øke flyten langs  $p$  med denne flaskehals-restkapasiteten

FORD-FULKERSON( $G, s, t$ )

- 1 **for** each edge  $(u, v) \in G.E$
- 2      $(u, v).f = 0$
- 3 **while** there is a path  $p$  from  $s$  to  $t$  in  $G_f$
- 4      $c_f(p) = \min \{c_f(u, v) : (u, v) \text{ is in } p\}$
- 5     **for** each edge  $(u, v)$  in  $p$
- 6         **if**  $(u, v) \in E$

$G$	flyttnett
$s$	kilde
$t$	sluk
$u$	node
$v$	node
$f$	flyt
$G_f$	restnett
$p$	$s \rightsquigarrow t$ i $G_f$
$c_f$	restkap.

Foroverkant med restkapasitet?

FORD-FULKERSON( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 while there is a path  $p$  from  $s$  to  $t$  in  $G_f$ 
4    $c_f(p) = \min \{c_f(u, v) : (u, v) \text{ is in } p\}$ 
5   for each edge  $(u, v)$  in  $p$ 
6     if  $(u, v) \in E$ 
7        $(u, v).f = (u, v).f + c_f(p)$ 
```

$G$	flyttnett
$s$	kilde
$t$	sluk
$u$	node
$v$	node
$f$	flyt
$G_f$	restnett
$p$	$s \rightsquigarrow t$ i $G_f$
$c_f$	restkap.

Da kan flyten økes langs kanten

FORD-FULKERSON( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 while there is a path  $p$  from  $s$  to  $t$  in  $G_f$ 
4    $c_f(p) = \min \{c_f(u, v) : (u, v) \text{ is in } p\}$ 
5   for each edge  $(u, v)$  in  $p$ 
6     if  $(u, v) \in E$ 
7        $(u, v).f = (u, v).f + c_f(p)$ 
8     else  $(v, u).f = (v, u).f - c_f(p)$ 
```

$G$	flyttnett
$s$	kilde
$t$	sluk
$u$	node
$v$	node
$f$	flyt
$G_f$	restnett
$p$	$s \rightsquigarrow t$ i $G_f$
$c_f$	restkap.

Baklengs: Opphev flyt; omdirigeres implisitt til neste kant i  $p$

Dette er ment å gi en mer konkret og detaljert forståelse av hvordan Edmonds-Karp fungerer, og hvordan den kan implementeres, men dette er kun et supplement for økt forståelse. Du trenger ikke «pugge» denne detaljerte varianten, eller huske akkurat hvordan den fungerer, så lenge du har skjønt og husker det som står i pensum.

- › Alternativ: «Flett inn» BFS
  - › Finn flaskehalsene underveis!
  - › Hold styr på hvor mye flyt vi får frem til hver node
  - › Traverser bare noder vi ikke har nådd frem til ennå
  - › Denne «implementasjonen» står ikke i boka

v.a

Mulig økning (*augmentation*)

Hvor mye mer flyt får vi til å sende fra  $s$  til  $v$ ?

v.a

Mulig økning (*augmentation*)

Gitt av flaskehalsen  $c_f(s \rightsquigarrow v)$  for en eller annen sti  $s \rightsquigarrow v$

*v.a*

Mulig økning (*augmentation*)

Etter traversering er  $t.a = c_f(p)$  for en forøkende sti  $p$  (eller 0)

Akkurat navnet «a» er bare et  
vilkårlig valg fra min side.

v.a

Mulig økning (*augmentation*)

Implementasjonsdetalj fra original-algoritmen; diskuteres ikke i boka

EDMONDS-KARP( $G, s, t$ )

$G$  flytnett  
 $s$  kilde  
 $t$  sluk

Bruker BFS for å finne forøkende sti. Atskillig mer detaljert . . .

EDMONDS-KARP( $G, s, t$ )

1 **for** each edge  $(u, v) \in G.E$

$G$  flytnett  
 $s$  kilde  
 $t$  sluk  
 $u$  node  
 $v$  node

EDMONDS-KARP( $G, s, t$ )

- 1 **for** each edge  $(u, v) \in G.E$
- 2        $(u, v).f = 0$

G flytnett  
s kilde  
t sluk  
u node  
v node  
 $f$  flyt

EDMONDS-KARP( $G, s, t$ )

- 1 **for** each edge  $(u, v) \in G.E$
- 2      $(u, v).f = 0$
- 3 **repeat** → until  $t.a == 0$

G flytnett  
s kilde  
t sluk  
u node  
v node  
f flyt  
a økning

Gjenta til vi ikke får mer flyt frem til  $t$

EDMONDS-KARP( $G, s, t$ )

- 1 **for** each edge  $(u, v) \in G.E$
- 2      $(u, v).f = 0$
- 3 **repeat** → until  $t.a == 0$
- 4     **for** each vertex  $u \in G.V$

G flytnett  
s kilde  
t sluk  
u node  
v node  
f flyt  
a økning

I hver iterasjon . . .

**EDMONDS-KARP( $G, s, t$ )**

- 1 **for** each edge  $(u, v) \in G.E$
- 2      $(u, v).f = 0$
- 3 **repeat** → until  $t.a == 0$
- 4     **for** each vertex  $u \in G.V$
- 5          $u.a = 0$  → reaching  $u$  in  $G_f$

$G$  flytnett  
 $s$  kilde  
 $t$  sluk  
 $u$  node  
 $v$  node  
 $f$  flyt  
 $a$  økning

Hvor mye mer flyt klarer vi å få frem til  $u$ ?

**EDMONDS-KARP( $G, s, t$ )**

- 1 **for** each edge  $(u, v) \in G.E$
- 2      $(u, v).f = 0$
- 3 **repeat** → until  $t.a == 0$
- 4     **for** each vertex  $u \in G.V$
- 5          $u.a = 0$  → reaching  $u$  in  $G_f$
- 6          $u.\pi = \text{NIL}$

$G$  flytnett  
 $s$  kilde  
 $t$  sluk  
 $u$  node  
 $v$  node  
 $f$  flyt  
 $a$  økning

Dette er forgjengeren i BFS-treet

**EDMONDS-KARP( $G, s, t$ )**

- 1 **for** each edge  $(u, v) \in G.E$
- 2      $(u, v).f = 0$
- 3 **repeat** → until  $t.a == 0$
- 4     **for** each vertex  $u \in G.V$
- 5          $u.a = 0$  → reaching  $u$  in  $G_f$
- 6          $u.\pi = \text{NIL}$
- 7      $s.a = \infty$

$G$  flytnett  
 $s$  kilde  
 $t$  sluk  
 $u$  node  
 $v$  node  
 $f$  flyt  
 $a$  økning

Alltid uendelig mye mer flyt hos  $s$ . Vi vil sende den videre

EDMONDS-KARP( $G, s, t$ )

- 1 **for** each edge  $(u, v) \in G.E$
- 2      $(u, v).f = 0$
- 3 **repeat** → until  $t.a == 0$
- 4     **for** each vertex  $u \in G.V$
- 5          $u.a = 0$  → reaching  $u$  in  $G_f$
- 6          $u.\pi = \text{NIL}$
- 7      $s.a = \infty$
- 8      $Q = \emptyset$

G	flytnett
s	kilde
t	sluk
u	node
v	node
f	flyt
a	økning
Q	kø

FIFO-kø til bredde-først-søk i  $G_f$

**EDMONDS-KARP( $G, s, t$ )**

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat → until  $t.a == 0$ 
4    for each vertex  $u \in G.V$ 
5       $u.a = 0$  → reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )

```

$G$	flytnett
$s$	kilde
$t$	sluk
$u$	node
$v$	node
$f$	flyt
$a$	økning
$Q$	kø

Vi traverserer fra  $s$ , og leter etter  $t$

**EDMONDS-KARP( $G, s, t$ )**

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat → until  $t.a == 0$ 
4    for each vertex  $u \in G.V$ 
5       $u.a = 0$  → reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     ...

```

$G$	flytnett
$s$	kilde
$t$	sluk
$u$	node
$v$	node
$f$	flyt
$a$	økning
$Q$	kø

EDMONDS-KARP( $G, s, t$ )

9

...

$G$  flytnett  
 $s$  kilde  
 $t$  sluk

EDMONDS-KARP( $G, s, t$ )

9        ...

10      **while**  $t.a == 0$  and  $Q \neq \emptyset$

G flytnett  
s kilde  
t sluk  
a økning  
Q kø

Her begynner BFS. Avbrytes hvis vi finner  $t$

```
EDMONDS-KARP(G, s, t)
```

```
9     ...
```

```
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11         $u = \text{DEQUEUE}(Q)$ 
```

G	flytnett
s	kilde
t	sluk
a	økning
Q	kø
u	node

Neste node vi skal besøke

EDMONDS-KARP( $G, s, t$ )

```

9     ...
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11         $u = \text{DEQUEUE}(Q)$ 
12        for all edges  $(u, v), (v, u) \in G.E$ 
```

G	flytnett
s	kilde
t	sluk
a	økning
Q	kø
u	node
v	node

For alle potensielle naboer i  $G_f$ : Se etter positiv restkapasitet

EDMONDS-KARP( $G, s, t$ )

```

9     ...
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11         $u = \text{DEQUEUE}(Q)$ 
12        for all edges  $(u, v), (v, u) \in G.E$ 
13            if  $(u, v) \in G.E$ 
```

G	flytnett
s	kilde
t	sluk
a	økning
Q	kø
u	node
v	node

Fremoverkant?

EDMONDS-KARP( $G, s, t$ )

```

9     ...
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11         $u = \text{DEQUEUE}(Q)$ 
12        for all edges  $(u, v), (v, u) \in G.E$ 
13            if  $(u, v) \in G.E$ 
14                 $c_f(u, v) = c(u, v) - (u, v).f$ 
```

$G$	flytnett
$s$	kilde
$t$	sluk
$a$	økning
$Q$	kø
$u$	node
$v$	node
$c$	kapasitet
$c_f$	restkap.
$f$	flyt

Restkapasitet = mulig økning av flyt langs kanten

EDMONDS-KARP( $G, s, t$ )

```

9     ...
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11         $u = \text{DEQUEUE}(Q)$ 
12        for all edges  $(u, v), (v, u) \in G.E$ 
13            if  $(u, v) \in G.E$ 
14                 $c_f(u, v) = c(u, v) - (u, v).f$ 
15            else  $c_f(u, v) = (v, u).f$ 
```

$G$	flytnett
$s$	kilde
$t$	sluk
$a$	økning
$Q$	$k\emptyset$
$u$	node
$v$	node
$c$	kapasitet
$c_f$	restkap.
$f$	flyt

Bakoverkant: Restkapasitet = mulig oppheving/omdirigering

EDMONDS-KARP( $G, s, t$ )

```

9     ...
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11         $u = \text{DEQUEUE}(Q)$ 
12        for all edges  $(u, v), (v, u) \in G.E$ 
13            if  $(u, v) \in G.E$ 
14                 $c_f(u, v) = c(u, v) - (u, v).f$ 
15            else  $c_f(u, v) = (v, u).f$ 
16            if  $c_f(u, v) > 0$  and  $v.a == 0$ 
```

$G$	flytnett
$s$	kilde
$t$	sluk
$a$	økning
$Q$	kø
$u$	node
$v$	node
$c$	kapasitet
$c_f$	restkap.
$f$	flyt
$a$	økning

Restkapasitet  $\iff$  kant i  $G_f$ . Ingen flytøkning  $\iff$  ubesøkt (hvit)

EDMONDS-KARP( $G, s, t$ )

```

9     ...
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11         $u = \text{DEQUEUE}(Q)$ 
12        for all edges  $(u, v), (v, u) \in G.E$ 
13            if  $(u, v) \in G.E$ 
14                 $c_f(u, v) = c(u, v) - (u, v).f$ 
15            else  $c_f(u, v) = (v, u).f$ 
16            if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                 $v.a = \min(u.a, c_f(u, v))$ 
```

$G$	flytnett
$s$	kilde
$t$	sluk
$a$	økning
$Q$	$\text{k}\emptyset$
$u$	node
$v$	node
$c$	kapasitet
$c_f$	restkap.
$f$	flyt
$a$	økning

Vi har med oss  $u.a$  ekstra flyt; får maks  $c_f(u, v)$  gjennom  $(u, v)$

EDMONDS-KARP( $G, s, t$ )

```

9     ...
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11         $u = \text{DEQUEUE}(Q)$ 
12        for all edges  $(u, v), (v, u) \in G.E$ 
13            if  $(u, v) \in G.E$ 
14                 $c_f(u, v) = c(u, v) - (u, v).f$ 
15            else  $c_f(u, v) = (v, u).f$ 
16            if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                 $v.a = \min(u.a, c_f(u, v))$ 
18                 $v.\pi = u$ 
```

$G$	flytnett
$s$	kilde
$t$	sluk
$a$	økning
$Q$	$\emptyset$
$u$	node
$v$	node
$c$	kapasitet
$c_f$	restkap.
$f$	flyt
$a$	økning
$\pi$	forgjenger

Hvor kom vi fra da vi oppdaget  $v$ ?

EDMONDS-KARP( $G, s, t$ )

```

9     ...
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11         $u = \text{DEQUEUE}(Q)$ 
12        for all edges  $(u, v), (v, u) \in G.E$ 
13            if  $(u, v) \in G.E$ 
14                 $c_f(u, v) = c(u, v) - (u, v).f$ 
15            else  $c_f(u, v) = (v, u).f$ 
16            if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                 $v.a = \min(u.a, c_f(u, v))$ 
18                 $v.\pi = u$ 
19                 $\text{ENQUEUE}(Q, v)$ 
```

$G$	flytnett
$s$	kilde
$t$	sluk
$a$	økning
$Q$	kø
$u$	node
$v$	node
$c$	kapasitet
$c_f$	restkap.
$f$	flyt
$a$	økning
$\pi$	forgjenger

Så vi husker å traversere  $v$  senere, og sende flyt videre fra den

EDMONDS-KARP( $G, s, t$ )

```

9   ...
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19         $\text{ENQUEUE}(Q, v)$ 
20    ...

```

G	flytnett
s	kilde
t	sluk
a	økning
Q	kø
u	node
v	node
c	kapasitet
$c_f$	restkap.
f	flyt
a	økning
$\pi$	forgjenger

EDMONDS-KARP( $G, s, t$ )

19

...

$G$	flytnett
$s$	kilde
$t$	sluk

Slutt på **while**: Har fått frem flyt til  $t$  eller gått tom for noder

EDMONDS-KARP( $G, s, t$ )

19            $\dots$

20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a = c_f(p)$

G flytnett  
s kilde  
t sluk  
u node  
v node  
 $\pi$  forgjenger

Vi har fått frem  $t.a$  ekstra flyt, kun begrenset av flaskehalsen  $c_f(p)$

EDMONDS-KARP( $G, s, t$ )

19                $\dots$

20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a = c_f(p)$

21       **while**  $u \neq \text{NIL}$

$G$  flytnett  
 $s$  kilde  
 $t$  sluk  
 $u$  node  
 $v$  node  
 $\pi$  forgjenger

Tilbakesporing: Følg  $\pi$ -feltene tilbake fra  $t$  til  $s$

EDMONDS-KARP( $G, s, t$ )

19                $\dots$

20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a = c_f(p)$

21       **while**  $u \neq \text{NIL}$

22               **if**  $(u, v) \in G.E$

G flytnett  
s kilde  
t sluk  
u node  
v node  
 $\pi$  forgjenger

Fremoverkant?

EDMONDS-KARP( $G, s, t$ )

19                $\dots$

20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a = c_f(p)$

21       **while**  $u \neq \text{NIL}$

22               **if**  $(u, v) \in G.E$

23                $(u, v).f = (u, v).f + t.a$

$G$	flytnett
$s$	kilde
$t$	sluk
$u$	node
$v$	node
$\pi$	forgjenger
$f$	flyt
$a$	økning

Øk flyten med  $t.a$ , dvs.  $c_f(p)$

EDMONDS-KARP( $G, s, t$ )

19                $\dots$

20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a = c_f(p)$

21       **while**  $u \neq \text{NIL}$

22               **if**  $(u, v) \in G.E$

23                $(u, v).f = (u, v).f + t.a$

24               **else**  $(v, u).f = (v, u).f - t.a$

$G$	flytnett
$s$	kilde
$t$	sluk
$u$	node
$v$	node
$\pi$	forgjenger
$f$	flyt
$a$	økning

Bakoverkant: Opphev og omdirigér  $t.a$ , dvs.  $c_f(p)$

EDMONDS-KARP( $G, s, t$ )

19                $\dots$

20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a = c_f(p)$

21       **while**  $u \neq \text{NIL}$

22               **if**  $(u, v) \in G.E$

23                $(u, v).f = (u, v).f + t.a$

24               **else**  $(v, u).f = (v, u).f - t.a$

25        $u, v = u.\pi, u$

$G$	flytnett
$s$	kilde
$t$	sluk
$u$	node
$v$	node
$\pi$	forgjenger
$f$	flyt
$a$	økning

Gå ett skritt bakover langs  $p$

EDMONDS-KARP( $G, s, t$ )

19                $\dots$

20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a = c_f(p)$

21       **while**  $u \neq \text{NIL}$

22               **if**  $(u, v) \in G.E$

23                $(u, v).f = (u, v).f + t.a$

24               **else**  $(v, u).f = (v, u).f - t.a$

25        $u, v = u.\pi, u$

26 **until**  $t.a == 0$

$G$	flytnett
$s$	kilde
$t$	sluk
$u$	node
$v$	node
$\pi$	forgjenger
$f$	flyt
$a$	økning

Ingen forøkende sti  $p$  funnet; flyten er maksimal

maks-flyt → edmonds-karp

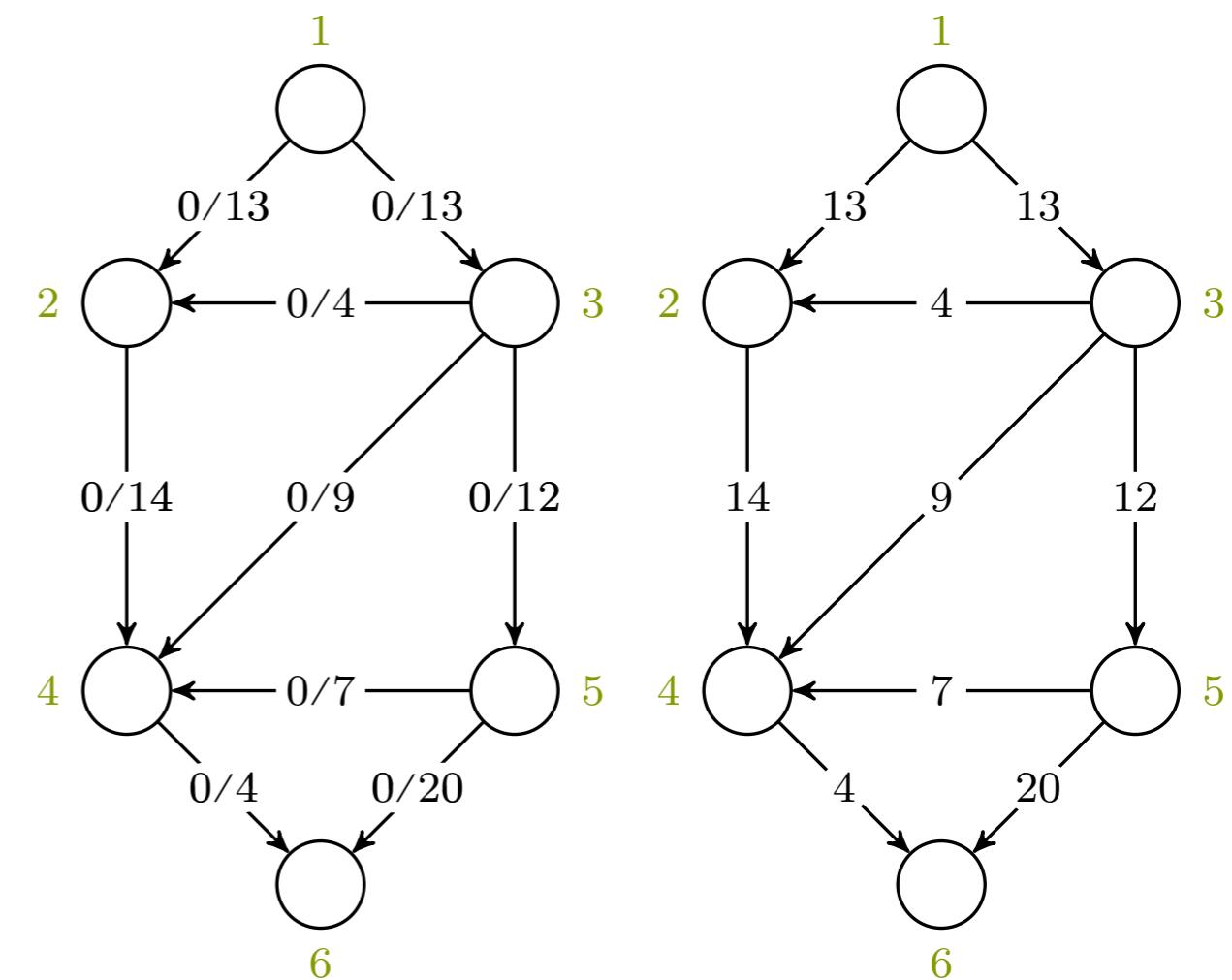
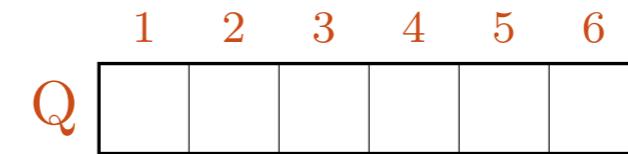
EDMONDS-KARP( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7    $s.a = \infty$ 
8    $Q = \emptyset$ 
9   ENQUEUE( $Q, s$ )
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20     $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21    while  $u \neq \text{NIL}$ 
22      if  $(u, v) \in G.E$ 
23         $(u, v).f = (u, v).f + t.a$ 
24      else  $(v, u).f = (v, u).f - t.a$ 
25       $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = -, -$

Merk: Selv i den fulle simuleringen så dropper jeg her noen detaljer etter hvert (og hopper forbi noen løkker, etc.).



Node 1 er kilde, node 6 er sluk.

flyt

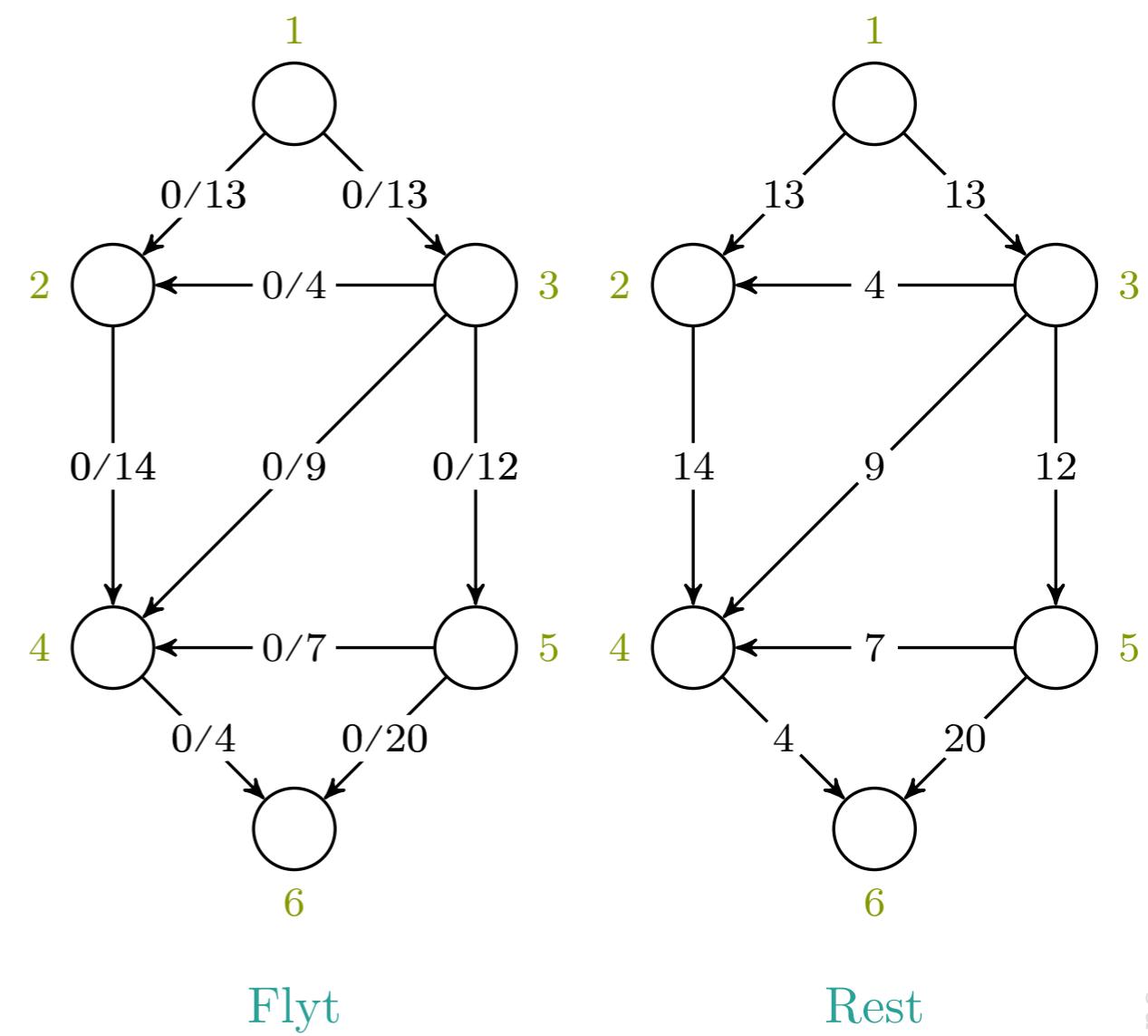
Rest

EDMONDS-KARP( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7    $s.a = \infty$ 
8    $Q = \emptyset$ 
9   ENQUEUE( $Q, s$ )
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20     $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21    while  $u \neq \text{NIL}$ 
22      if  $(u, v) \in G.E$ 
23         $(u, v).f = (u, v).f + t.a$ 
24      else  $(v, u).f = (v, u).f - t.a$ 
25     $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = -, -$

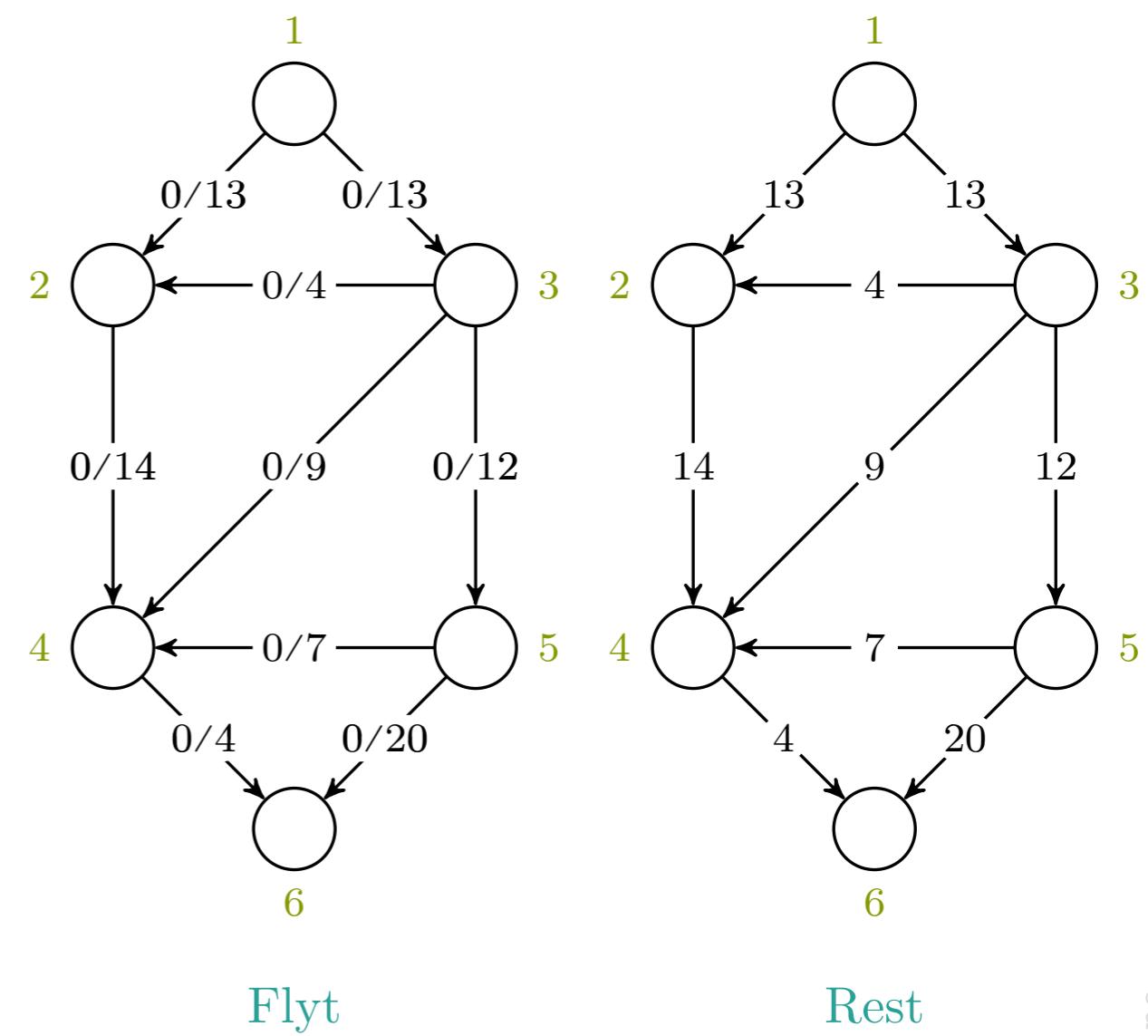


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7     $s.a = \infty$ 
8     $Q = \emptyset$ 
9    ENQUEUE( $Q, s$ )
10   while  $t.a == 0$  and  $Q \neq \emptyset$ 
11      $u = \text{DEQUEUE}(Q)$ 
12     for all edges  $(u, v), (v, u) \in G.E$ 
13       if  $(u, v) \in G.E$ 
14          $c_f(u, v) = c(u, v) - (u, v).f$ 
15       else  $c_f(u, v) = (v, u).f$ 
16       if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17          $v.a = \min(u.a, c_f(u, v))$ 
18          $v.\pi = u$ 
19         ENQUEUE( $Q, v$ )
20      $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25      $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = -, -$

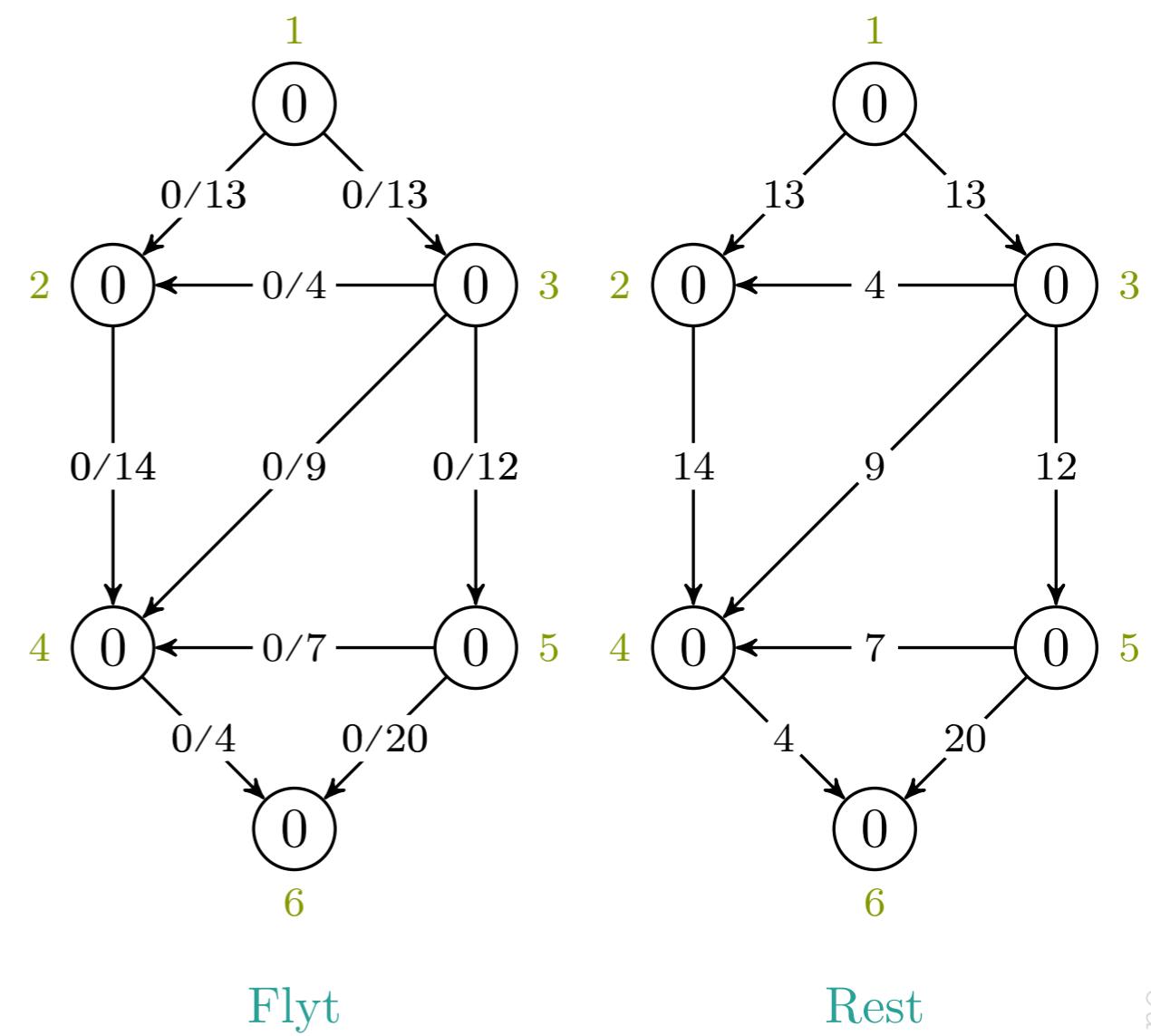


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2       $(u, v).f = 0$ 
3  repeat
4      for each vertex  $u \in G.V$ 
5           $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6           $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11          $u = \text{DEQUEUE}(Q)$ 
12         for all edges  $(u, v), (v, u) \in G.E$ 
13             if  $(u, v) \in G.E$ 
14                  $c_f(u, v) = c(u, v) - (u, v).f$ 
15             else  $c_f(u, v) = (v, u).f$ 
16             if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                  $v.a = \min(u.a, c_f(u, v))$ 
18                  $v.\pi = u$ 
19                 ENQUEUE( $Q, v$ )
20              $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21             while  $u \neq \text{NIL}$ 
22                 if  $(u, v) \in G.E$ 
23                      $(u, v).f = (u, v).f + t.a$ 
24                 else  $(v, u).f = (v, u).f - t.a$ 
25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = -, -$



maks-flyt › edmonds-karp

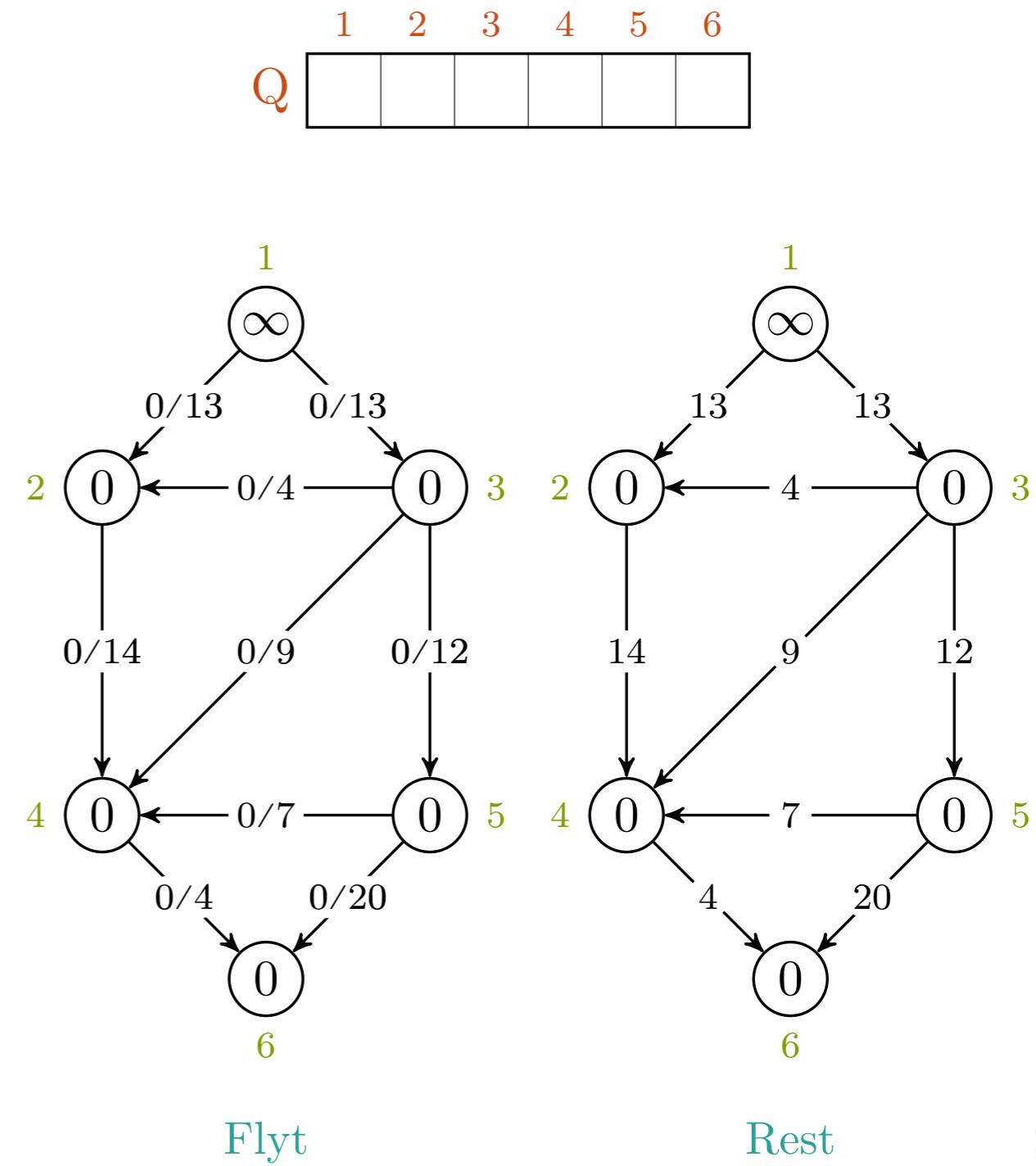
EDMONDS-KARP(G, s, t)

```

1   for each edge  $(u, v) \in G.E$ 
2        $(u, v).f = 0$ 
3   repeat
4       for each vertex  $u \in G.V$ 
5            $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6            $u.\pi = \text{NIL}$ 
7            $s.a = \infty$ 
8            $Q = \emptyset$ 
9           ENQUEUE( $Q, s$ )
10          while  $t.a == 0$  and  $Q \neq \emptyset$ 
11               $u = \text{DEQUEUE}(Q)$ 
12              for all edges  $(u, v), (v, u) \in G.E$ 
13                  if  $(u, v) \in G.E$ 
14                       $c_f(u, v) = c(u, v) - (u, v).f$ 
15                  else  $c_f(u, v) = (v, u).f$ 
16                  if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                       $v.a = \min(u.a, c_f(u, v))$ 
18                       $v.\pi = u$ 
19                      ENQUEUE( $Q, v$ )
20           $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21          while  $u \neq \text{NIL}$ 
22              if  $(u, v) \in G.E$ 
23                   $(u, v).f = (u, v).f + t.a$ 
24              else  $(v, u).f = (v, u).f - t.a$ 
25               $u, v = u.\pi, u$ 
26          until  $t.a == 0$ 

```

$$u, v = -, -$$

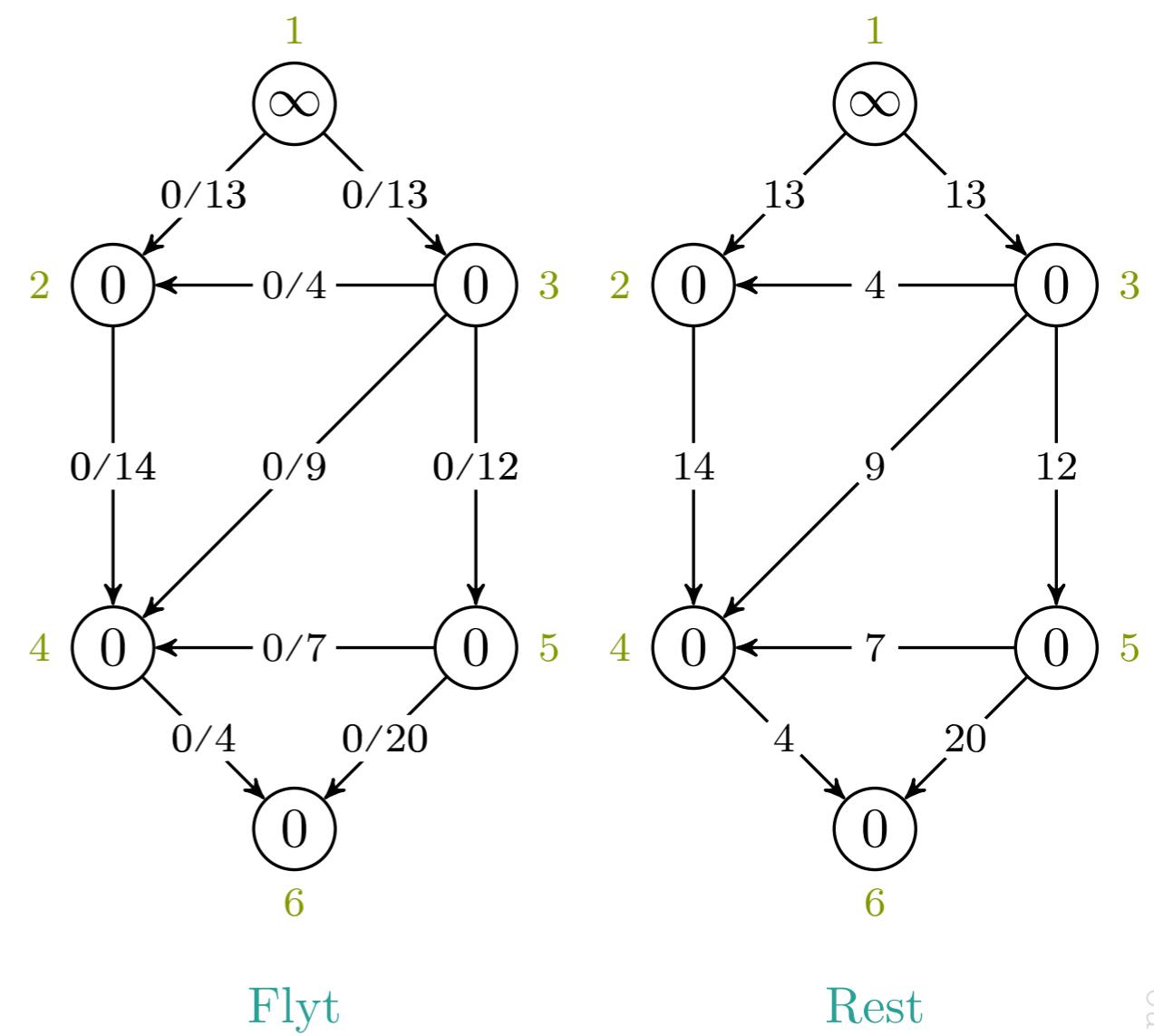


EDMONDS-KARP( $G, s, t$ )

```

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25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = -, -$

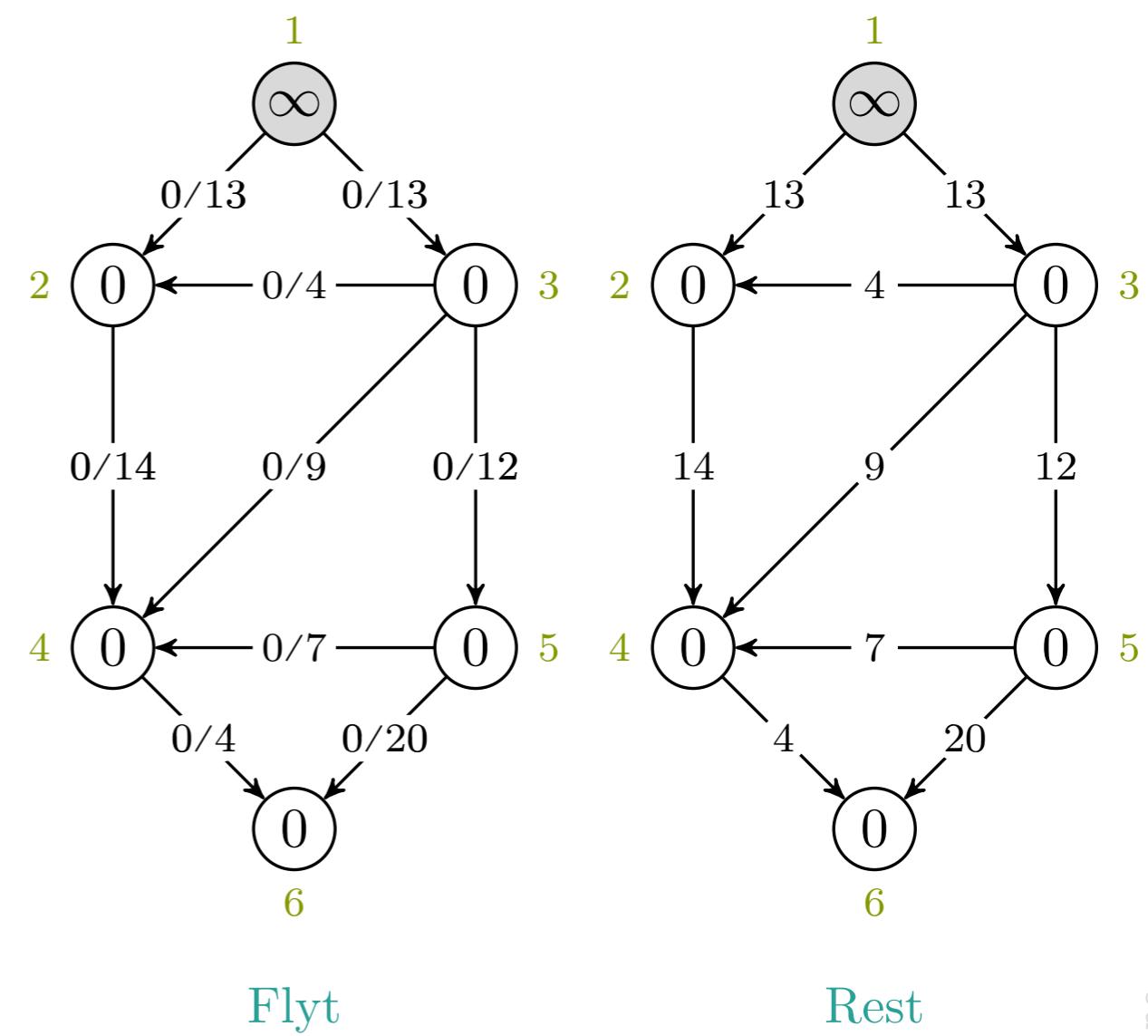


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```

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25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = -, -$

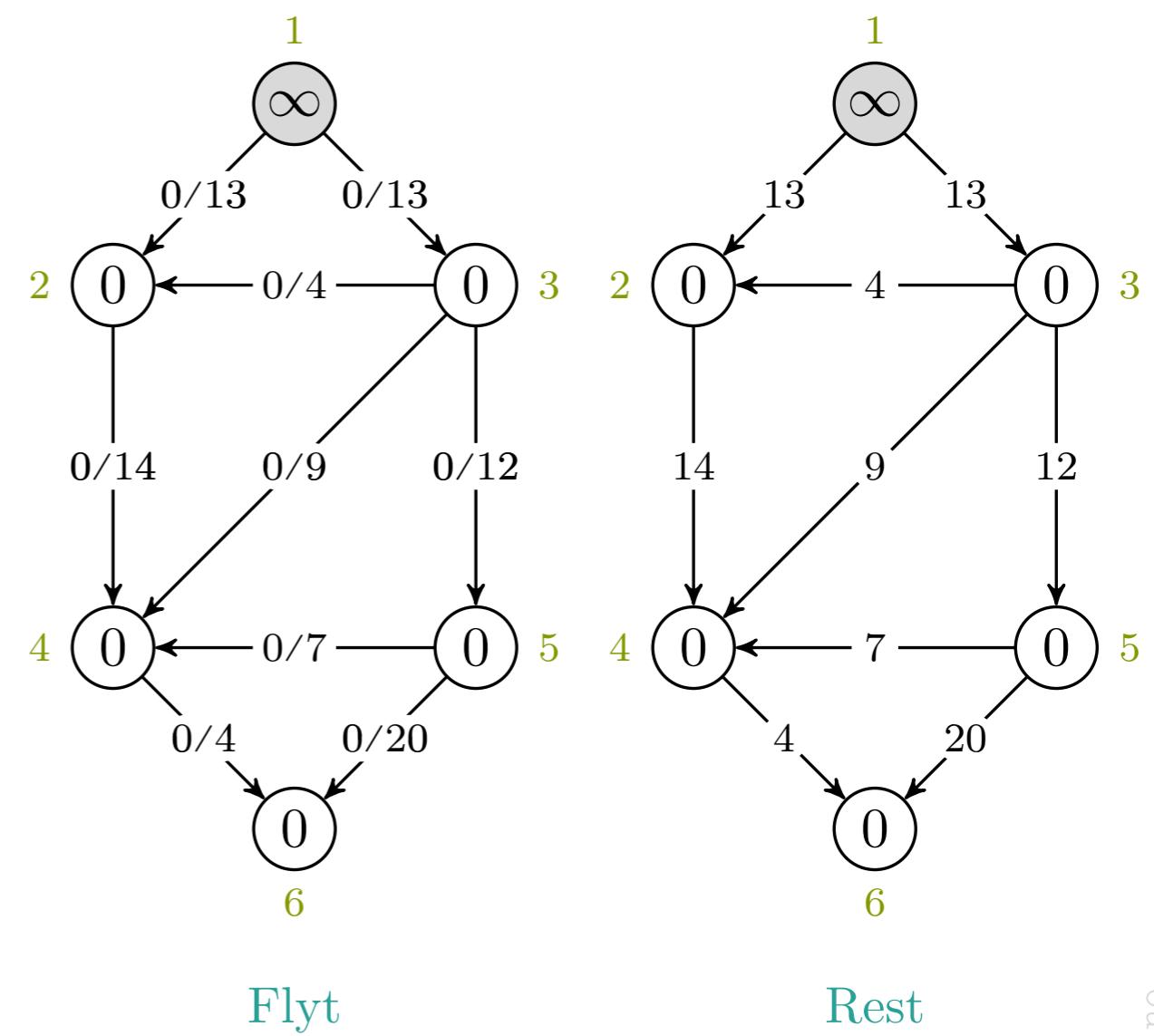


EDMONDS-KARP( $G, s, t$ )

```

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25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

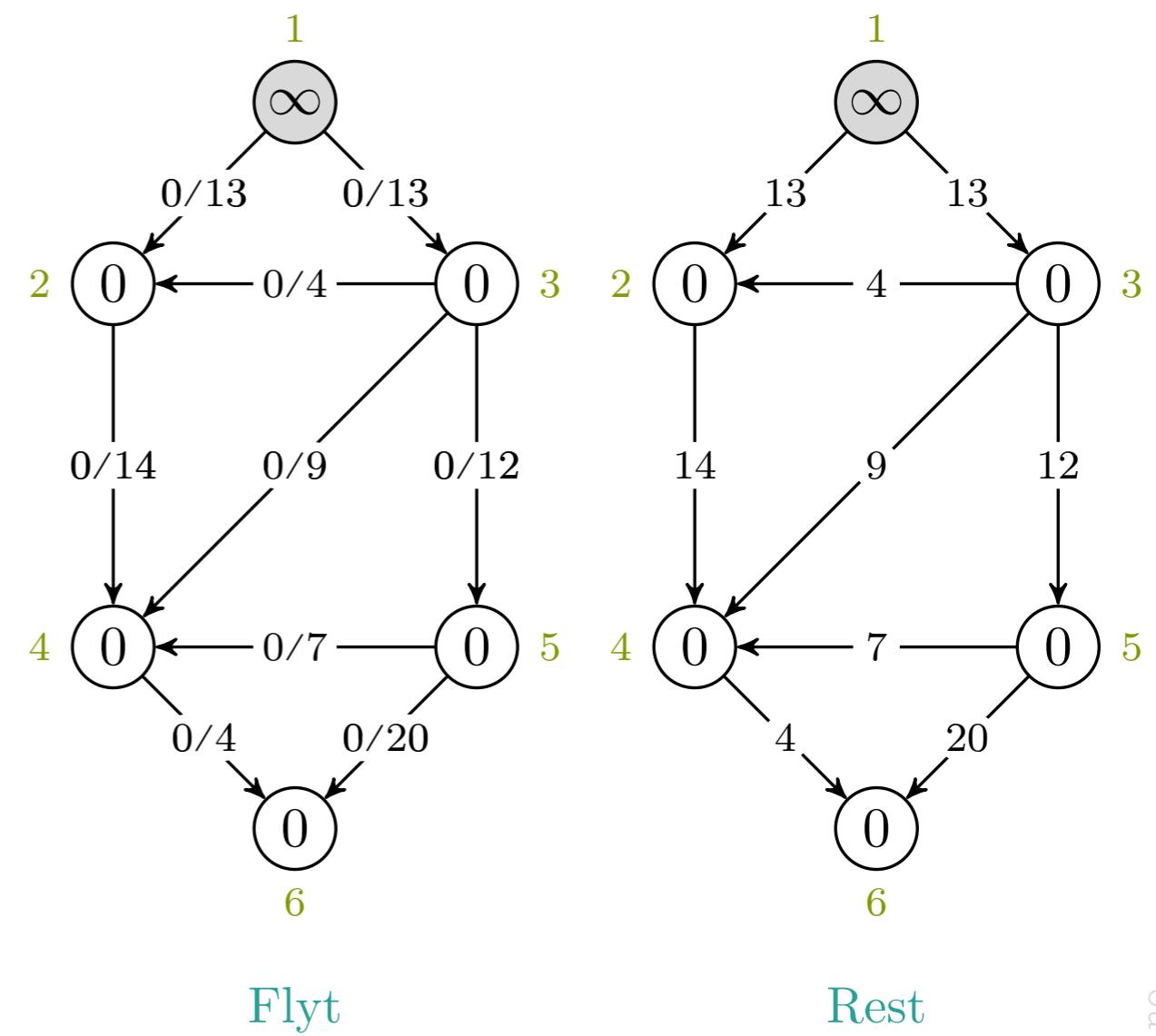
```

 $u, v = -, -$ 

EDMONDS-KARP( $G, s, t$ )

```

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25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

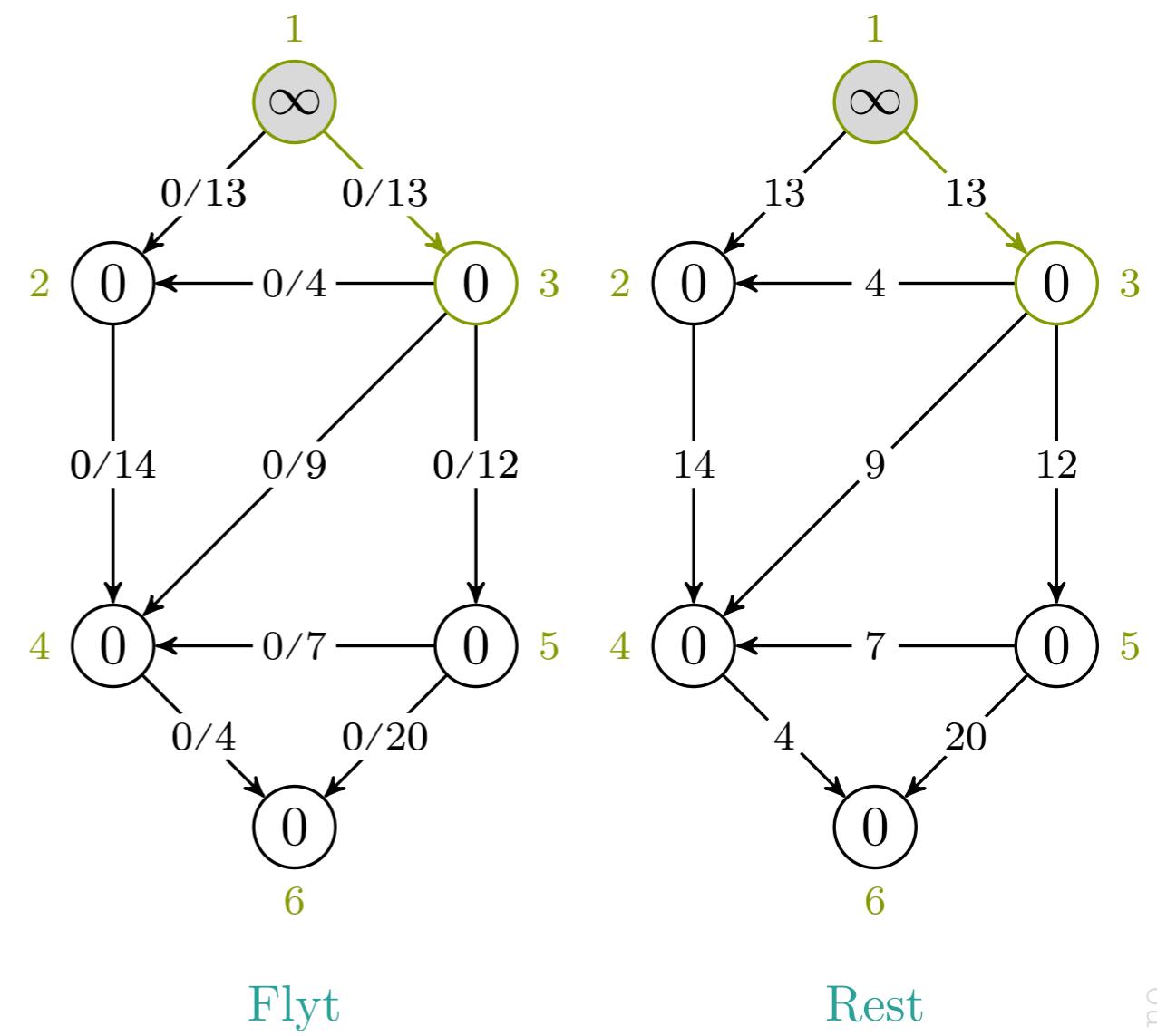
 $u, v = 1, -$ 

EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2       $(u, v).f = 0$ 
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26 until  $t.a == 0$ 
```

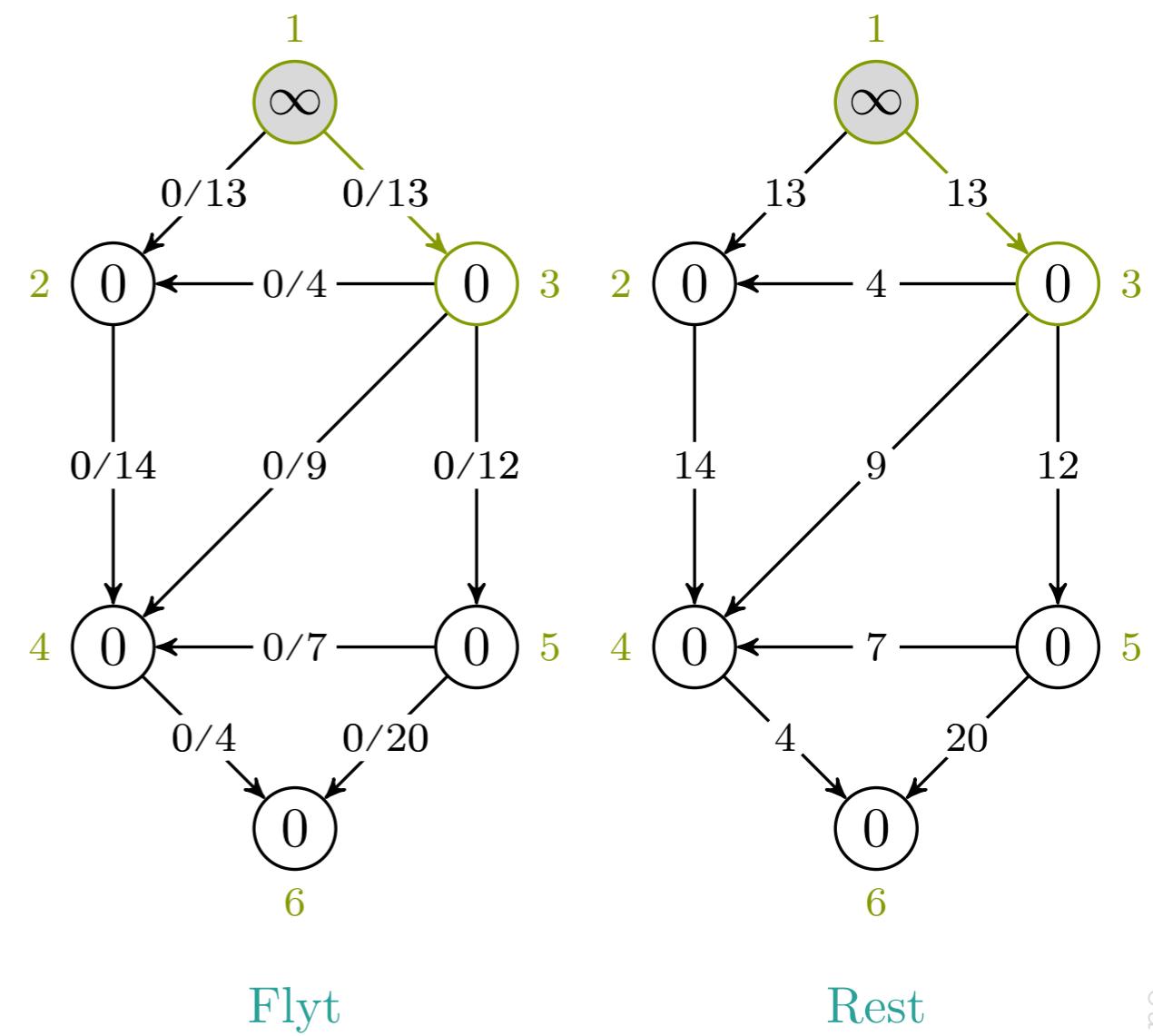
$u, v = 1, 3$



EDMONDS-KARP( $G, s, t$ )

```

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2     $(u, v).f = 0$ 
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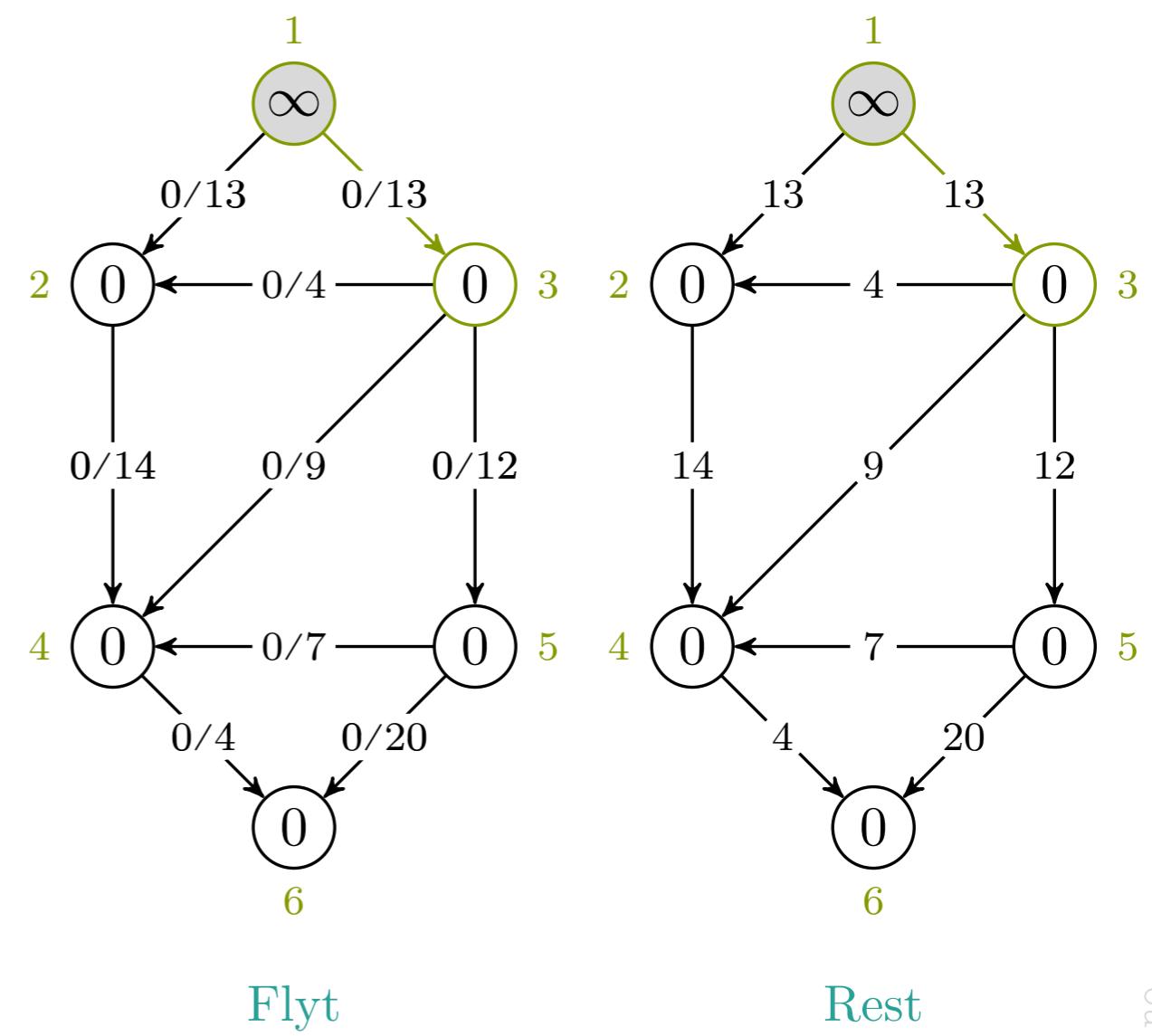
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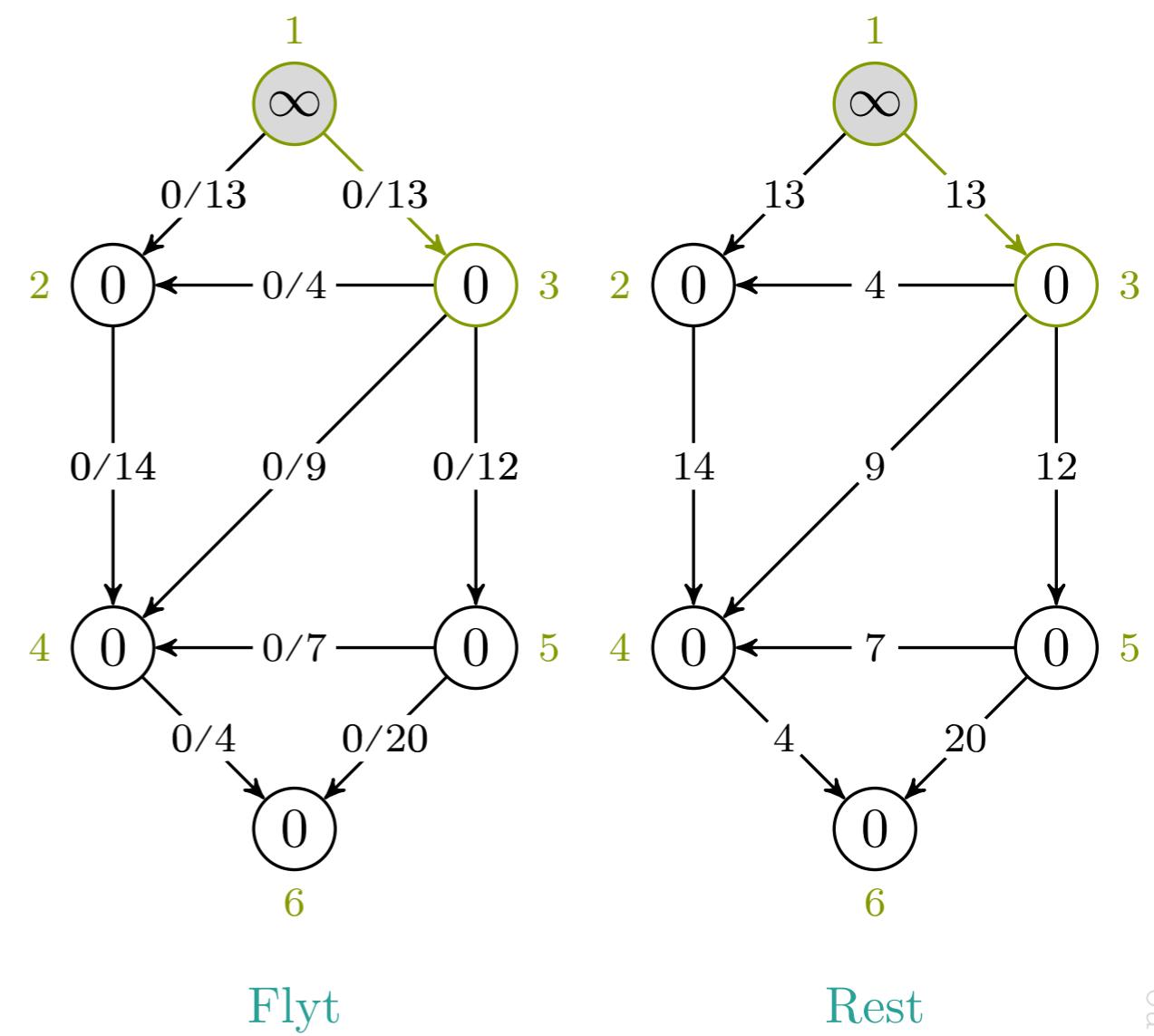
```

 $u, v = 1, 3$ 

EDMONDS-KARP( $G, s, t$ )

```

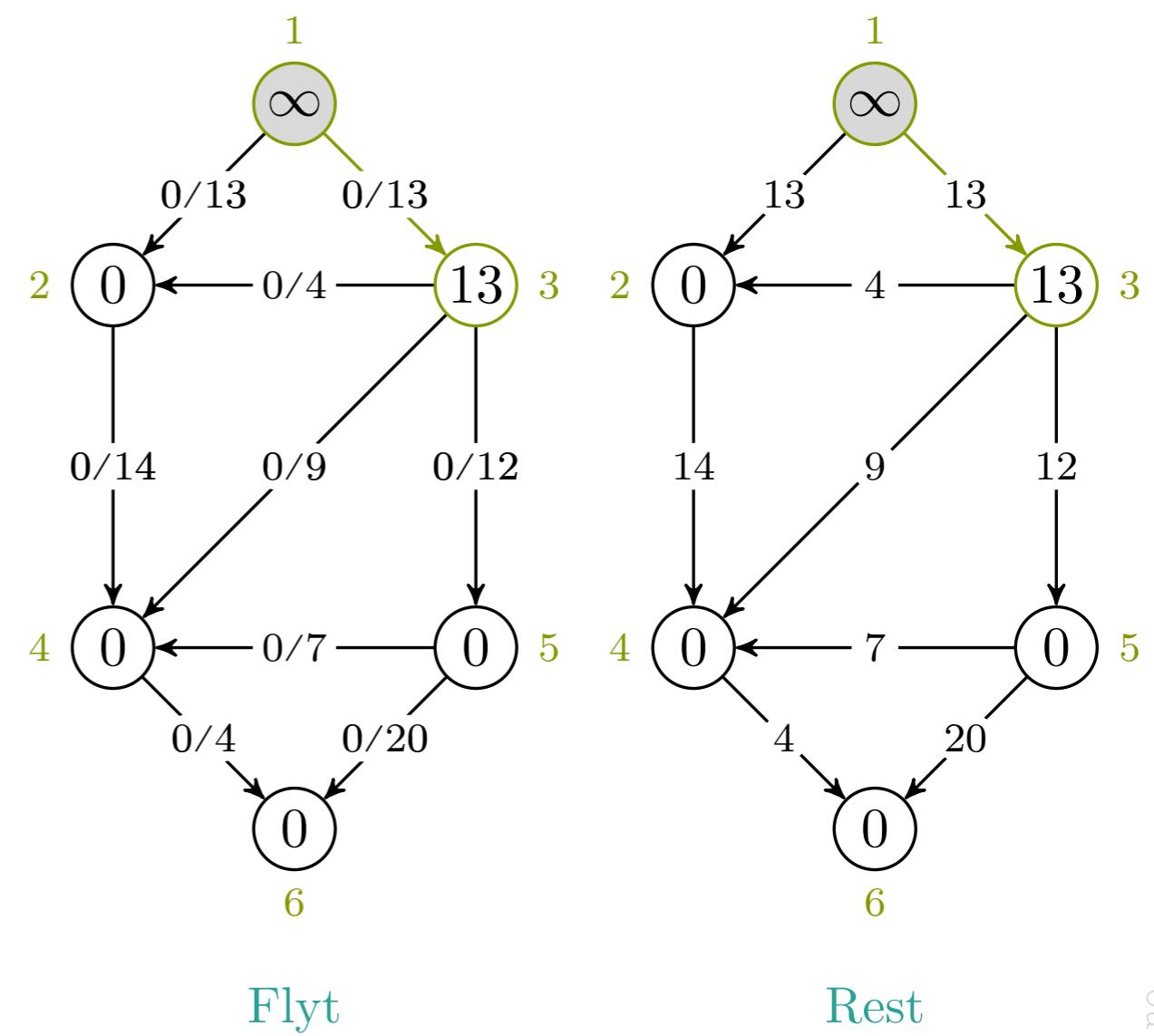
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 $u, v = 1, 3$ 

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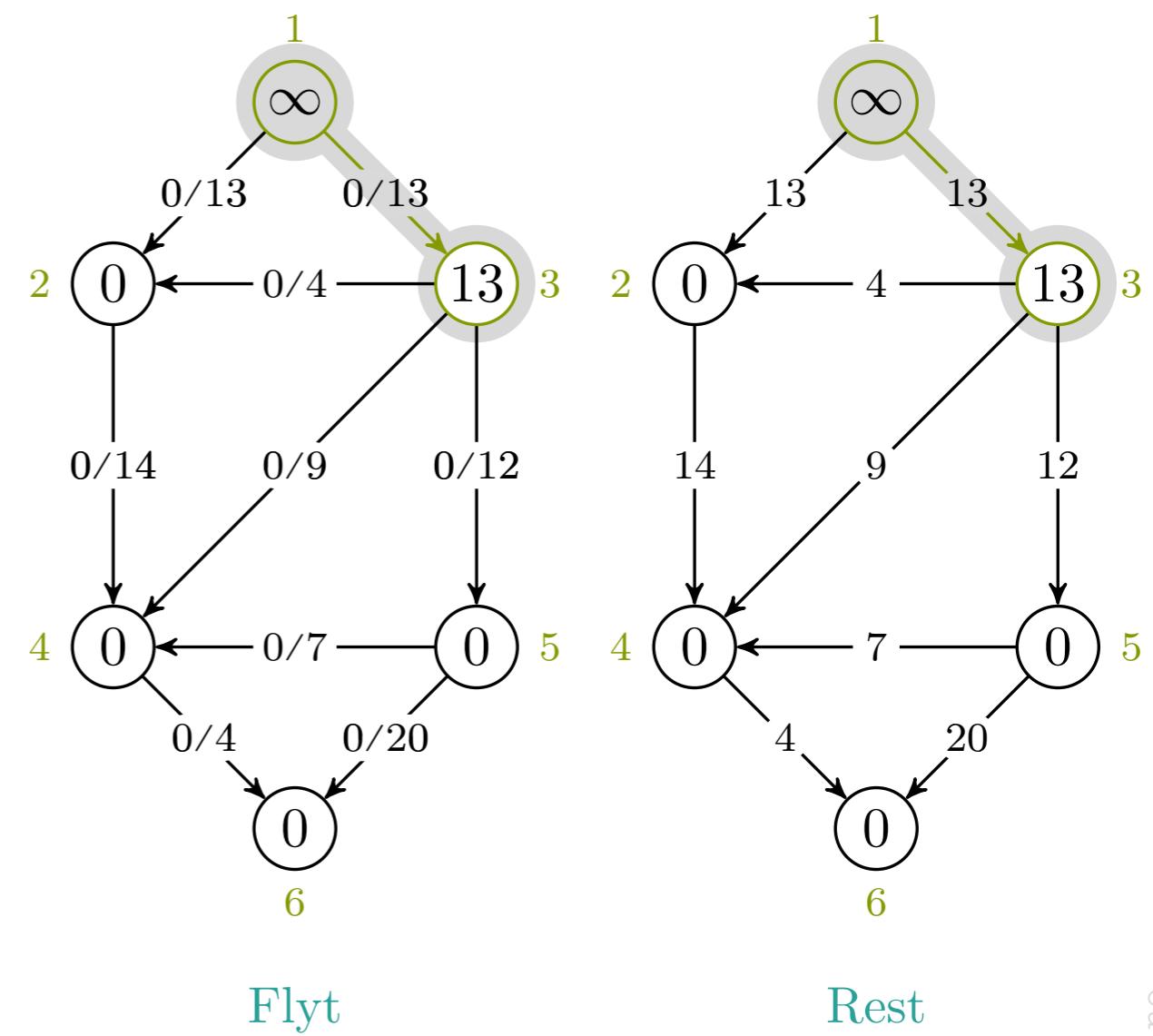
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26 until  $t.a == 0$ 
```

$u, v = 1, 3$

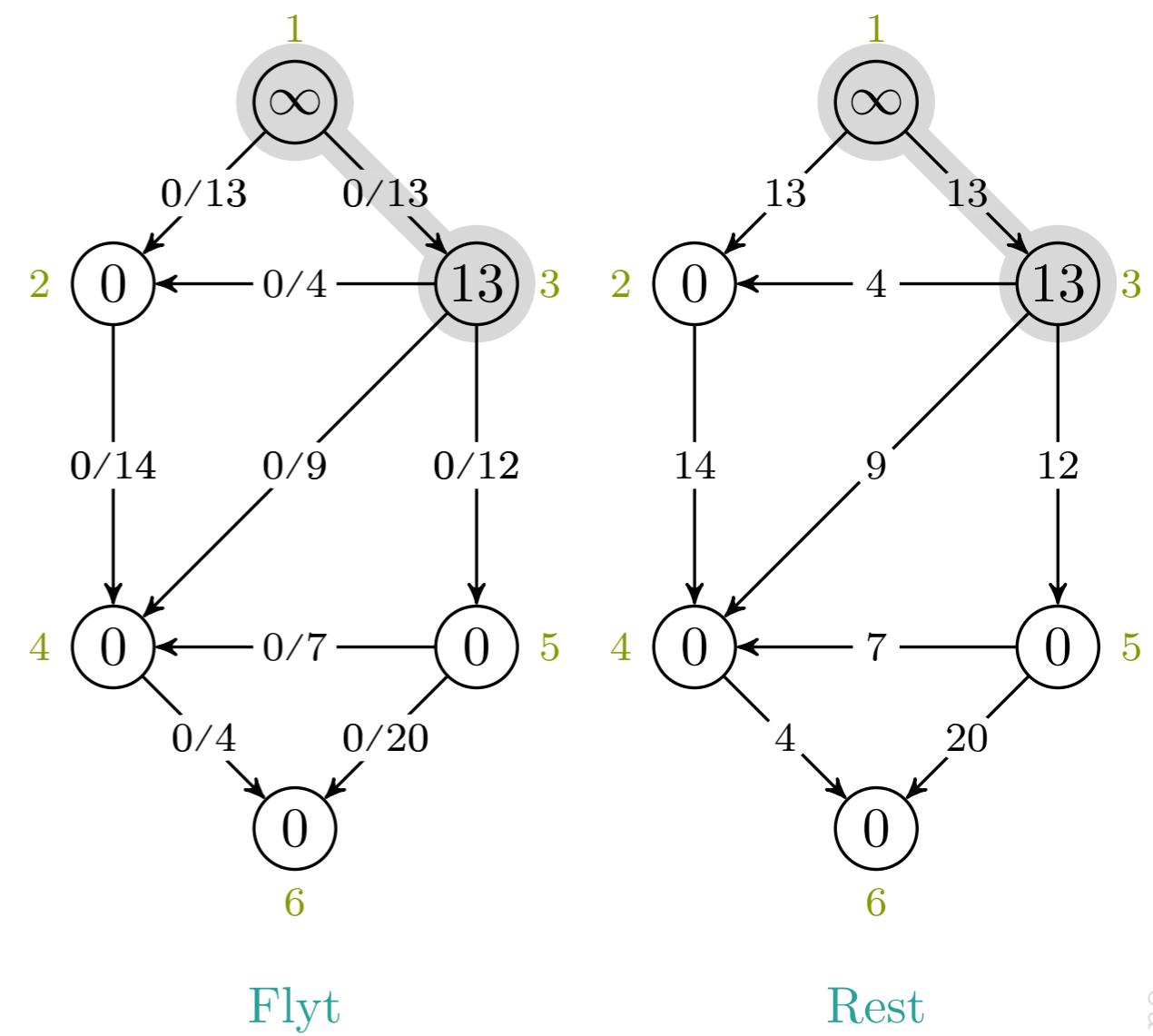


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```

$u, v = 1, -$

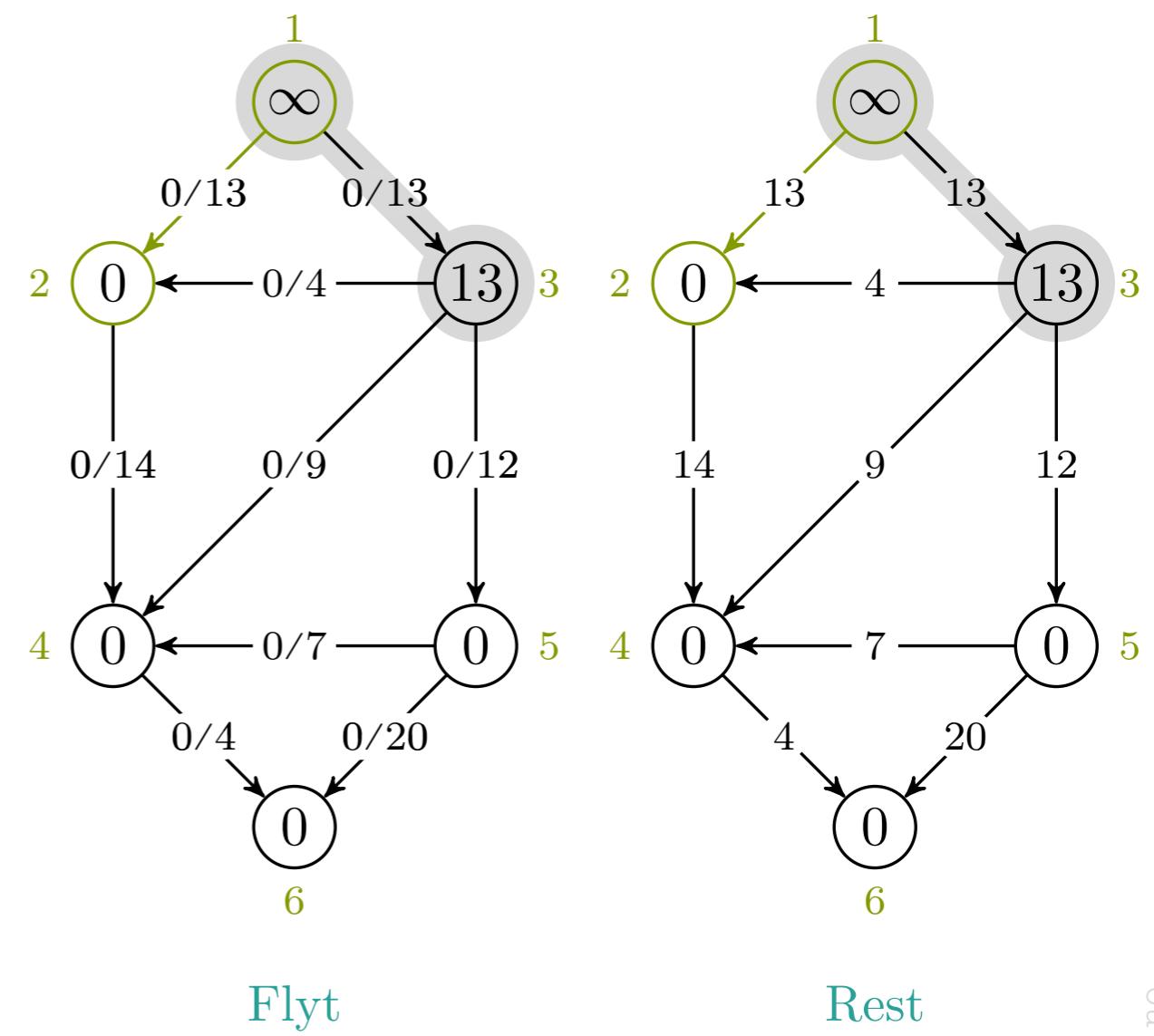


EDMONDS-KARP( $G, s, t$ )

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18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, 2$

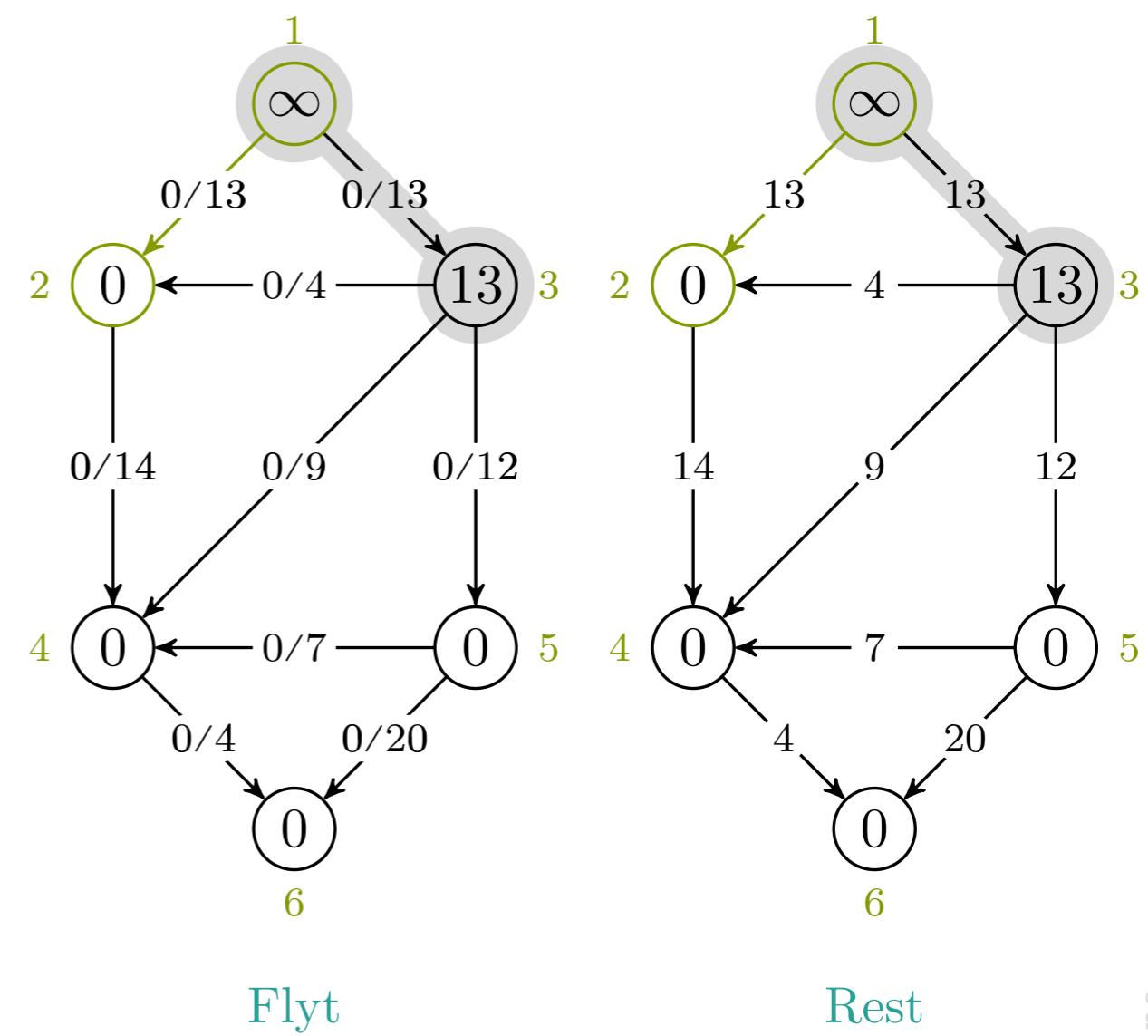


EDMONDS-KARP( $G, s, t$ )

```

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14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, 2$



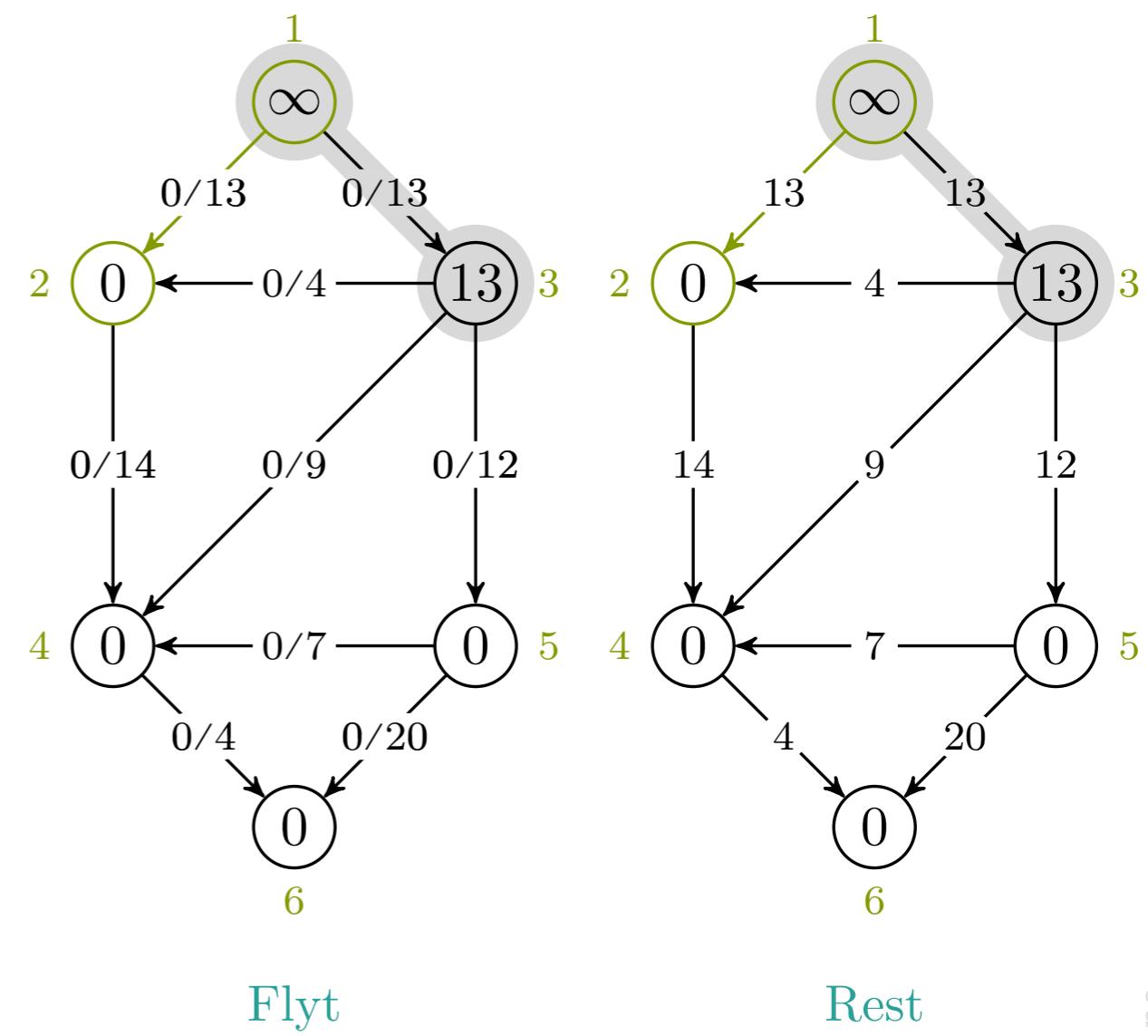
maks-flyt → edmonds-karp

EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2       $(u, v).f = 0$ 
3  repeat
4      for each vertex  $u \in G.V$ 
5           $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6           $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11          $u = \text{DEQUEUE}(Q)$ 
12         for all edges  $(u, v), (v, u) \in G.E$ 
13             if  $(u, v) \in G.E$ 
14                  $c_f(u, v) = c(u, v) - (u, v).f$ 
15             else  $c_f(u, v) = (v, u).f$ 
16             if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                  $v.a = \min(u.a, c_f(u, v))$ 
18                  $v.\pi = u$ 
19                 ENQUEUE( $Q, v$ )
20              $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21             while  $u \neq \text{NIL}$ 
22                 if  $(u, v) \in G.E$ 
23                      $(u, v).f = (u, v).f + t.a$ 
24                 else  $(v, u).f = (v, u).f - t.a$ 
25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, 2$



maks-flyt → edmonds-karp

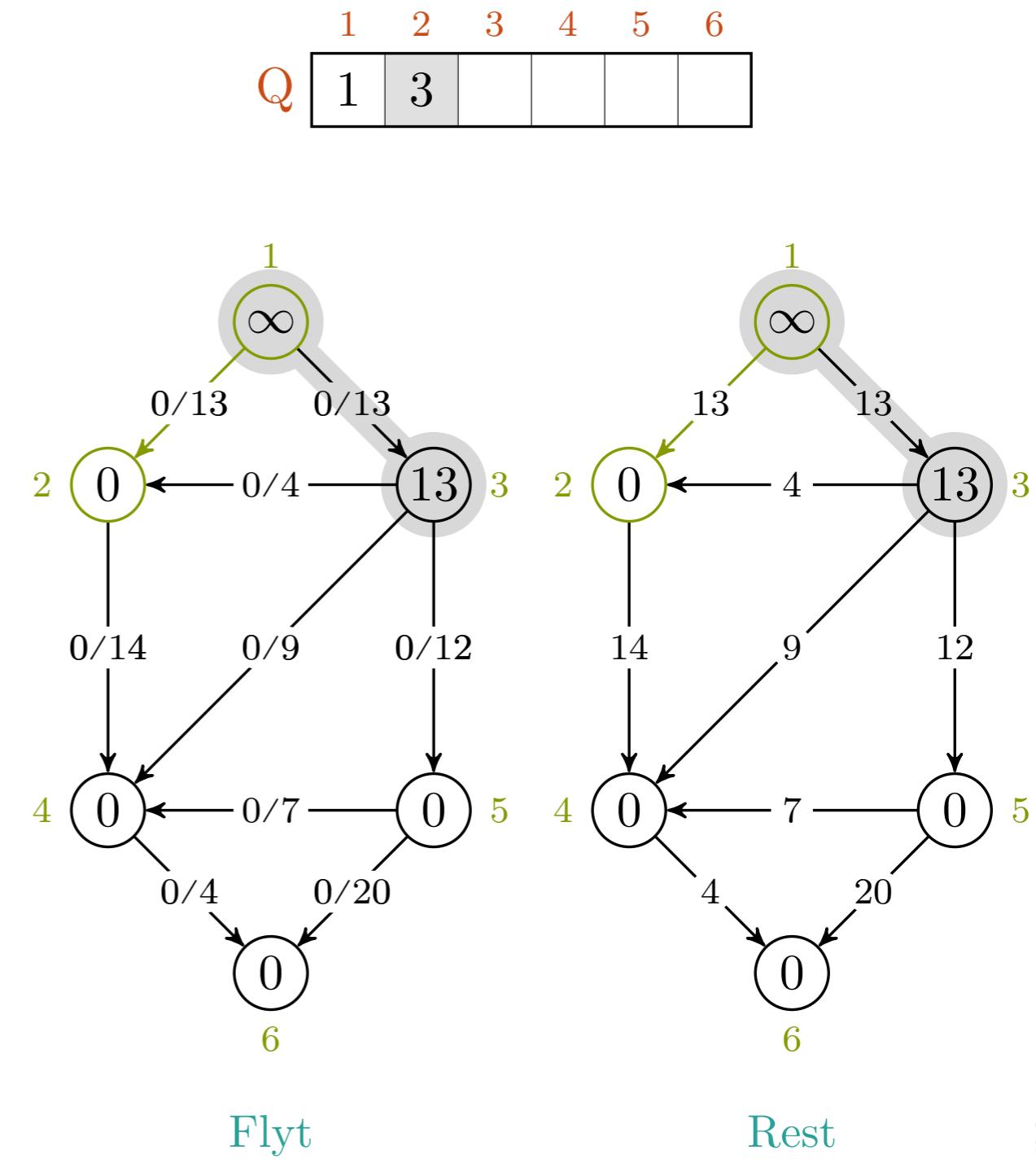
EDMONDS-KARP(G, s, t)

```

1   for each edge  $(u, v) \in G.E$ 
2      $(u, v).f = 0$ 
3   repeat
4     for each vertex  $u \in G.V$ 
5        $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6        $u.\pi = \text{NIL}$ 
7        $s.a = \infty$ 
8        $Q = \emptyset$ 
9       ENQUEUE( $Q, s$ )
10      while  $t.a == 0$  and  $Q \neq \emptyset$ 
11         $u = \text{DEQUEUE}(Q)$ 
12        for all edges  $(u, v), (v, u) \in G.E$ 
13          if  $(u, v) \in G.E$ 
14             $c_f(u, v) = c(u, v) - (u, v).f$ 
15          else  $c_f(u, v) = (v, u).f$ 
16          if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17             $v.a = \min(u.a, c_f(u, v))$ 
18             $v.\pi = u$ 
19            ENQUEUE( $Q, v$ )
20         $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f$ 
21        while  $u \neq \text{NIL}$ 
22          if  $(u, v) \in G.E$ 
23             $(u, v).f = (u, v).f + t.a$ 
24          else  $(v, u).f = (v, u).f - t.a$ 
25           $u, v = u.\pi, u$ 
26      until  $t.a == 0$ 

```

$$u, v = 1, 2$$

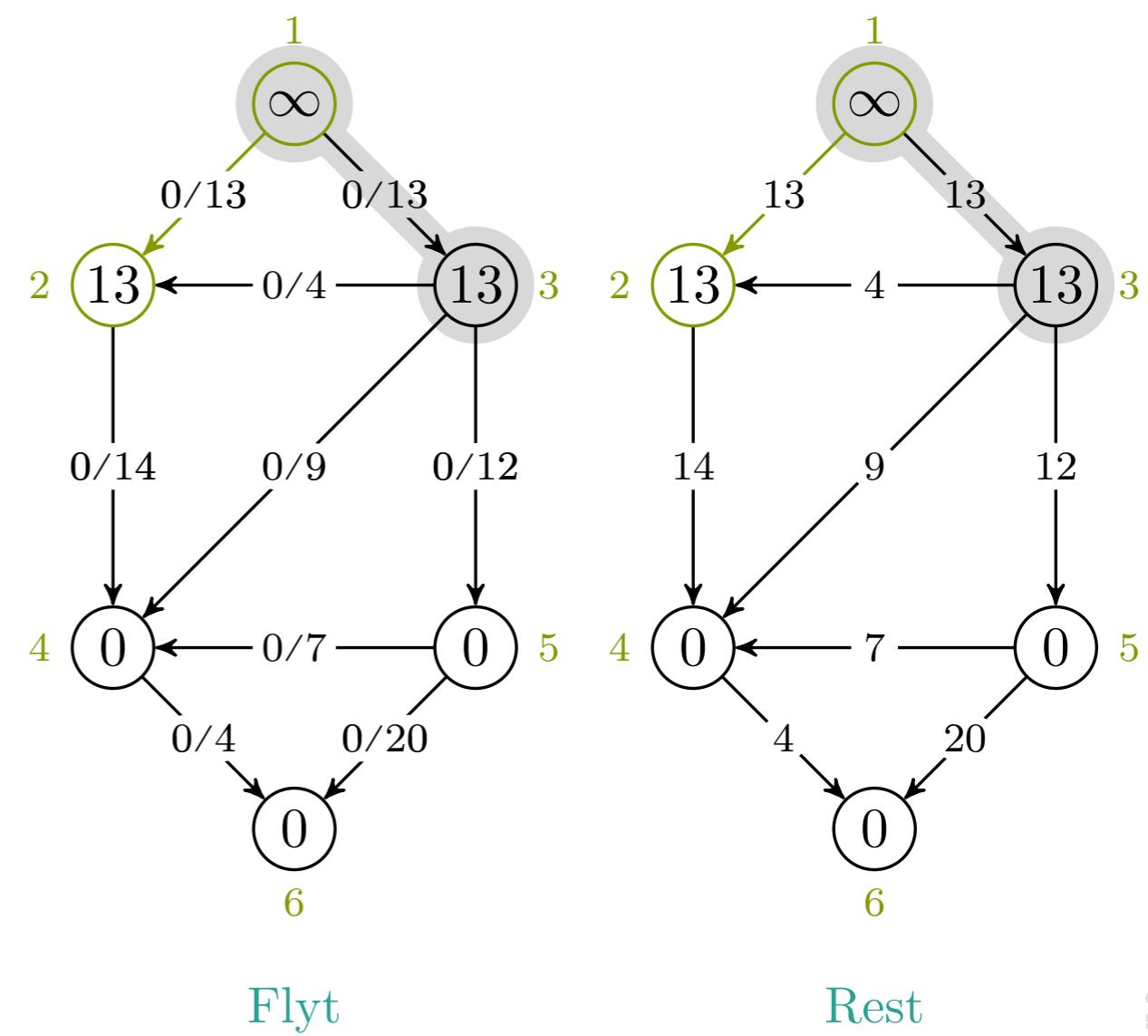


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, 2$

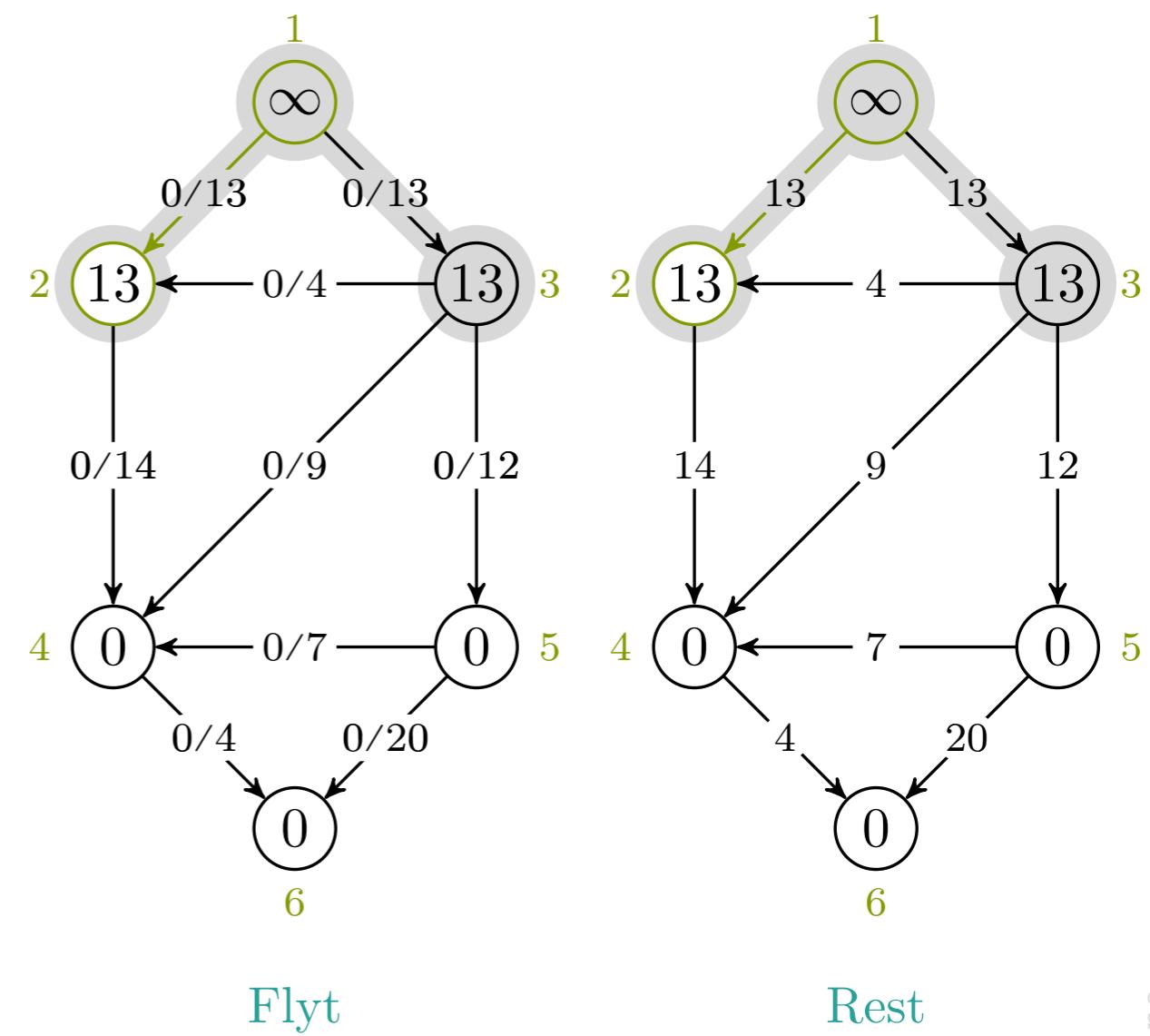


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25          $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, 2$

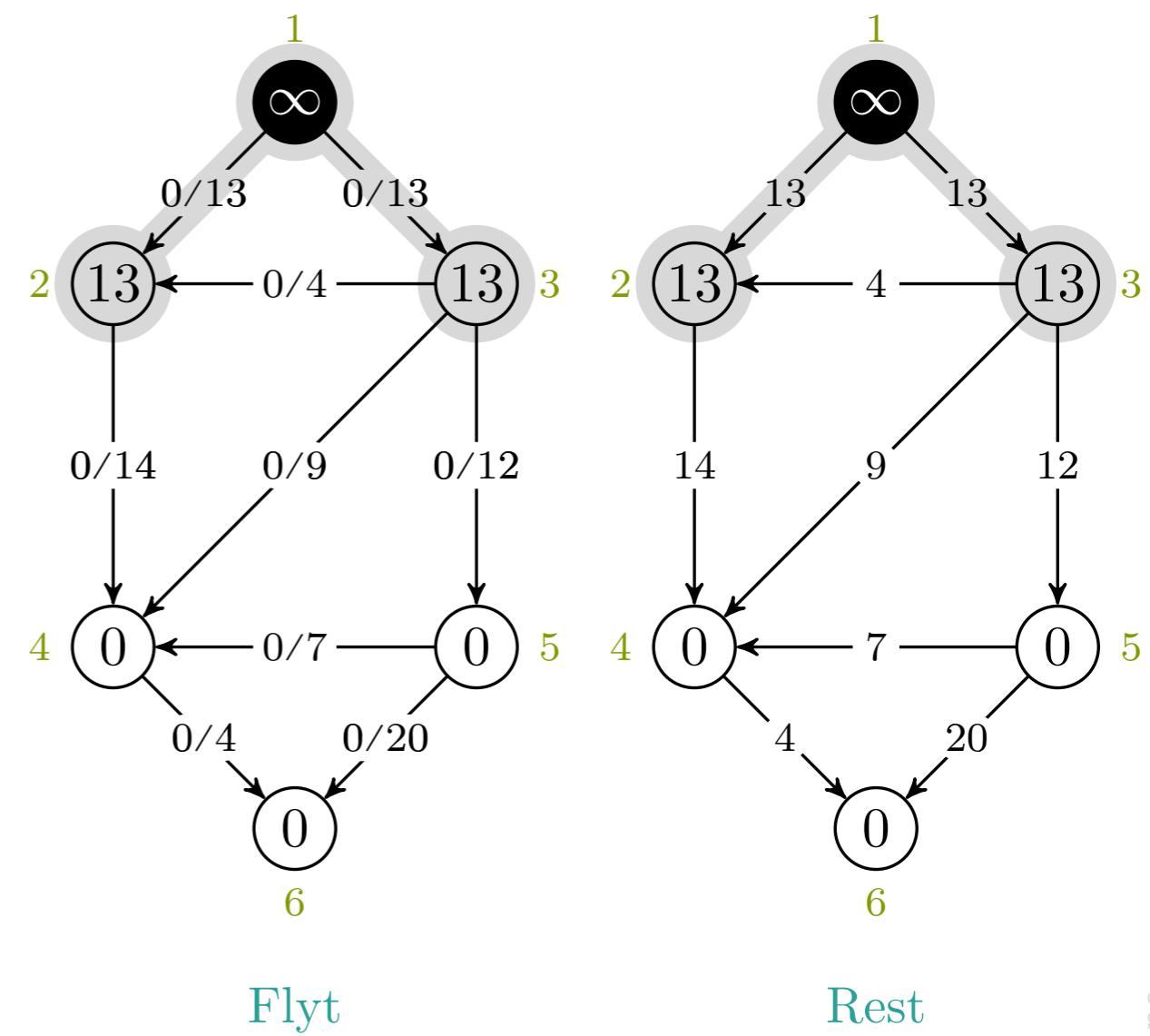


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, -$

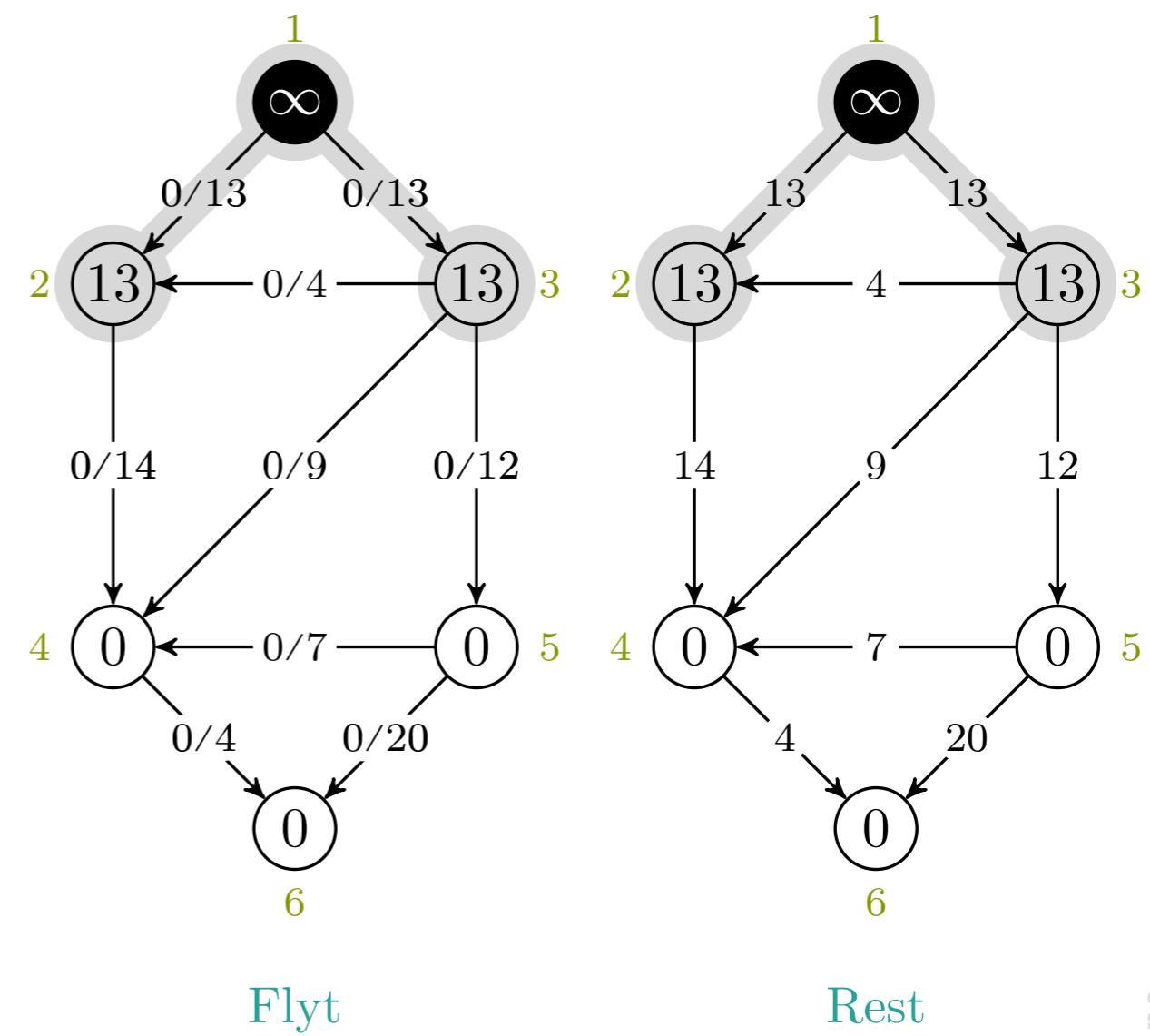


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, -$

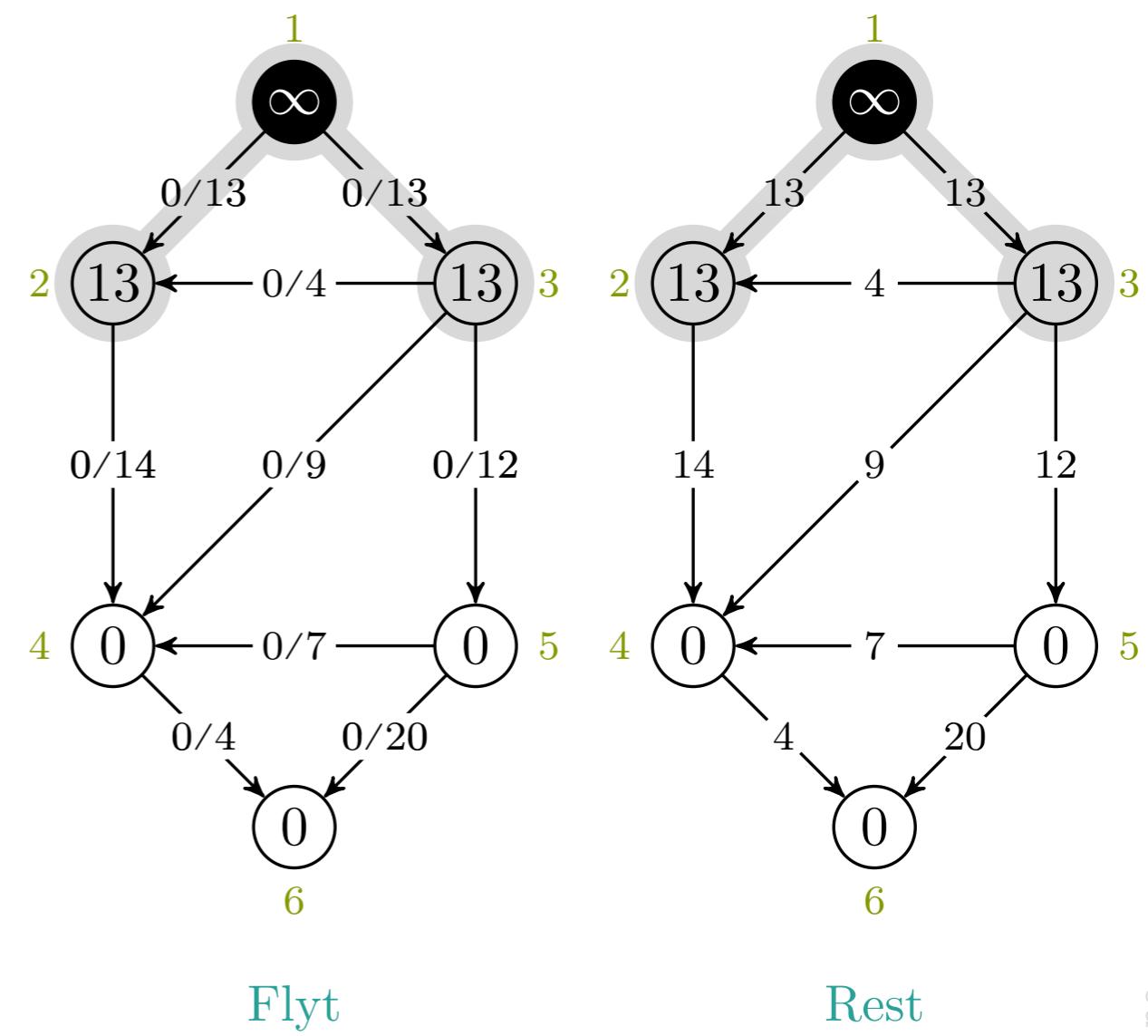


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, -$

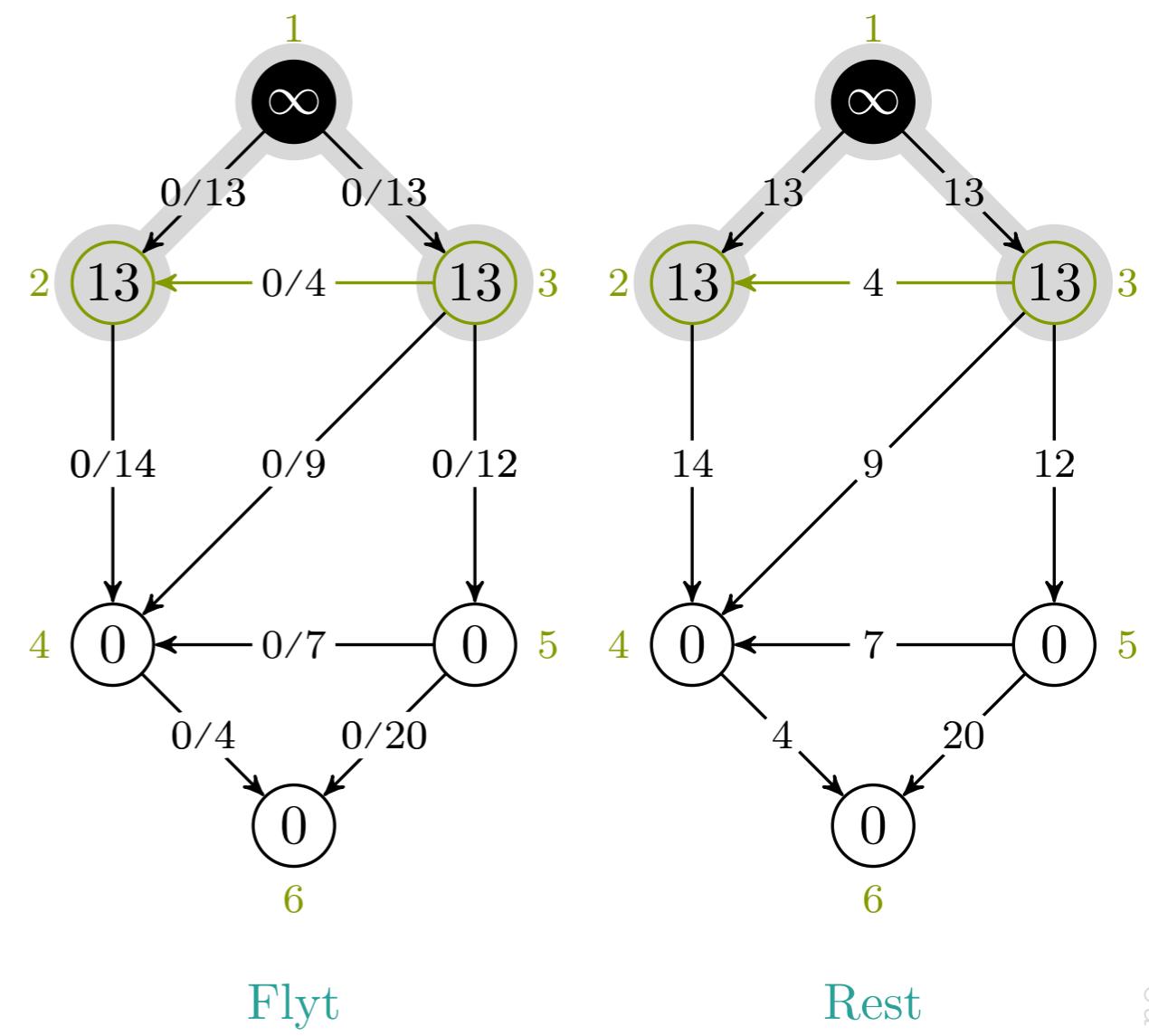


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 2$

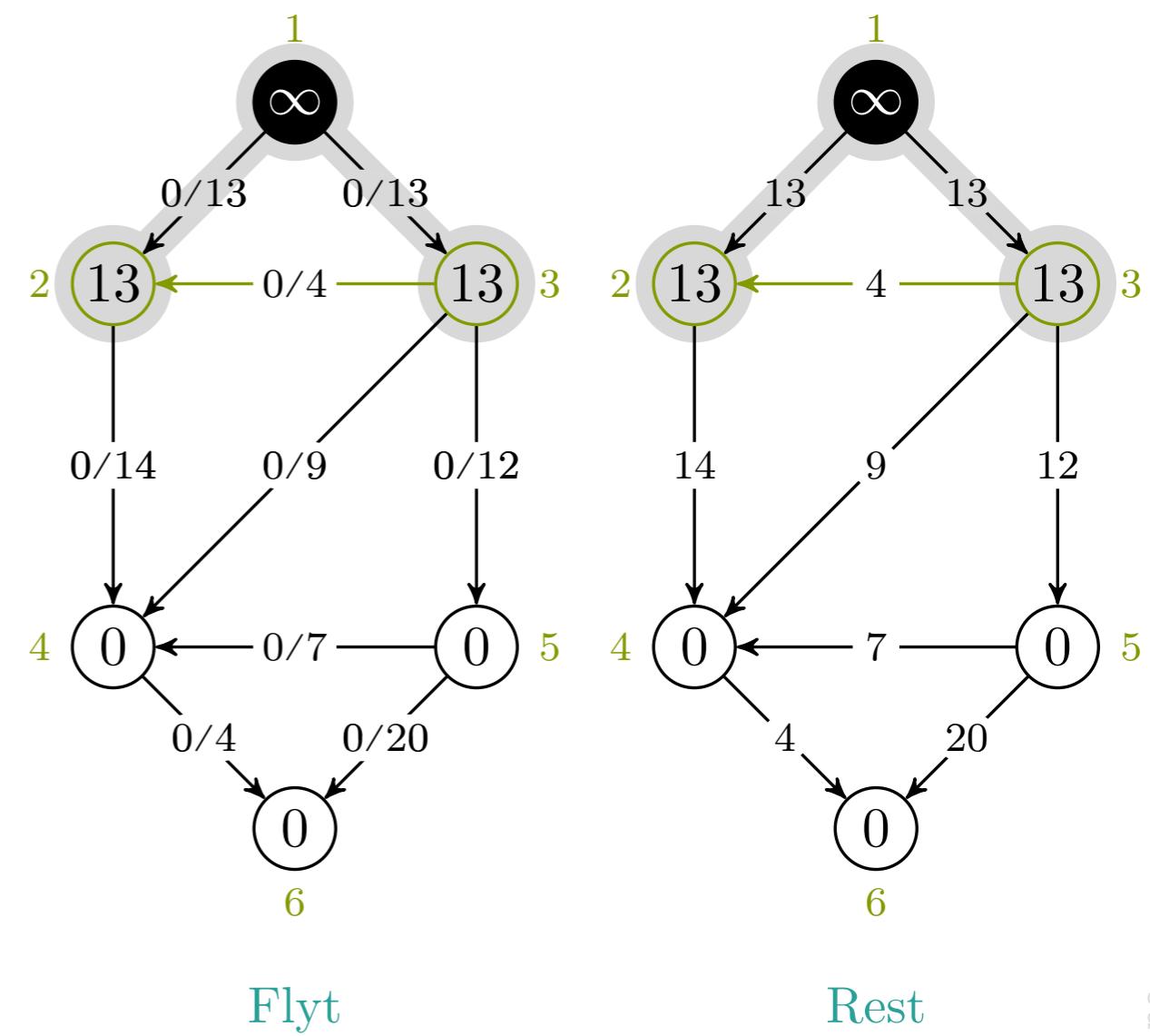


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7     $s.a = \infty$ 
8     $Q = \emptyset$ 
9    ENQUEUE( $Q, s$ )
10   while  $t.a == 0$  and  $Q \neq \emptyset$ 
11      $u = \text{DEQUEUE}(Q)$ 
12     for all edges  $(u, v), (v, u) \in G.E$ 
13       if  $(u, v) \in G.E$ 
14          $c_f(u, v) = c(u, v) - (u, v).f$ 
15       else  $c_f(u, v) = (v, u).f$ 
16       if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17          $v.a = \min(u.a, c_f(u, v))$ 
18          $v.\pi = u$ 
19         ENQUEUE( $Q, v$ )
20    $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21   while  $u \neq \text{NIL}$ 
22     if  $(u, v) \in G.E$ 
23        $(u, v).f = (u, v).f + t.a$ 
24     else  $(v, u).f = (v, u).f - t.a$ 
25    $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 2$

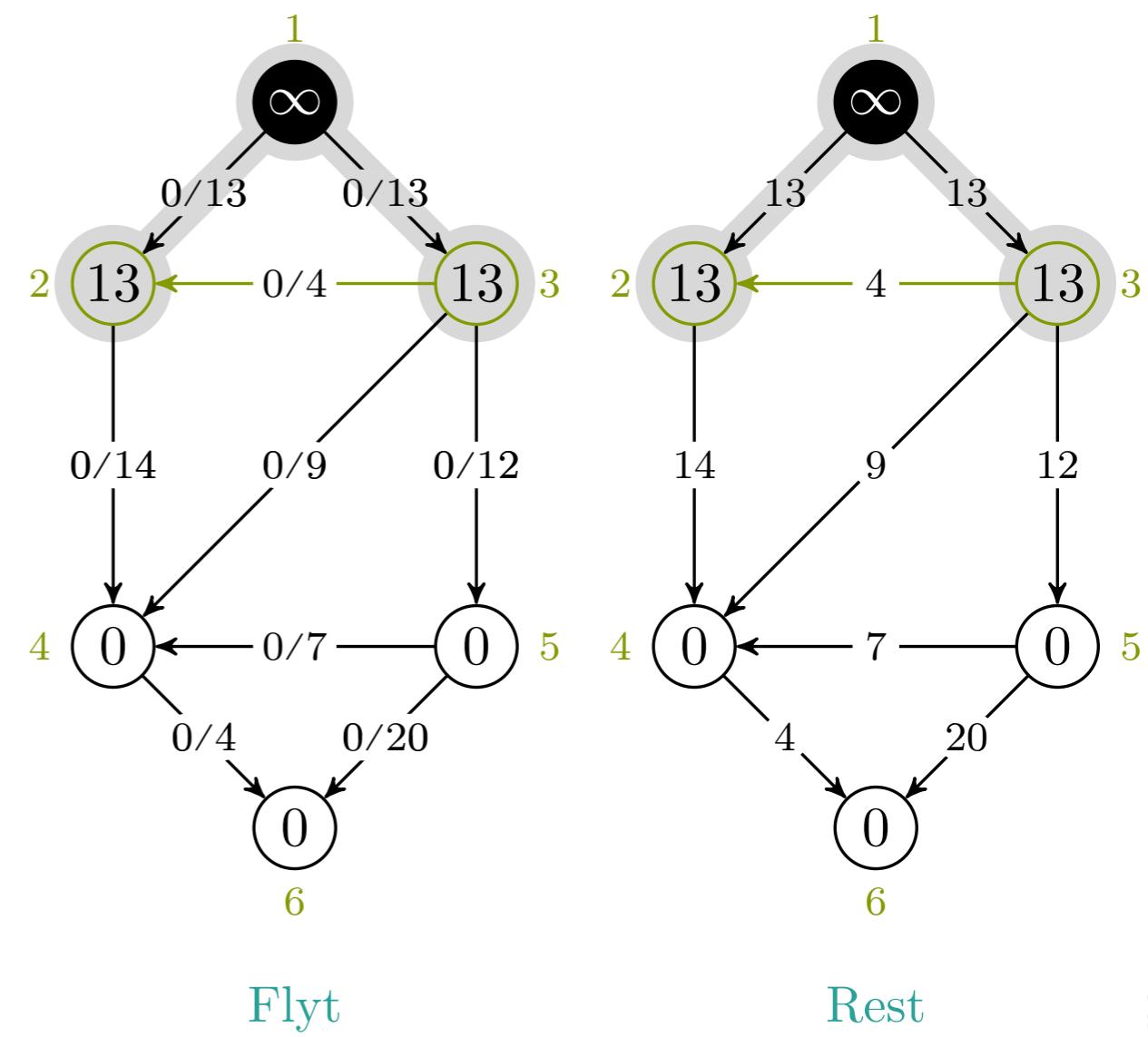


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 2$

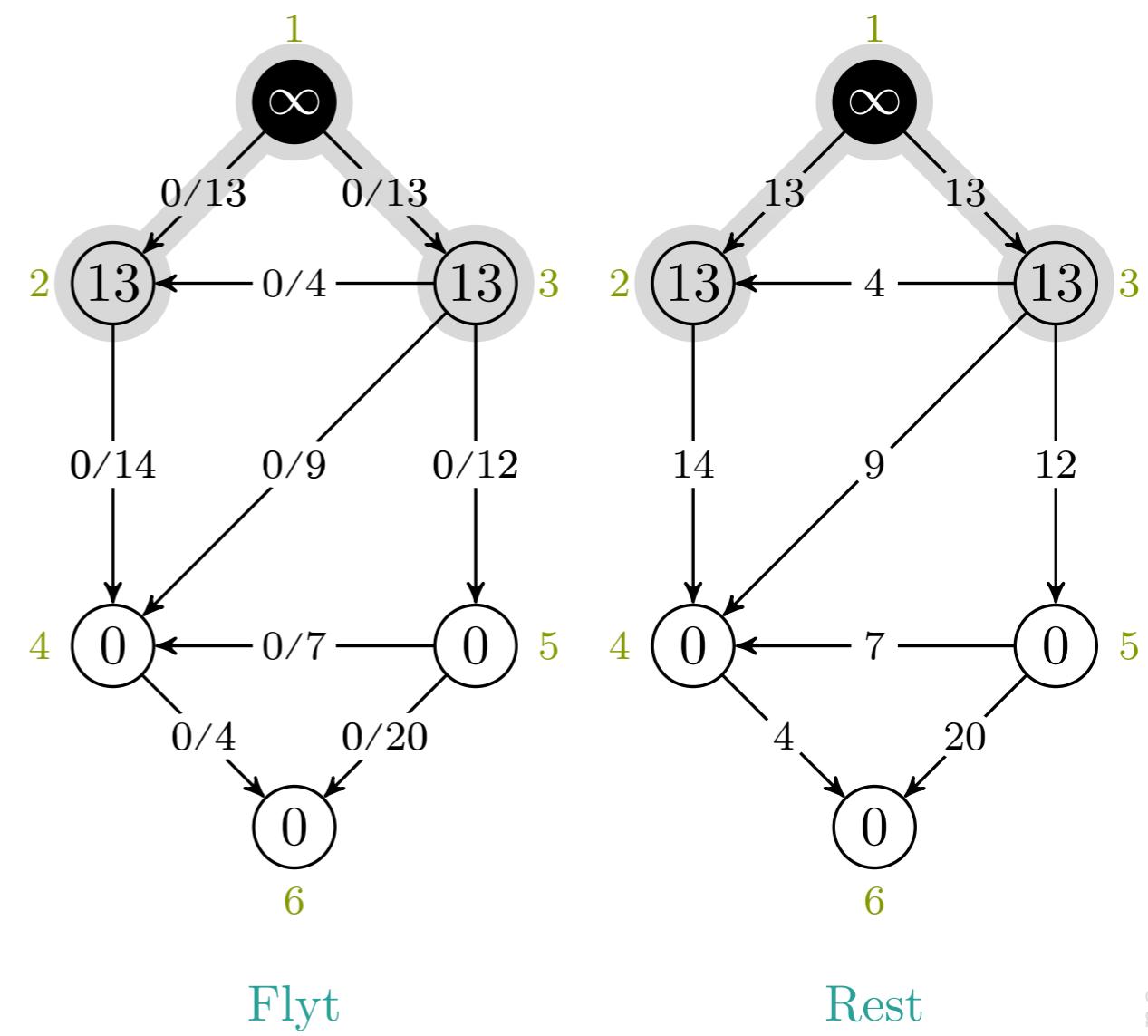


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, -$

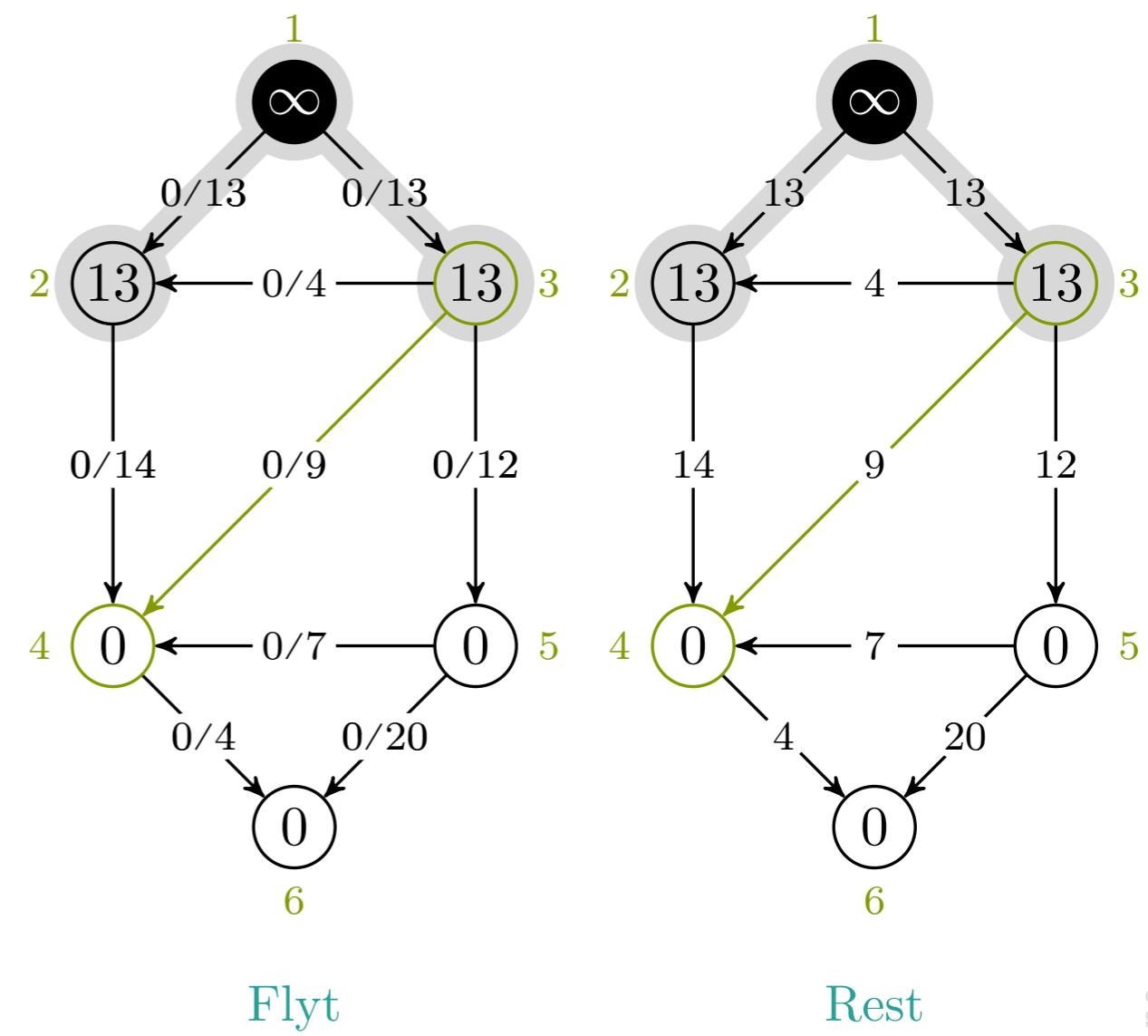


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 4$



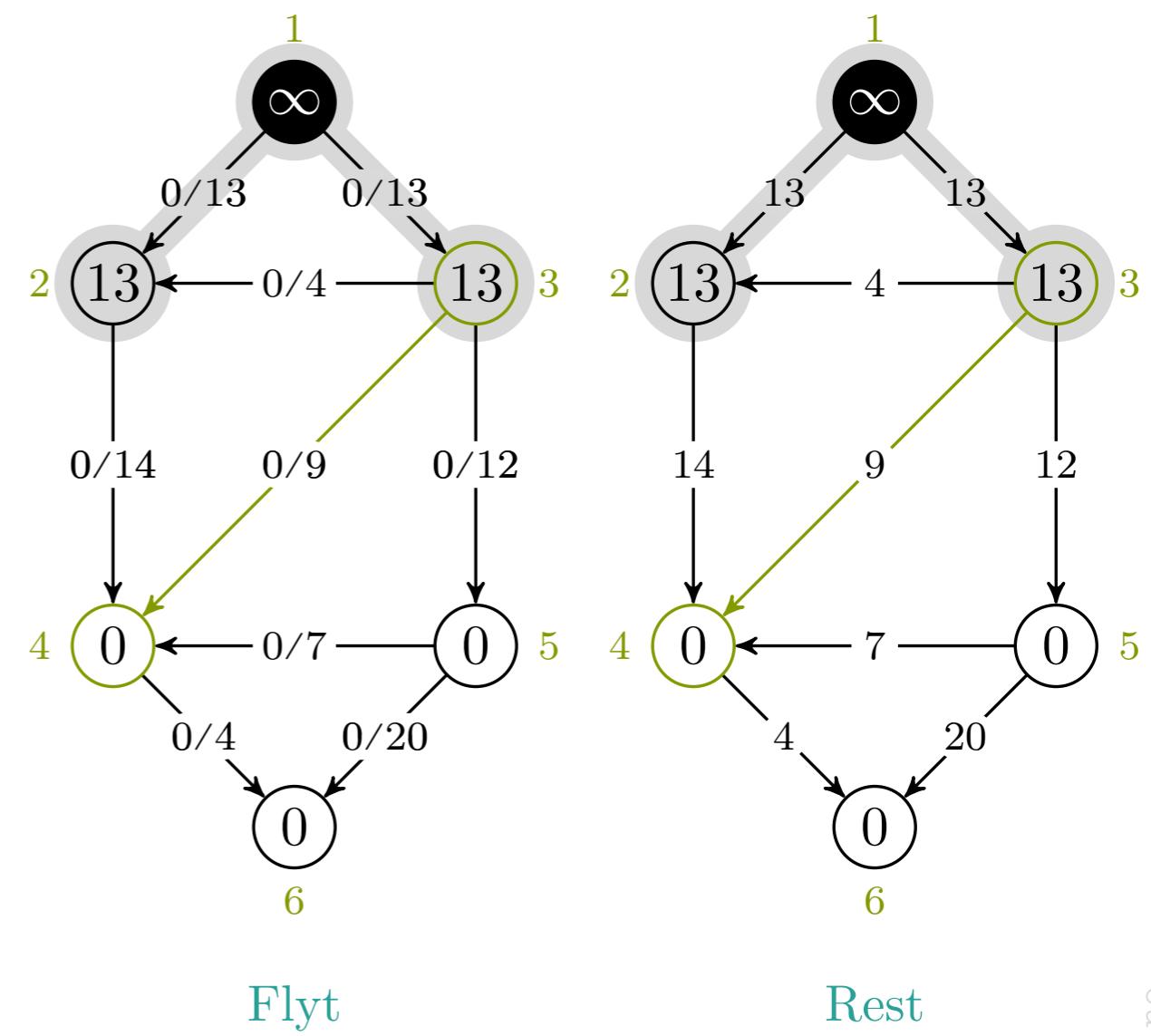
Rest

EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

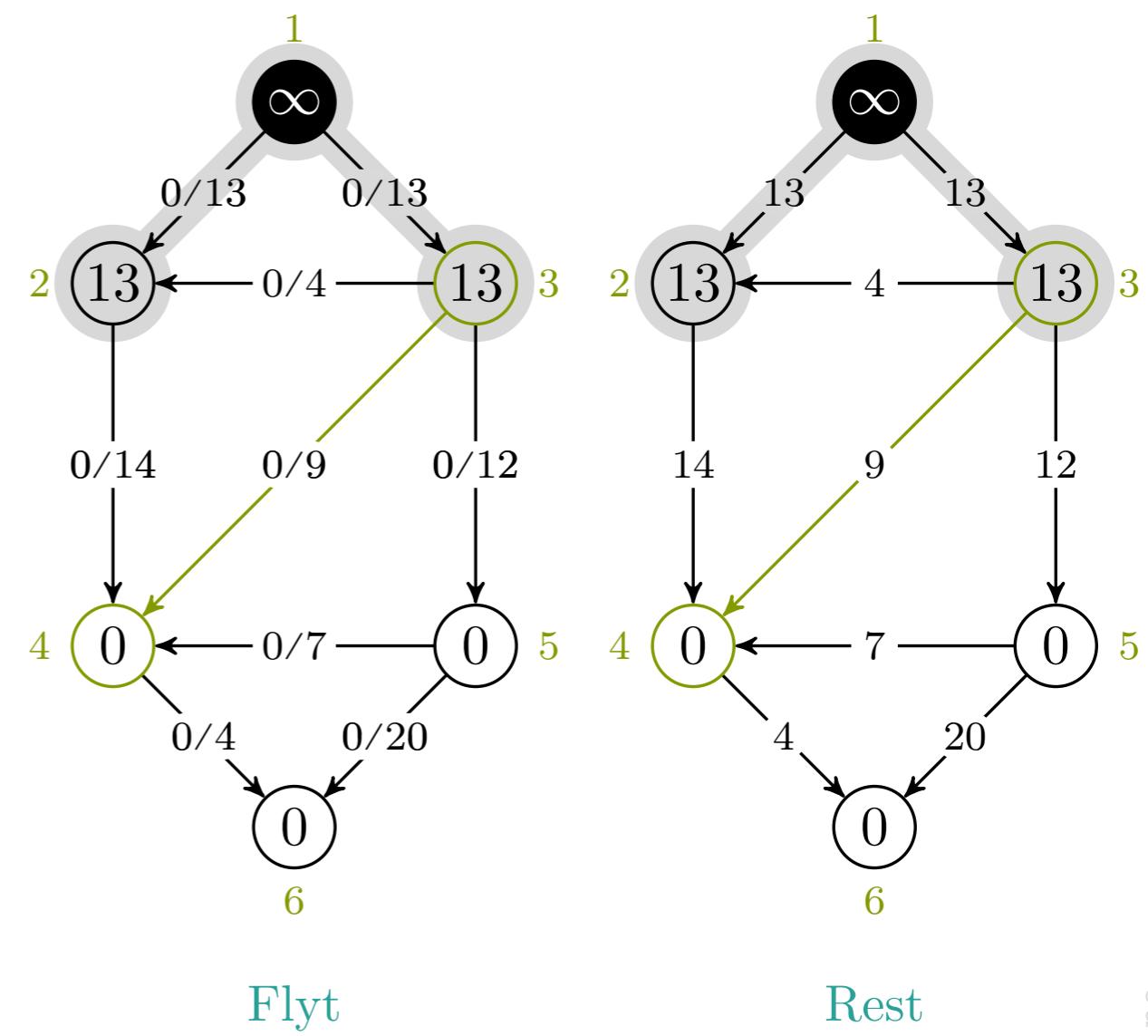
$u, v = 3, 4$



EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

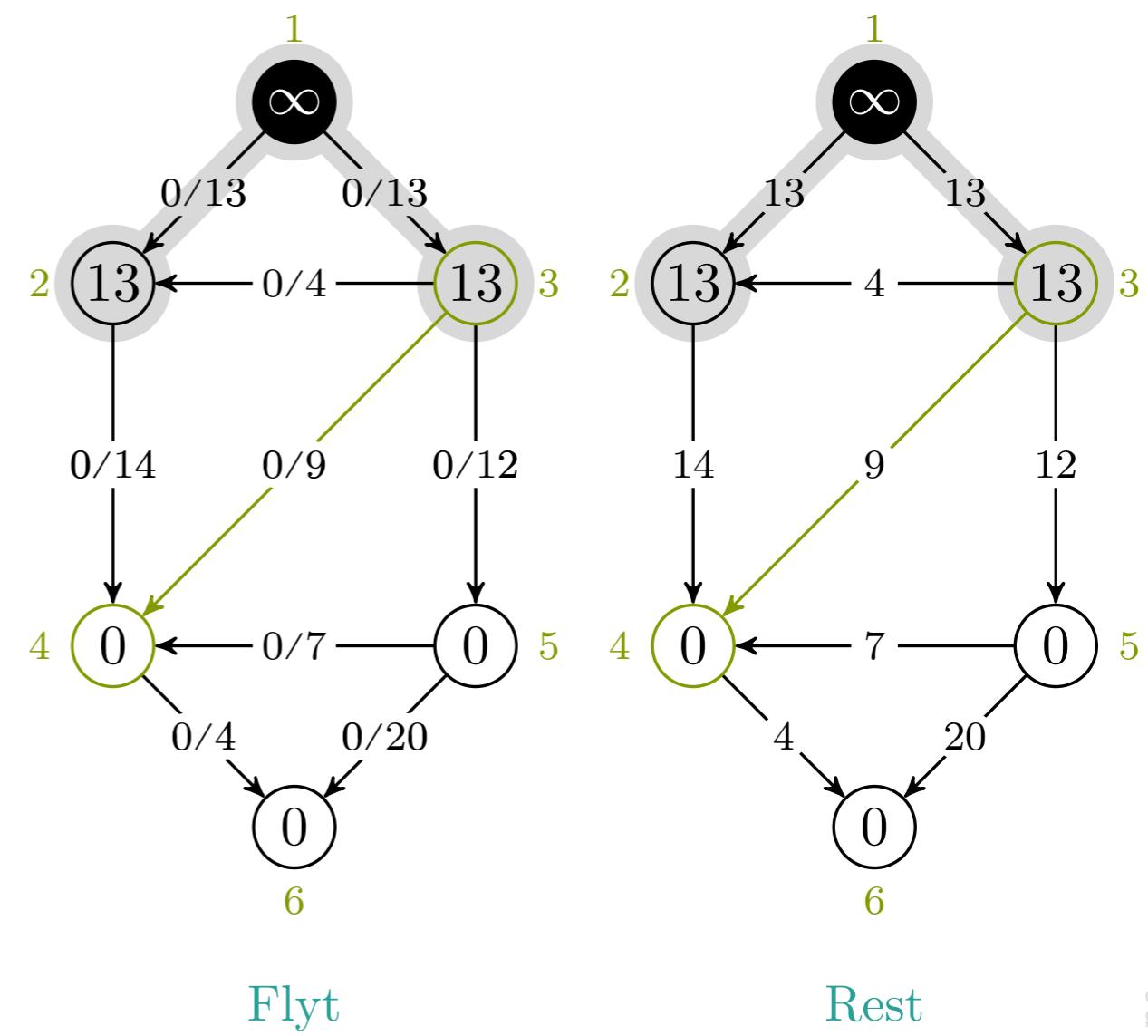
 $u, v = 3, 4$ 

EDMONDS-KARP( $G, s, t$ )

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25          $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 4$

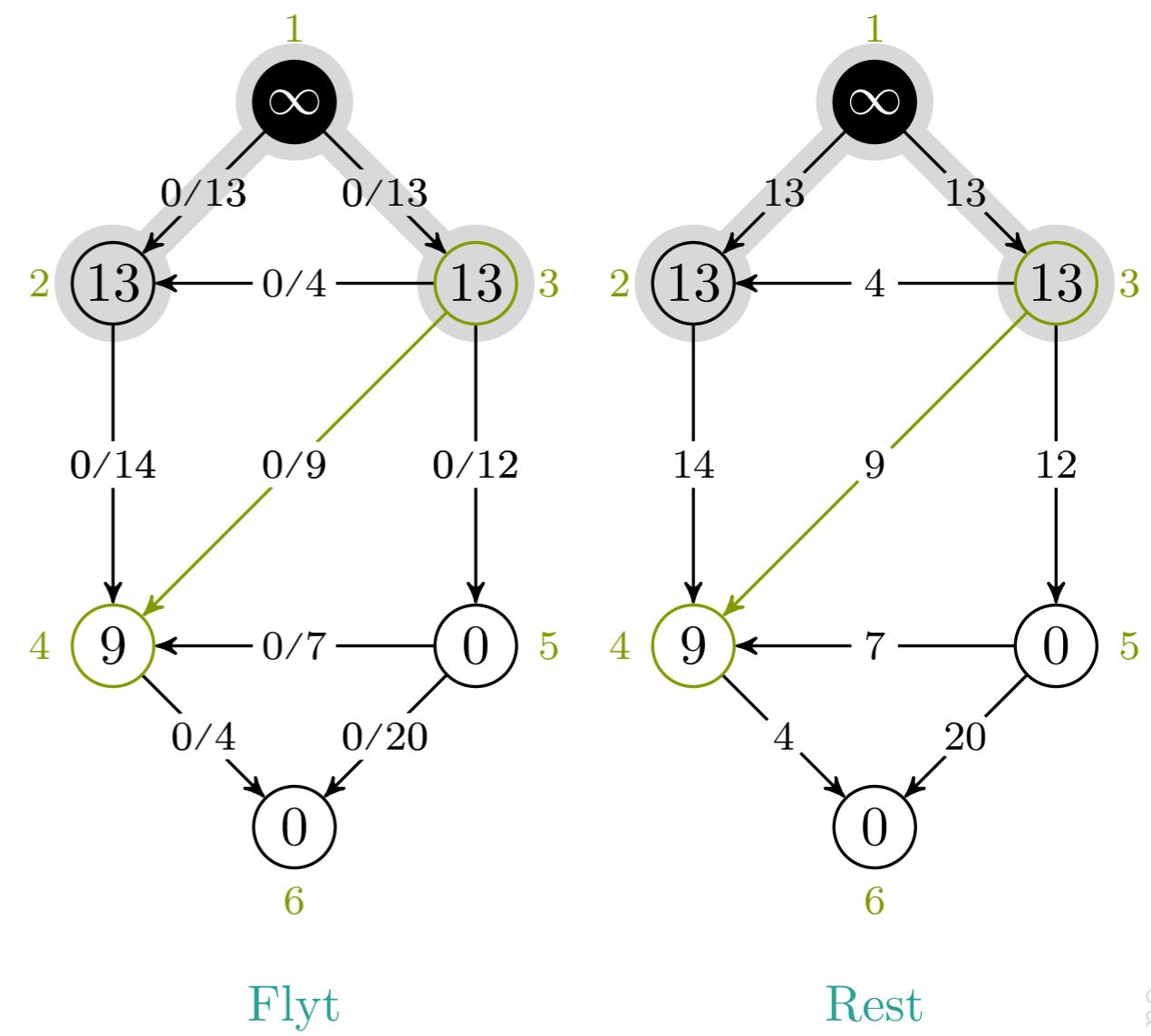


EDMONDS-KARP( $G, s, t$ )

```

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24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

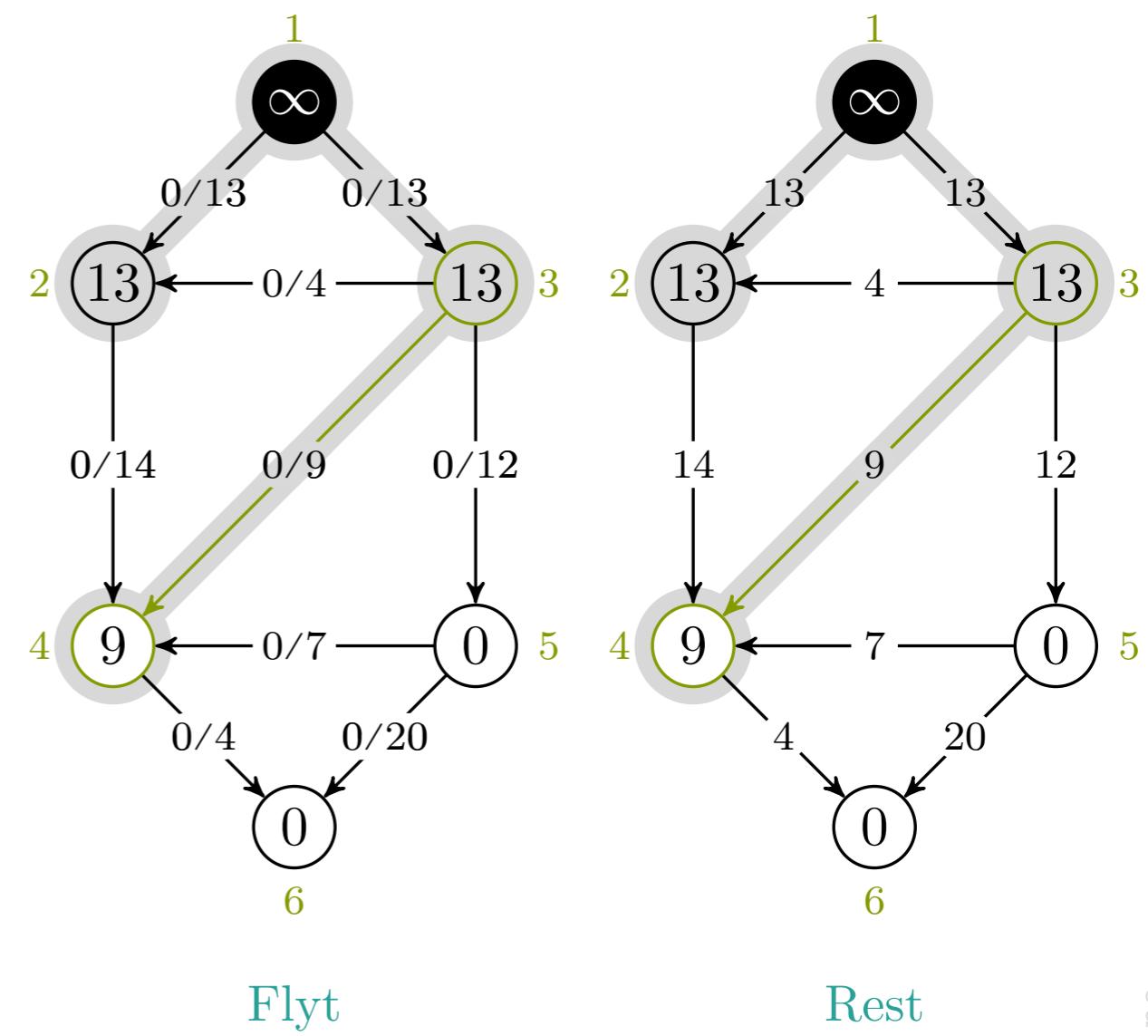
$u, v = 3, 4$



EDMONDS-KARP( $G, s, t$ )

```

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25          $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = 3, 4$ 

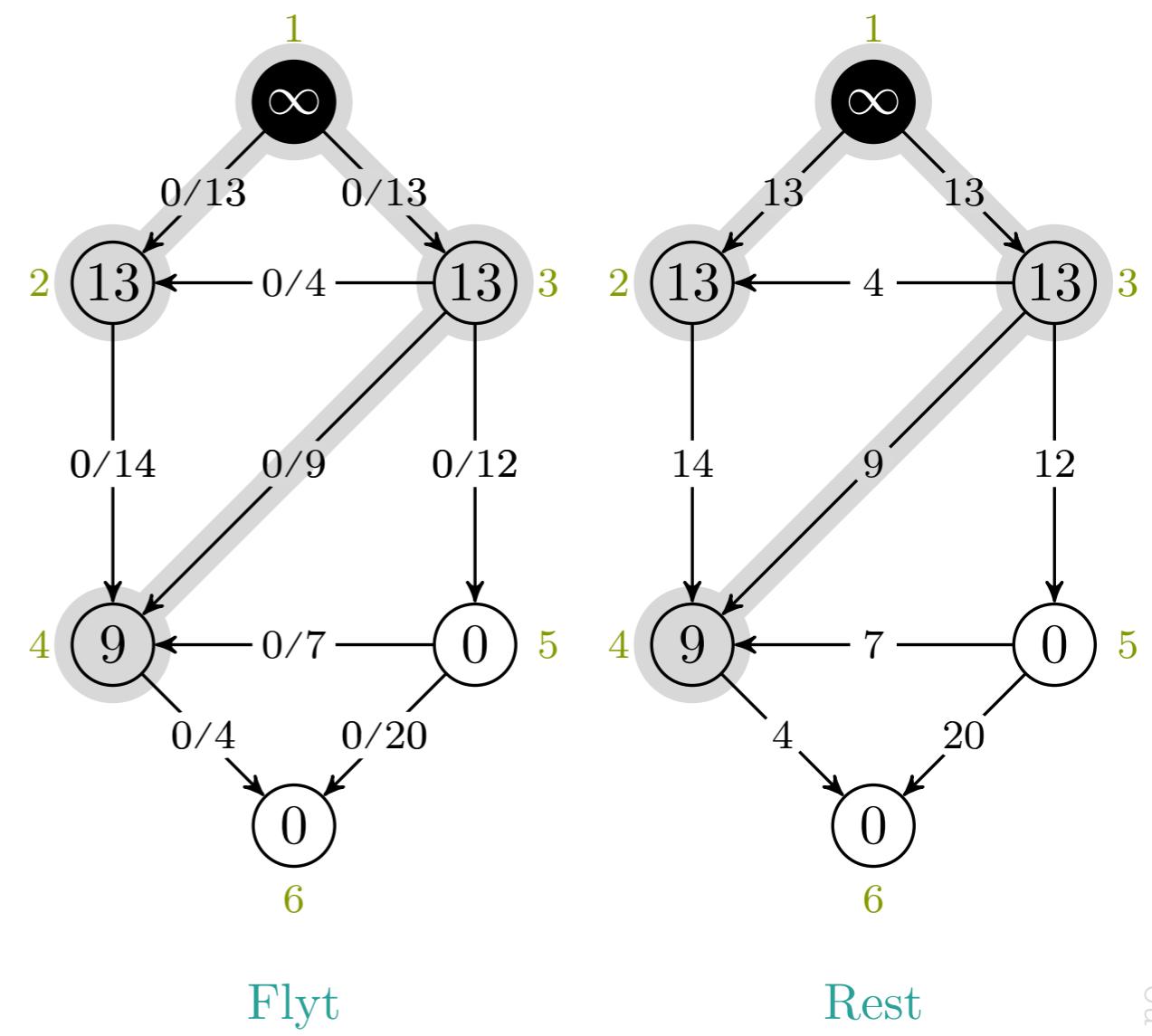
maks-flyt → edmonds-karp

EDMONDS-KARP( $G, s, t$ )

```

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23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, -$

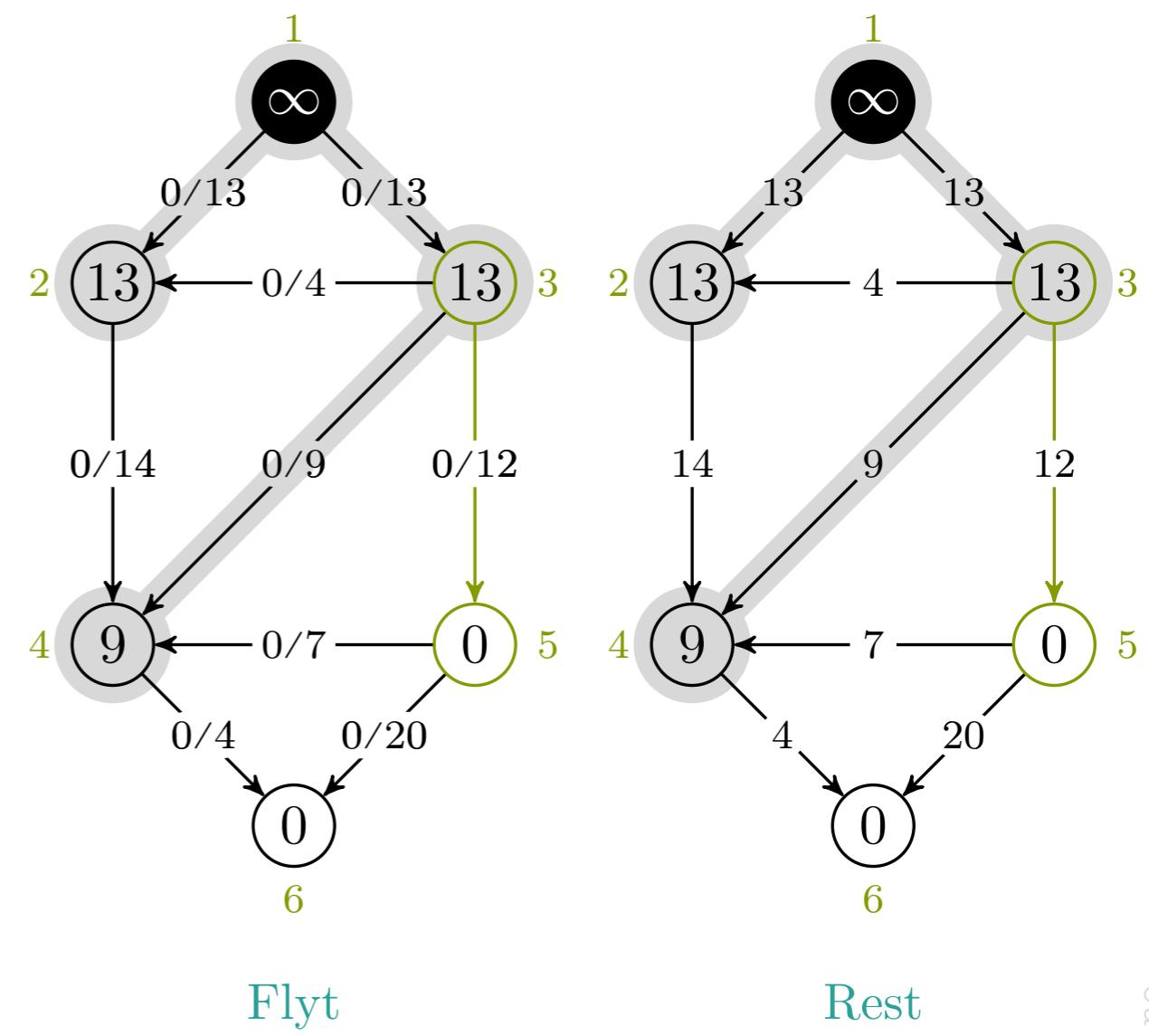


EDMONDS-KARP( $G, s, t$ )

```

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25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 5$

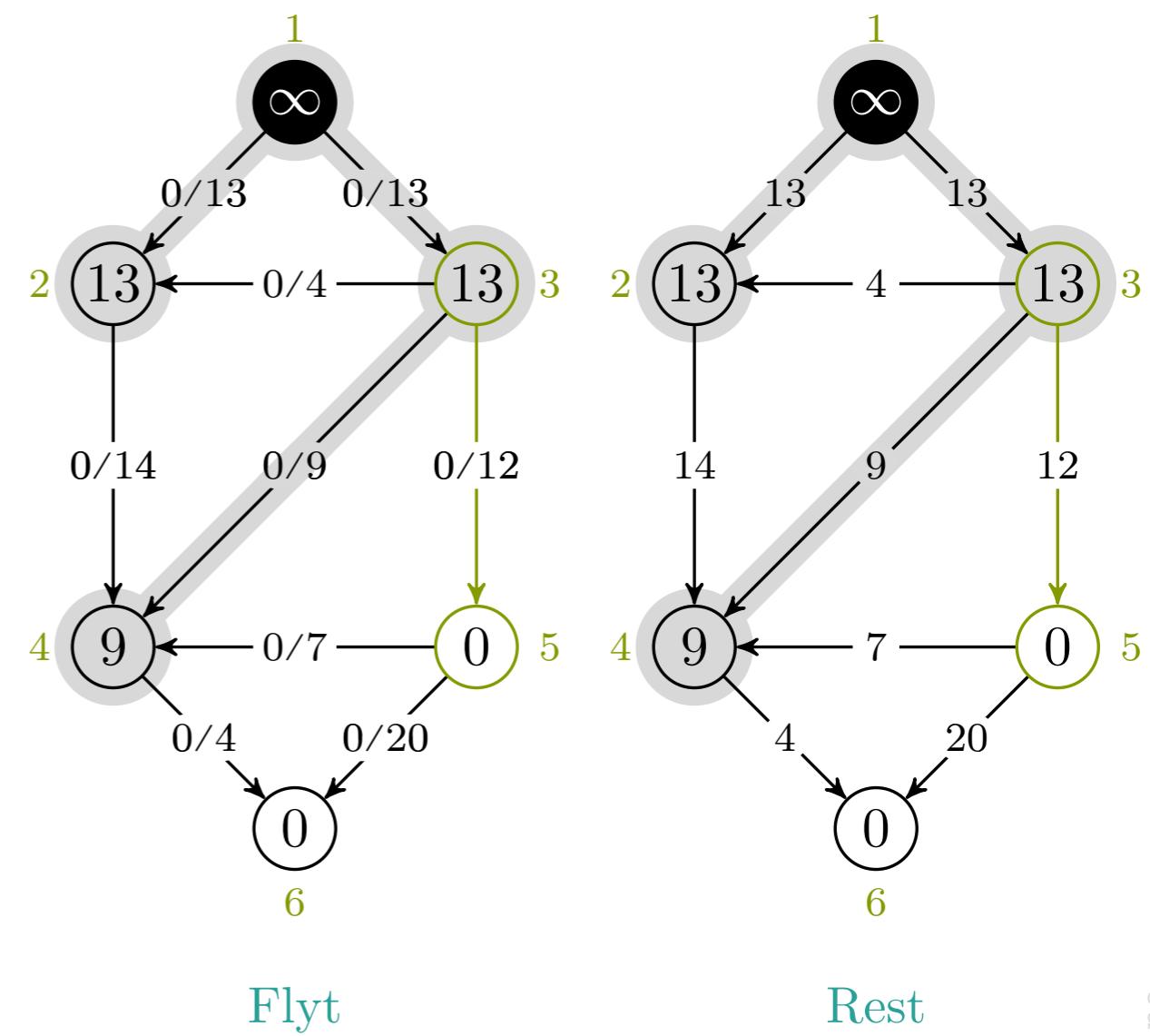


EDMONDS-KARP( $G, s, t$ )

```

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18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

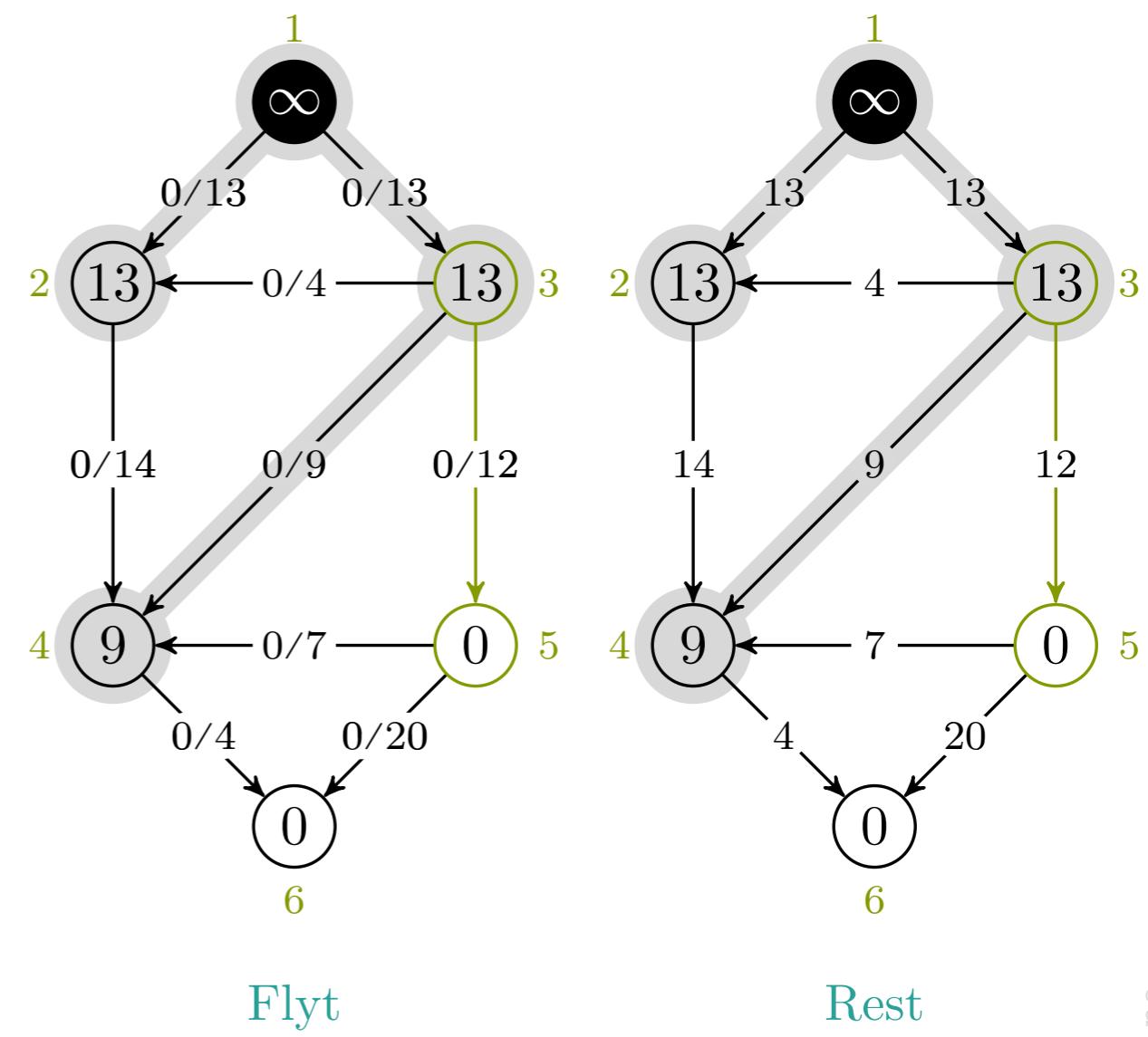
$u, v = 3, 5$



EDMONDS-KARP( $G, s, t$ )

```

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```

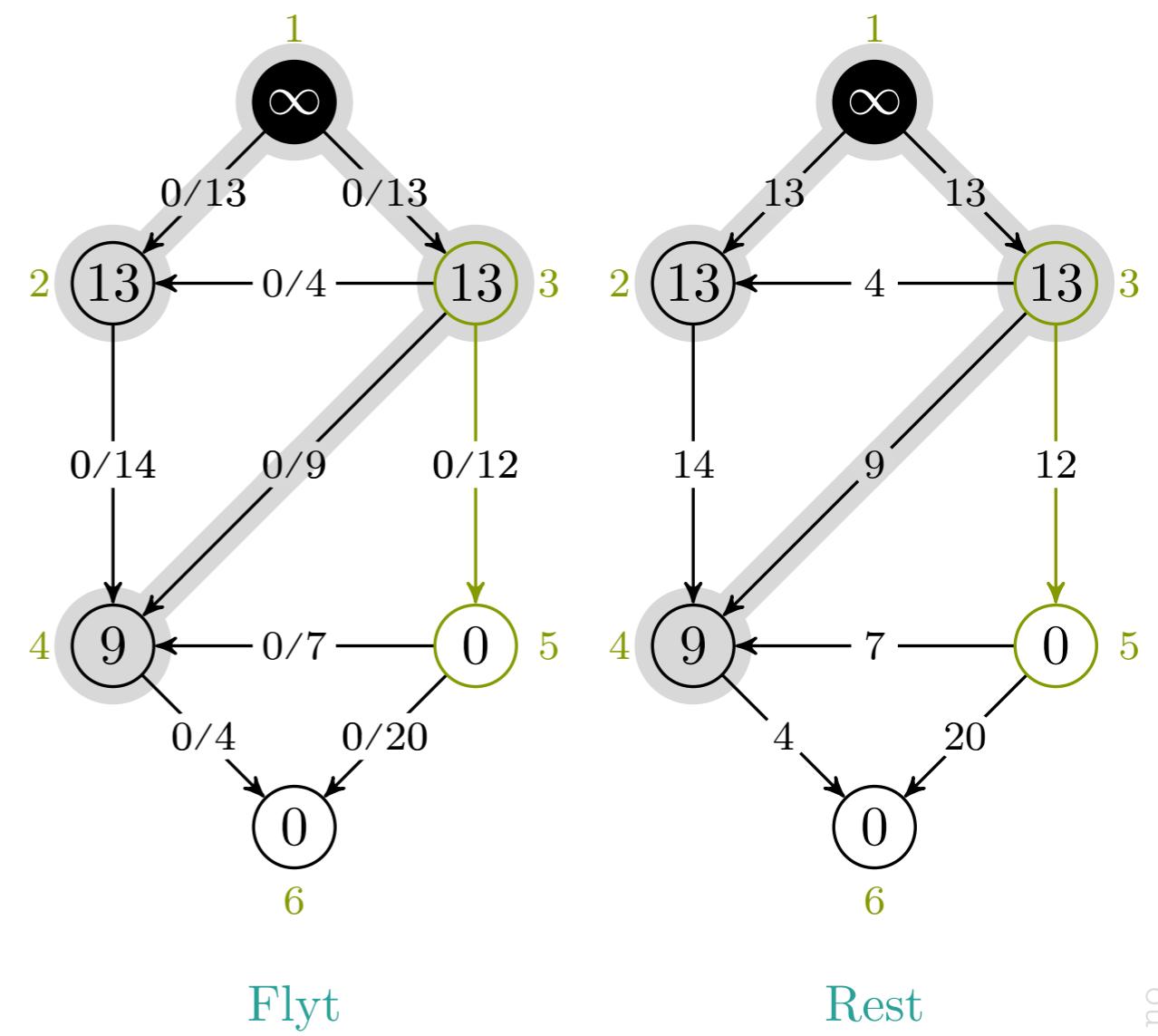
 $u, v = 3, 5$ 

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```

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```

$u, v = 3, 5$

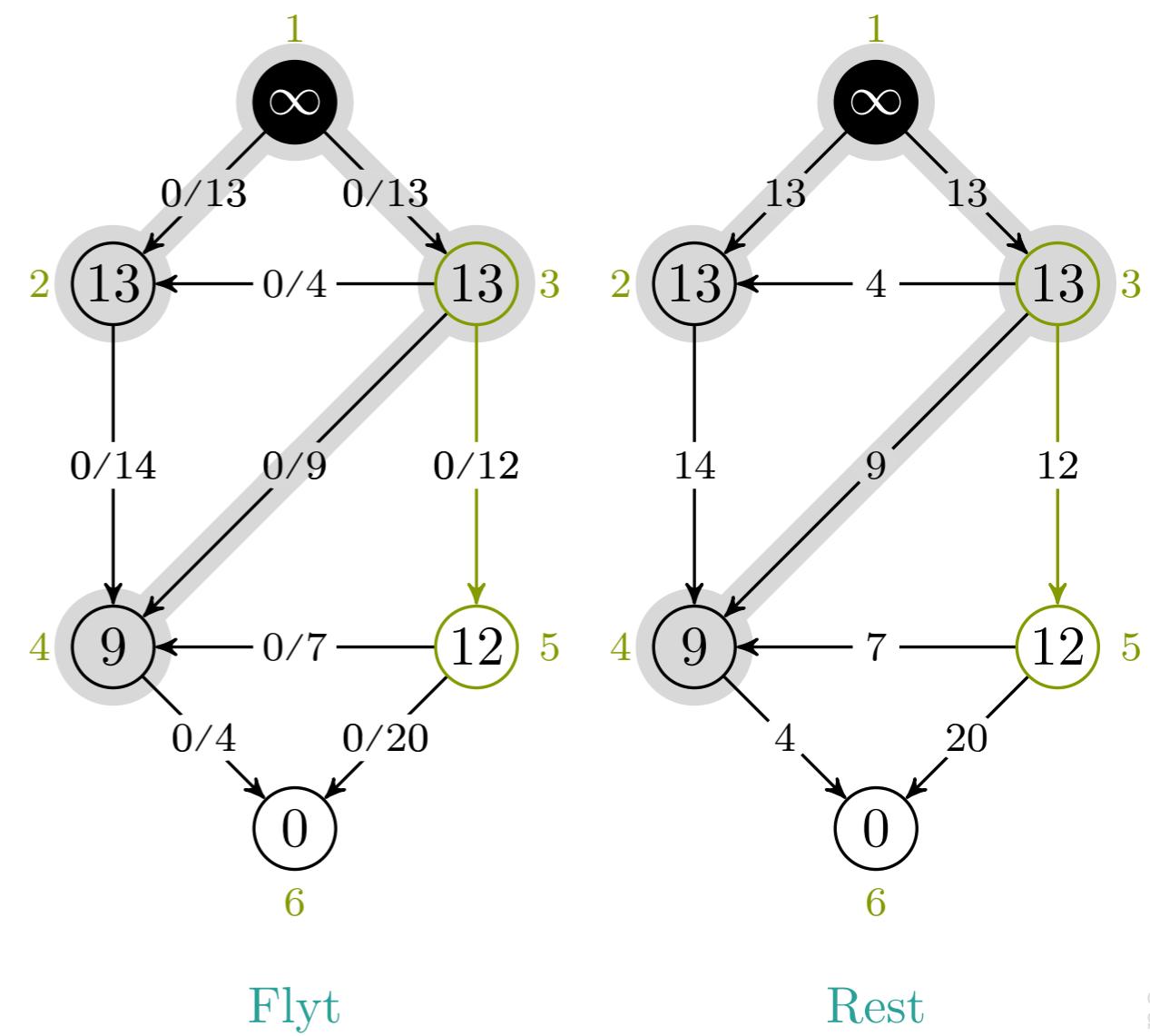


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26 until  $t.a == 0$ 
```

$u, v = 3, 5$



maks-flyt → edmonds-karp

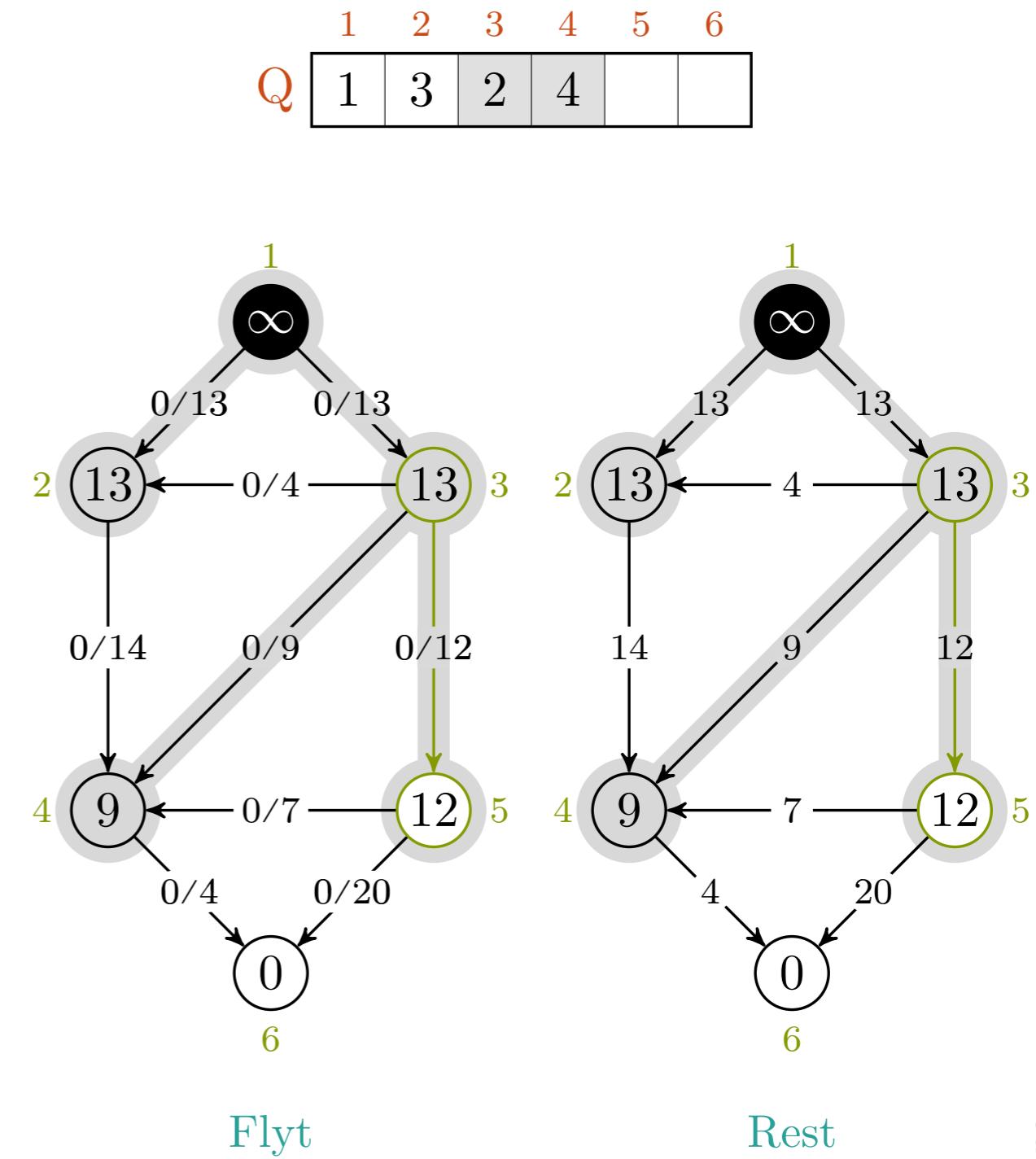
## EDMONDS-KARP(G, s, t)

```

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23                      $(u, v).f = (u, v).f + t.a$ 
24                 else  $(v, u).f = (v, u).f - t.a$ 
25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

$$u, v = 3, 5$$



maks-flyt → edmonds-karp

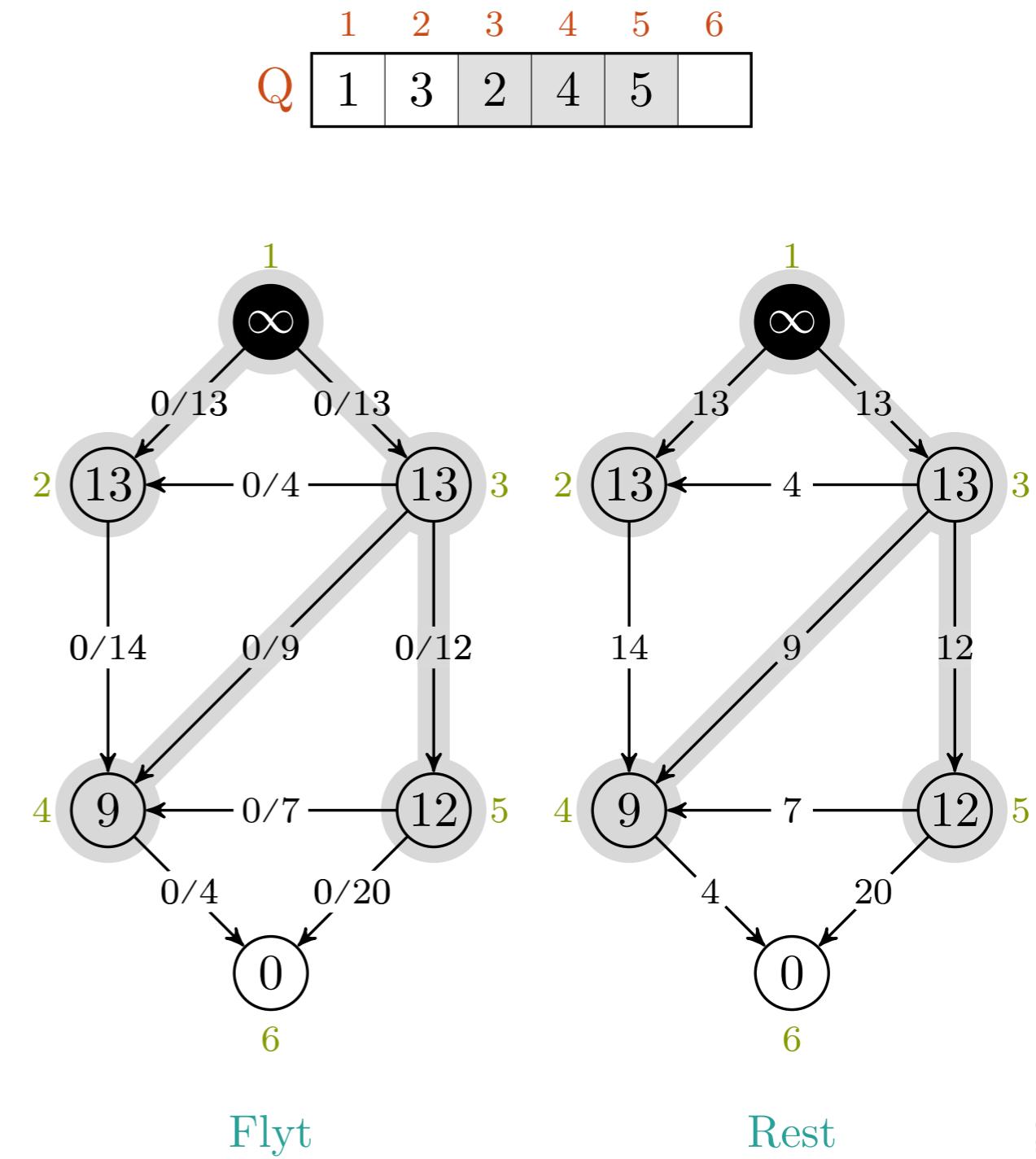
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```

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24              else  $(v, u).f = (v, u).f - t.a$ 
25               $u, v = u.\pi, u$ 
26          until  $t.a == 0$ 

```

$$u, v = 3, -$$

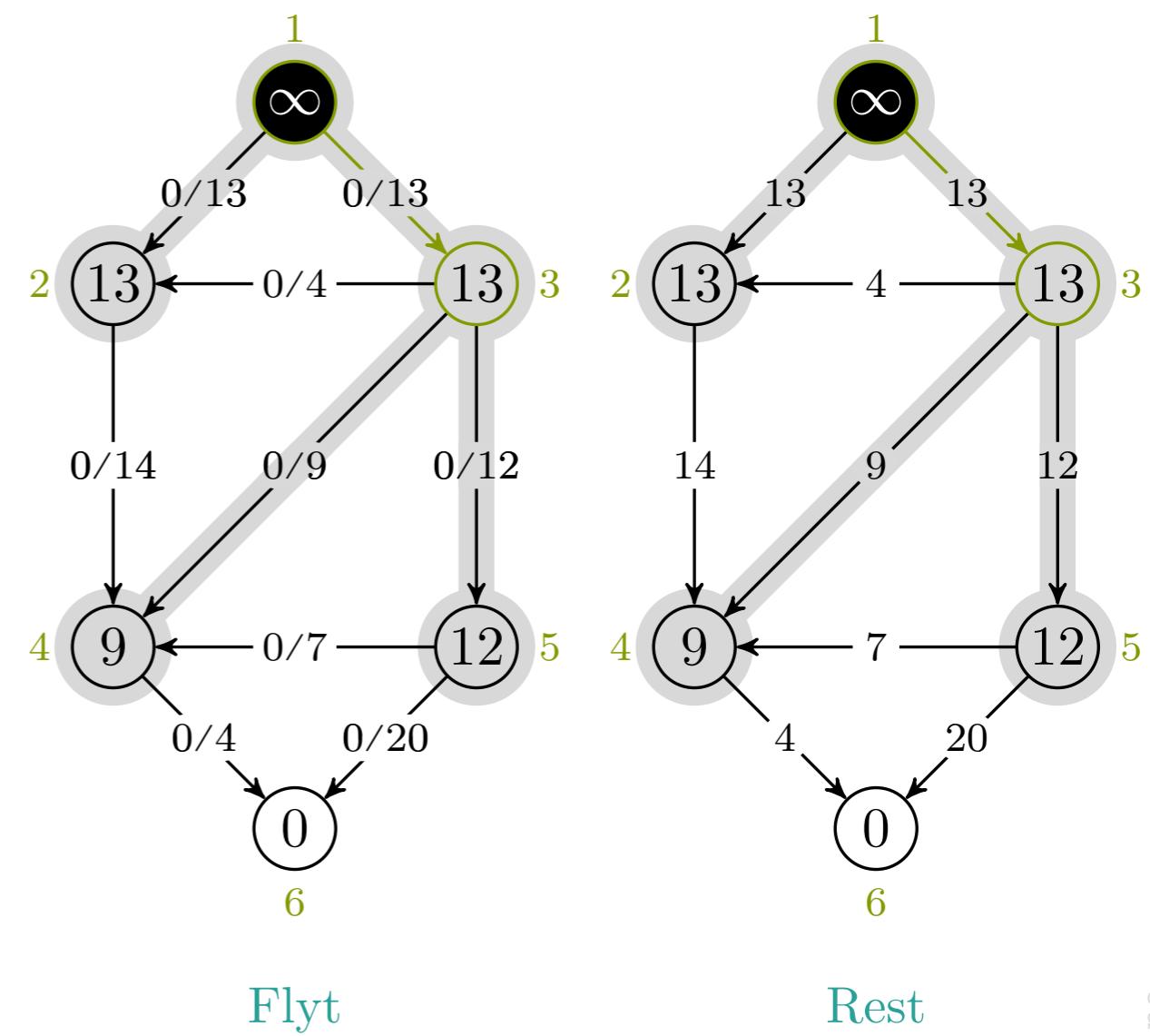


EDMONDS-KARP( $G, s, t$ )

```

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12       for all edges  $(u, v), (v, u) \in G.E$ 
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21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 1$

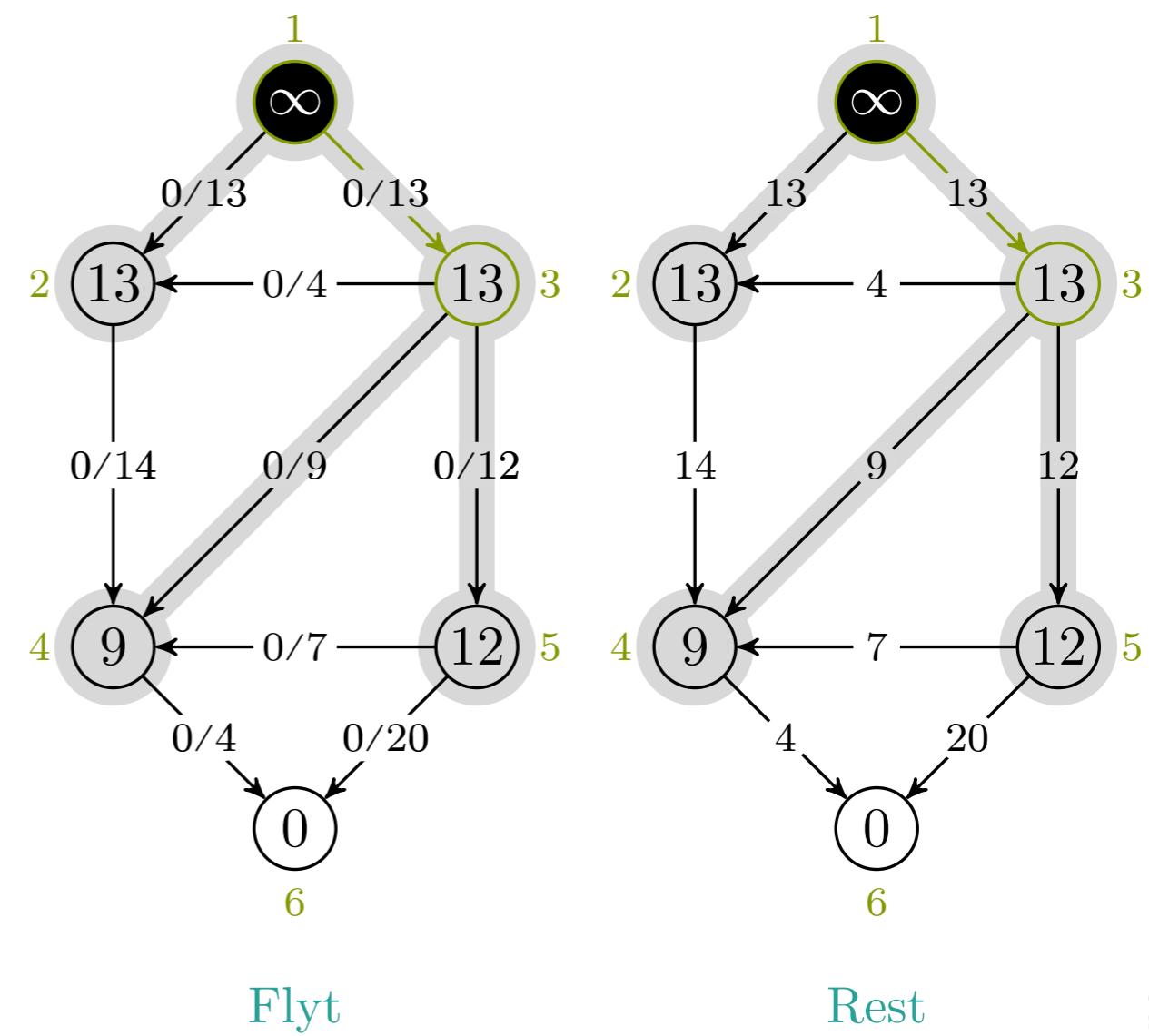


EDMONDS-KARP( $G, s, t$ )

```

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13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 1$

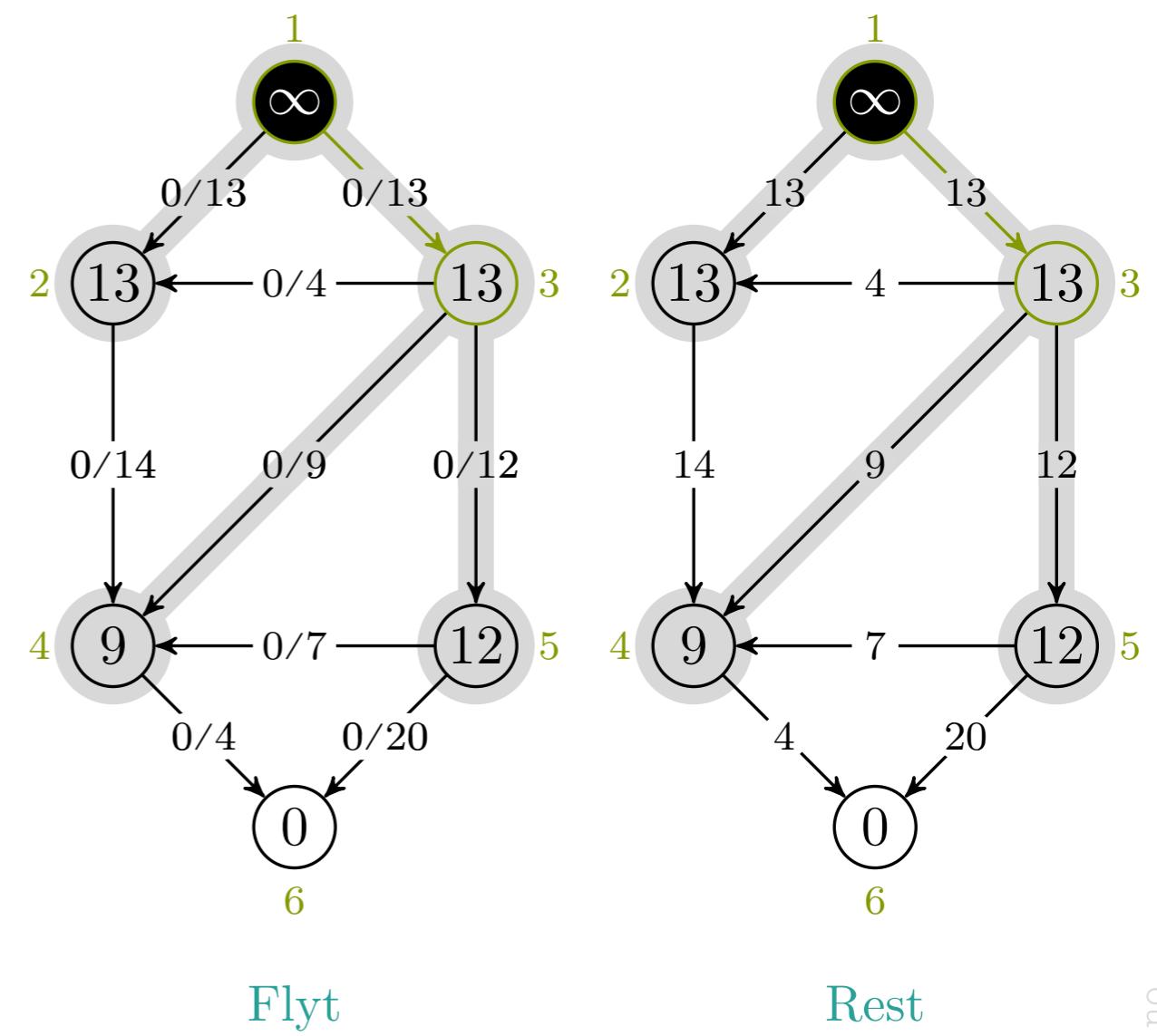


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 1$

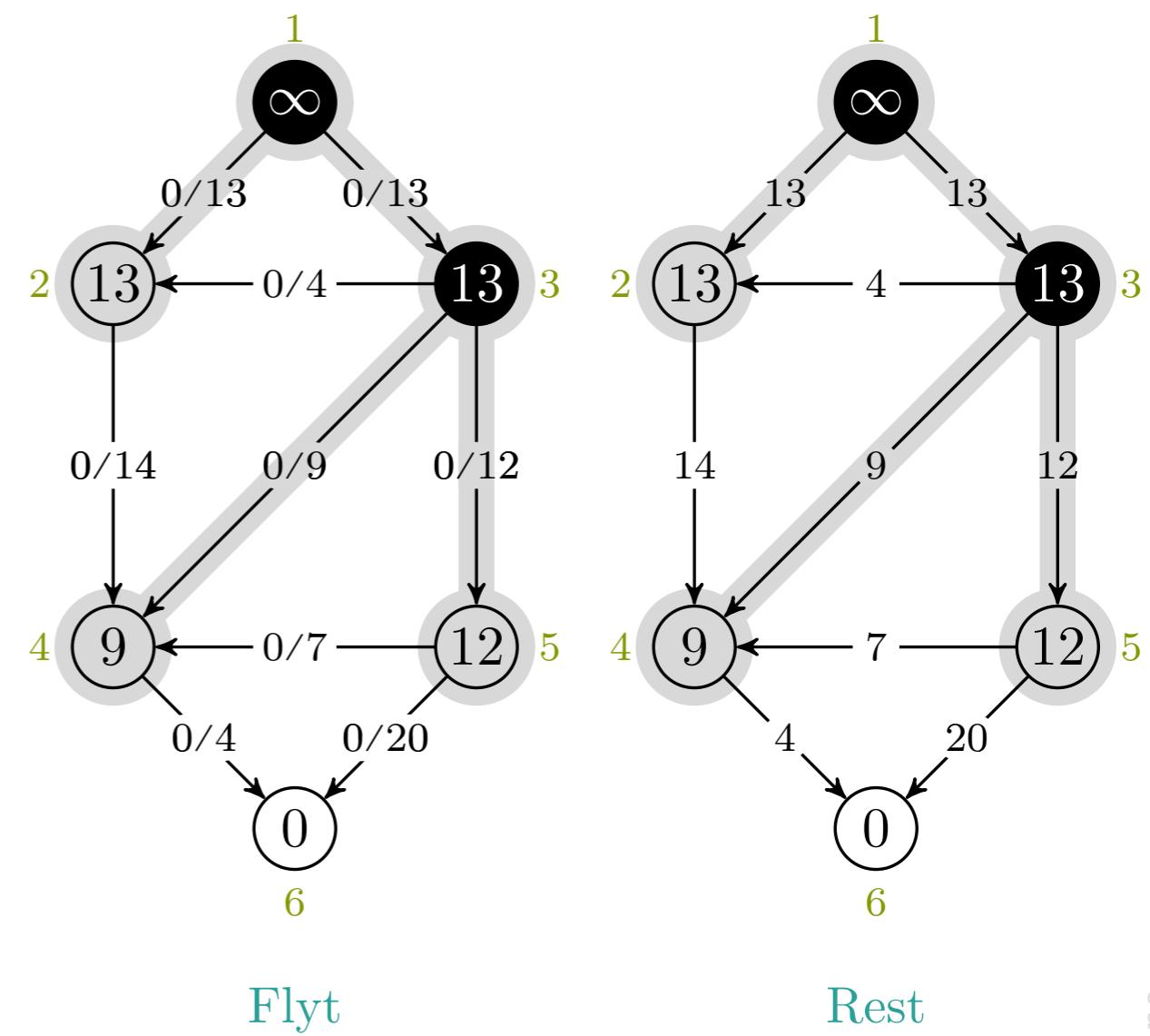


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
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10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
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12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
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22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, -$

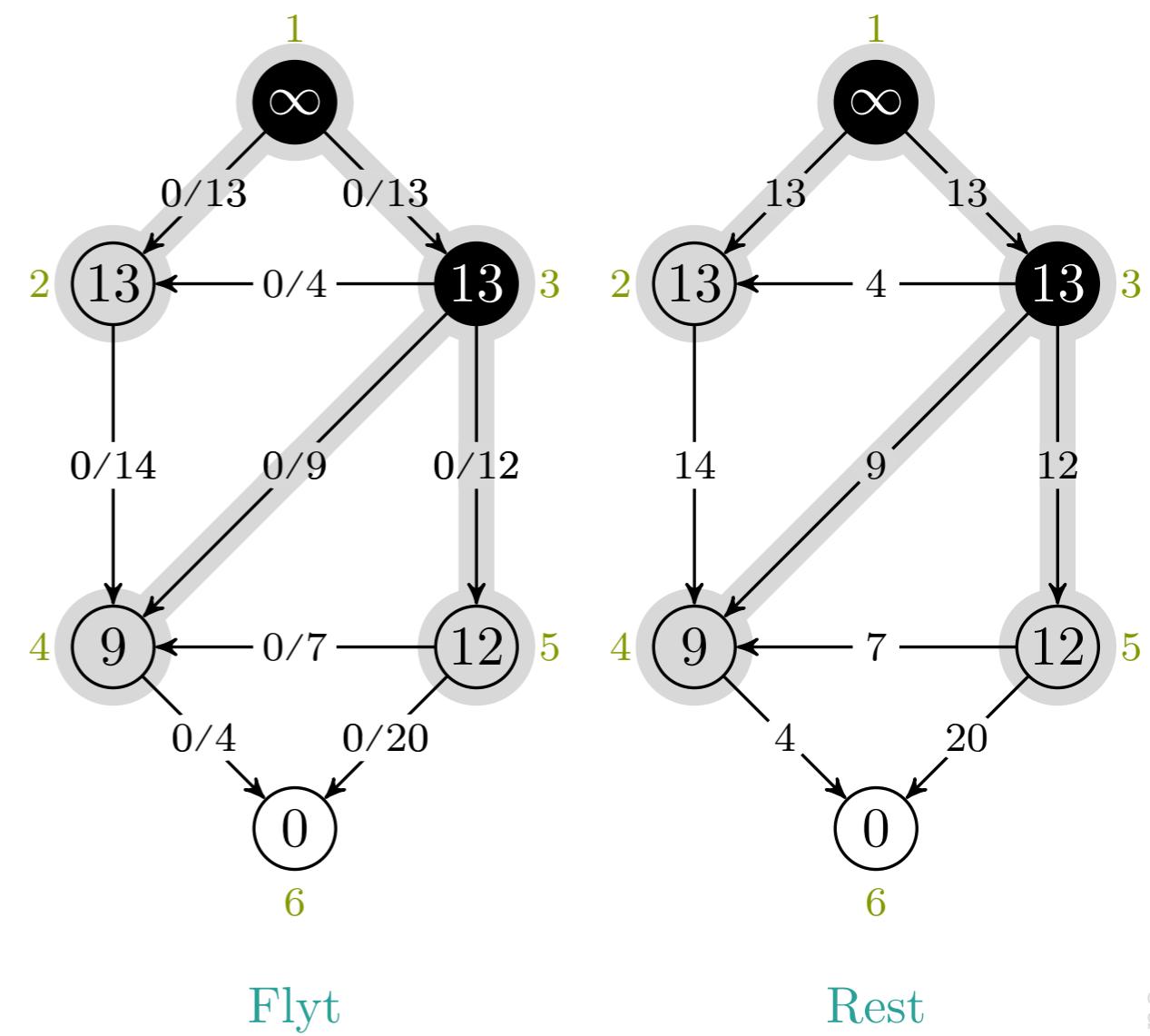


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
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3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, -$

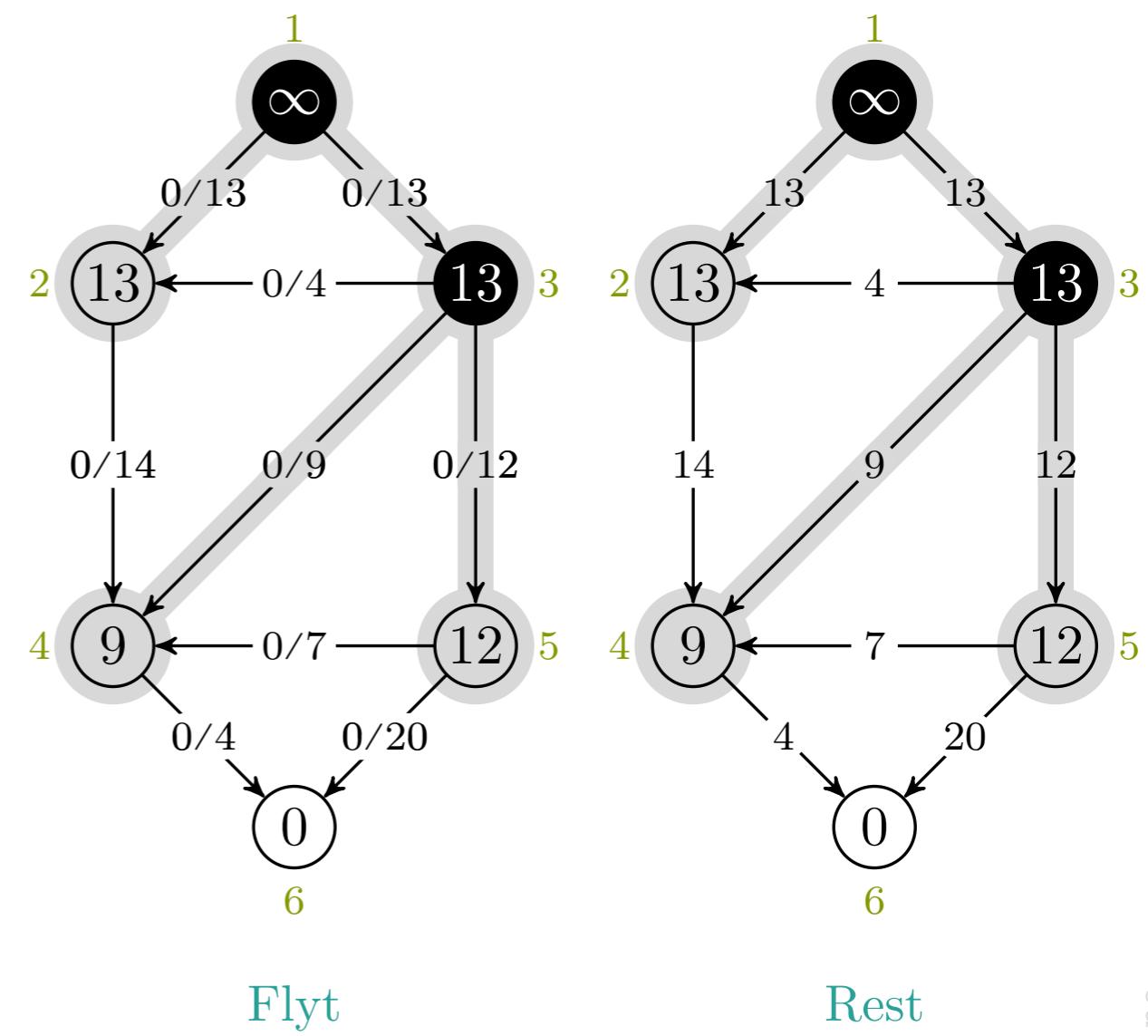


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

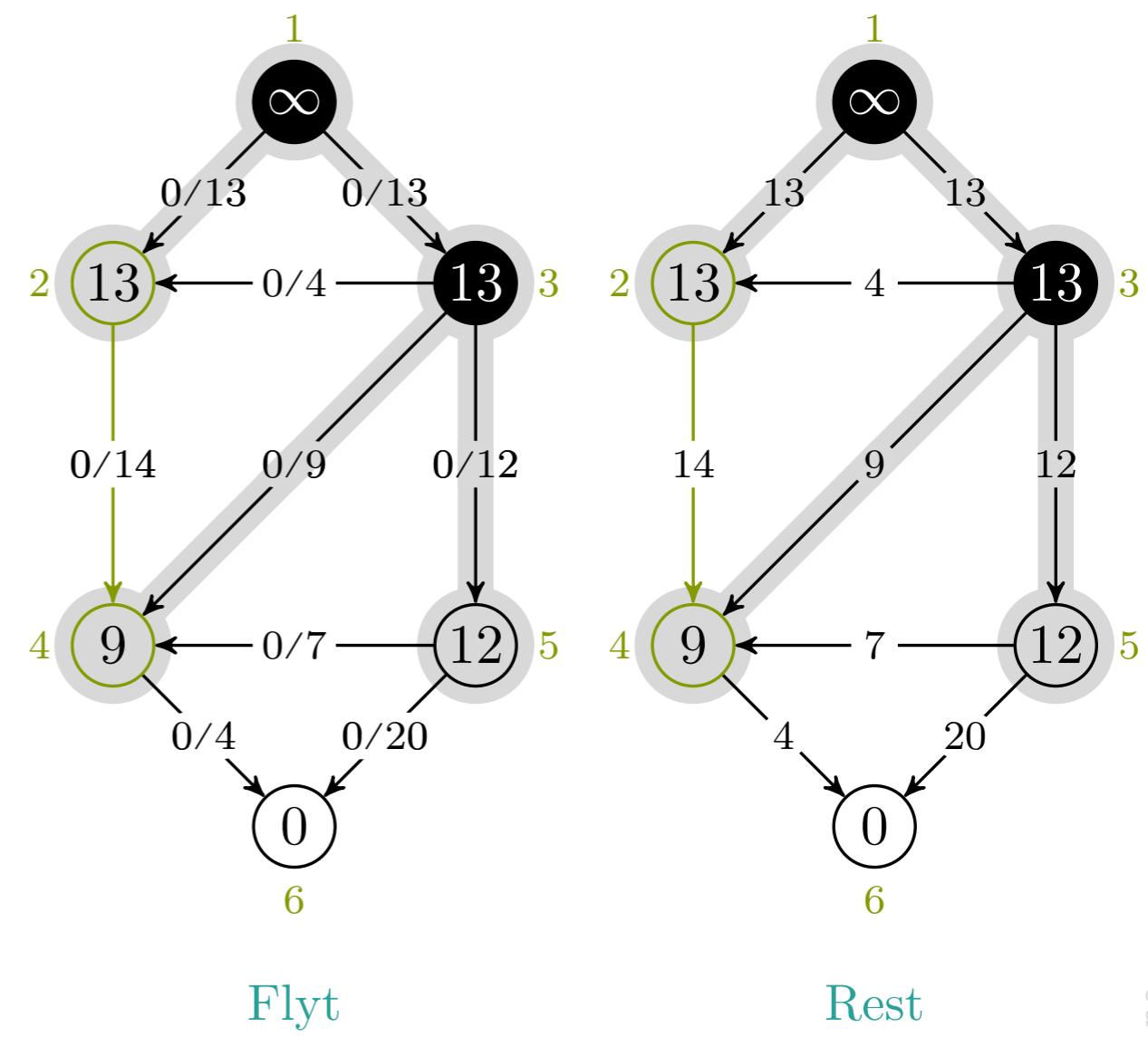
$u, v = 2, -$



EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

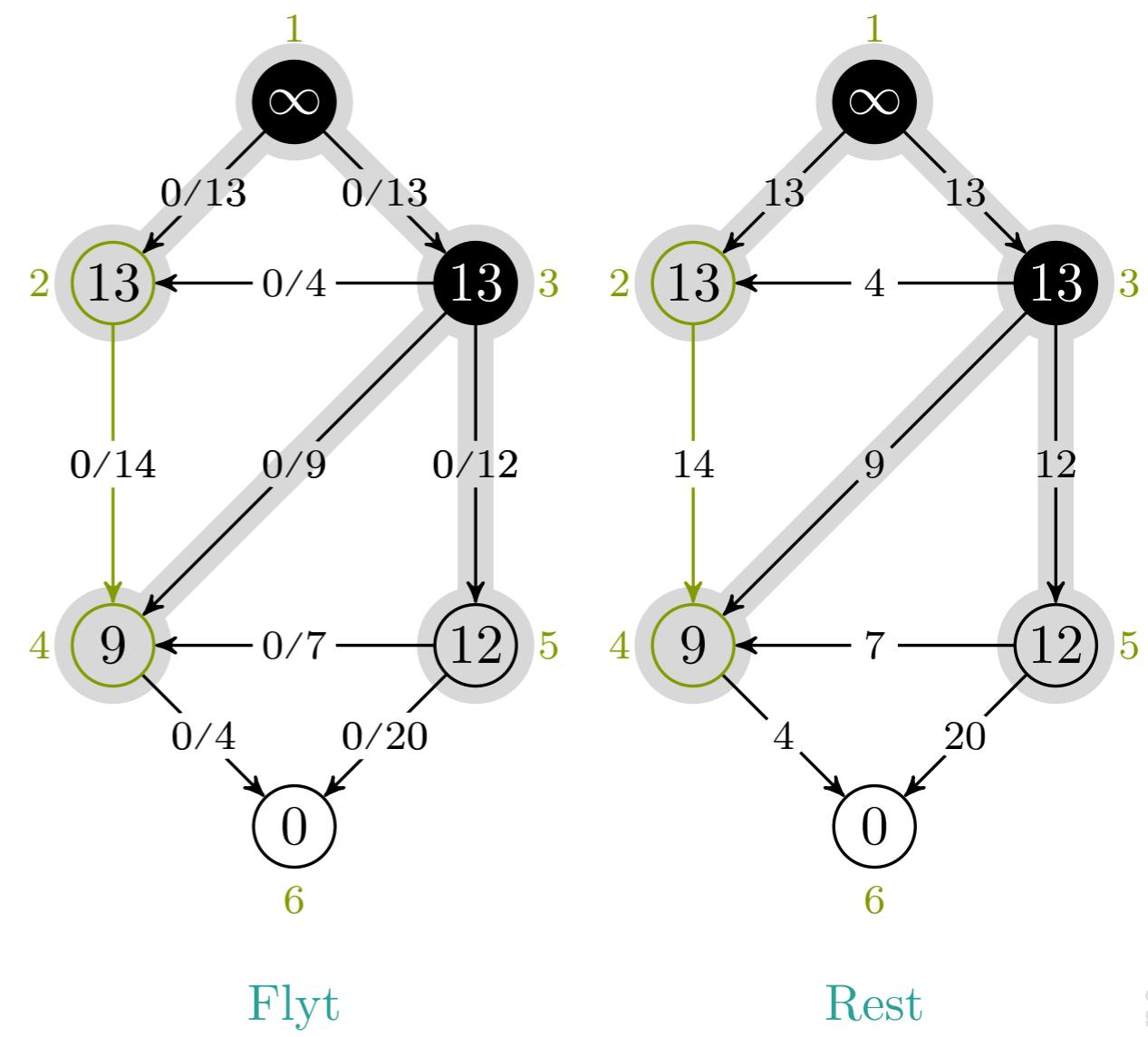
 $u, v = 2, 4$ 

EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, 4$



maks-flyt → edmonds-karp

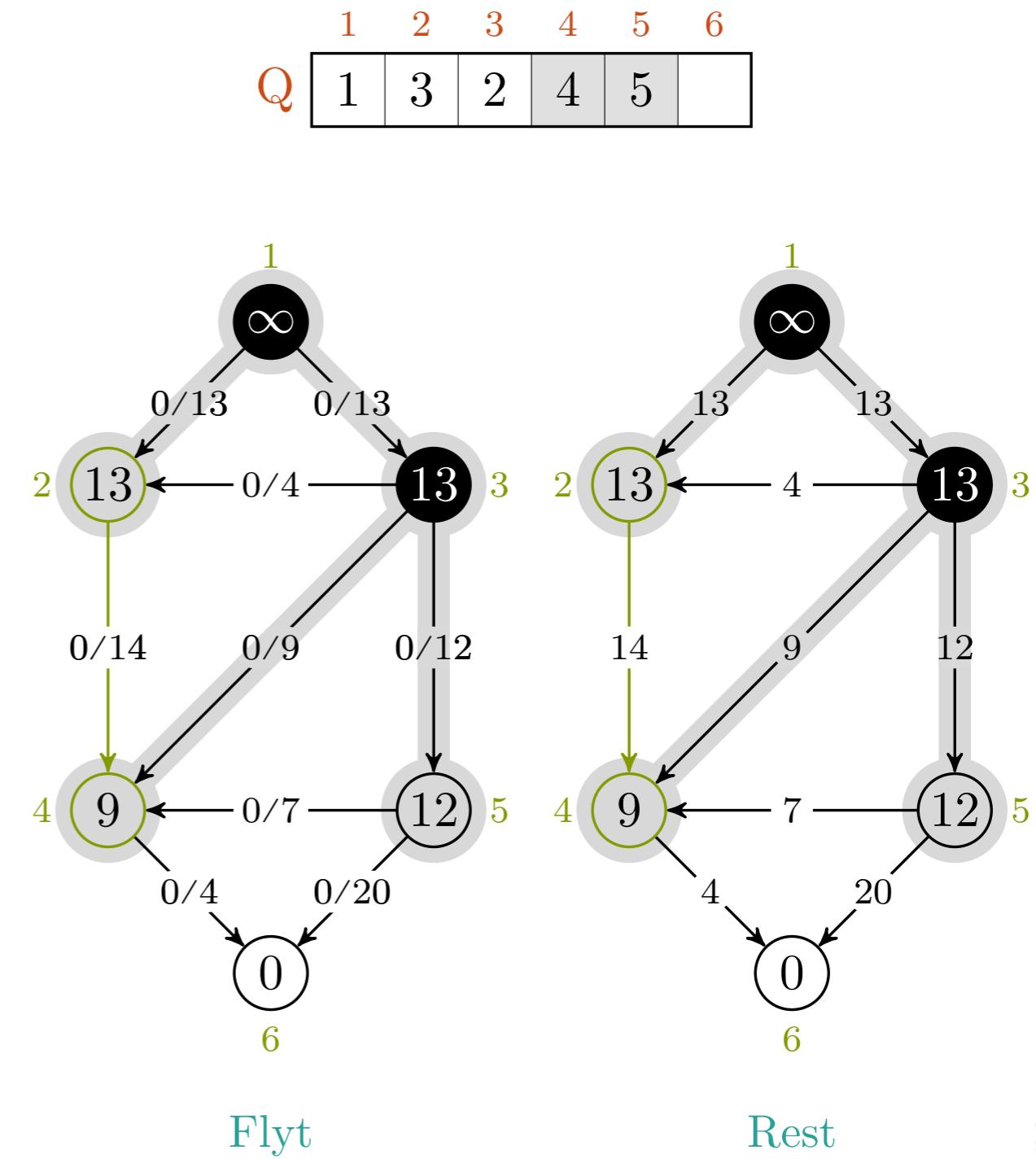
### EDMONDS-KARP(G, s, t)

```

1   for each edge  $(u, v) \in G.E$ 
2        $(u, v).f = 0$ 
3   repeat
4       for each vertex  $u \in G.V$ 
5            $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6            $u.\pi = NIL$ 
7            $s.a = \infty$ 
8            $Q = \emptyset$ 
9           ENQUEUE( $Q, s$ )
10      while  $t.a == 0$  and  $Q \neq \emptyset$ 
11           $u = DEQUEUE(Q)$ 
12          for all edges  $(u, v), (v, u) \in G.E$ 
13              if  $(u, v) \in G.E$ 
14                   $c_f(u, v) = c(u, v) - (u, v).f$ 
15              else  $c_f(u, v) = (v, u).f$ 
16              if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                   $v.a = \min(u.a, c_f(u, v))$ 
18                   $v.\pi = u$ 
19                  ENQUEUE( $Q, v$ )
20       $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(t)$ 
21      while  $u \neq NIL$ 
22          if  $(u, v) \in G.E$ 
23               $(u, v).f = (u, v).f + t.a$ 
24          else  $(v, u).f = (v, u).f - t.a$ 
25           $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

$$u, v = 2, 4$$



maks-flyt → edmonds-karp

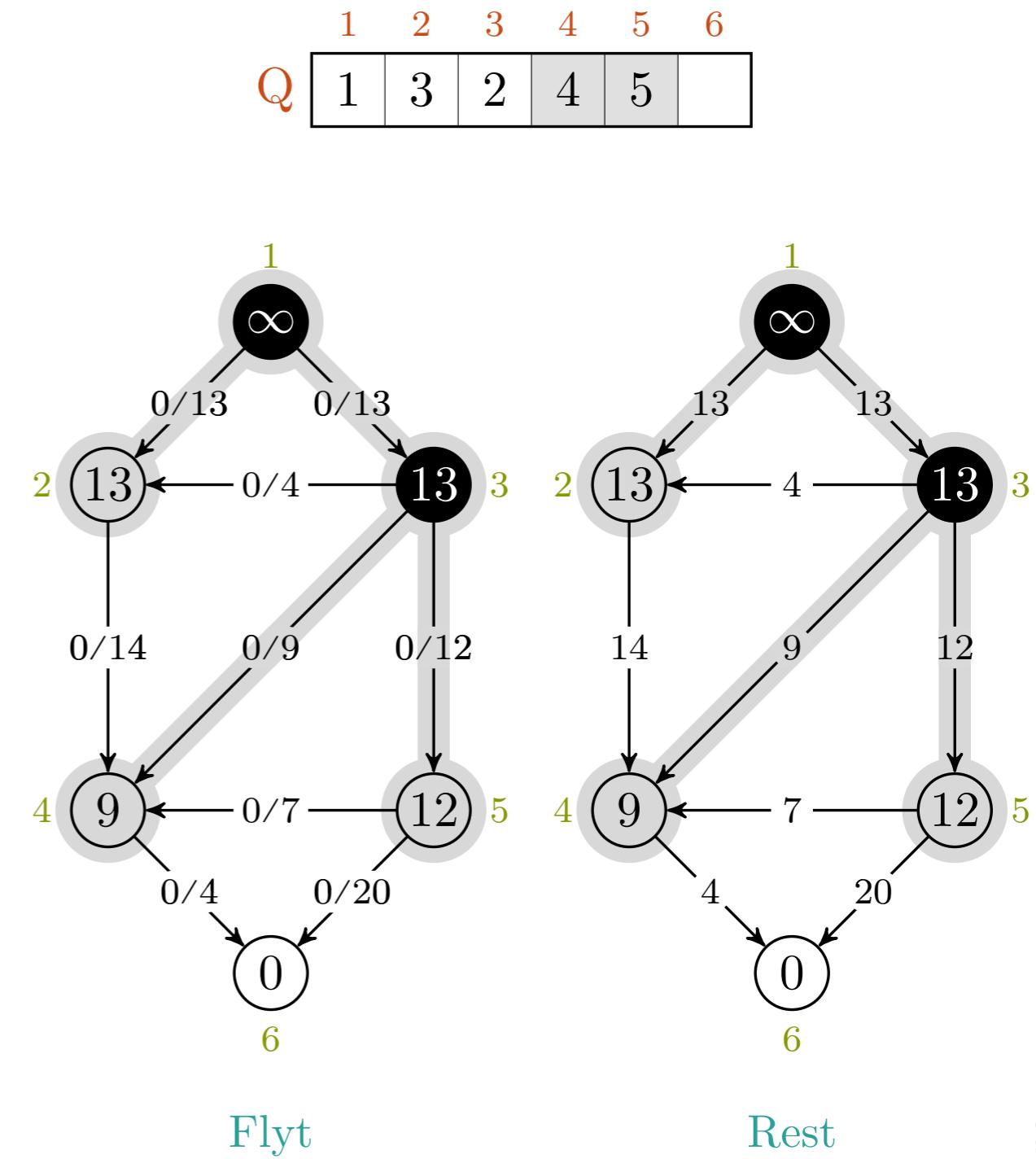
## EDMONDS-KARP(G, s, t)

```

1   for each edge  $(u, v) \in G.E$ 
2        $(u, v).f = 0$ 
3   repeat
4       for each vertex  $u \in G.V$ 
5            $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6            $u.\pi = NIL$ 
7            $s.a = \infty$ 
8            $Q = \emptyset$ 
9           ENQUEUE( $Q, s$ )
10      while  $t.a == 0$  and  $Q \neq \emptyset$ 
11           $u = DEQUEUE(Q)$ 
12          for all edges  $(u, v), (v, u) \in G.E$ 
13              if  $(u, v) \in G.E$ 
14                   $c_f(u, v) = c(u, v) - (u, v).f$ 
15              else  $c_f(u, v) = (v, u).f$ 
16              if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                   $v.a = \min(u.a, c_f(u, v))$ 
18                   $v.\pi = u$ 
19                  ENQUEUE( $Q, v$ )
20       $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f$ 
21      while  $u \neq NIL$ 
22          if  $(u, v) \in G.E$ 
23               $(u, v).f = (u, v).f + t.a$ 
24          else  $(v, u).f = (v, u).f - t.a$ 
25           $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

$$u, v = 2, -$$

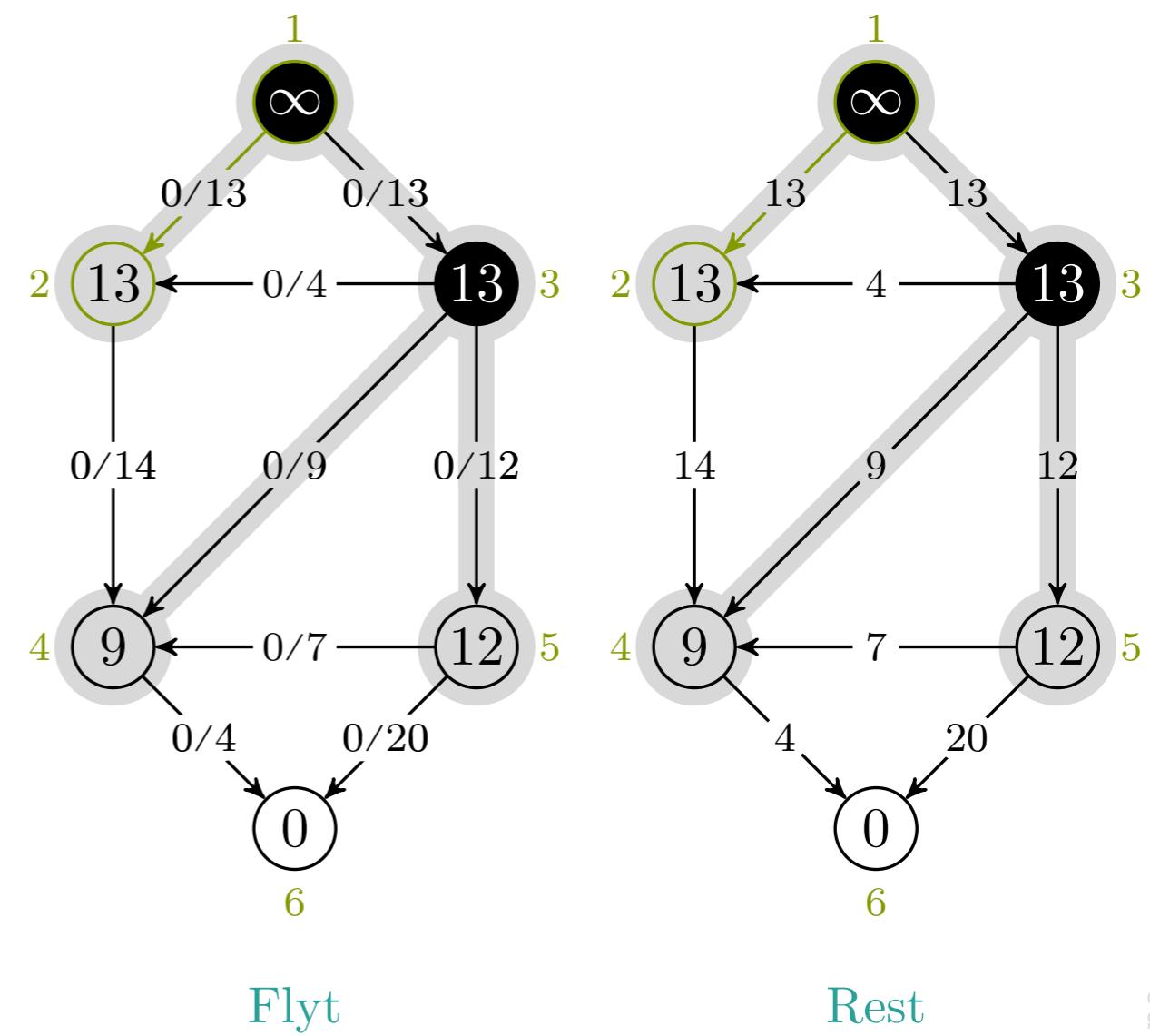


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
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19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
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23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, 1$

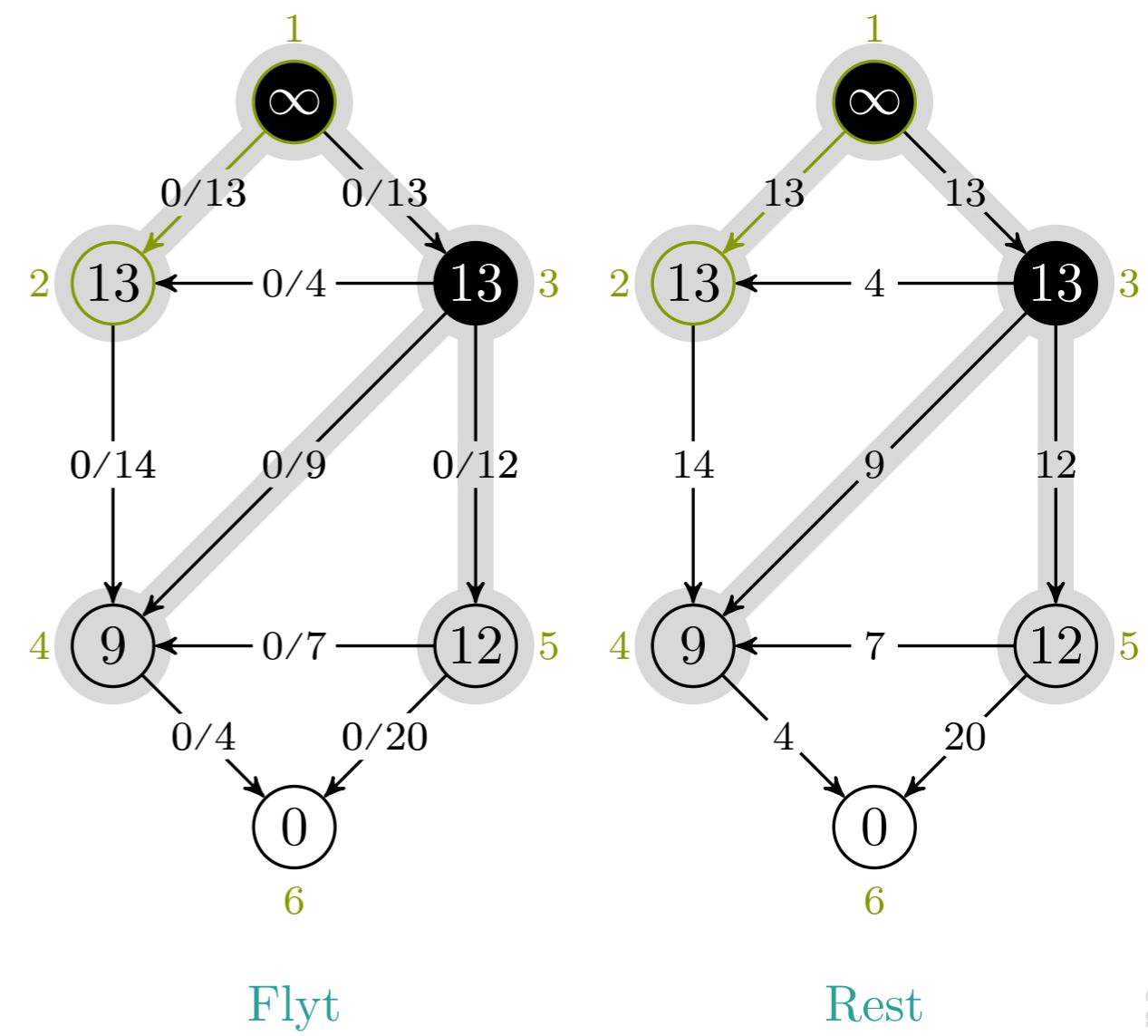


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
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4    for each vertex  $u \in G.V$ 
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25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, 1$

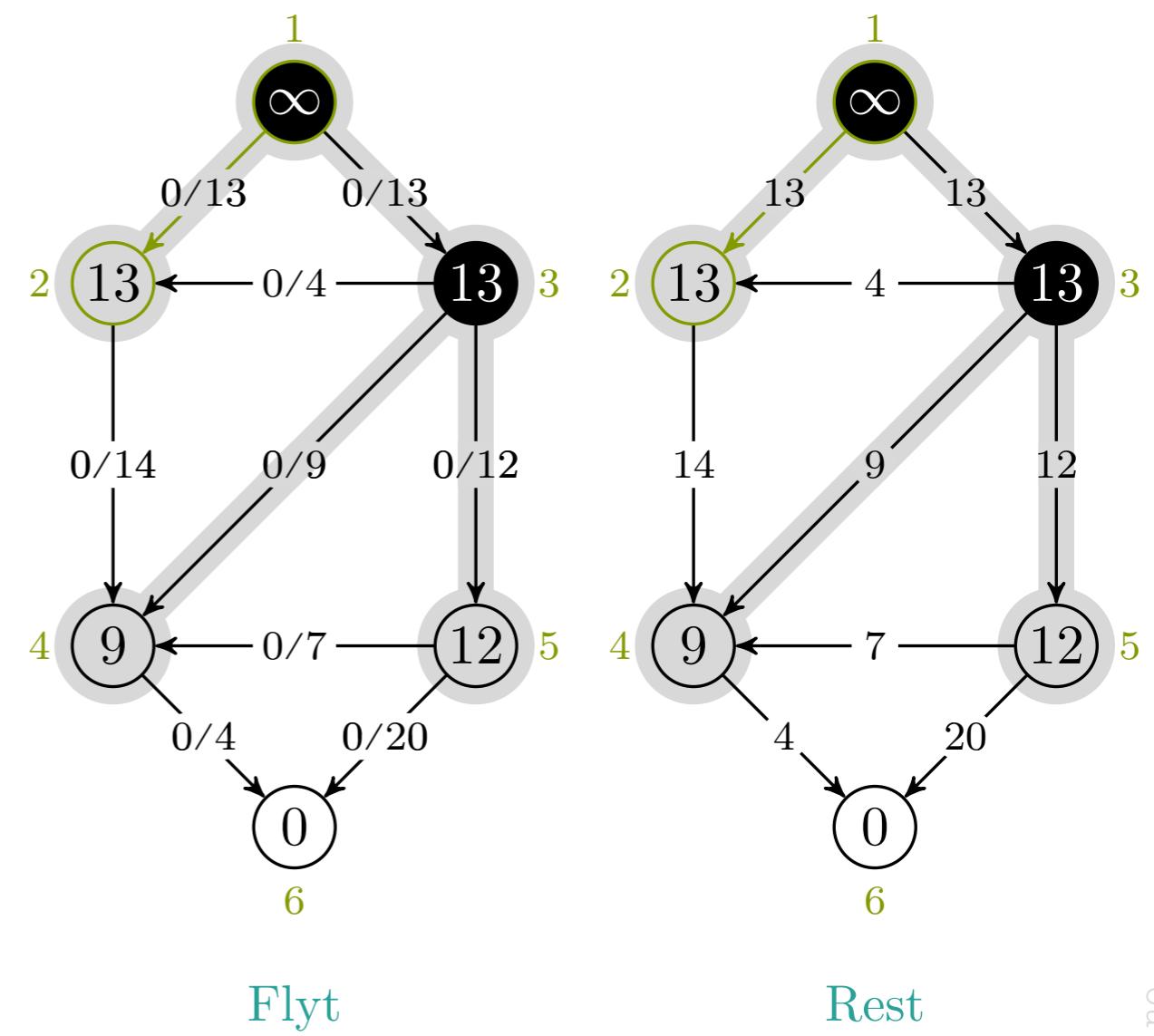


EDMONDS-KARP( $G, s, t$ )

```

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22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, 1$

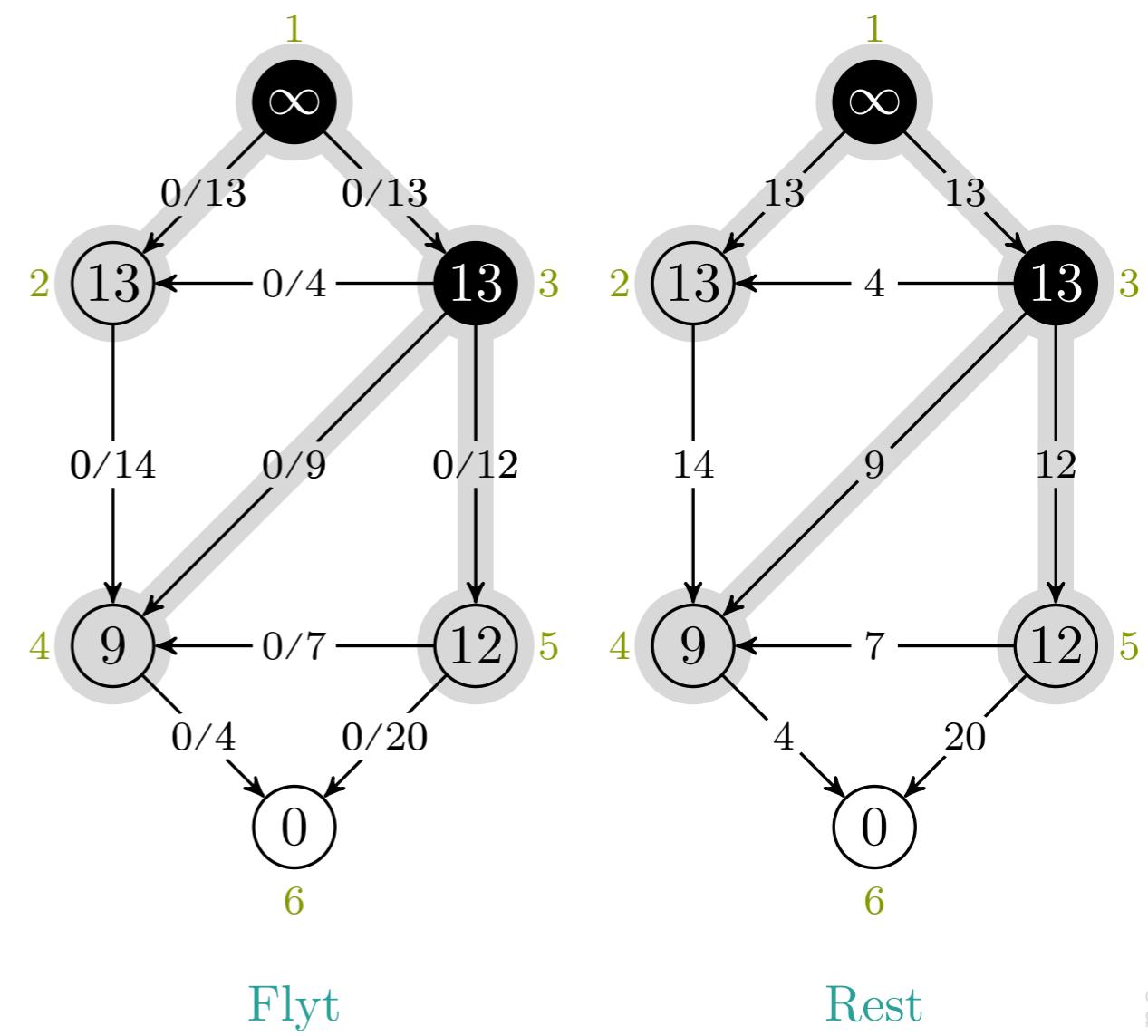


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
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8       $Q = \emptyset$ 
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17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, -$

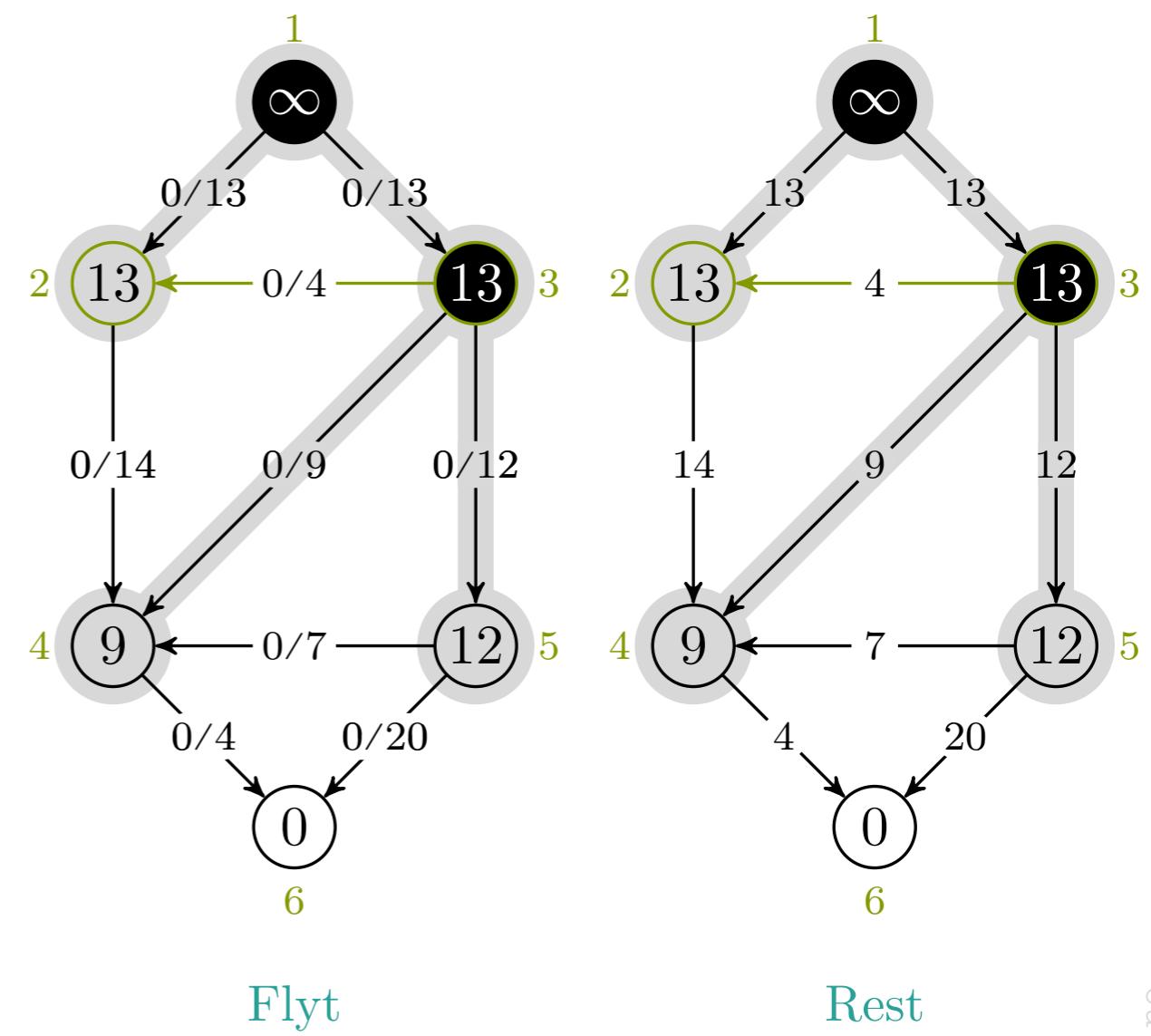


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, 3$

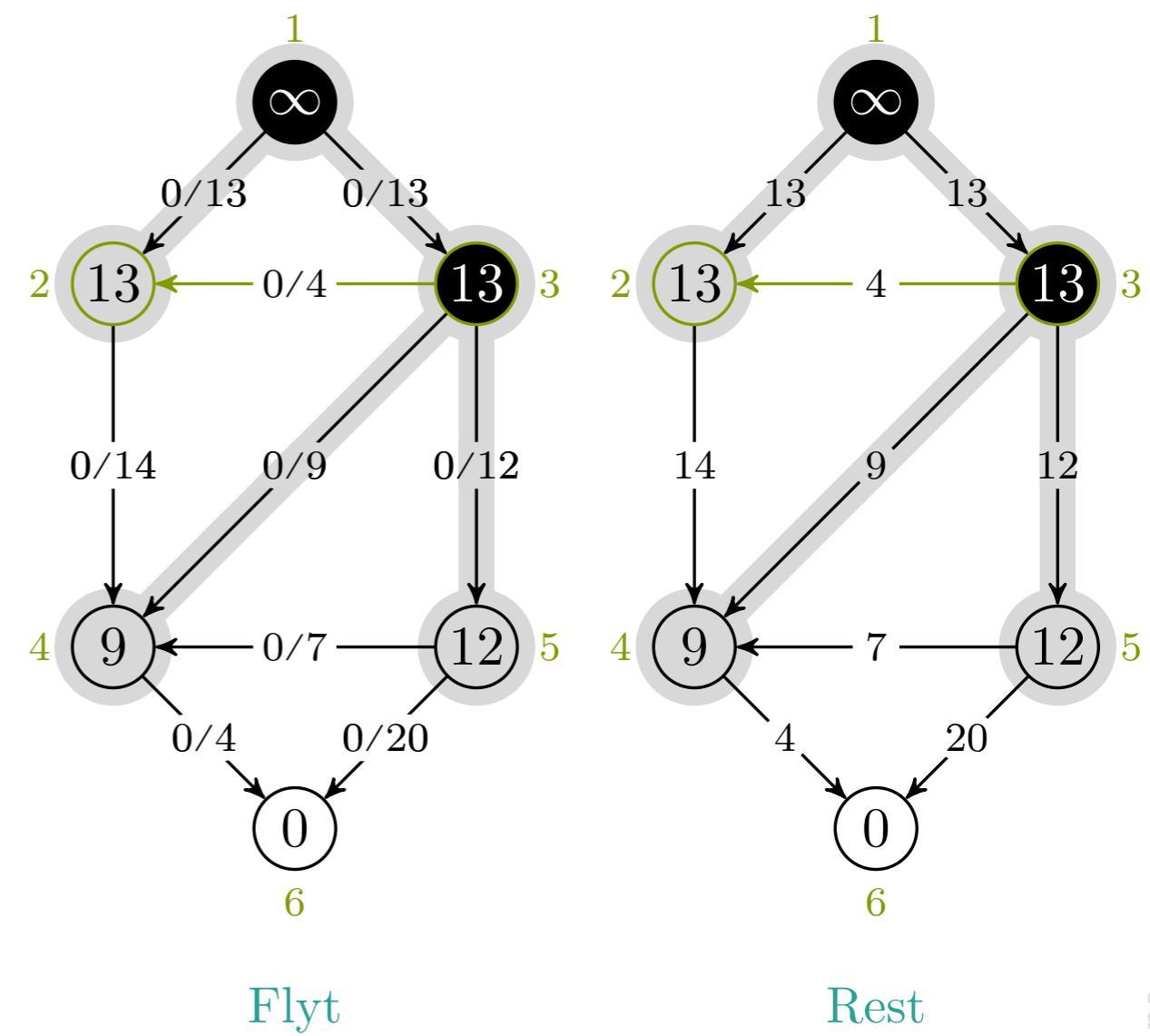


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, 3$

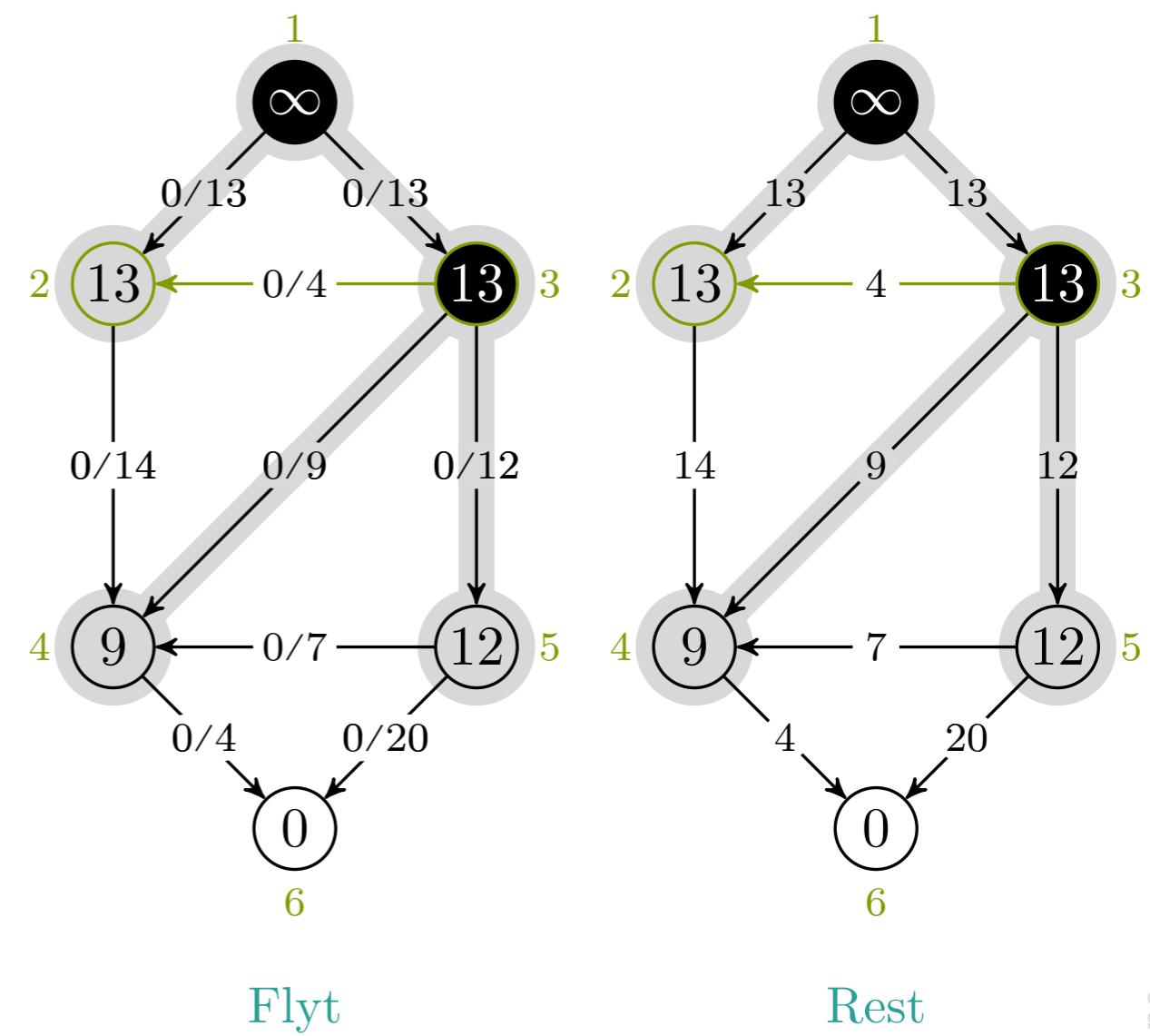


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$u, v = 2, 3$

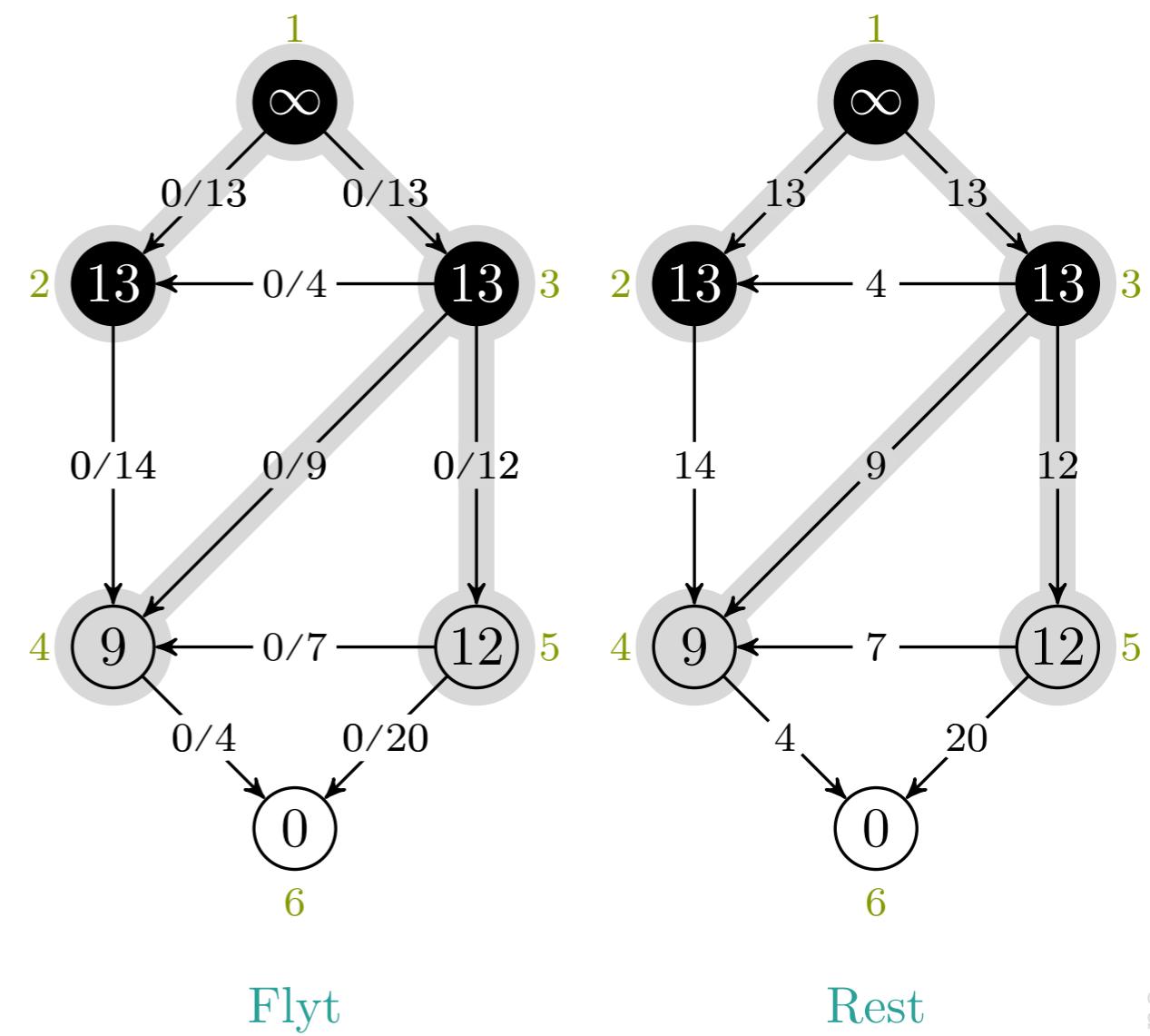


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```

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25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, -$

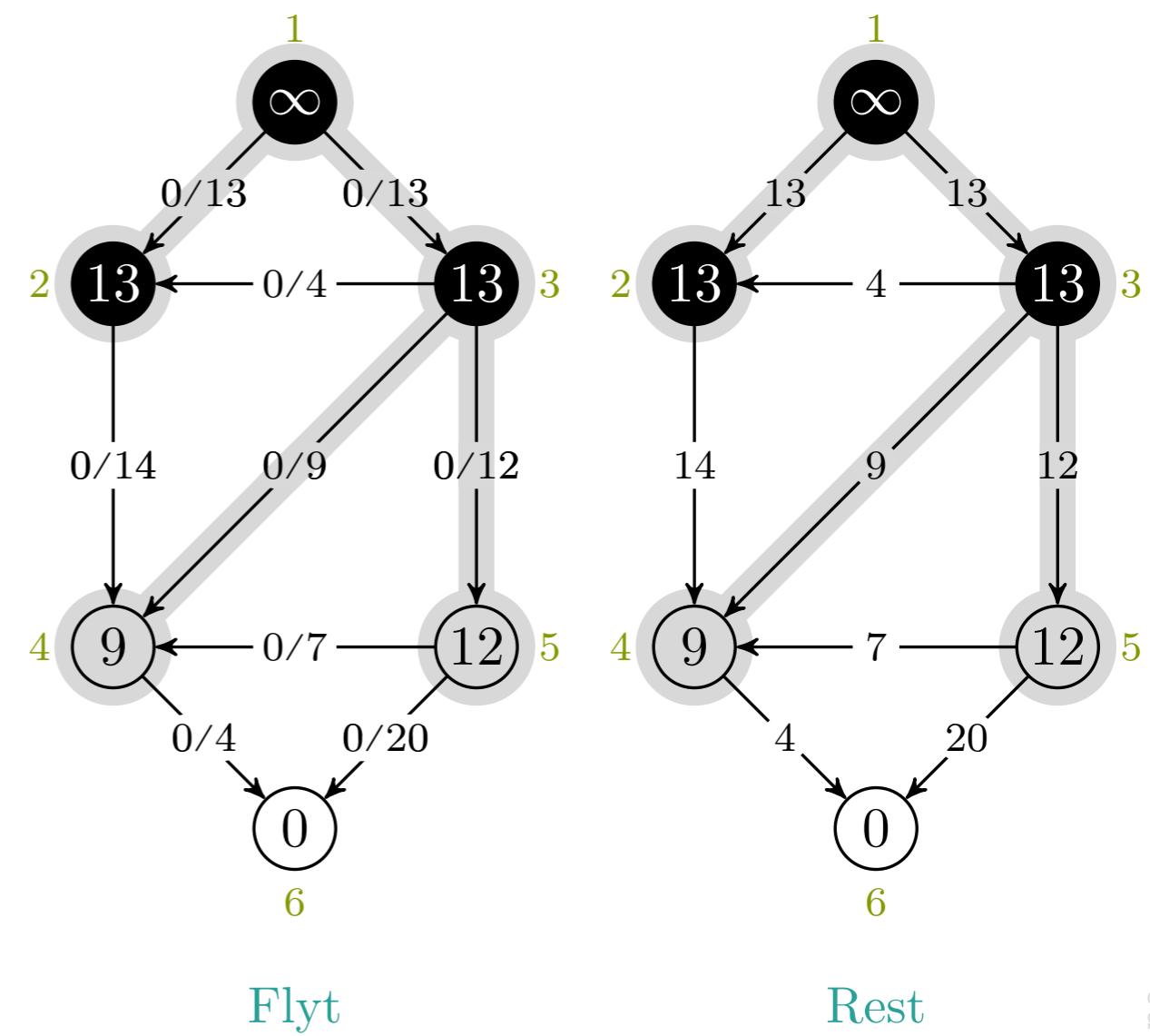


EDMONDS-KARP( $G, s, t$ )

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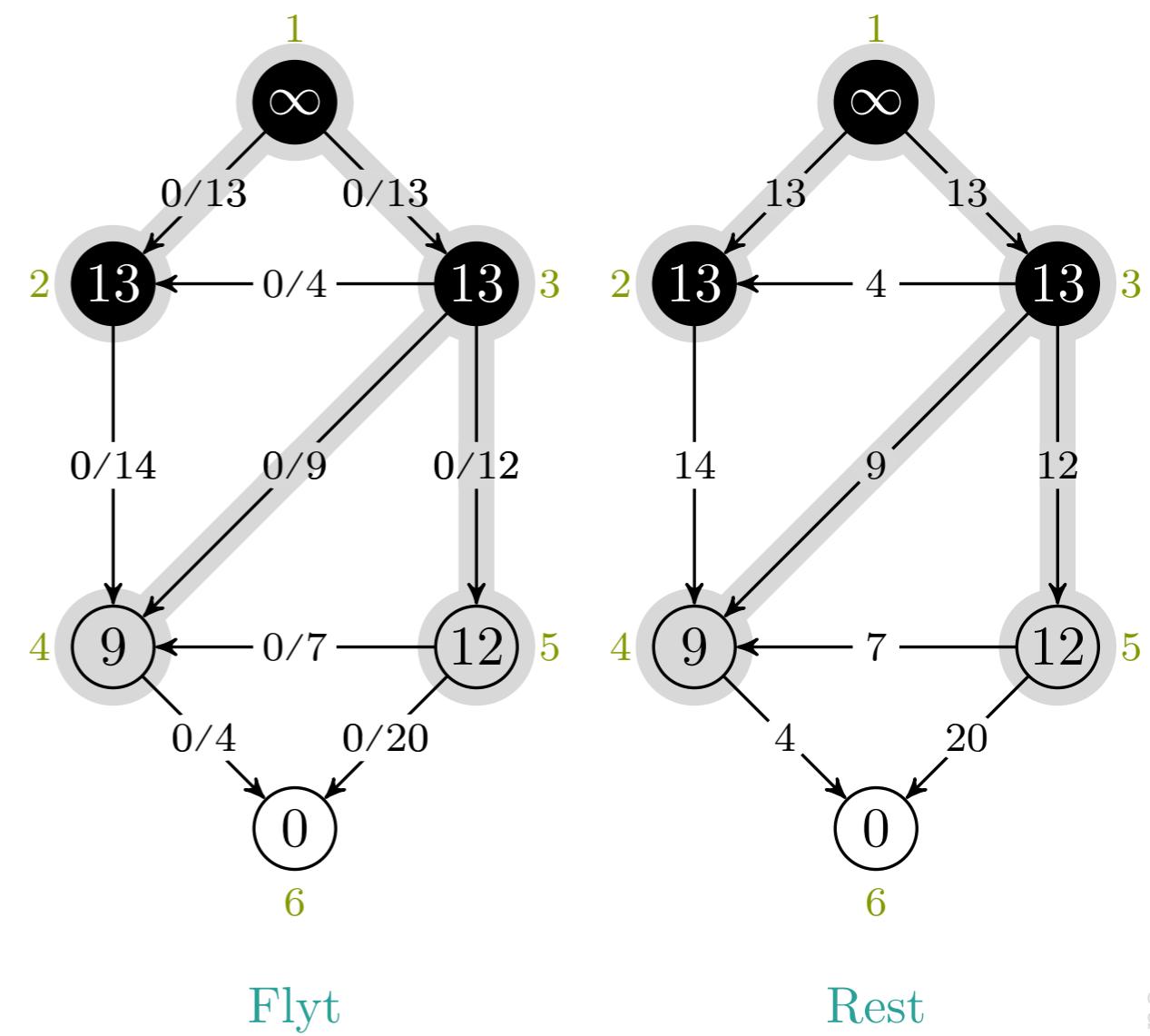
maks-flyt → edmonds-karp

EDMONDS-KARP( $G, s, t$ )

```

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25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, -$

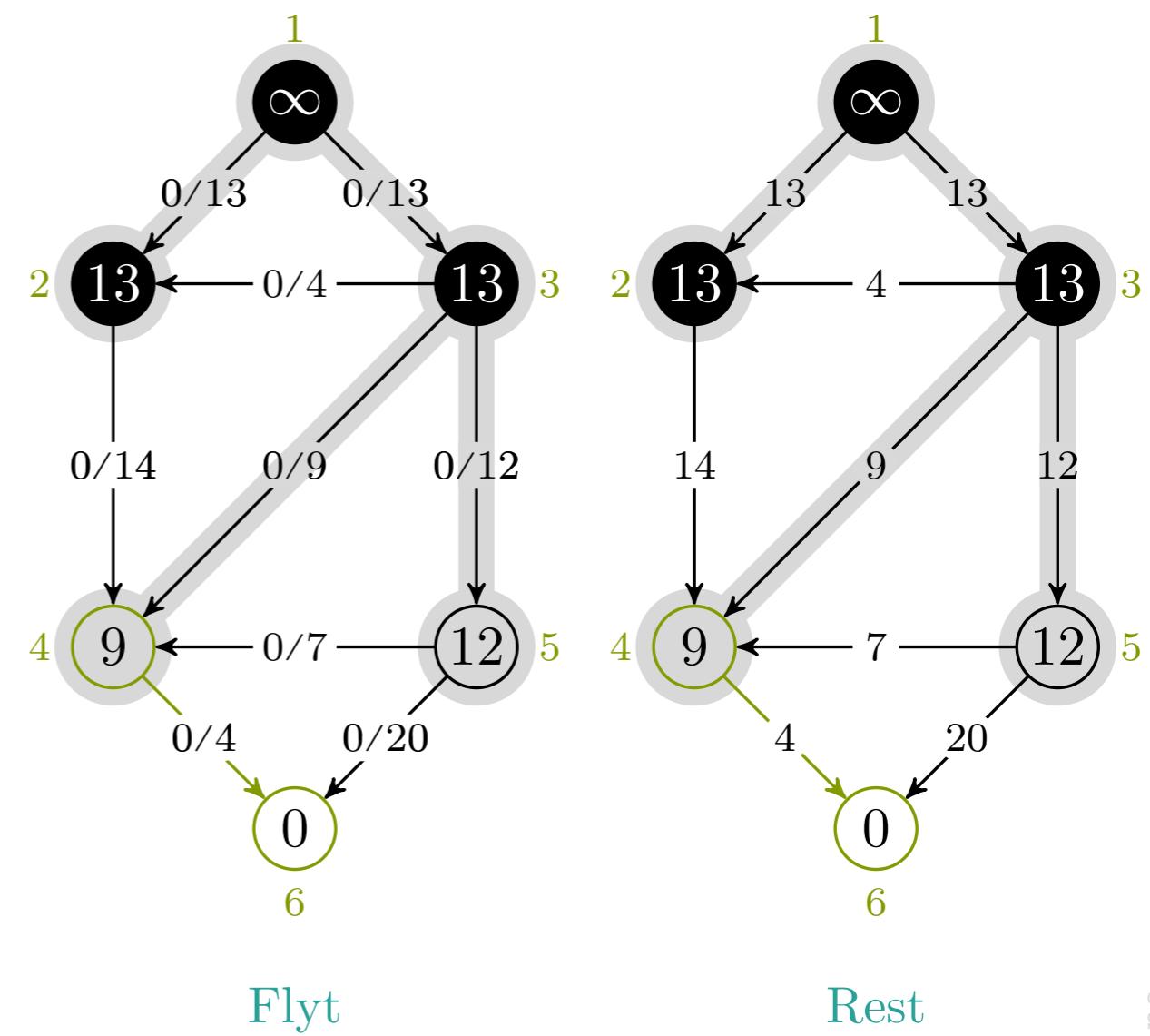


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
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26 until  $t.a == 0$ 
```

$u, v = 4, 6$

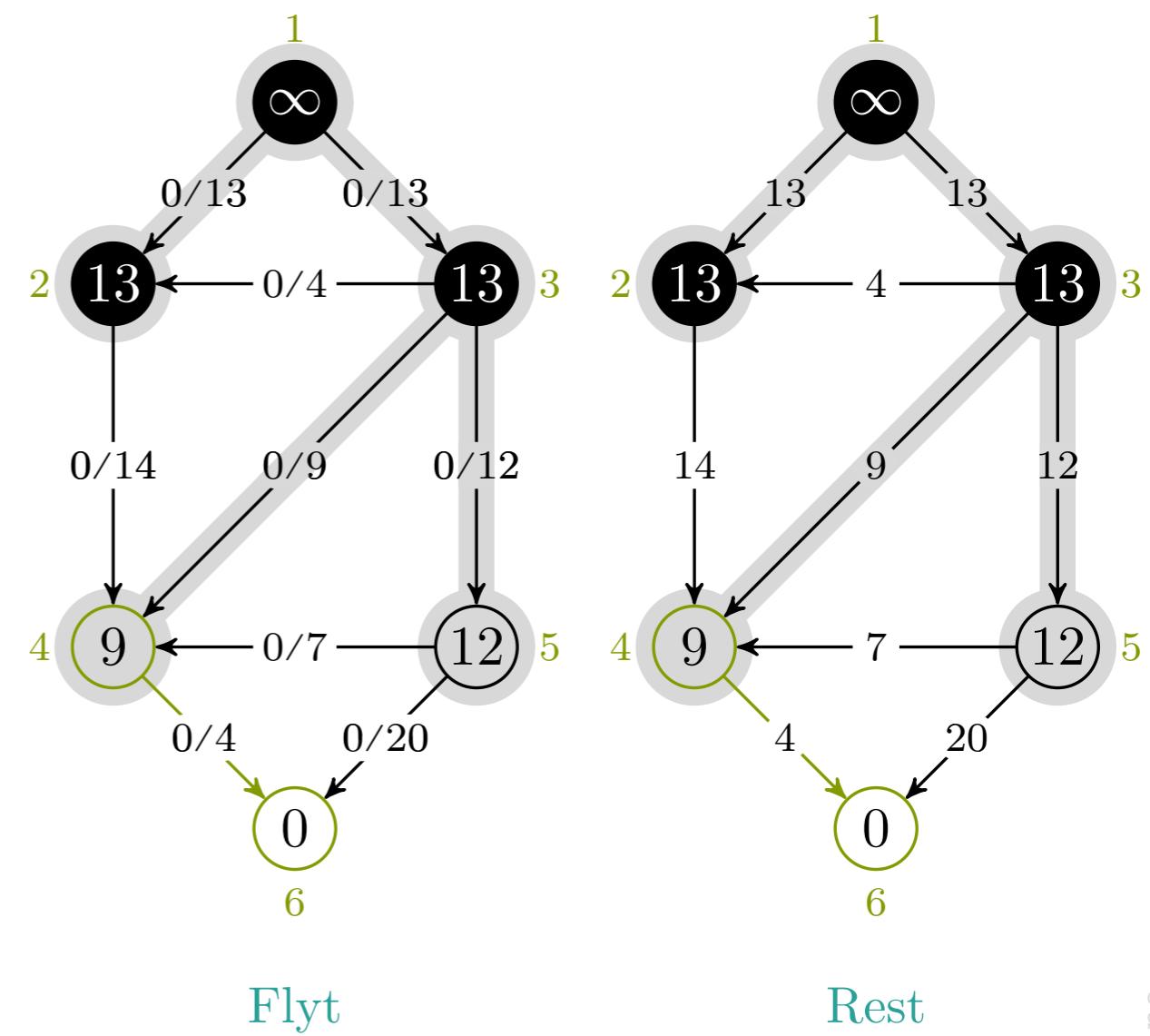


EDMONDS-KARP( $G, s, t$ )

```

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```

$u, v = 4, 6$

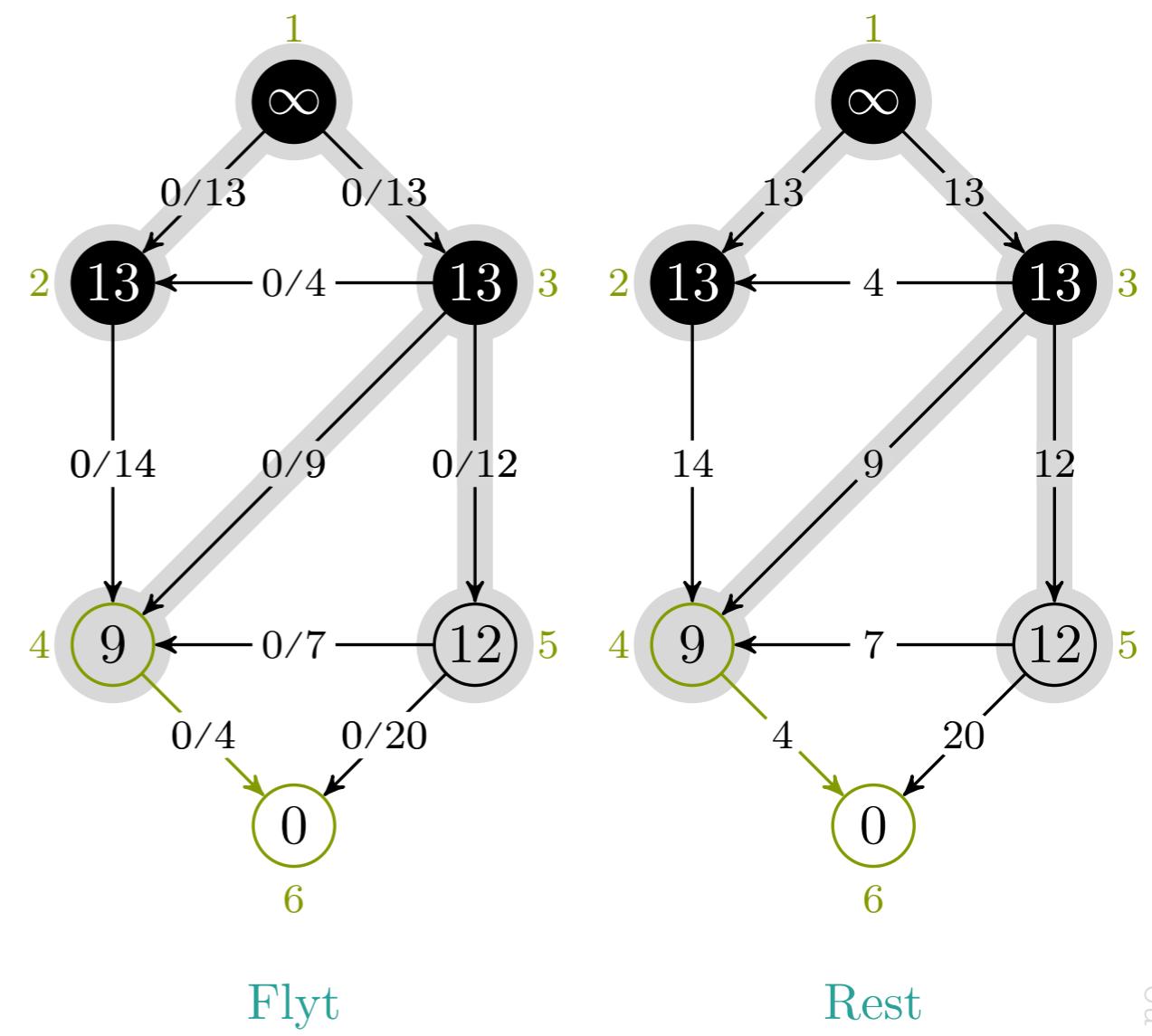


EDMONDS-KARP( $G, s, t$ )

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```

$u, v = 4, 6$

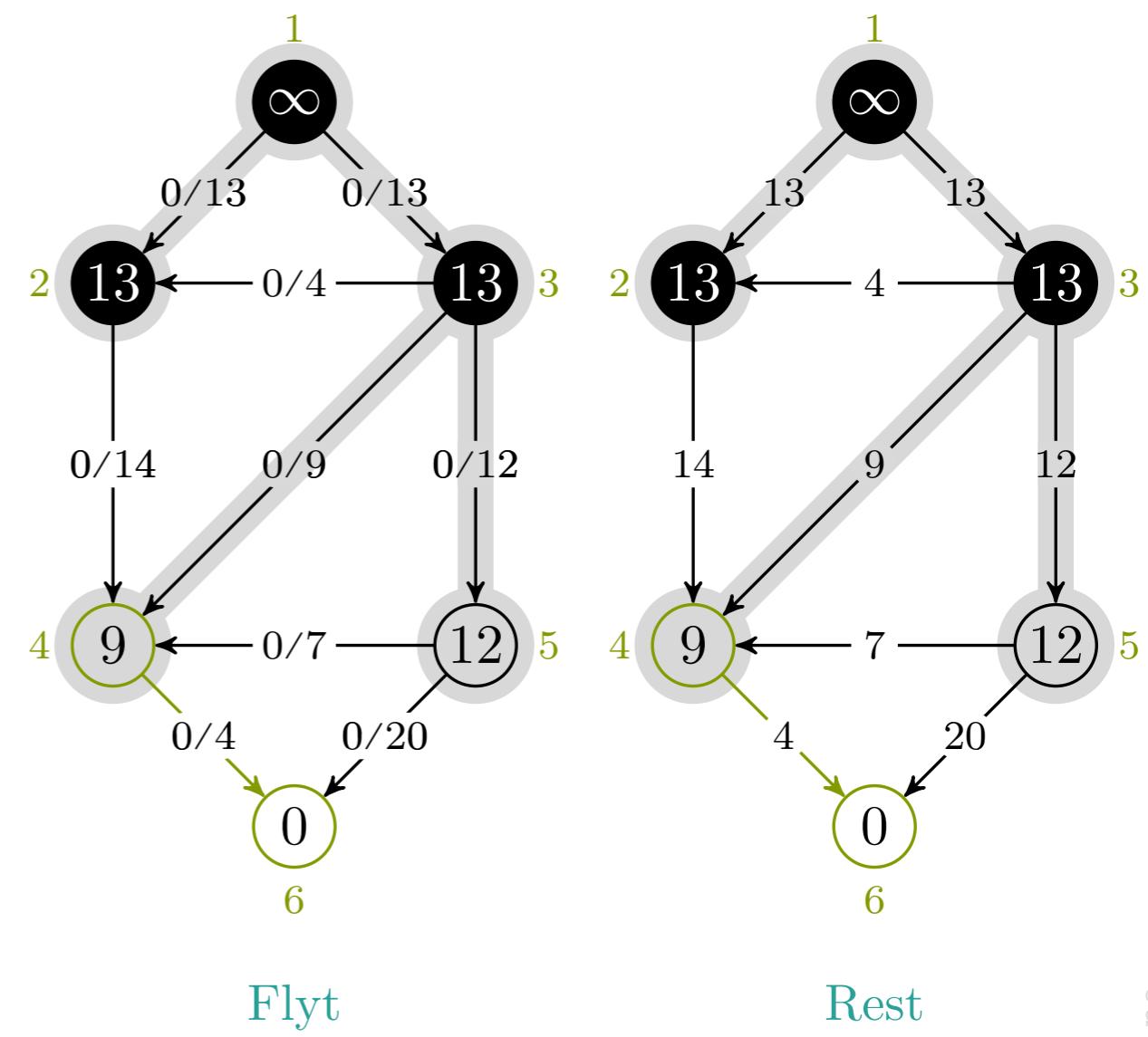


EDMONDS-KARP( $G, s, t$ )

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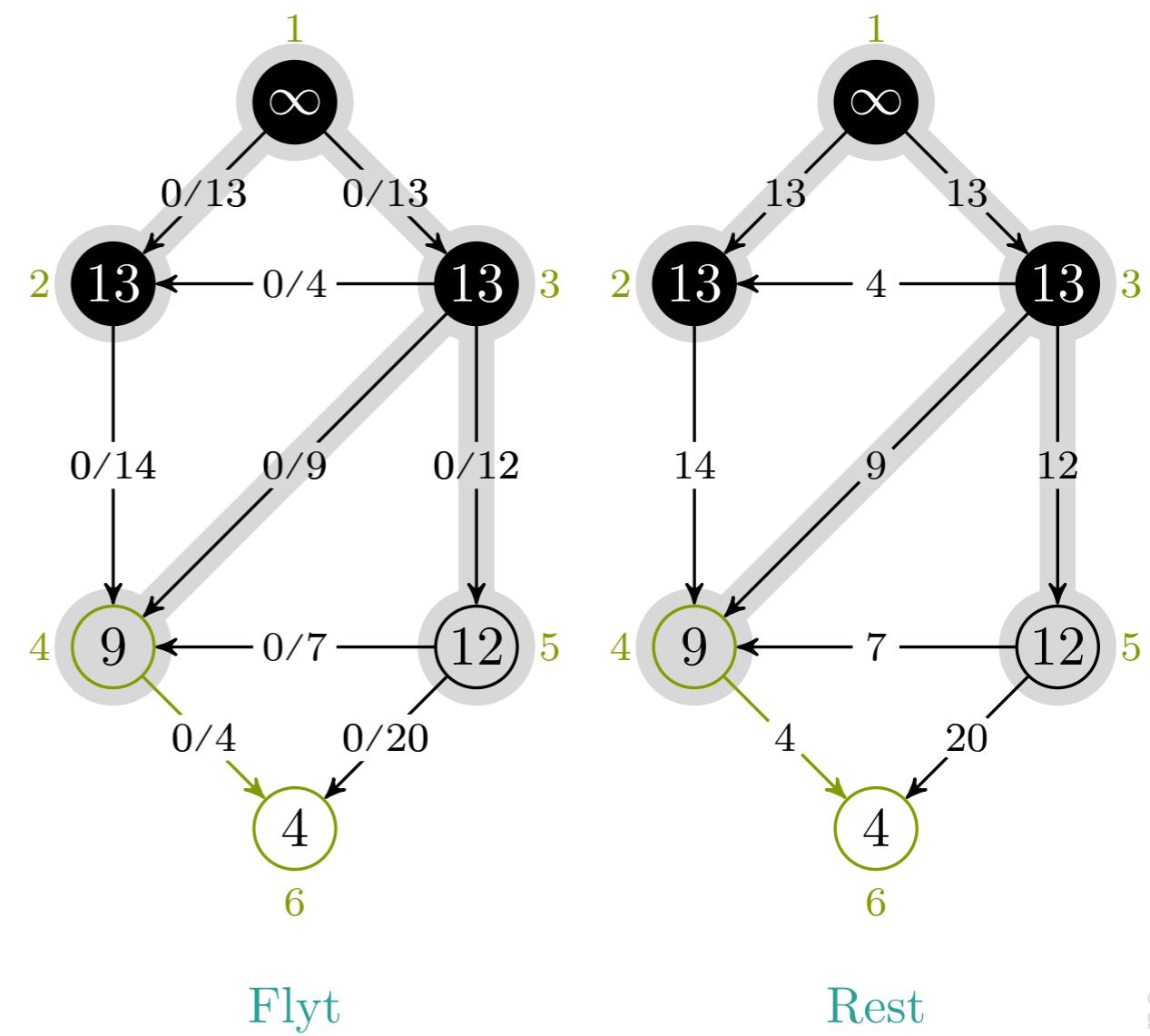


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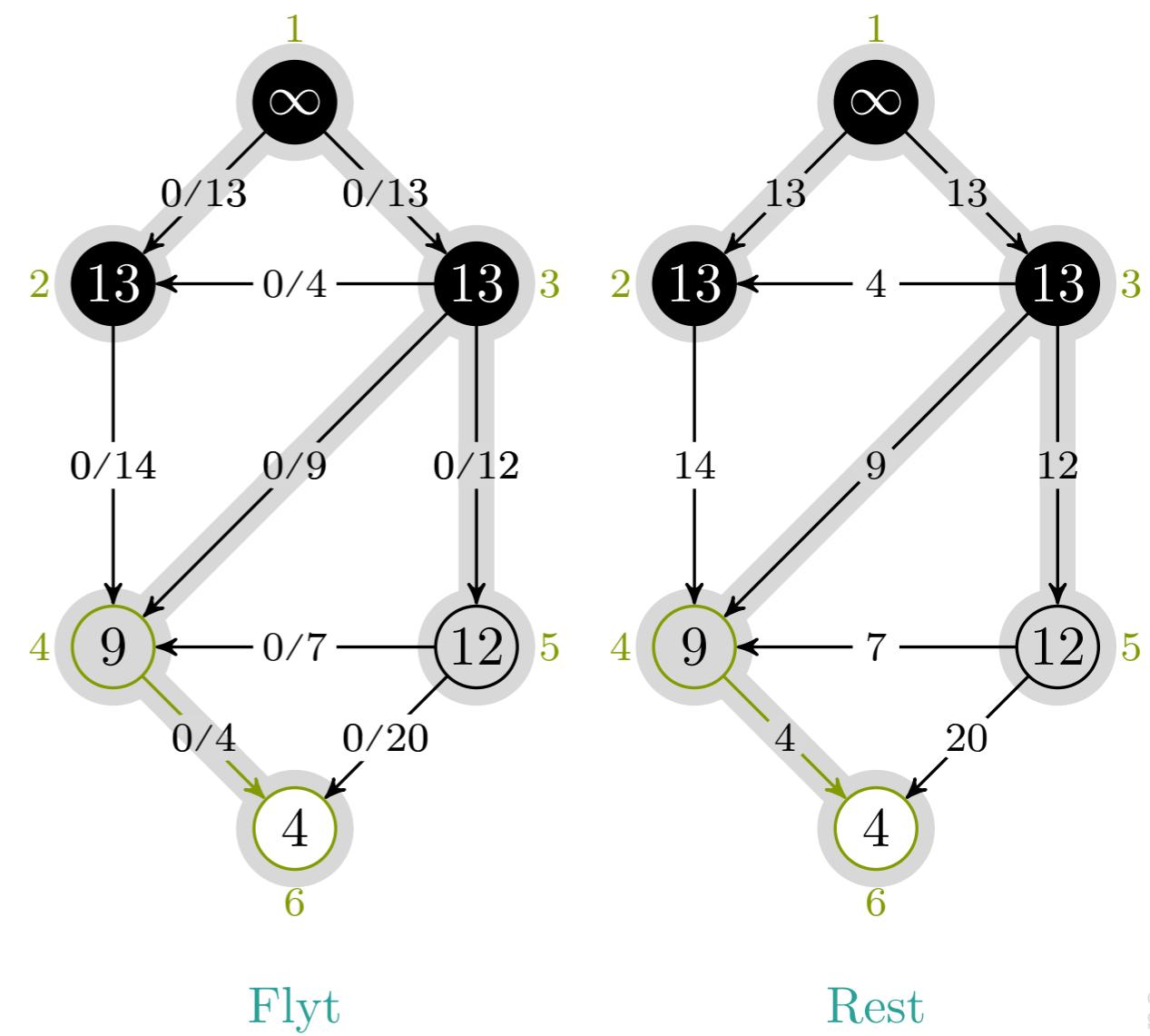


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26 until  $t.a == 0$ 
```

$u, v = 4, 6$

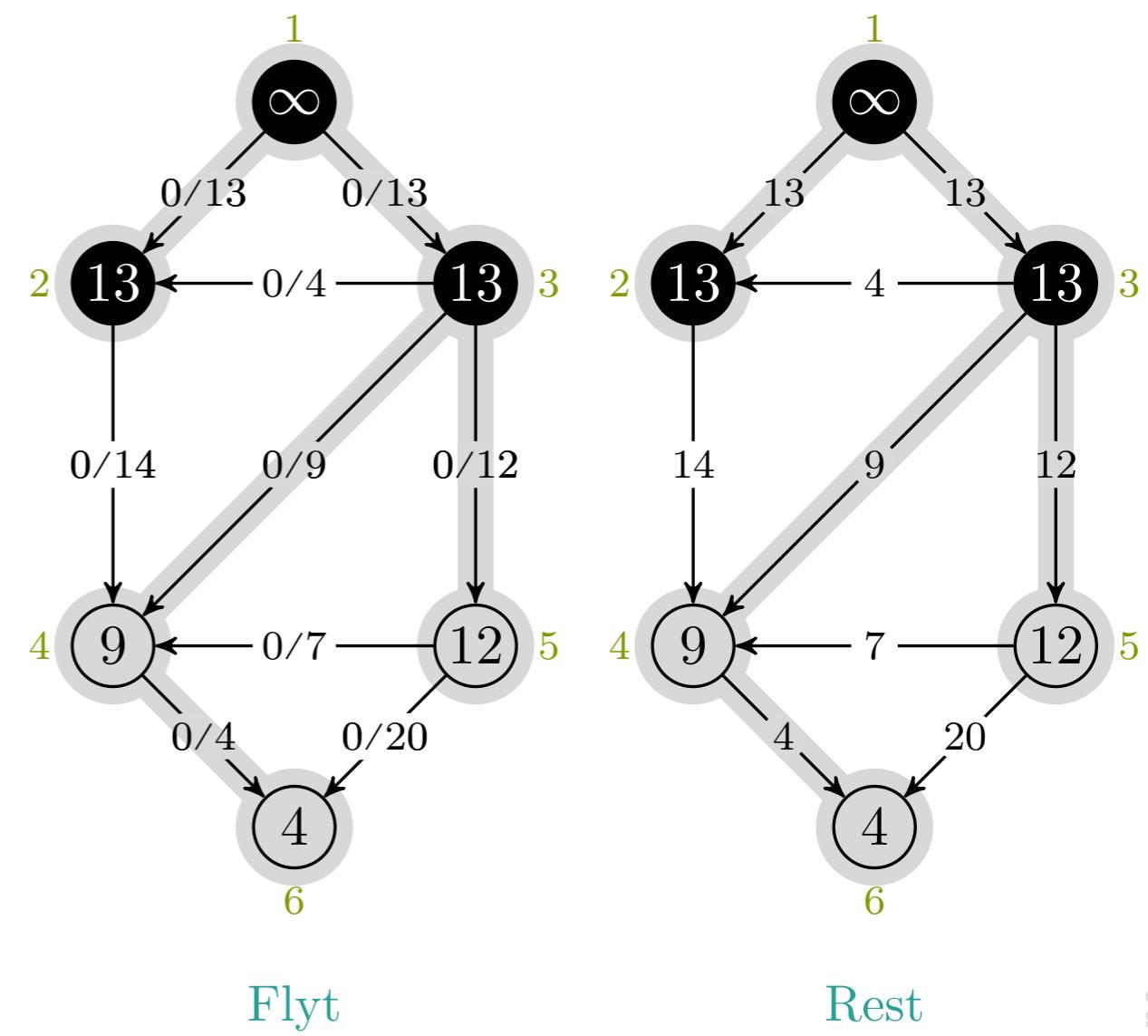


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```

$u, v = 4, -$

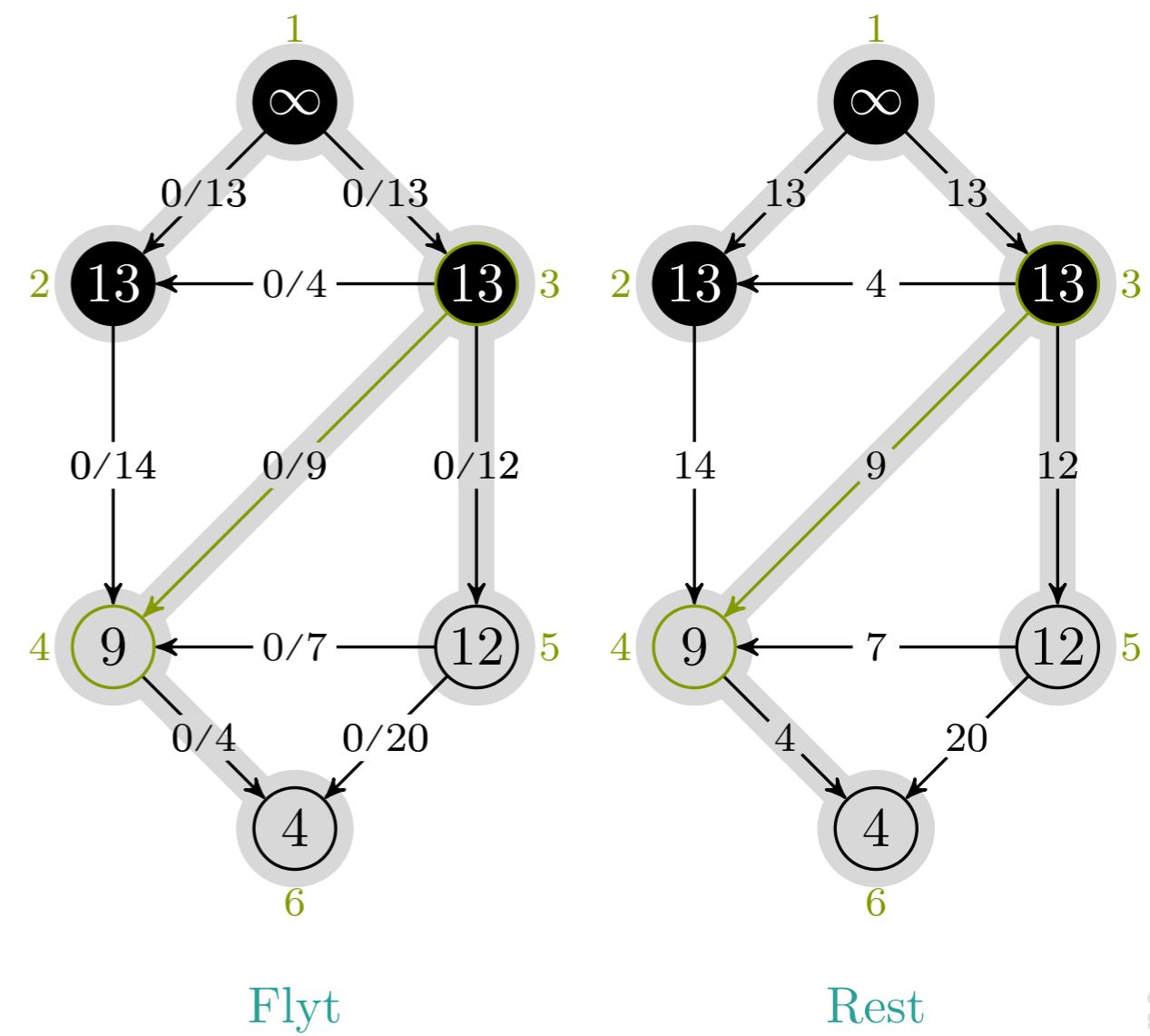


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20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 3$

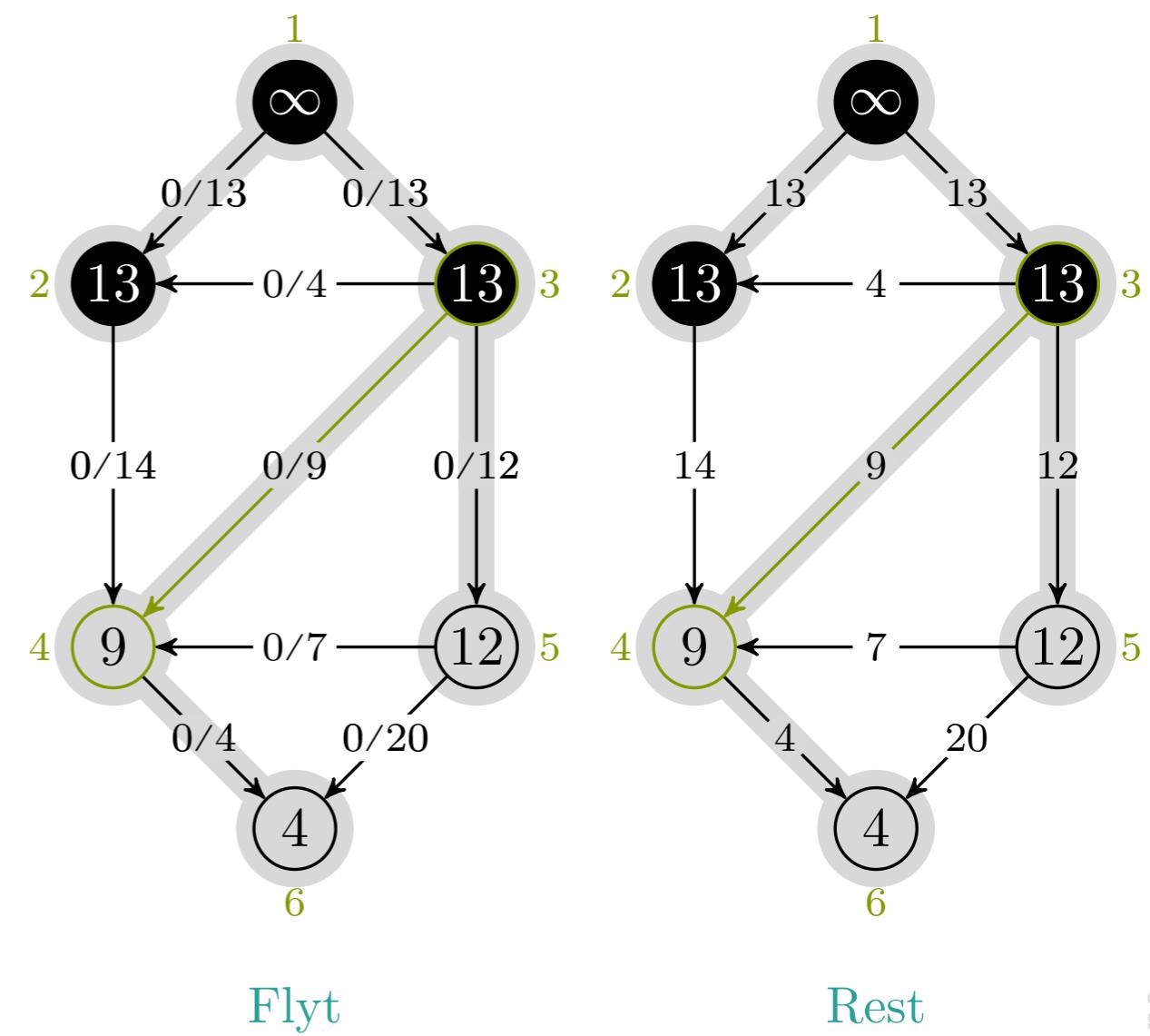


EDMONDS-KARP( $G, s, t$ )

```

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14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 3$

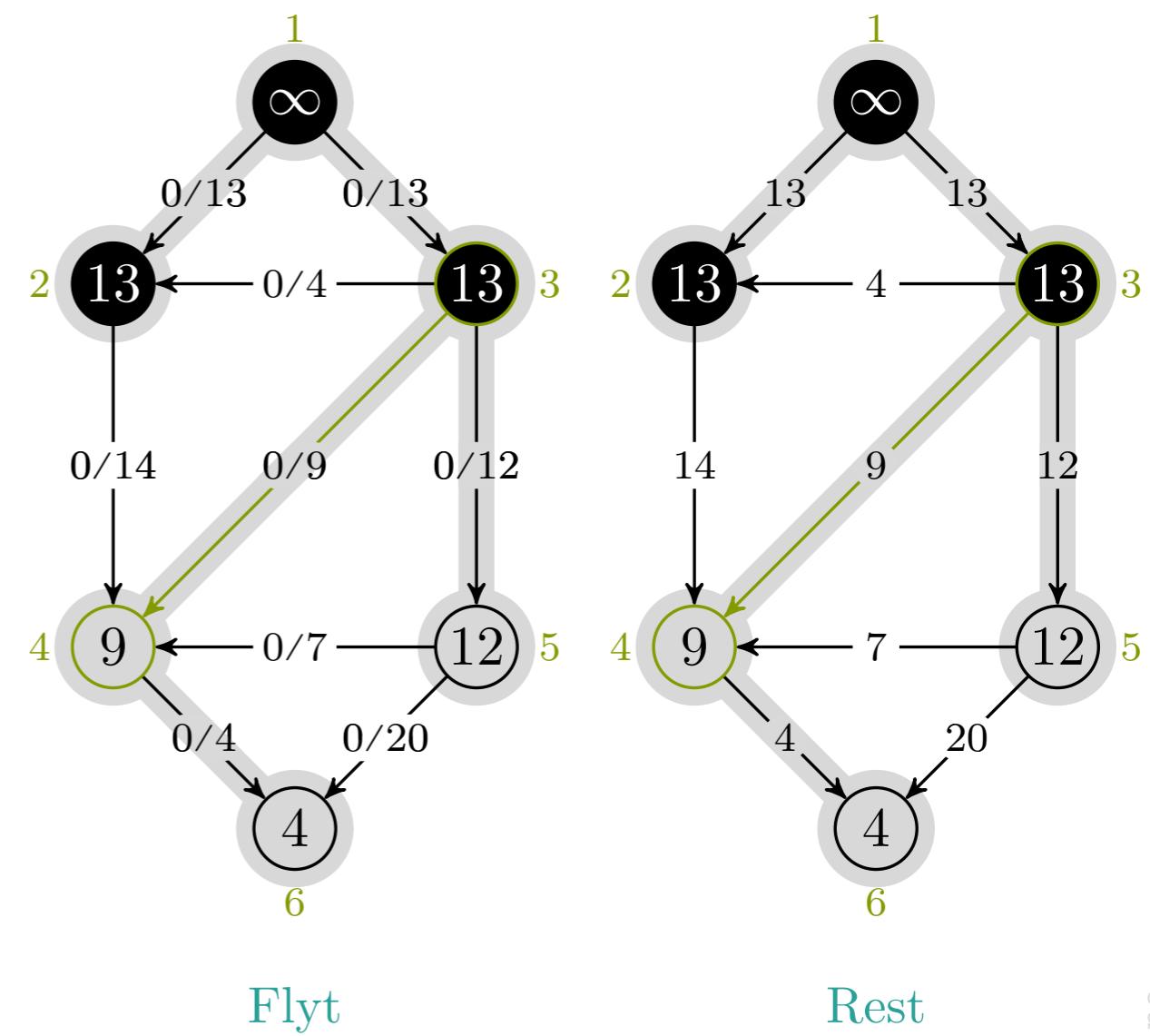


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 3$



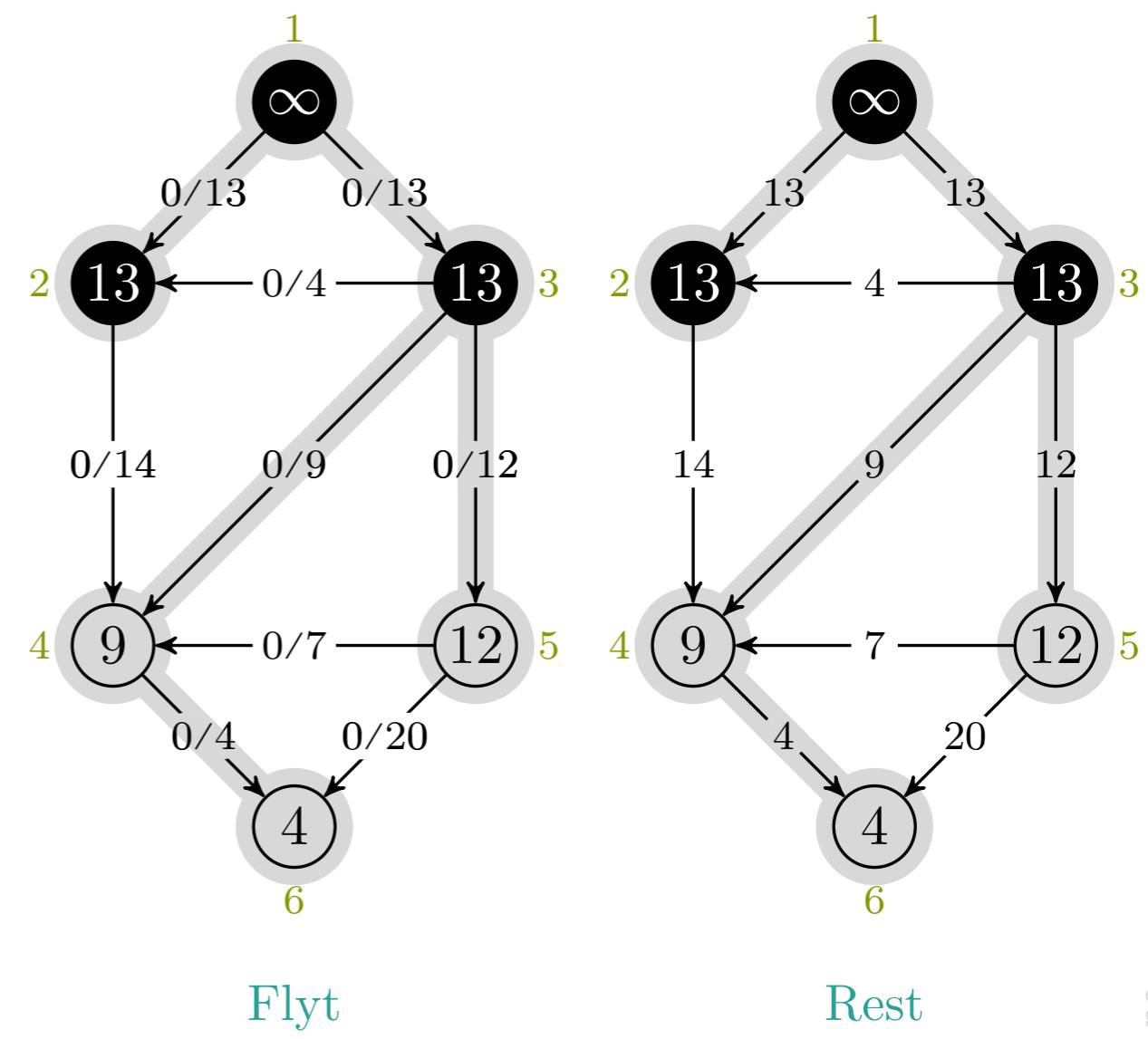
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, -$

Q	1	2	3	4	5	6
	1	3	2	4	5	6

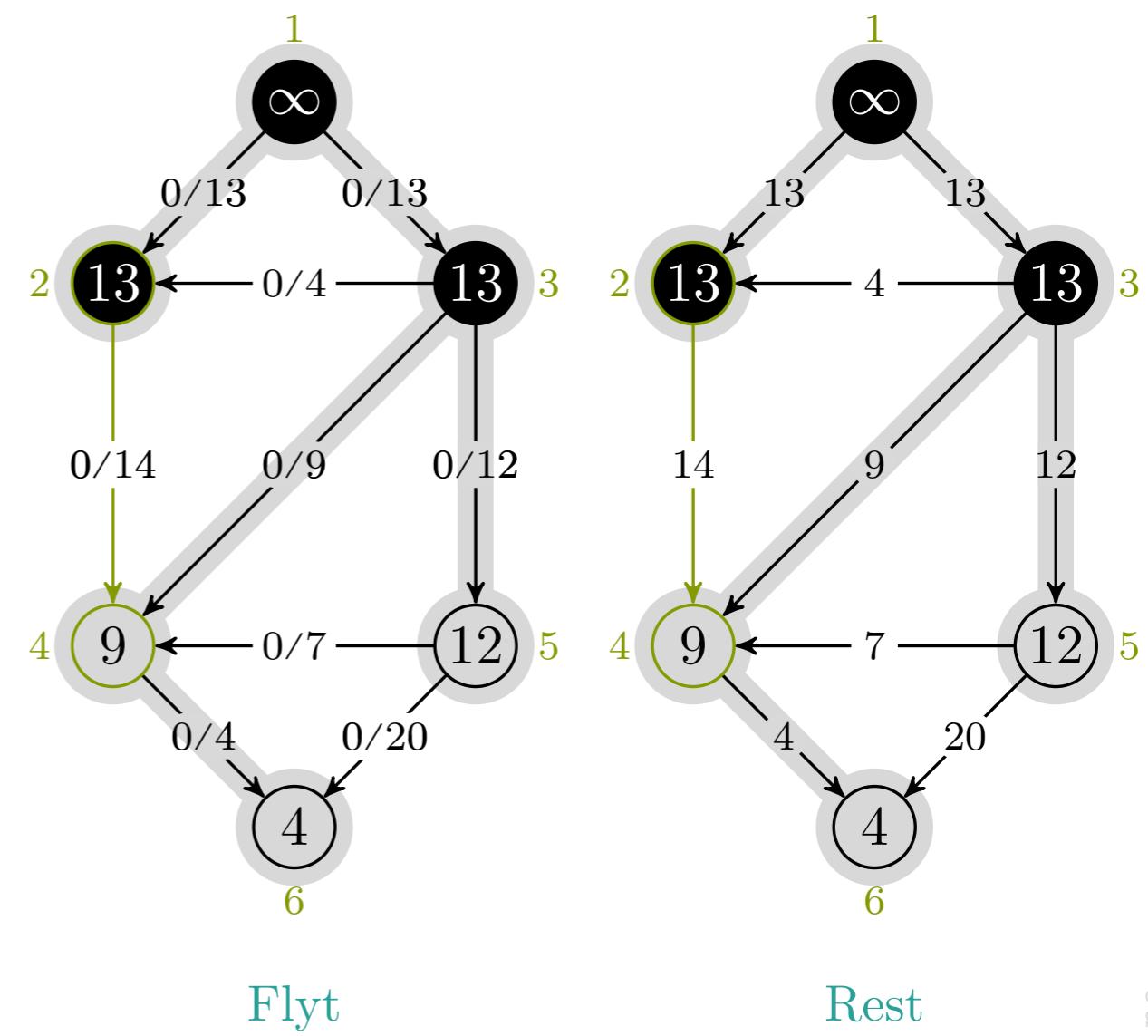


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 2$

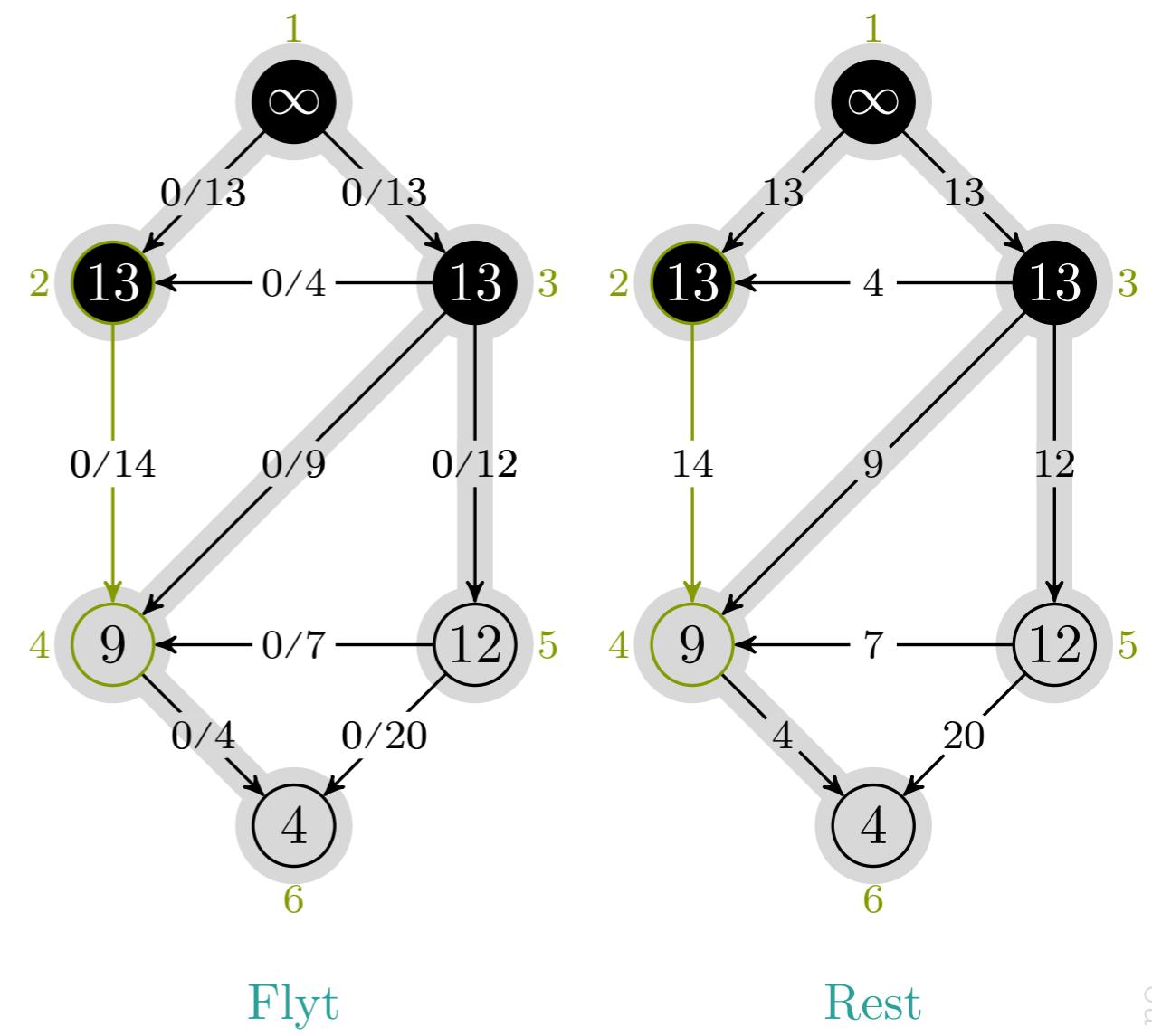


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 2$

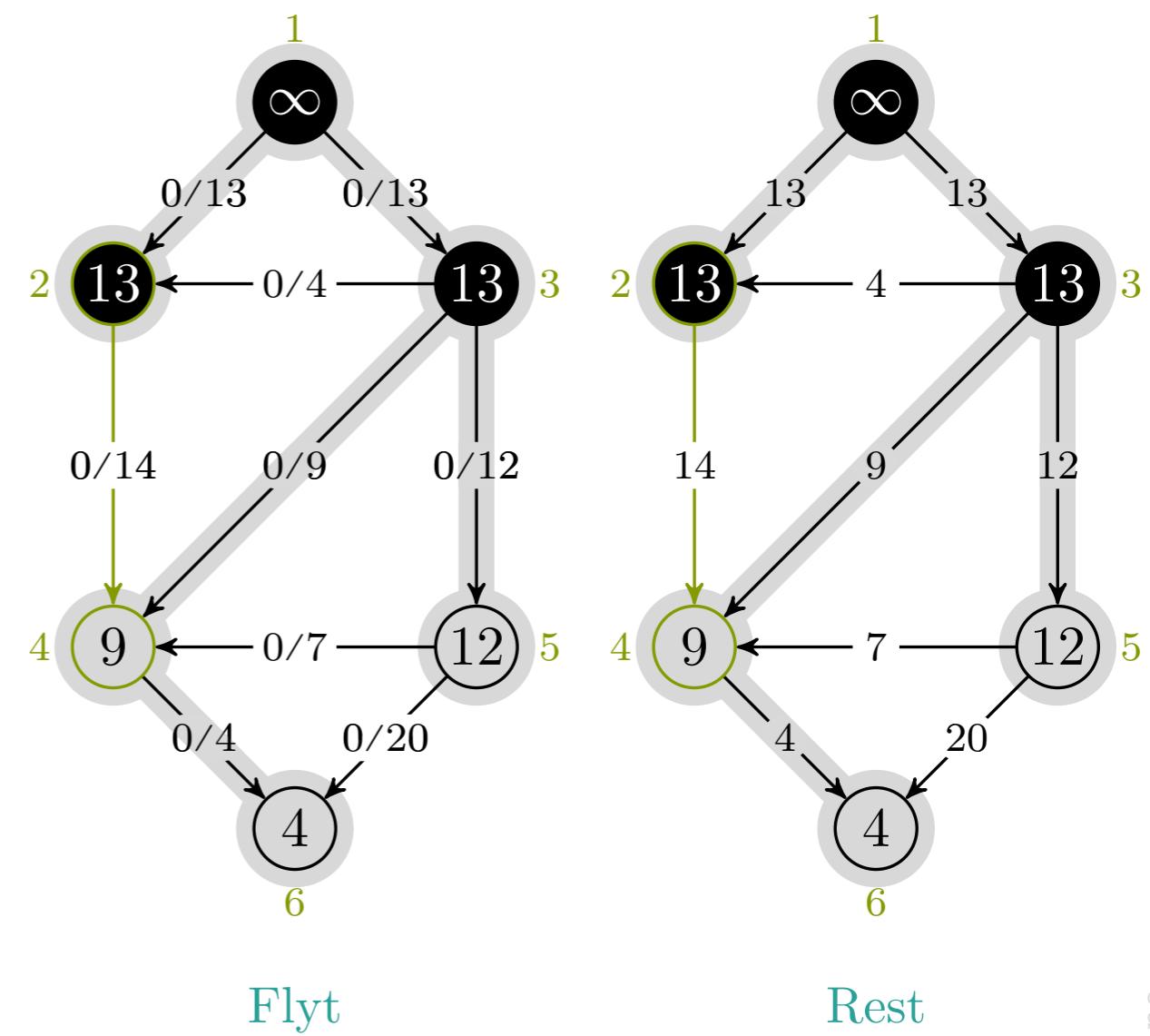


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 2$

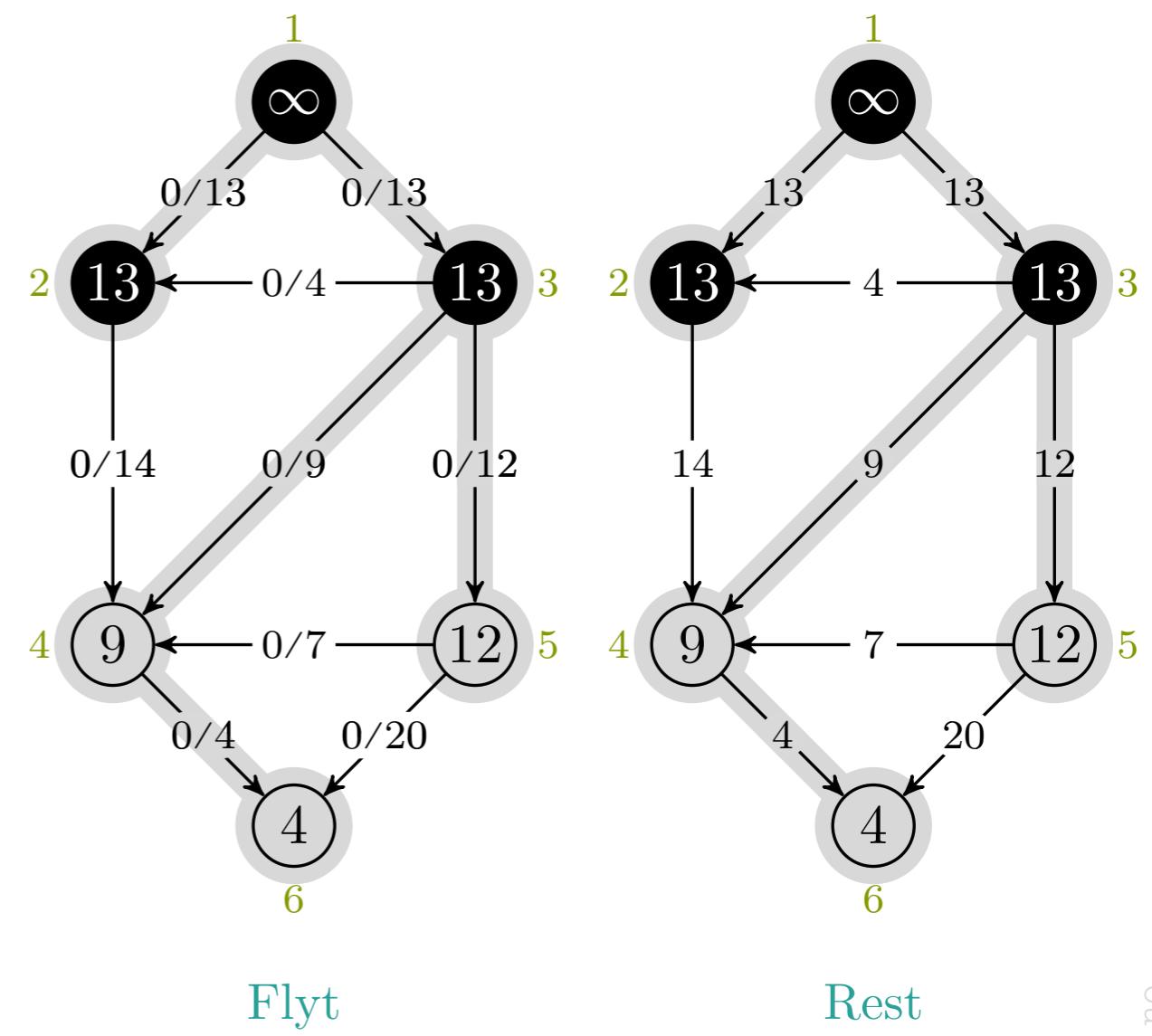


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
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22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, -$



maks-flyt → edmonds-karp

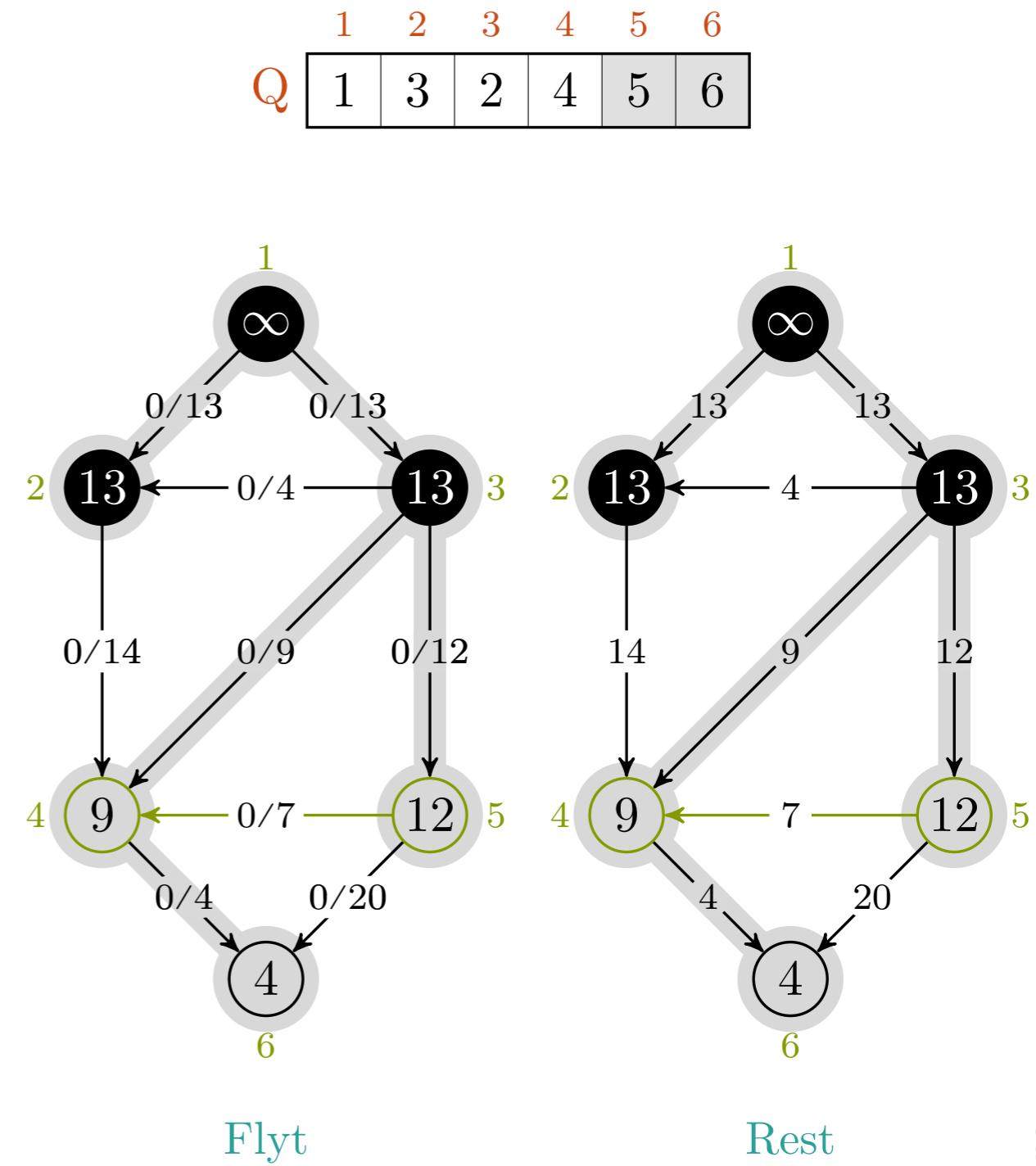
## EDMONDS-KARP(G, s, t)

```

1   for each edge  $(u, v) \in G.E$ 
2        $(u, v).f = 0$ 
3   repeat
4       for each vertex  $u \in G.V$ 
5            $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6            $u.\pi = NIL$ 
7            $s.a = \infty$ 
8            $Q = \emptyset$ 
9           ENQUEUE( $Q, s$ )
10      while  $t.a == 0$  and  $Q \neq \emptyset$ 
11           $u = DEQUEUE(Q)$ 
12          for all edges  $(u, v), (v, u) \in G.E$ 
13              if  $(u, v) \in G.E$ 
14                   $c_f(u, v) = c(u, v) - (u, v).f$ 
15              else  $c_f(u, v) = (v, u).f$ 
16              if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                   $v.a = \min(u.a, c_f(u, v))$ 
18                   $v.\pi = u$ 
19                  ENQUEUE( $Q, v$ )
20           $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21          while  $u \neq NIL$ 
22              if  $(u, v) \in G.E$ 
23                   $(u, v).f = (u, v).f + t.a$ 
24              else  $(v, u).f = (v, u).f - t.a$ 
25               $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

$$u, v = 4, 5$$

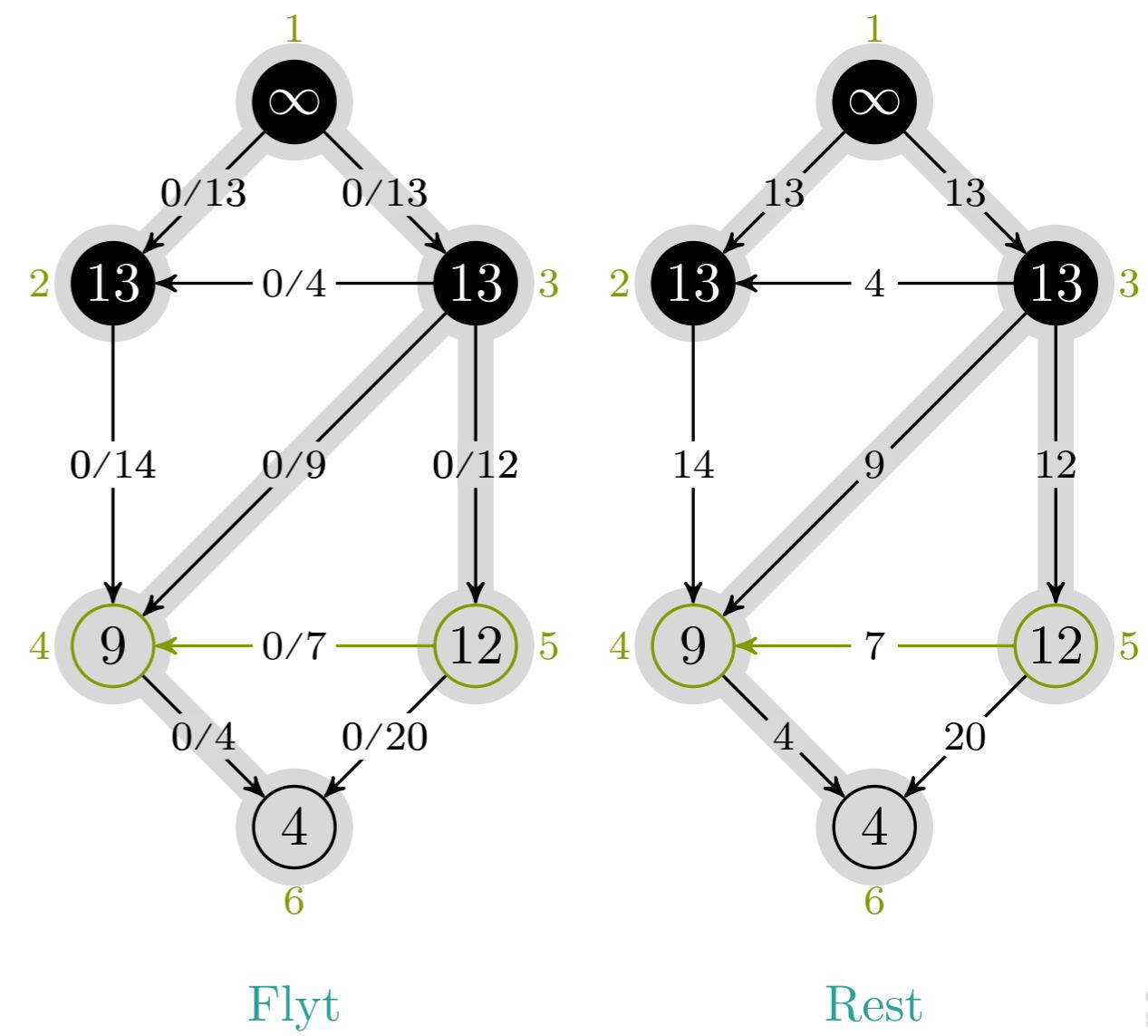


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
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14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 5$



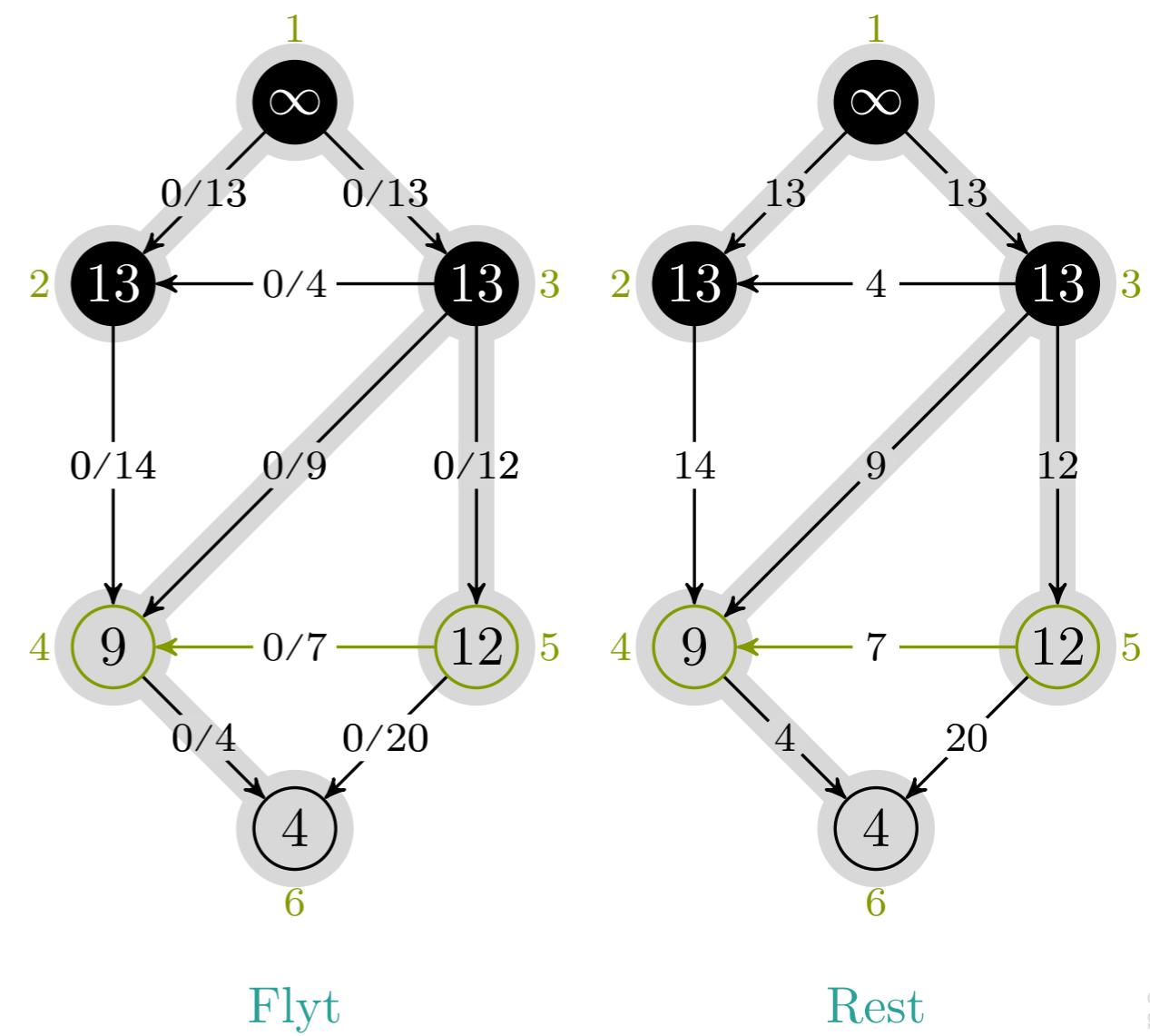
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
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16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
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20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
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24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 5$

Q	1	2	3	4	5	6
	1	3	2	4	5	6



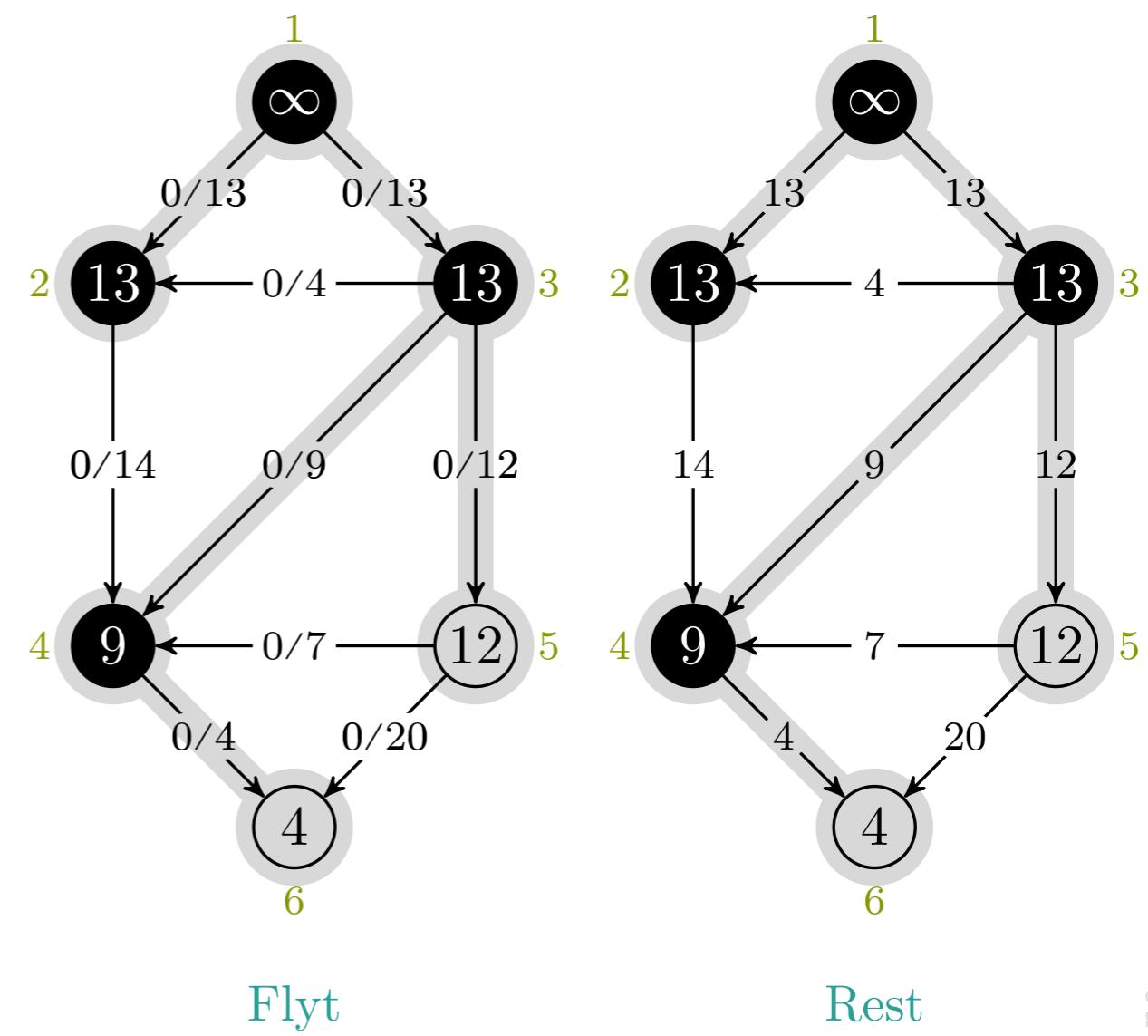
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
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24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, -$

Q	1	2	3	4	5	6
	1	3	2	4	5	6

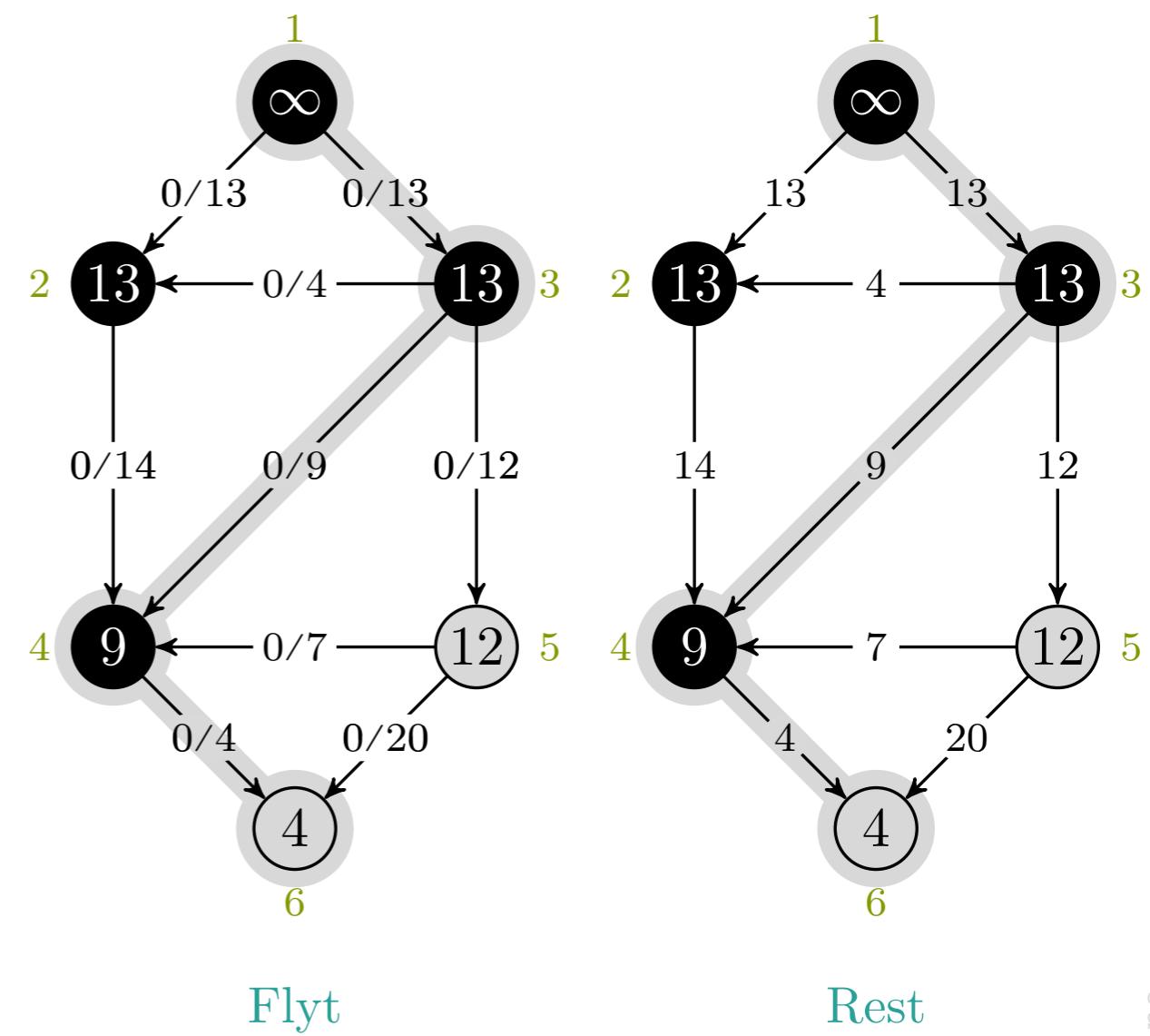


EDMONDS-KARP( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
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15      else  $c_f(u, v) = (v, u).f$ 
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18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
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22      if  $(u, v) \in G.E$ 
23         $(u, v).f = (u, v).f + t.a$ 
24      else  $(v, u).f = (v, u).f - t.a$ 
25     $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = -, -$

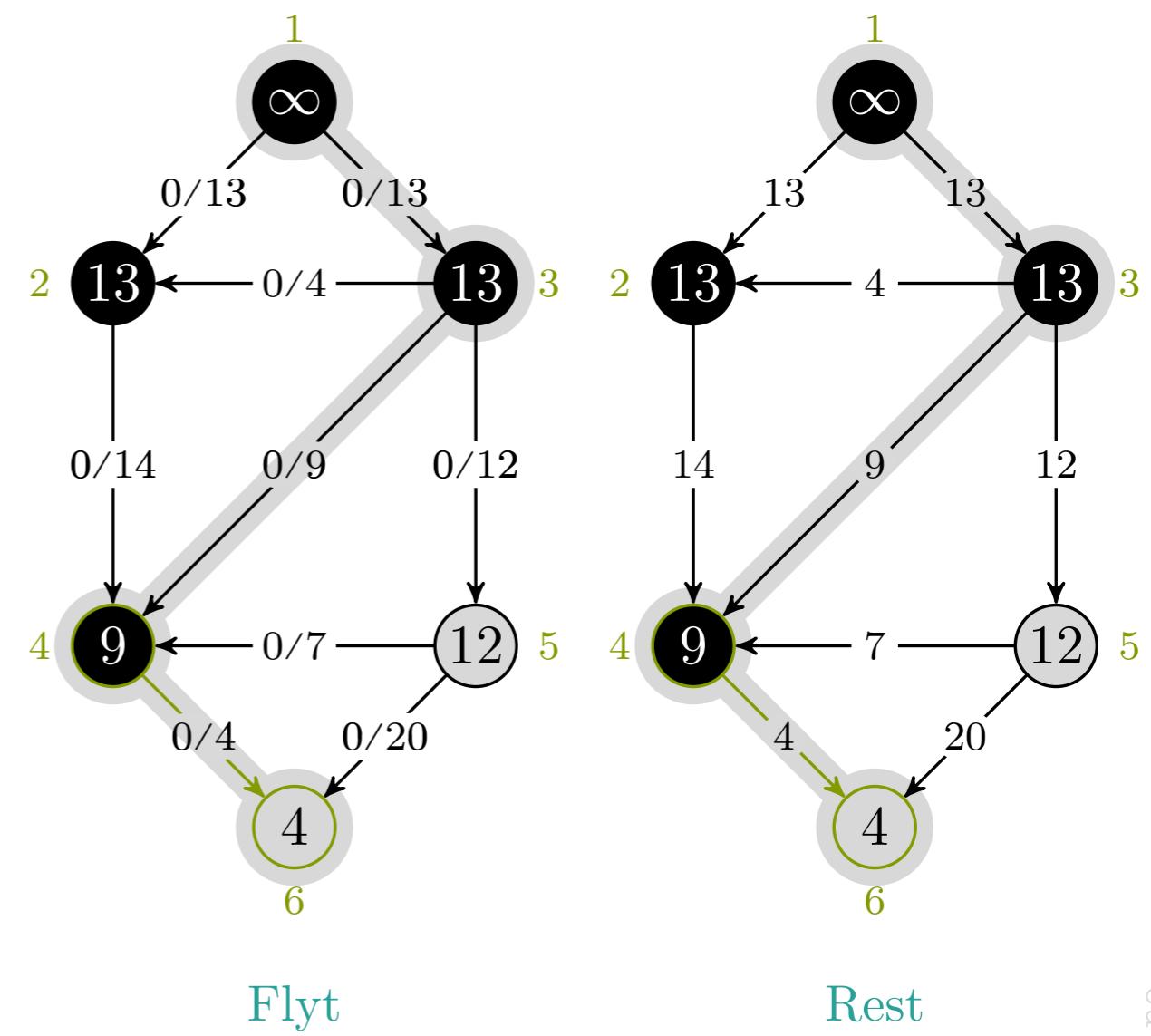


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
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10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
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20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 6$

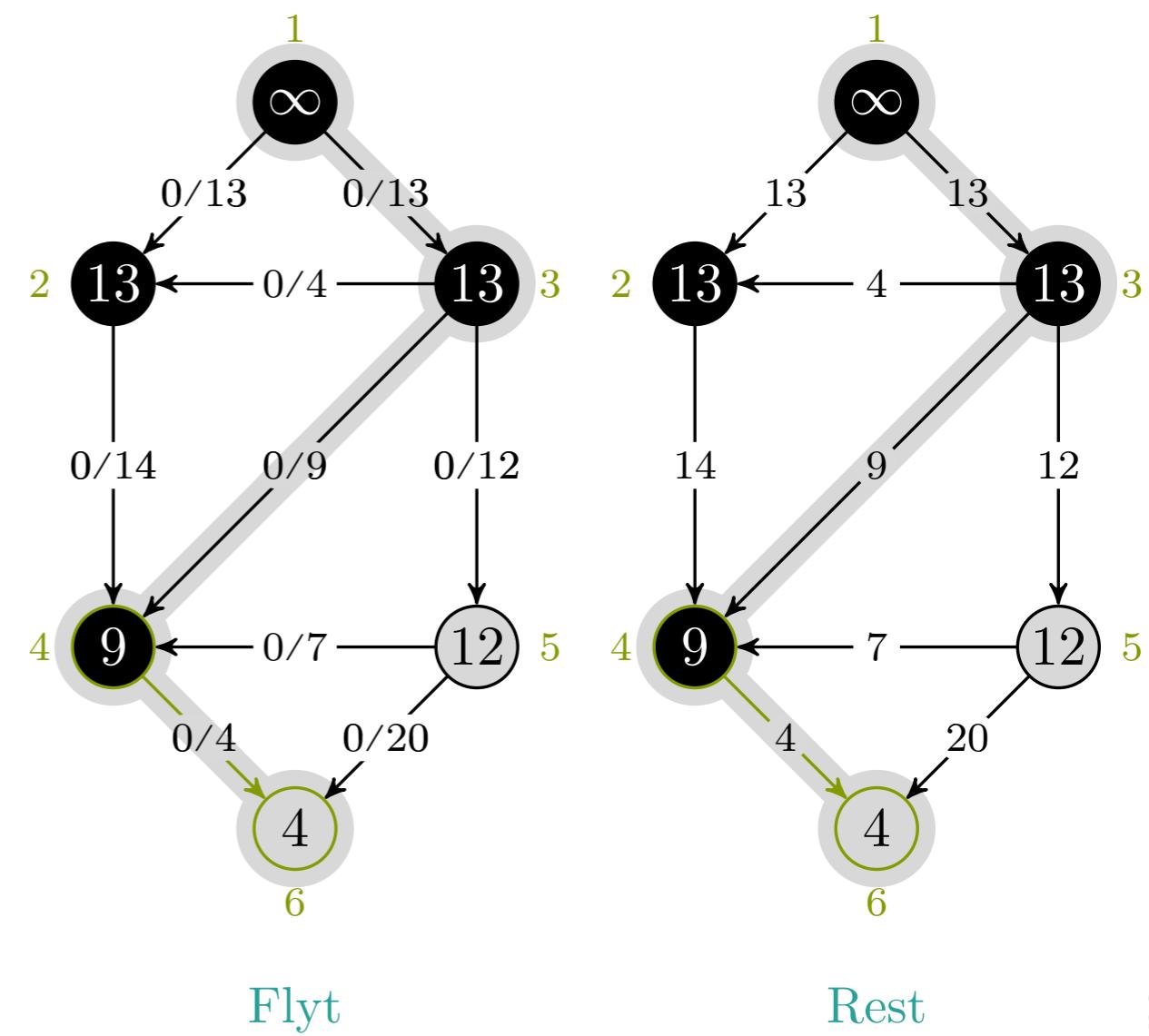


```
EDMONDS-KARP(G, s, t)
```

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7    $s.a = \infty$ 
8    $Q = \emptyset$ 
9   ENQUEUE(Q, s)
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE(Q, v)
20     $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21    while  $u \neq \text{NIL}$ 
22      if  $(u, v) \in G.E$ 
23         $(u, v).f = (u, v).f + t.a$ 
24      else  $(v, u).f = (v, u).f - t.a$ 
25     $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 6$

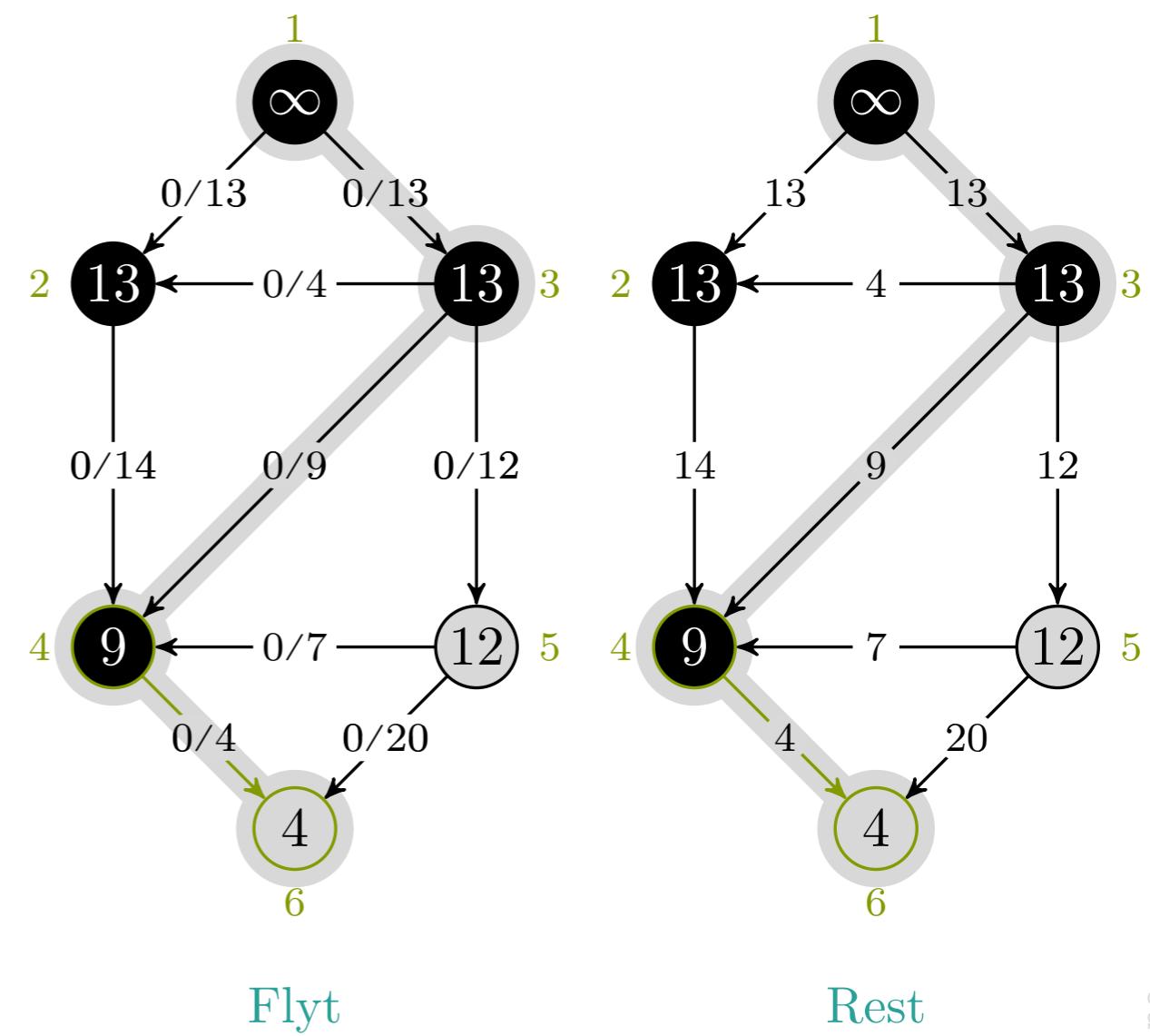


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 6$

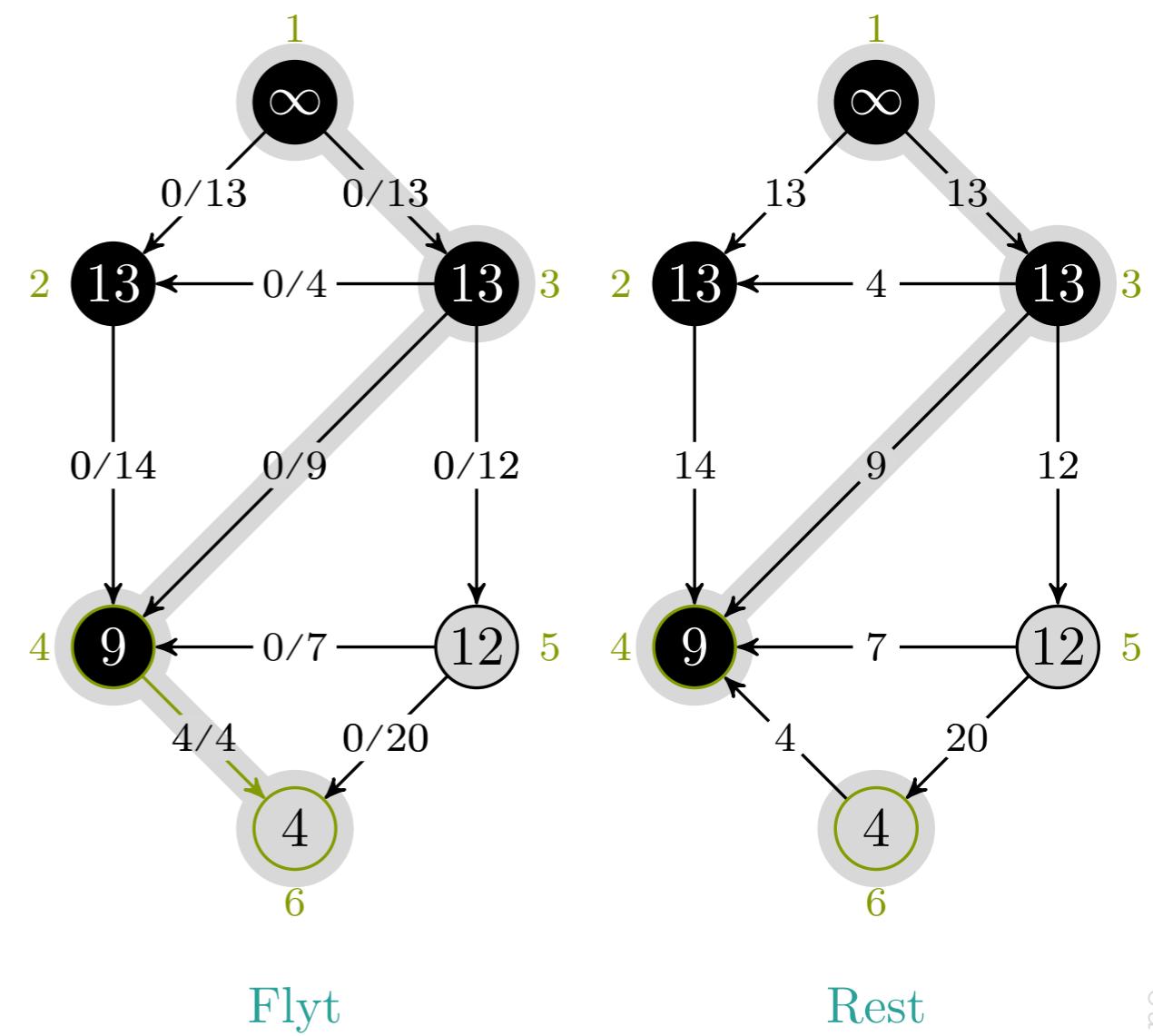


EDMONDS-KARP( $G, s, t$ )

```

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25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 6$

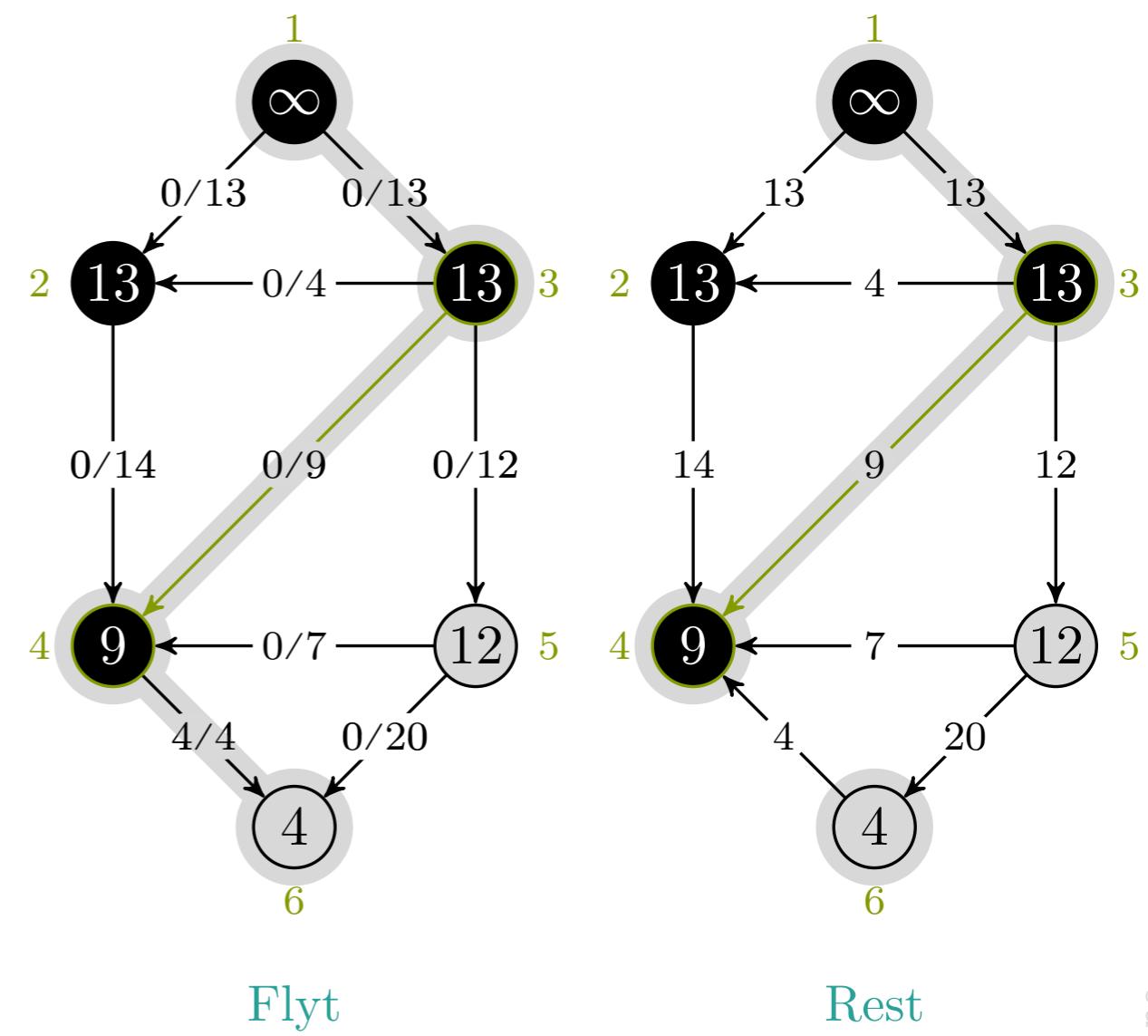


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```

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24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 4$



maks-flyt → edmonds-karp

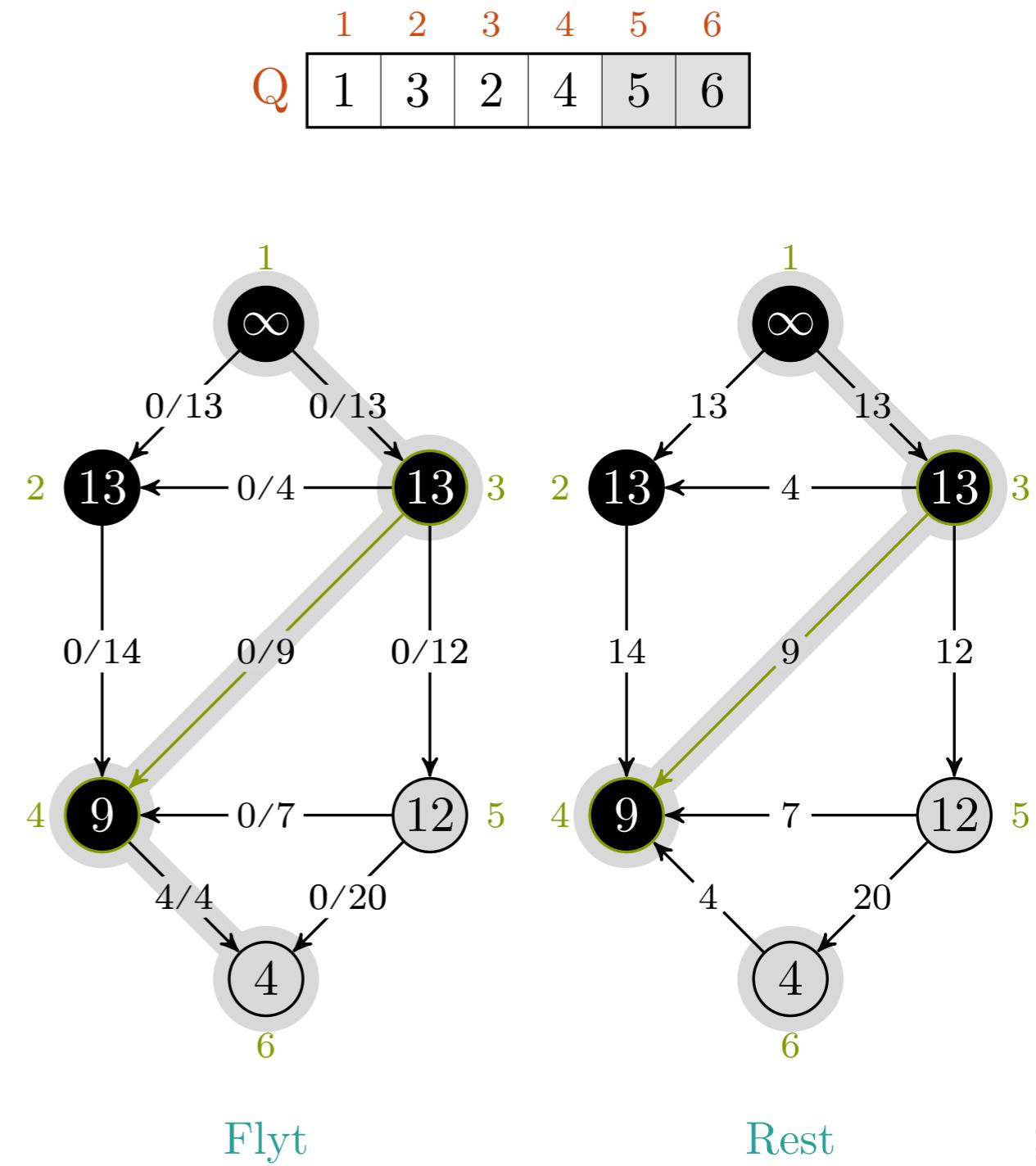
## EDMONDS-KARP(G, s, t)

```

1   for each edge  $(u, v) \in G.E$ 
2        $(u, v).f = 0$ 
3   repeat
4       for each vertex  $u \in G.V$ 
5            $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6            $u.\pi = NIL$ 
7            $s.a = \infty$ 
8            $Q = \emptyset$ 
9           ENQUEUE( $Q, s$ )
10          while  $t.a == 0$  and  $Q \neq \emptyset$ 
11               $u = DEQUEUE(Q)$ 
12              for all edges  $(u, v), (v, u) \in G.E$ 
13                  if  $(u, v) \in G.E$ 
14                       $c_f(u, v) = c(u, v) - (u, v).f$ 
15                  else  $c_f(u, v) = (v, u).f$ 
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17                       $v.a = \min(u.a, c_f(u, v))$ 
18                       $v.\pi = u$ 
19                      ENQUEUE( $Q, v$ )
20           $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21          while  $u \neq NIL$ 
22              if  $(u, v) \in G.E$ 
23                   $(u, v).f = (u, v).f + t.a$ 
24              else  $(v, u).f = (v, u).f - t.a$ 
25               $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

$$u, v = 3, 4$$

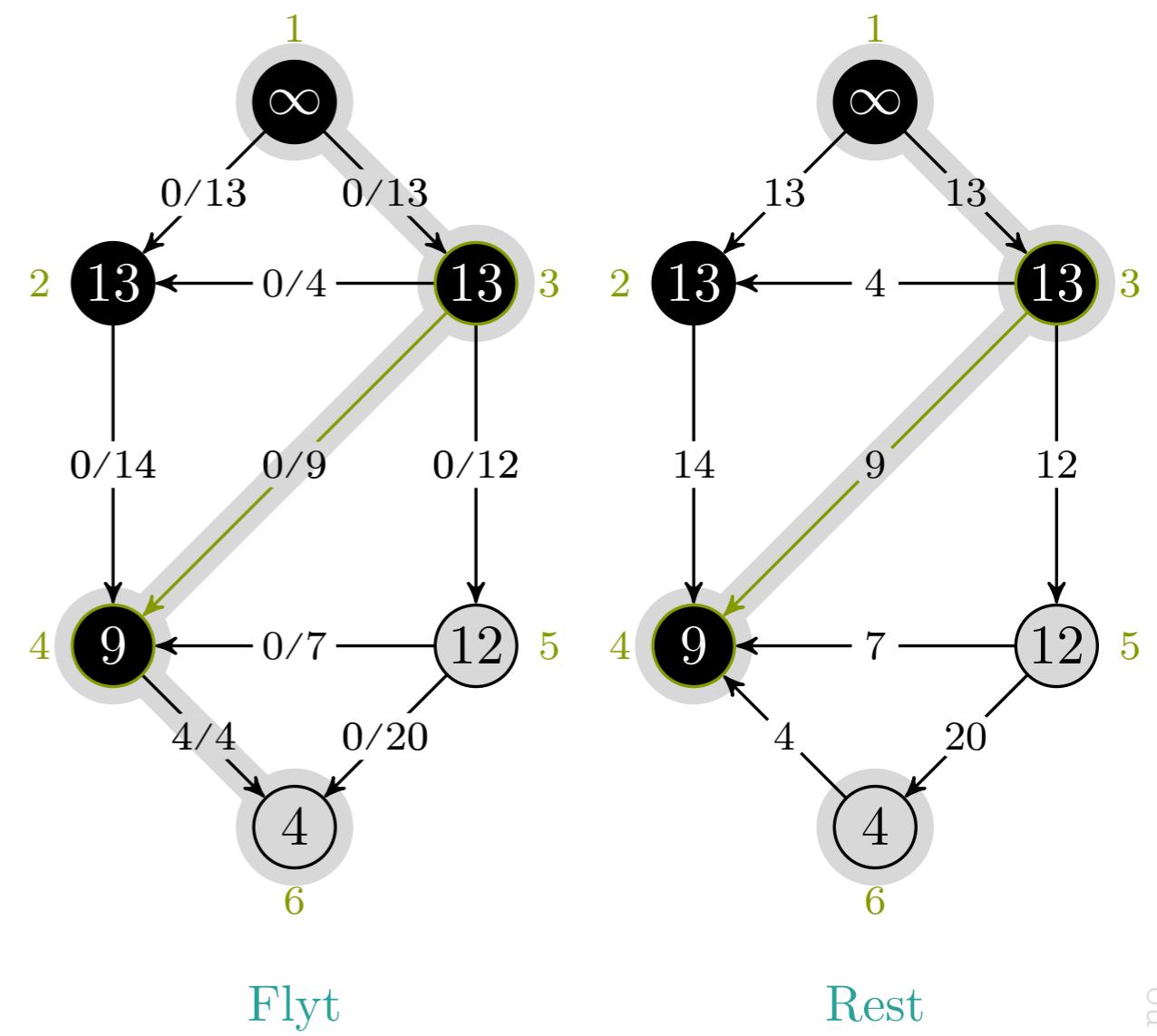


EDMONDS-KARP( $G, s, t$ )

```

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2     $(u, v).f = 0$ 
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4    for each vertex  $u \in G.V$ 
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25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 4$

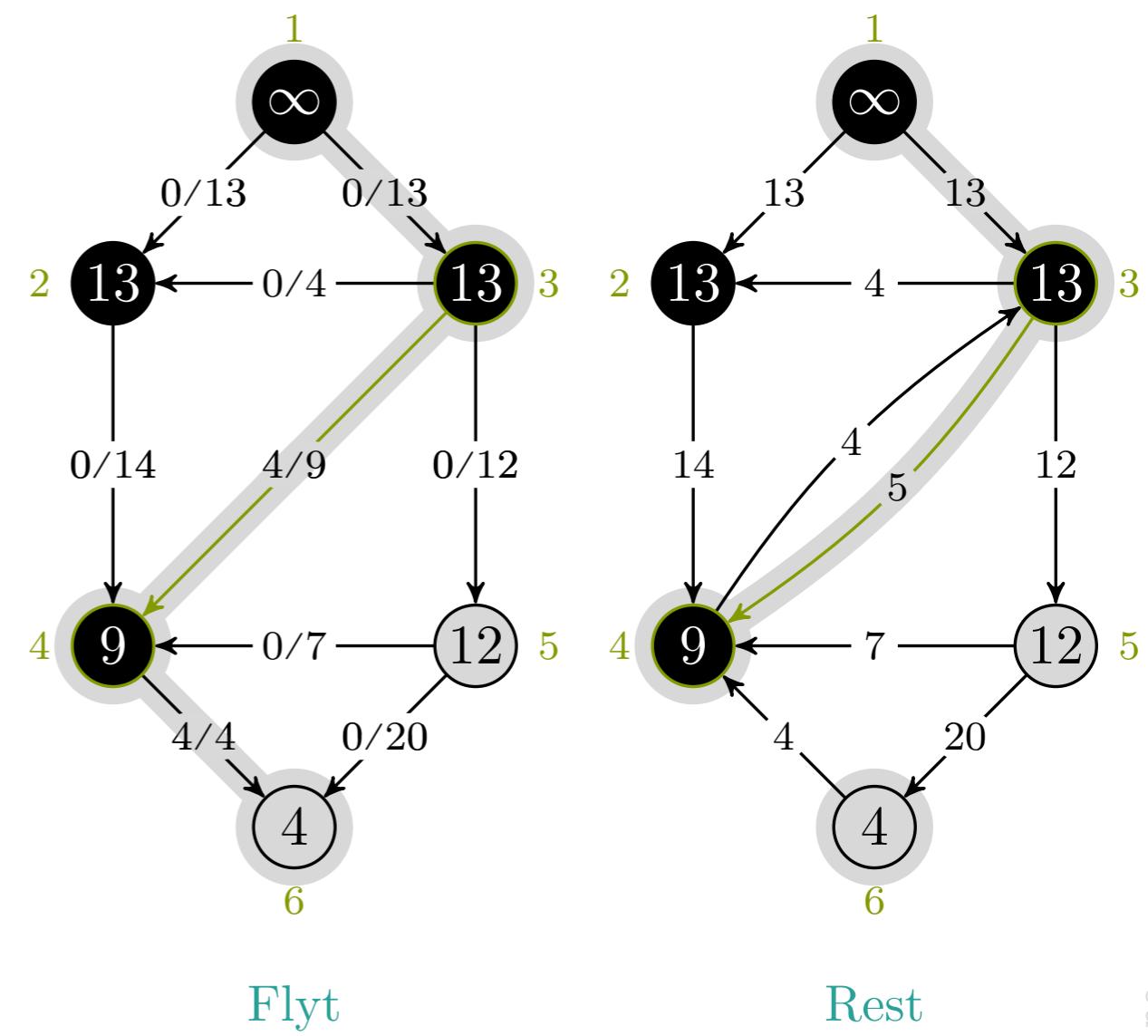


```
EDMONDS-KARP(G, s, t)
```

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
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25     $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 4$

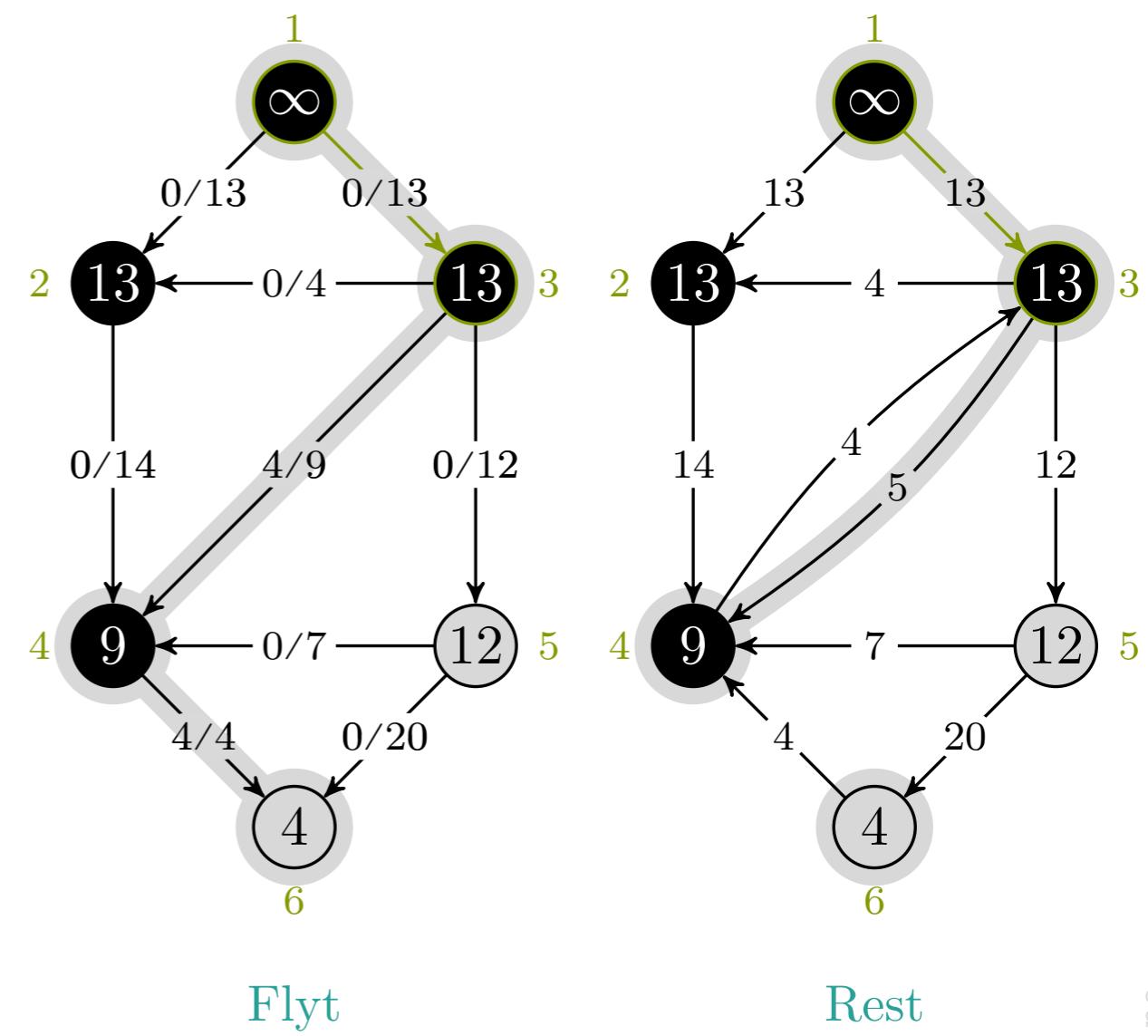


EDMONDS-KARP( $G, s, t$ )

```

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2     $(u, v).f = 0$ 
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25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, 3$

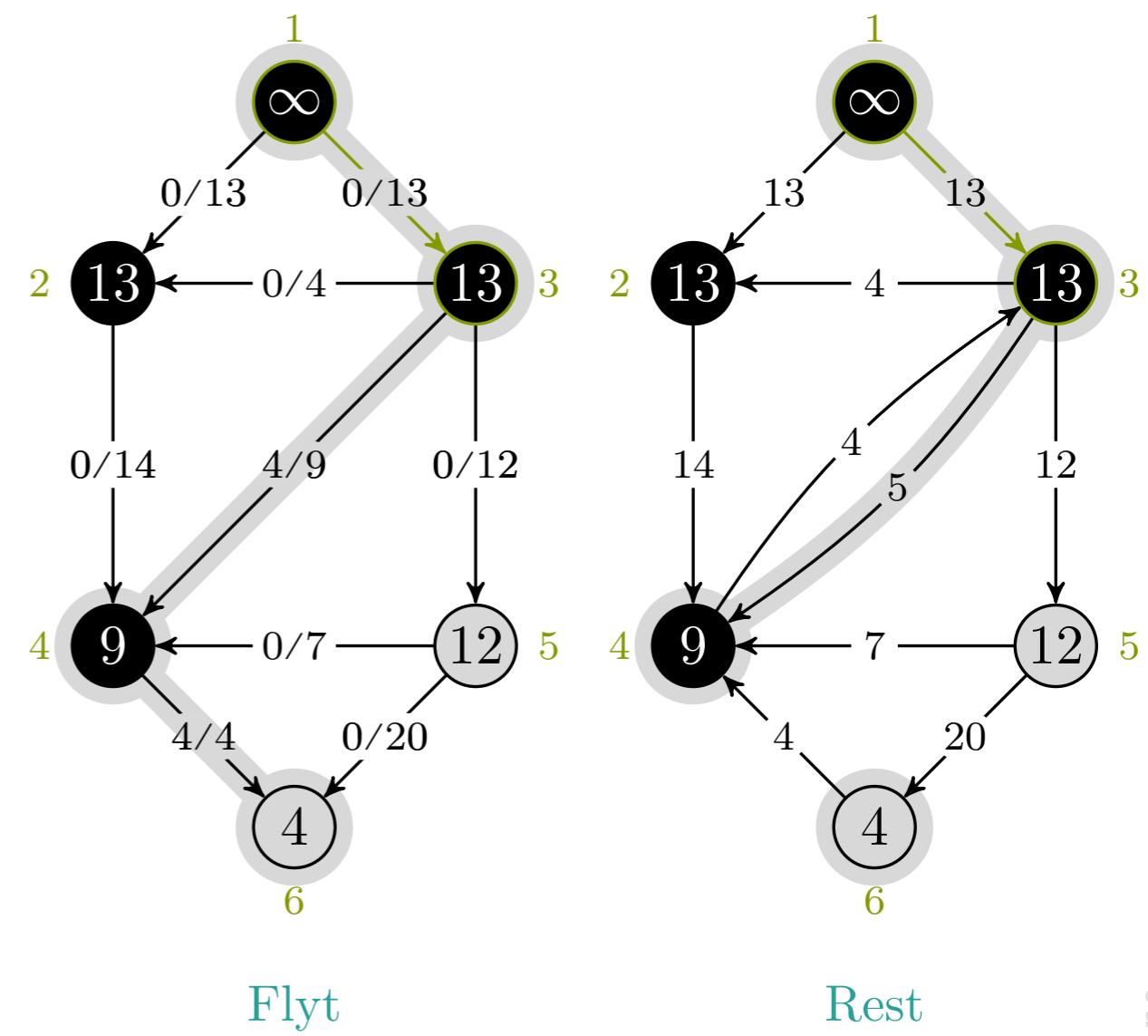


EDMONDS-KARP( $G, s, t$ )

```

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4    for each vertex  $u \in G.V$ 
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22       if  $(u, v) \in G.E$ 
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25        $u, v = u.\pi, u$ 
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```

$u, v = 1, 3$

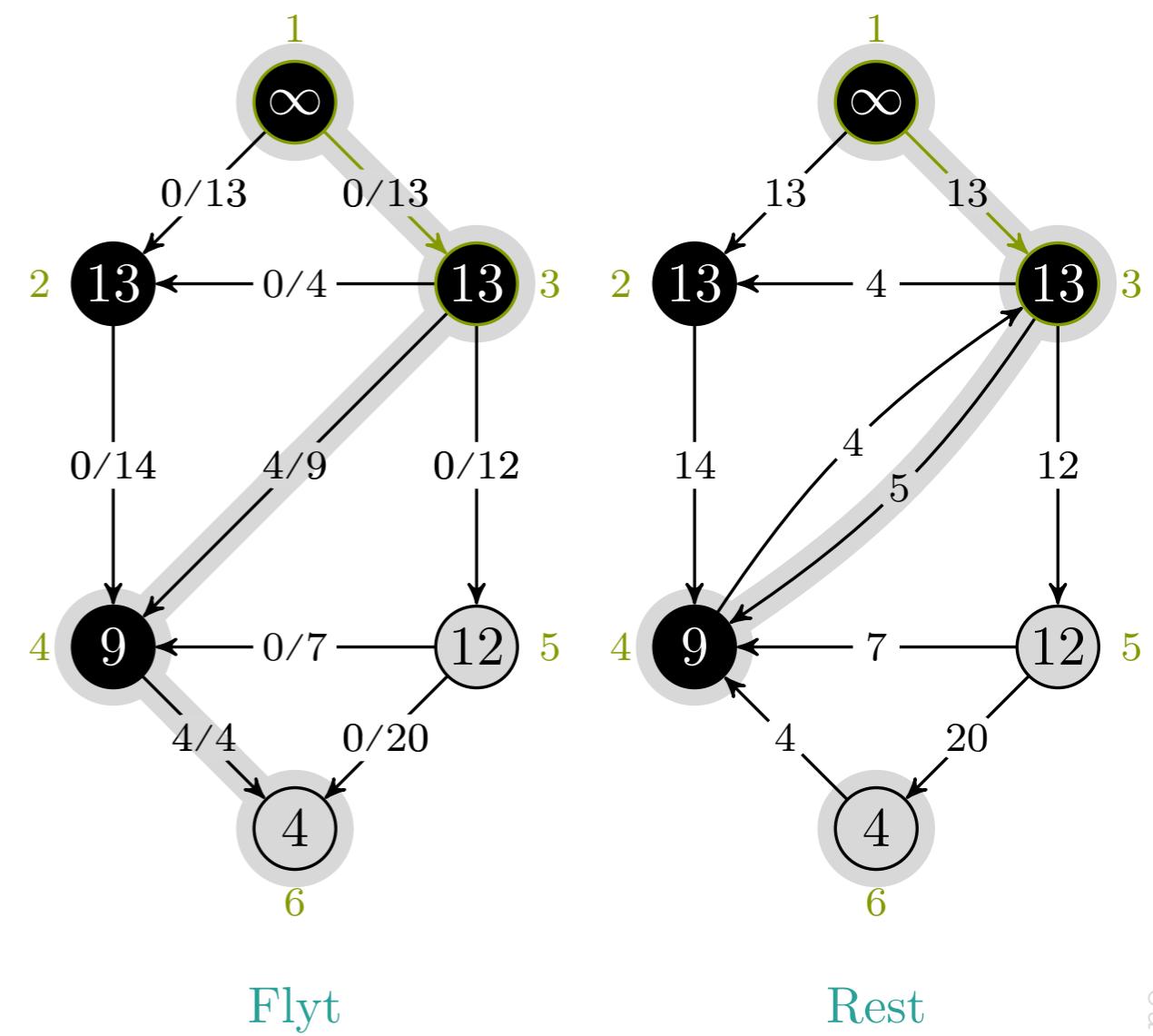


EDMONDS-KARP( $G, s, t$ )

```

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```

$u, v = 1, 3$

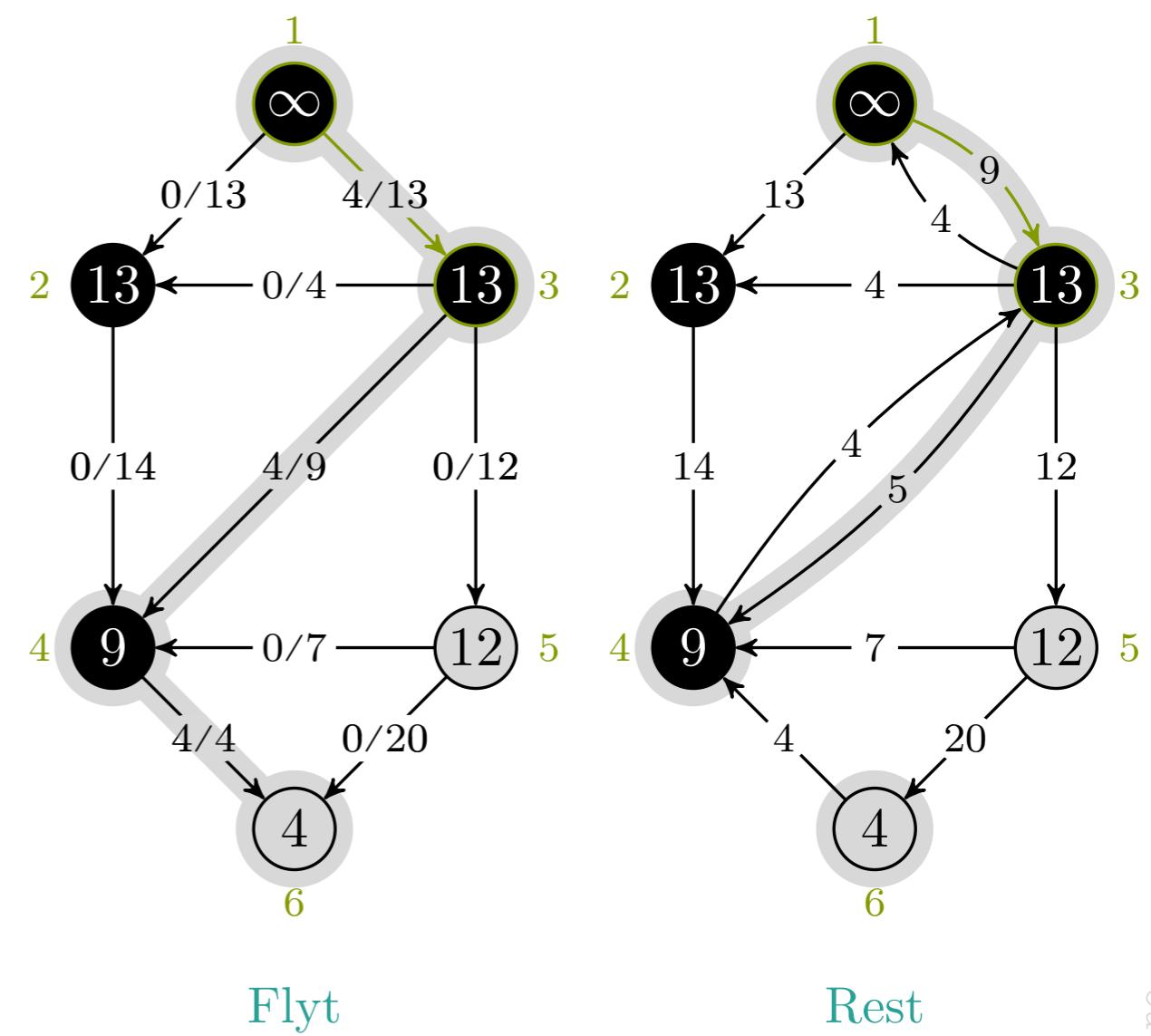


```
EDMONDS-KARP(G, s, t)
```

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$u, v = 1, 3$

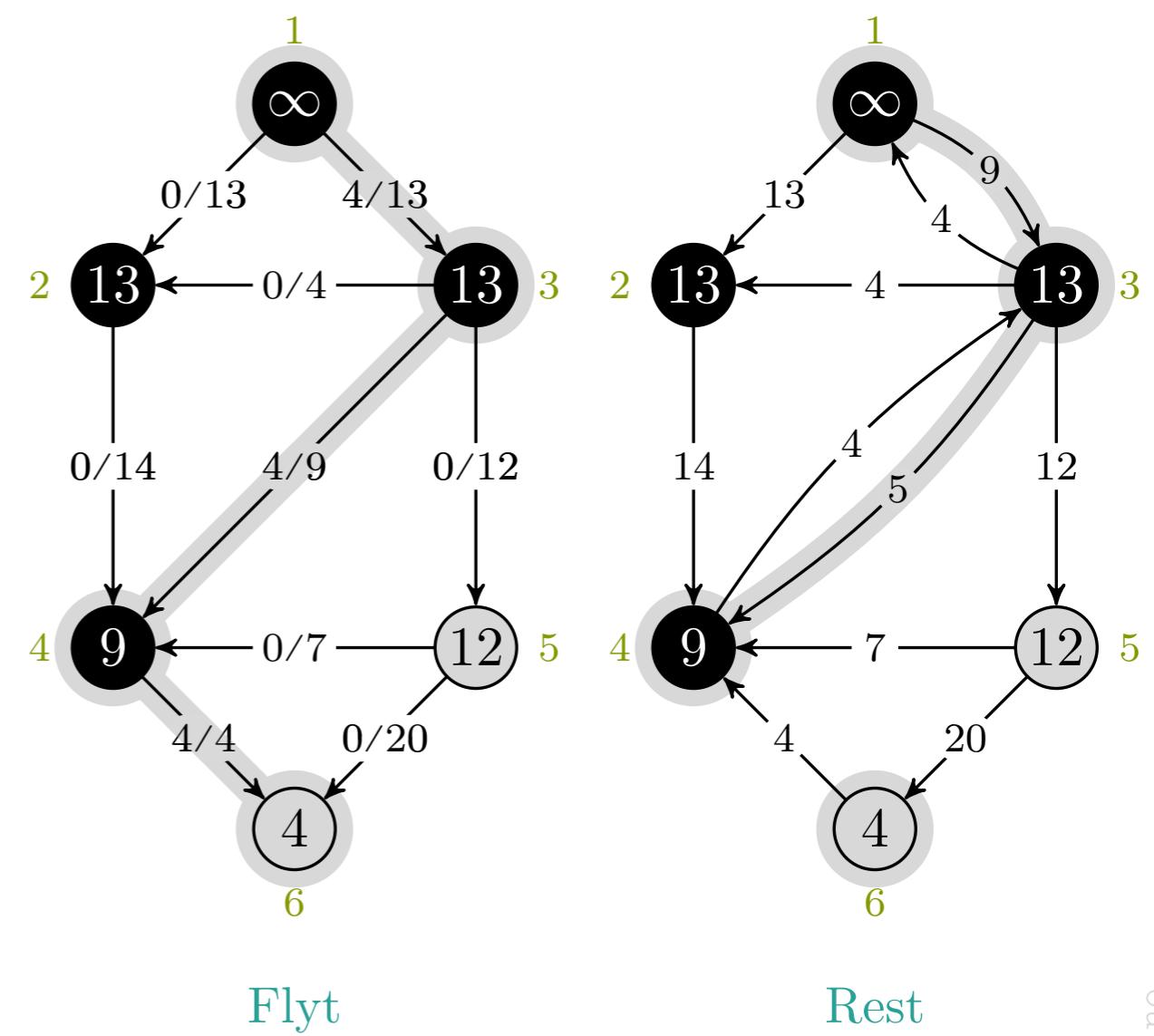


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24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$

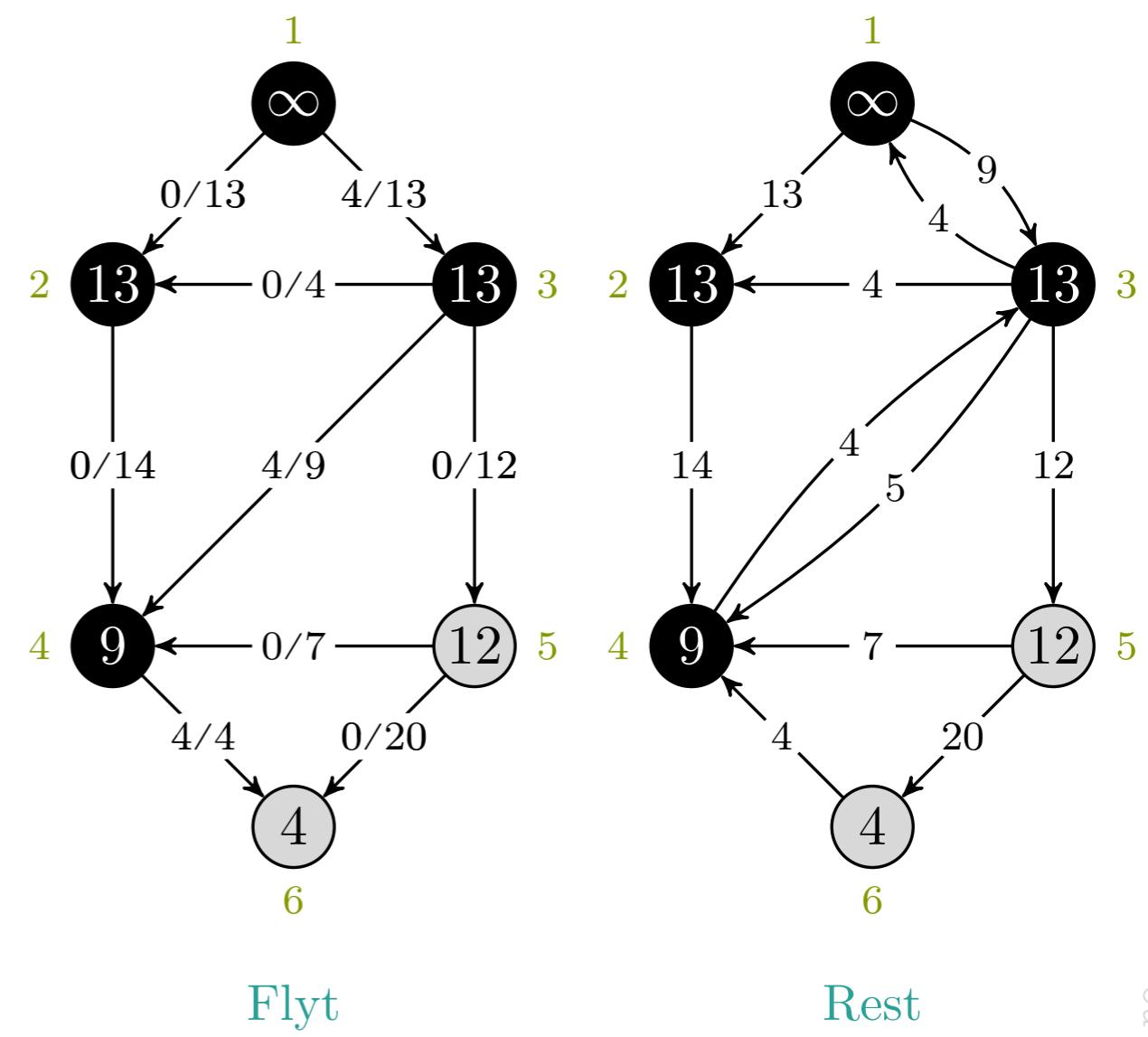


EDMONDS-KARP( $G, s, t$ )

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```

$u, v = \text{NIL}, 1$



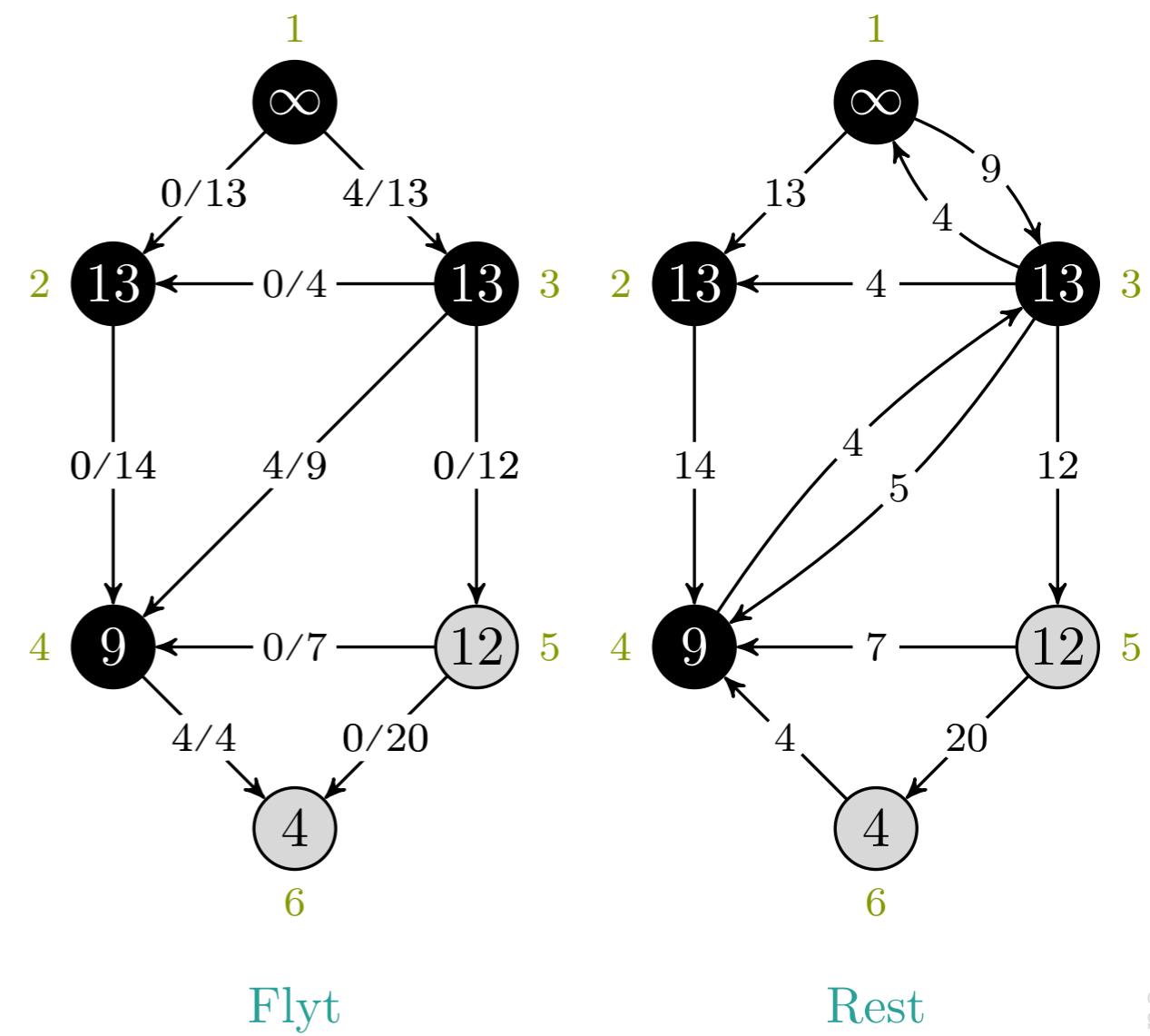
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```

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24          else  $(v, u).f = (v, u).f - t.a$ 
25           $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$

Q	1	2	3	4	5	6
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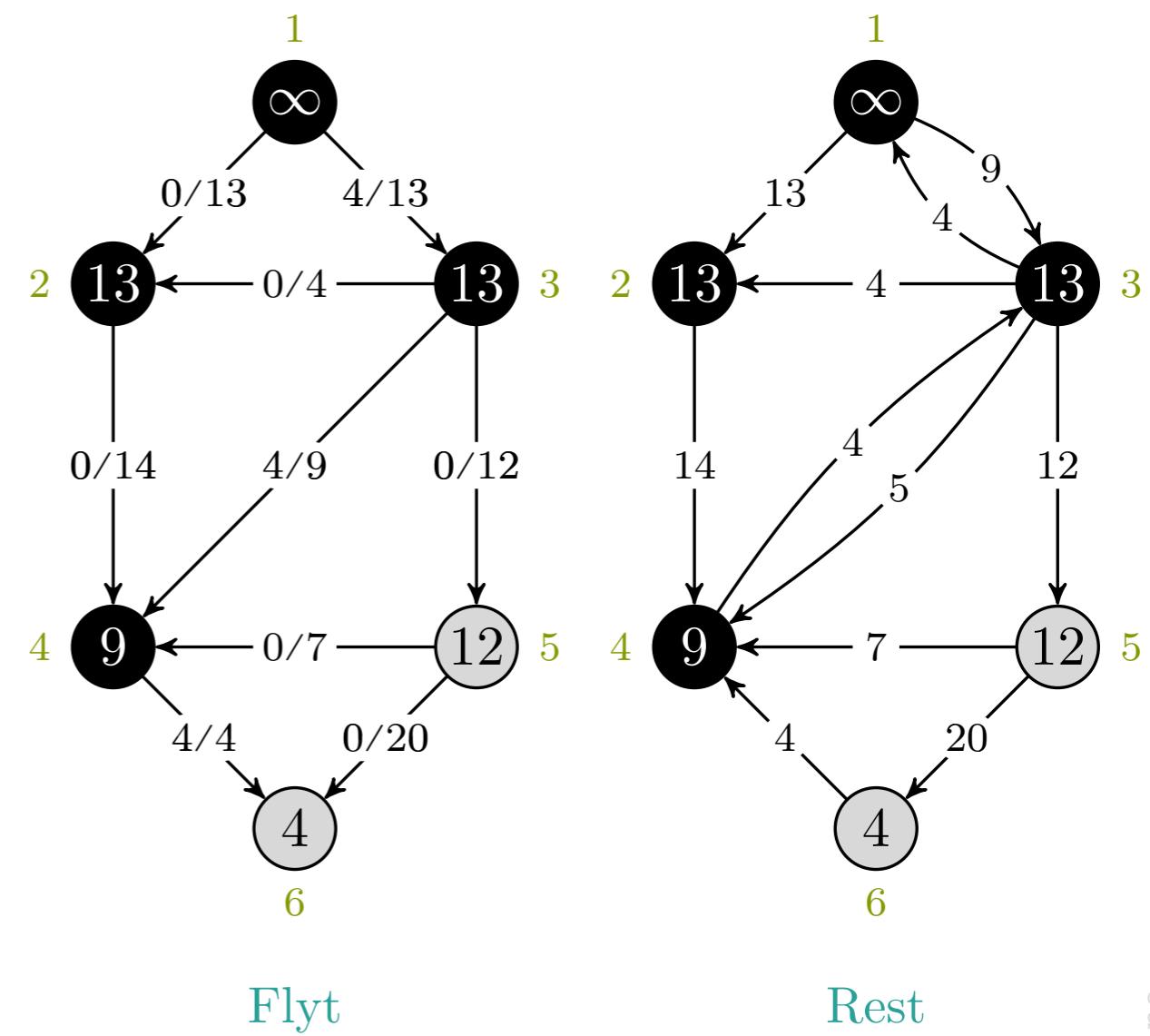


EDMONDS-KARP( $G, s, t$ )

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13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20     $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21    while  $u \neq \text{NIL}$ 
22      if  $(u, v) \in G.E$ 
23         $(u, v).f = (u, v).f + t.a$ 
24      else  $(v, u).f = (v, u).f - t.a$ 
25     $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$

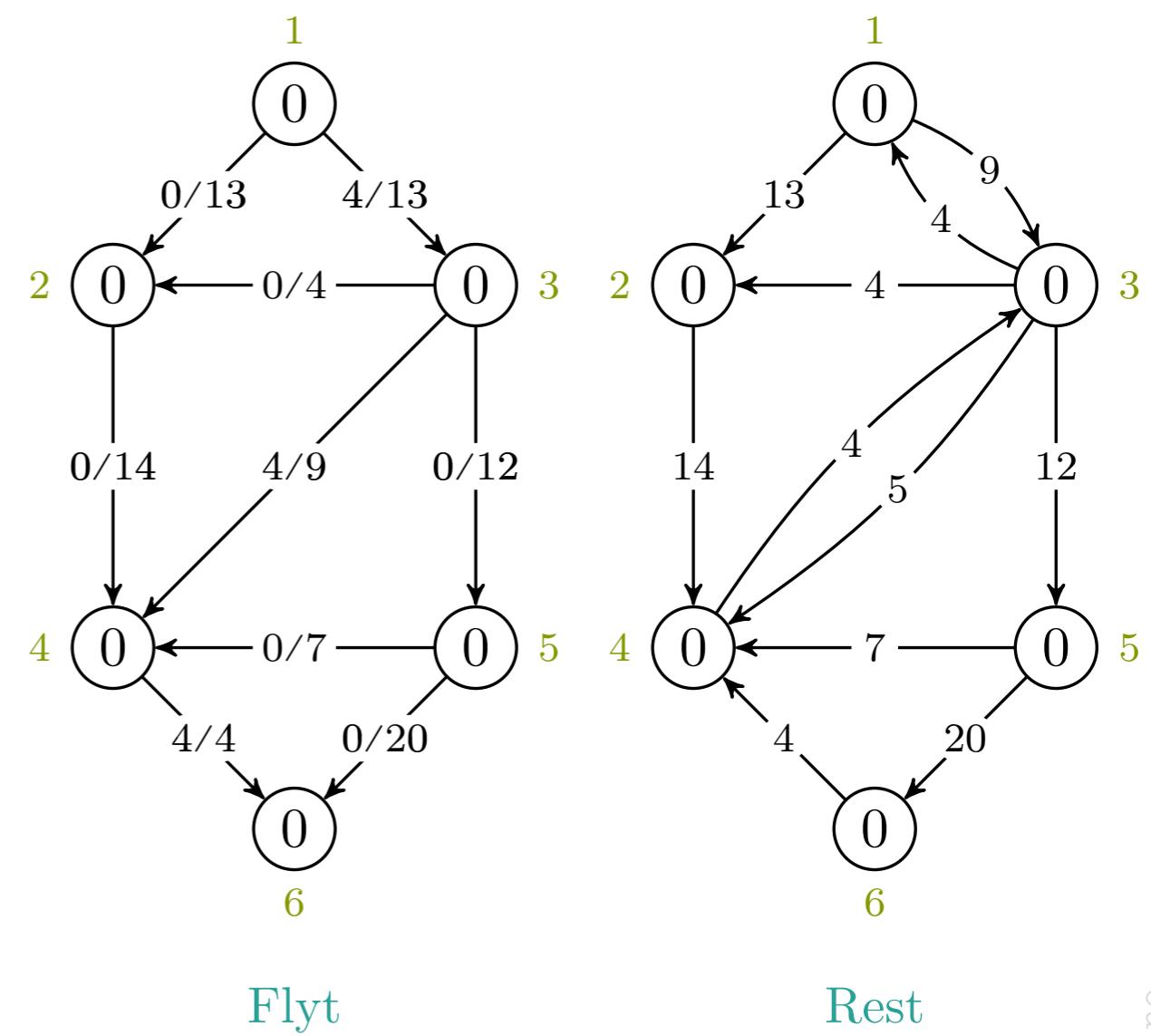


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7     $s.a = \infty$ 
8     $Q = \emptyset$ 
9    ENQUEUE( $Q, s$ )
10   while  $t.a == 0$  and  $Q \neq \emptyset$ 
11      $u = \text{DEQUEUE}(Q)$ 
12     for all edges  $(u, v), (v, u) \in G.E$ 
13       if  $(u, v) \in G.E$ 
14          $c_f(u, v) = c(u, v) - (u, v).f$ 
15       else  $c_f(u, v) = (v, u).f$ 
16       if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17          $v.a = \min(u.a, c_f(u, v))$ 
18          $v.\pi = u$ 
19         ENQUEUE( $Q, v$ )
20      $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
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23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25      $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$

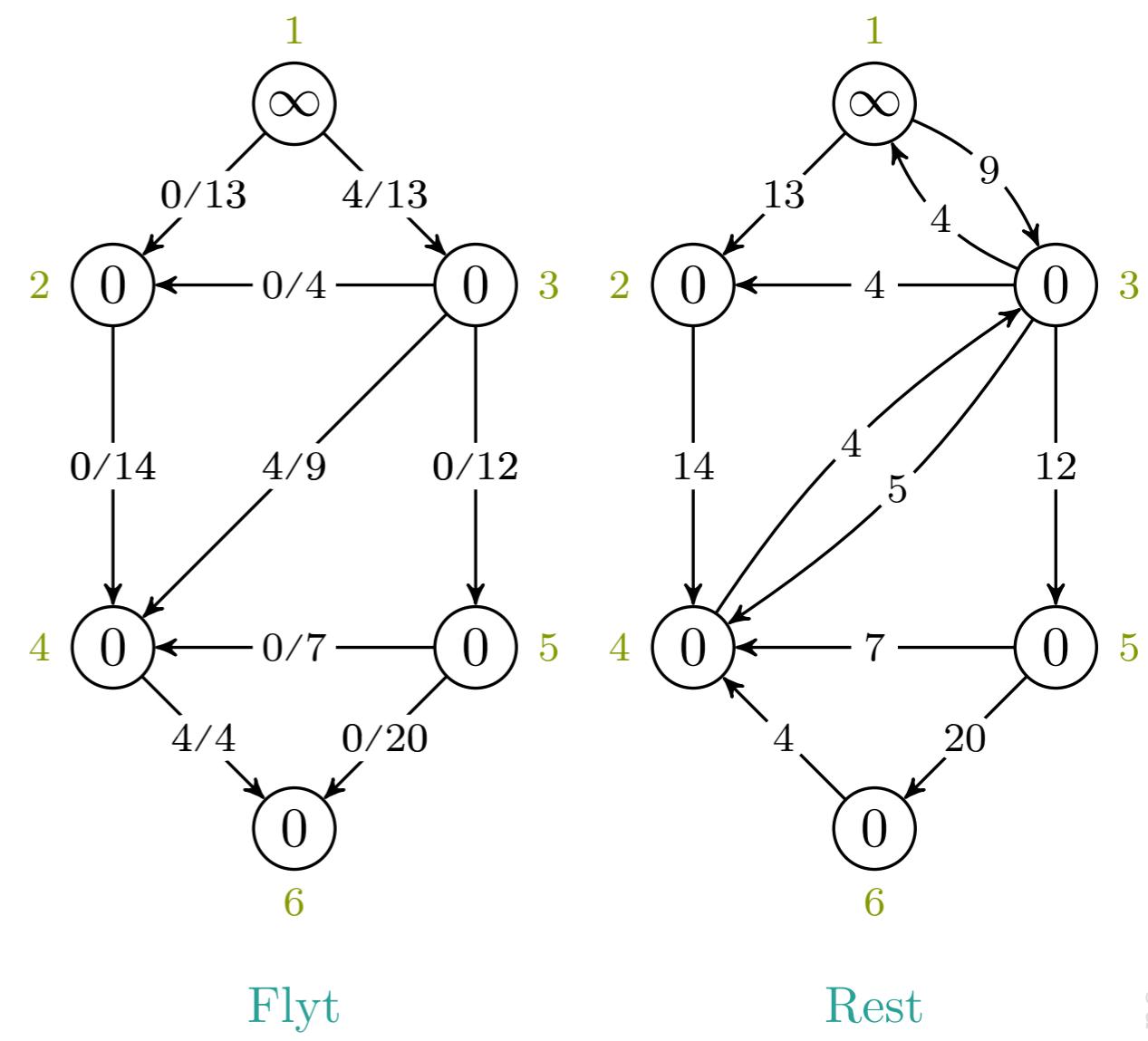


EDMONDS-KARP( $G, s, t$ )

```

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2       $(u, v).f = 0$ 
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13             if  $(u, v) \in G.E$ 
14                  $c_f(u, v) = c(u, v) - (u, v).f$ 
15             else  $c_f(u, v) = (v, u).f$ 
16             if  $c_f(u, v) > 0$  and  $v.a == 0$ 
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18                  $v.\pi = u$ 
19                 ENQUEUE( $Q, v$ )
20              $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21             while  $u \neq \text{NIL}$ 
22                 if  $(u, v) \in G.E$ 
23                      $(u, v).f = (u, v).f + t.a$ 
24                 else  $(v, u).f = (v, u).f - t.a$ 
25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$

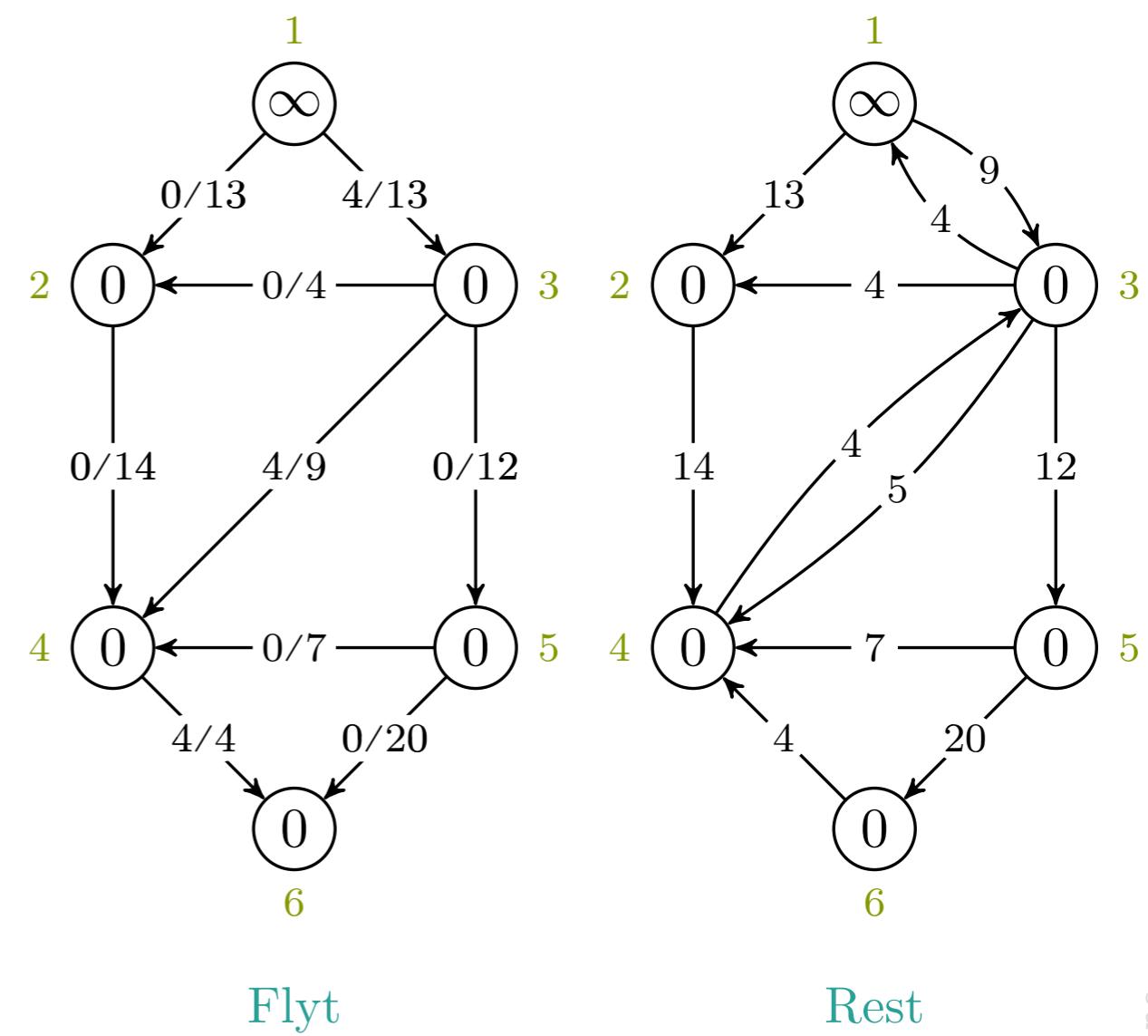


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2       $(u, v).f = 0$ 
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6           $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11          $u = \text{DEQUEUE}(Q)$ 
12         for all edges  $(u, v), (v, u) \in G.E$ 
13             if  $(u, v) \in G.E$ 
14                  $c_f(u, v) = c(u, v) - (u, v).f$ 
15             else  $c_f(u, v) = (v, u).f$ 
16             if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                  $v.a = \min(u.a, c_f(u, v))$ 
18                  $v.\pi = u$ 
19                 ENQUEUE( $Q, v$ )
20              $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21             while  $u \neq \text{NIL}$ 
22                 if  $(u, v) \in G.E$ 
23                      $(u, v).f = (u, v).f + t.a$ 
24                 else  $(v, u).f = (v, u).f - t.a$ 
25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$



EDMONDS-KARP( $G, s, t$ )

```

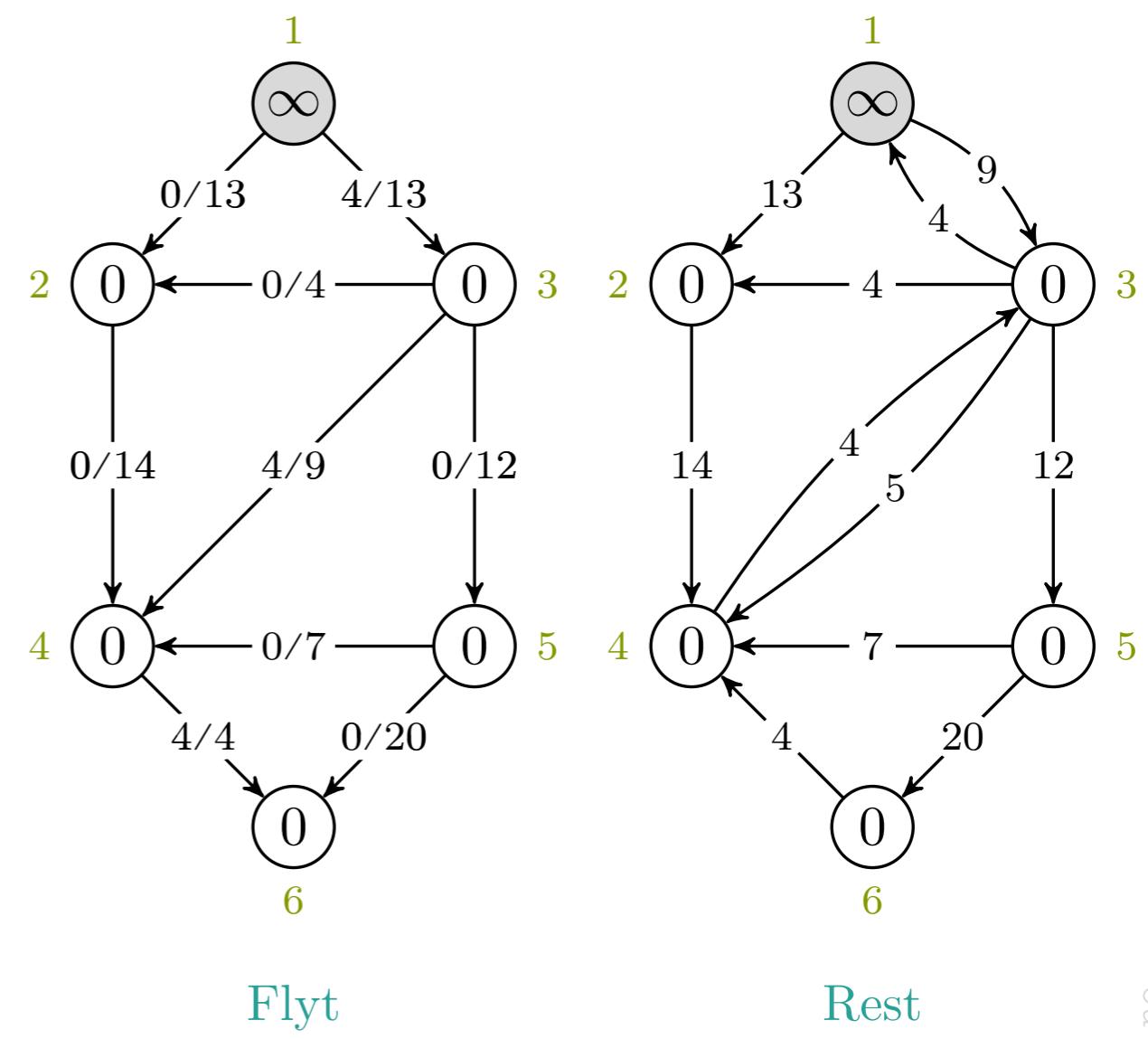
1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
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19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

 $u, v = \text{NIL}, 1$ 

1    2    3    4    5    6

<b>Q</b>	1	3	2	4	5	6
----------	---	---	---	---	---	---

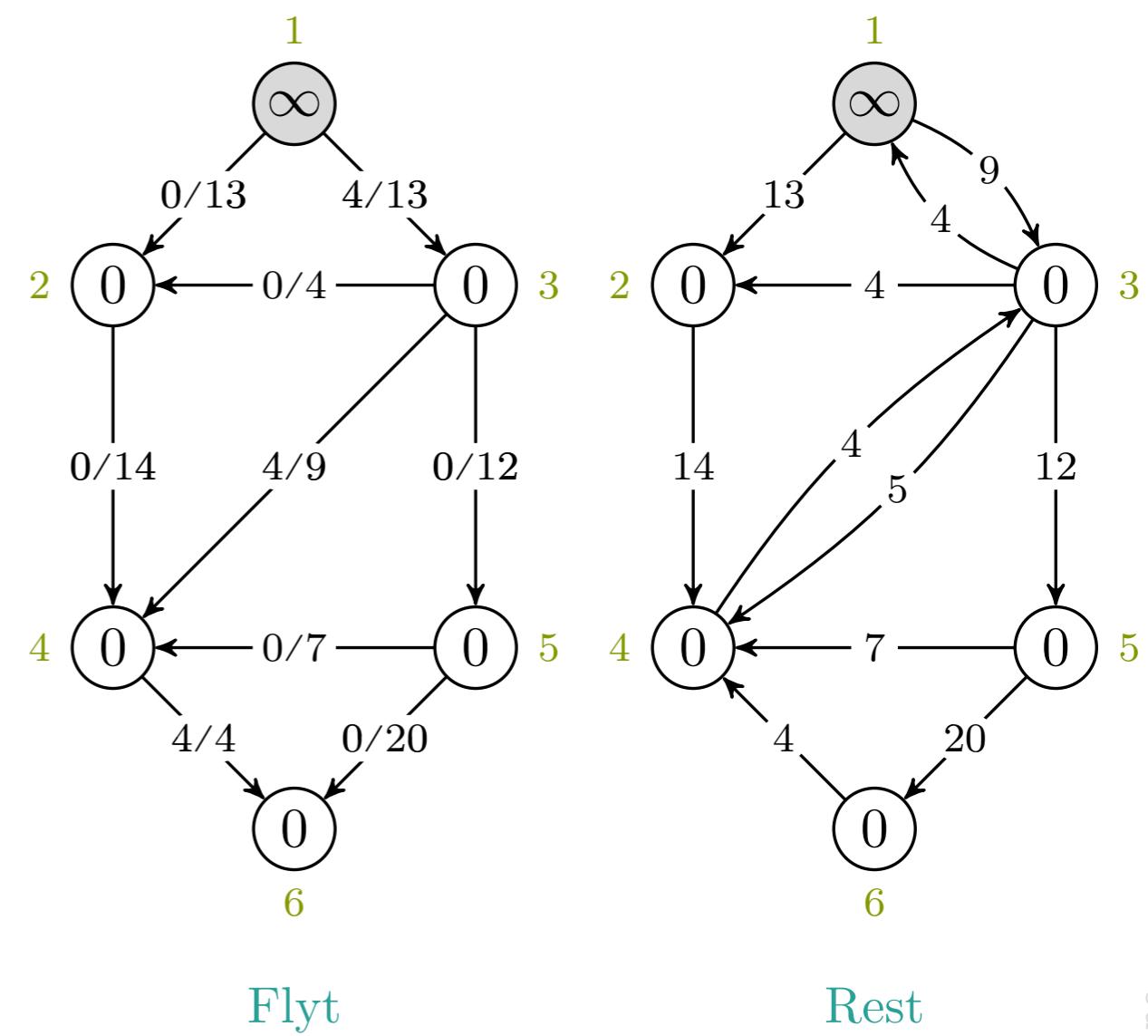


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

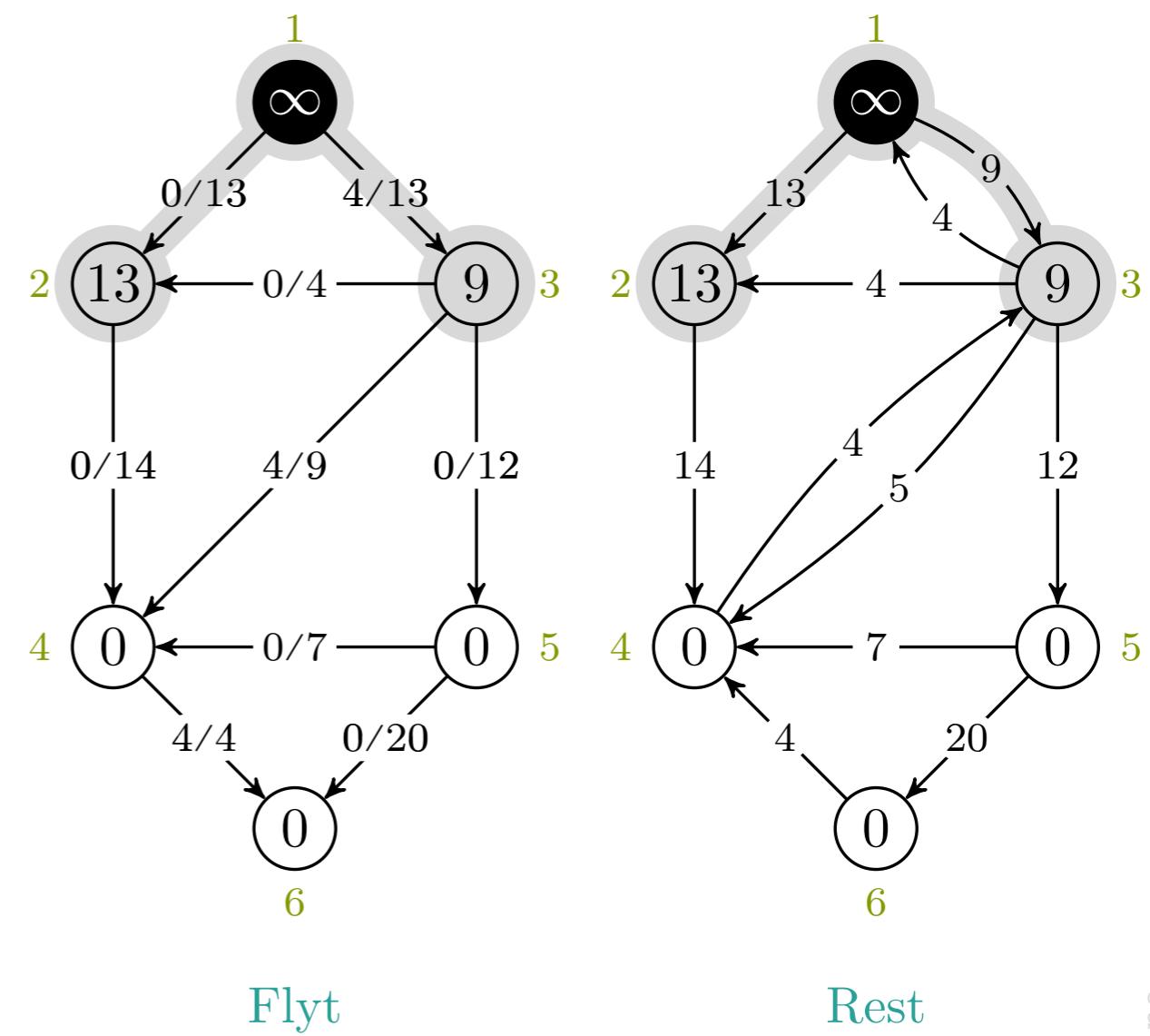
 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
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24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, -$

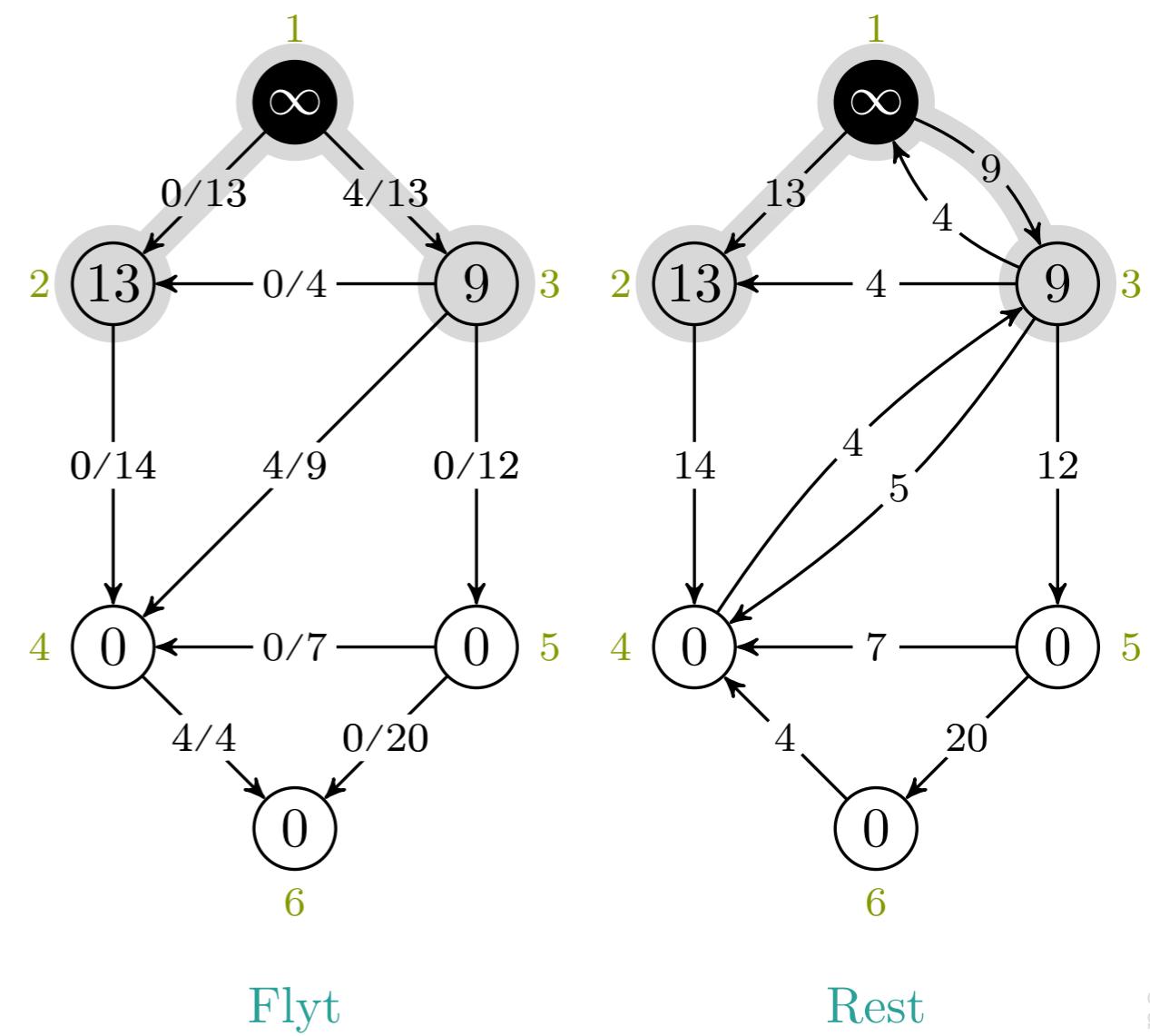


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
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17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
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22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, -$

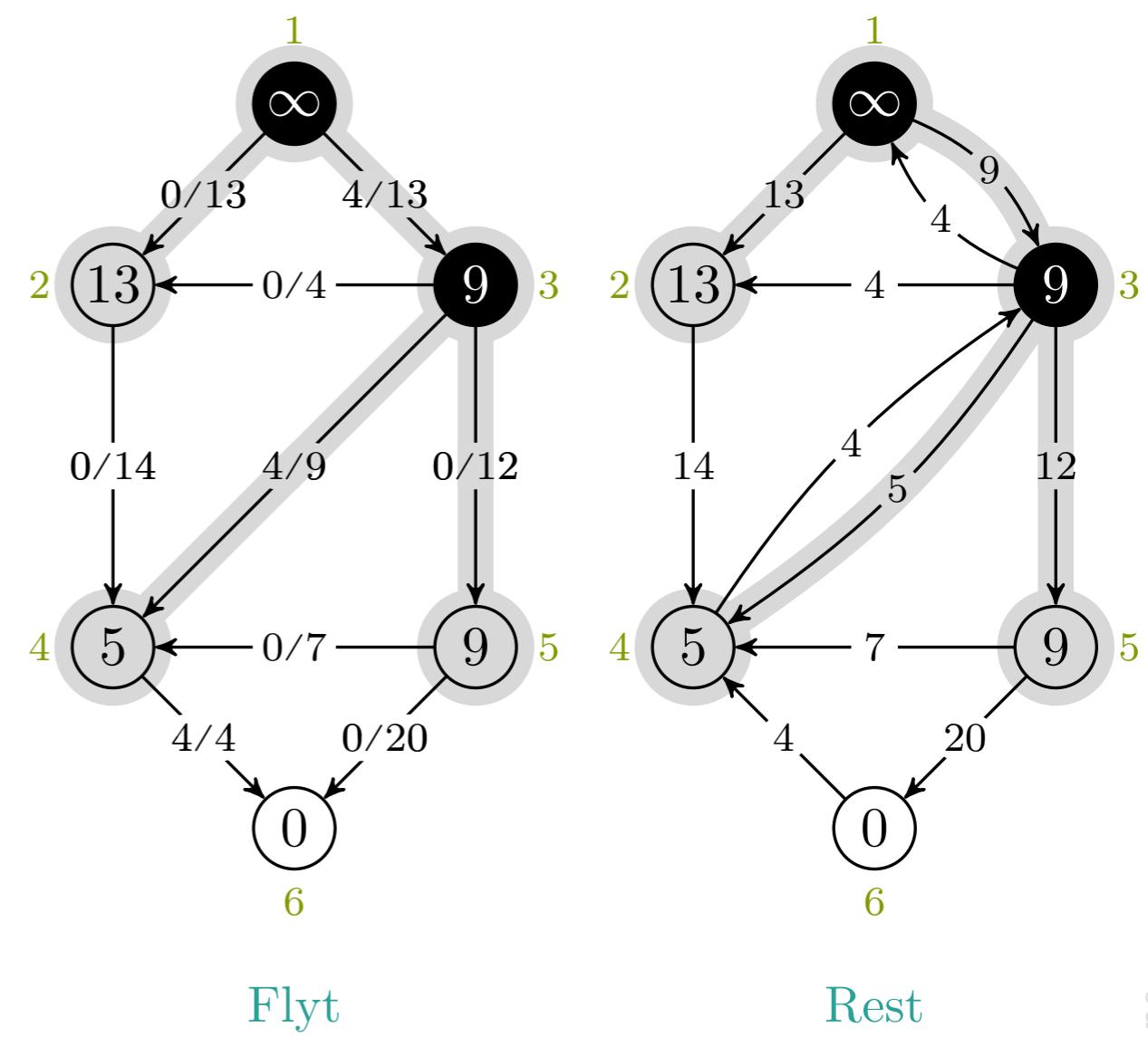


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
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9      ENQUEUE( $Q, s$ )
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11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
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16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
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25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, -$

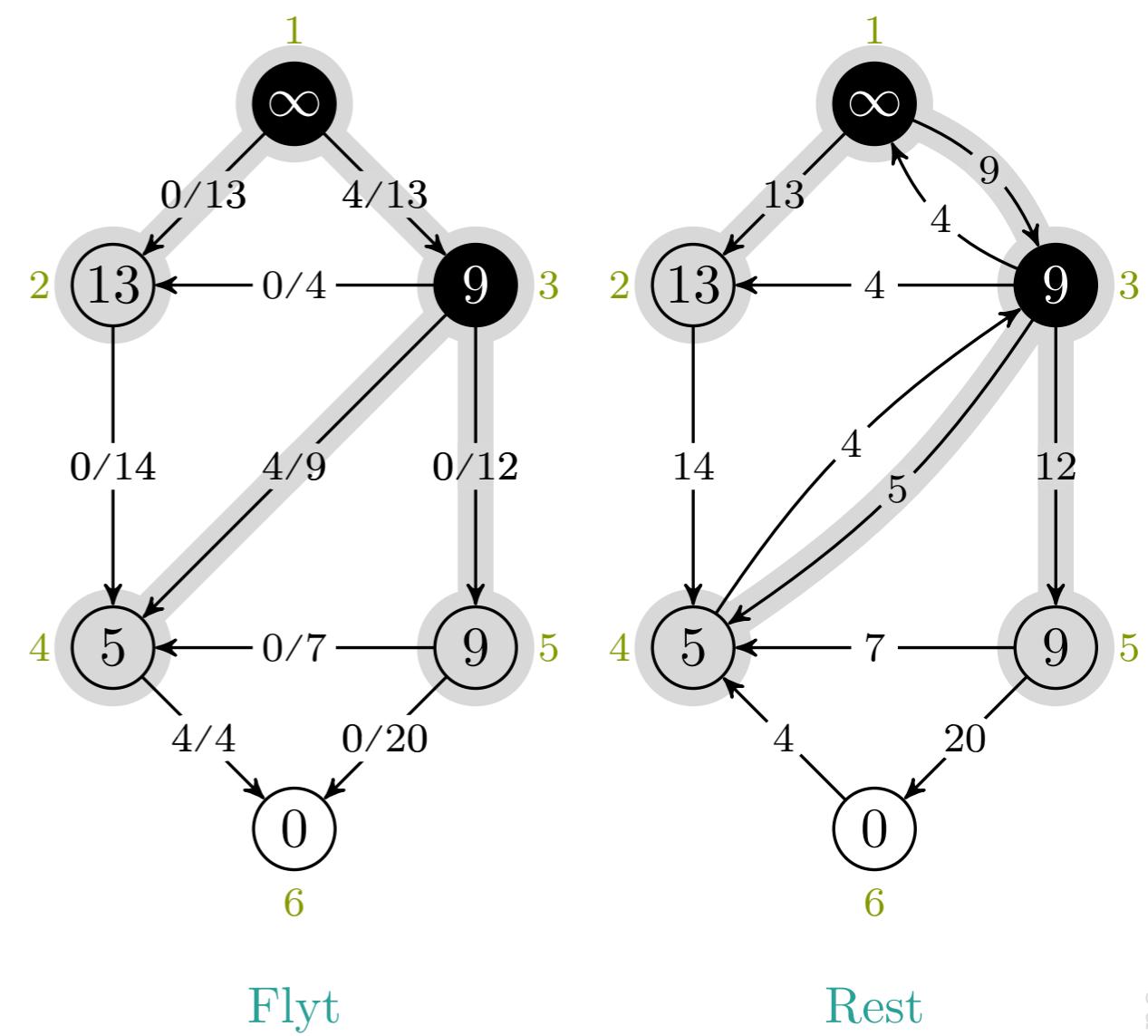


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
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16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, -$

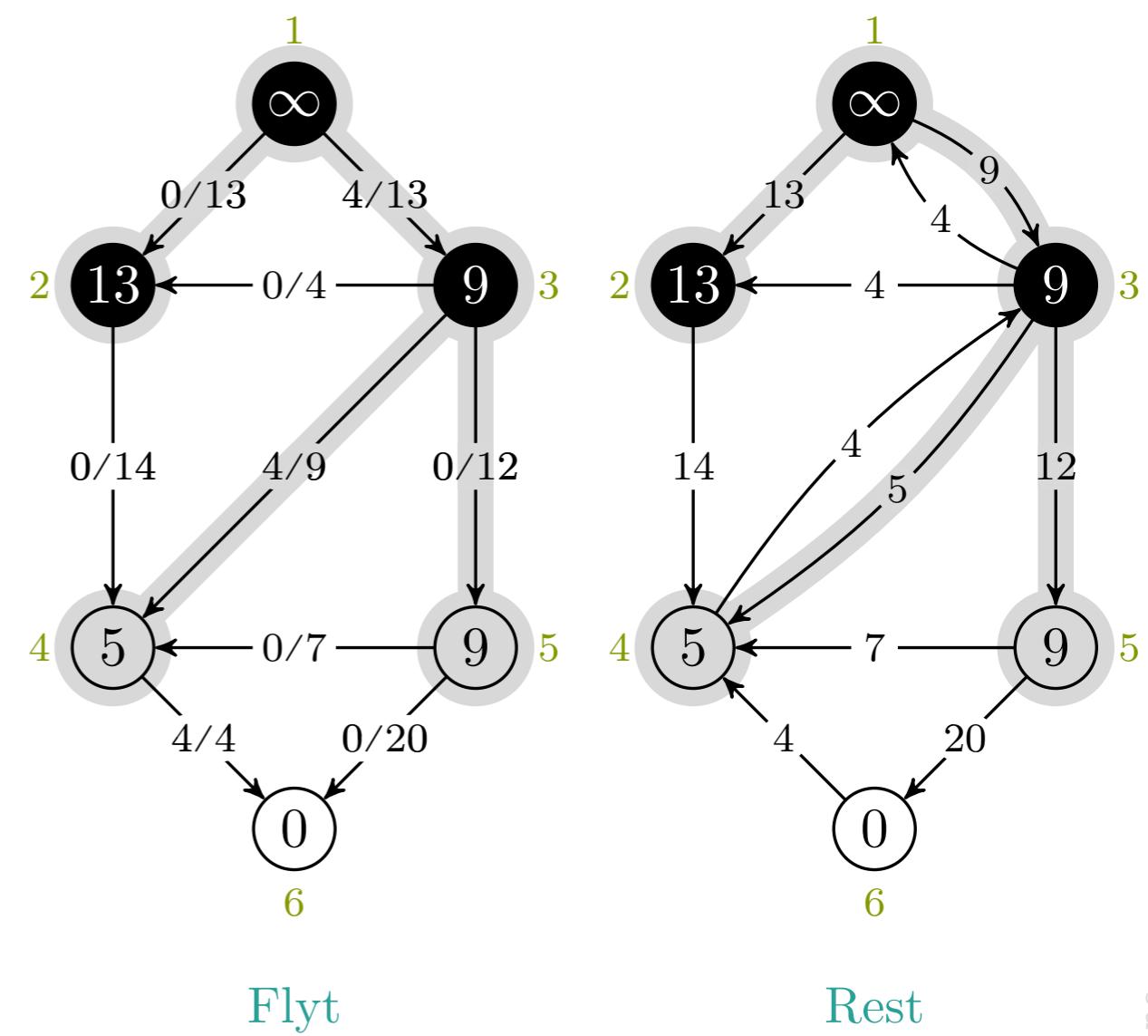


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
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4    for each vertex  $u \in G.V$ 
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9      ENQUEUE( $Q, s$ )
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23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, -$

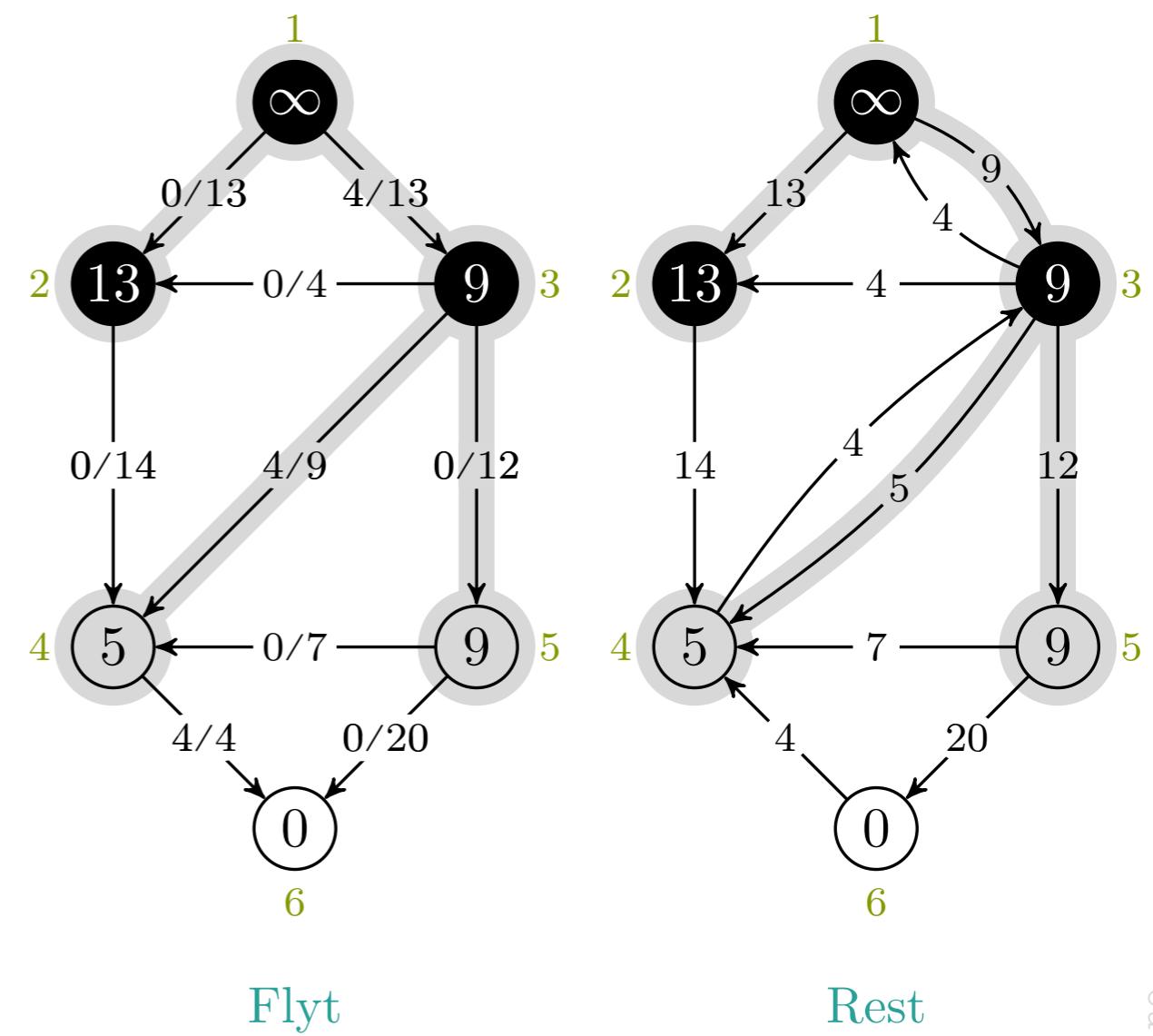


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25          $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, -$

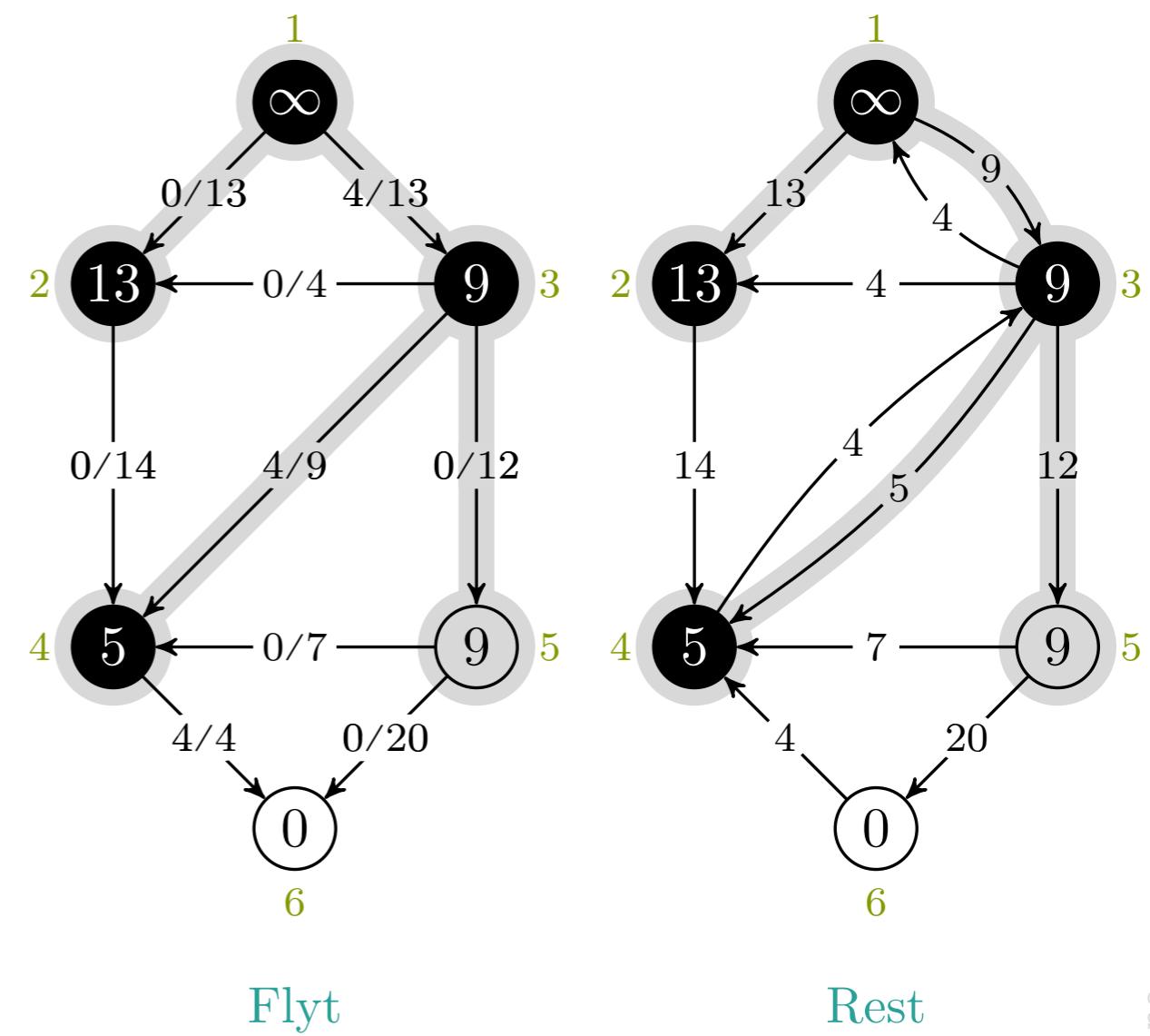


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
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20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
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22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, -$

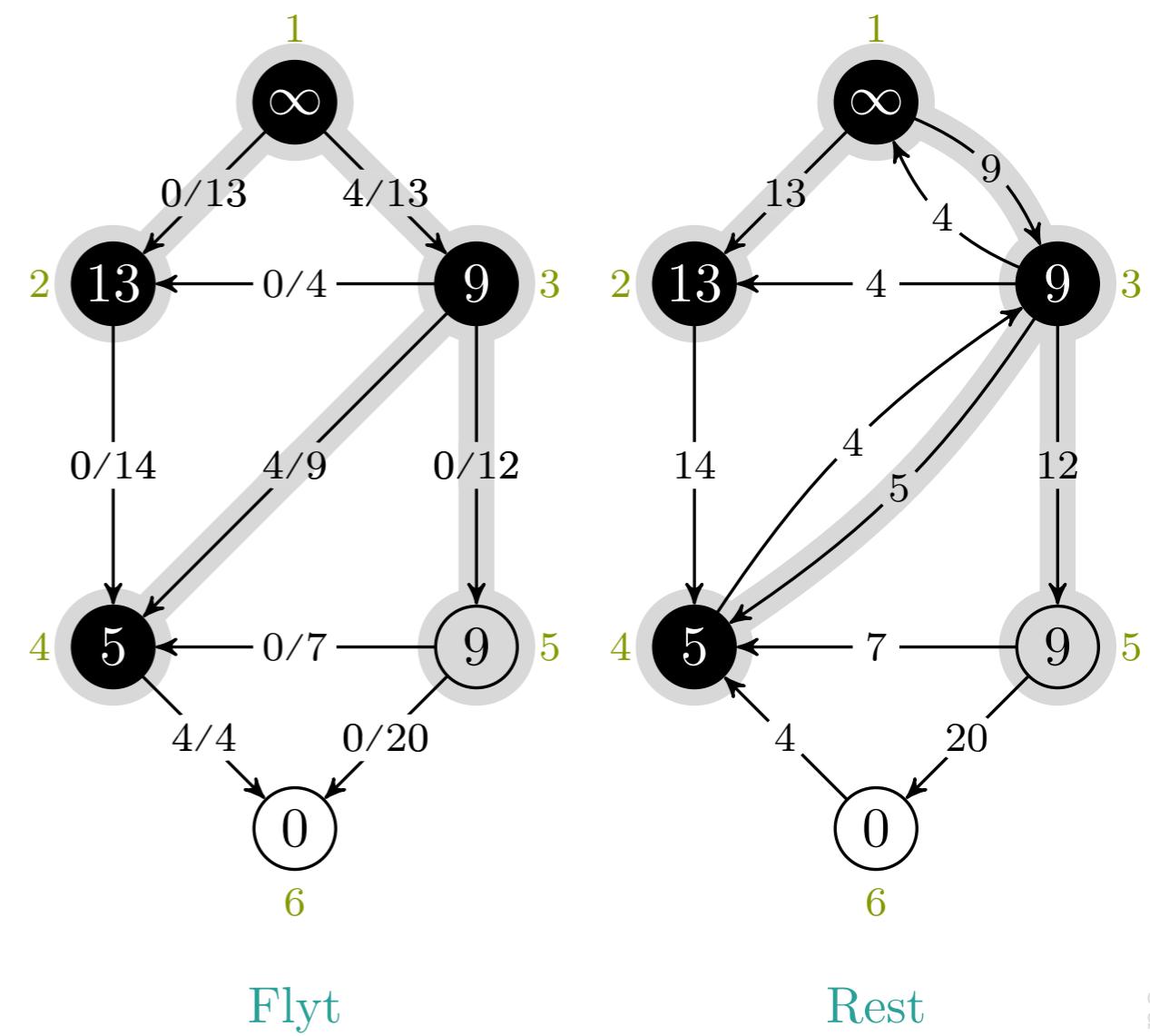


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25          $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, -$

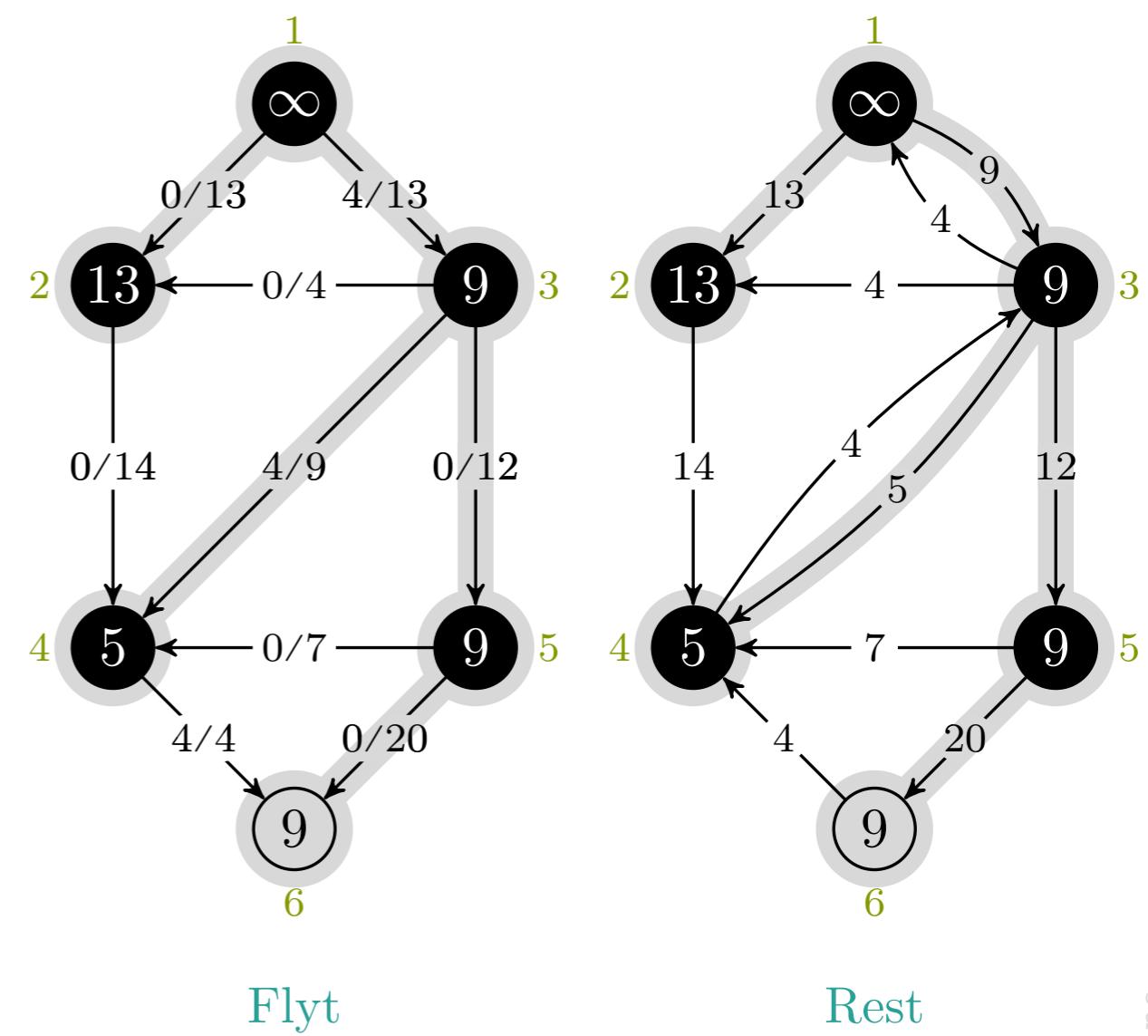


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 5, -$

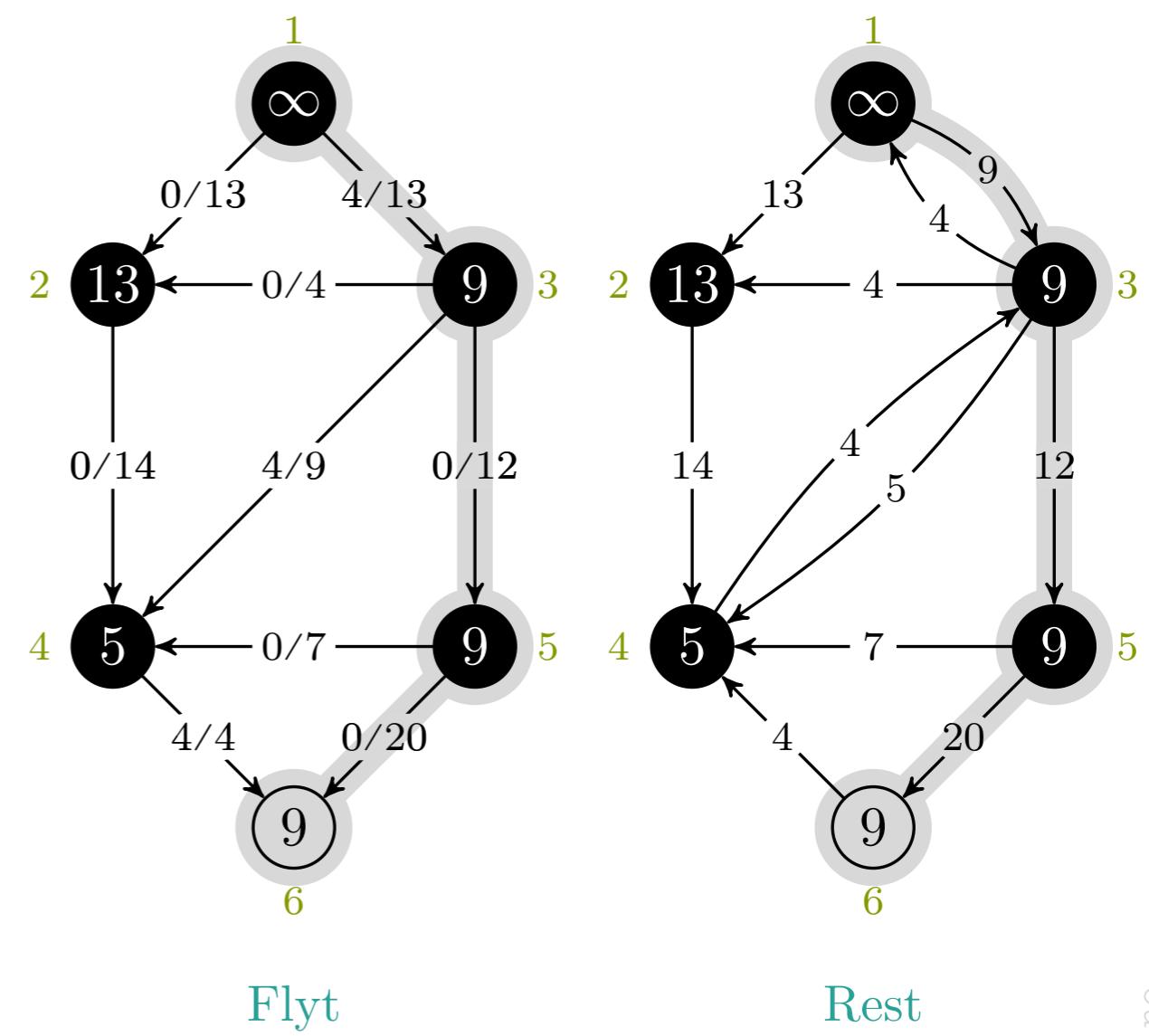


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
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13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
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18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
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22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = -, -$

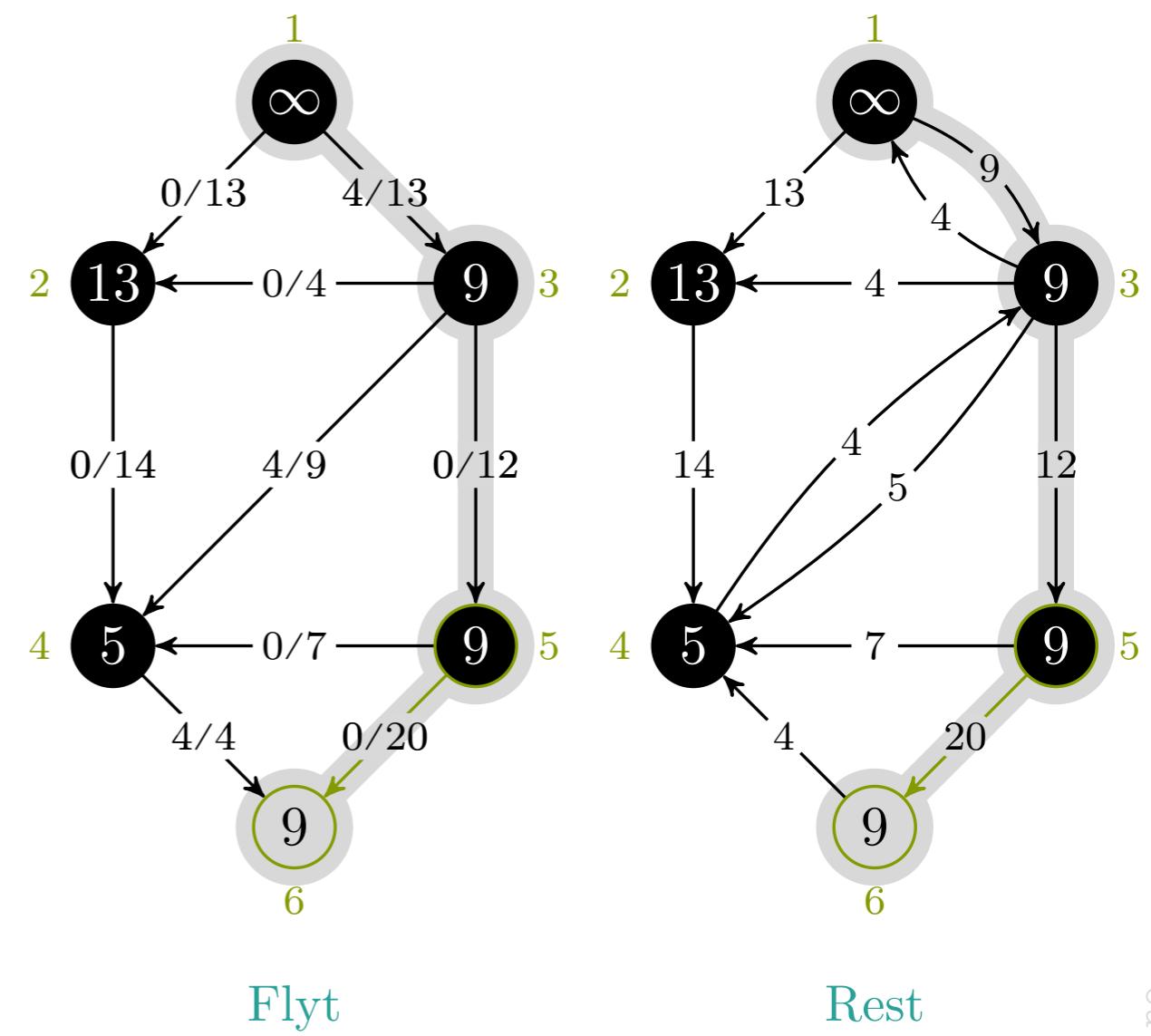


EDMONDS-KARP( $G, s, t$ )

```

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3 repeat
4   for each vertex  $u \in G.V$ 
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7    $s.a = \infty$ 
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13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
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18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20     $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21  while  $u \neq \text{NIL}$ 
22    if  $(u, v) \in G.E$ 
23       $(u, v).f = (u, v).f + t.a$ 
24    else  $(v, u).f = (v, u).f - t.a$ 
25     $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 5, 6$

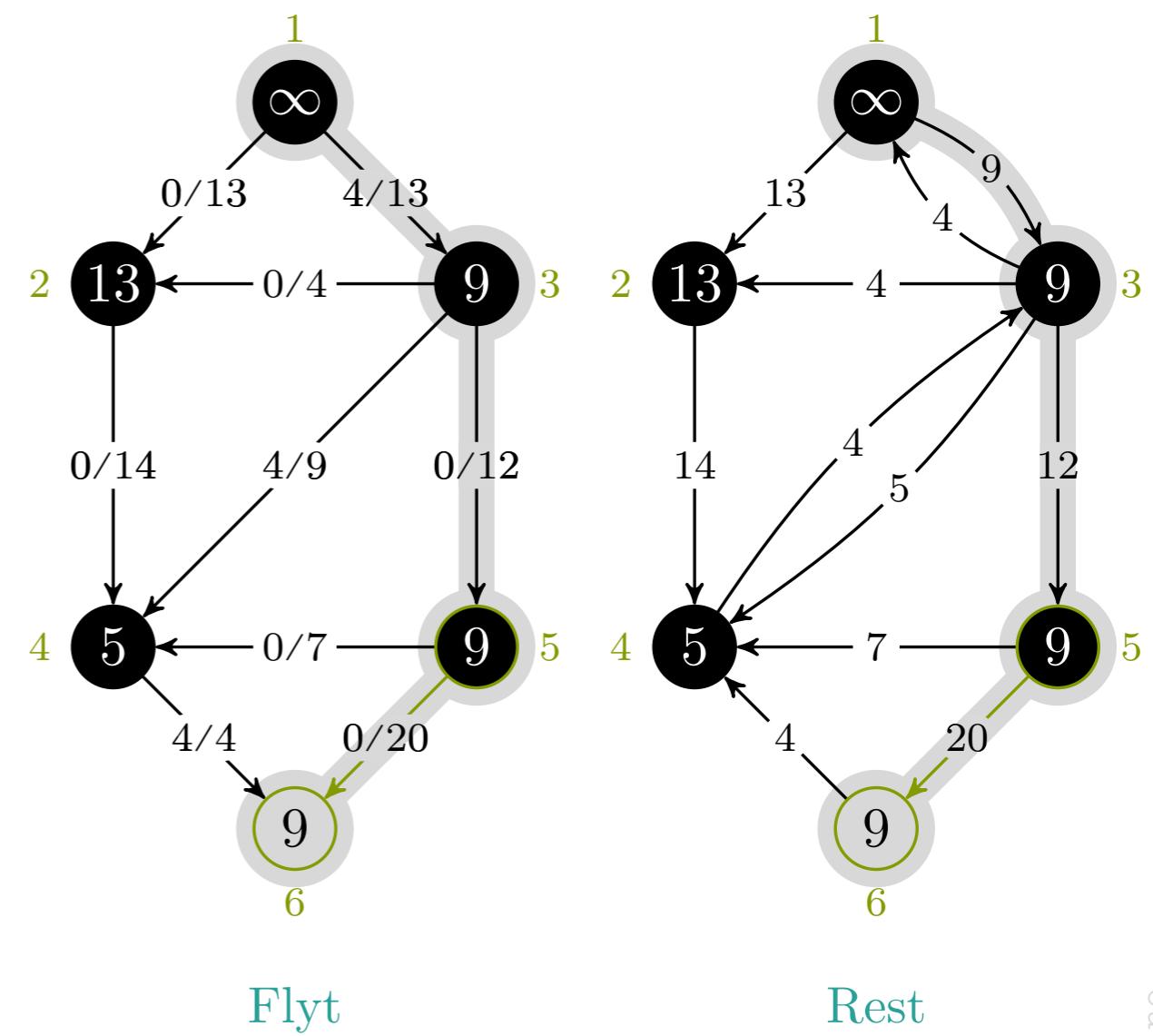


EDMONDS-KARP( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7    $s.a = \infty$ 
8    $Q = \emptyset$ 
9   ENQUEUE( $Q, s$ )
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20     $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21    while  $u \neq \text{NIL}$ 
22      if  $(u, v) \in G.E$ 
23         $(u, v).f = (u, v).f + t.a$ 
24      else  $(v, u).f = (v, u).f - t.a$ 
25     $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 5, 6$



maks-flyt › edmonds-karp

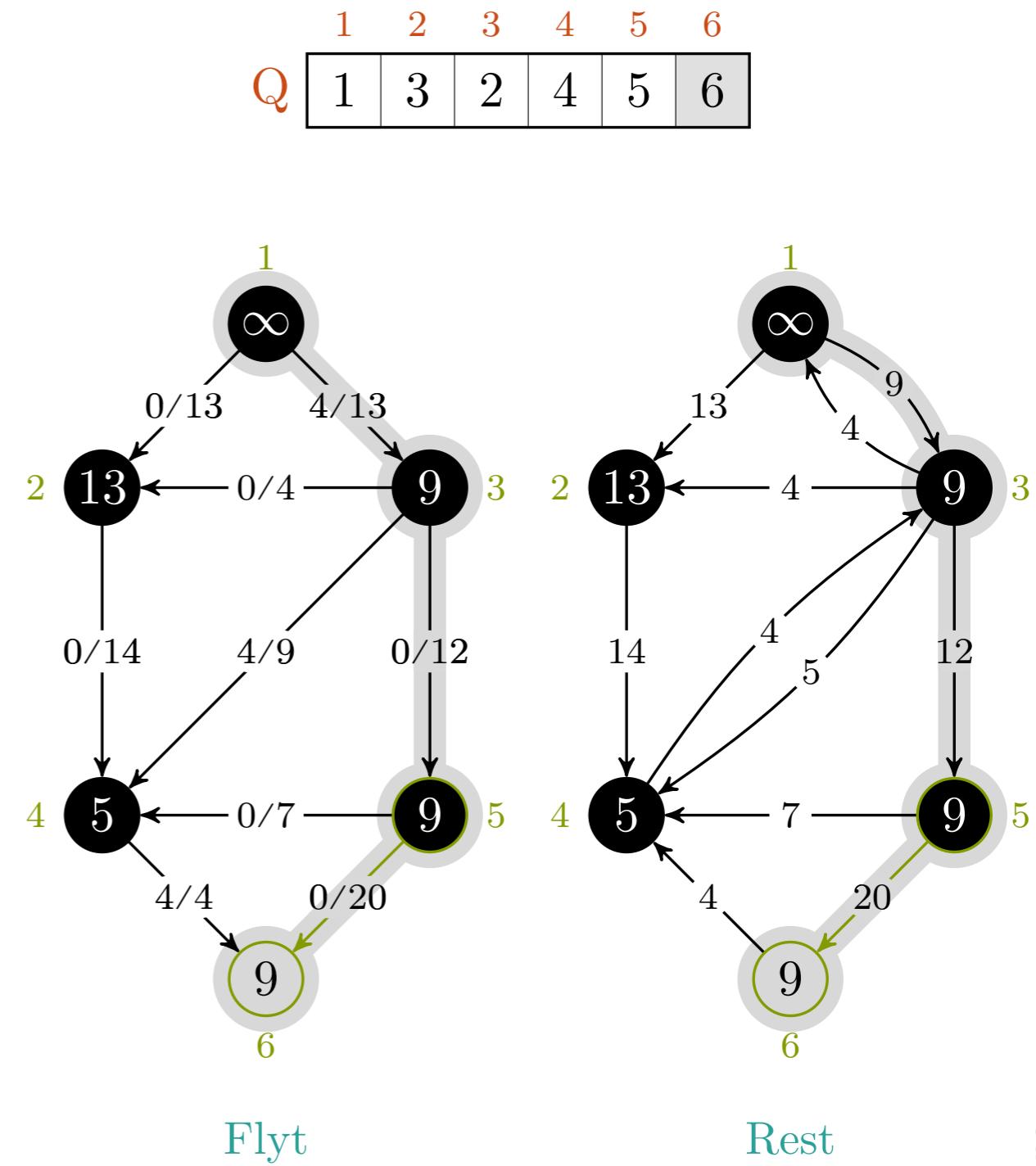
## EDMONDS-KARP(G, s, t)

```

1  for each edge  $(u, v) \in G.E$ 
2       $(u, v).f = 0$ 
3  repeat
4      for each vertex  $u \in G.V$ 
5           $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6           $u.\pi = NIL$ 
7           $s.a = \infty$ 
8           $Q = \emptyset$ 
9          ENQUEUE( $Q, s$ )
10         while  $t.a == 0$  and  $Q \neq \emptyset$ 
11              $u = DEQUEUE(Q)$ 
12             for all edges  $(u, v), (v, u) \in G.E$ 
13                 if  $(u, v) \in G.E$ 
14                      $c_f(u, v) = c(u, v) - (u, v).f$ 
15                 else  $c_f(u, v) = (v, u).f$ 
16                 if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                      $v.a = \min(u.a, c_f(u, v))$ 
18                      $v.\pi = u$ 
19                     ENQUEUE( $Q, v$ )
20              $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21             while  $u \neq NIL$ 
22                 if  $(u, v) \in G.E$ 
23                      $(u, v).f = (u, v).f + t.a$ 
24                 else  $(v, u).f = (v, u).f - t.a$ 
25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

$$u, v = 5, 6$$

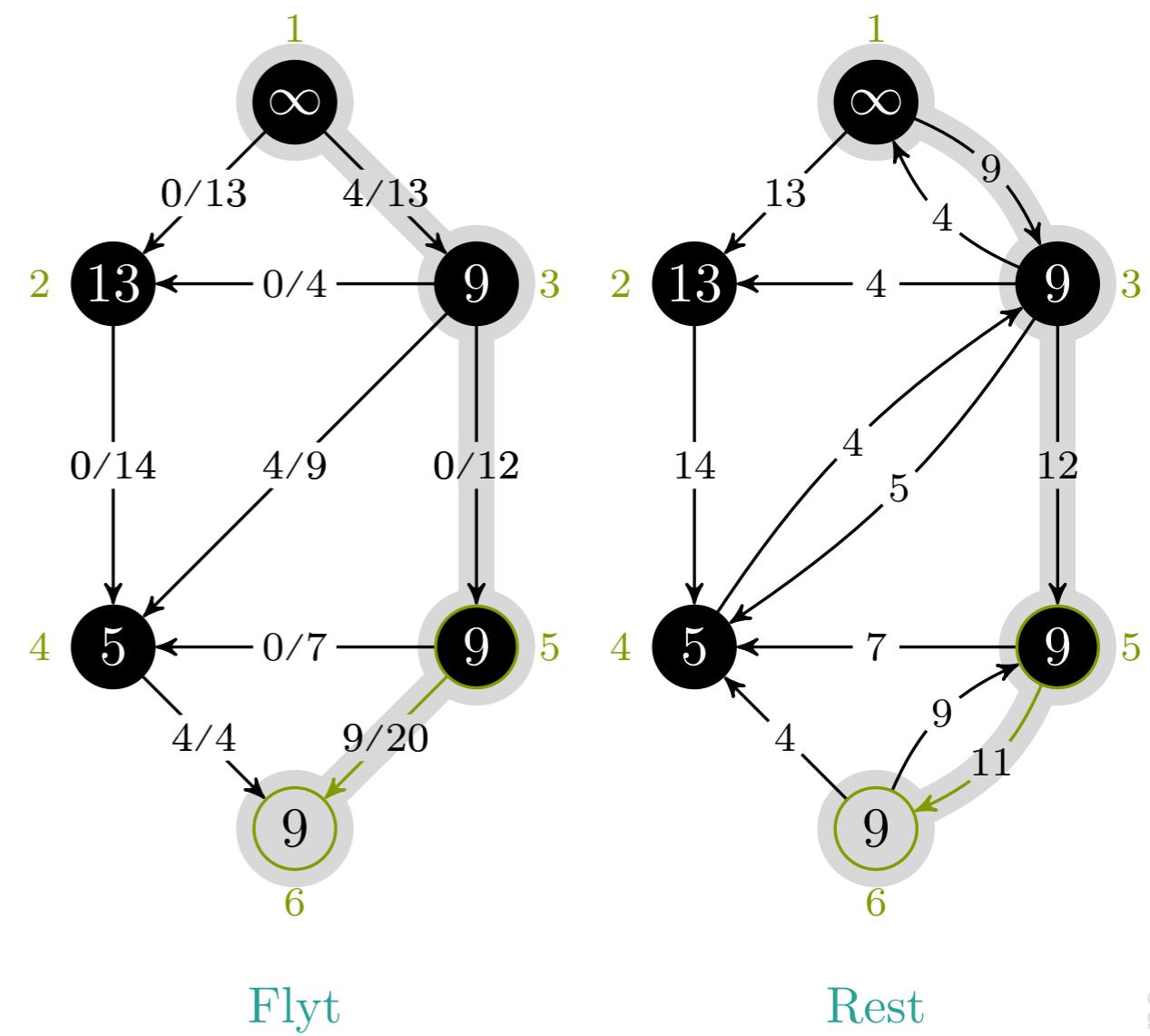


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
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24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 5, 6$

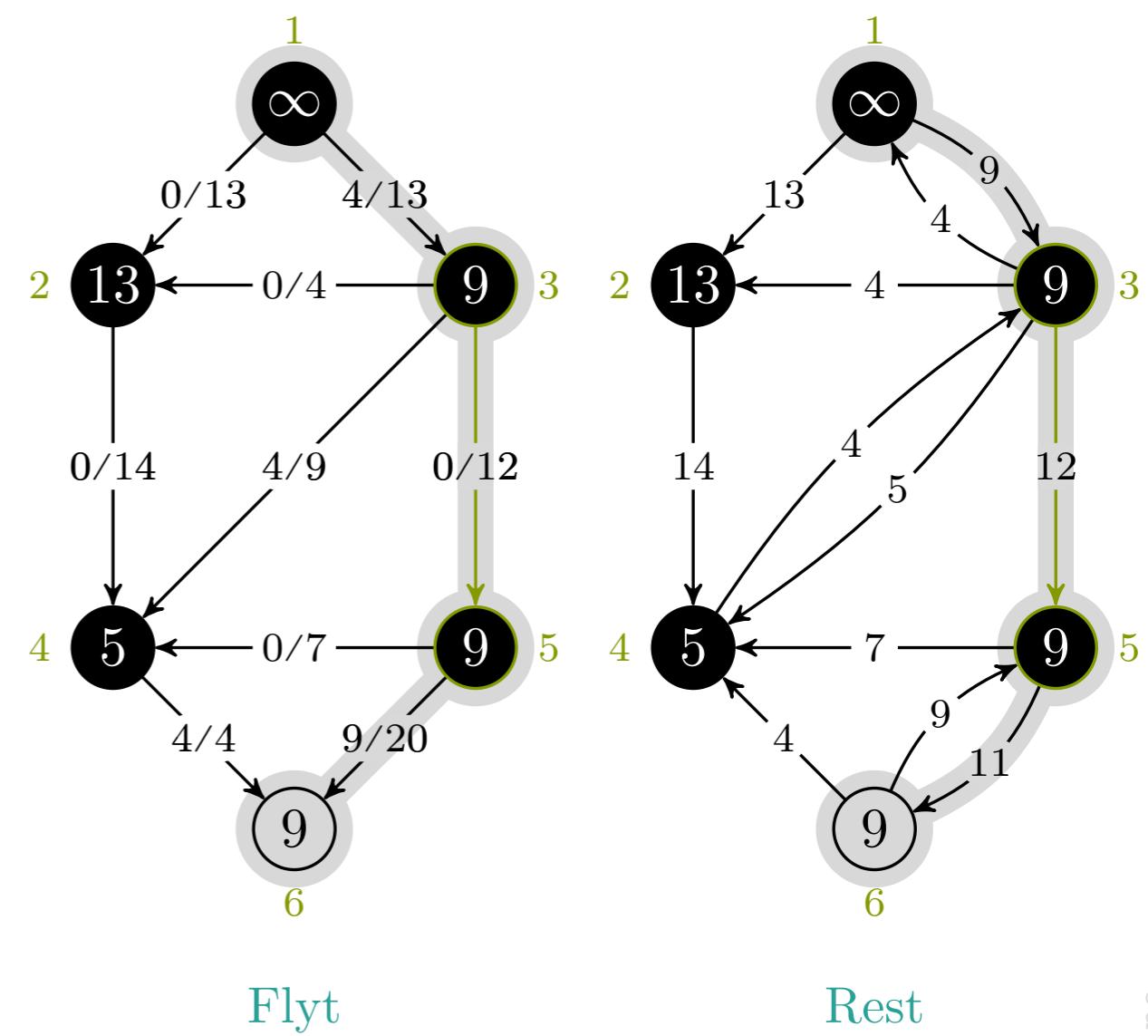


EDMONDS-KARP( $G, s, t$ )

```

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2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
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20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
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23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 5$

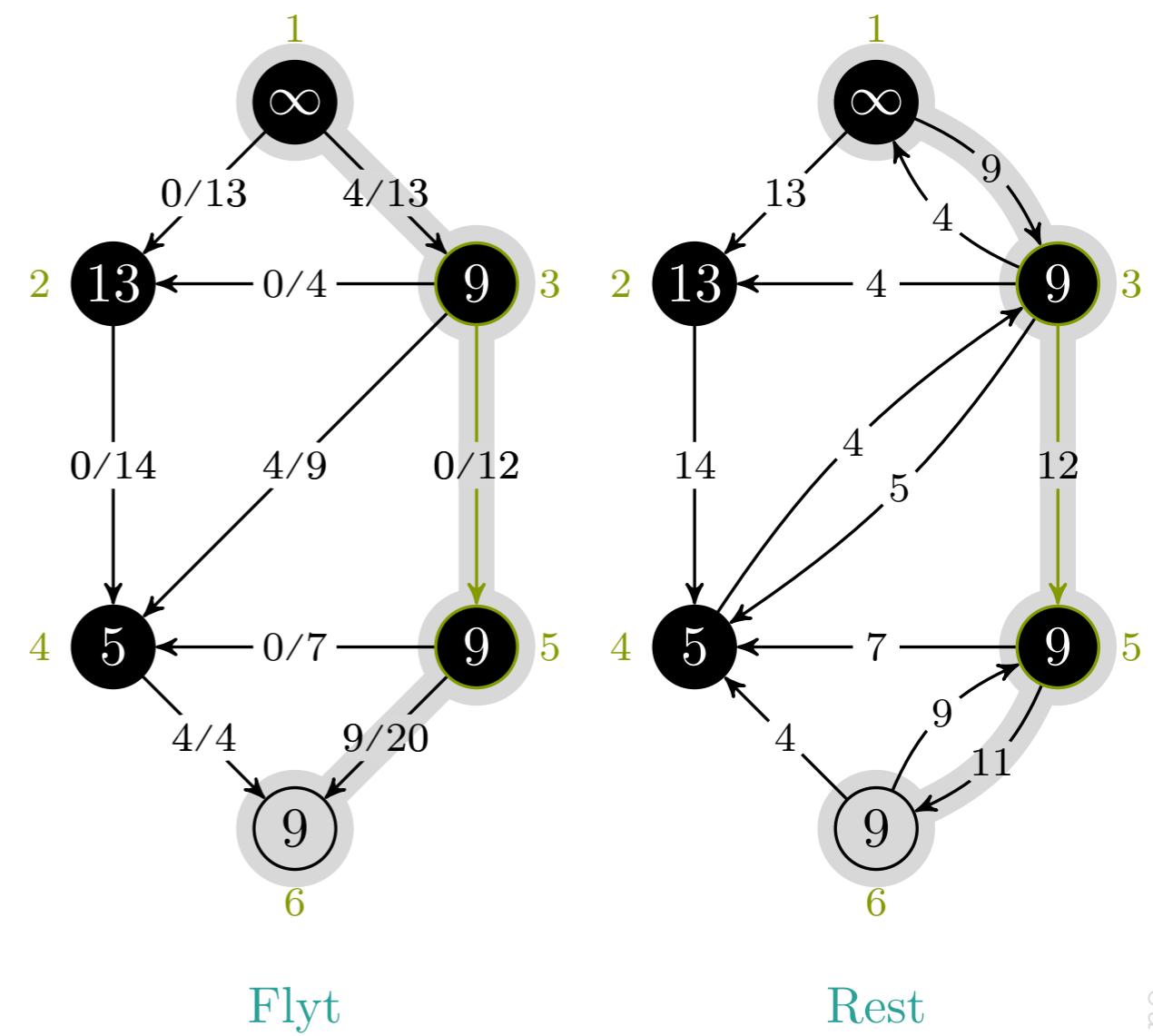


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
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17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 5$

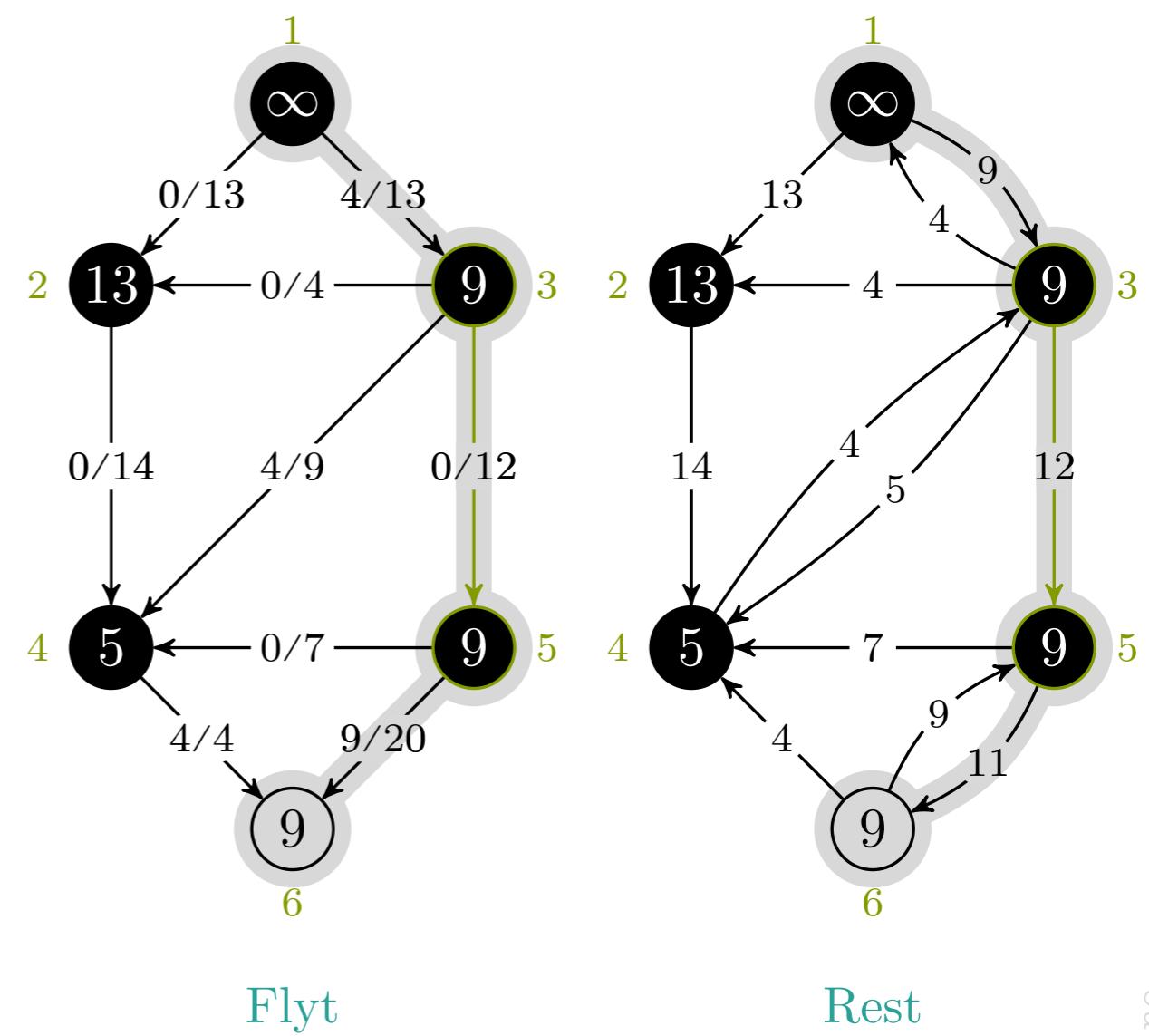


EDMONDS-KARP( $G, s, t$ )

```

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11        $u = \text{DEQUEUE}(Q)$ 
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24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 5$



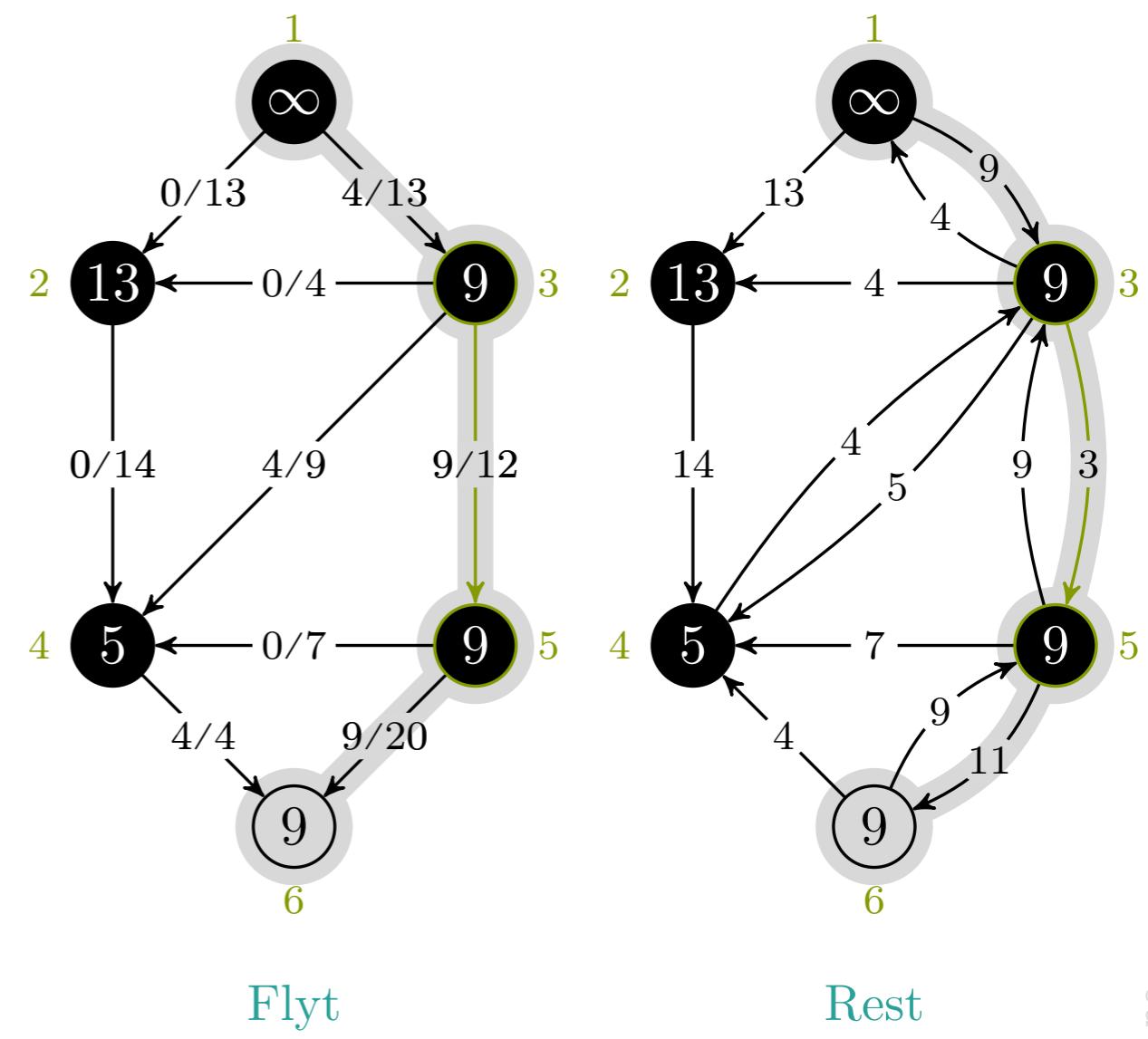
```
EDMONDS-KARP(G, s, t)
```

```

1 for each edge  $(u, v) \in G.E$ 
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24      else  $(v, u).f = (v, u).f - t.a$ 
25     $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 5$

Q	1	2	3	4	5	6
	1	3	2	4	5	6



## EDMONDS-KARP(G, s, t)

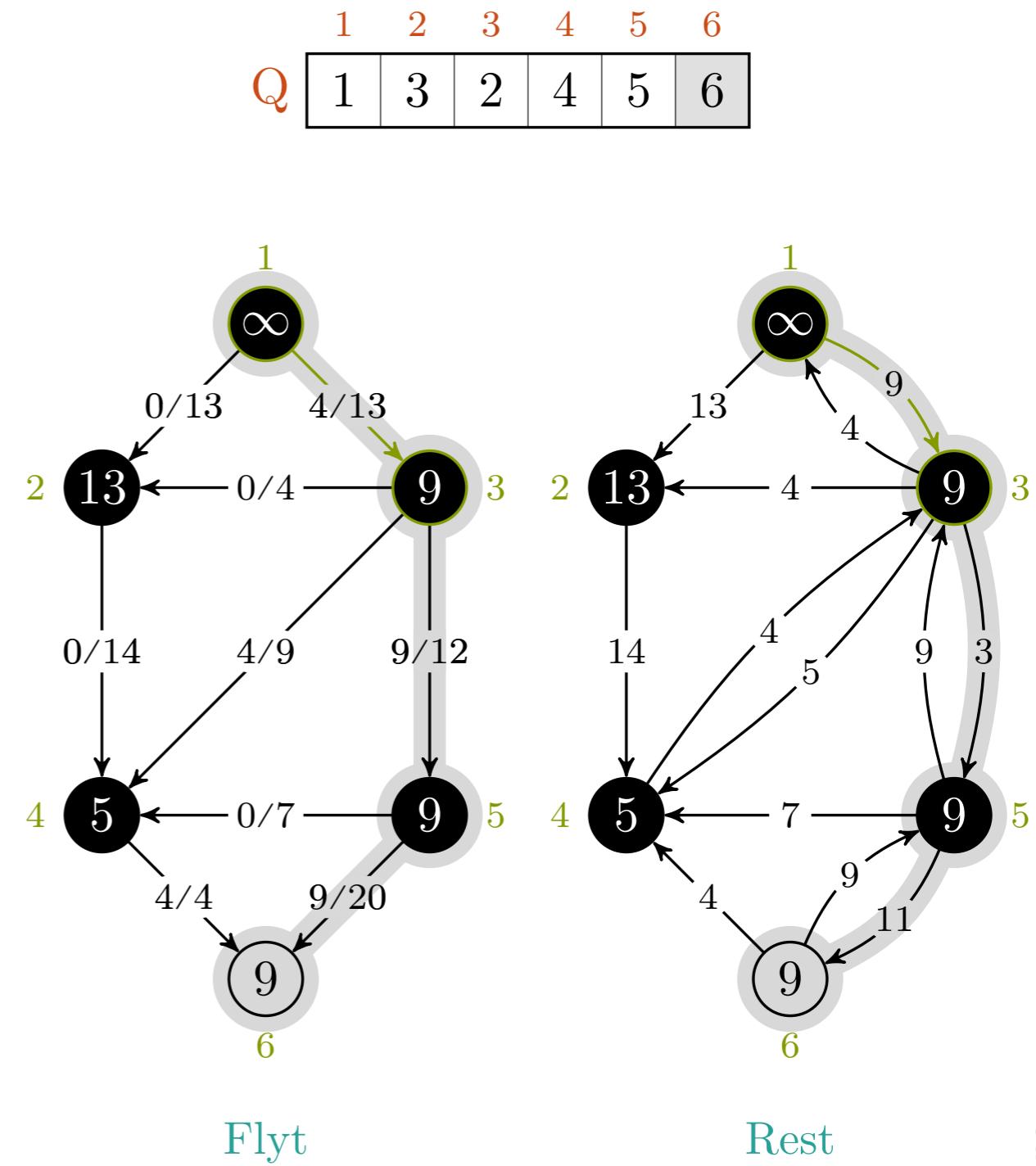
```

1  for each edge  $(u, v) \in G.E$ 
2       $(u, v).f = 0$ 
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23                      $(u, v).f = (u, v).f + t.a$ 
24                 else  $(v, u).f = (v, u).f - t.a$ 
25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

$$u, v = 1, 3$$

maks-flyt → edmonds-karp

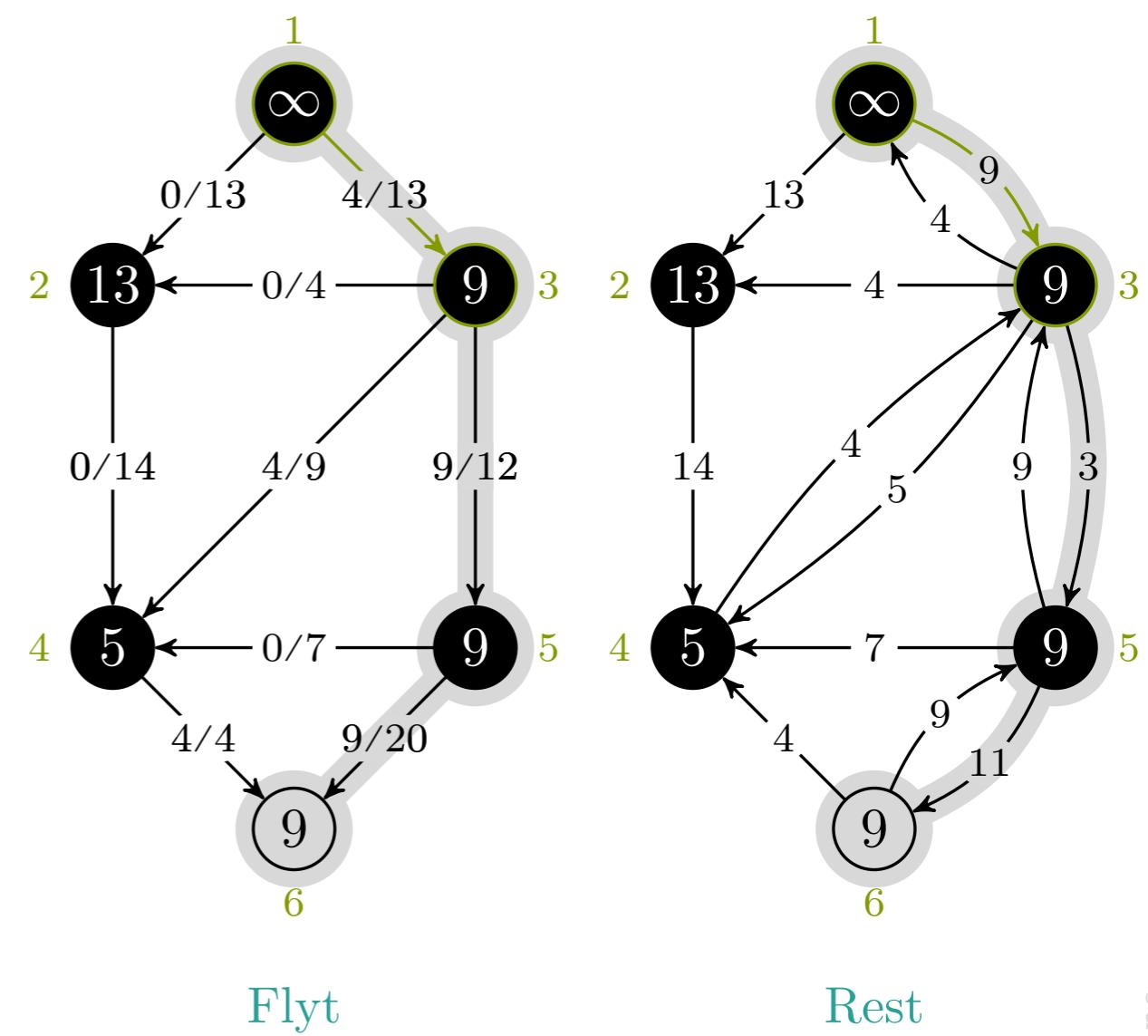


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
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13         if  $(u, v) \in G.E$ 
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15         else  $c_f(u, v) = (v, u).f$ 
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17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, 3$



maks-flyt › edmonds-karp

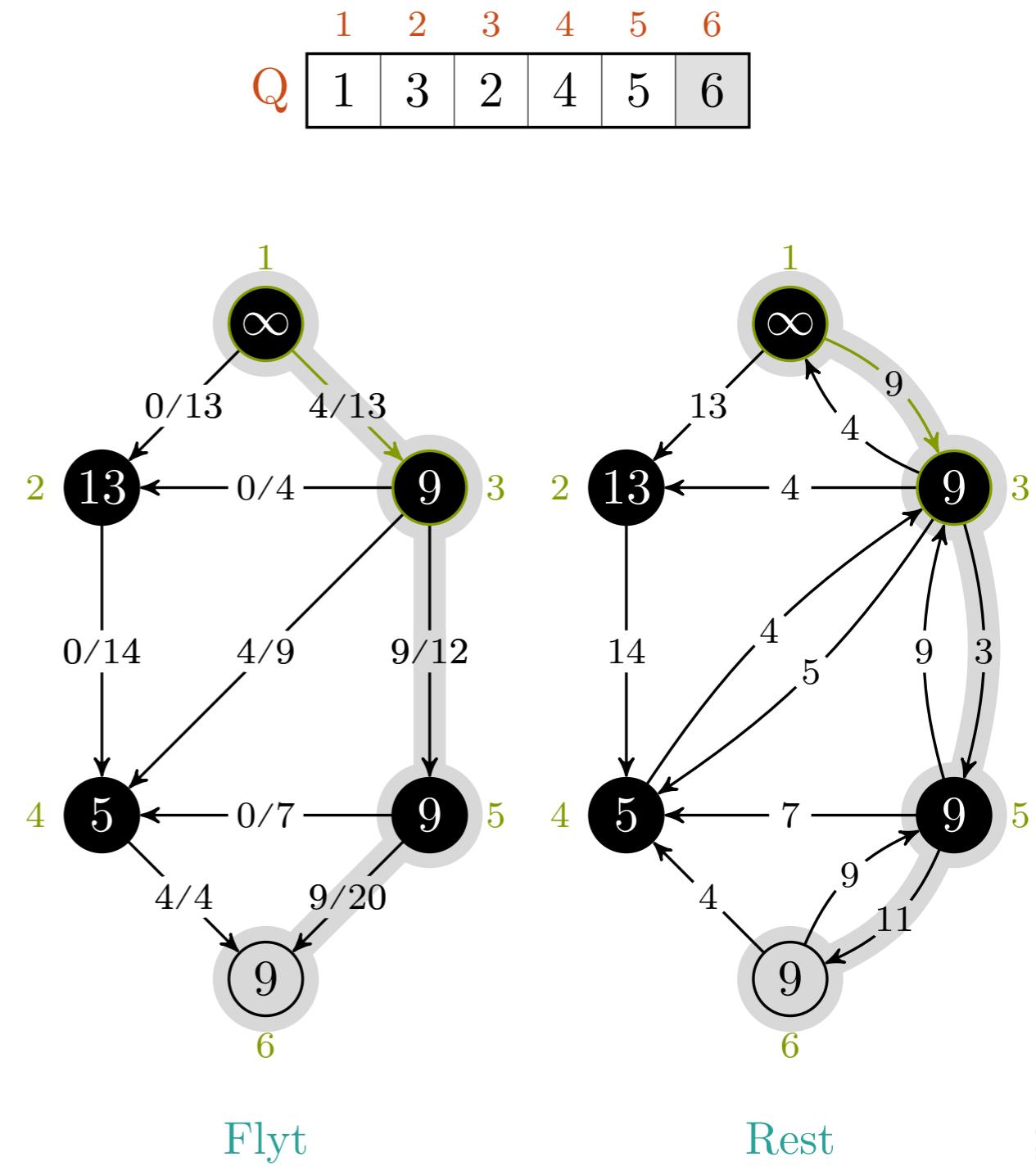
## EDMONDS-KARP(G, s, t)

```

1  for each edge  $(u, v) \in G.E$ 
2       $(u, v).f = 0$ 
3  repeat
4      for each vertex  $u \in G.V$ 
5           $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6           $u.\pi = NIL$ 
7           $s.a = \infty$ 
8           $Q = \emptyset$ 
9          ENQUEUE( $Q, s$ )
10         while  $t.a == 0$  and  $Q \neq \emptyset$ 
11              $u = DEQUEUE(Q)$ 
12             for all edges  $(u, v), (v, u) \in G.E$ 
13                 if  $(u, v) \in G.E$ 
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15                 else  $c_f(u, v) = (v, u).f$ 
16                 if  $c_f(u, v) > 0$  and  $v.a == 0$ 
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18                      $v.\pi = u$ 
19                     ENQUEUE( $Q, v$ )
20              $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21             while  $u \neq NIL$ 
22                 if  $(u, v) \in G.E$ 
23                      $(u, v).f = (u, v).f + t.a$ 
24                 else  $(v, u).f = (v, u).f - t.a$ 
25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

$$u, v = 1, 3$$

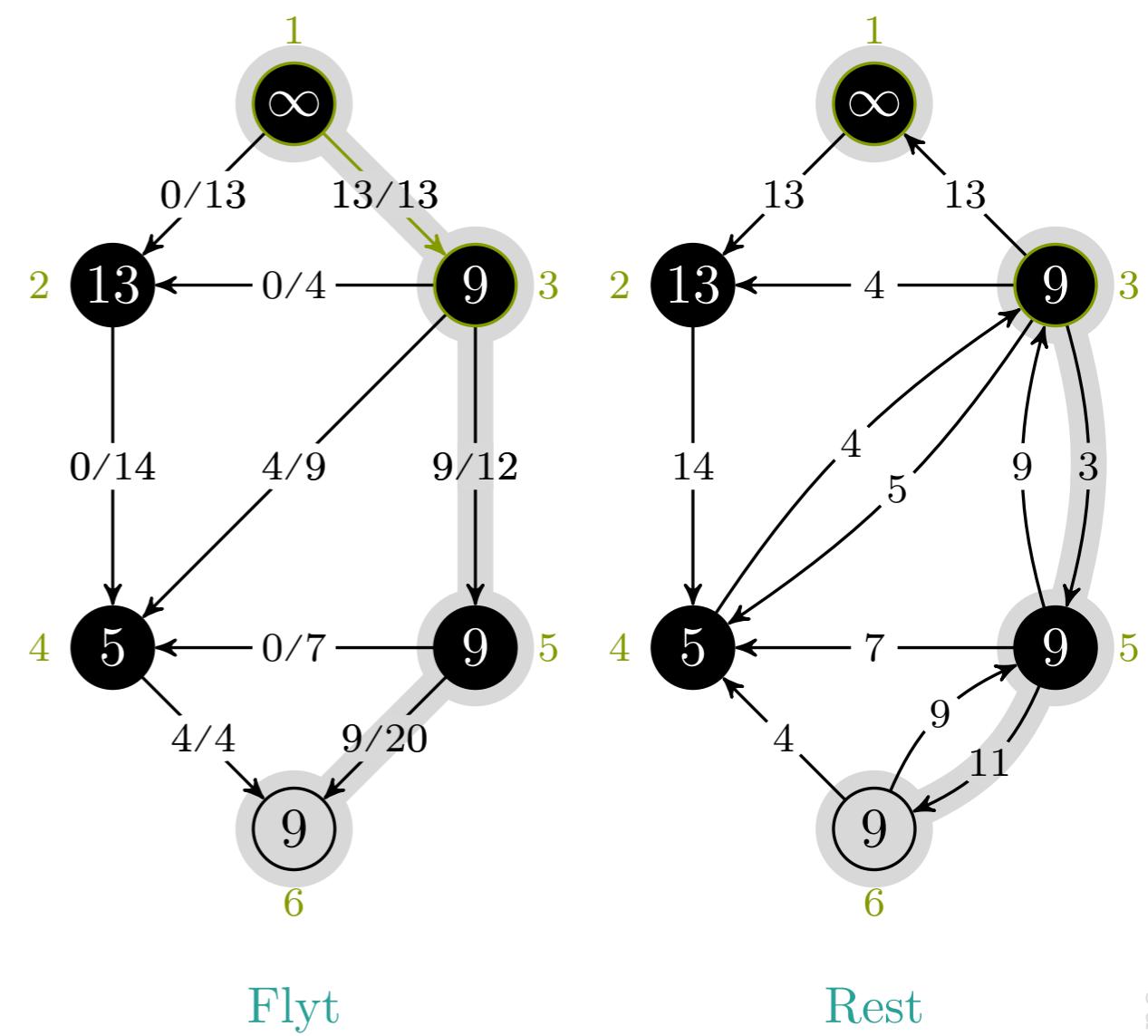


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
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13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25      $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, 3$

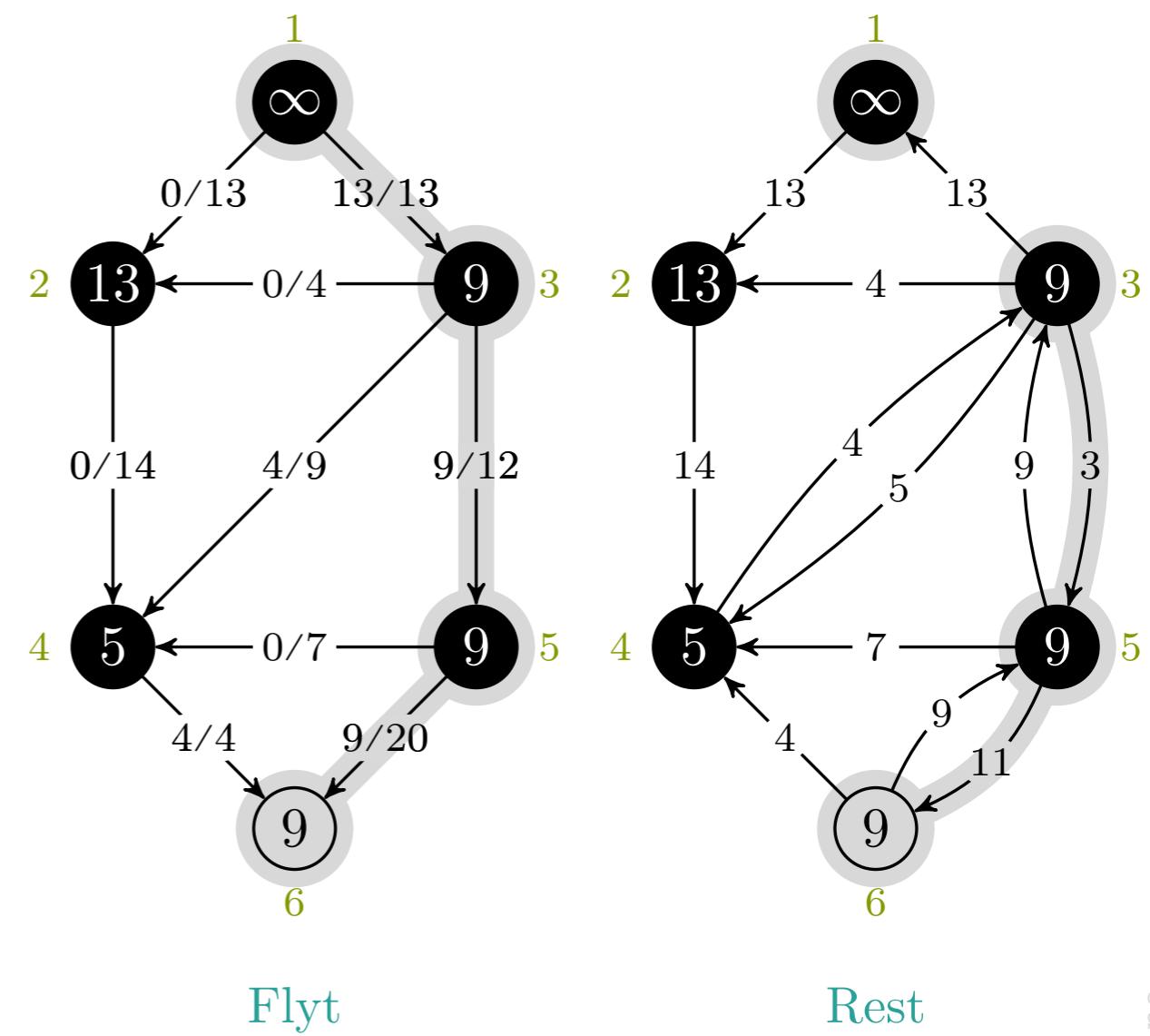


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$

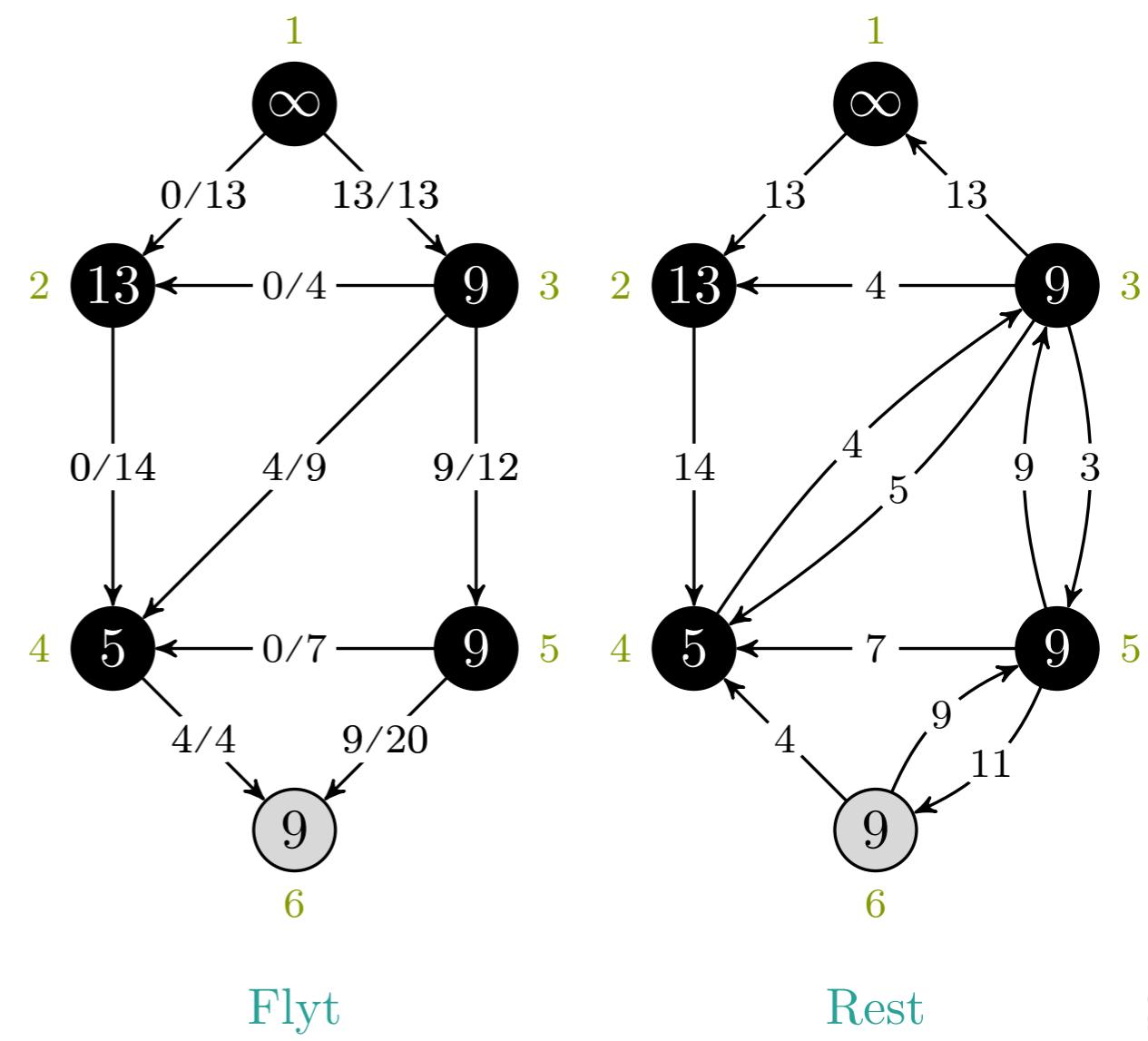


EDMONDS-KARP( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7    $s.a = \infty$ 
8    $Q = \emptyset$ 
9   ENQUEUE( $Q, s$ )
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20       $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21      while  $u \neq \text{NIL}$ 
22        if  $(u, v) \in G.E$ 
23           $(u, v).f = (u, v).f + t.a$ 
24        else  $(v, u).f = (v, u).f - t.a$ 
25       $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$



EDMONDS-KARP( $G, s, t$ )

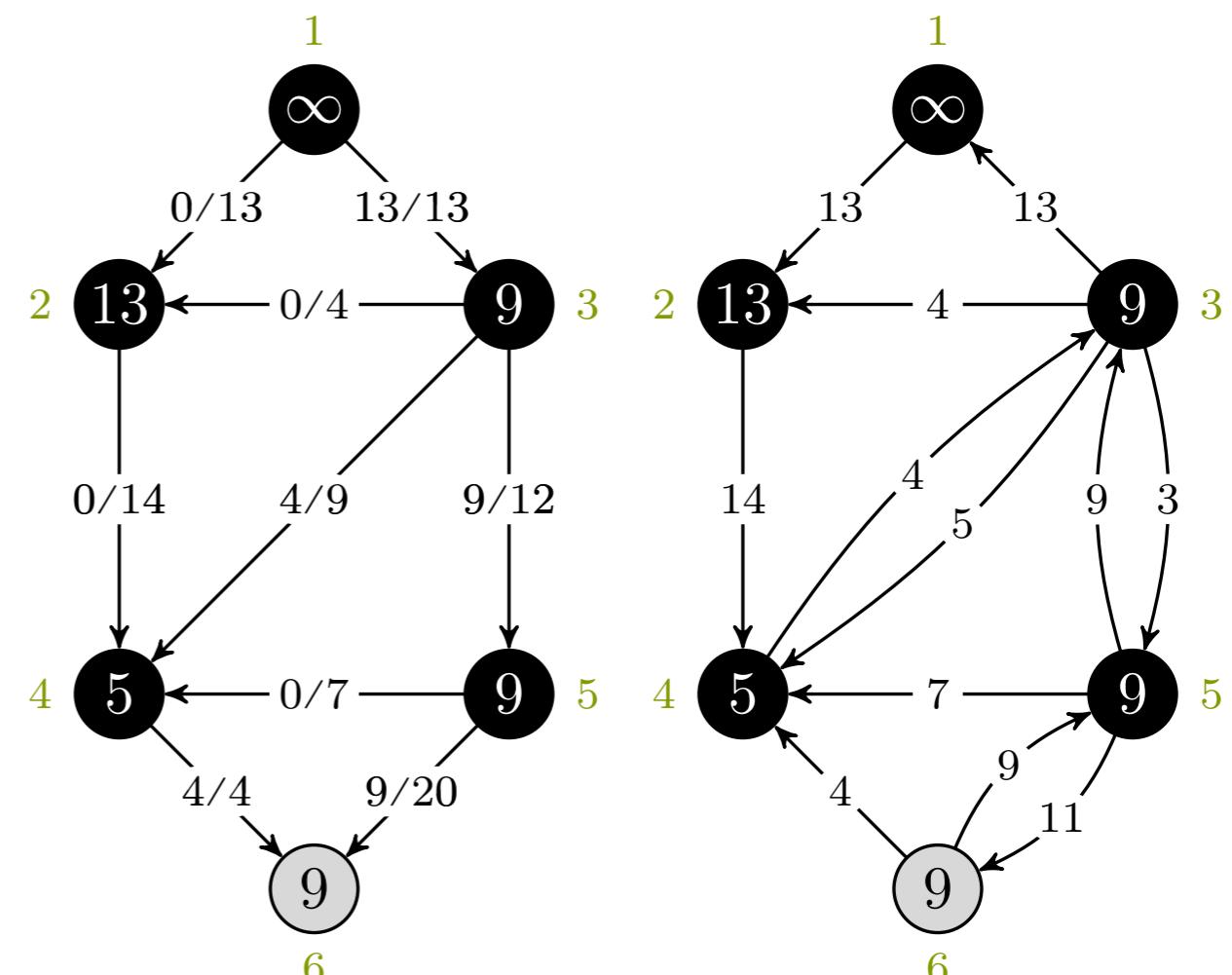
```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7      $s.a = \infty$ 
8      $Q = \emptyset$ 
9     ENQUEUE( $Q, s$ )
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11       $u = \text{DEQUEUE}(Q)$ 
12      for all edges  $(u, v), (v, u) \in G.E$ 
13        if  $(u, v) \in G.E$ 
14           $c_f(u, v) = c(u, v) - (u, v).f$ 
15        else  $c_f(u, v) = (v, u).f$ 
16        if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17           $v.a = \min(u.a, c_f(u, v))$ 
18           $v.\pi = u$ 
19          ENQUEUE( $Q, v$ )
20         $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21        while  $u \neq \text{NIL}$ 
22          if  $(u, v) \in G.E$ 
23             $(u, v).f = (u, v).f + t.a$ 
24          else  $(v, u).f = (v, u).f - t.a$ 
25           $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = \text{NIL}, 1$ 

1    2    3    4    5    6

<b>Q</b>	1	3	2	4	5	6
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Flyt

Rest

maks-flyt › edmonds-karp

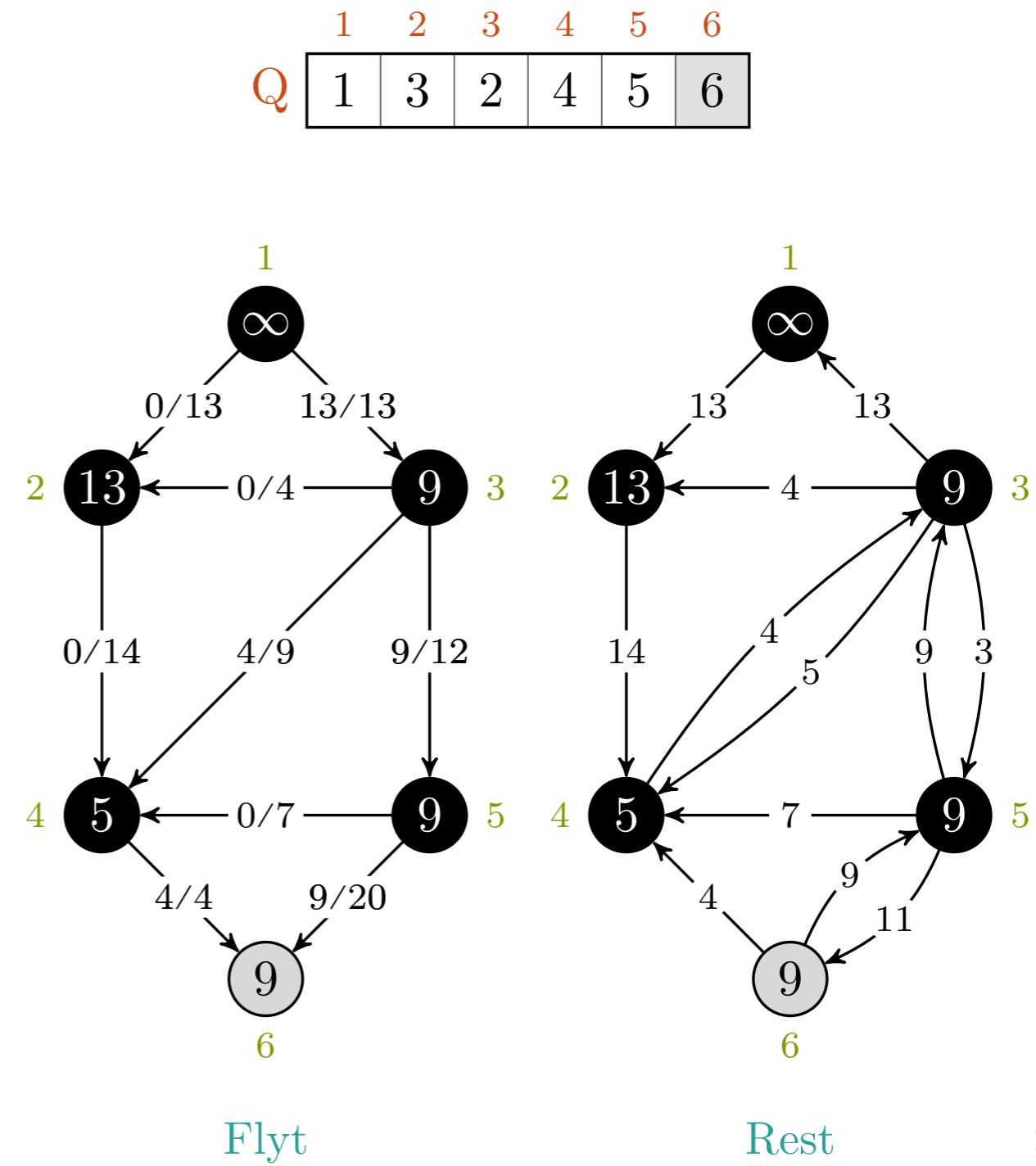
## EDMONDS-KARP(G, s, t)

```

1   for each edge  $(u, v) \in G.E$ 
2        $(u, v).f = 0$ 
3   repeat
4       for each vertex  $u \in G.V$ 
5            $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6            $u.\pi = \text{NIL}$ 
7            $s.a = \infty$ 
8            $Q = \emptyset$ 
9           ENQUEUE( $Q, s$ )
10          while  $t.a == 0$  and  $Q \neq \emptyset$ 
11               $u = \text{DEQUEUE}(Q)$ 
12              for all edges  $(u, v), (v, u) \in G.E$ 
13                  if  $(u, v) \in G.E$ 
14                       $c_f(u, v) = c(u, v) - (u, v).f$ 
15                  else  $c_f(u, v) = (v, u).f$ 
16                  if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                       $v.a = \min(u.a, c_f(u, v))$ 
18                       $v.\pi = u$ 
19                      ENQUEUE( $Q, v$ )
20           $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21          while  $u \neq \text{NIL}$ 
22              if  $(u, v) \in G.E$ 
23                   $(u, v).f = (u, v).f + t.a$ 
24              else  $(v, u).f = (v, u).f - t.a$ 
25               $u, v = u.\pi, u$ 
26          until  $t.a == 0$ 

```

$u, v = \text{NIL}, 1$



EDMONDS-KARP( $G, s, t$ )

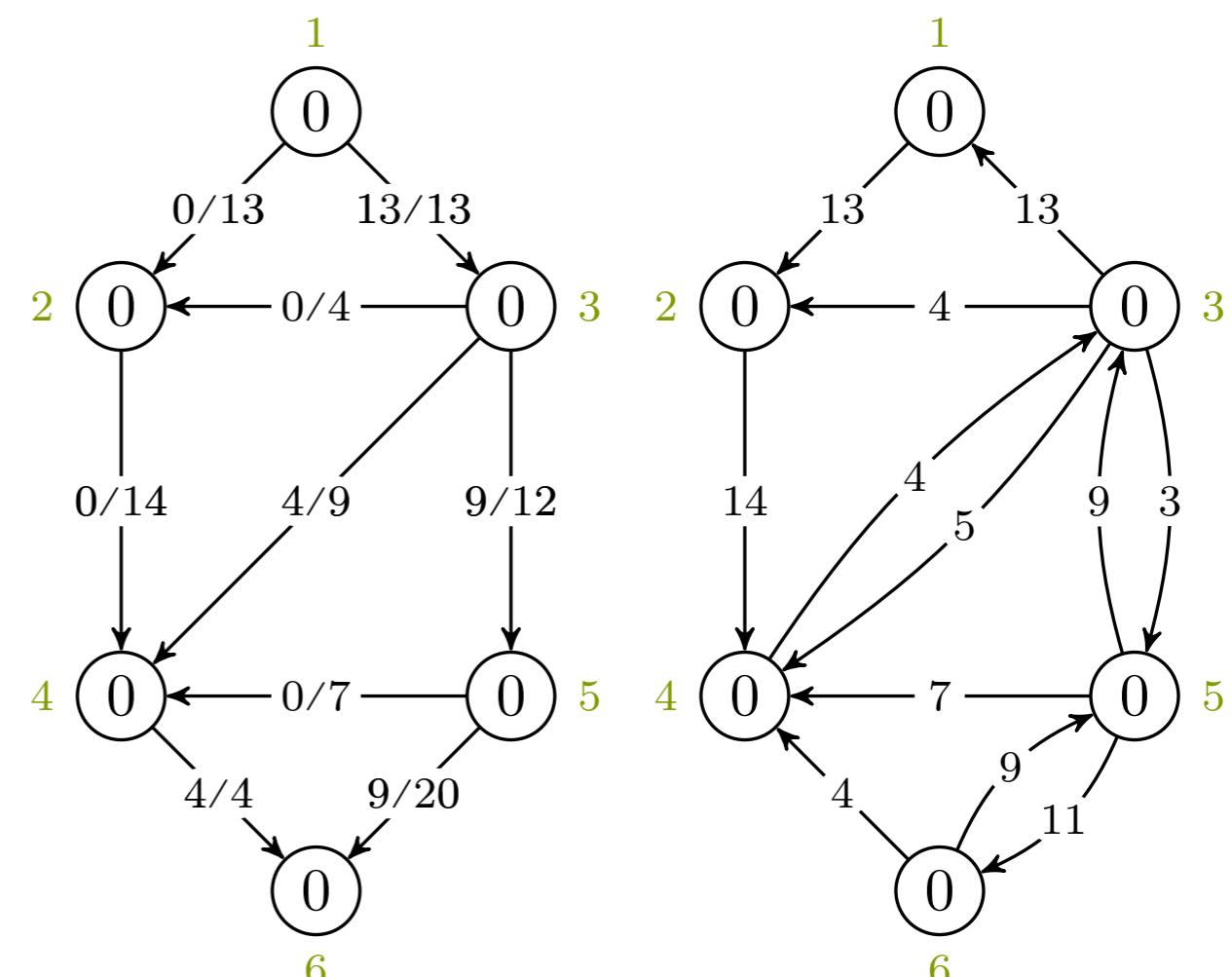
```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7    $s.a = \infty$ 
8    $Q = \emptyset$ 
9   ENQUEUE( $Q, s$ )
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20     $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21    while  $u \neq \text{NIL}$ 
22      if  $(u, v) \in G.E$ 
23         $(u, v).f = (u, v).f + t.a$ 
24      else  $(v, u).f = (v, u).f - t.a$ 
25       $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = \text{NIL}, 1$ 

1    2    3    4    5    6

<b>Q</b>	1	3	2	4	5	6
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Flyt

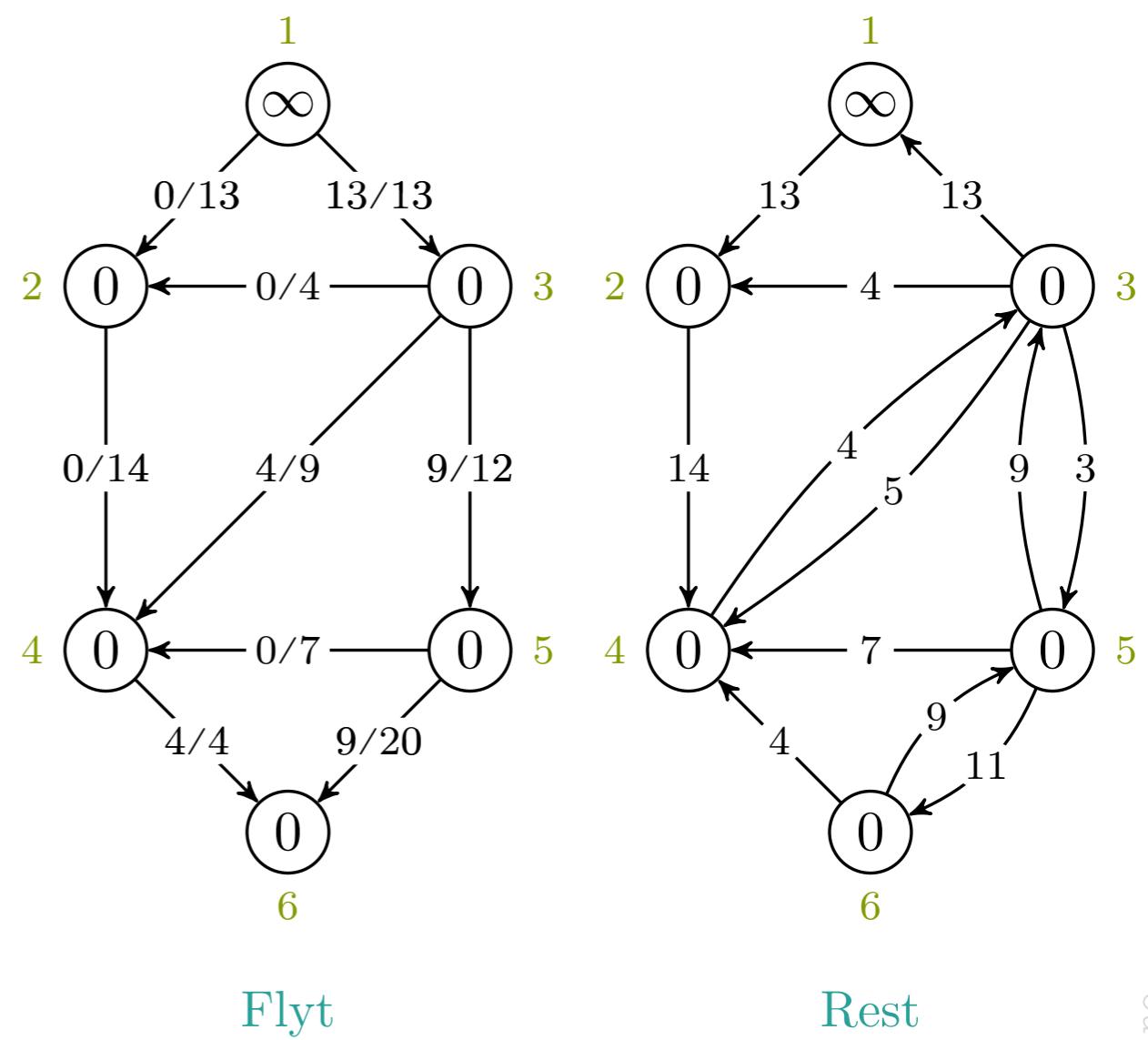
Rest

EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2       $(u, v).f = 0$ 
3  repeat
4      for each vertex  $u \in G.V$ 
5           $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6           $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11          $u = \text{DEQUEUE}(Q)$ 
12         for all edges  $(u, v), (v, u) \in G.E$ 
13             if  $(u, v) \in G.E$ 
14                  $c_f(u, v) = c(u, v) - (u, v).f$ 
15             else  $c_f(u, v) = (v, u).f$ 
16             if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                  $v.a = \min(u.a, c_f(u, v))$ 
18                  $v.\pi = u$ 
19                 ENQUEUE( $Q, v$ )
20              $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21             while  $u \neq \text{NIL}$ 
22                 if  $(u, v) \in G.E$ 
23                      $(u, v).f = (u, v).f + t.a$ 
24                 else  $(v, u).f = (v, u).f - t.a$ 
25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$

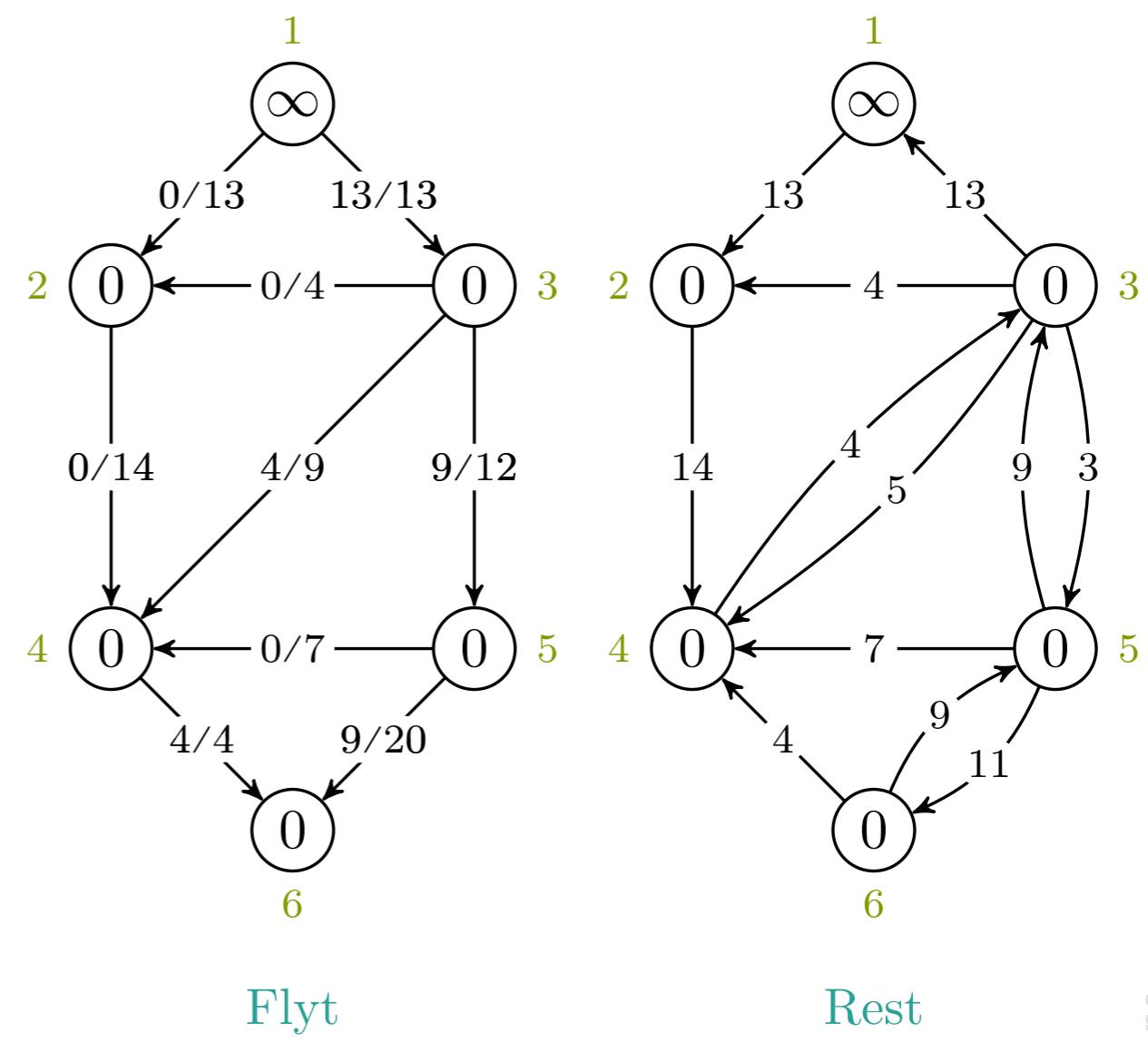


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2       $(u, v).f = 0$ 
3  repeat
4      for each vertex  $u \in G.V$ 
5           $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6           $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
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11          $u = \text{DEQUEUE}(Q)$ 
12         for all edges  $(u, v), (v, u) \in G.E$ 
13             if  $(u, v) \in G.E$ 
14                  $c_f(u, v) = c(u, v) - (u, v).f$ 
15             else  $c_f(u, v) = (v, u).f$ 
16             if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                  $v.a = \min(u.a, c_f(u, v))$ 
18                  $v.\pi = u$ 
19                 ENQUEUE( $Q, v$ )
20              $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21             while  $u \neq \text{NIL}$ 
22                 if  $(u, v) \in G.E$ 
23                      $(u, v).f = (u, v).f + t.a$ 
24                 else  $(v, u).f = (v, u).f - t.a$ 
25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$



EDMONDS-KARP( $G, s, t$ )

```

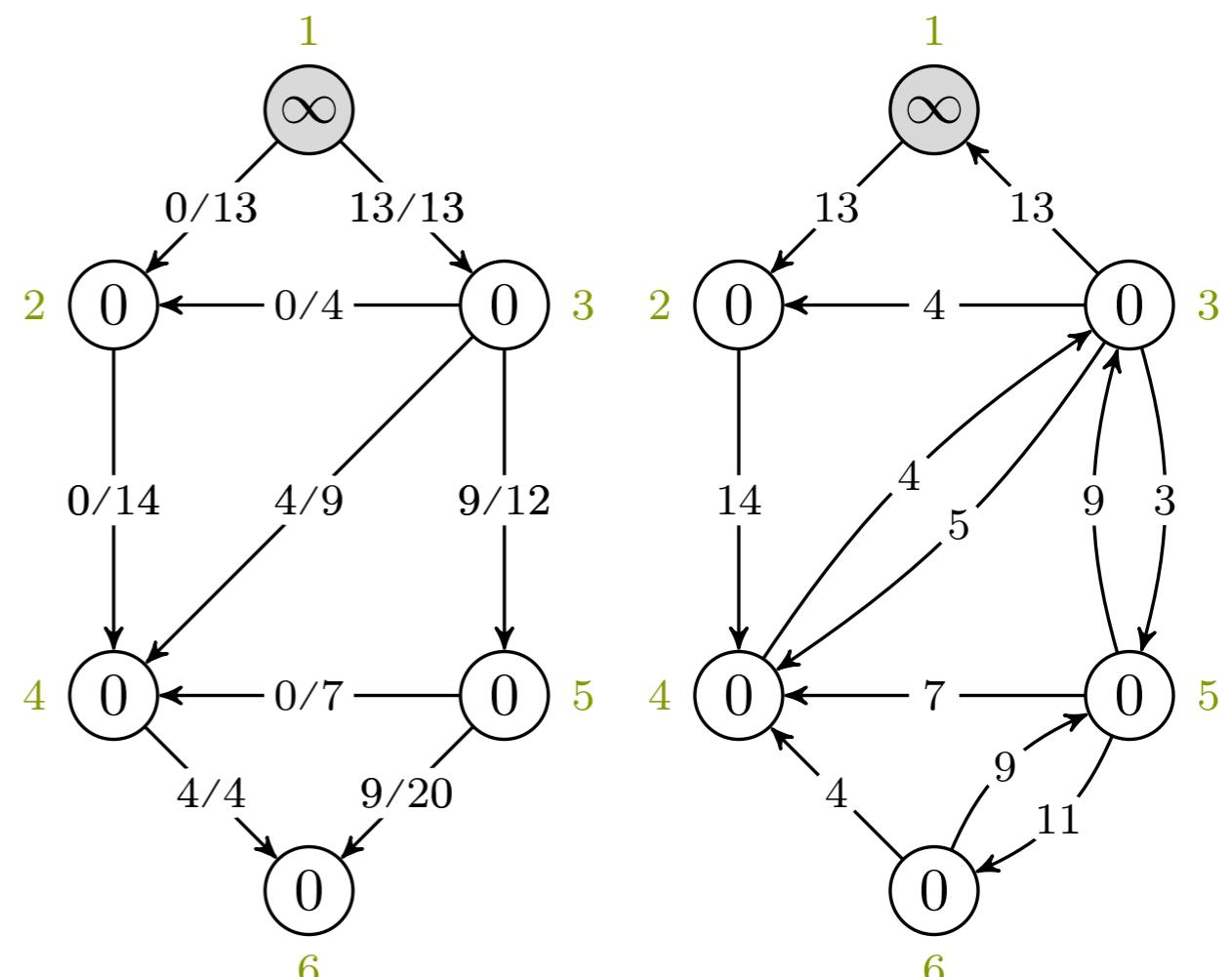
1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

 $u, v = \text{NIL}, 1$ 

1    2    3    4    5    6

<b>Q</b>	1	3	2	4	5	6
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Flyt

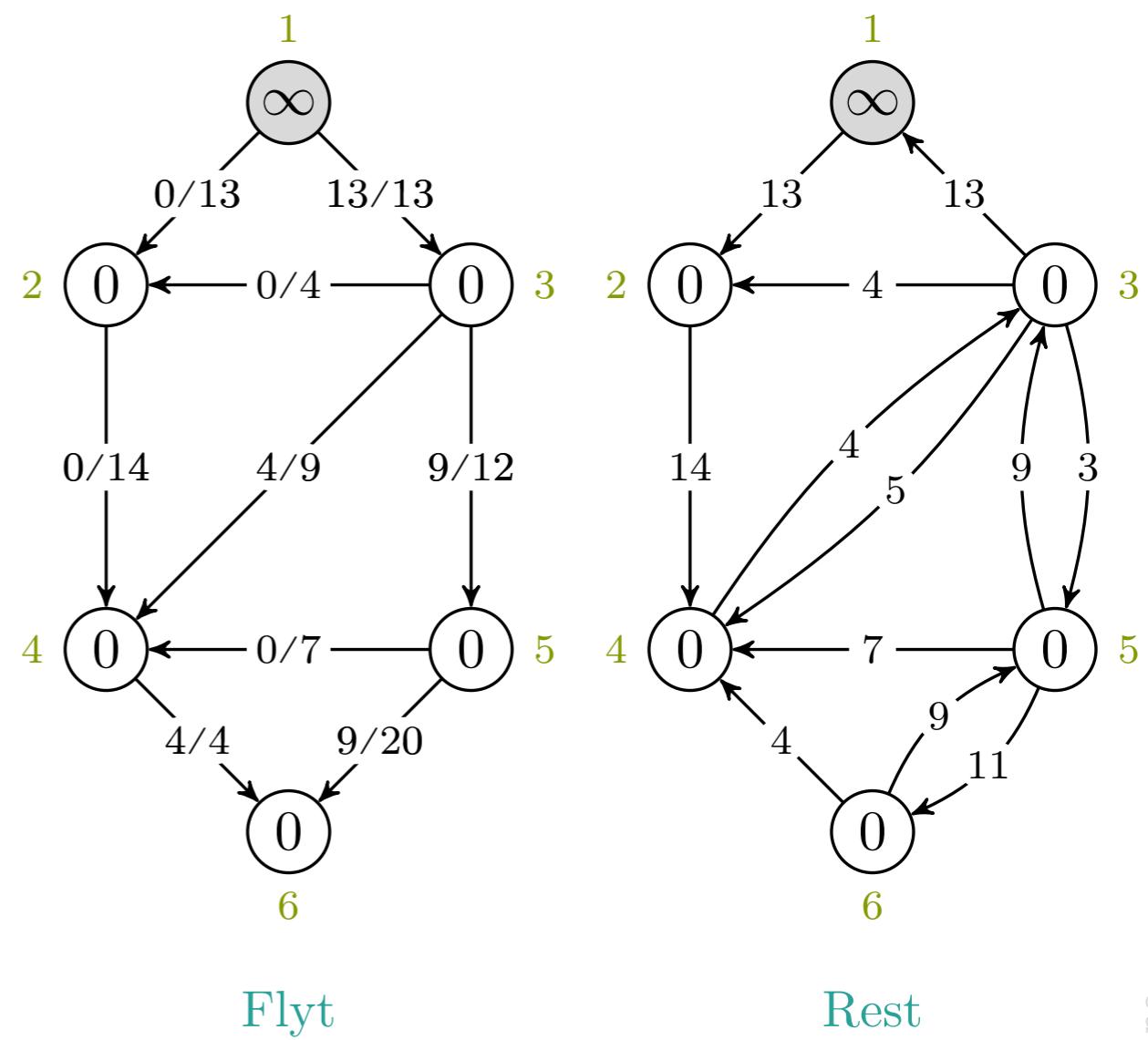
Rest

EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
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10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
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24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$



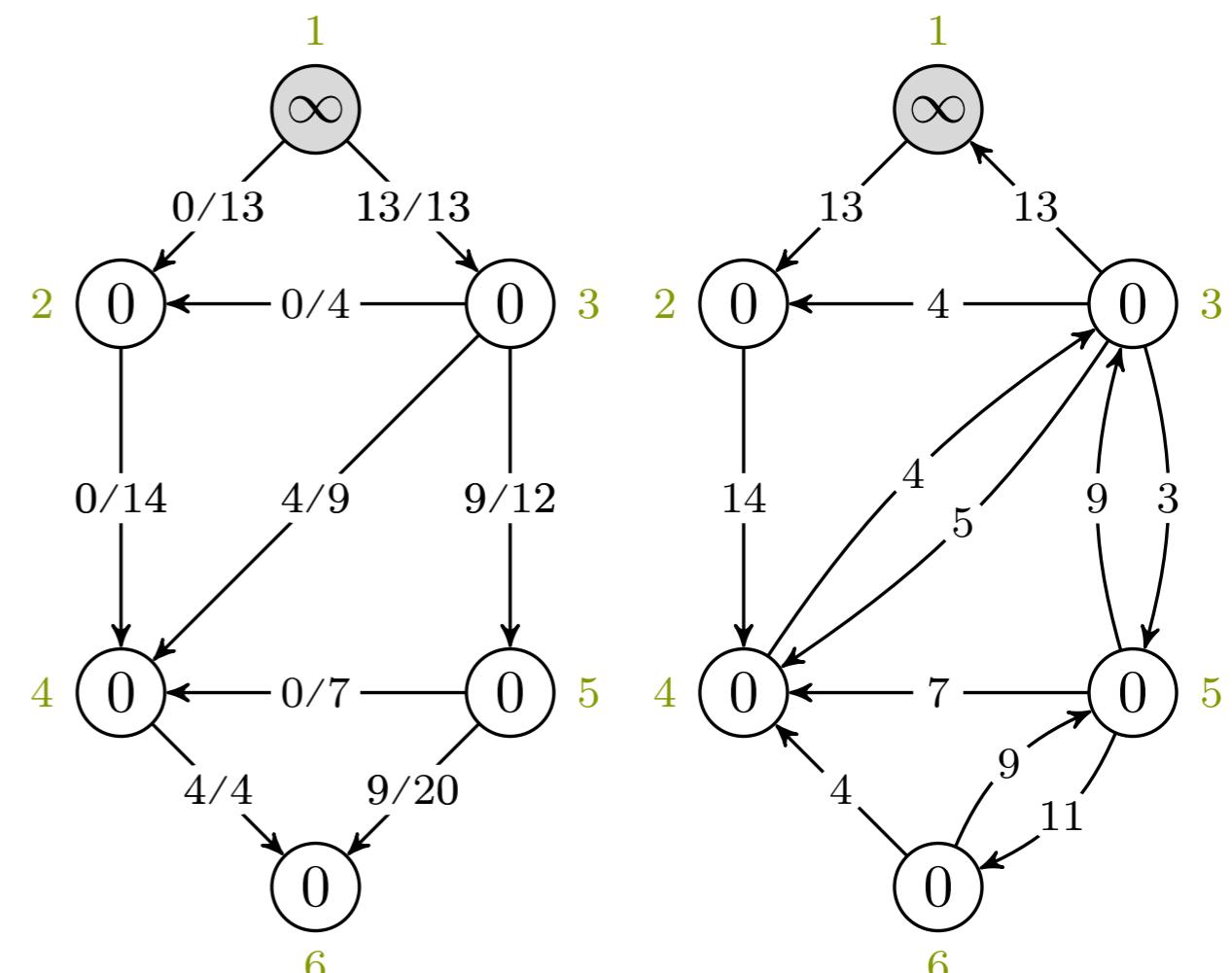
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
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16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
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23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = 1, 1$ 

Q	1	2	3	4	5	6
	1	3	2	4	5	6



Flyt

Rest

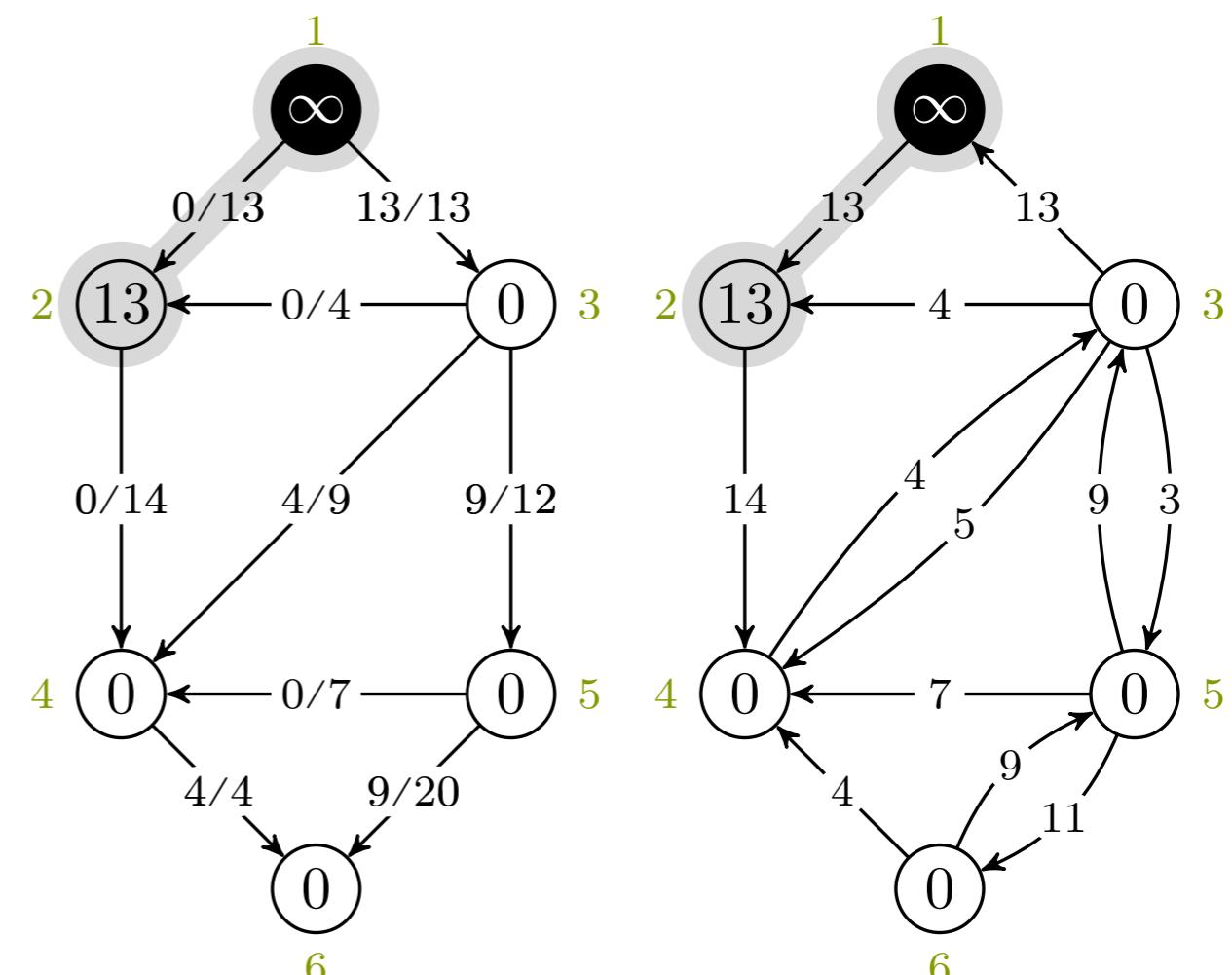
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
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25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, -$

Q	1	2	2	4	5	6
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Flyt

Rest

EDMONDS-KARP( $G, s, t$ )

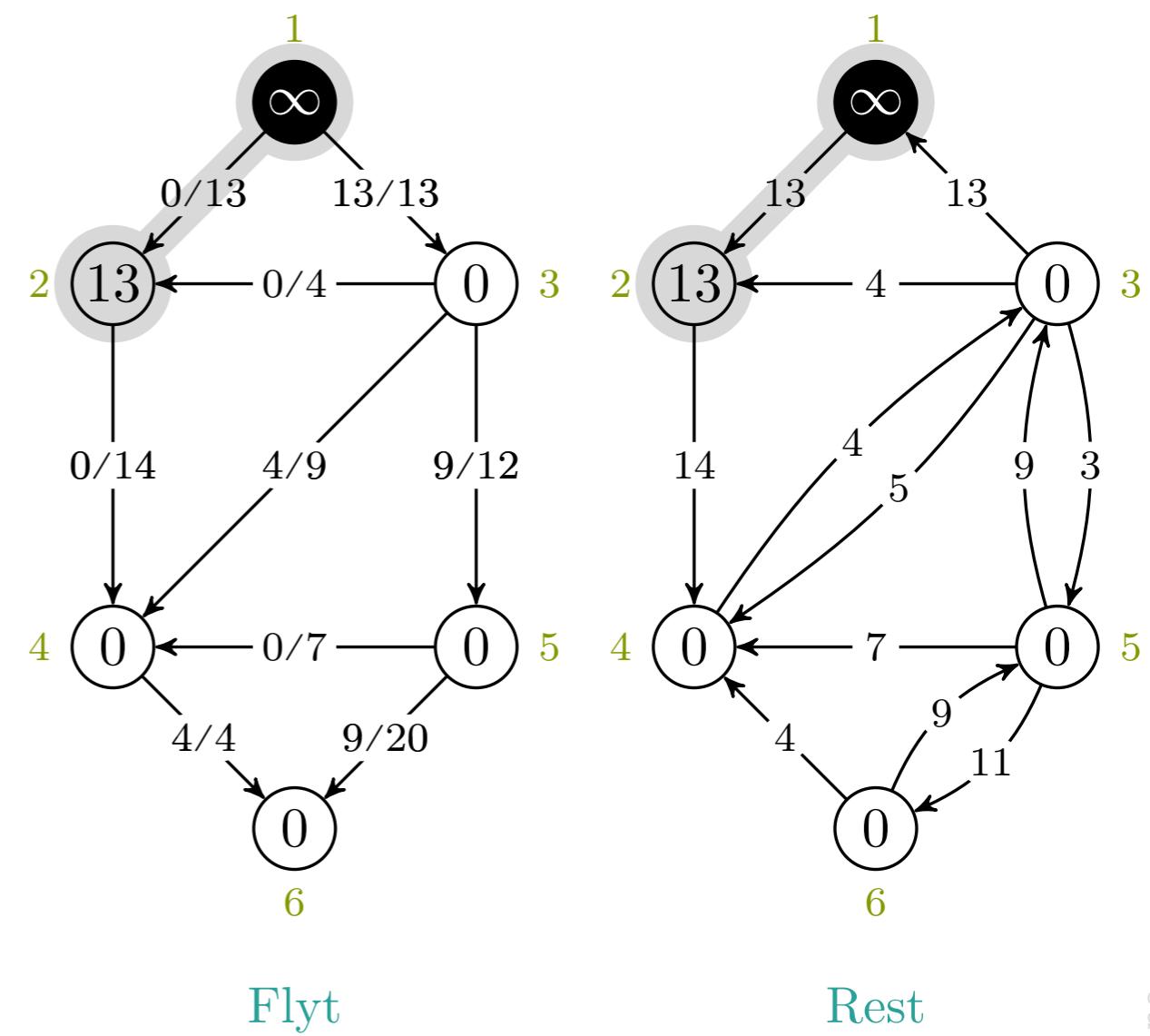
```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
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10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
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13         if  $(u, v) \in G.E$ 
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15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
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20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

 $u, v = 1, -$ 

Q	1	2	2	4	5	6
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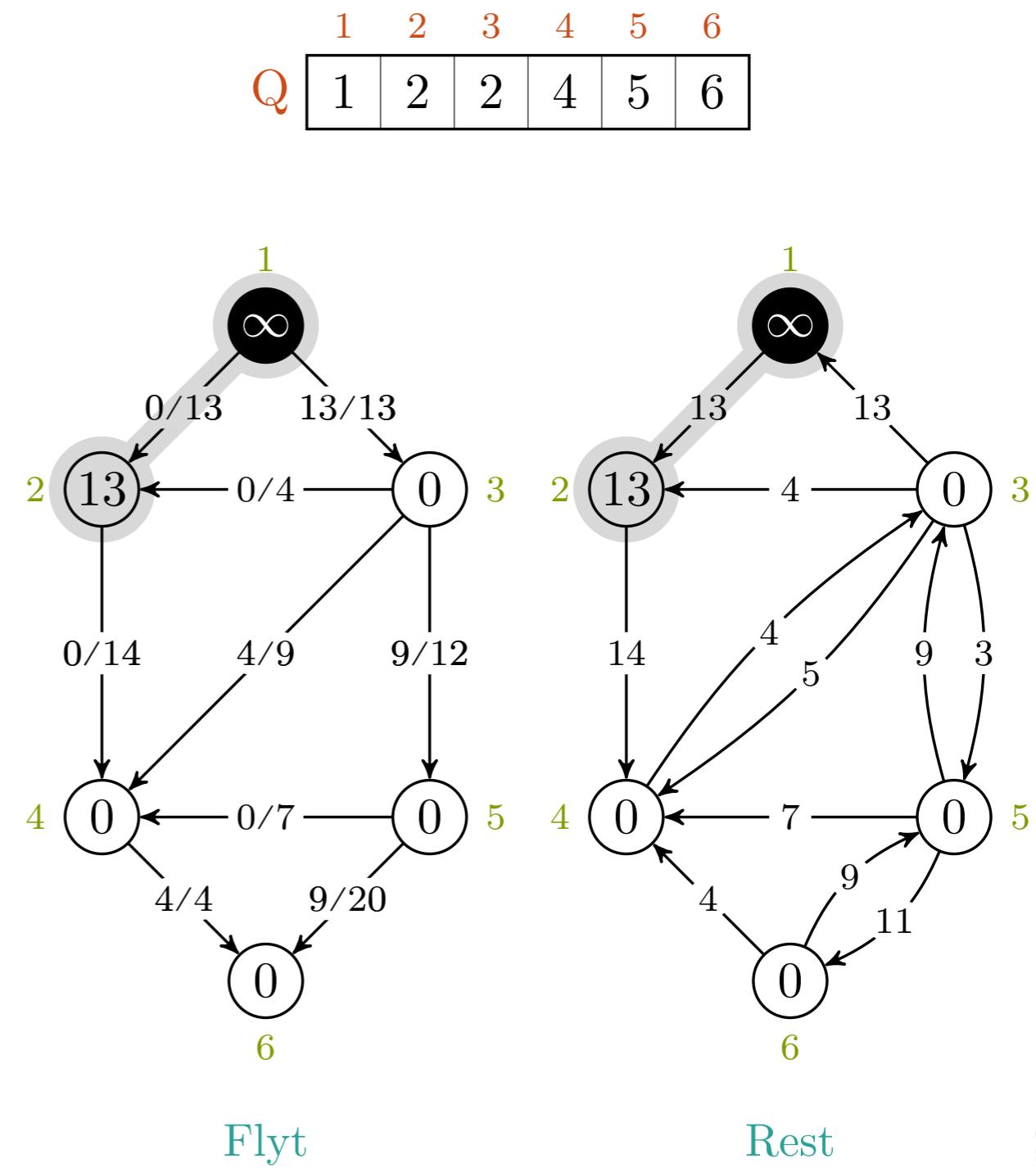


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, -$



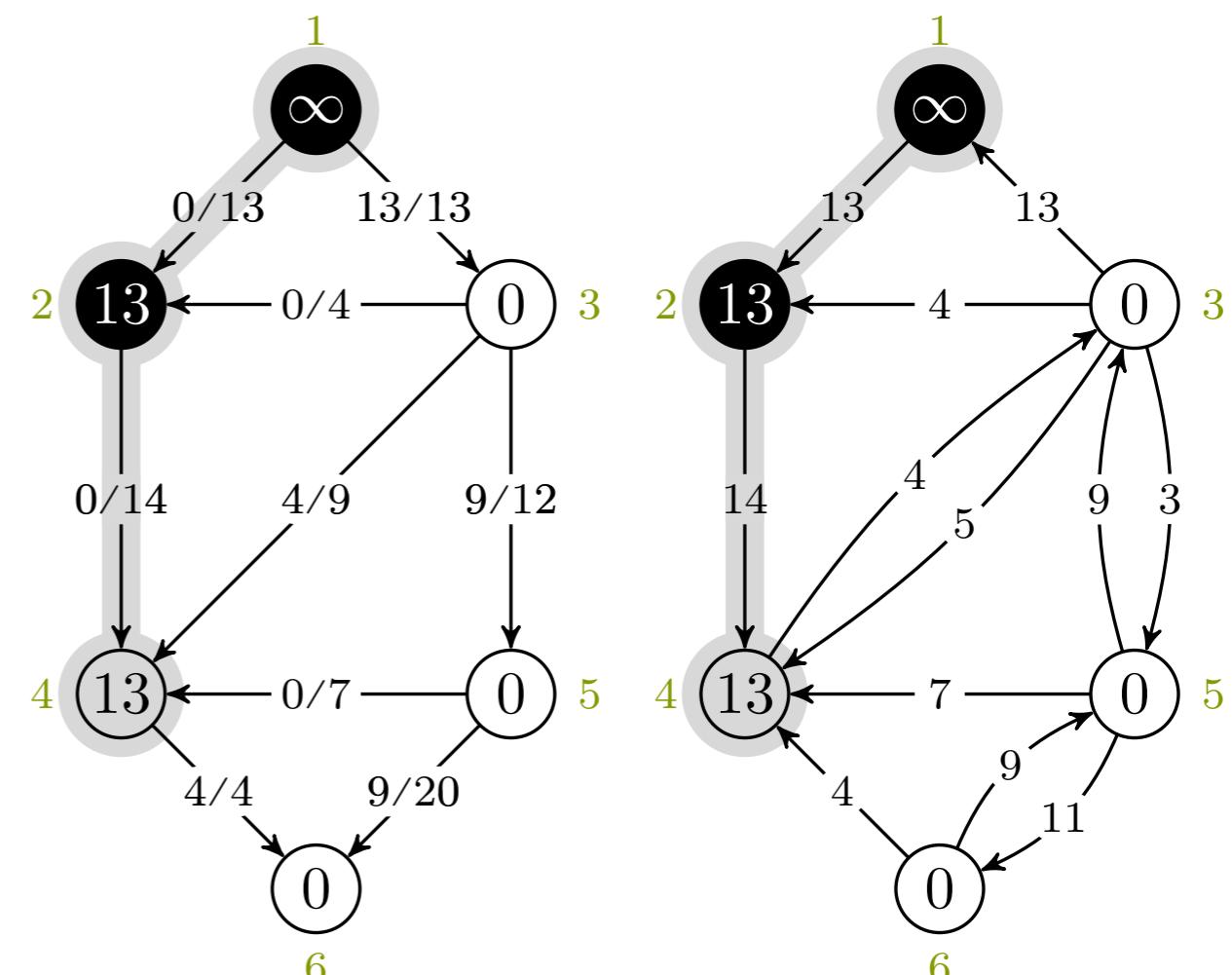
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = 2, -$ 

Q	1	2	4	4	5	6
---	---	---	---	---	---	---



Flyt

Rest

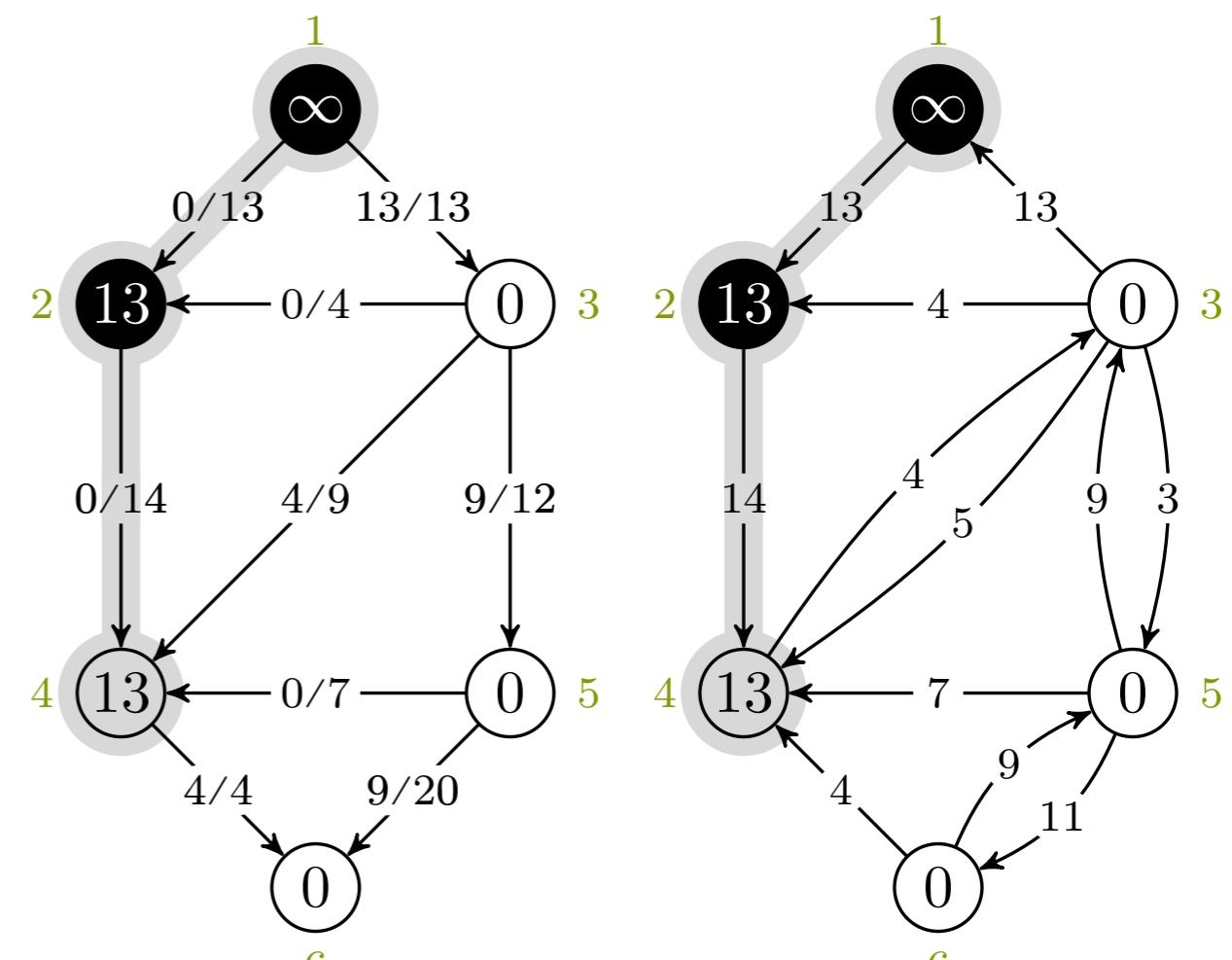
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = 2, -$ 

Q	1	2	4	4	5	6
---	---	---	---	---	---	---



Flyt

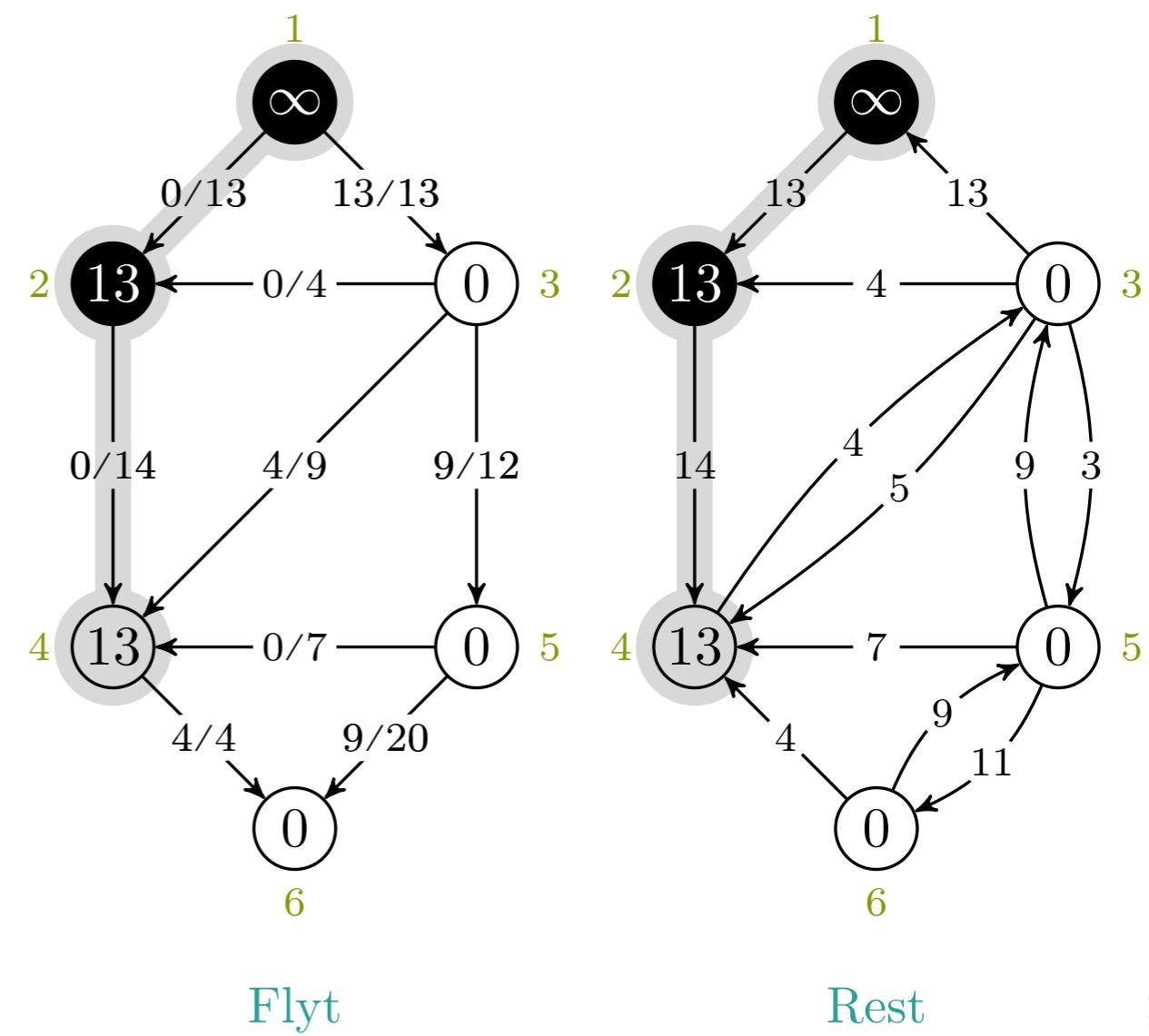
Rest

EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, -$



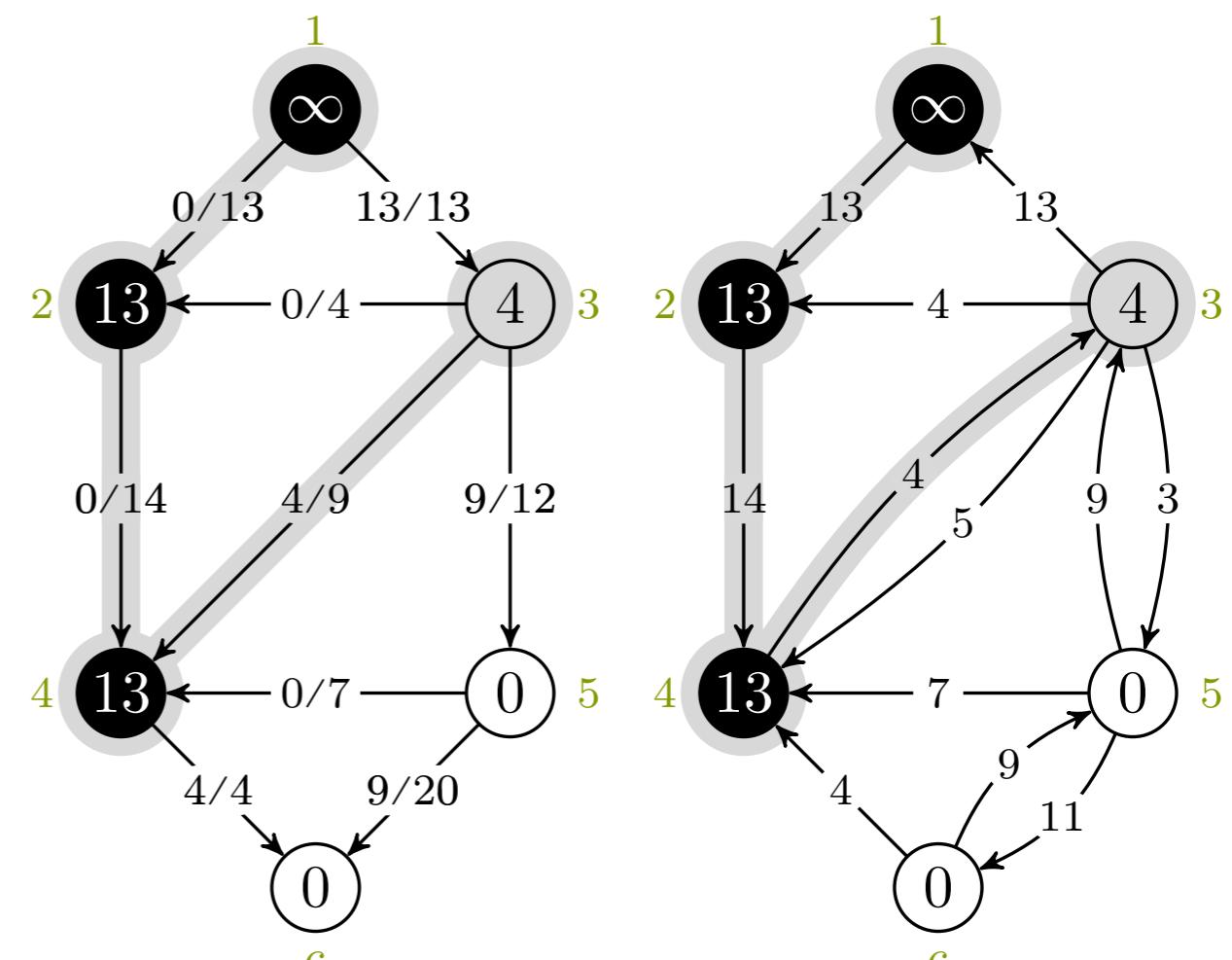
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, -$

Q	1	2	3	4	5	6
	1	2	4	3	5	6



Flyt

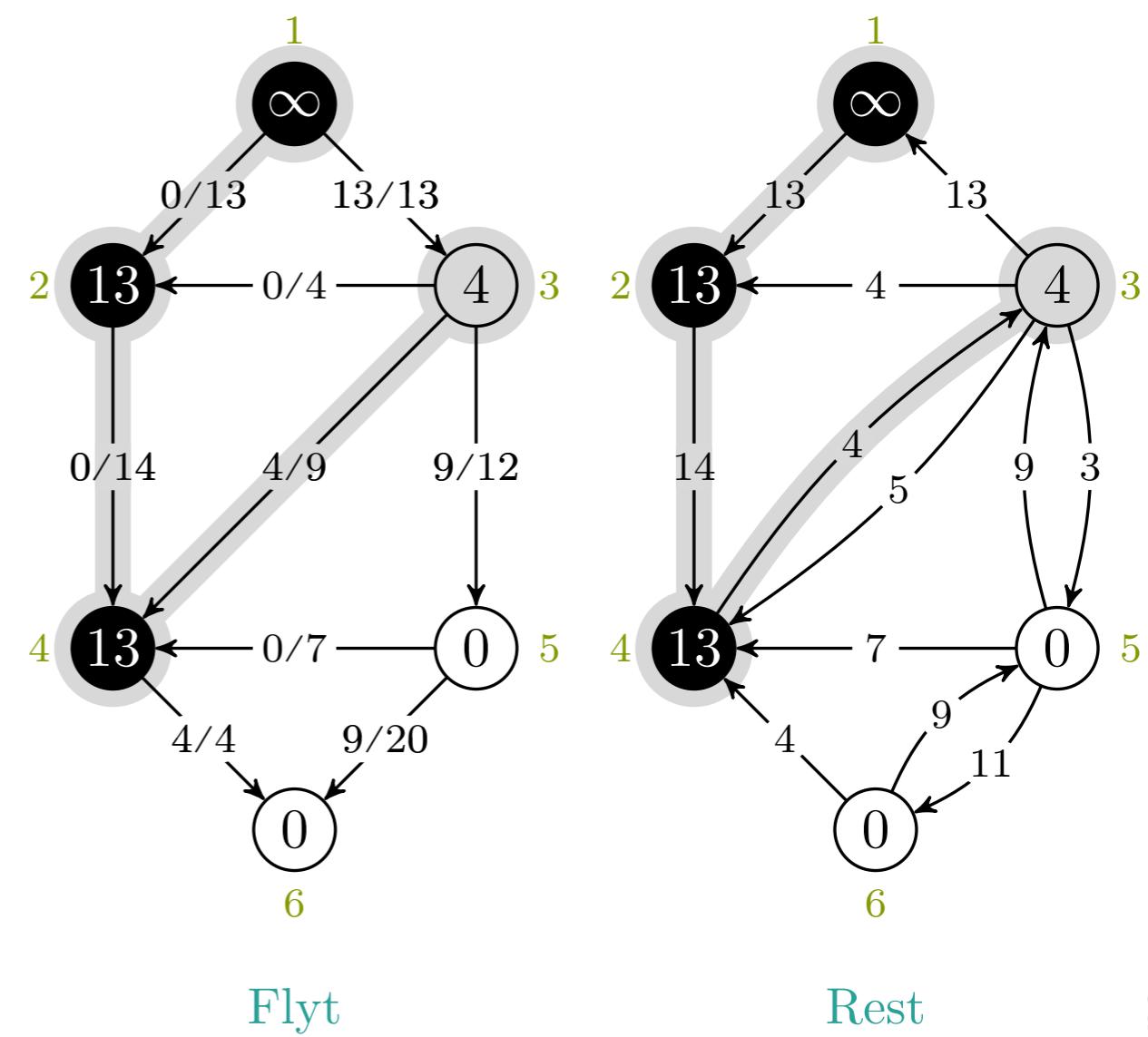
Rest

EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
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13         if  $(u, v) \in G.E$ 
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15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25          $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, -$

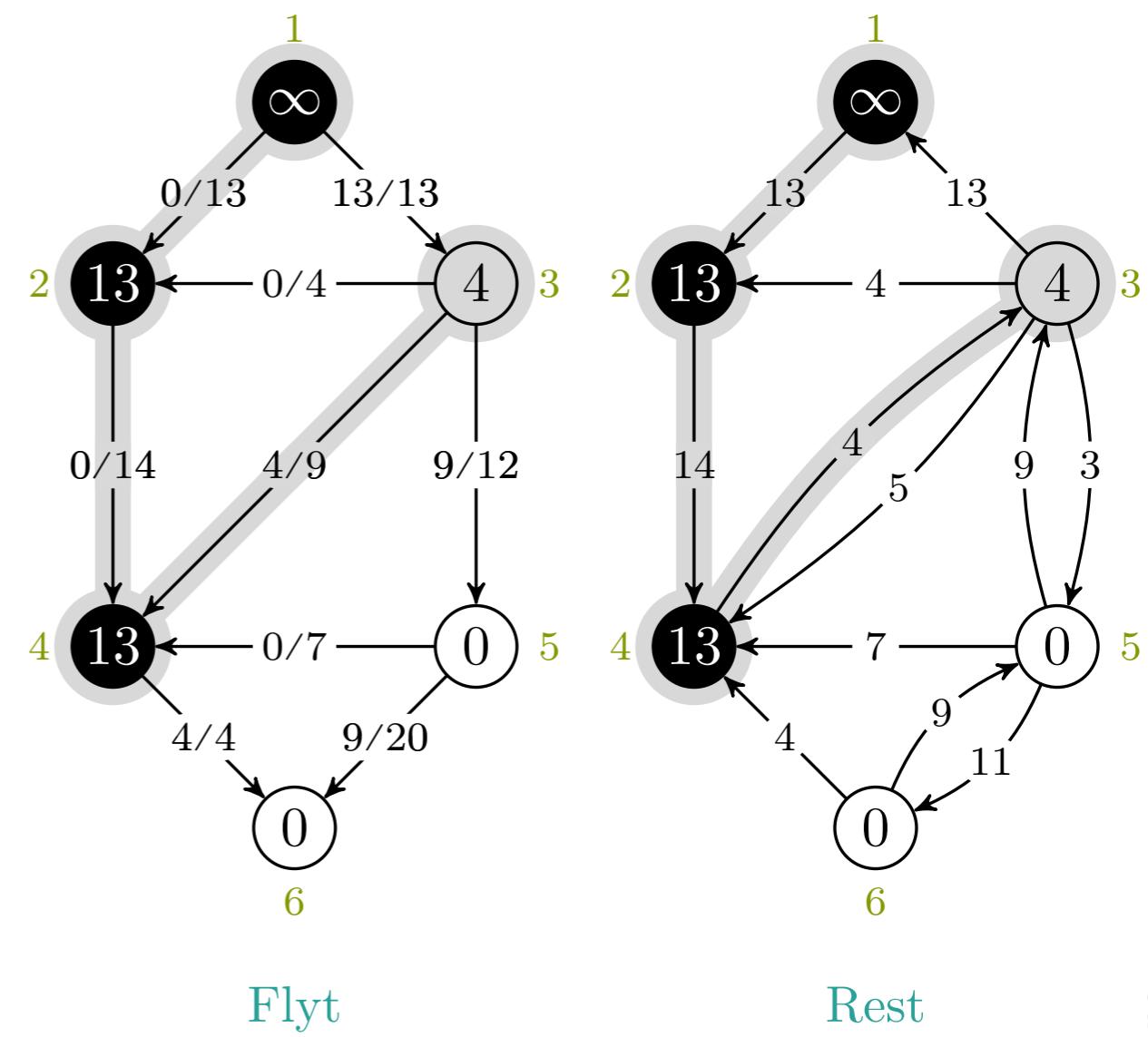


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
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20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
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23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, -$



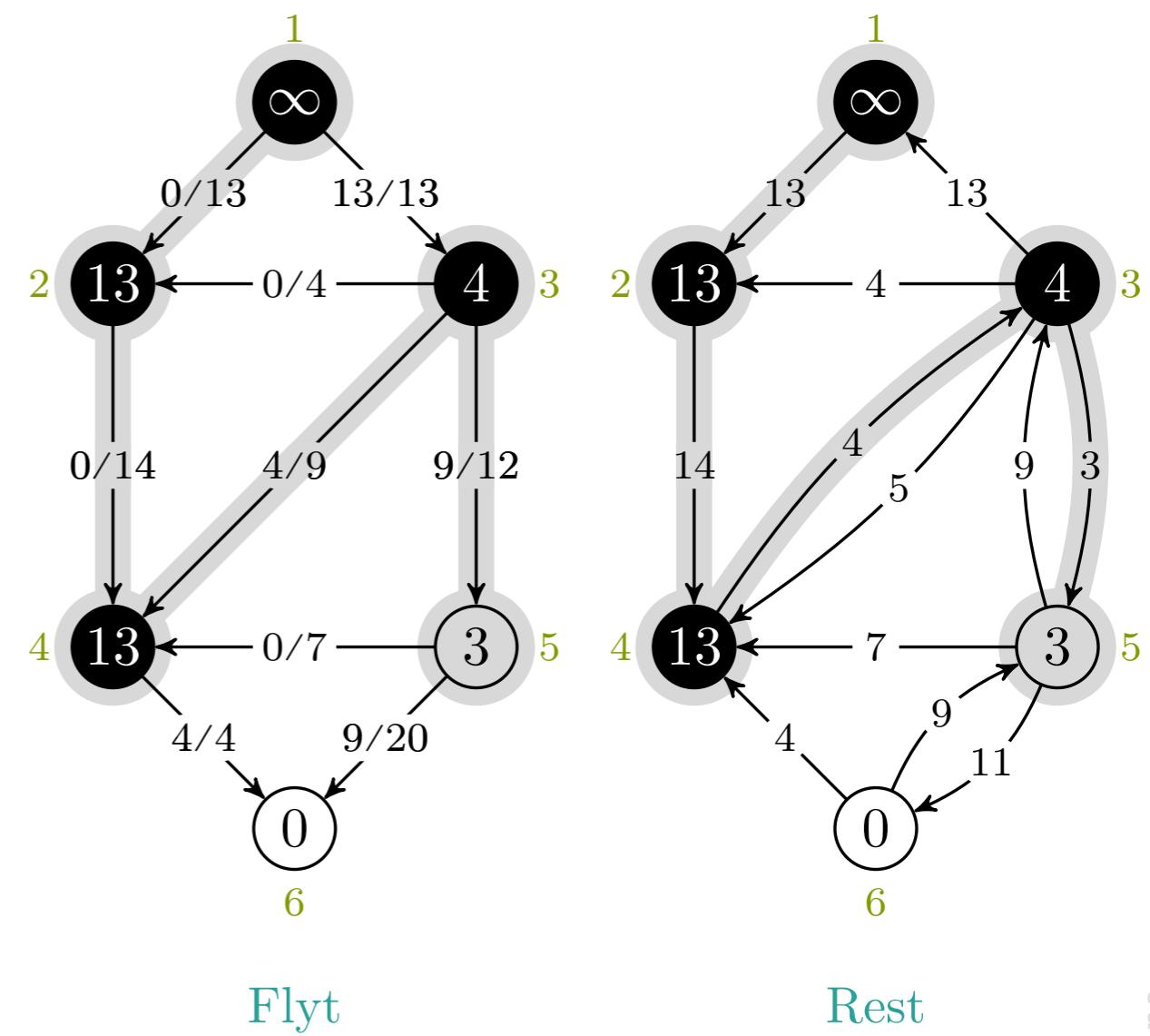
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21         while  $u \neq \text{NIL}$ 
22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, -$

Q	1	2	3	4	5	6
	1	2	4	3	5	6

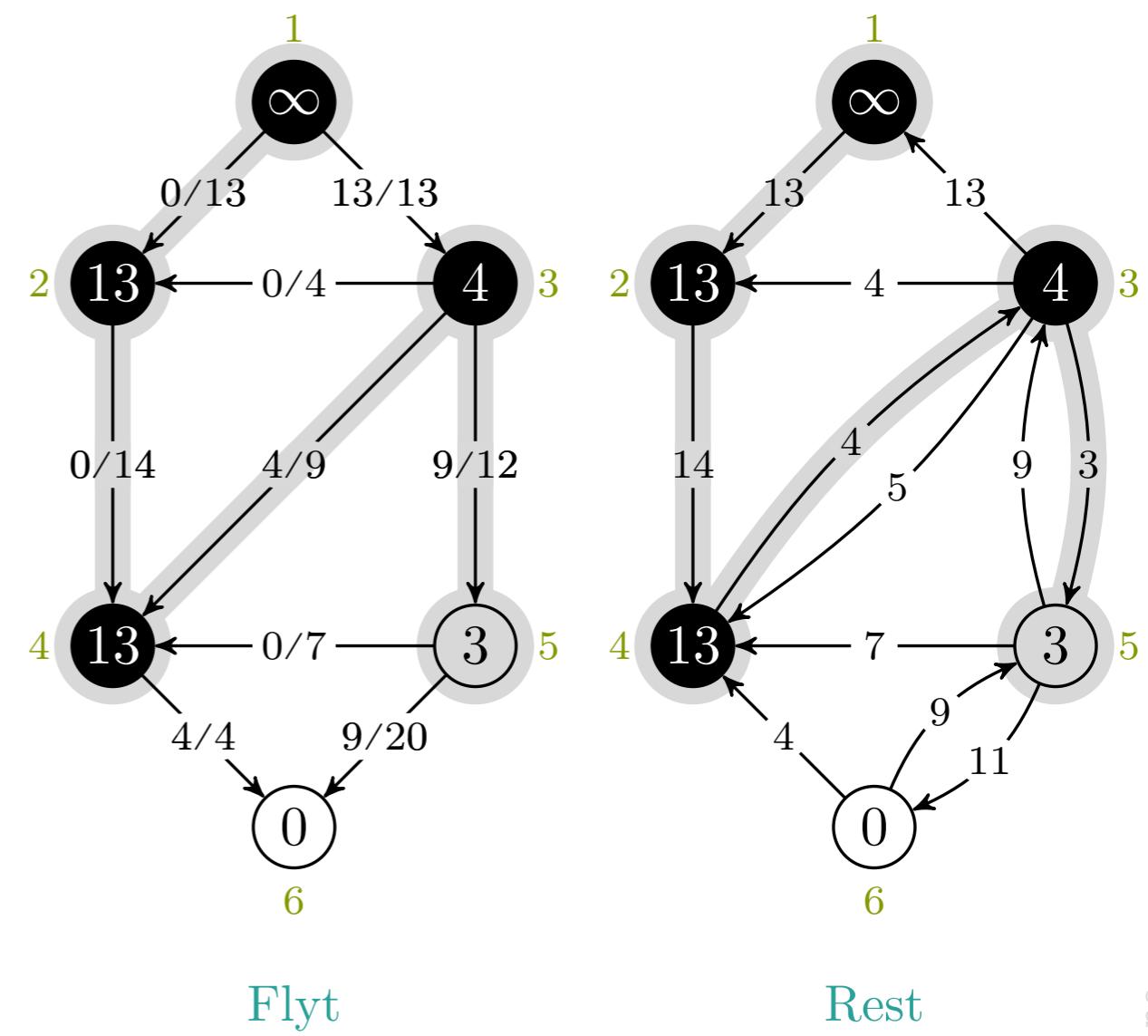


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
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22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25          $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, -$



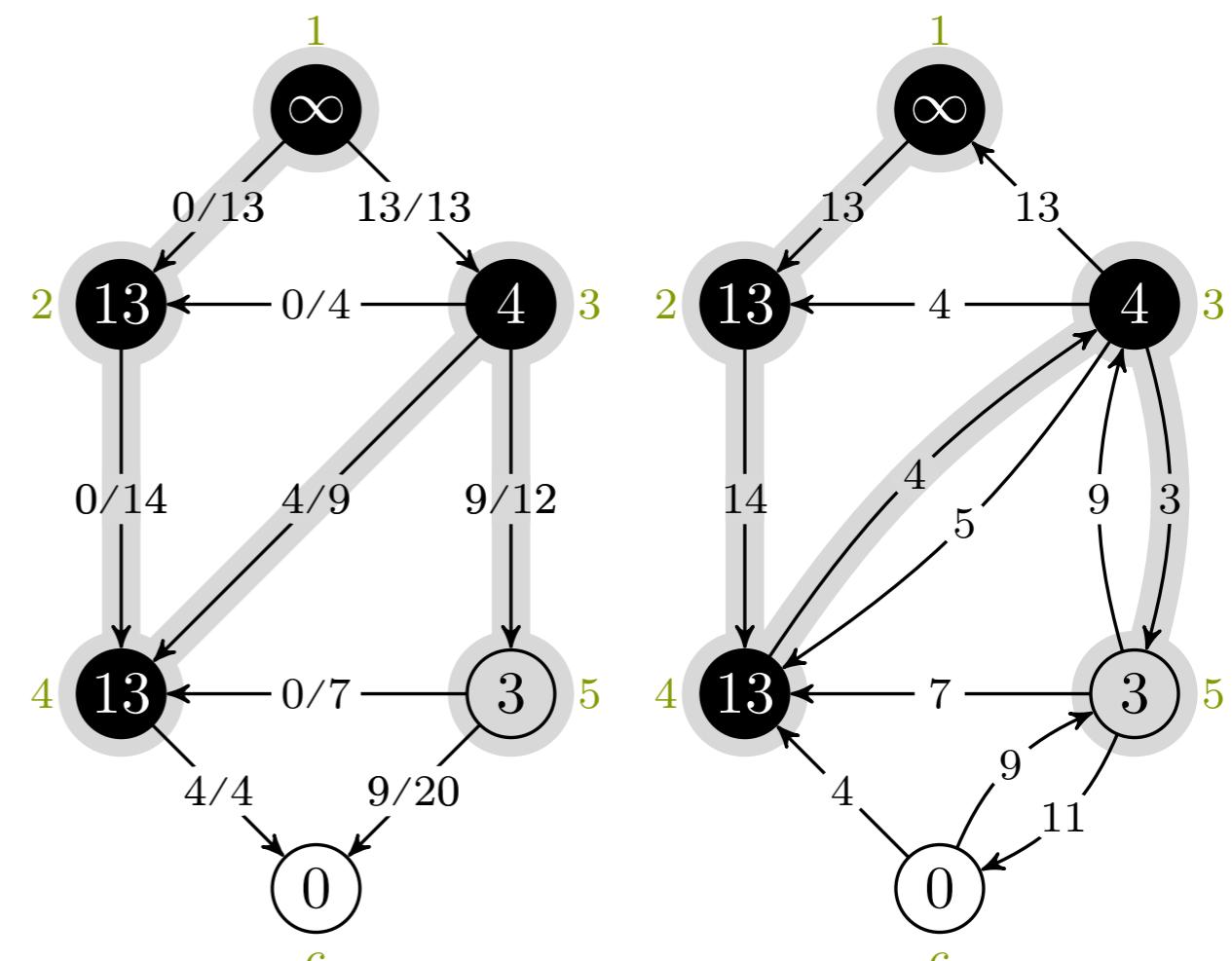
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
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4    for each vertex  $u \in G.V$ 
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23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = 5, -$ 

Q	1	2	3	4	5	6
	1	2	4	3	5	6



Flyt

Rest

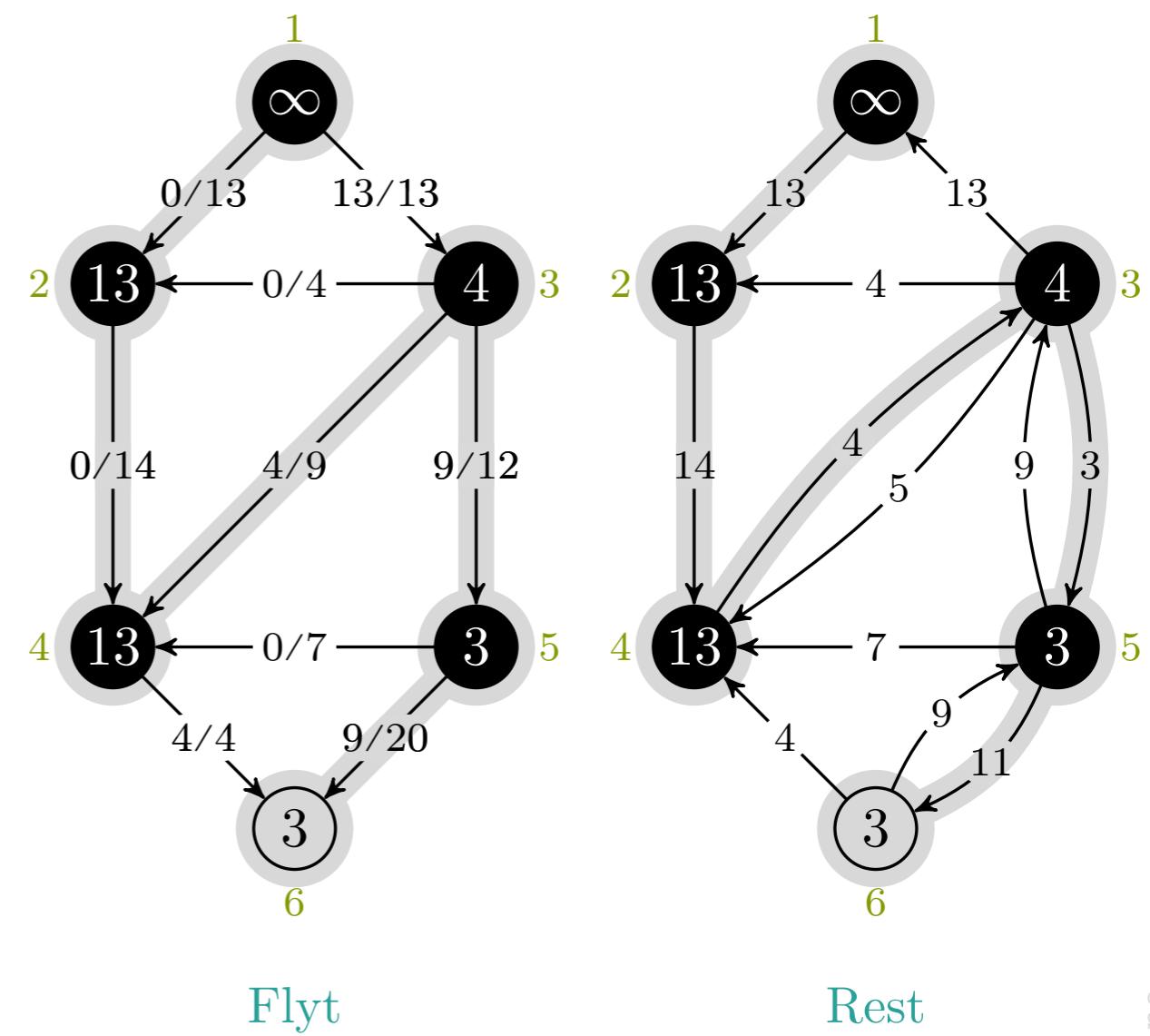
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
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9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
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17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
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20          $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
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22           if  $(u, v) \in G.E$ 
23              $(u, v).f = (u, v).f + t.a$ 
24           else  $(v, u).f = (v, u).f - t.a$ 
25            $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 5, -$

Q	1	2	3	4	5	6
	1	2	4	3	5	6

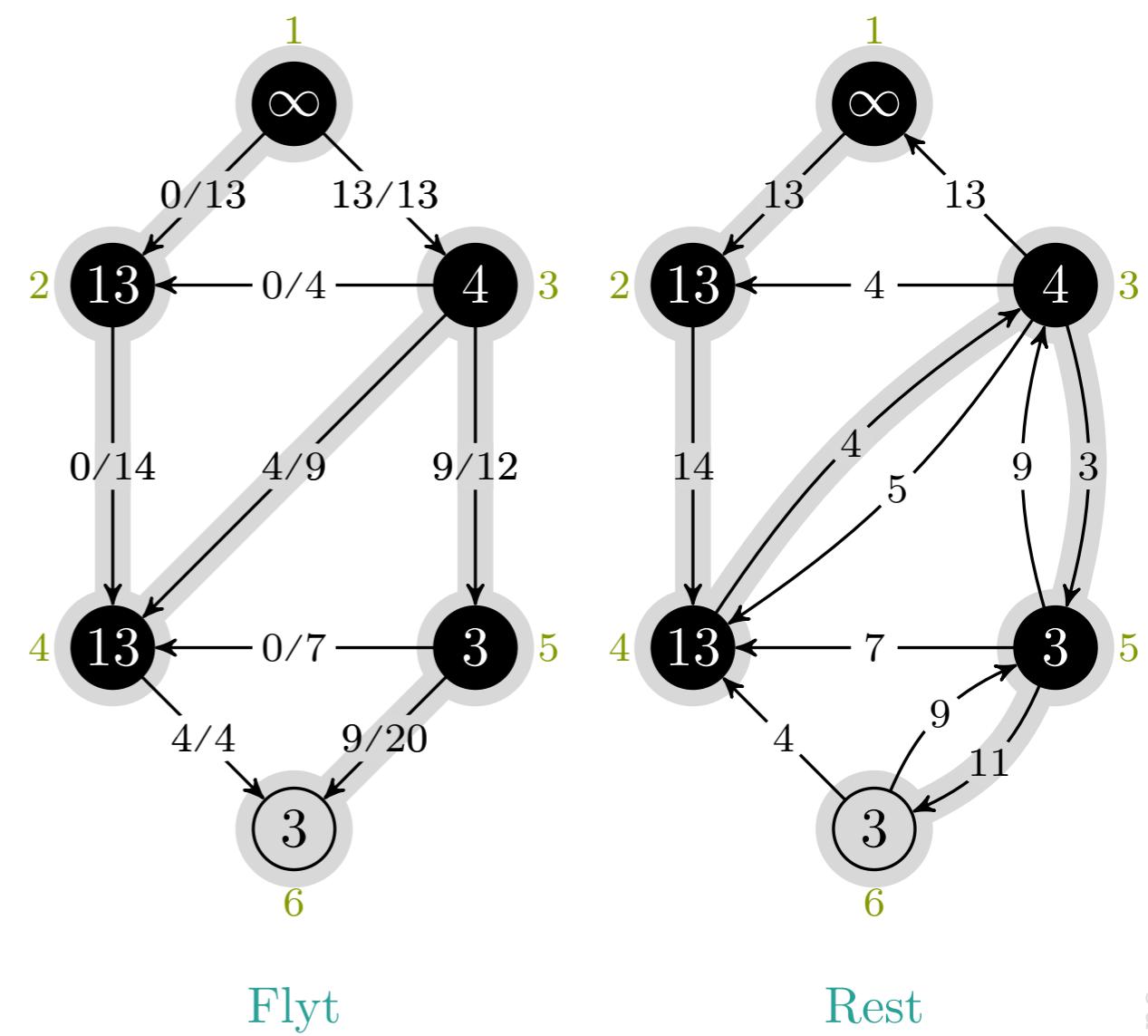


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
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18            $v.\pi = u$ 
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20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
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22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = -, -$

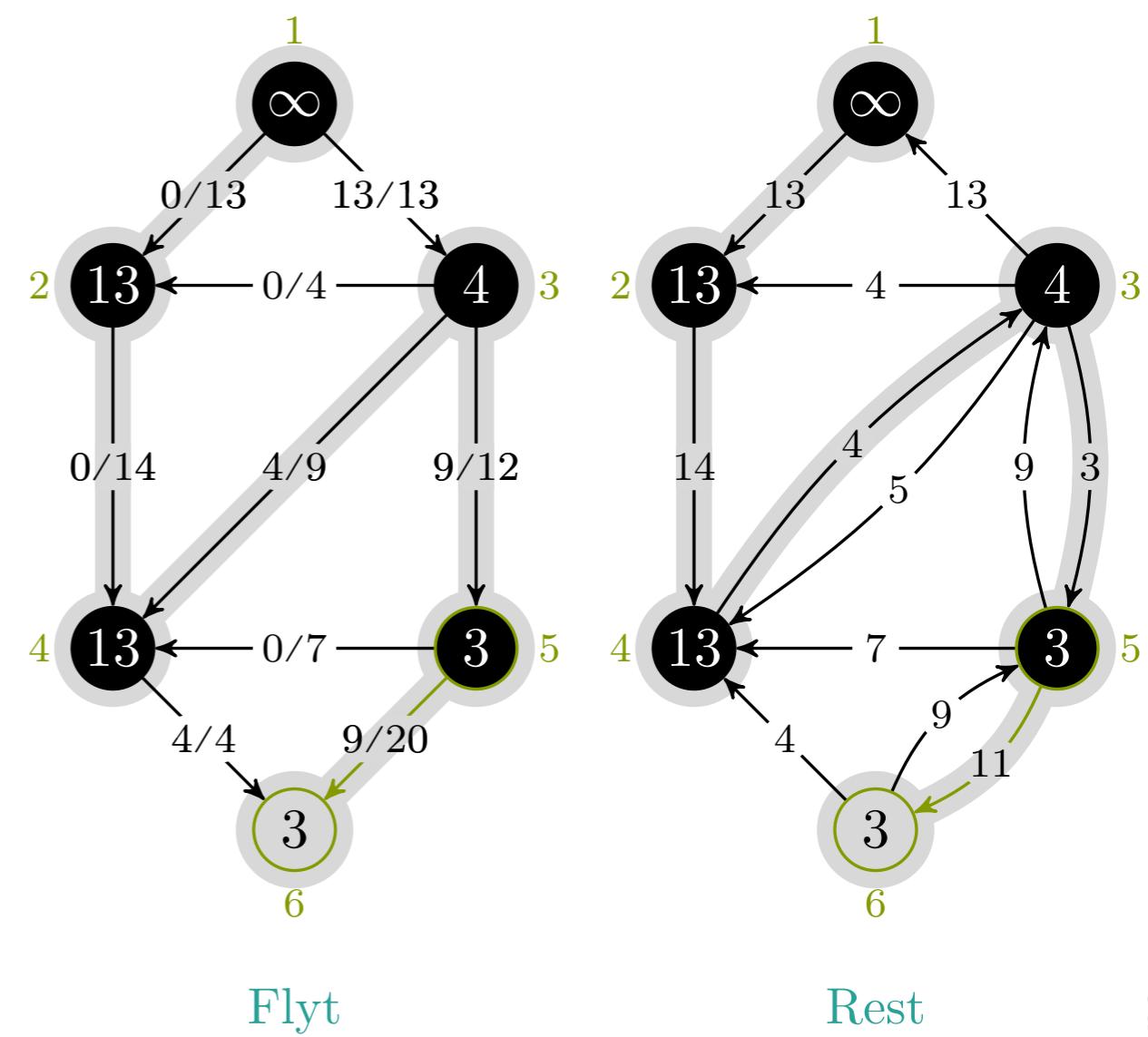


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
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15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 5, 6$

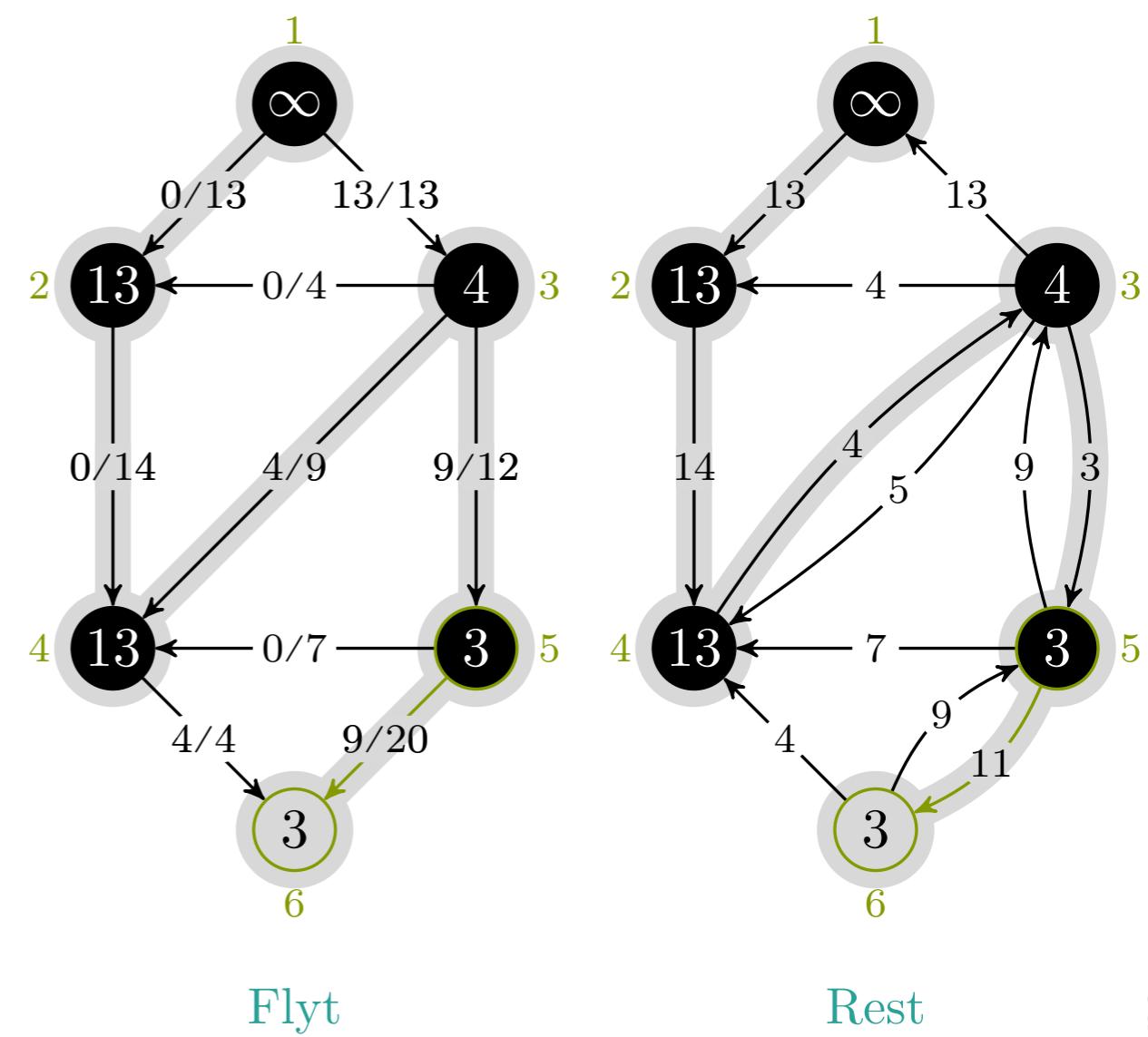


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
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8       $Q = \emptyset$ 
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10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
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18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 5, 6$

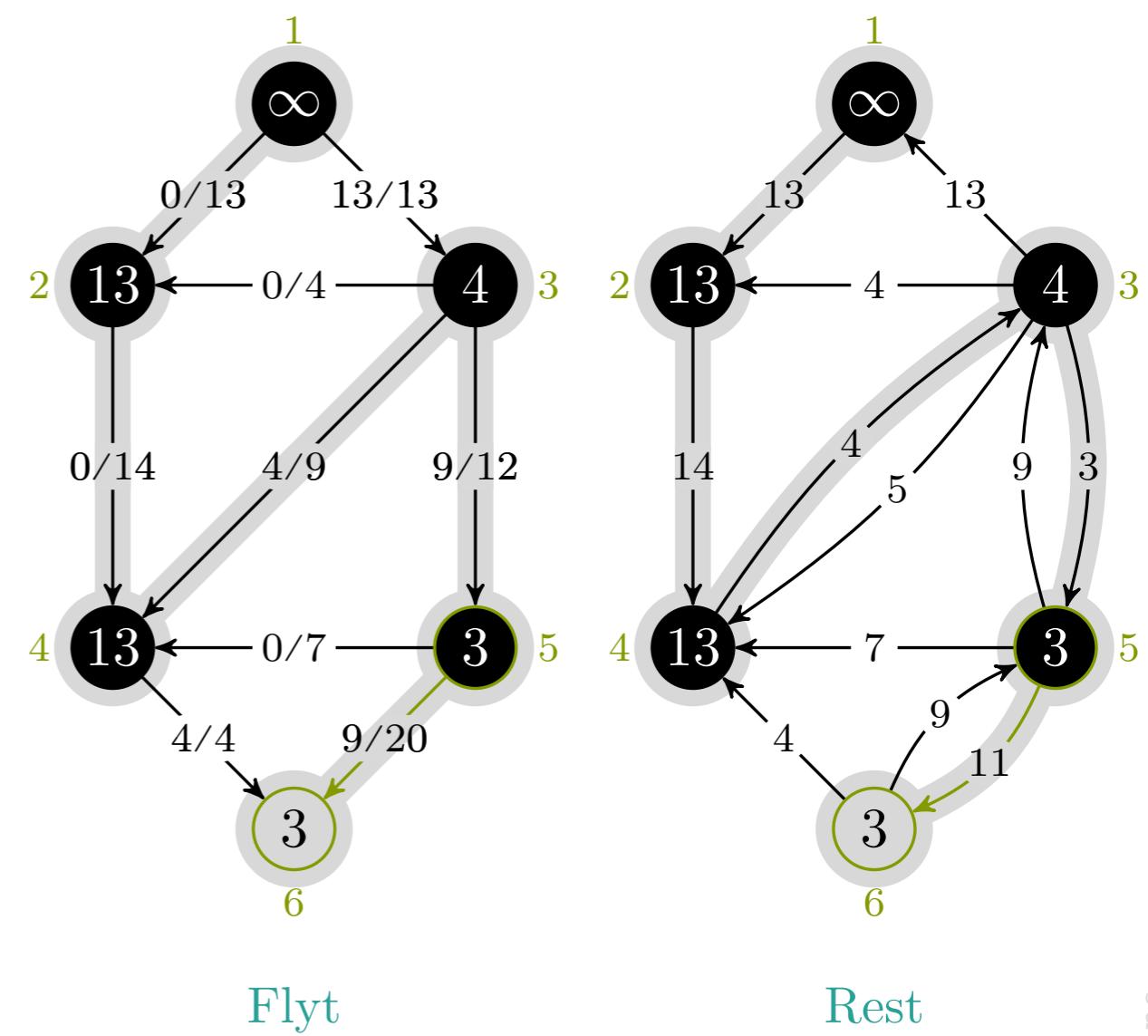


EDMONDS-KARP( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7    $s.a = \infty$ 
8    $Q = \emptyset$ 
9   ENQUEUE( $Q, s$ )
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20     $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21    while  $u \neq \text{NIL}$ 
22      if  $(u, v) \in G.E$ 
23         $(u, v).f = (u, v).f + t.a$ 
24      else  $(v, u).f = (v, u).f - t.a$ 
25     $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 5, 6$

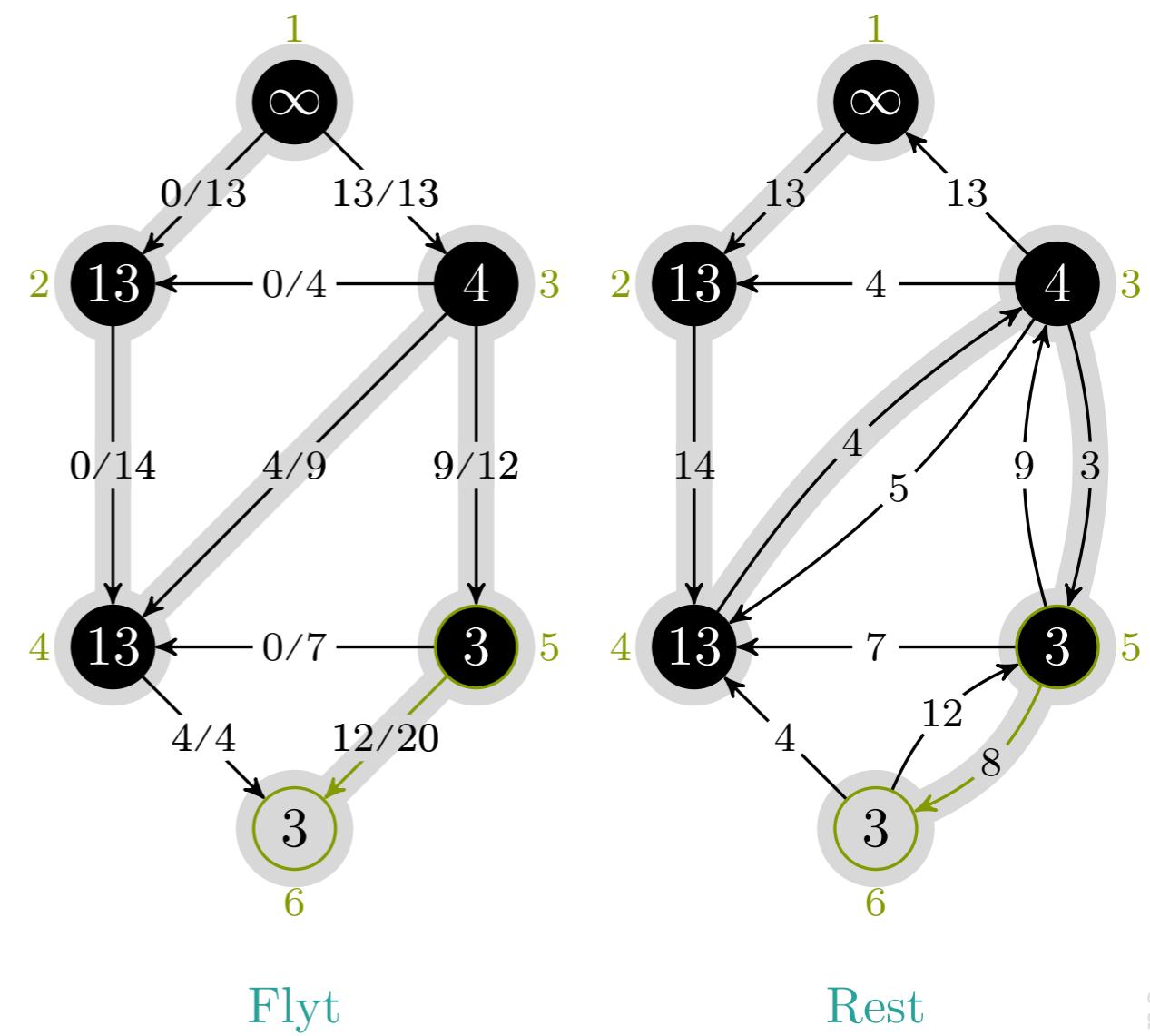


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
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7       $s.a = \infty$ 
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10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
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13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
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18            $v.\pi = u$ 
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23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 5, 6$

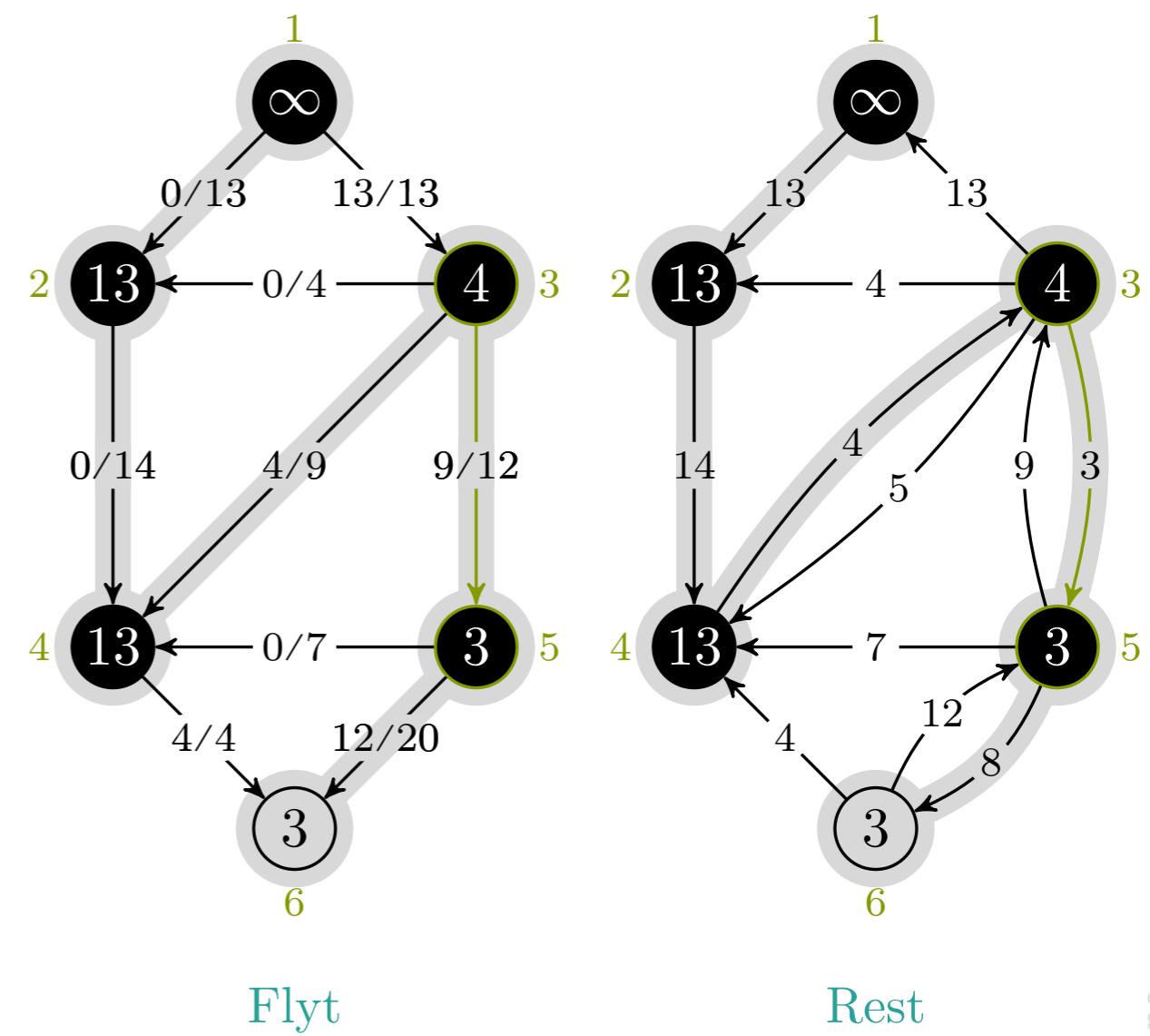


EDMONDS-KARP( $G, s, t$ )

```

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4    for each vertex  $u \in G.V$ 
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17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 5$

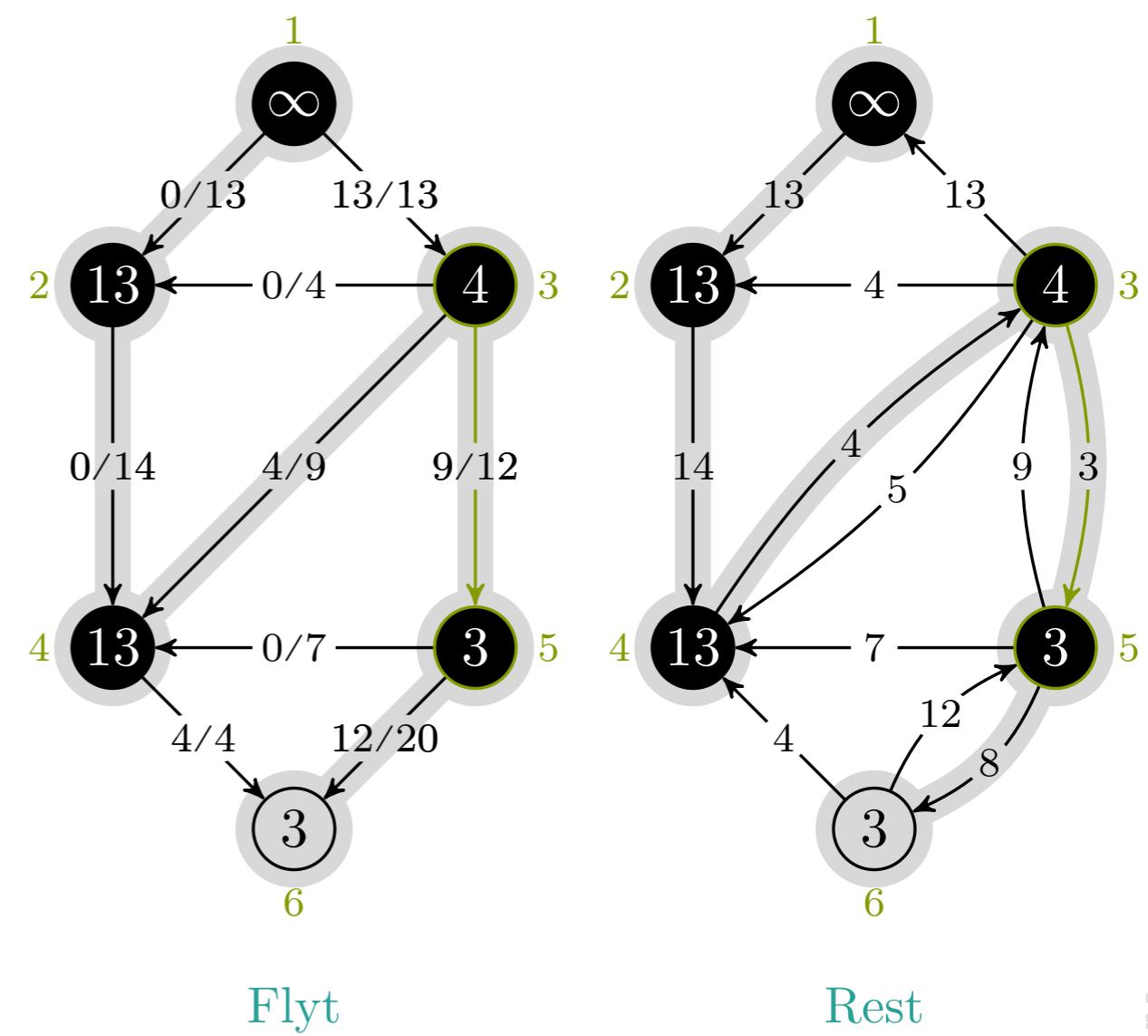


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 5$

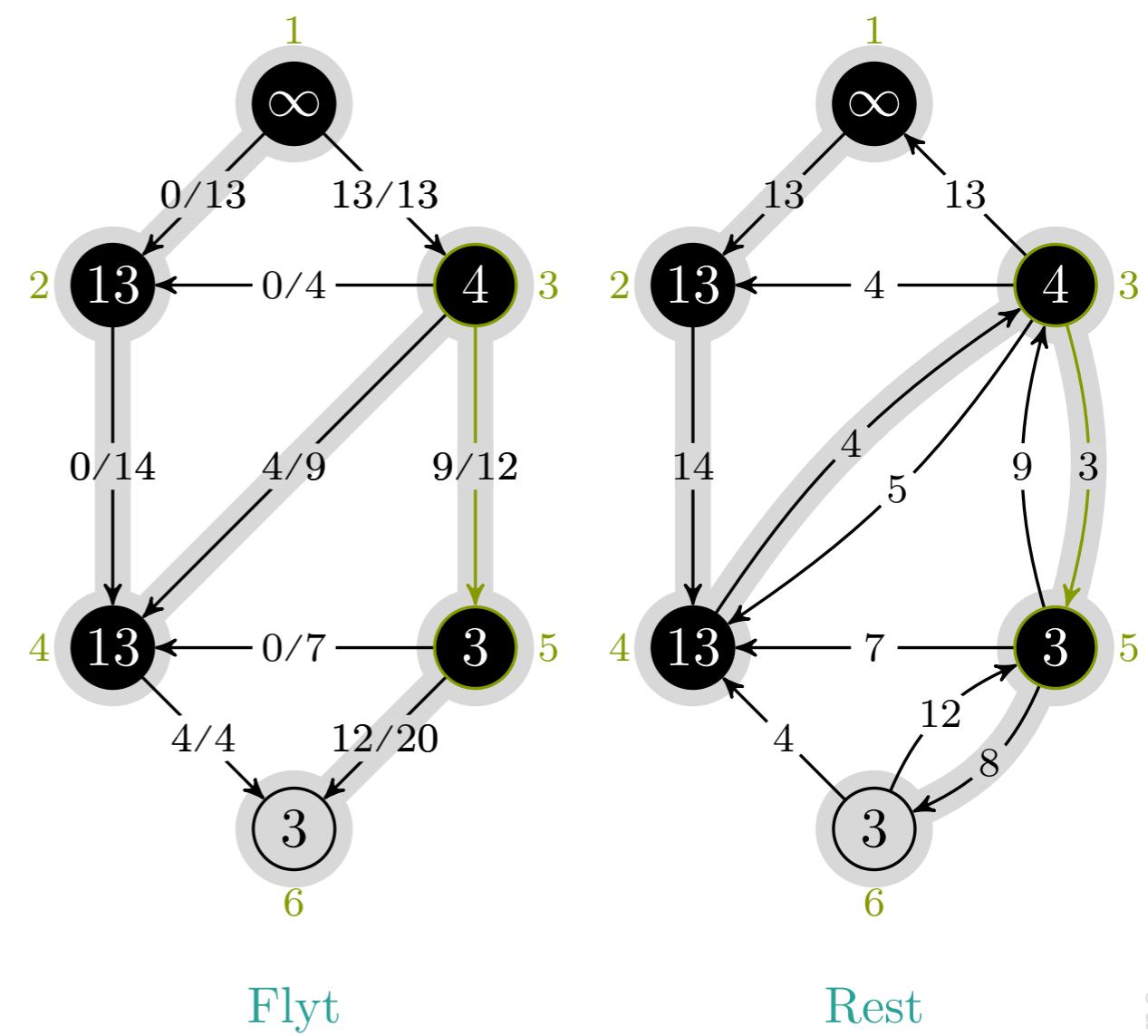


EDMONDS-KARP( $G, s, t$ )

```

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2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
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17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
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22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 5$

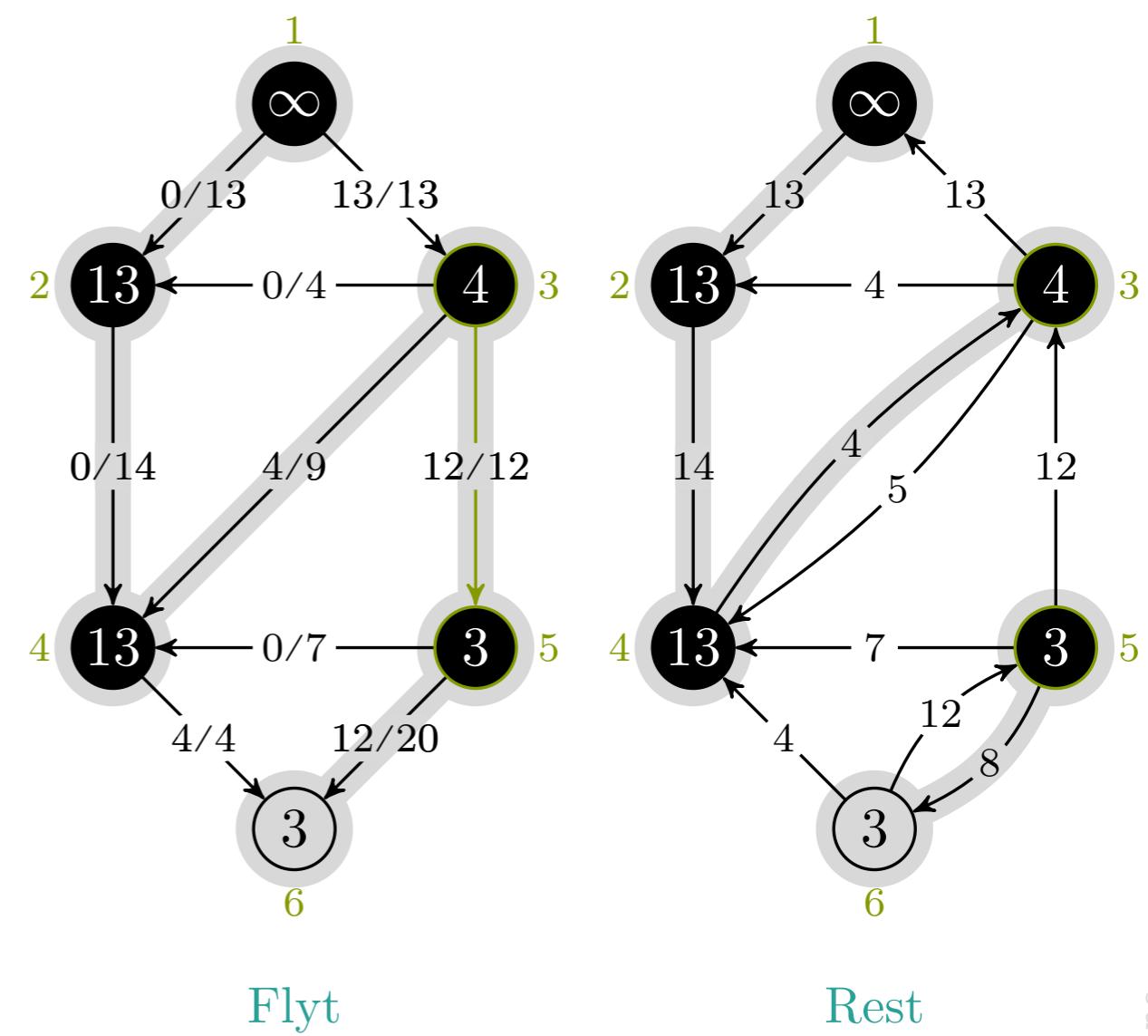


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
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25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 3, 5$

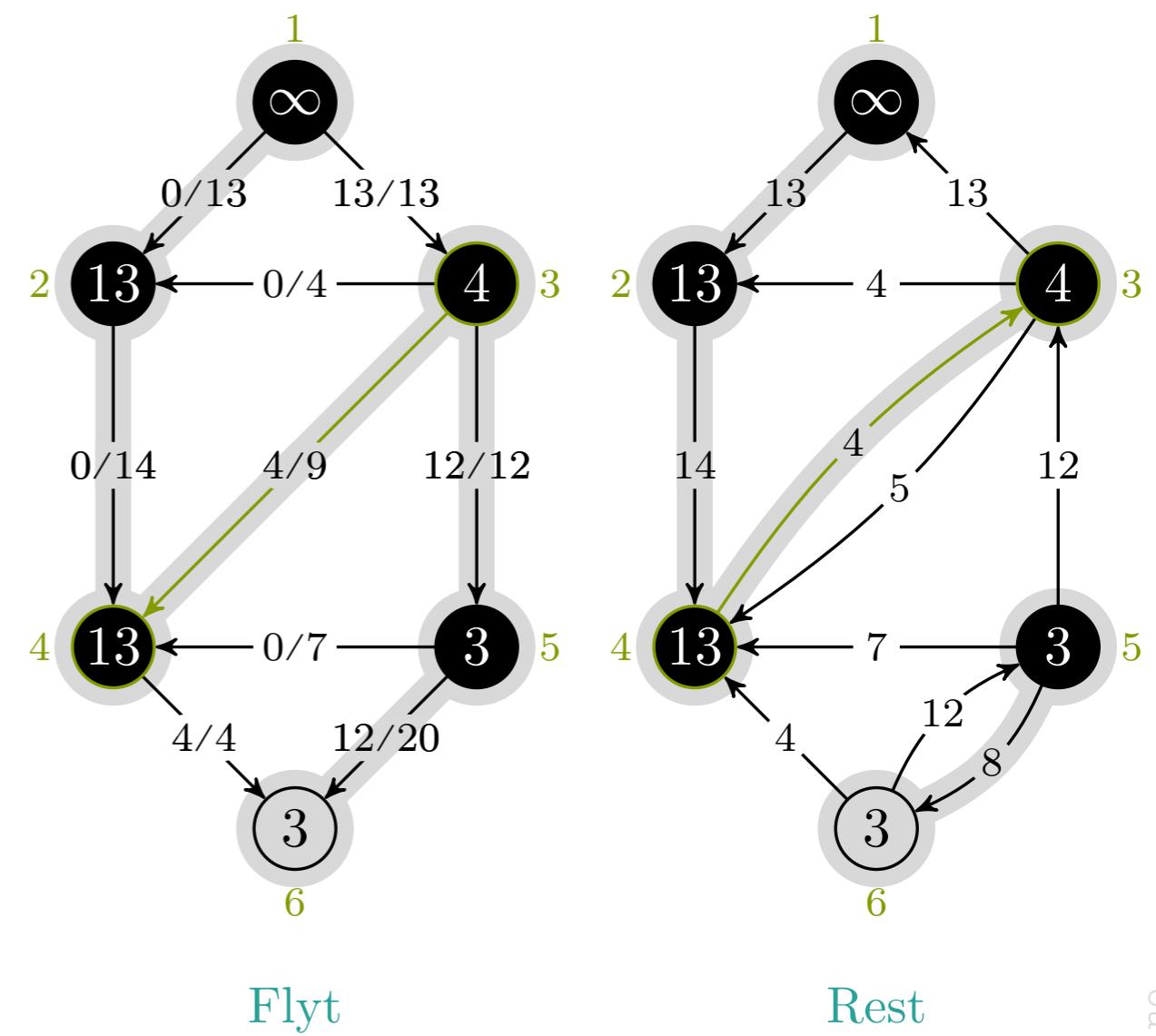


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
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24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 3$



maks-flyt → edmonds-karp

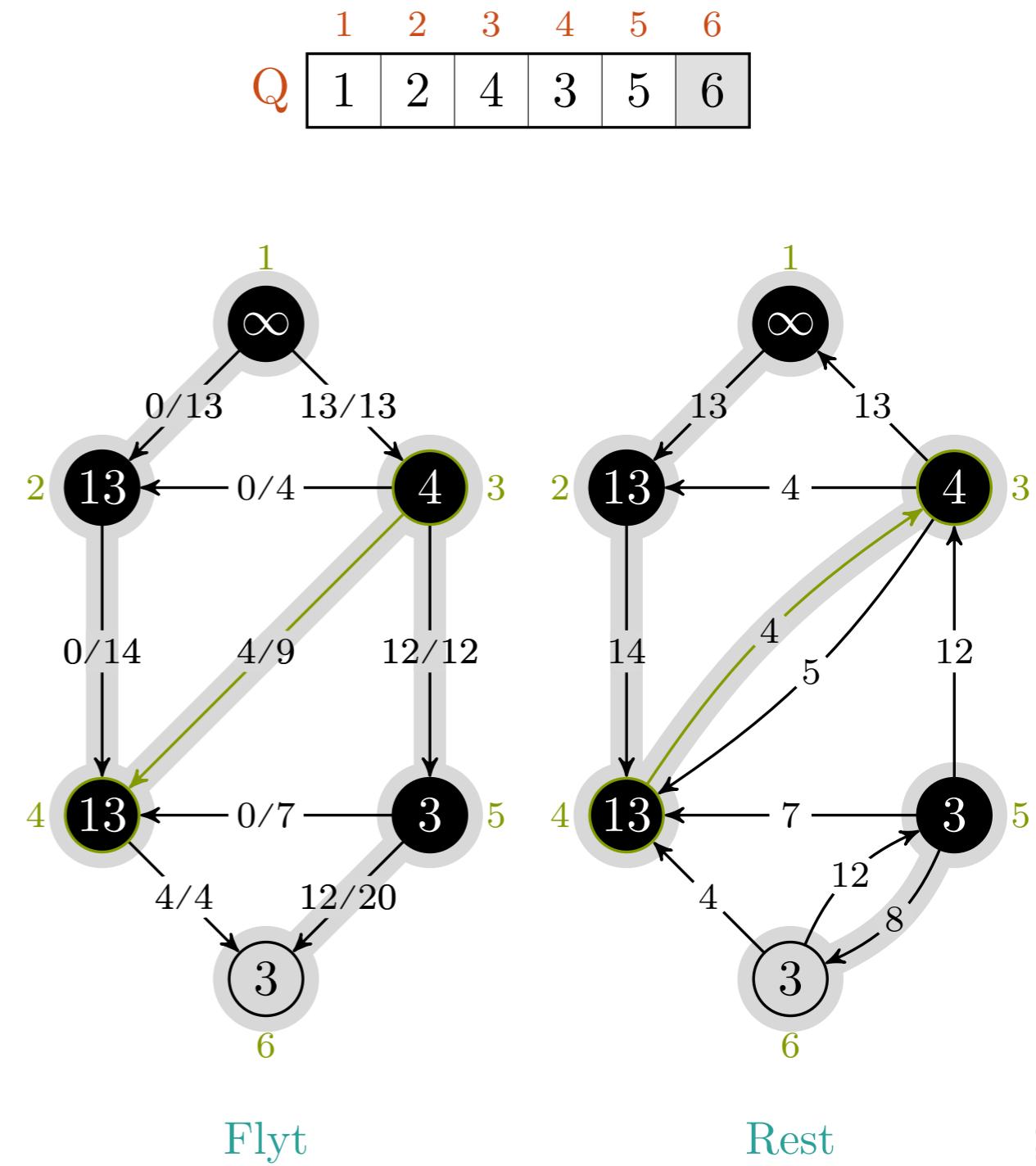
## EDMONDS-KARP(G, s, t)

```

1   for each edge  $(u, v) \in G.E$ 
2        $(u, v).f = 0$ 
3   repeat
4       for each vertex  $u \in G.V$ 
5            $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6            $u.\pi = NIL$ 
7            $s.a = \infty$ 
8            $Q = \emptyset$ 
9           ENQUEUE( $Q, s$ )
10          while  $t.a == 0$  and  $Q \neq \emptyset$ 
11               $u = DEQUEUE(Q)$ 
12              for all edges  $(u, v), (v, u) \in G.E$ 
13                  if  $(u, v) \in G.E$ 
14                       $c_f(u, v) = c(u, v) - (u, v).f$ 
15                  else  $c_f(u, v) = (v, u).f$ 
16                  if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                       $v.a = \min(u.a, c_f(u, v))$ 
18                       $v.\pi = u$ 
19                      ENQUEUE( $Q, v$ )
20           $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21          while  $u \neq NIL$ 
22              if  $(u, v) \in G.E$ 
23                   $(u, v).f = (u, v).f + t.a$ 
24              else  $(v, u).f = (v, u).f - t.a$ 
25               $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

$$u, v = 4, 3$$

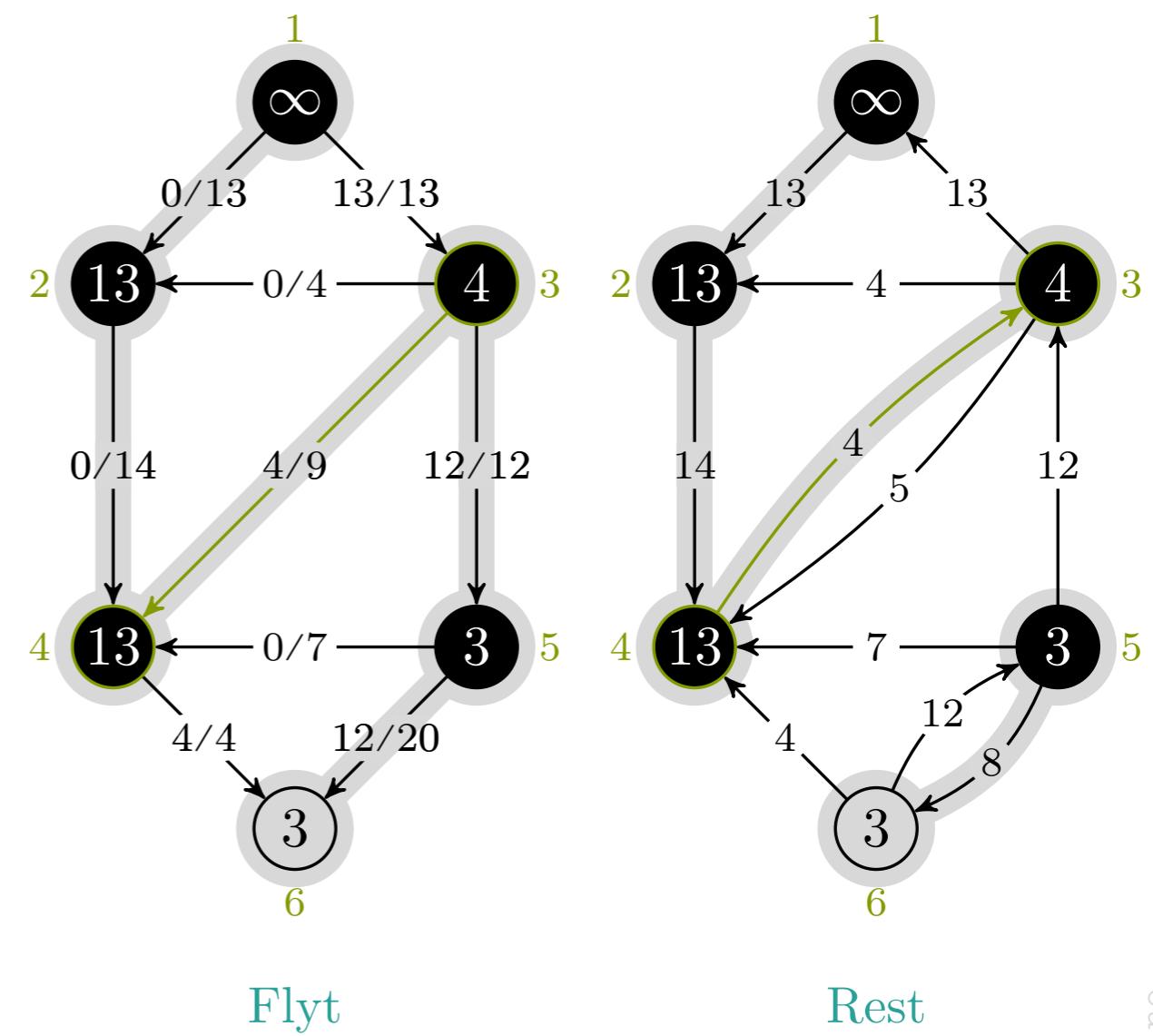


EDMONDS-KARP( $G, s, t$ )

```

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4    for each vertex  $u \in G.V$ 
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6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
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```

$u, v = 4, 3$

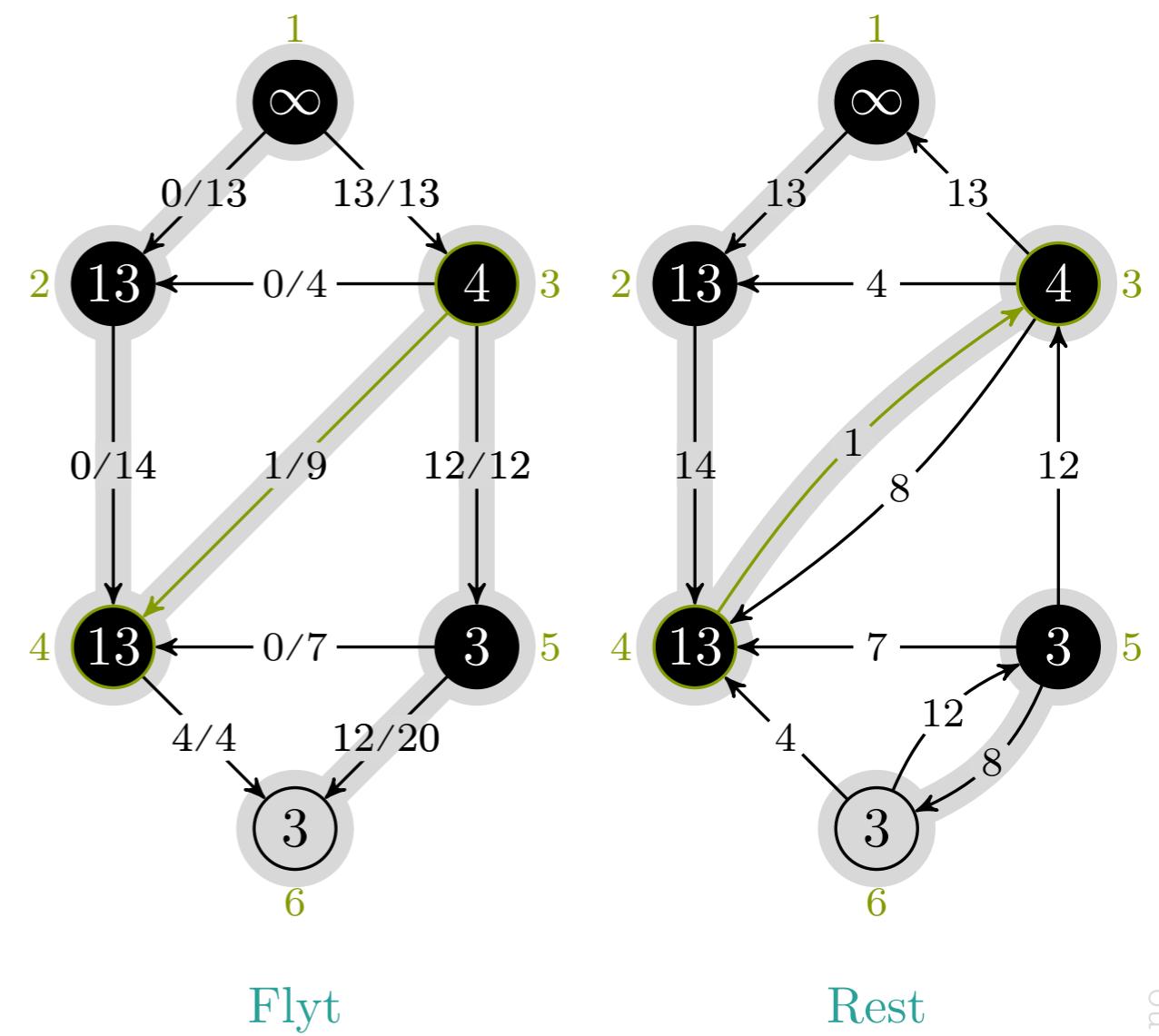


EDMONDS-KARP( $G, s, t$ )

```

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25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 4, 3$

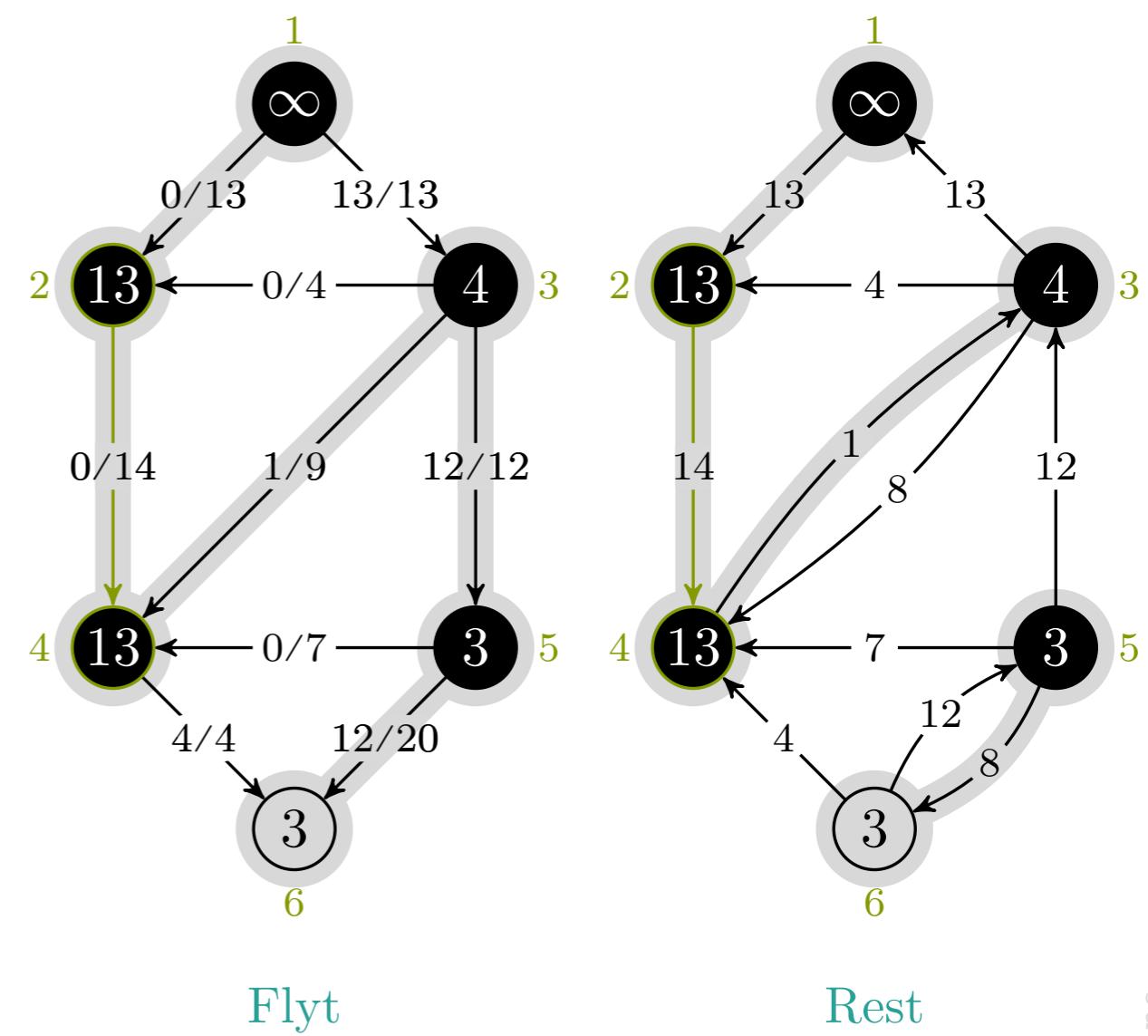


EDMONDS-KARP( $G, s, t$ )

```

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22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, 4$

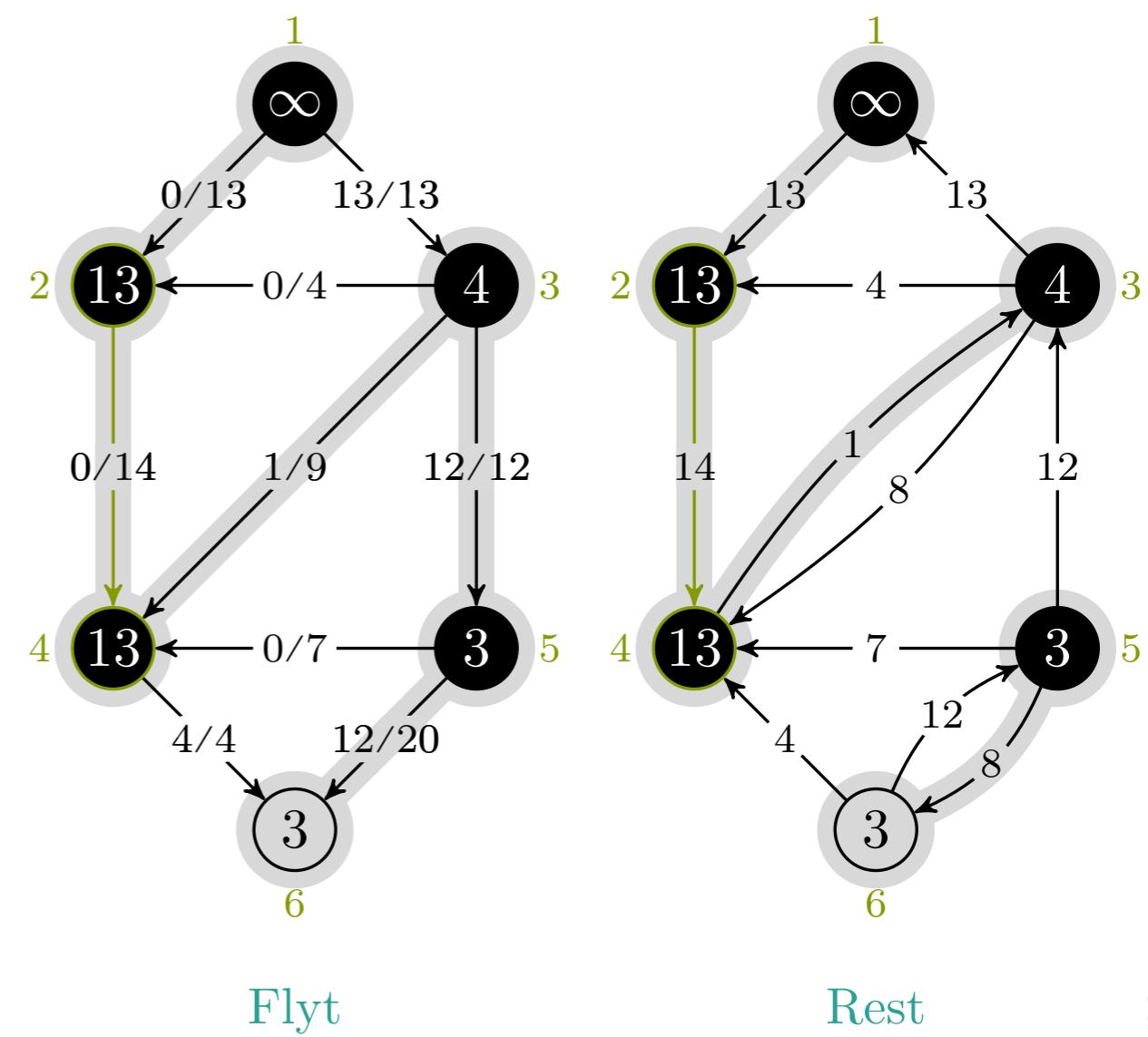


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, 4$

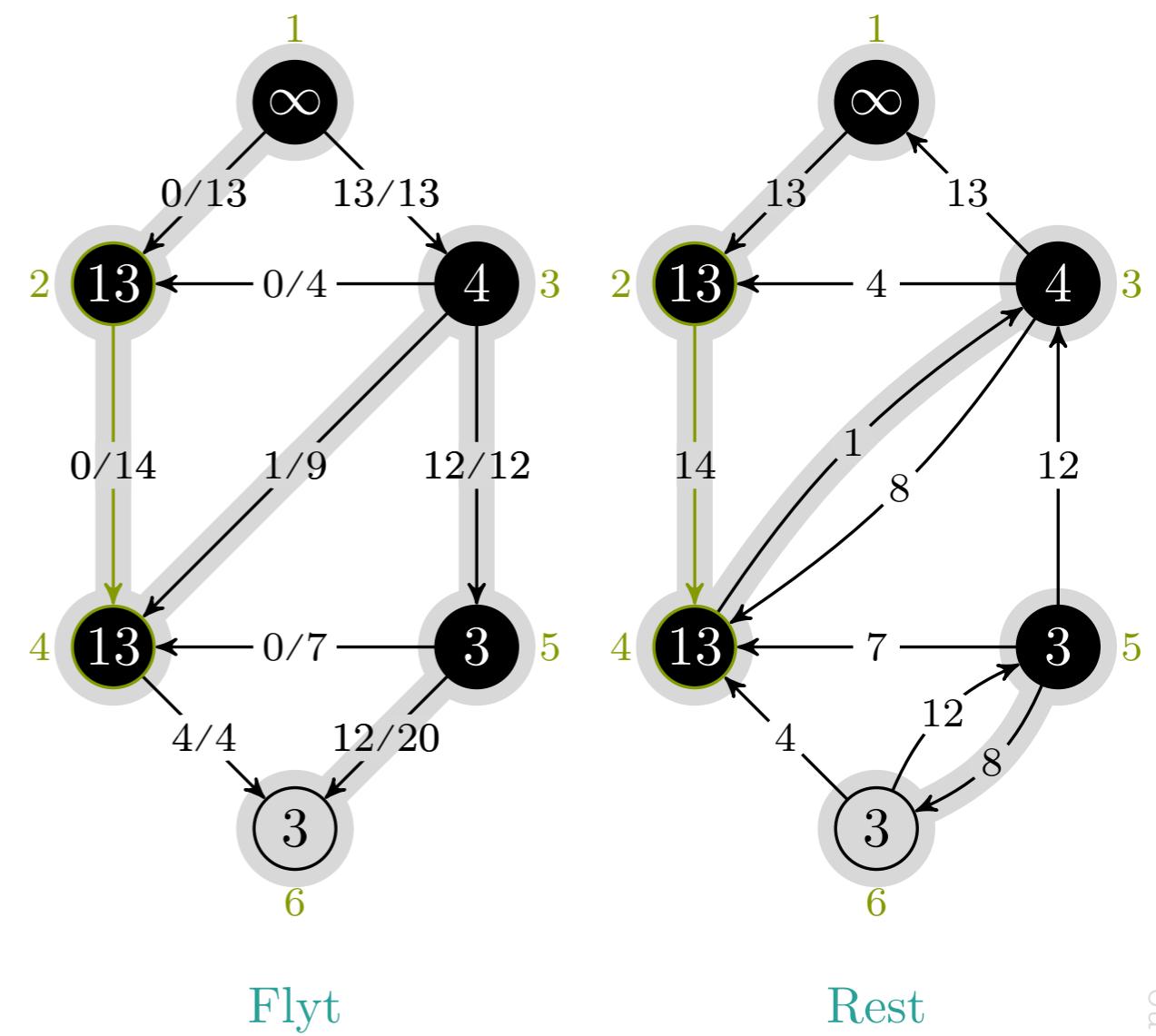


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
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4    for each vertex  $u \in G.V$ 
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6       $u.\pi = \text{NIL}$ 
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8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
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20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
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23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, 4$

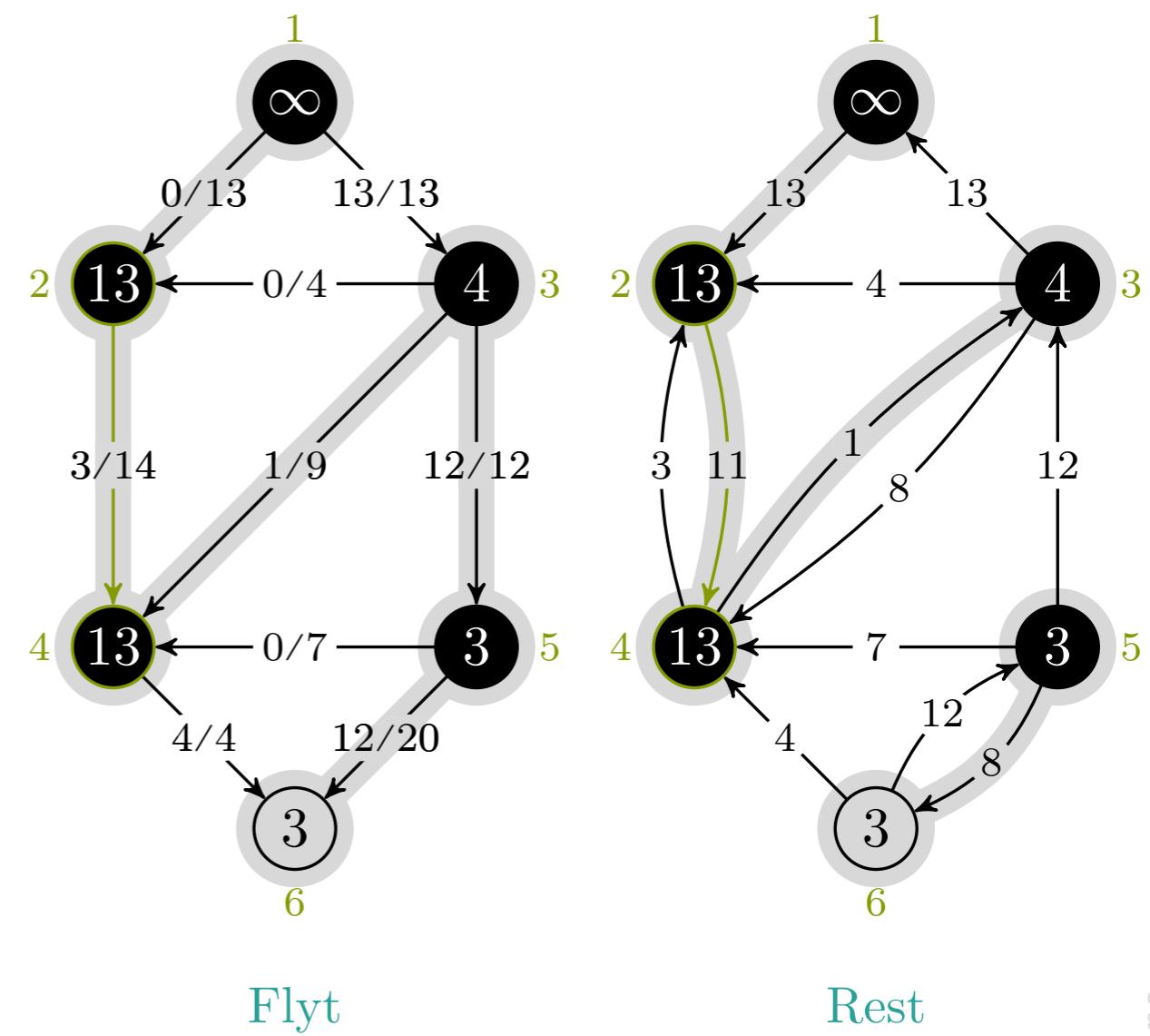


EDMONDS-KARP( $G, s, t$ )

```

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9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 2, 4$

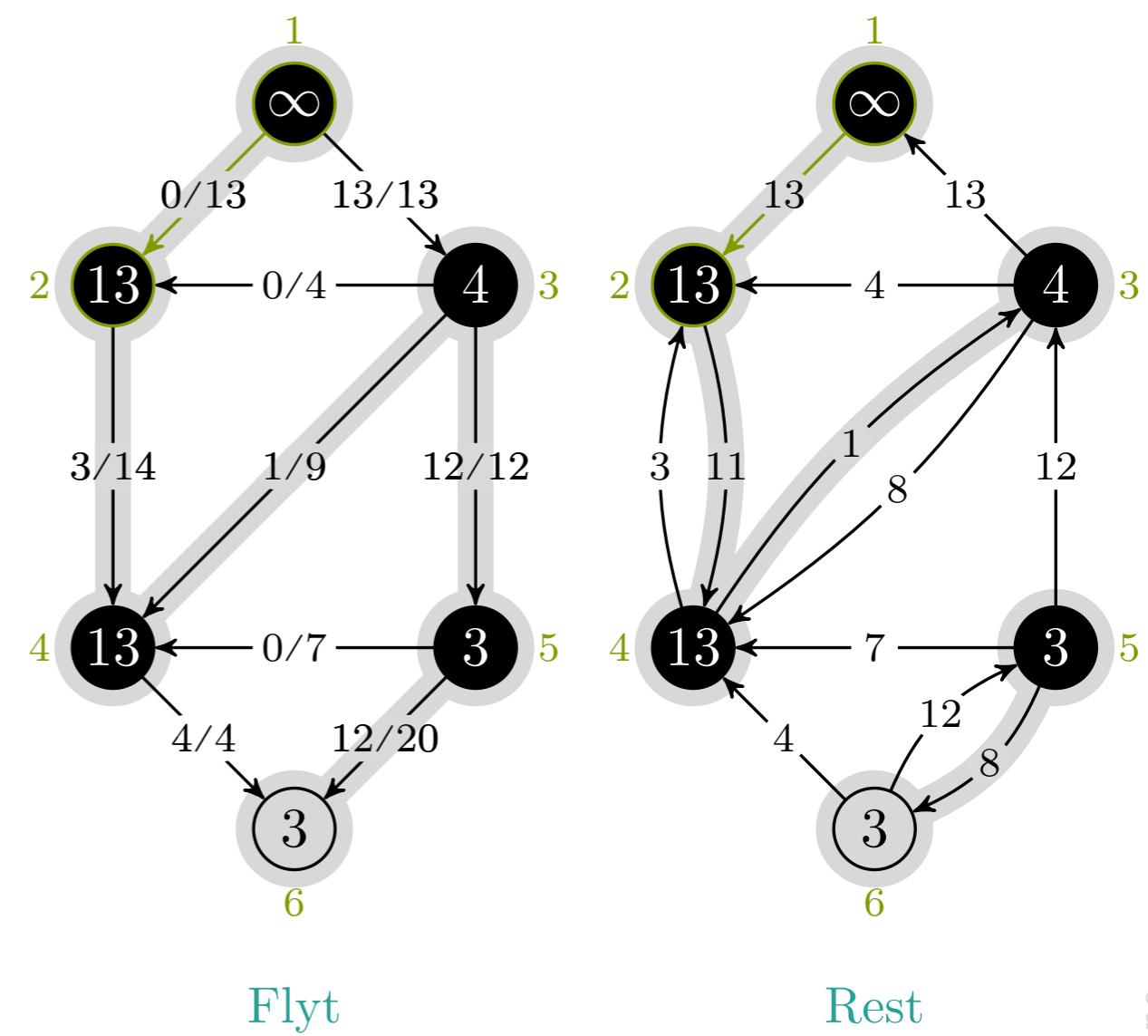


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, 2$

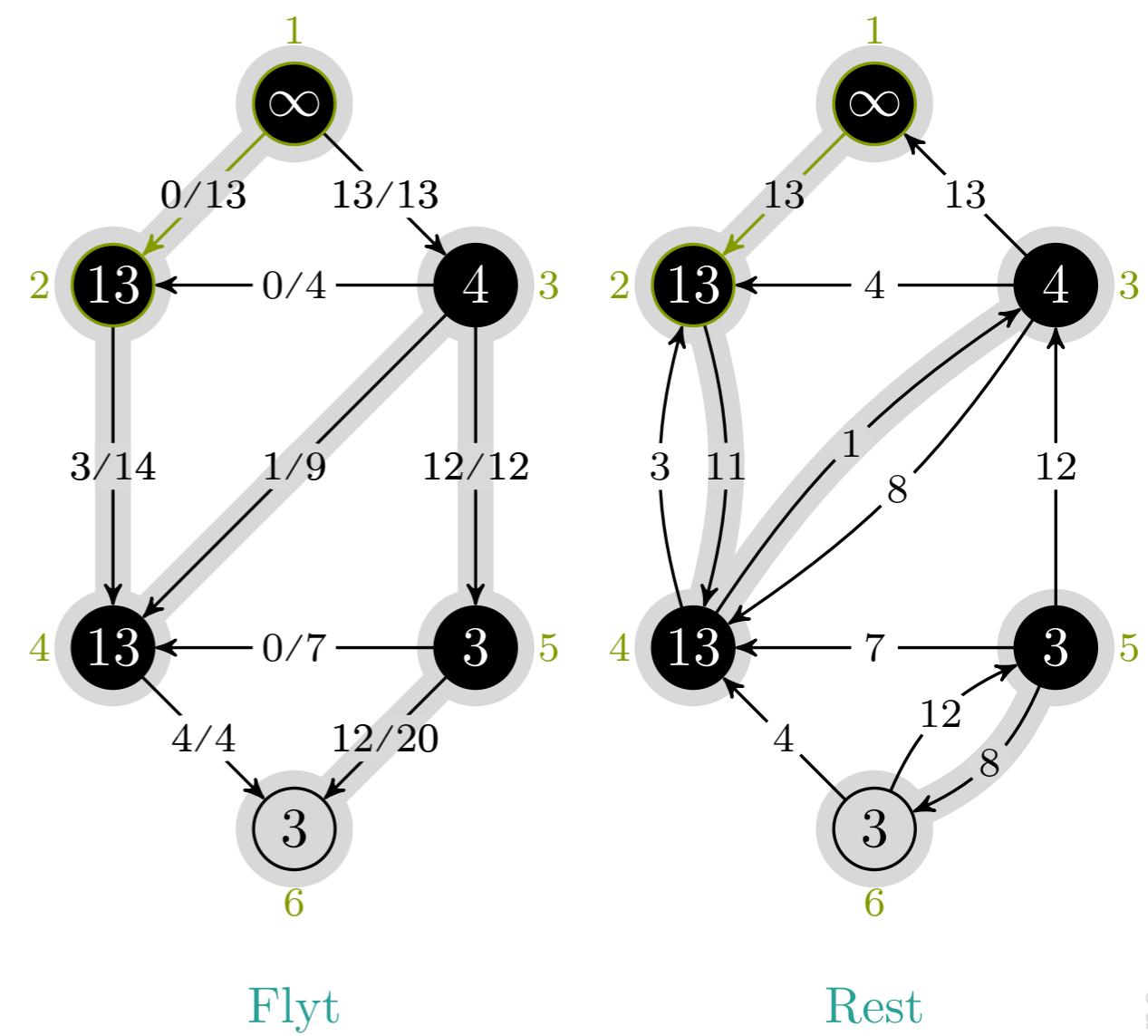


EDMONDS-KARP( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7    $s.a = \infty$ 
8    $Q = \emptyset$ 
9   ENQUEUE( $Q, s$ )
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20     $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21    while  $u \neq \text{NIL}$ 
22      if  $(u, v) \in G.E$ 
23         $(u, v).f = (u, v).f + t.a$ 
24      else  $(v, u).f = (v, u).f - t.a$ 
25     $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, 2$

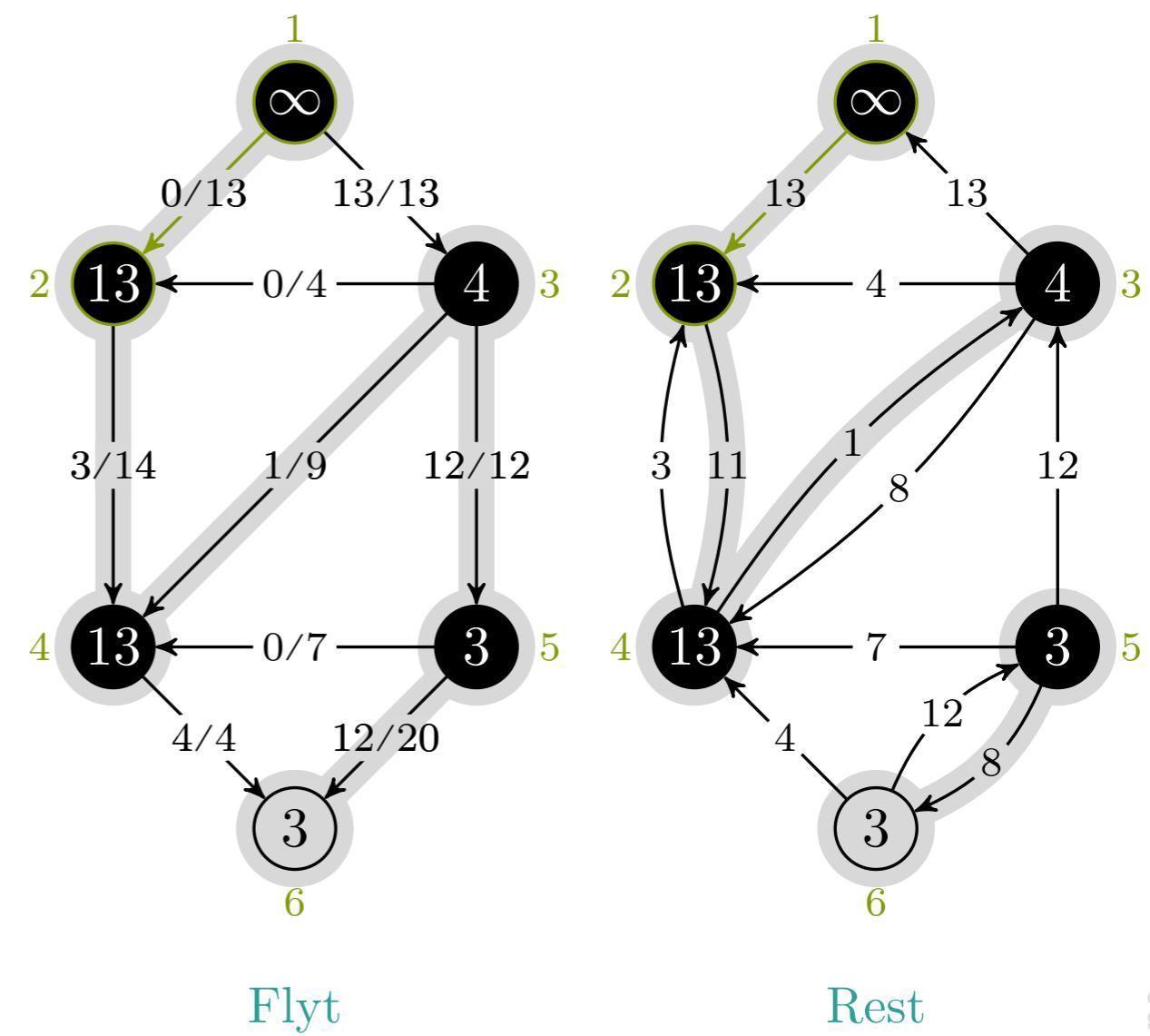


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, 2$

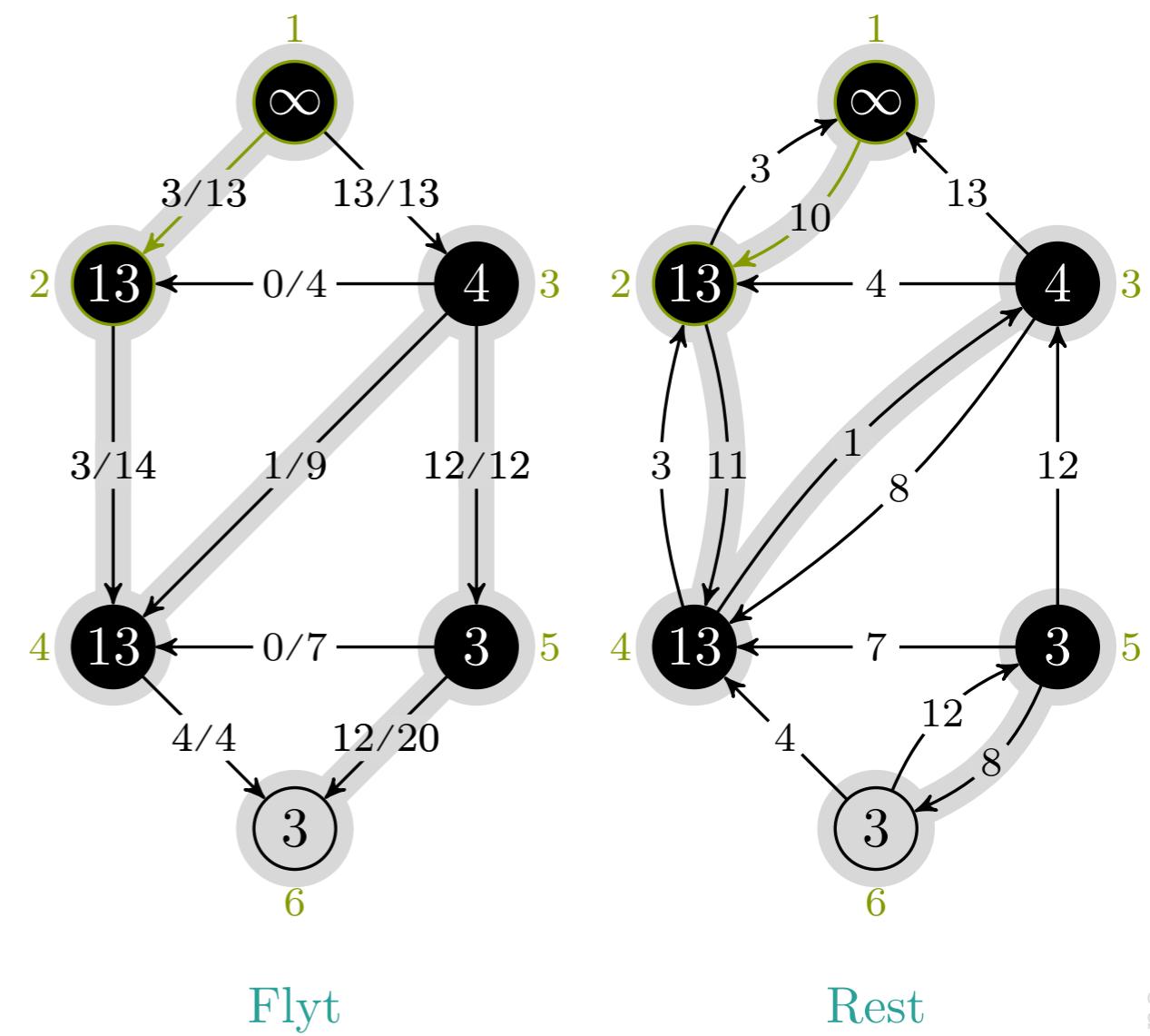


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = 1, 2$

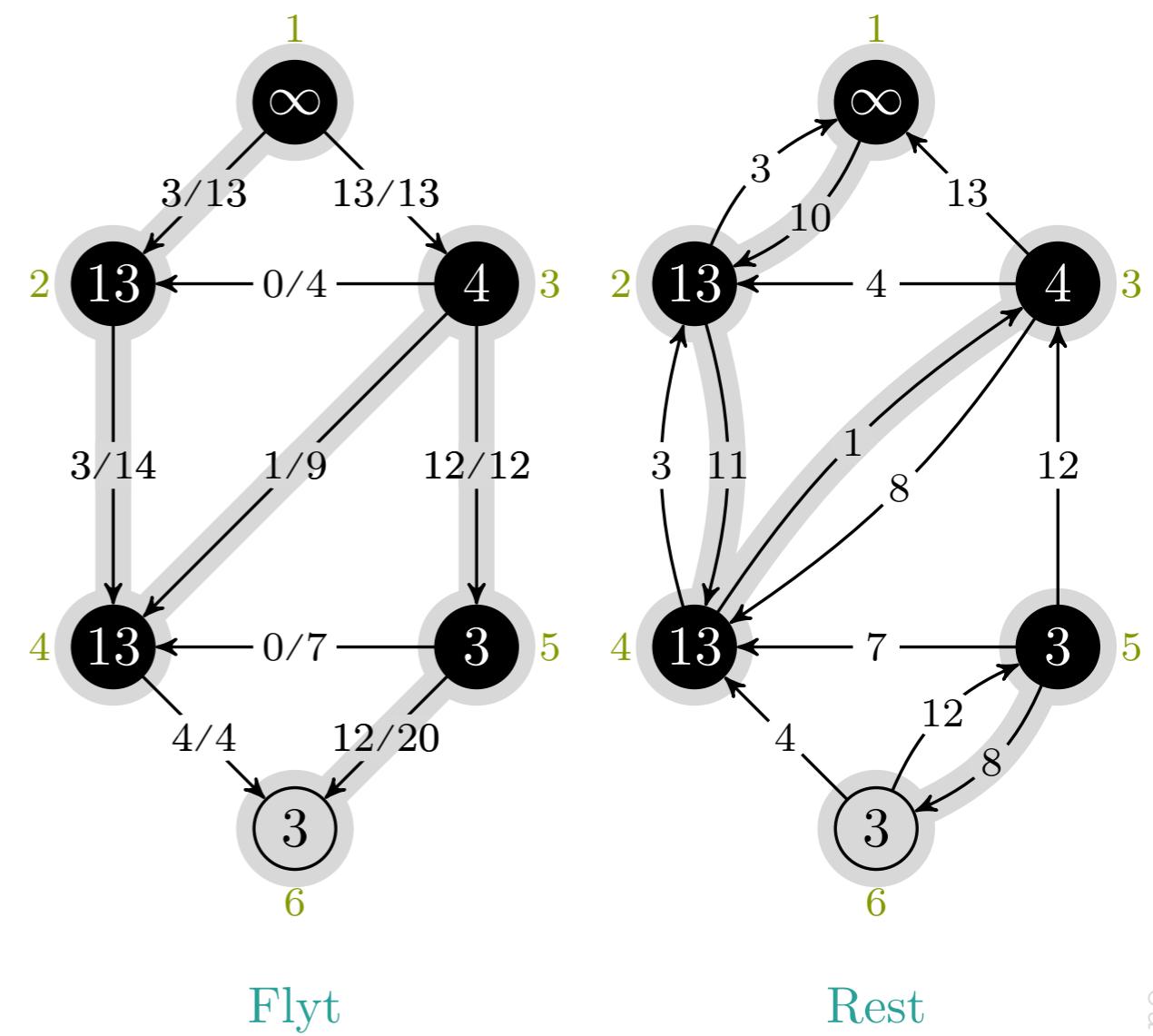


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$

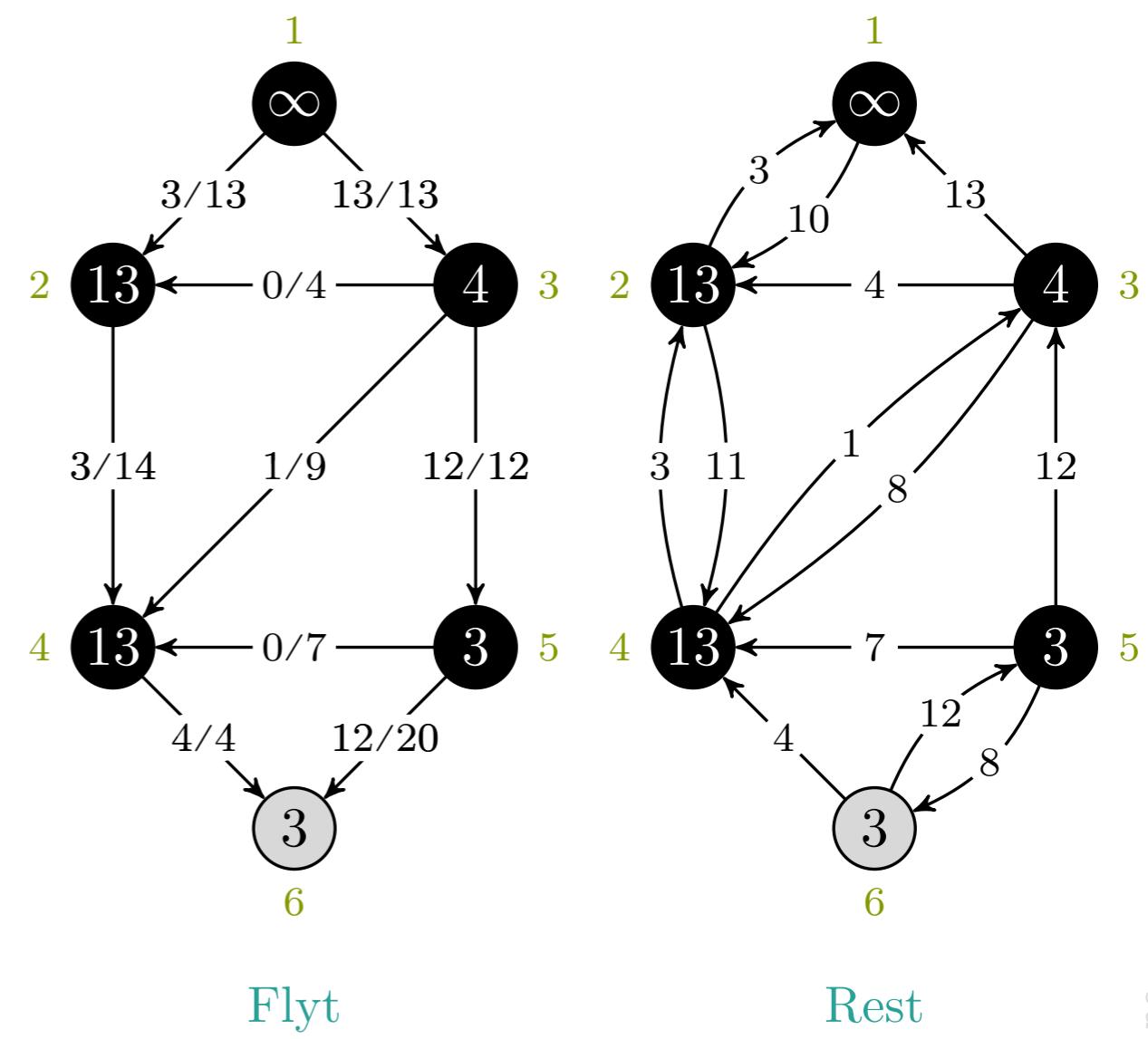


EDMONDS-KARP( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7    $s.a = \infty$ 
8    $Q = \emptyset$ 
9   ENQUEUE( $Q, s$ )
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20       $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21      while  $u \neq \text{NIL}$ 
22        if  $(u, v) \in G.E$ 
23           $(u, v).f = (u, v).f + t.a$ 
24        else  $(v, u).f = (v, u).f - t.a$ 
25       $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$

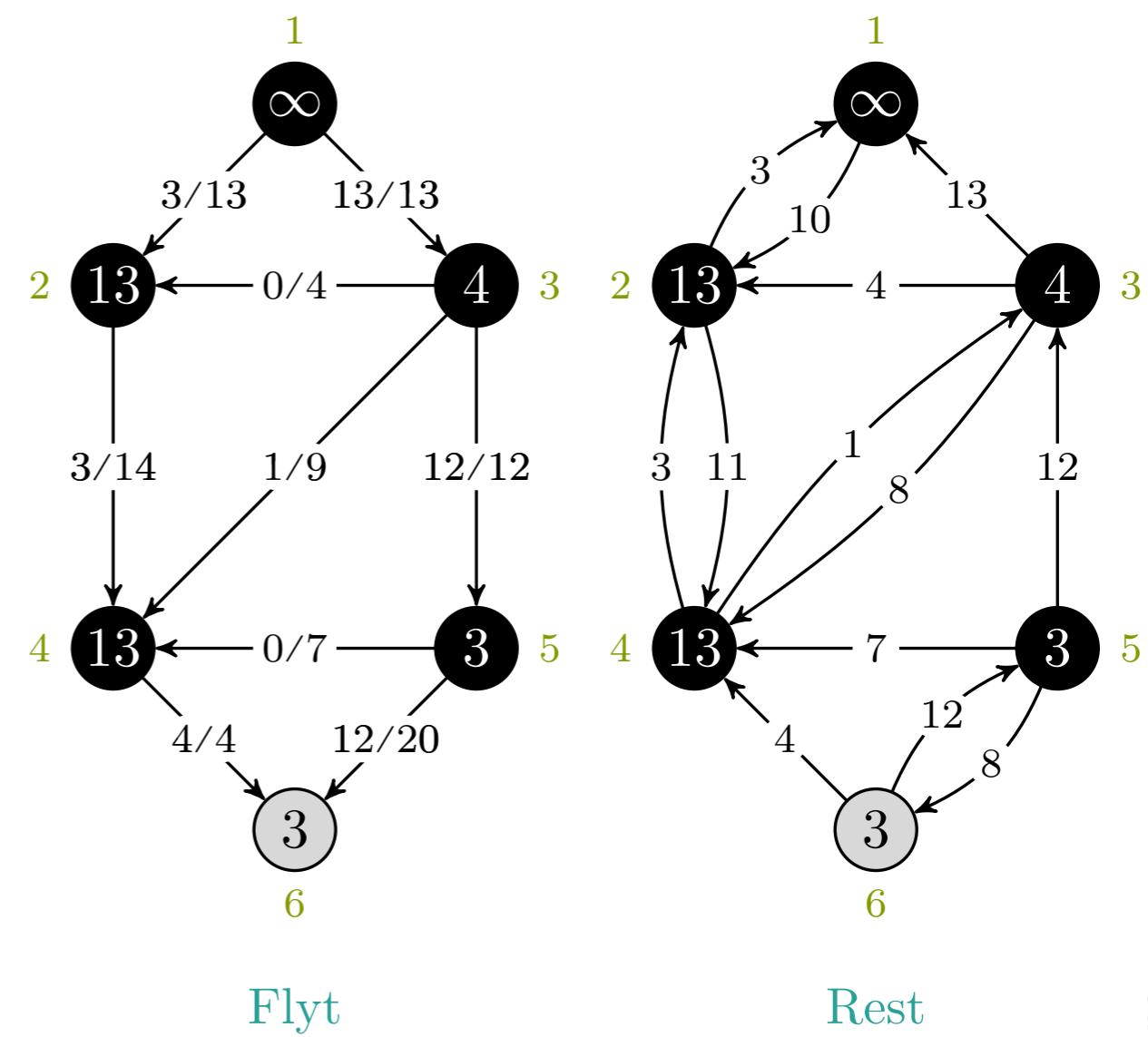


EDMONDS-KARP( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7      $s.a = \infty$ 
8      $Q = \emptyset$ 
9     ENQUEUE( $Q, s$ )
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11       $u = \text{DEQUEUE}(Q)$ 
12      for all edges  $(u, v), (v, u) \in G.E$ 
13        if  $(u, v) \in G.E$ 
14           $c_f(u, v) = c(u, v) - (u, v).f$ 
15        else  $c_f(u, v) = (v, u).f$ 
16        if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17           $v.a = \min(u.a, c_f(u, v))$ 
18           $v.\pi = u$ 
19          ENQUEUE( $Q, v$ )
20         $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21        while  $u \neq \text{NIL}$ 
22          if  $(u, v) \in G.E$ 
23             $(u, v).f = (u, v).f + t.a$ 
24          else  $(v, u).f = (v, u).f - t.a$ 
25           $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$



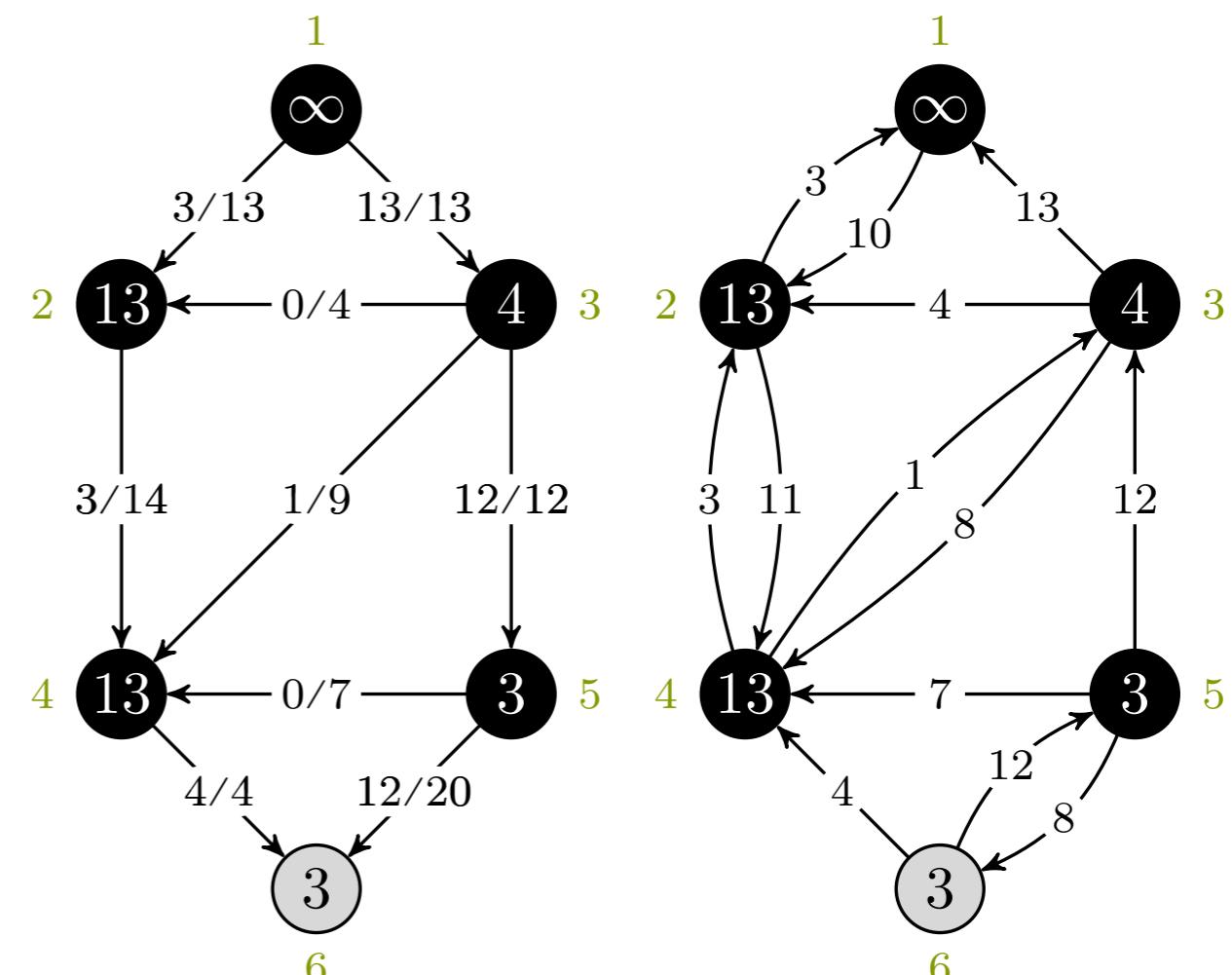
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20      $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$

Q	1	2	3	4	5	6
	1	2	4	3	5	6



Flyt

Rest

## EDMONDS-KARP(G, s, t)

```

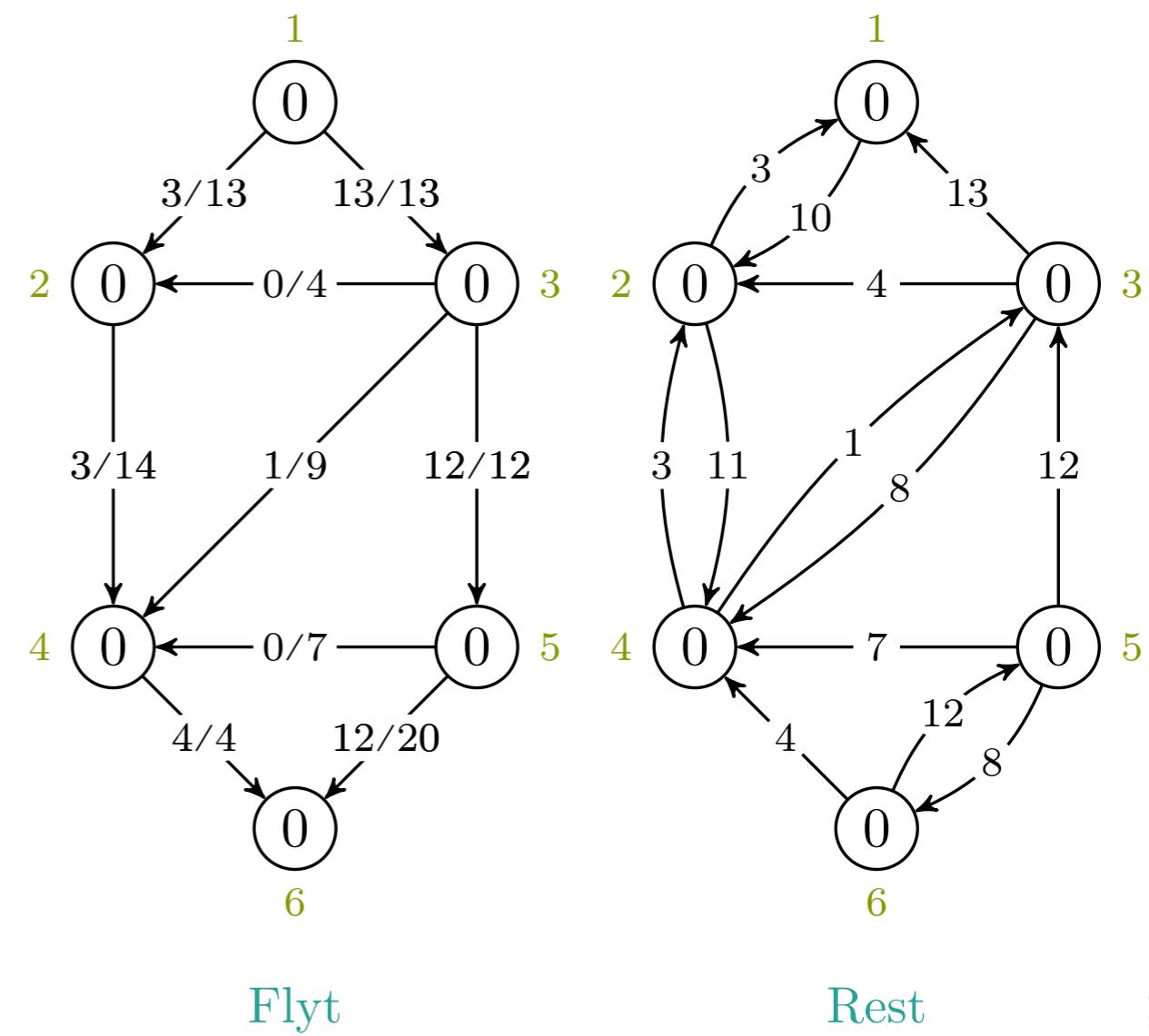
1   for each edge  $(u, v) \in G.E$ 
2        $(u, v).f = 0$ 
3   repeat
4       for each vertex  $u \in G.V$ 
5            $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6            $u.\pi = NIL$ 
7            $s.a = \infty$ 
8            $Q = \emptyset$ 
9           ENQUEUE( $Q, s$ )
10          while  $t.a == 0$  and  $Q \neq \emptyset$ 
11               $u = DEQUEUE(Q)$ 
12              for all edges  $(u, v), (v, u) \in G.E$ 
13                  if  $(u, v) \in G.E$ 
14                       $c_f(u, v) = c(u, v) - (u, v).f$ 
15                  else  $c_f(u, v) = (v, u).f$ 
16                  if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                       $v.a = \min(u.a, c_f(u, v))$ 
18                       $v.\pi = u$ 
19                      ENQUEUE( $Q, v$ )
20           $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21          while  $u \neq NIL$ 
22              if  $(u, v) \in G.E$ 
23                   $(u, v).f = (u, v).f + t.a$ 
24              else  $(v, u).f = (v, u).f - t.a$ 
25               $u, v = u.\pi, u$ 
26          until  $t.a == 0$ 

```

$u, v = \text{NIL}, 1$

maks-flyt → edmonds-karp

	1	2	3	4	5	6
Q	1	2	4	3	5	6



## EDMONDS-KARP(G, s, t)

```

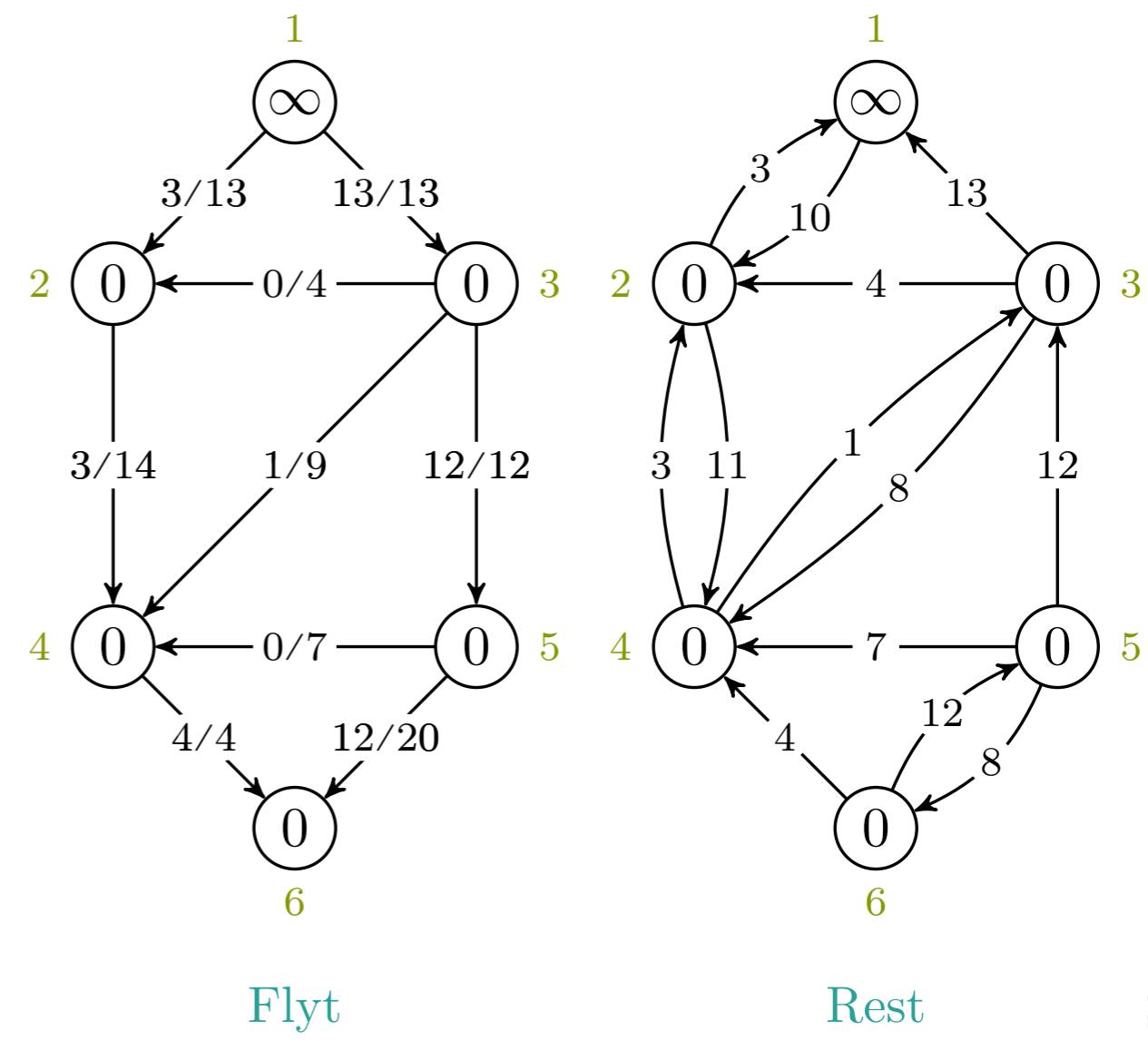
1   for each edge  $(u, v) \in G.E$ 
2        $(u, v).f = 0$ 
3   repeat
4       for each vertex  $u \in G.V$ 
5            $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6            $u.\pi = \text{NIL}$ 
7            $s.a = \infty$ 
8            $Q = \emptyset$ 
9           ENQUEUE( $Q, s$ )
10          while  $t.a == 0$  and  $Q \neq \emptyset$ 
11               $u = \text{DEQUEUE}(Q)$ 
12              for all edges  $(u, v), (v, u) \in G.E$ 
13                  if  $(u, v) \in G.E$ 
14                       $c_f(u, v) = c(u, v) - (u, v).f$ 
15                  else  $c_f(u, v) = (v, u).f$ 
16                  if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                       $v.a = \min(u.a, c_f(u, v))$ 
18                       $v.\pi = u$ 
19                      ENQUEUE( $Q, v$ )
20           $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21          while  $u \neq \text{NIL}$ 
22              if  $(u, v) \in G.E$ 
23                   $(u, v).f = (u, v).f + t.a$ 
24              else  $(v, u).f = (v, u).f - t.a$ 
25               $u, v = u.\pi, u$ 
26          until  $t.a == 0$ 

```

$u, v = \text{NIL}, 1$

maks-flyt → edmonds-karp

	1	2	3	4	5	6
Q	1	2	4	3	5	6

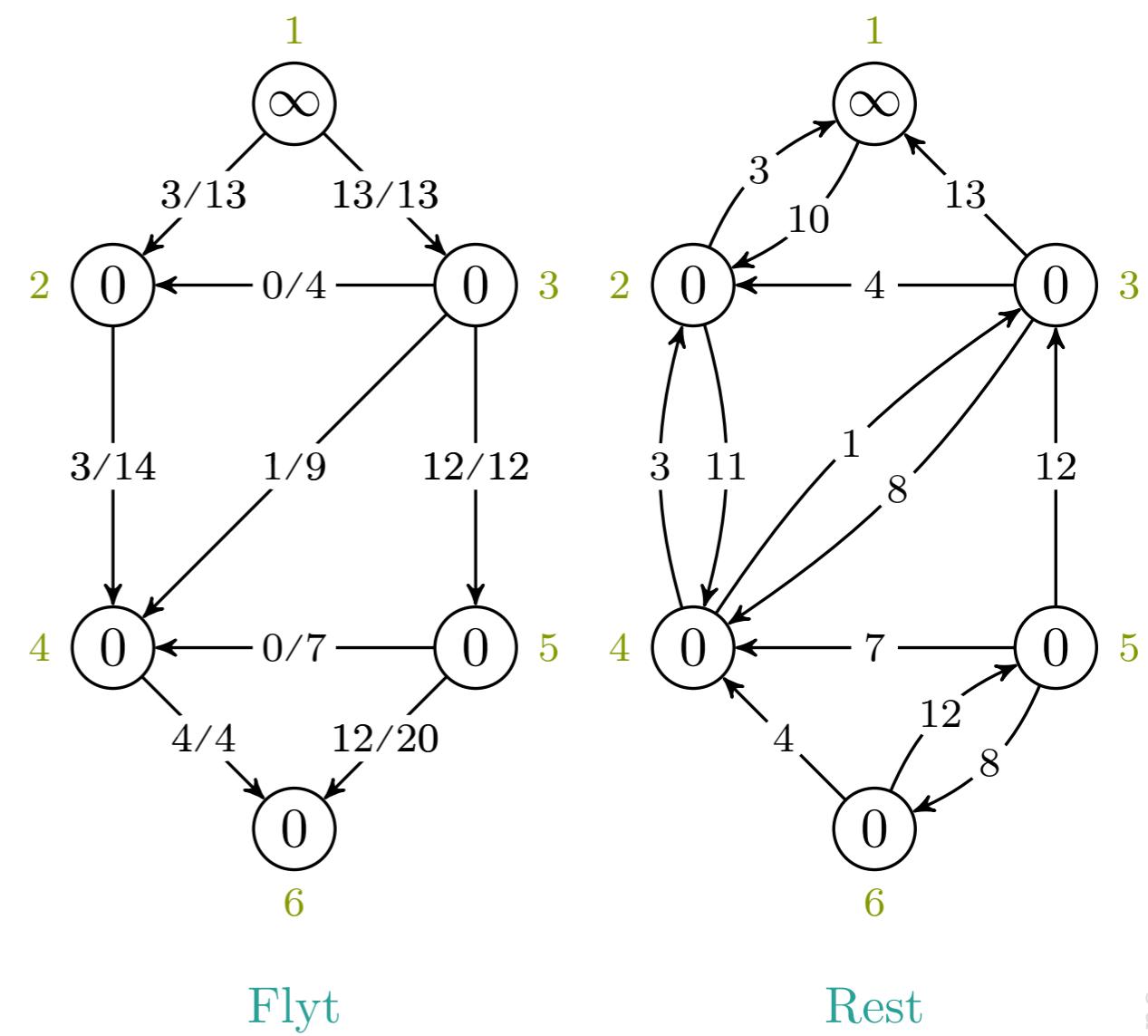


EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2       $(u, v).f = 0$ 
3  repeat
4      for each vertex  $u \in G.V$ 
5           $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6           $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11          $u = \text{DEQUEUE}(Q)$ 
12         for all edges  $(u, v), (v, u) \in G.E$ 
13             if  $(u, v) \in G.E$ 
14                  $c_f(u, v) = c(u, v) - (u, v).f$ 
15             else  $c_f(u, v) = (v, u).f$ 
16             if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17                  $v.a = \min(u.a, c_f(u, v))$ 
18                  $v.\pi = u$ 
19                 ENQUEUE( $Q, v$ )
20              $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21             while  $u \neq \text{NIL}$ 
22                 if  $(u, v) \in G.E$ 
23                      $(u, v).f = (u, v).f + t.a$ 
24                 else  $(v, u).f = (v, u).f - t.a$ 
25              $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 1$



EDMONDS-KARP( $G, s, t$ )

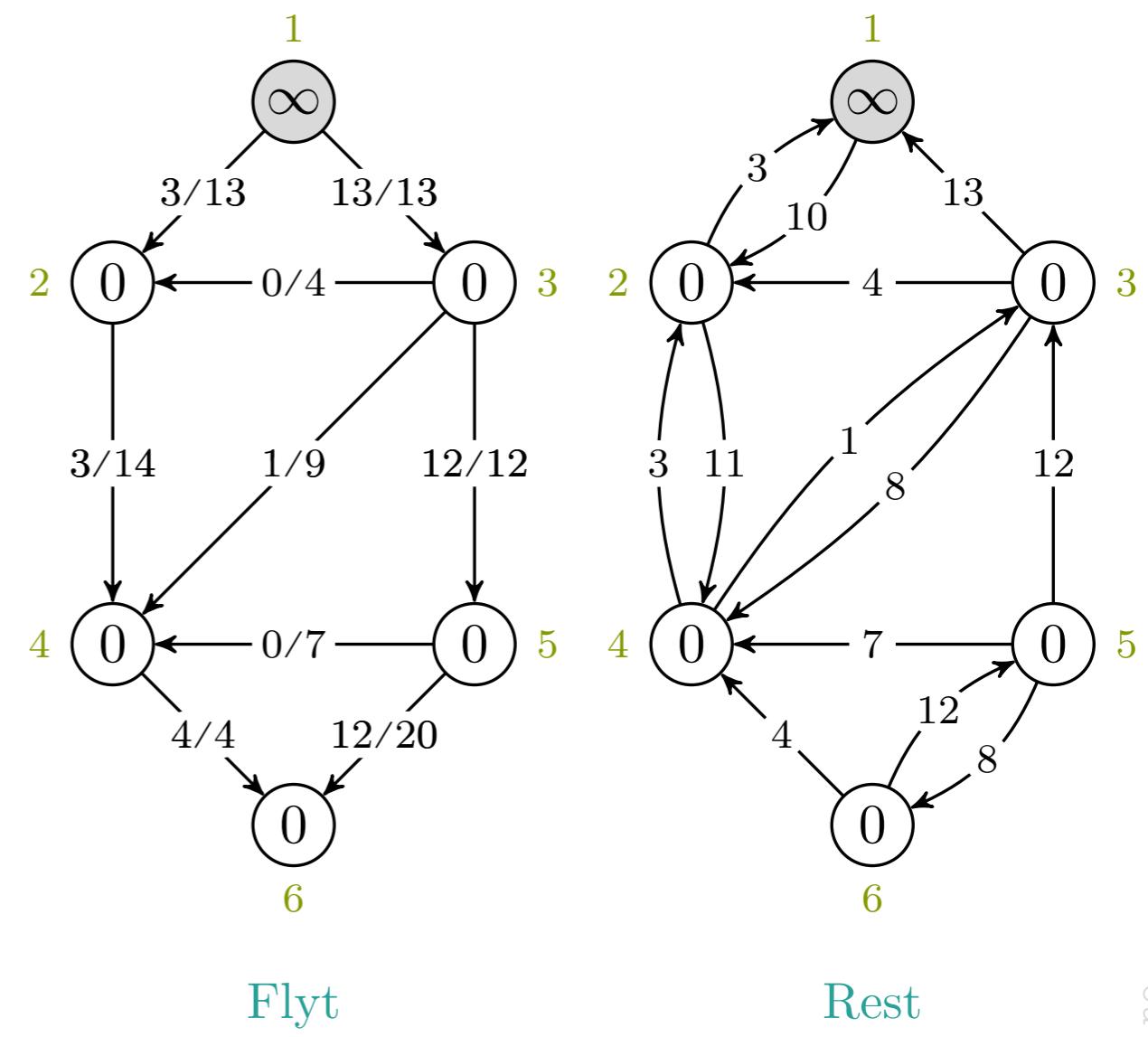
```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

 $u, v = \text{NIL}, 1$ 

Q	1	2	3	4	5	6
	1	2	4	3	5	6



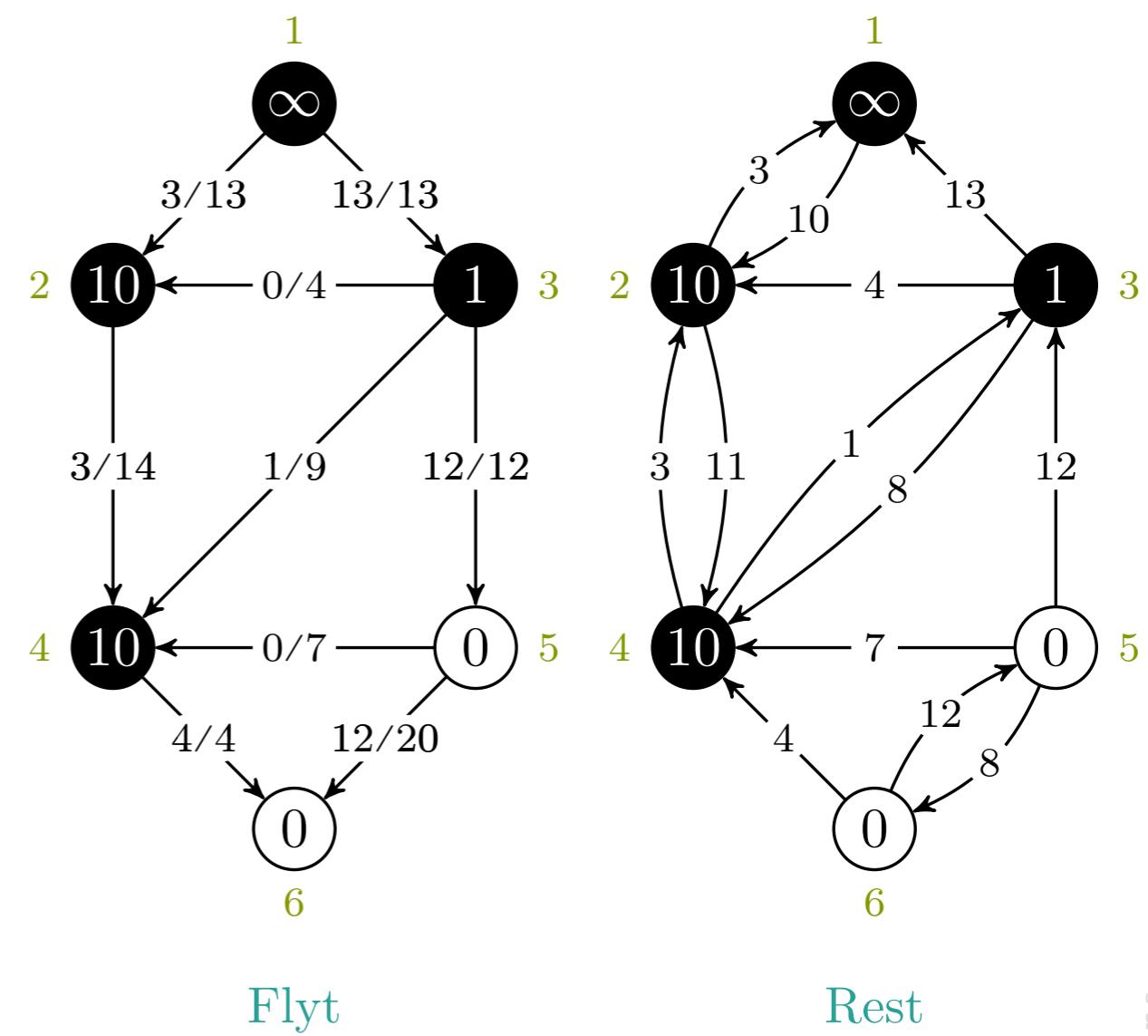
EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = -, -$

Q	1	2	3	4	5	6
1	2	4	3	5	6	

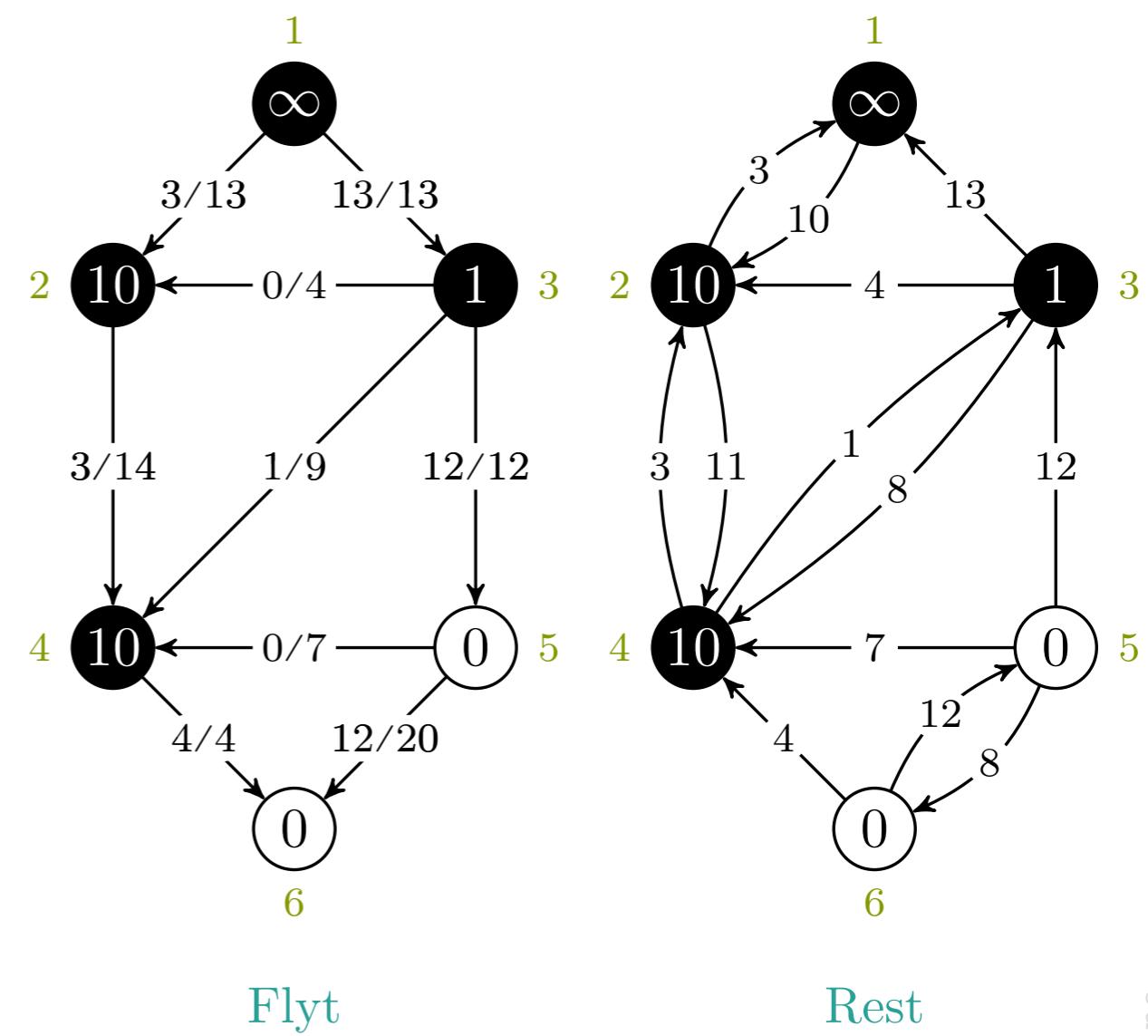


EDMONDS-KARP( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7    $s.a = \infty$ 
8    $Q = \emptyset$ 
9   ENQUEUE( $Q, s$ )
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20     $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21  while  $u \neq \text{NIL}$ 
22    if  $(u, v) \in G.E$ 
23       $(u, v).f = (u, v).f + t.a$ 
24    else  $(v, u).f = (v, u).f - t.a$ 
25     $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 6$

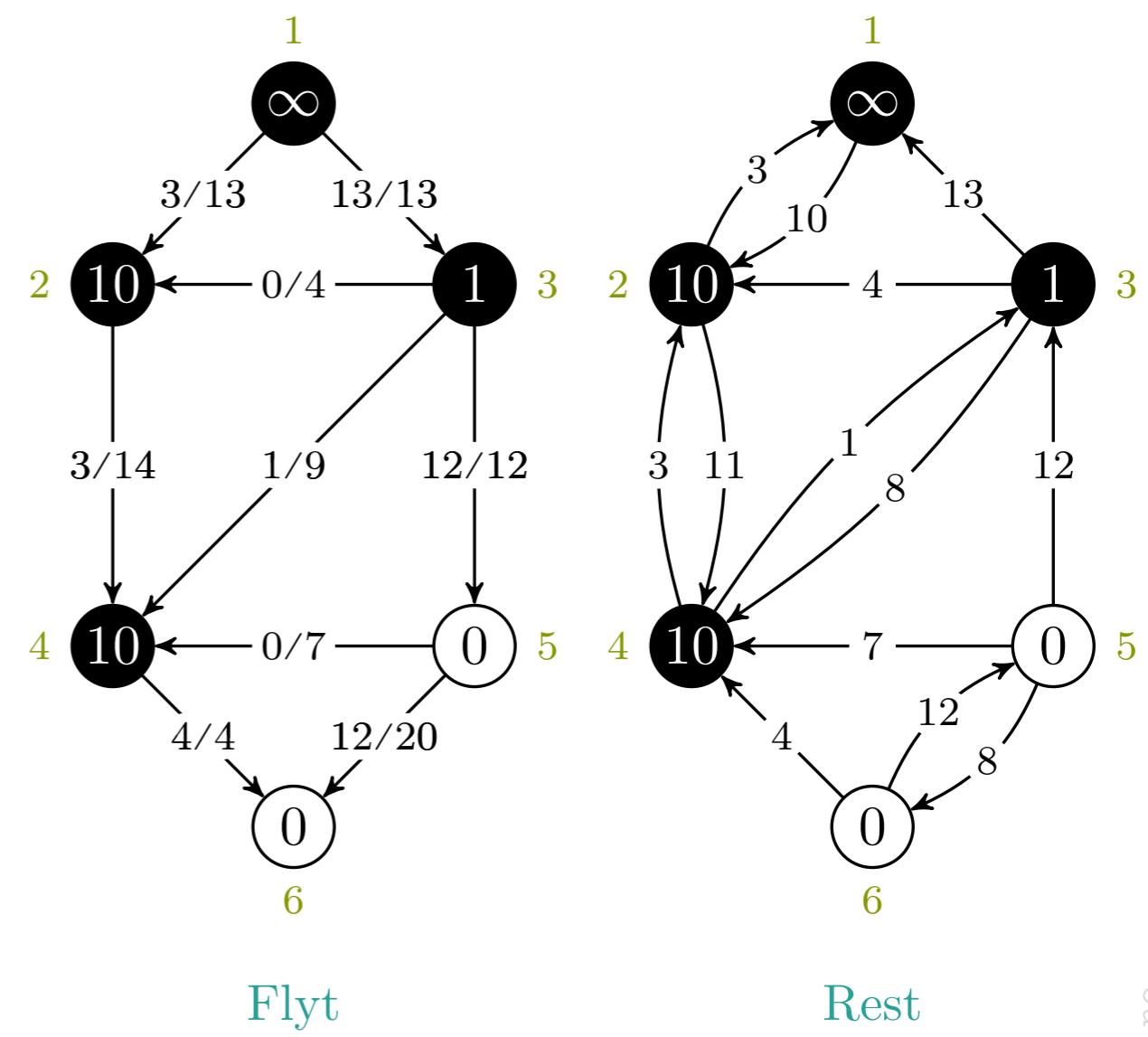


EDMONDS-KARP( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7    $s.a = \infty$ 
8    $Q = \emptyset$ 
9   ENQUEUE( $Q, s$ )
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20       $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21      while  $u \neq \text{NIL}$ 
22        if  $(u, v) \in G.E$ 
23           $(u, v).f = (u, v).f + t.a$ 
24        else  $(v, u).f = (v, u).f - t.a$ 
25       $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

$u, v = \text{NIL}, 6$



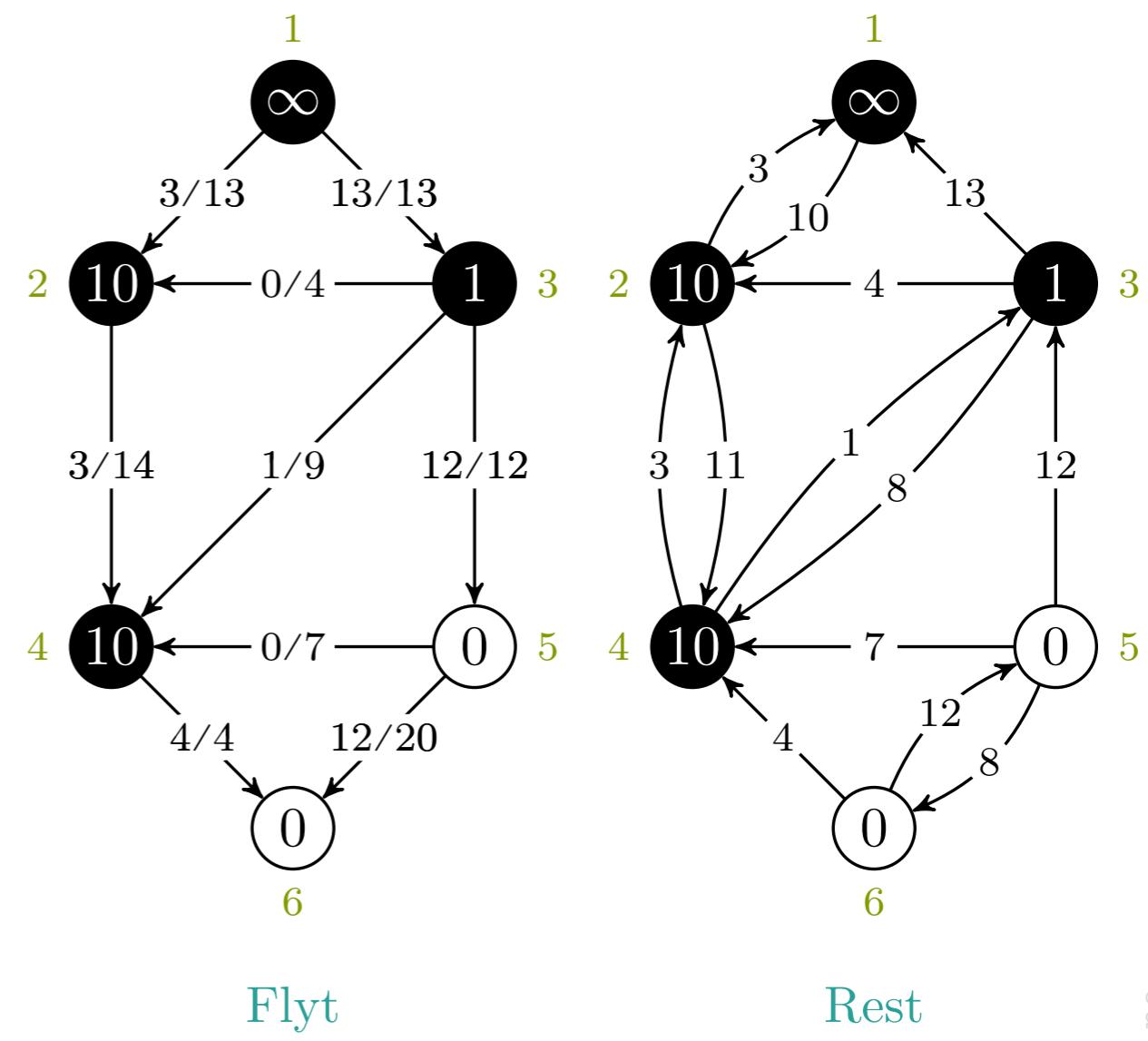
EDMONDS-KARP( $G, s, t$ )

```

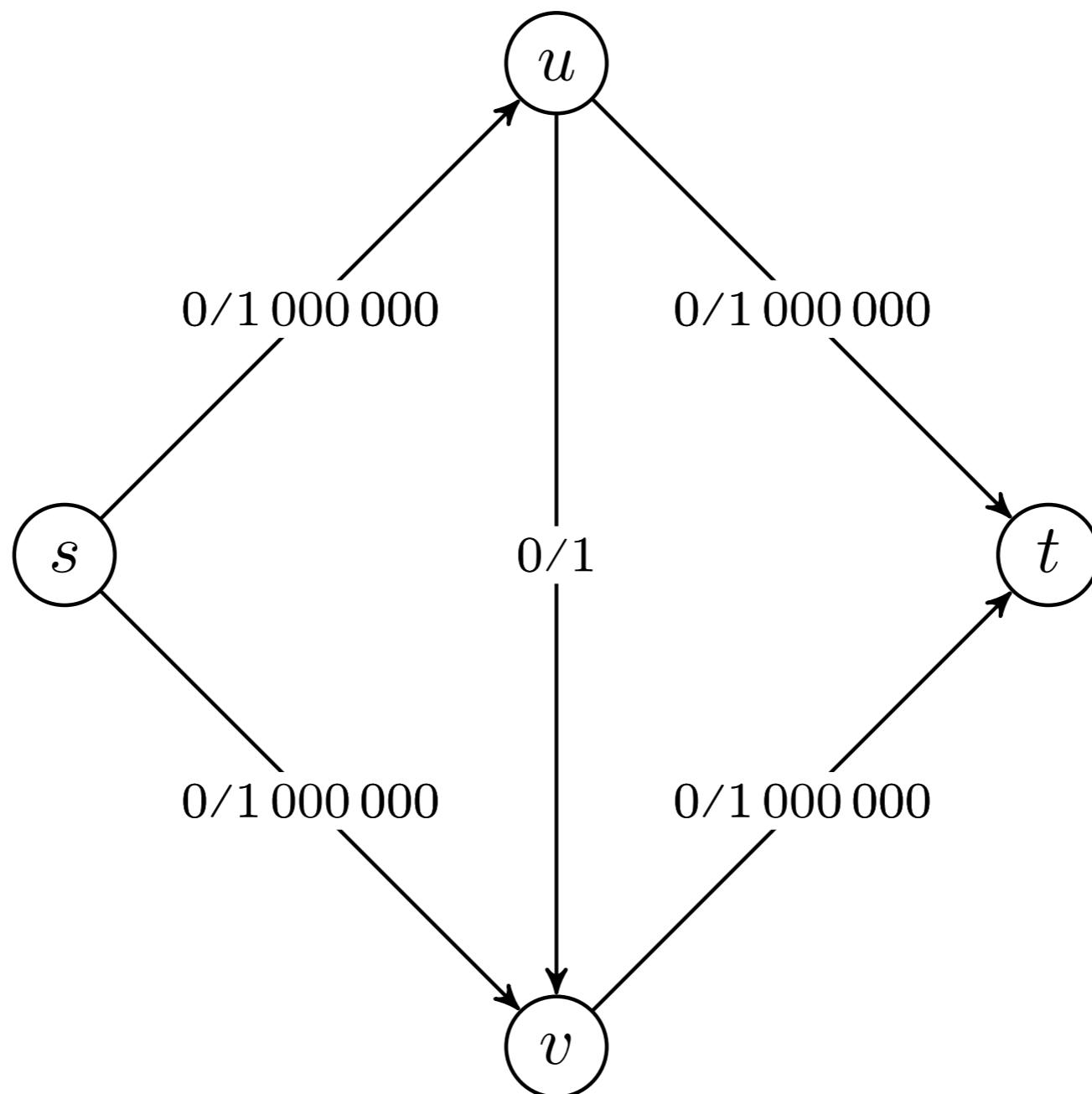
1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7    $s.a = \infty$ 
8    $Q = \emptyset$ 
9   ENQUEUE( $Q, s$ )
10  while  $t.a == 0$  and  $Q \neq \emptyset$ 
11     $u = \text{DEQUEUE}(Q)$ 
12    for all edges  $(u, v), (v, u) \in G.E$ 
13      if  $(u, v) \in G.E$ 
14         $c_f(u, v) = c(u, v) - (u, v).f$ 
15      else  $c_f(u, v) = (v, u).f$ 
16      if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17         $v.a = \min(u.a, c_f(u, v))$ 
18         $v.\pi = u$ 
19        ENQUEUE( $Q, v$ )
20     $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21    while  $u \neq \text{NIL}$ 
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24      else  $(v, u).f = (v, u).f - t.a$ 
25     $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

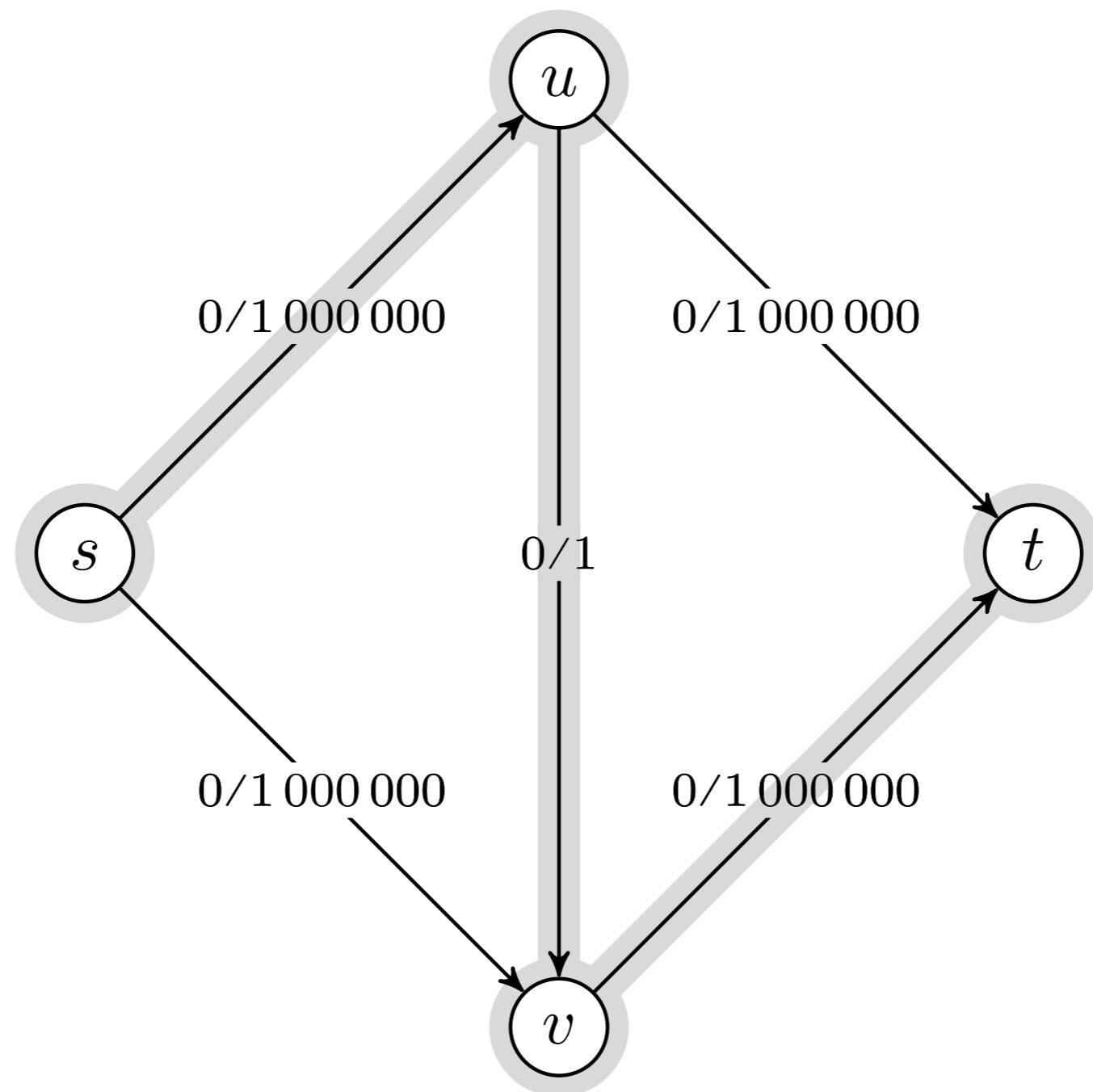
$u, v = -, -$

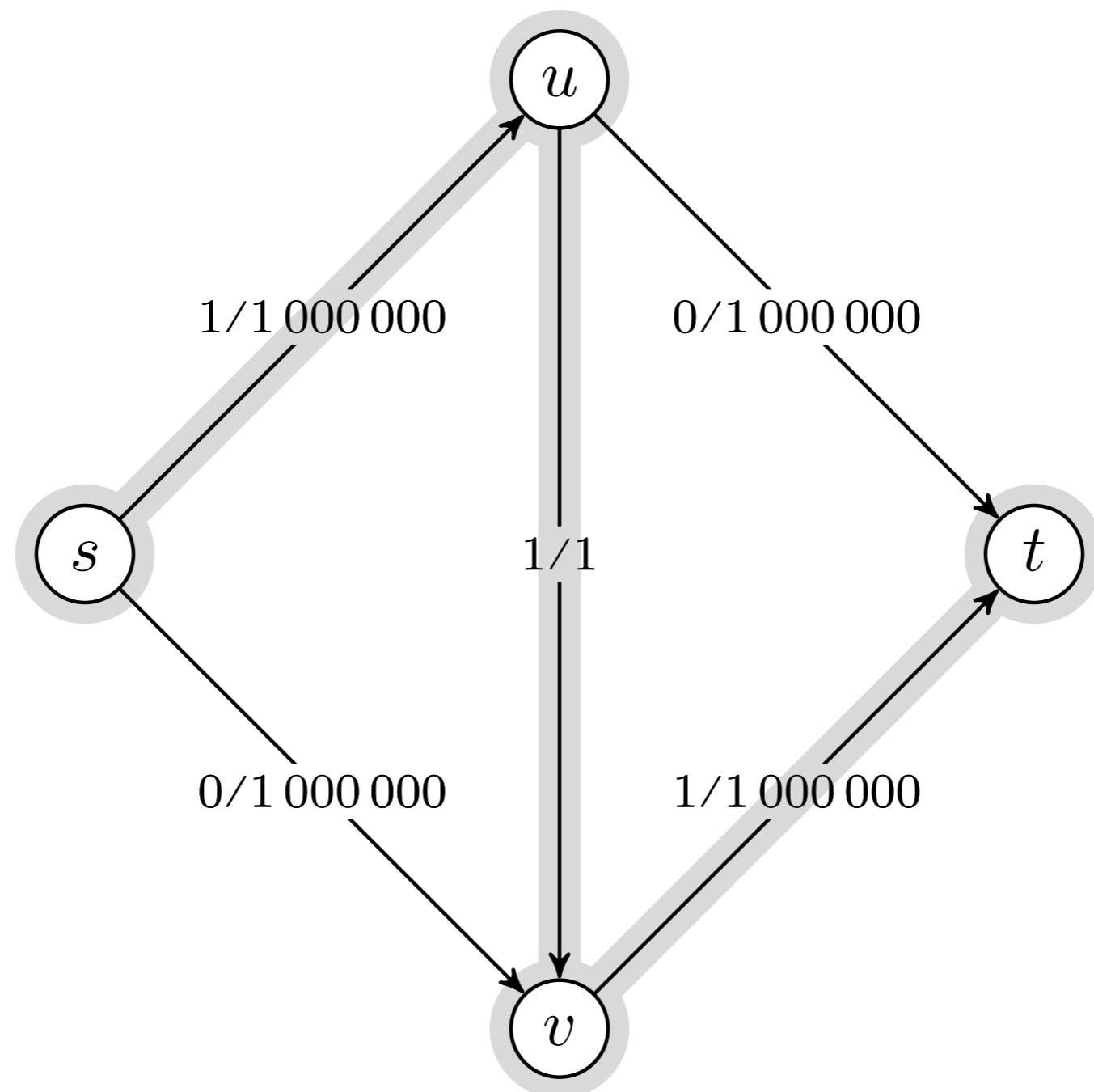
Q	1	2	3	4	5	6
	1	2	4	3	5	6

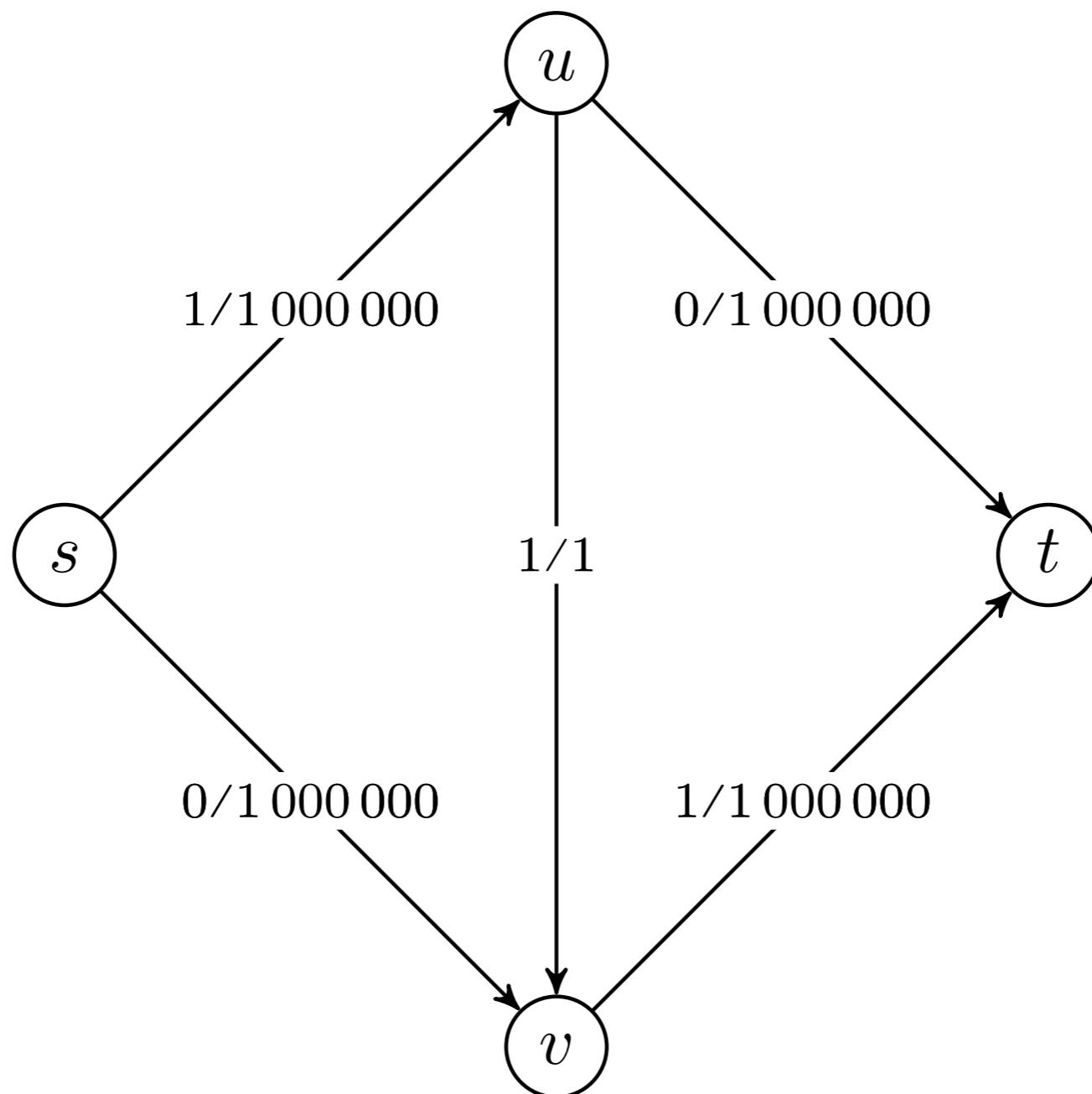


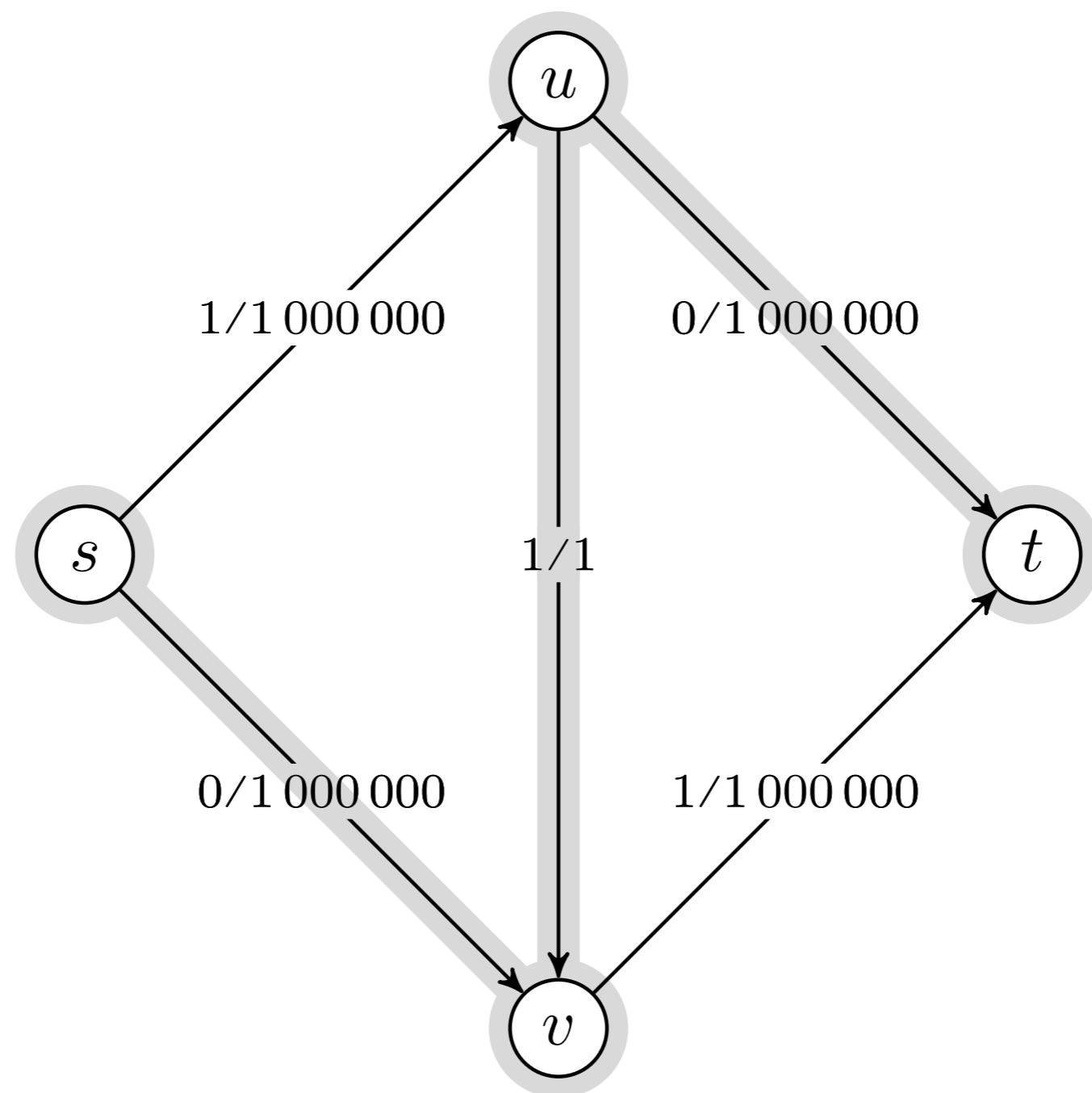
Kan gå ille uten BFS

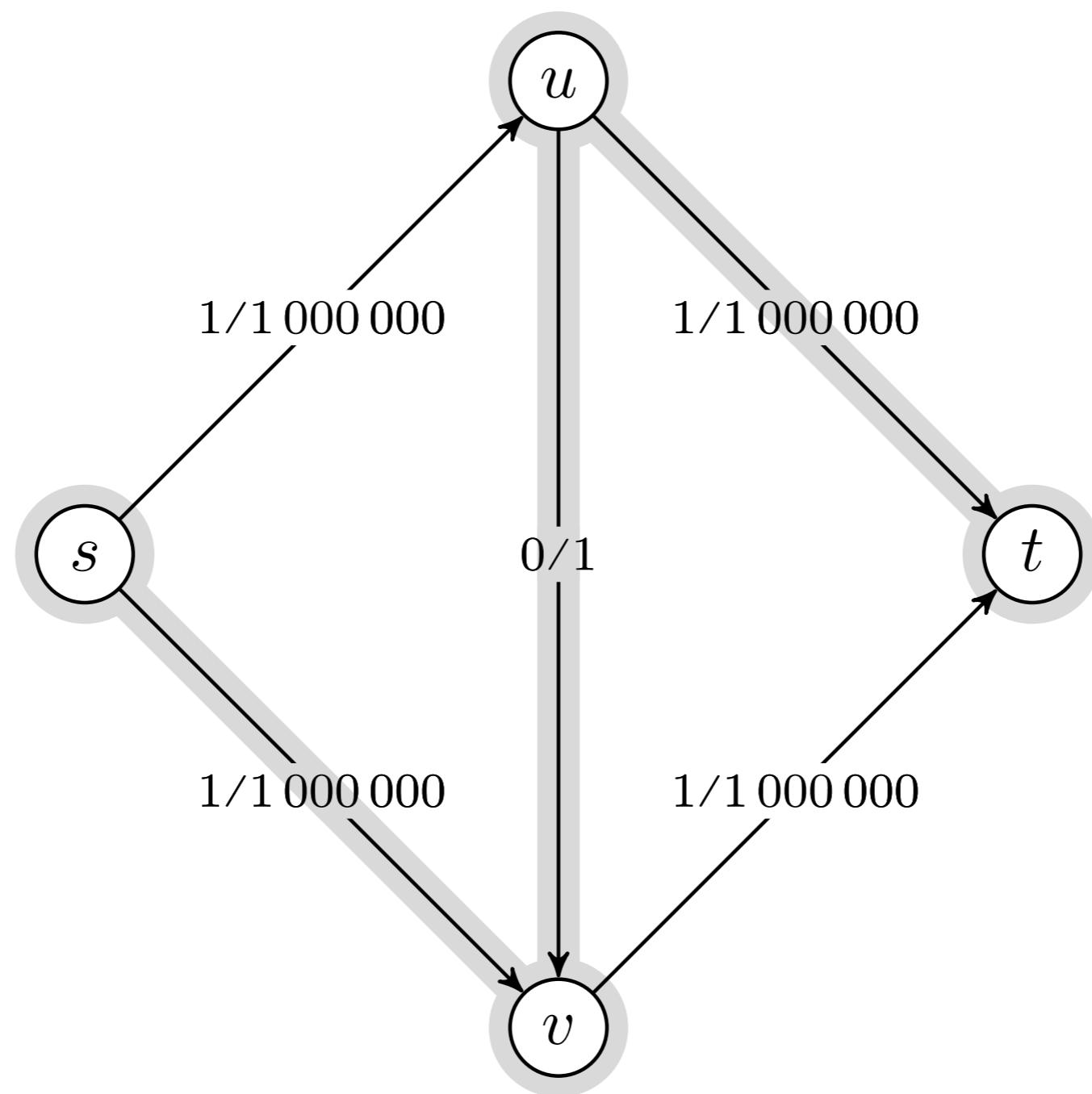


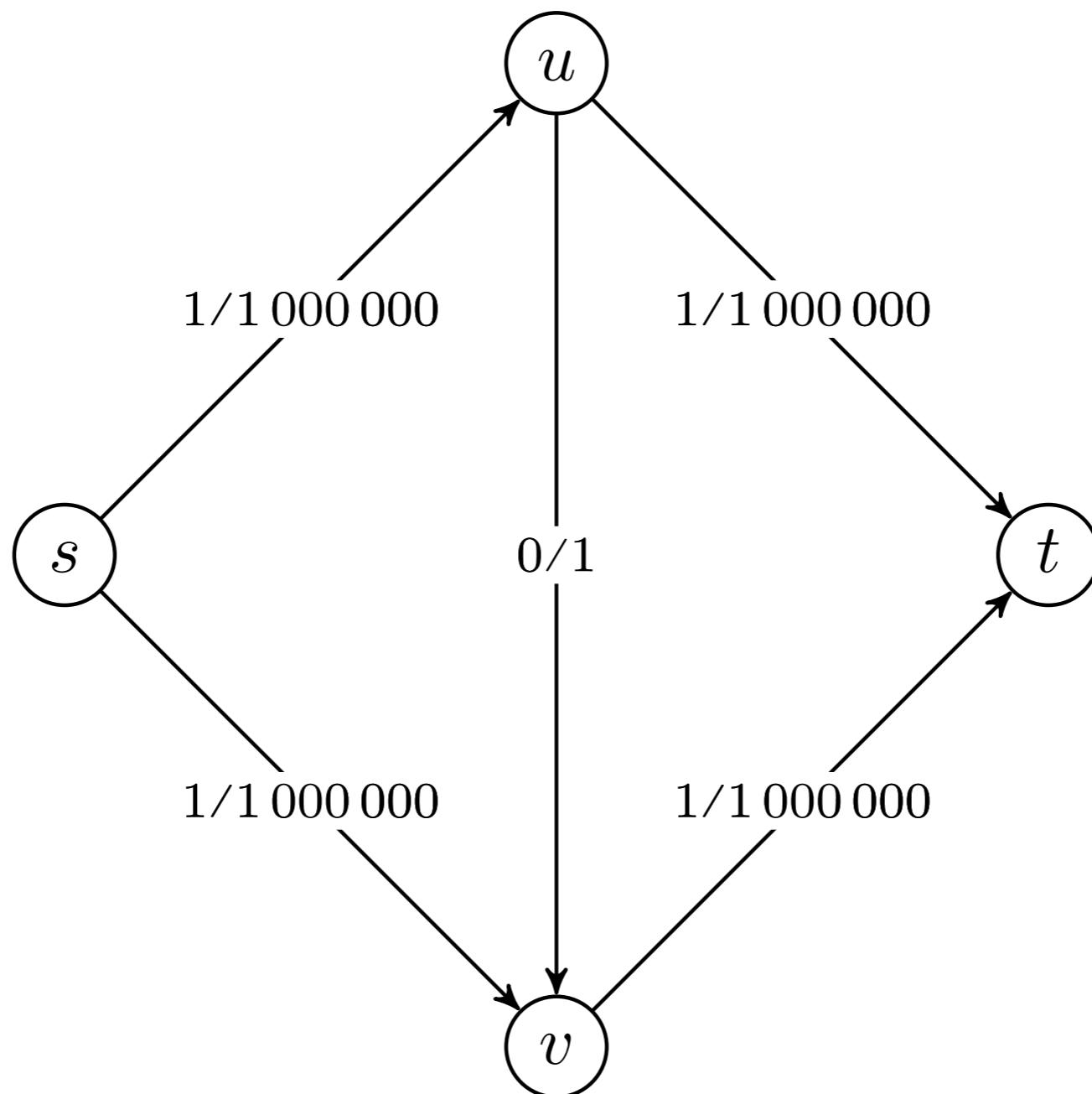


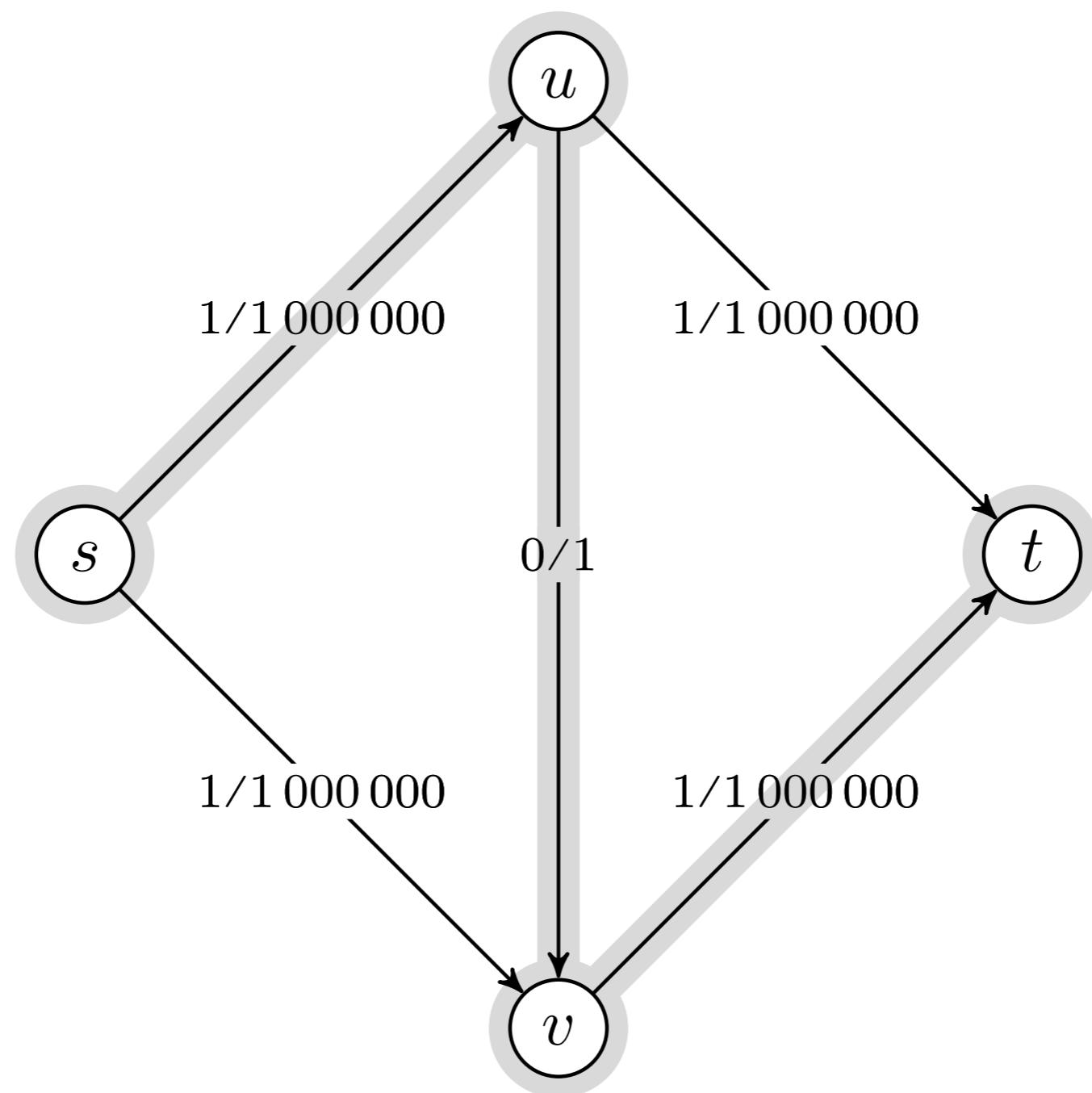


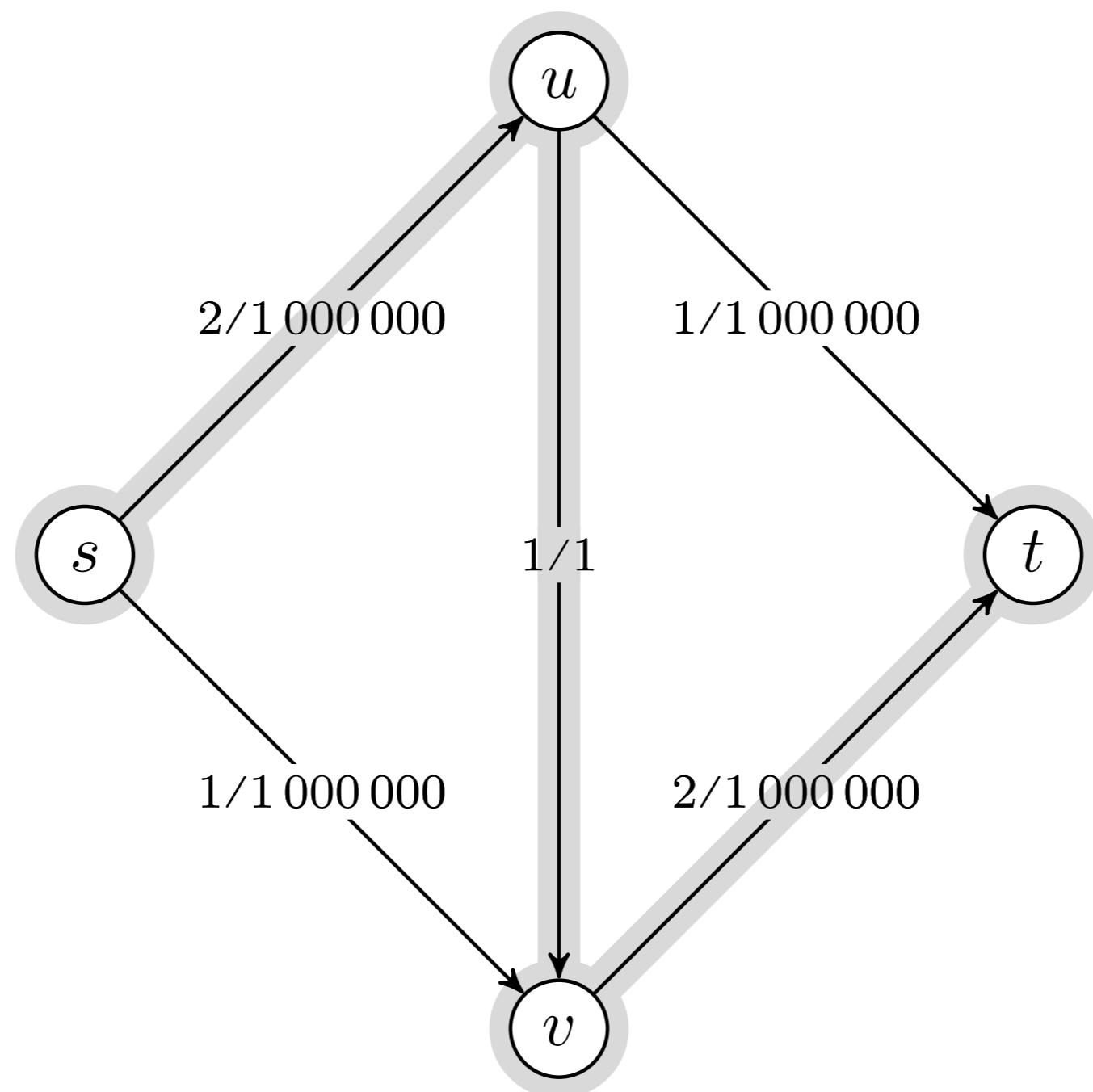


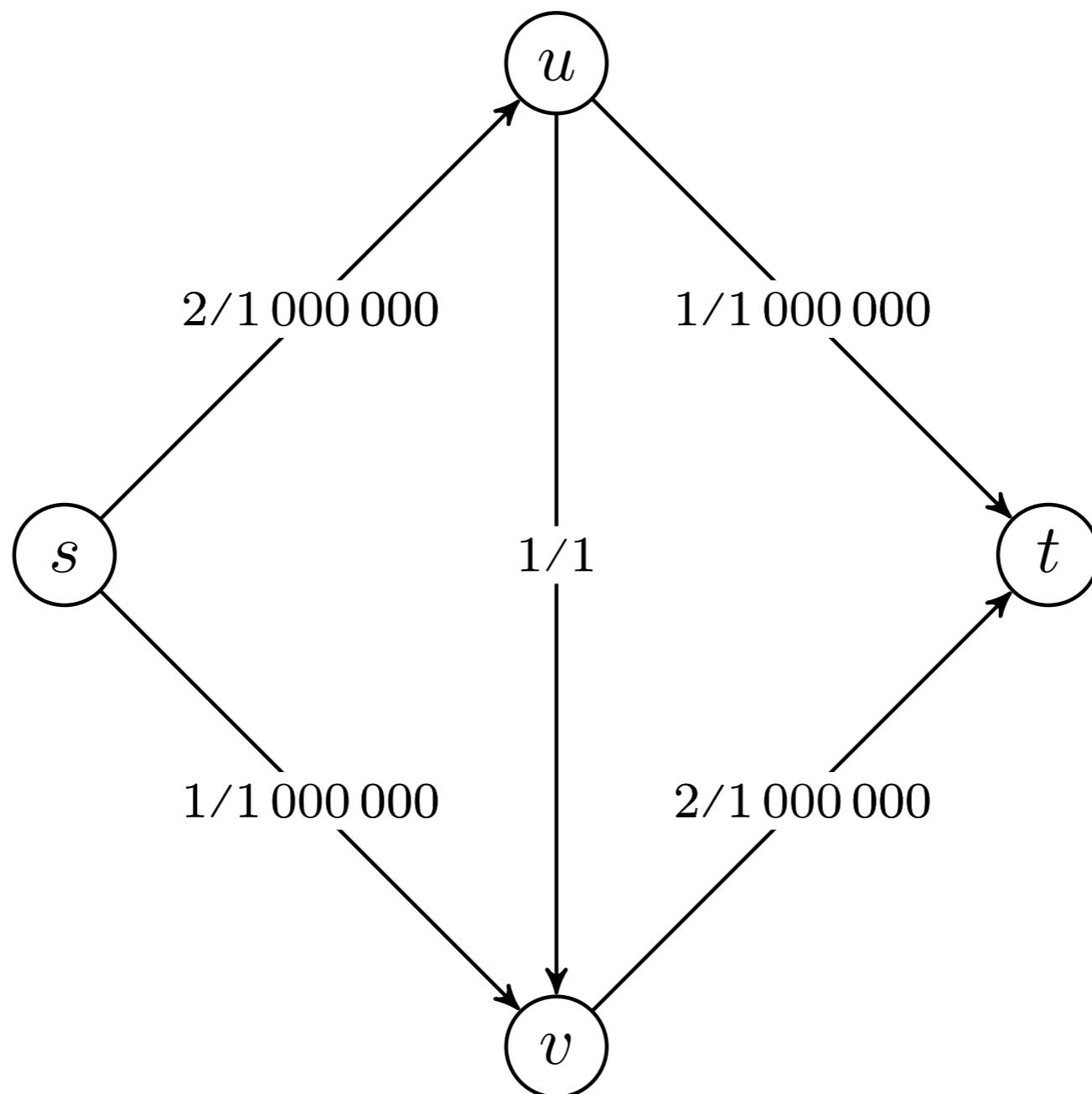


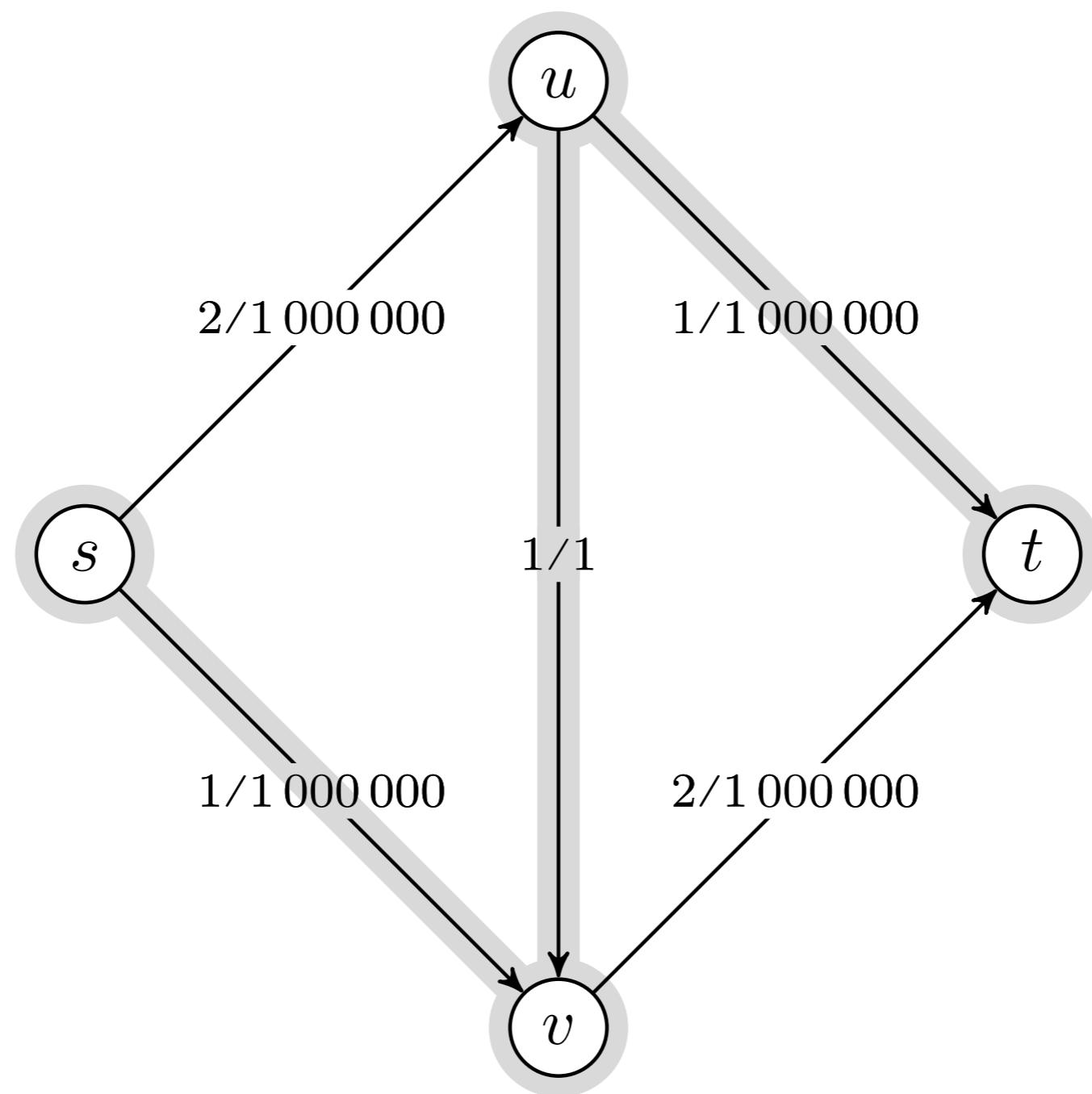


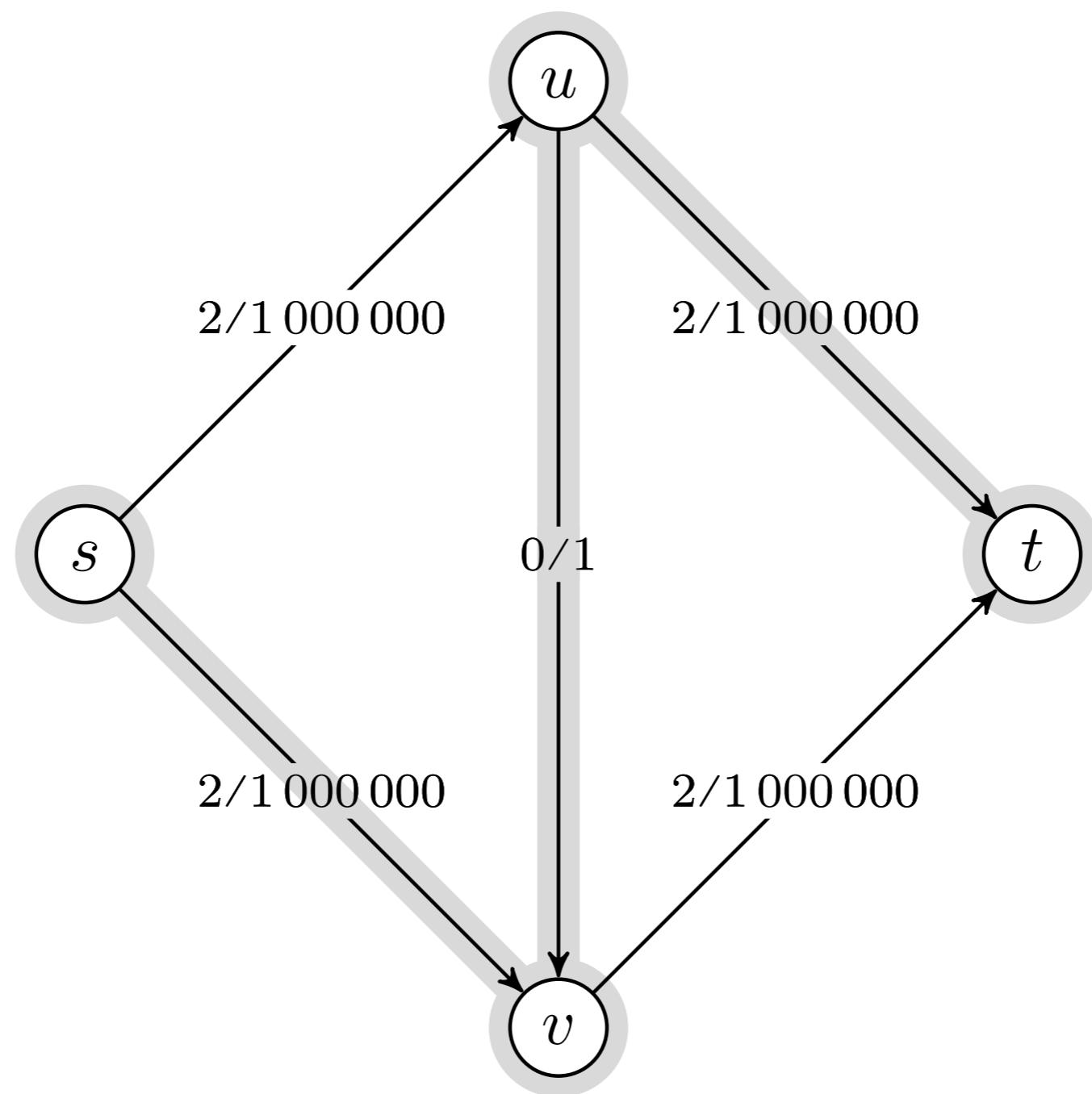


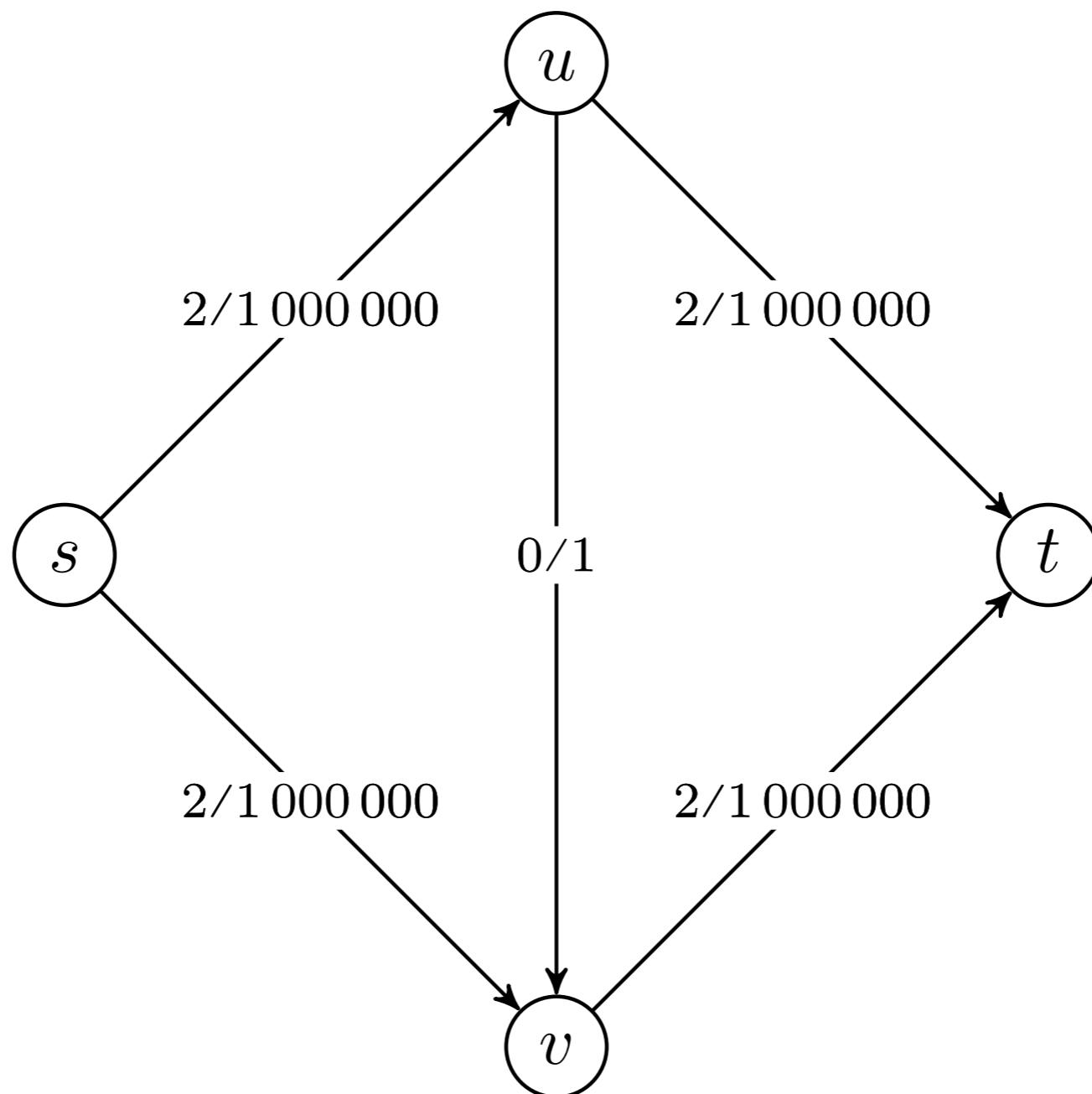


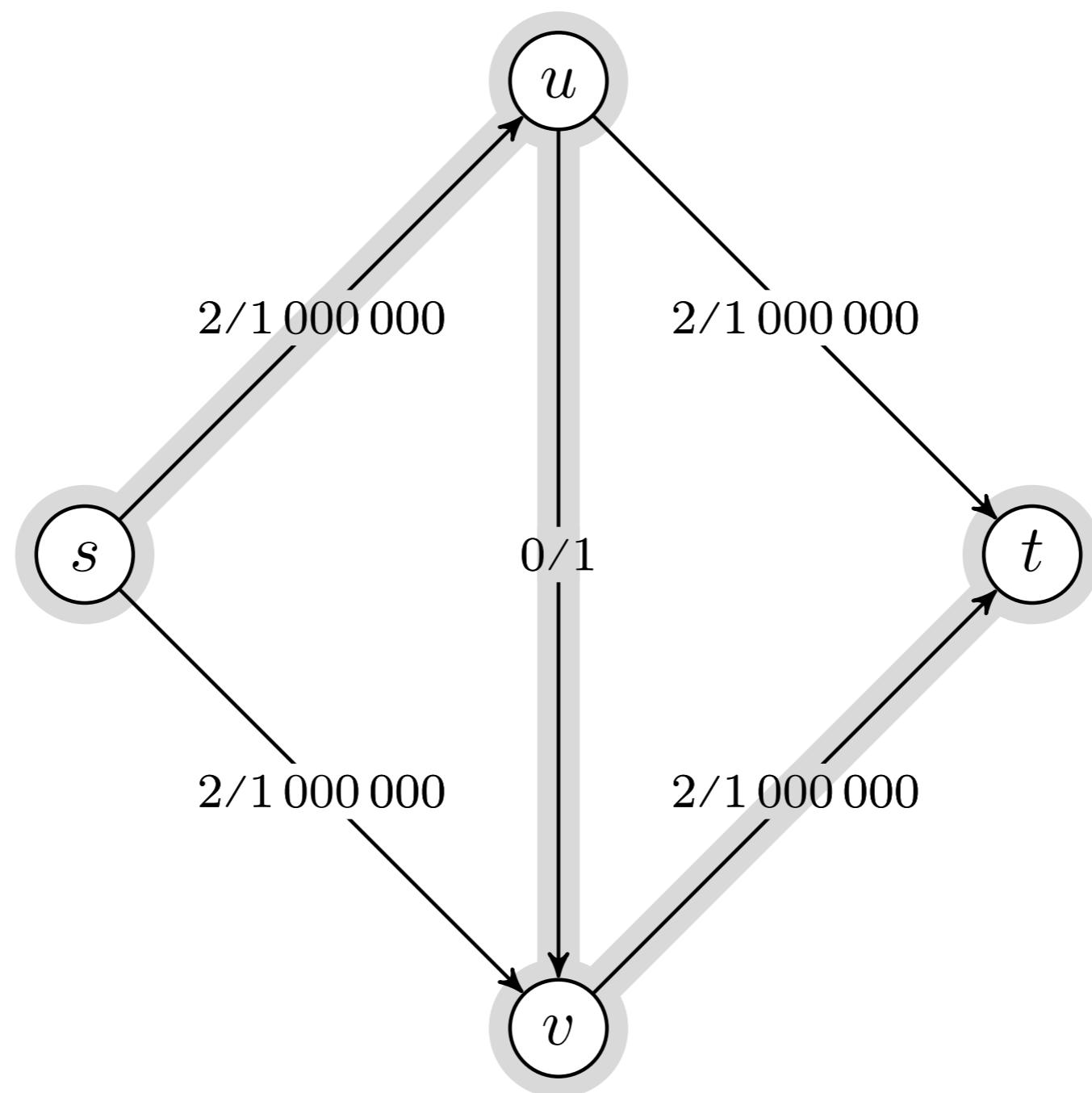


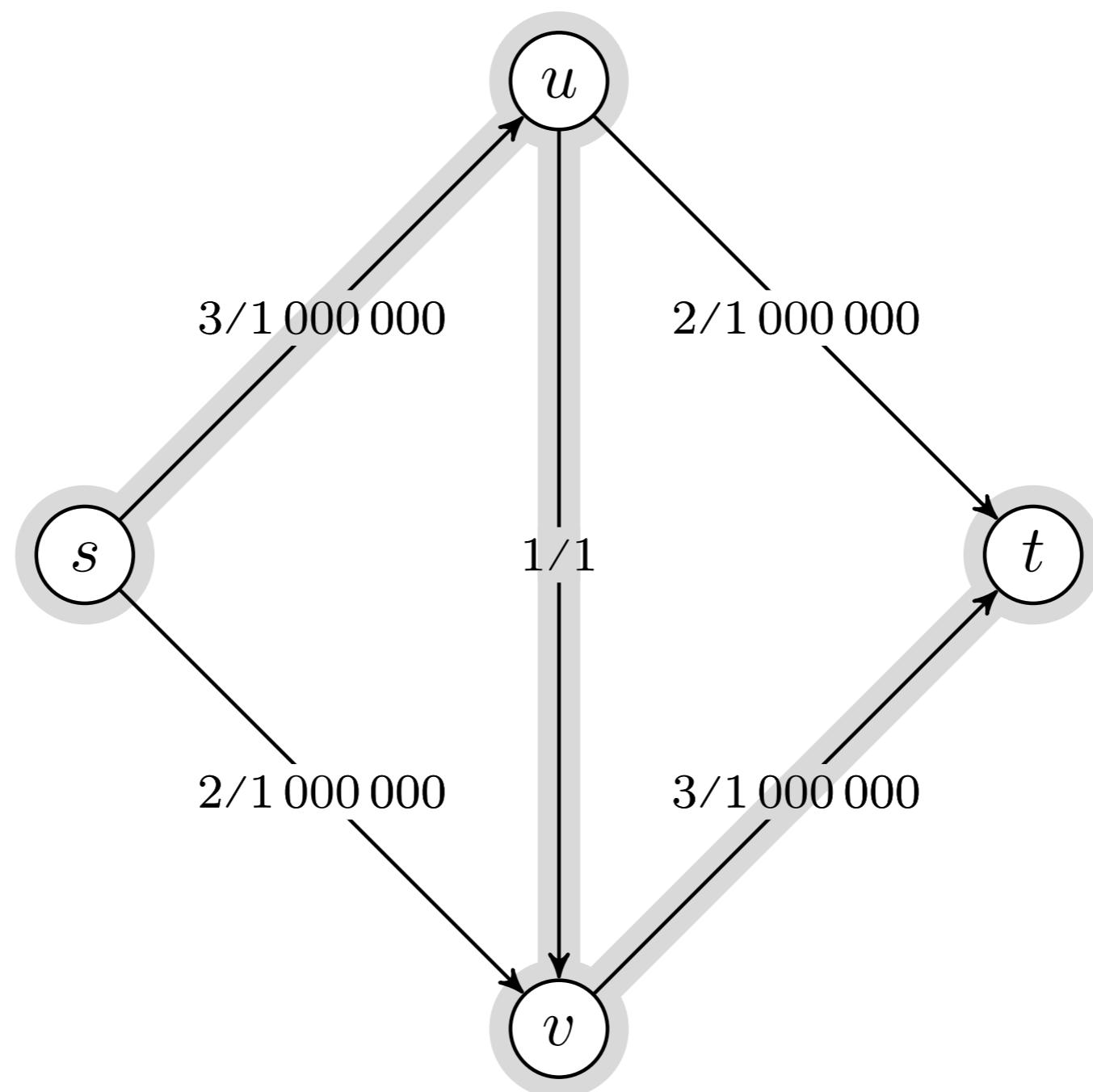


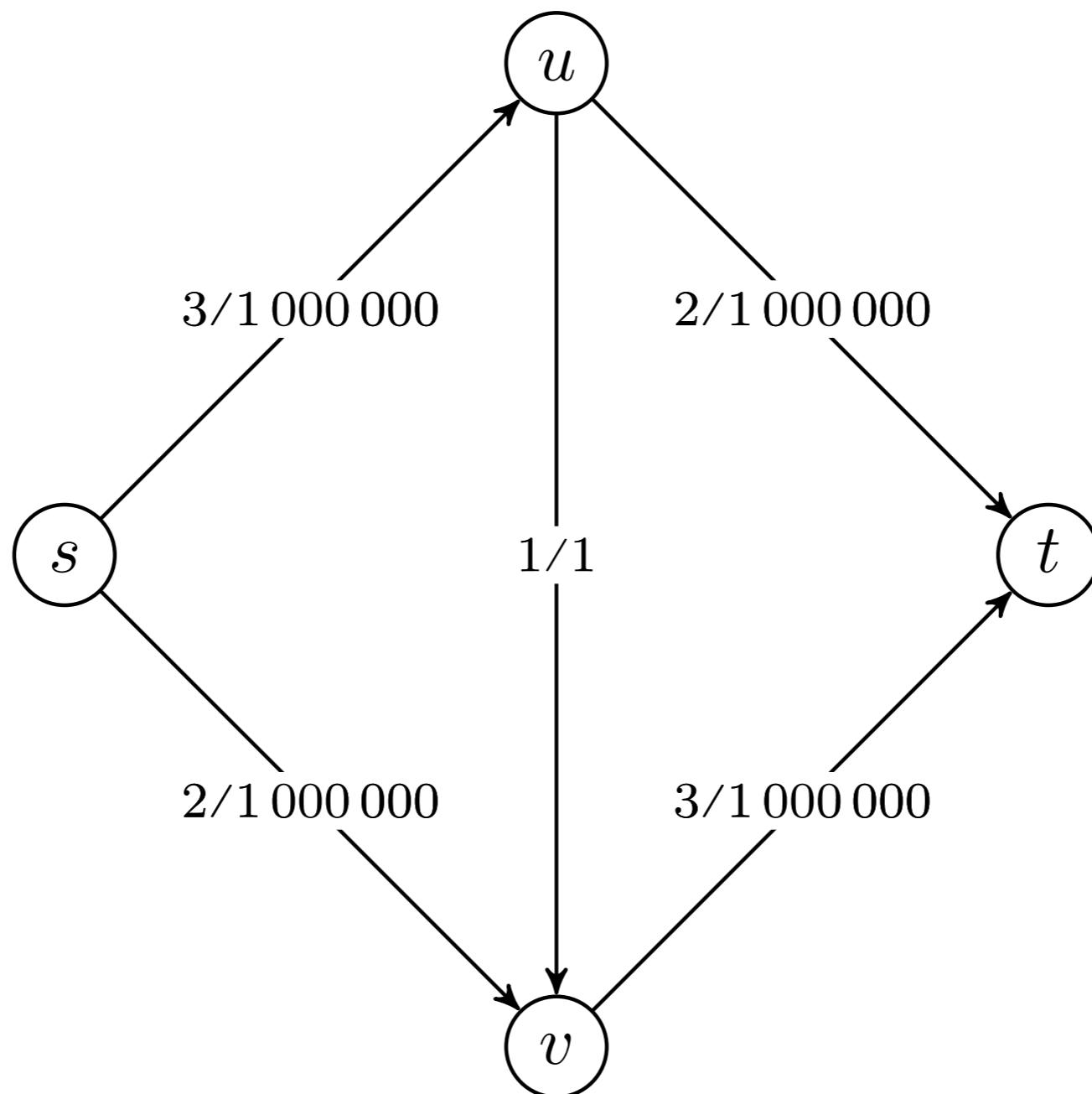


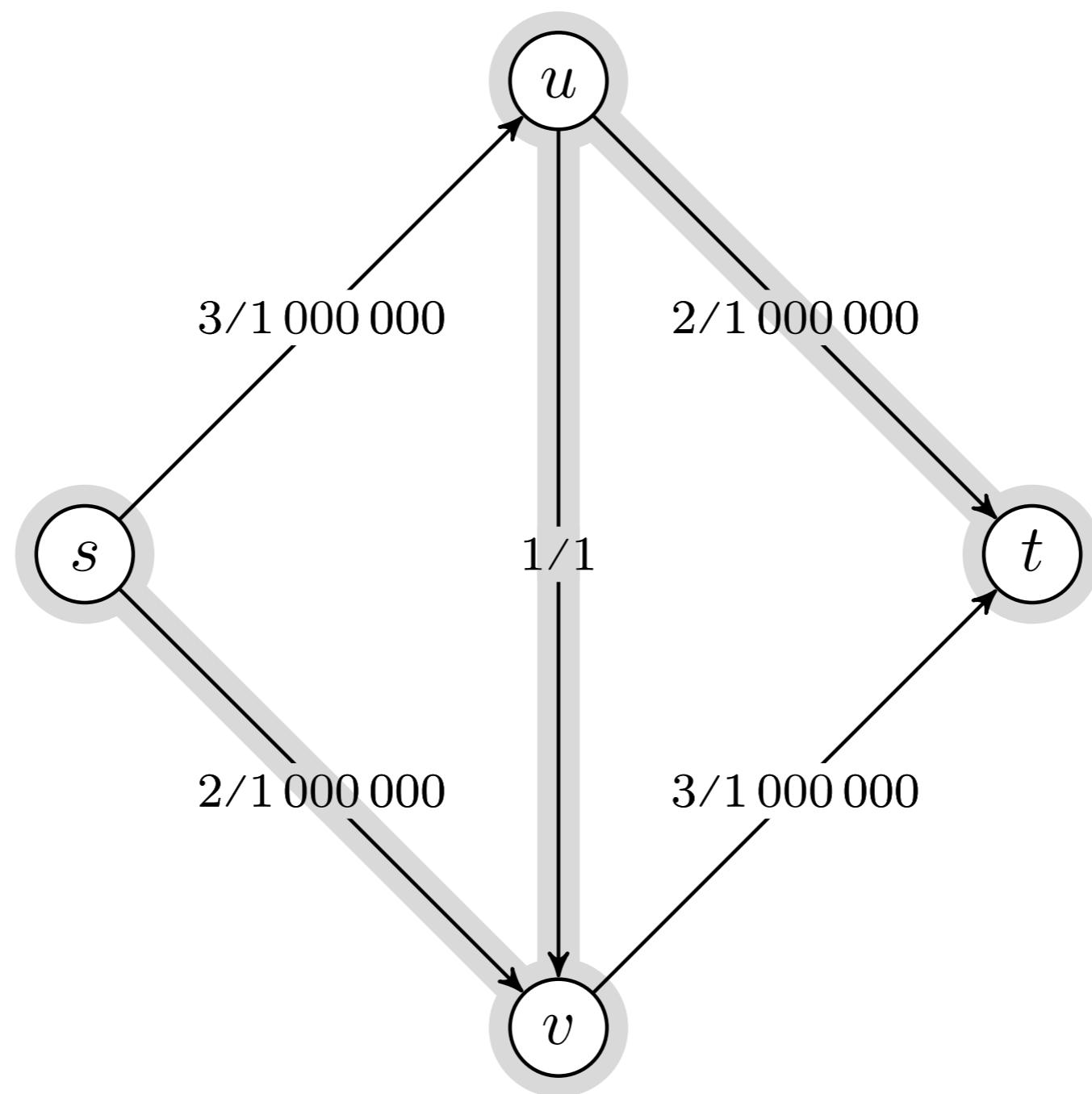


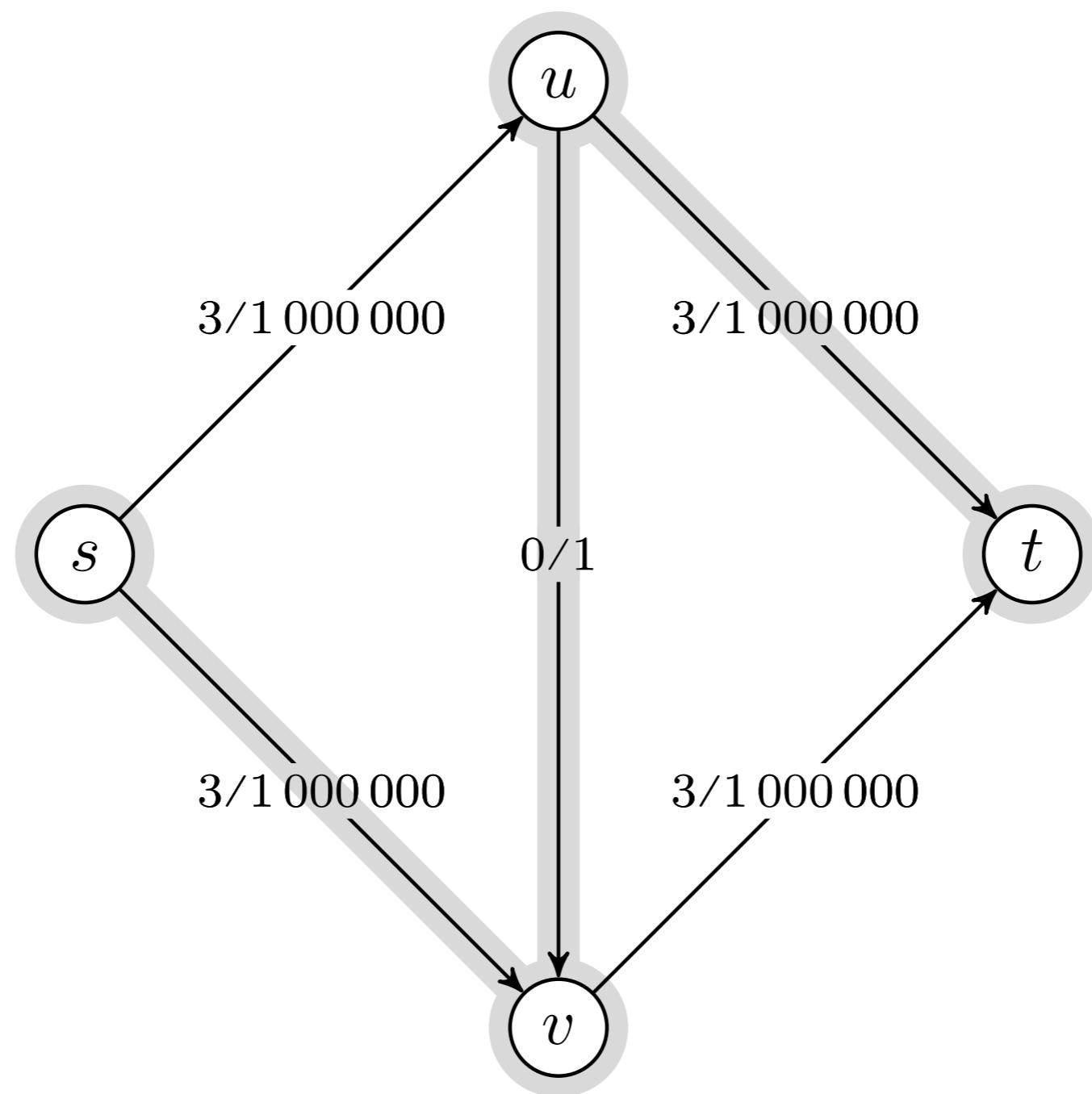


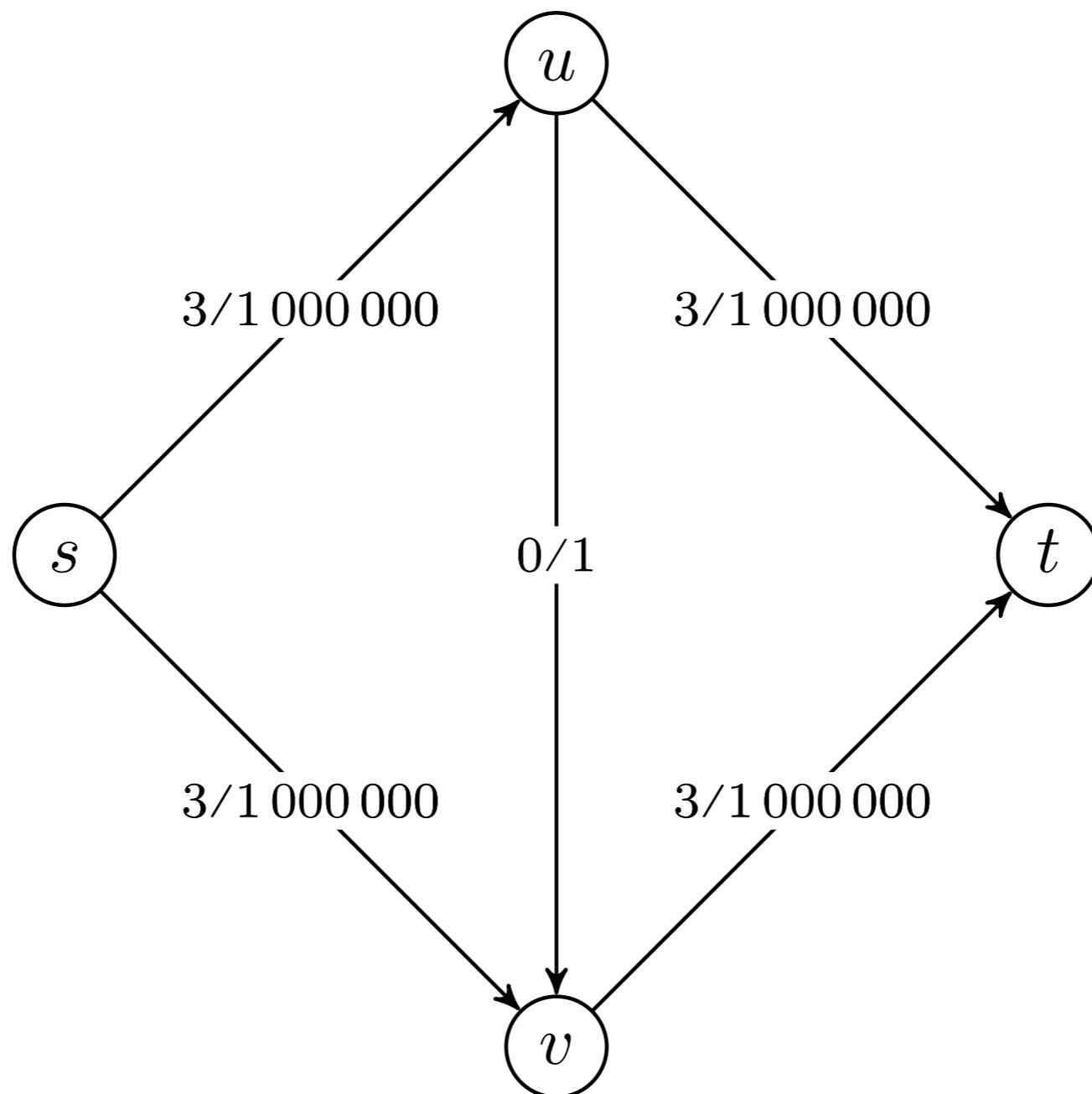


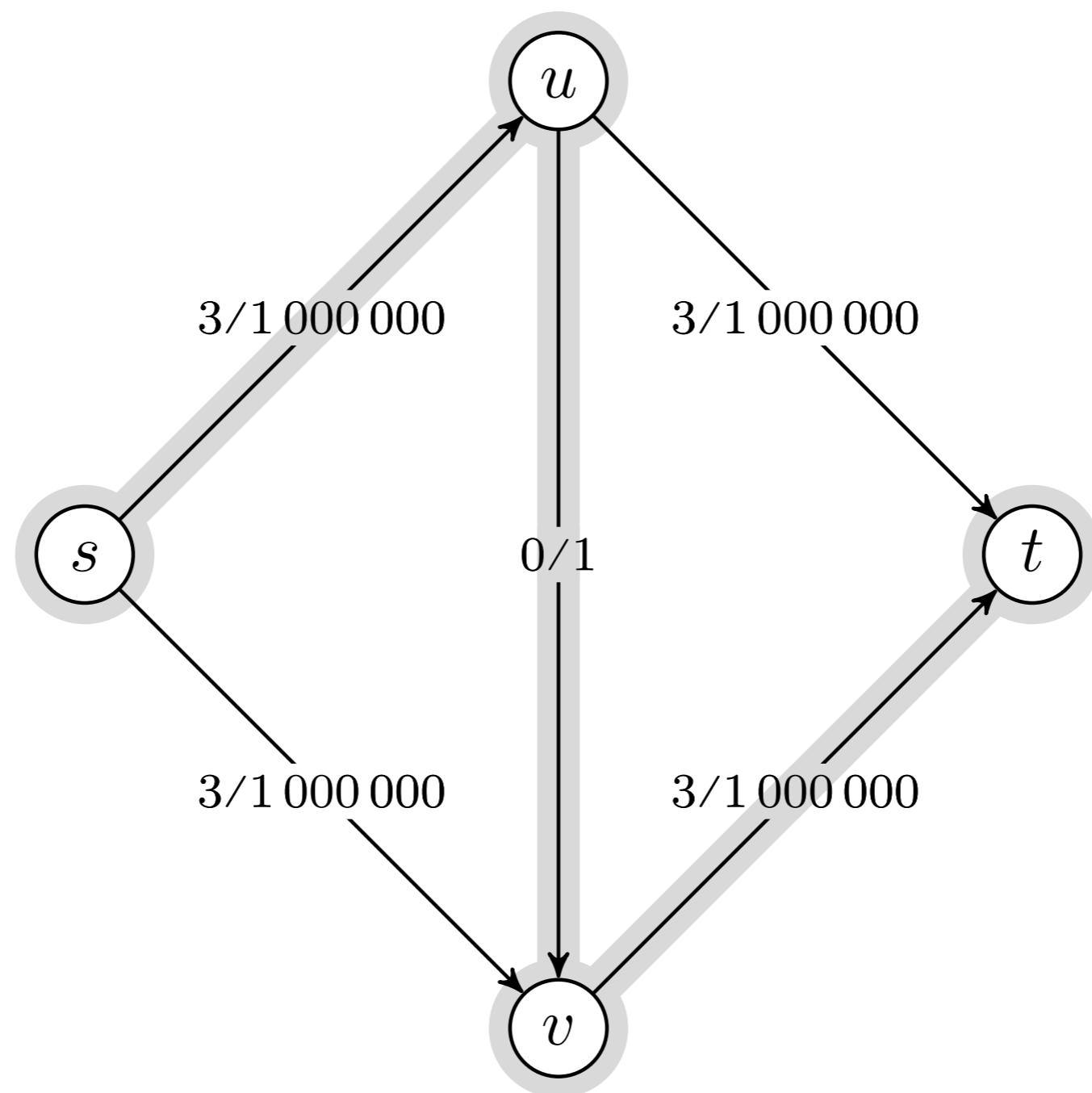


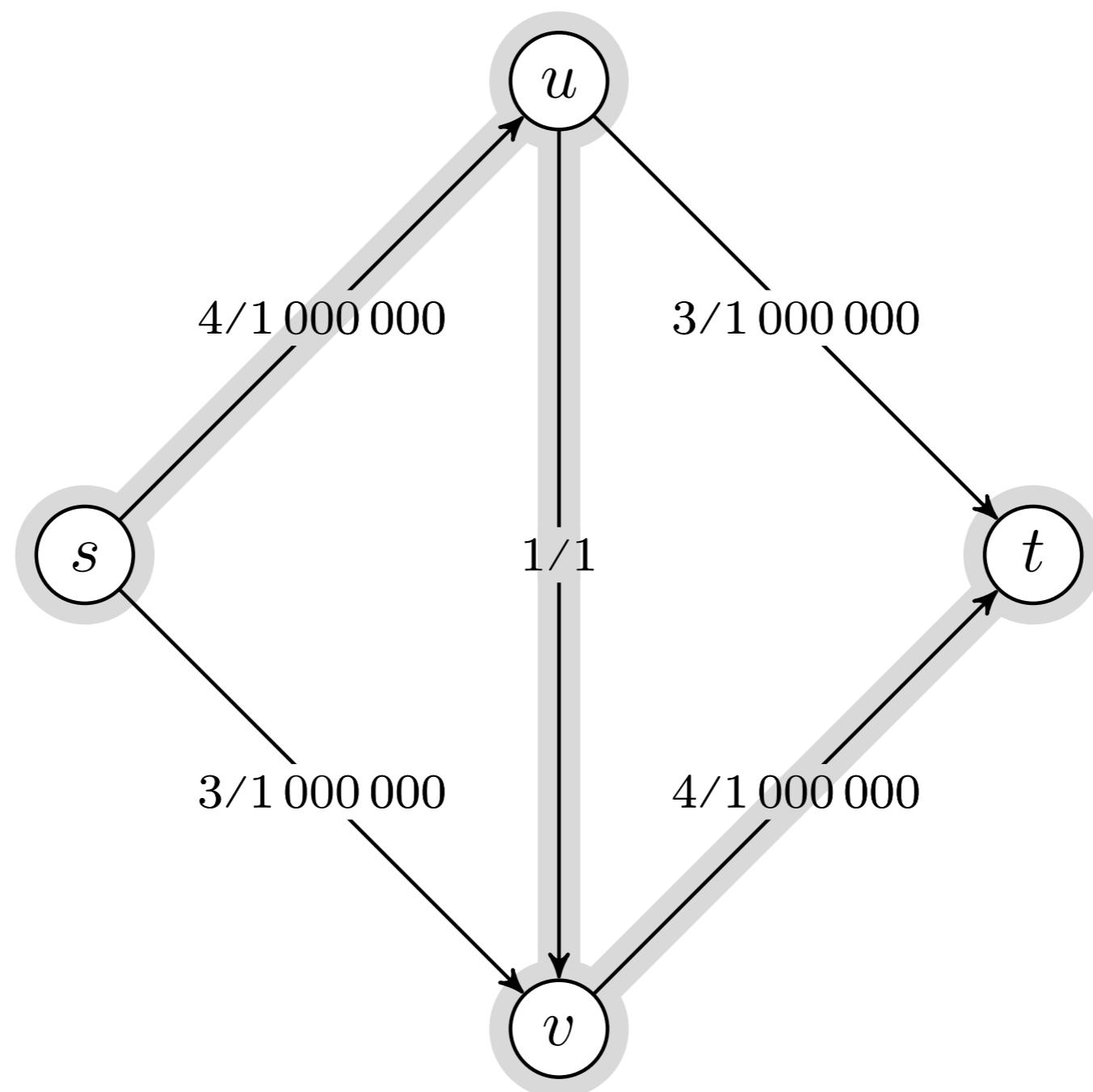


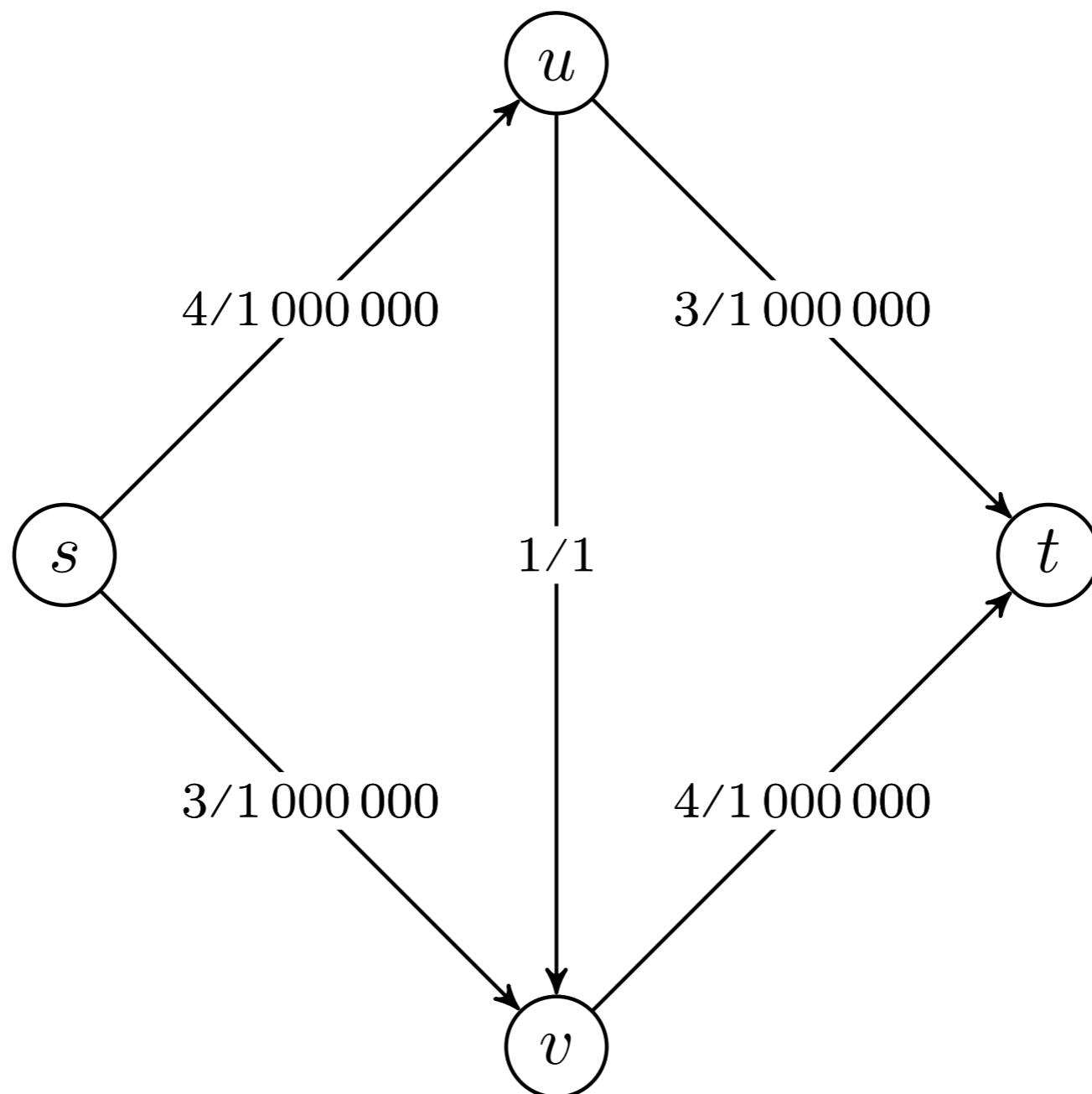


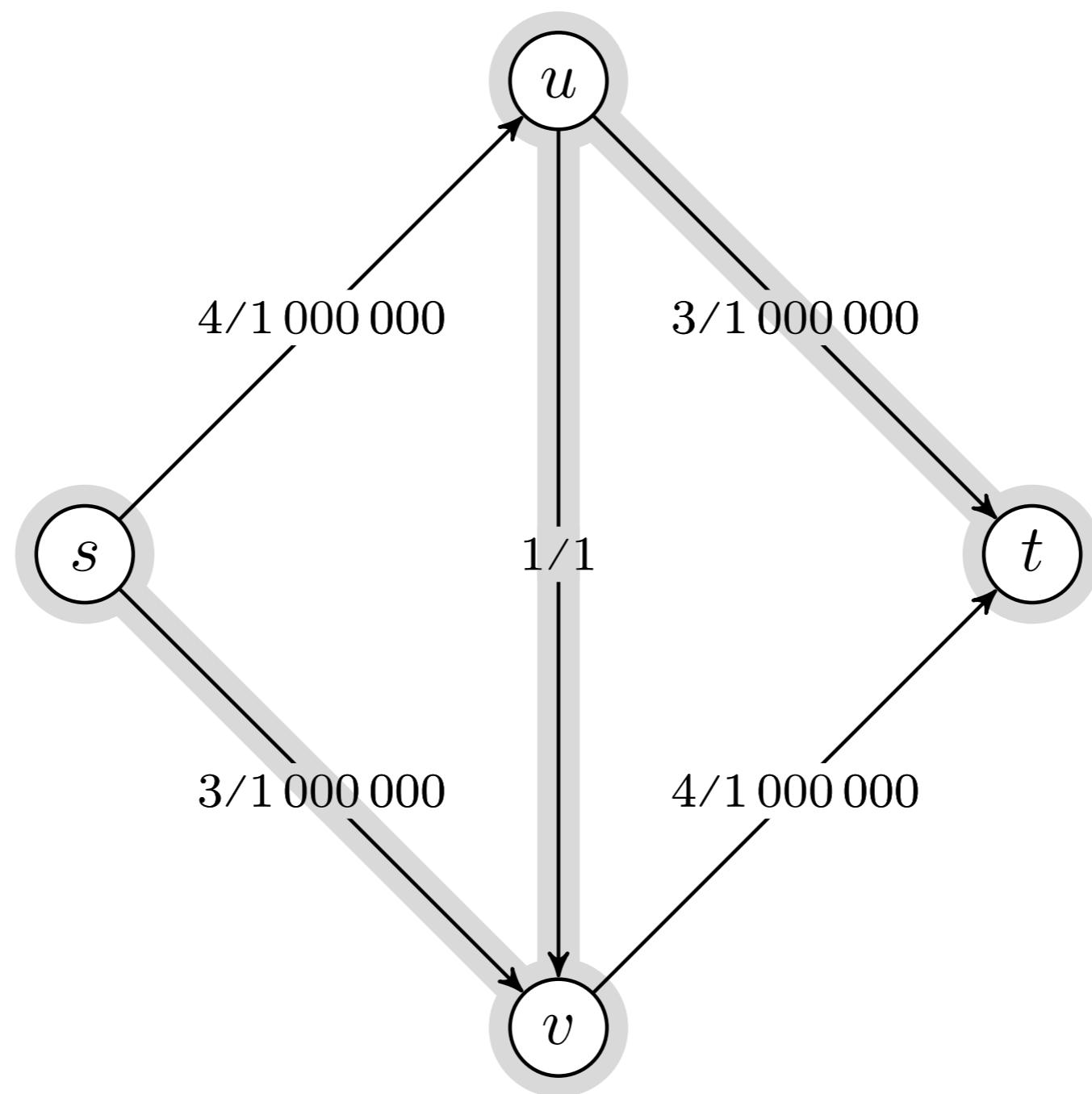


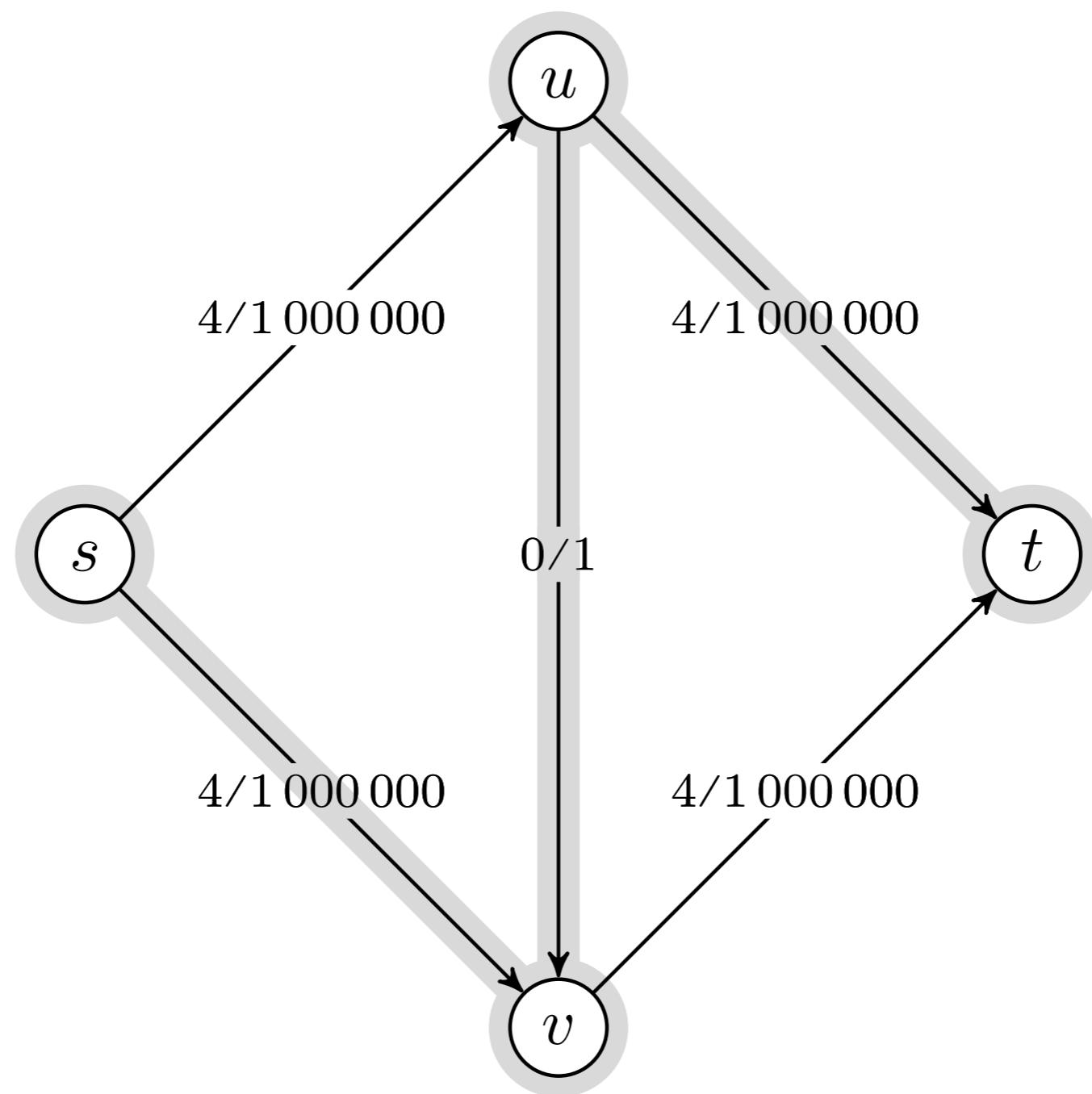


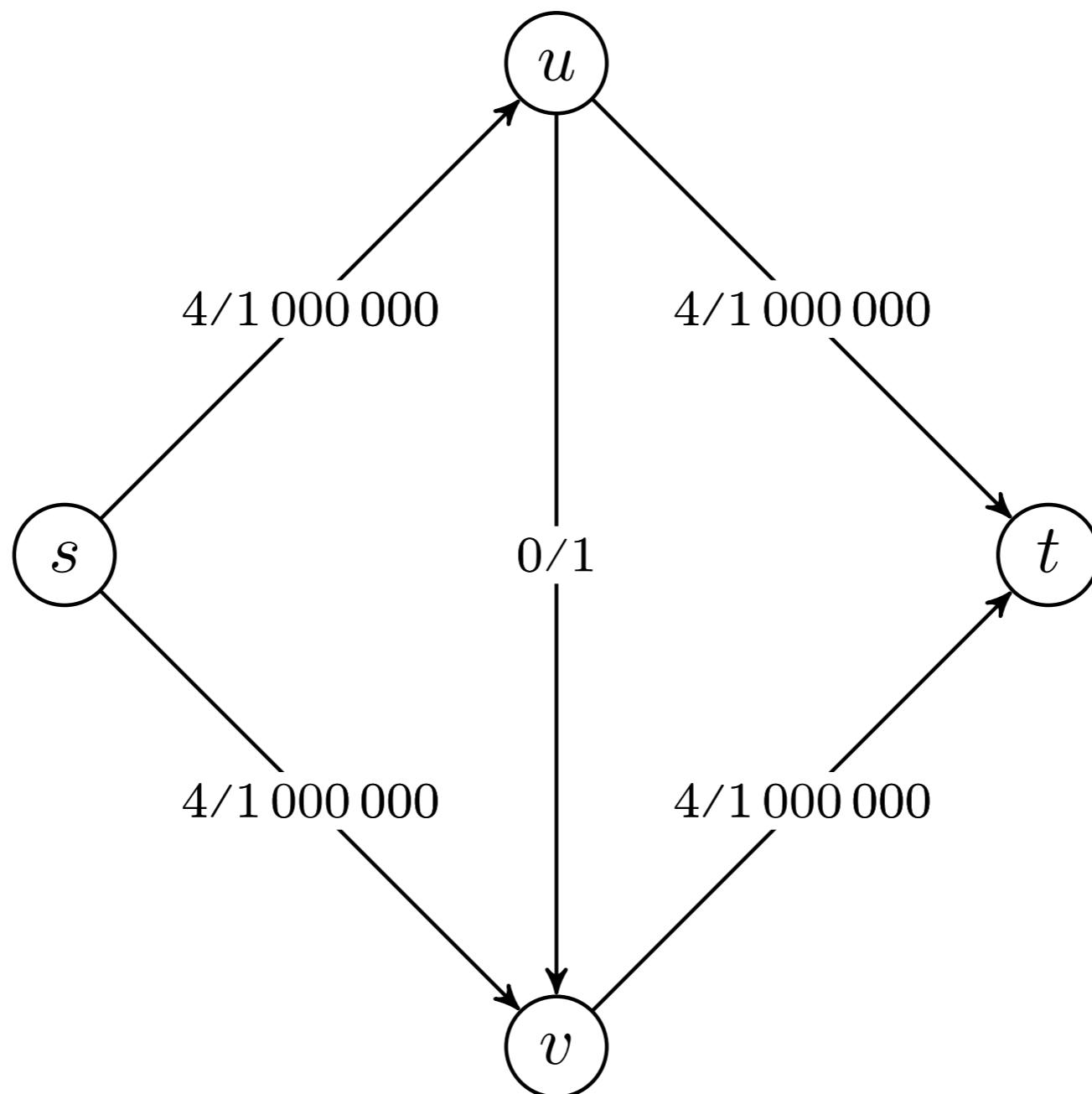


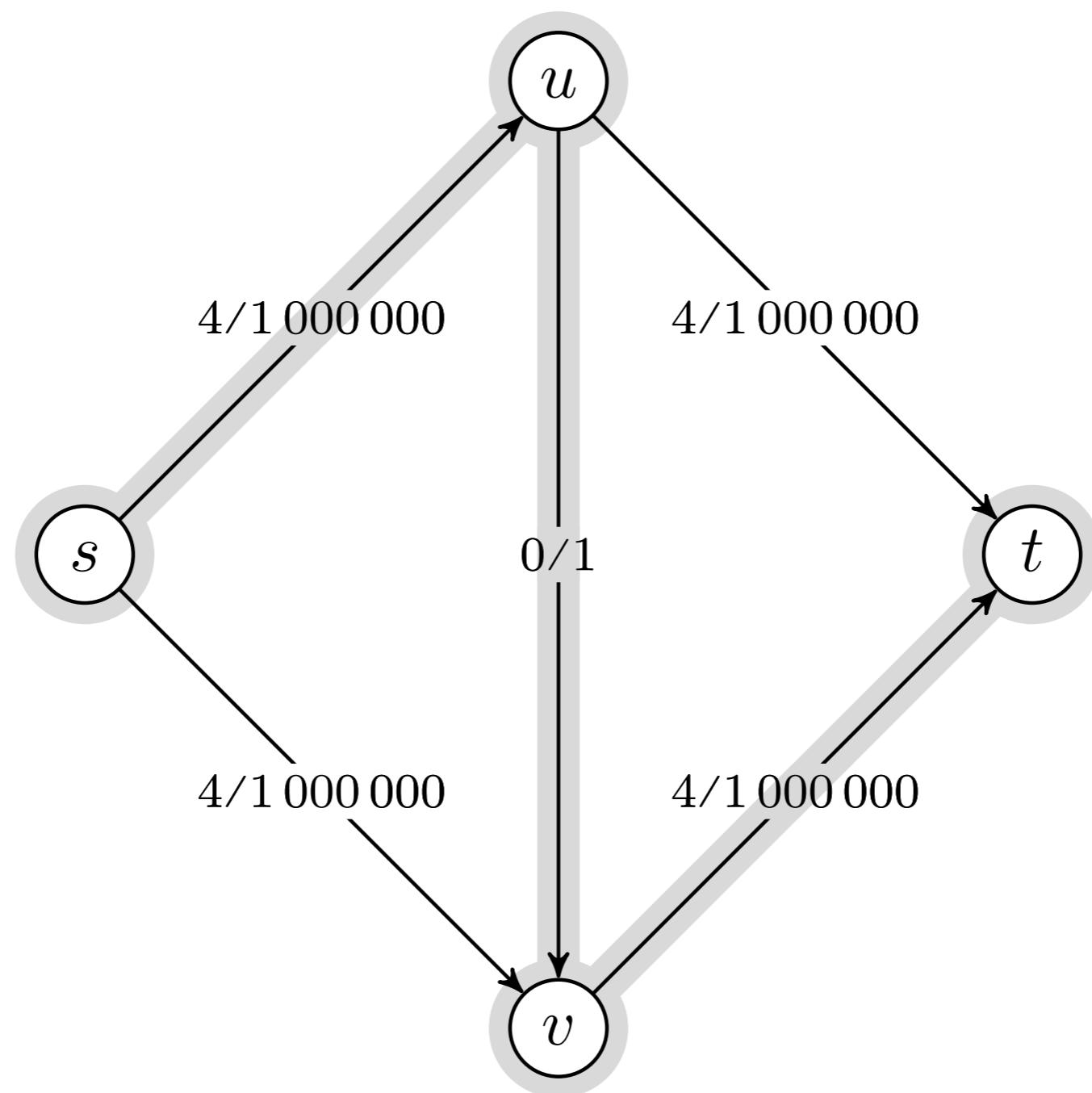


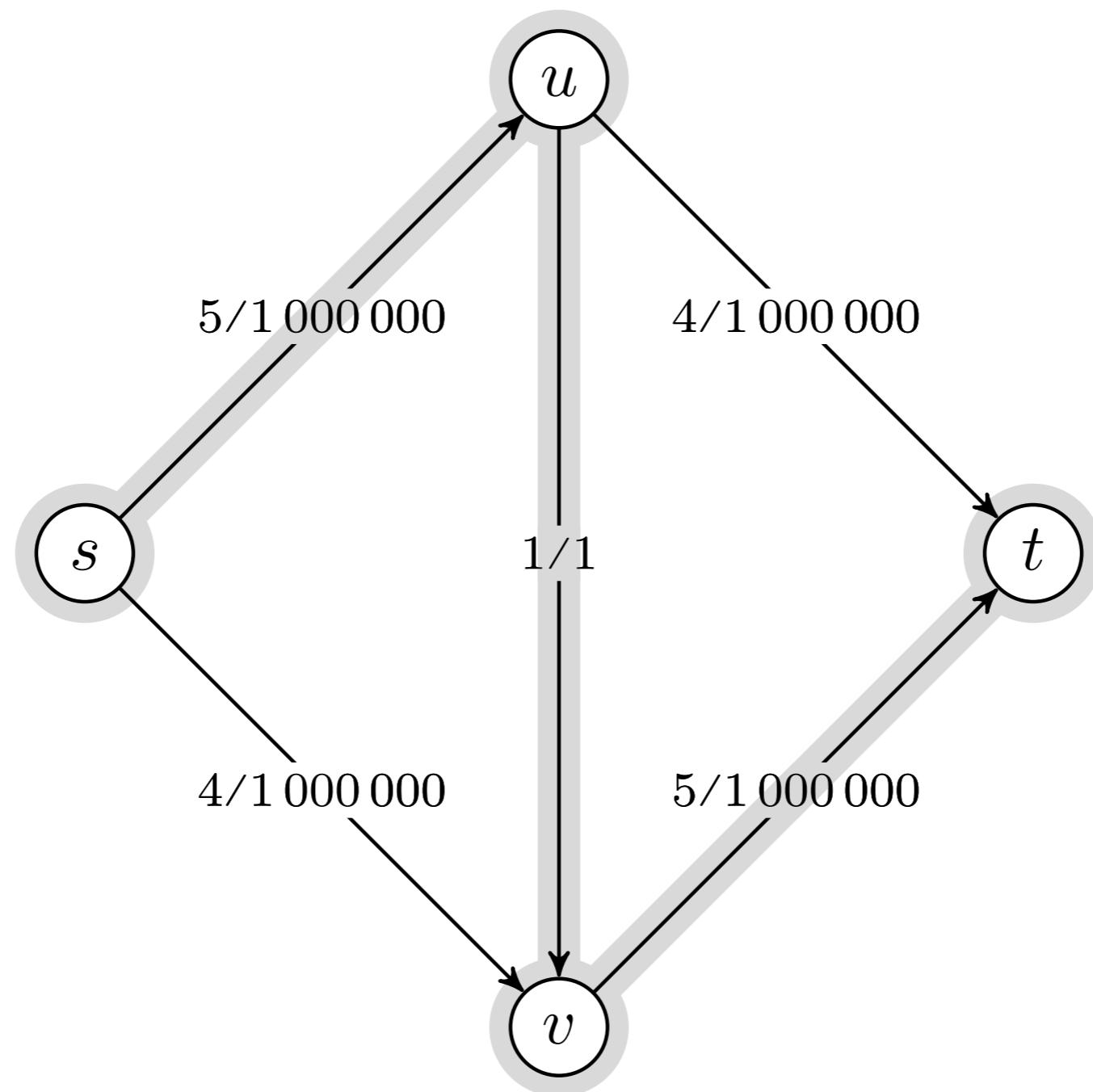


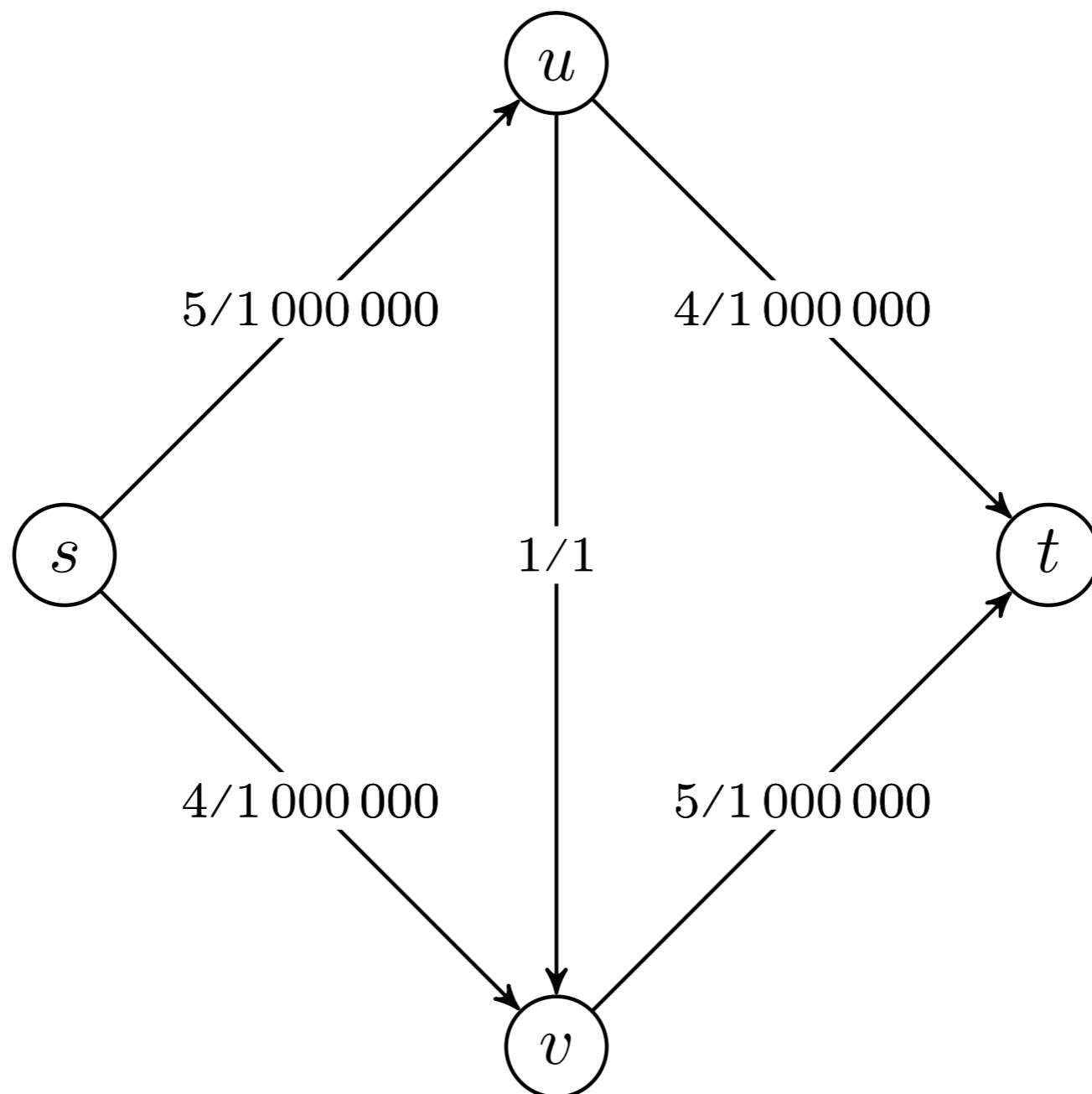


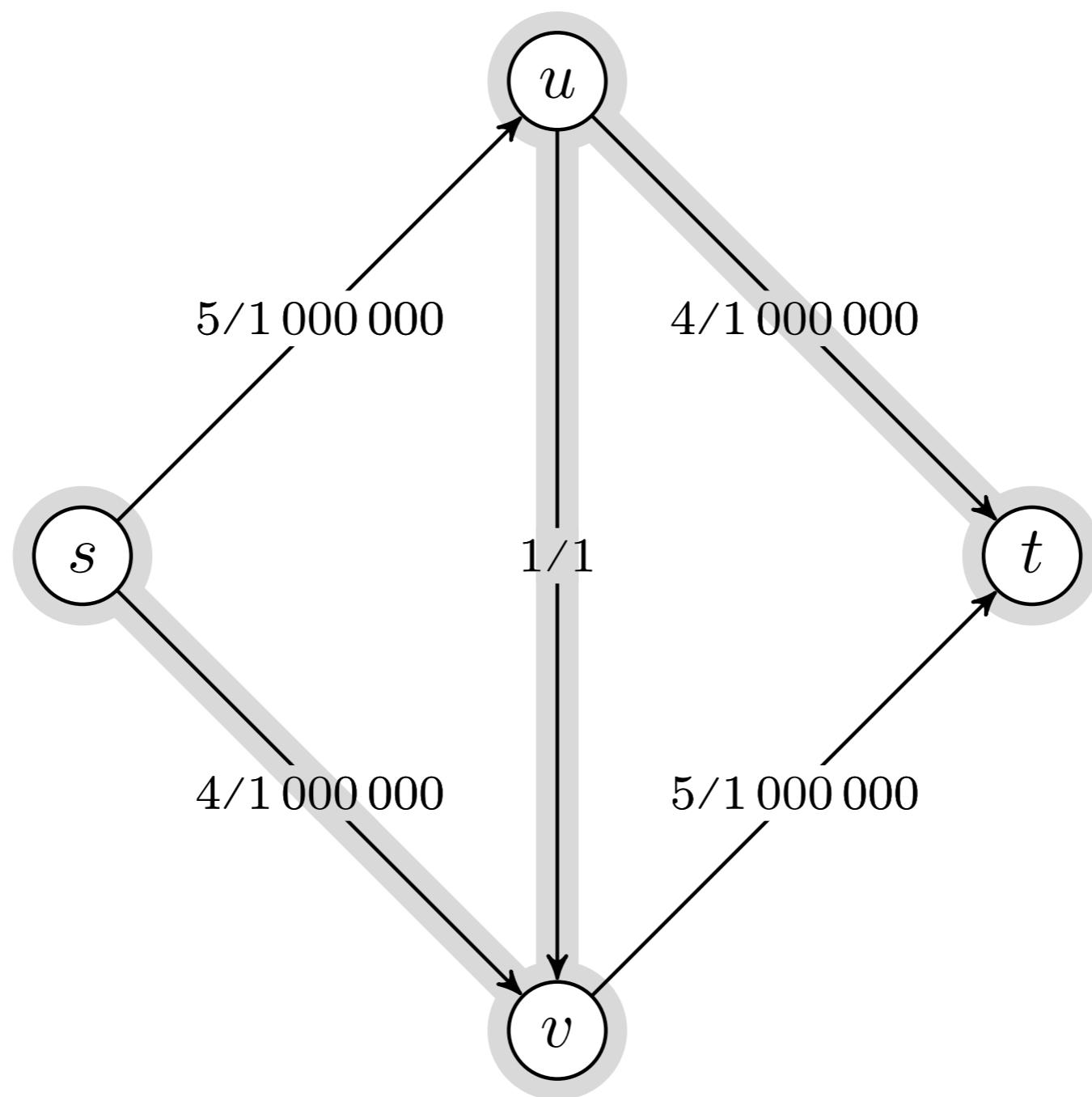


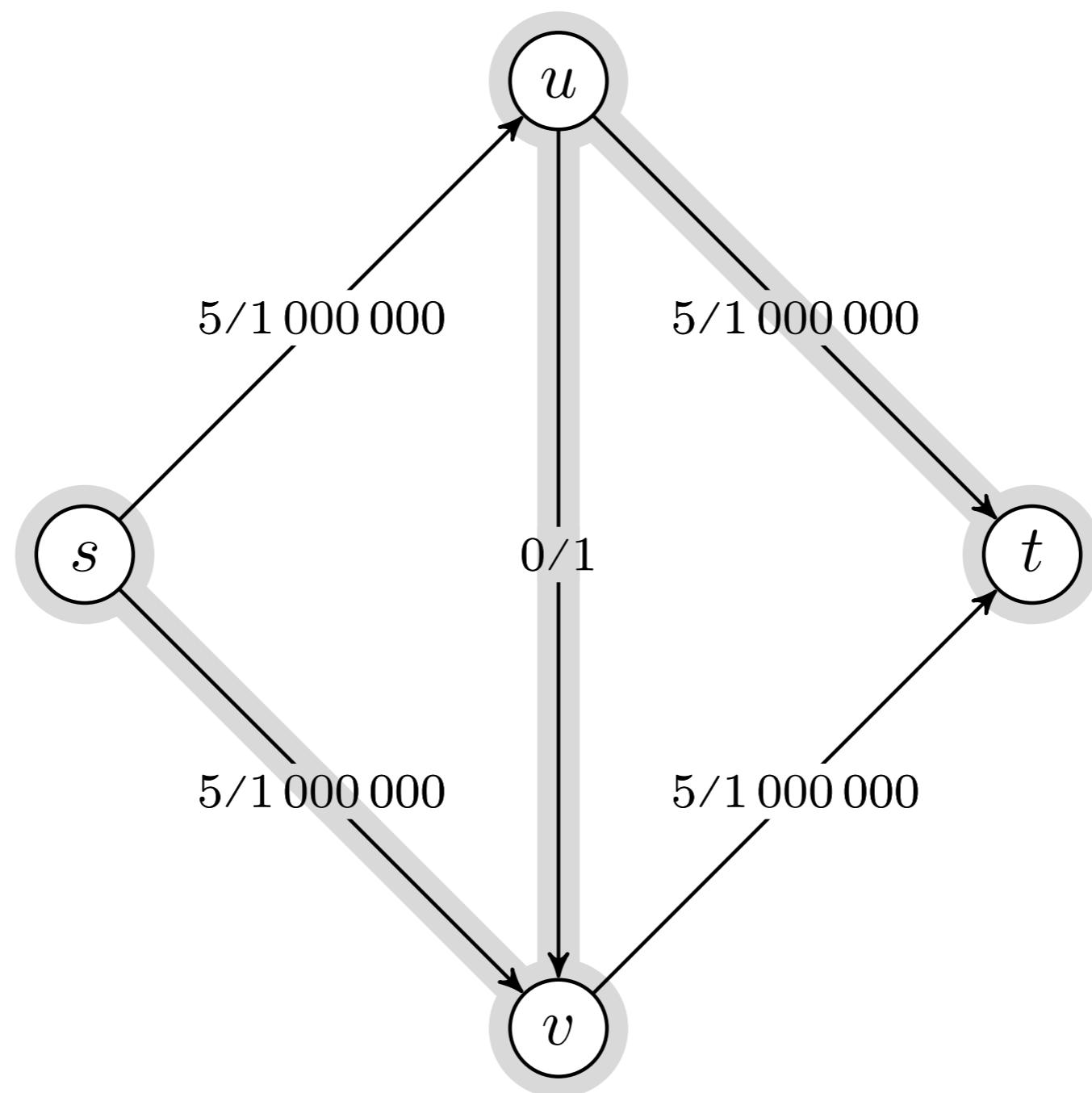


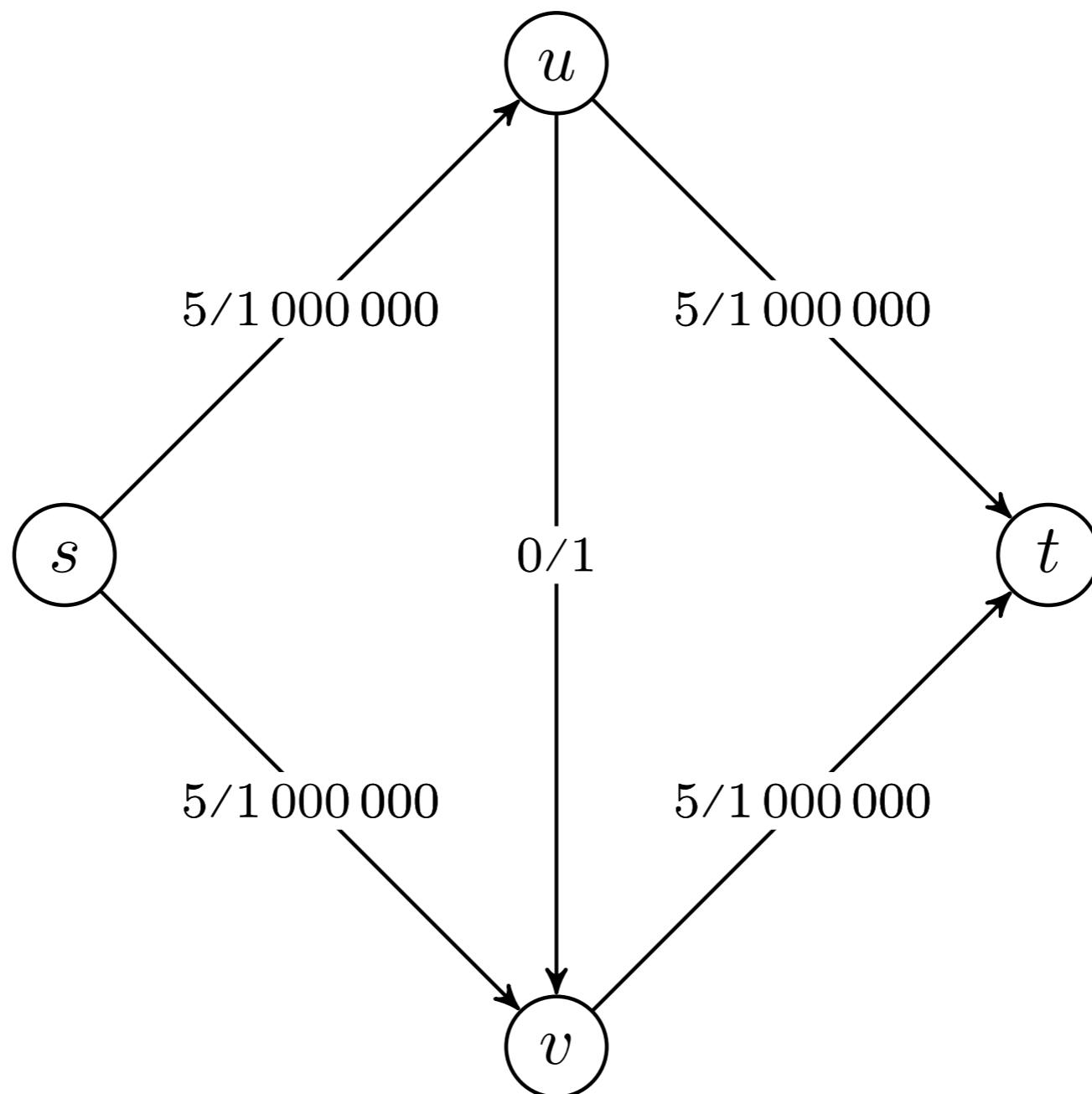


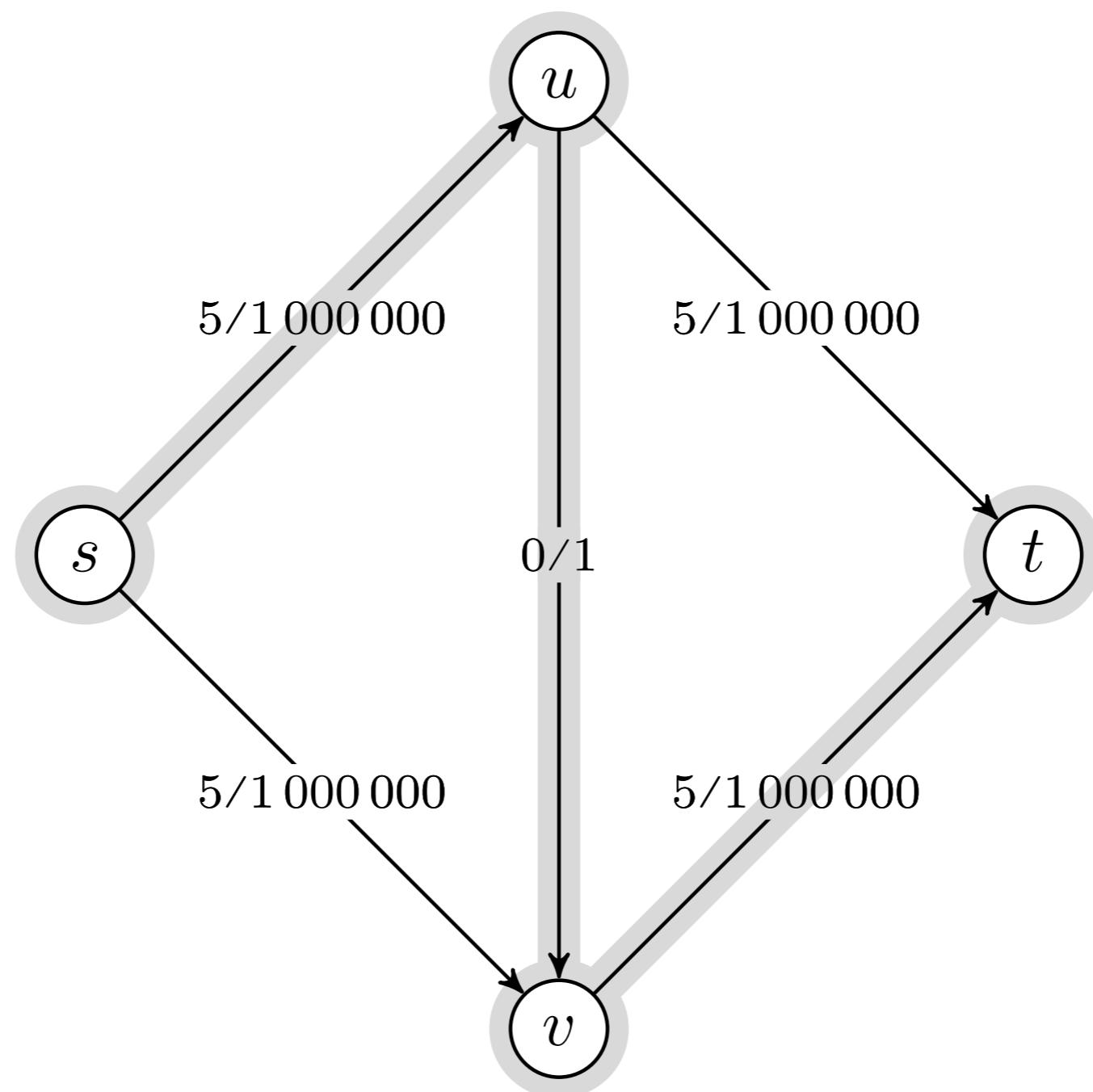


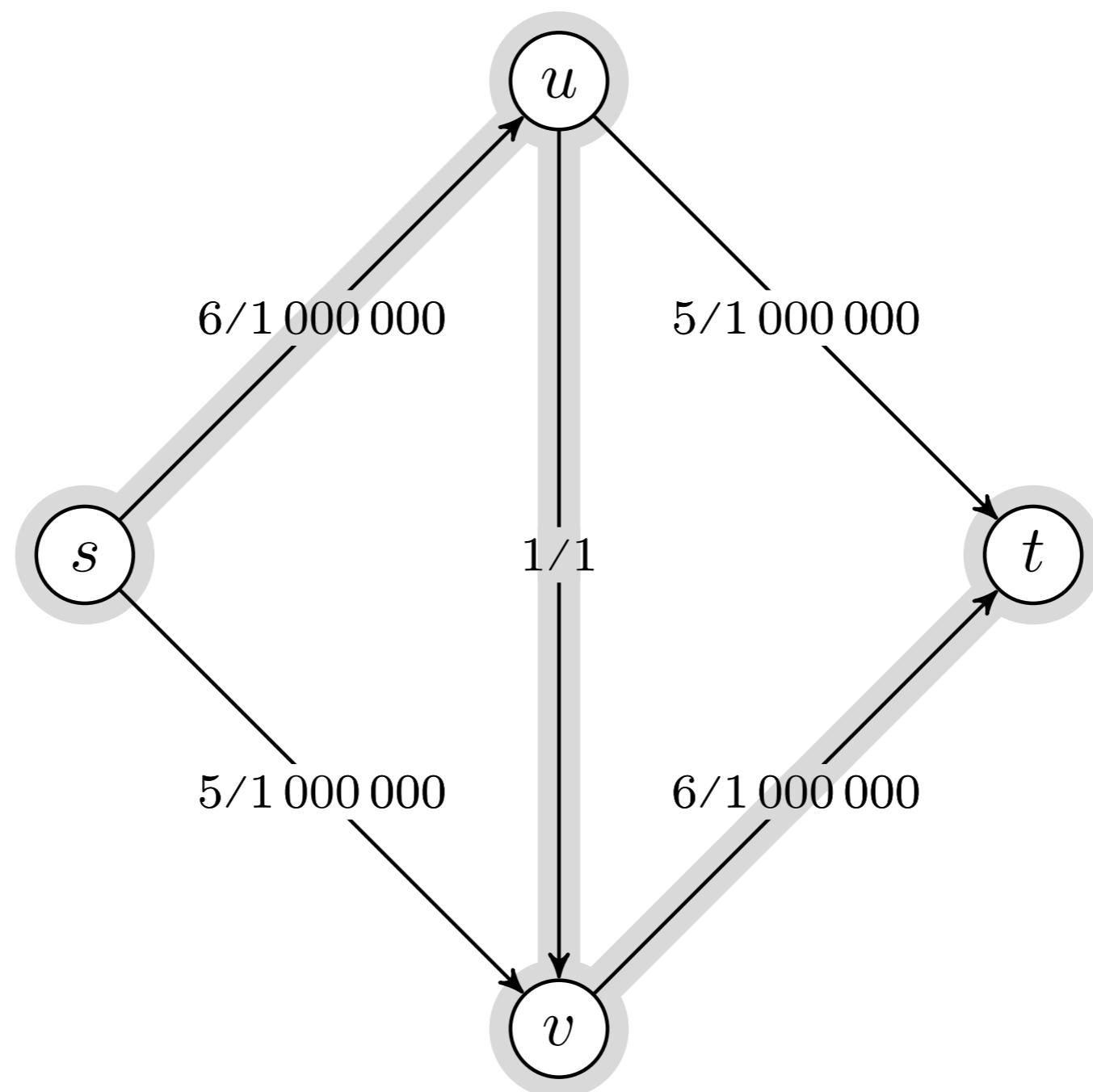


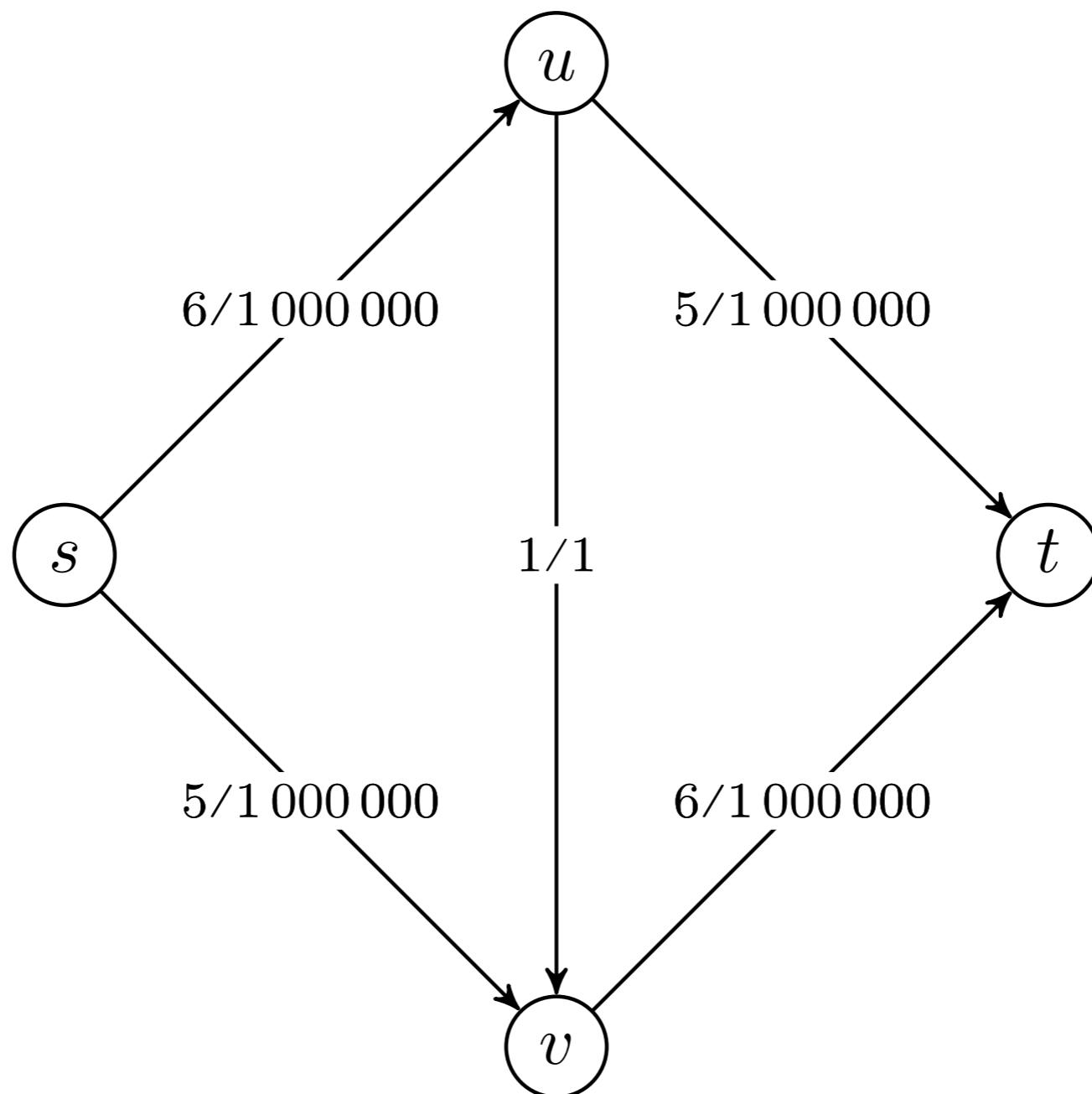


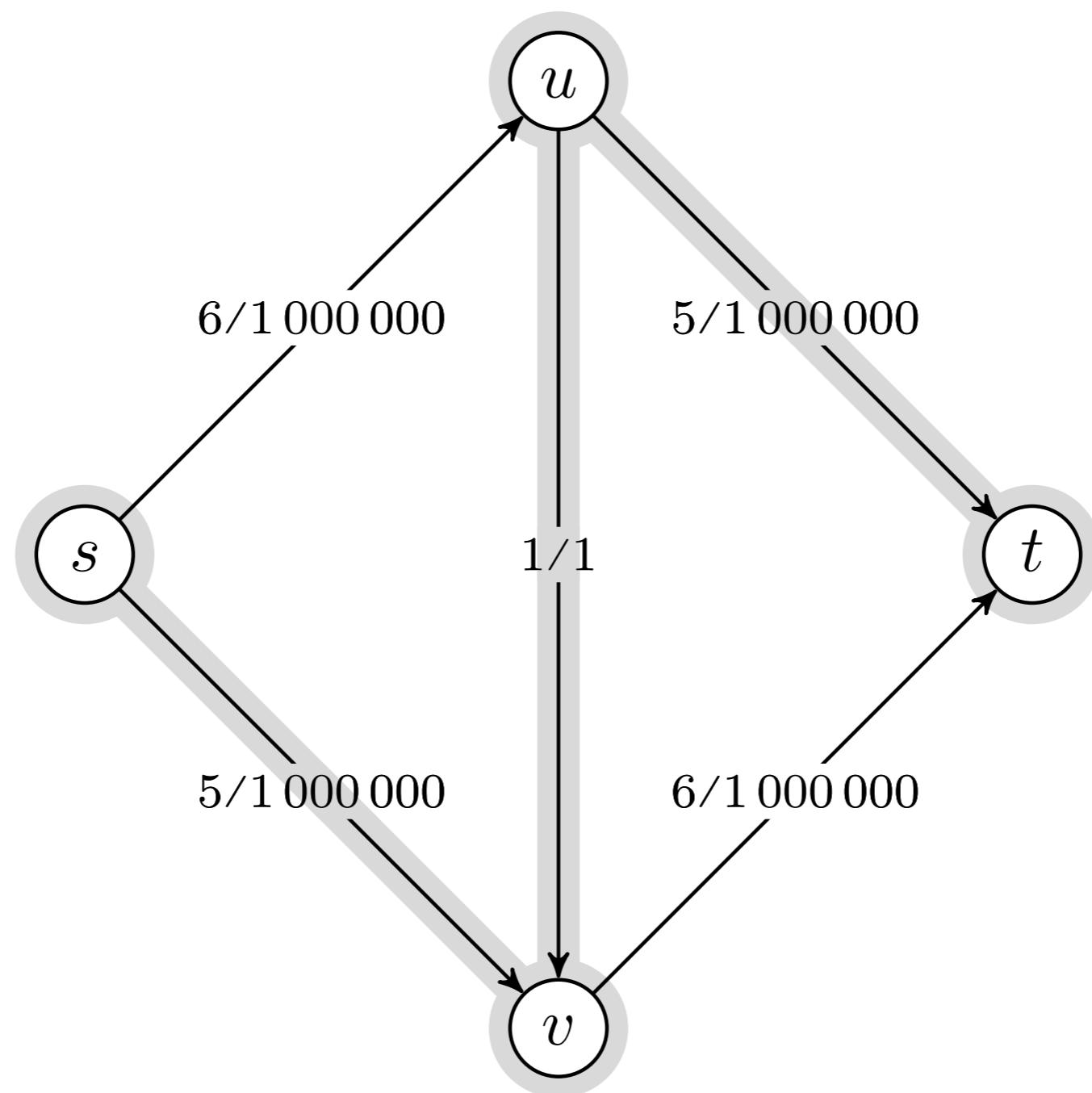


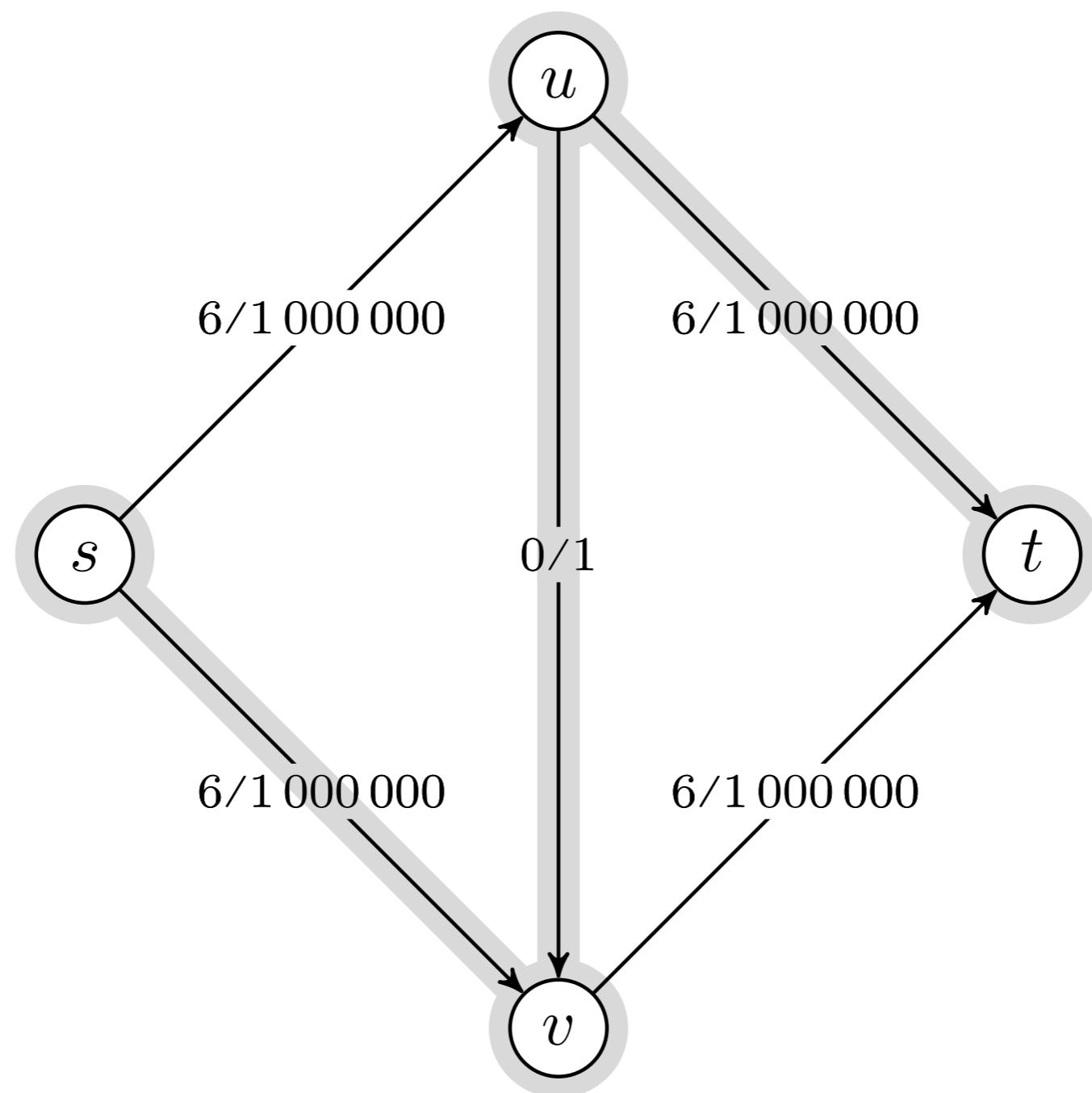


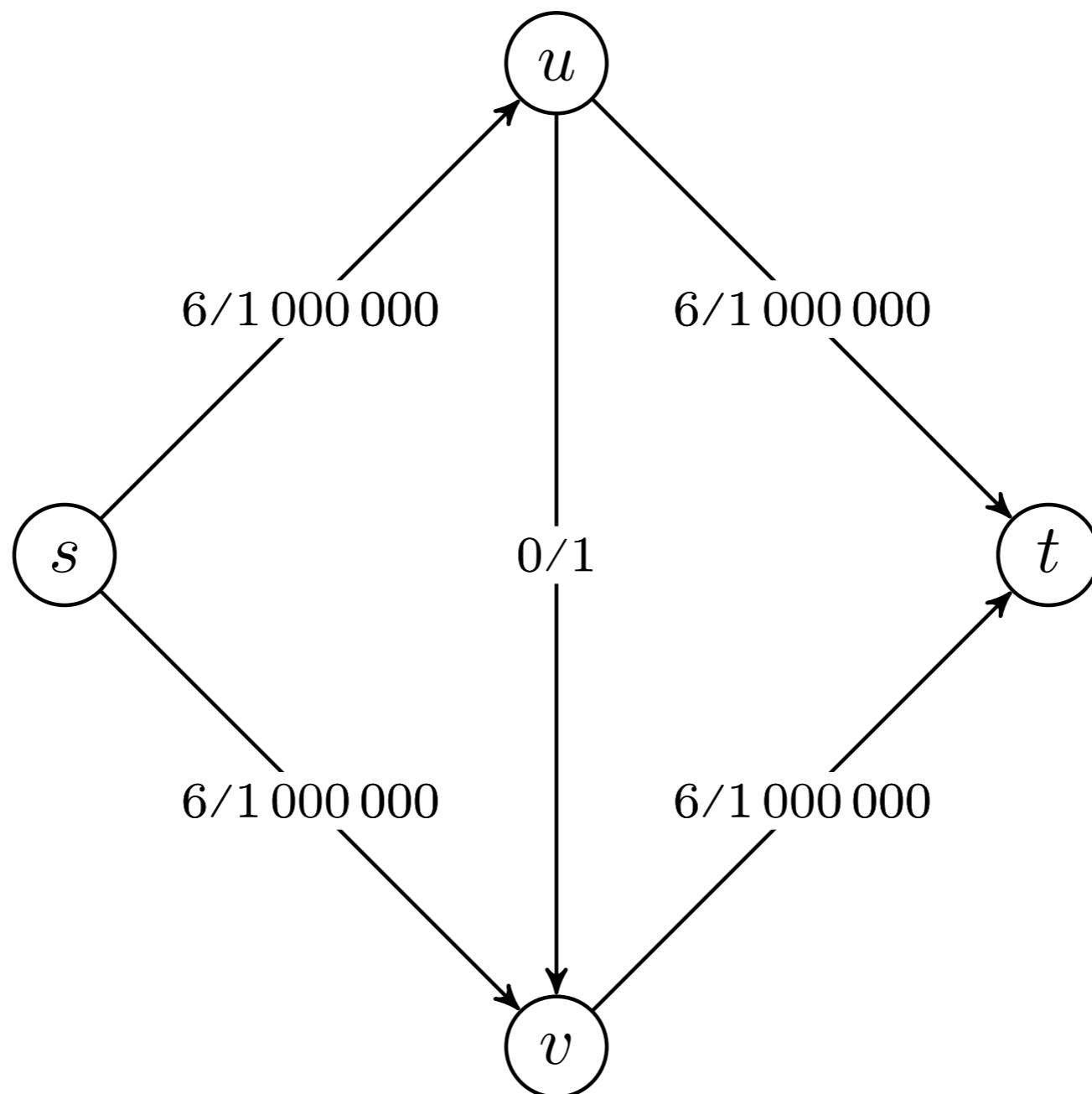


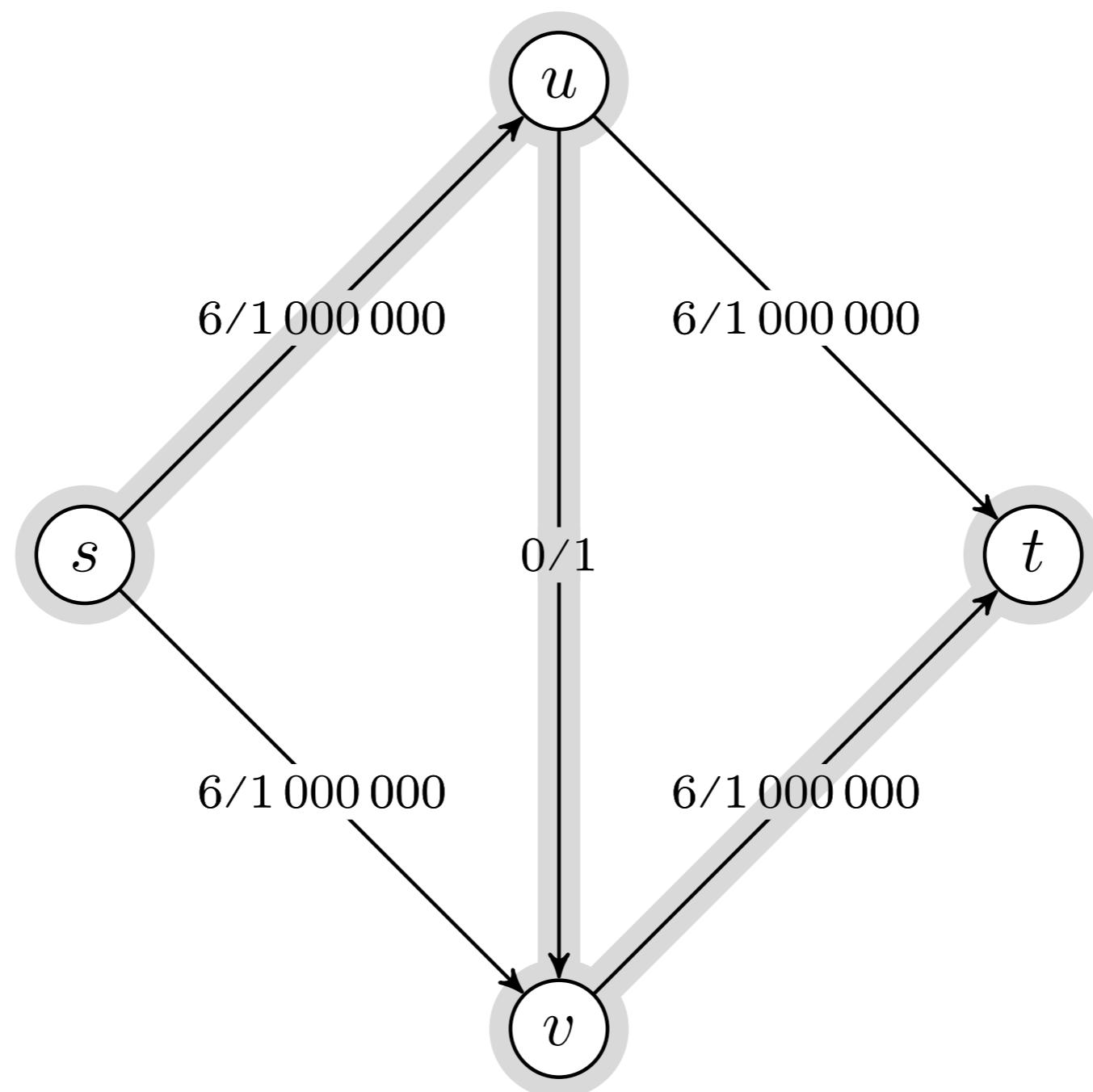


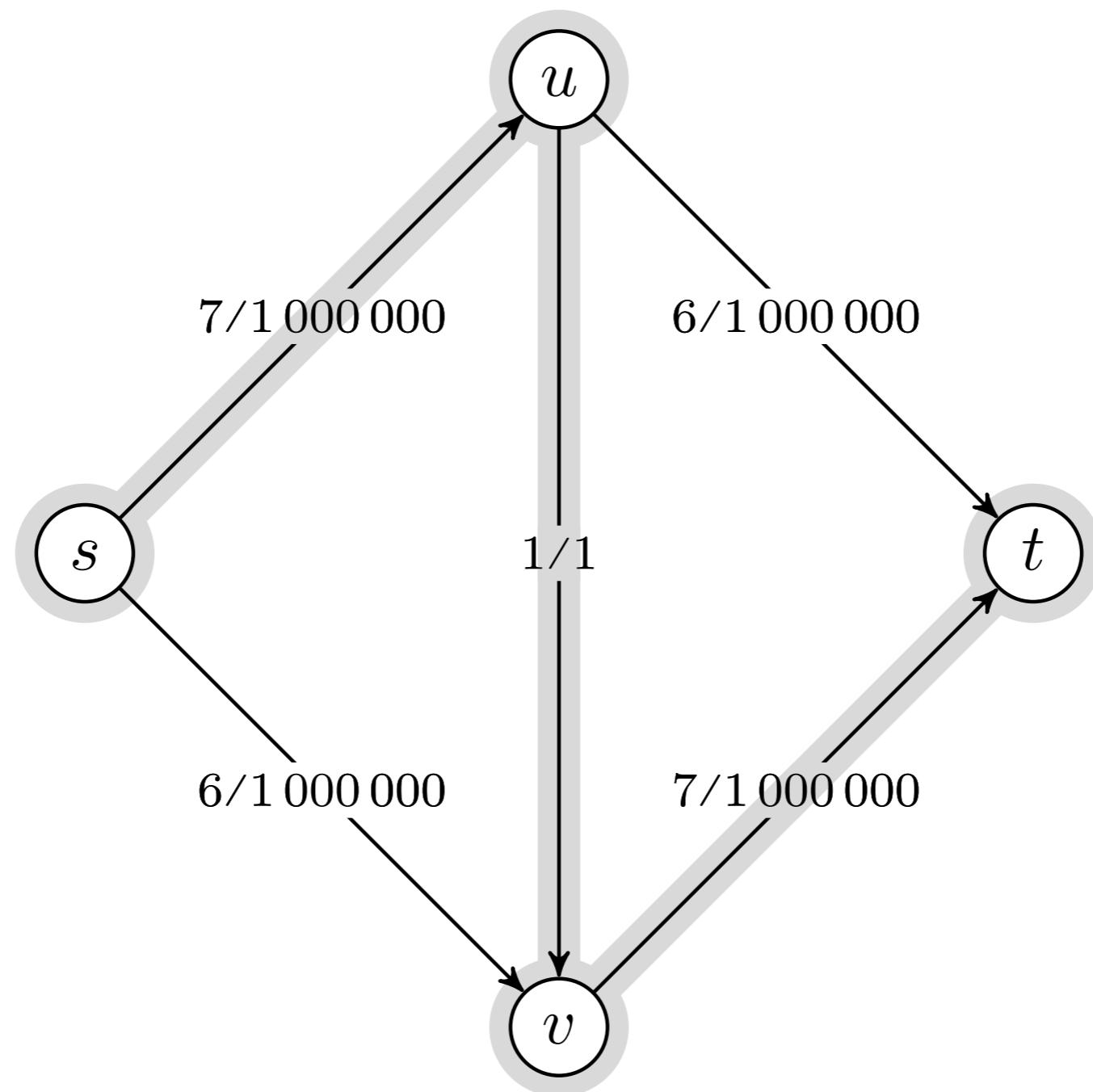


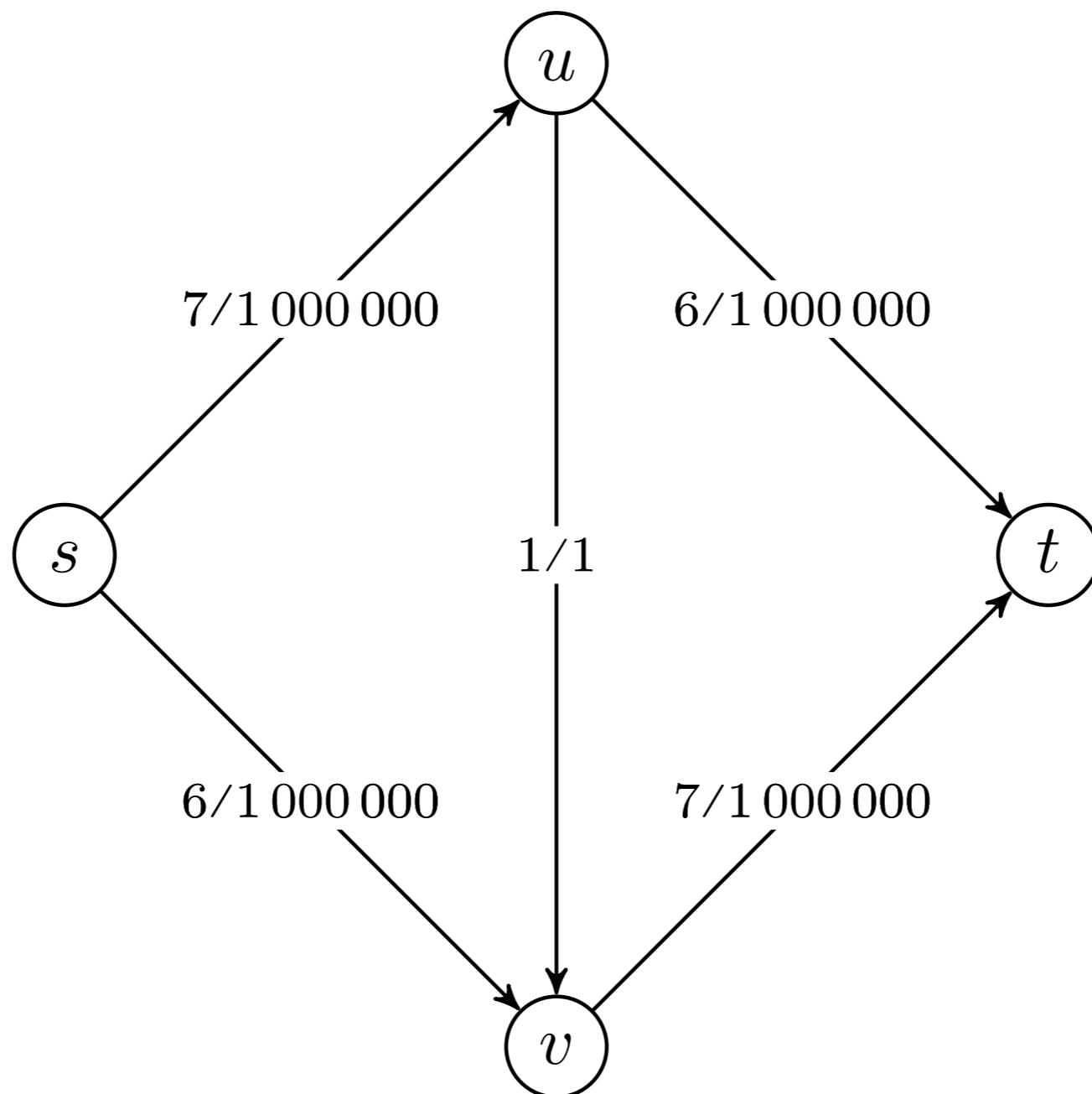


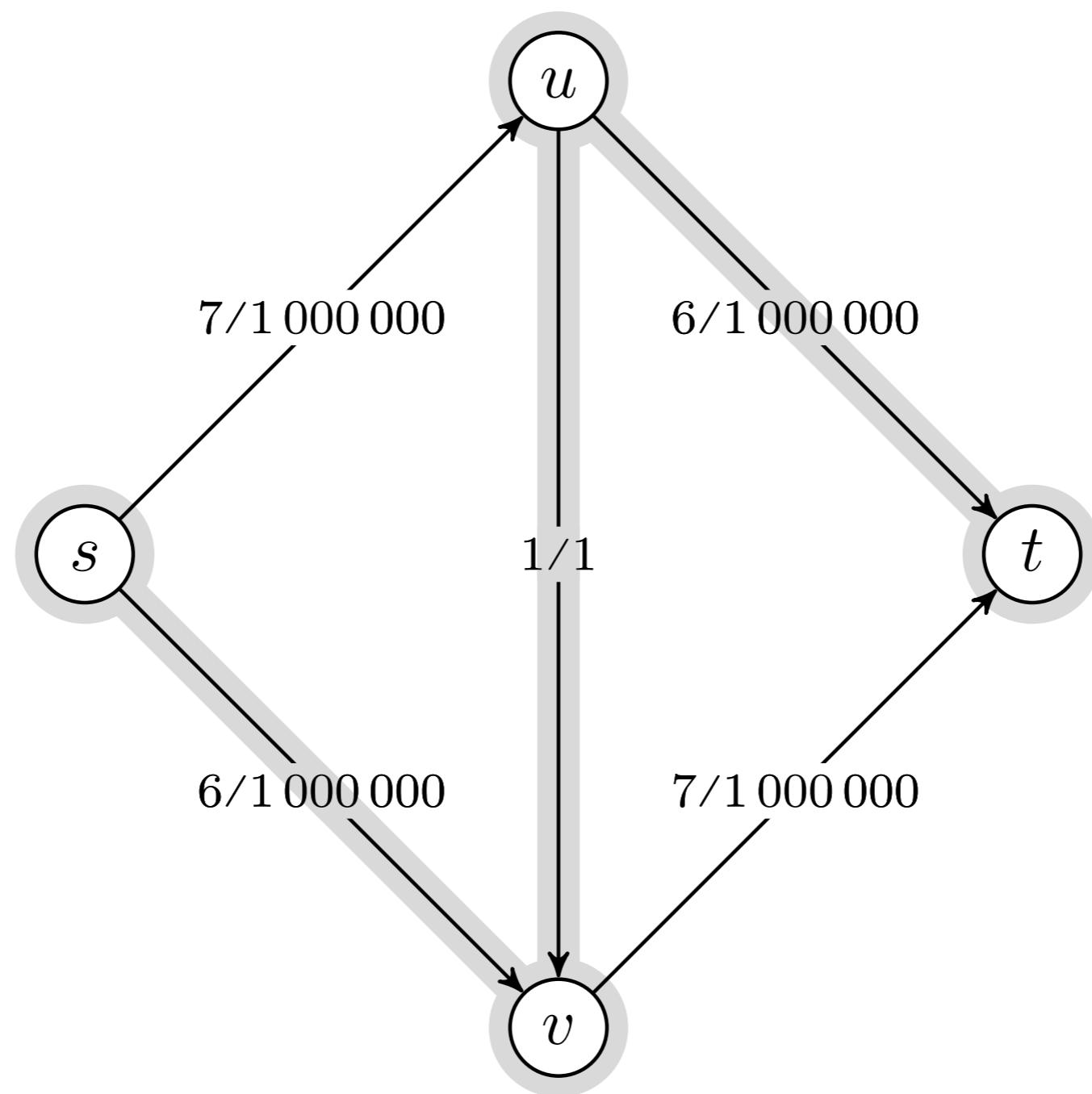


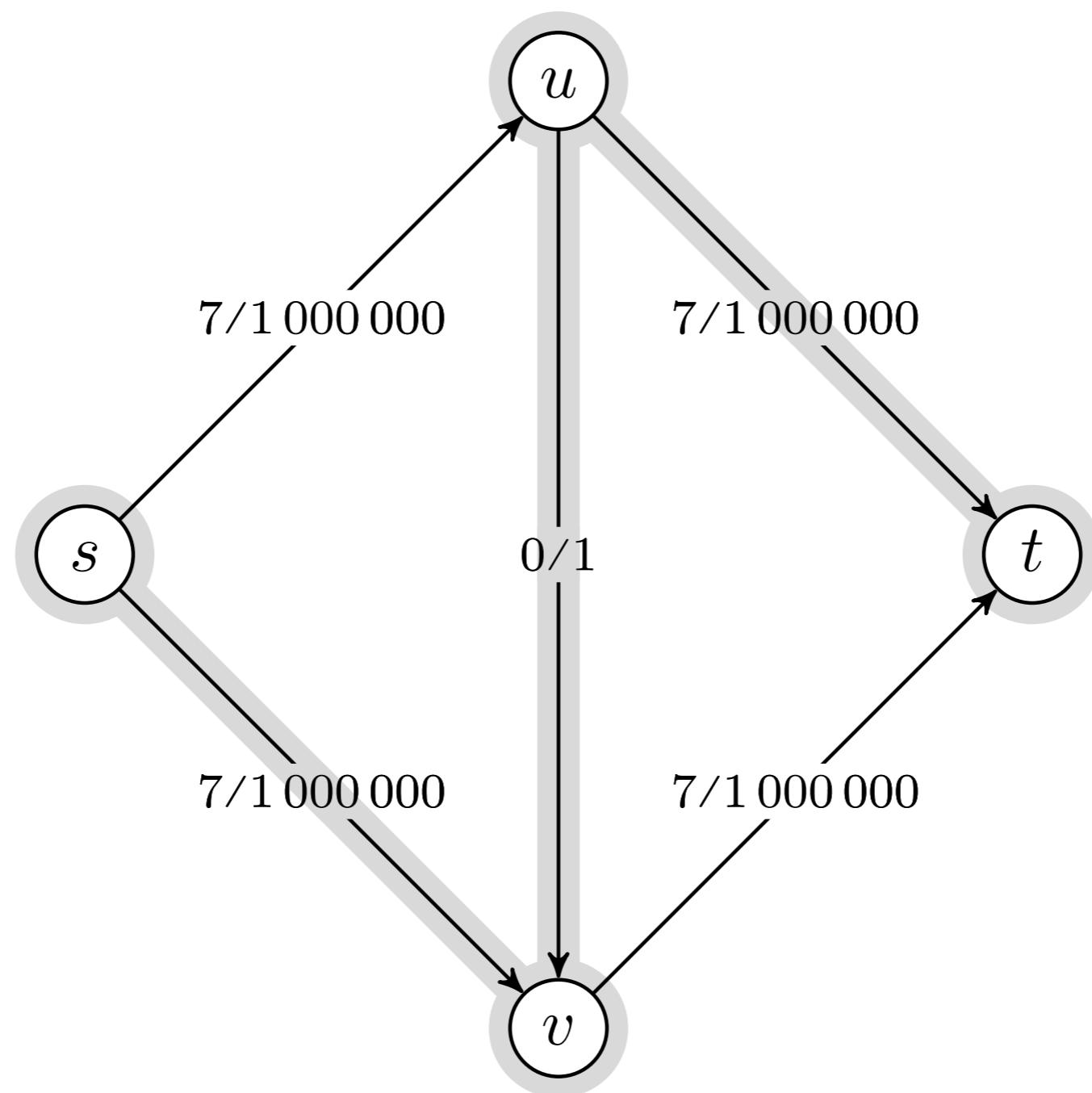


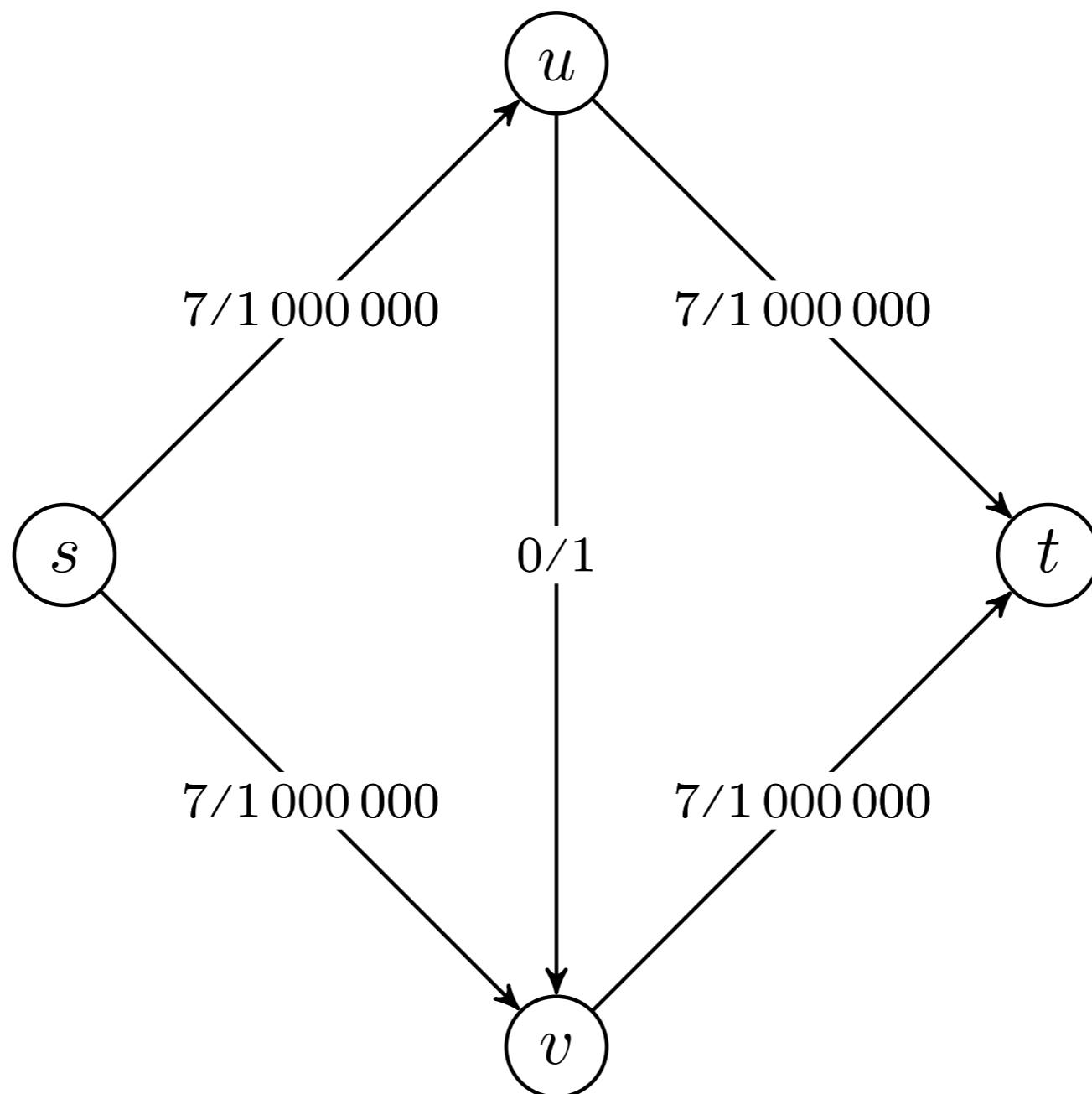


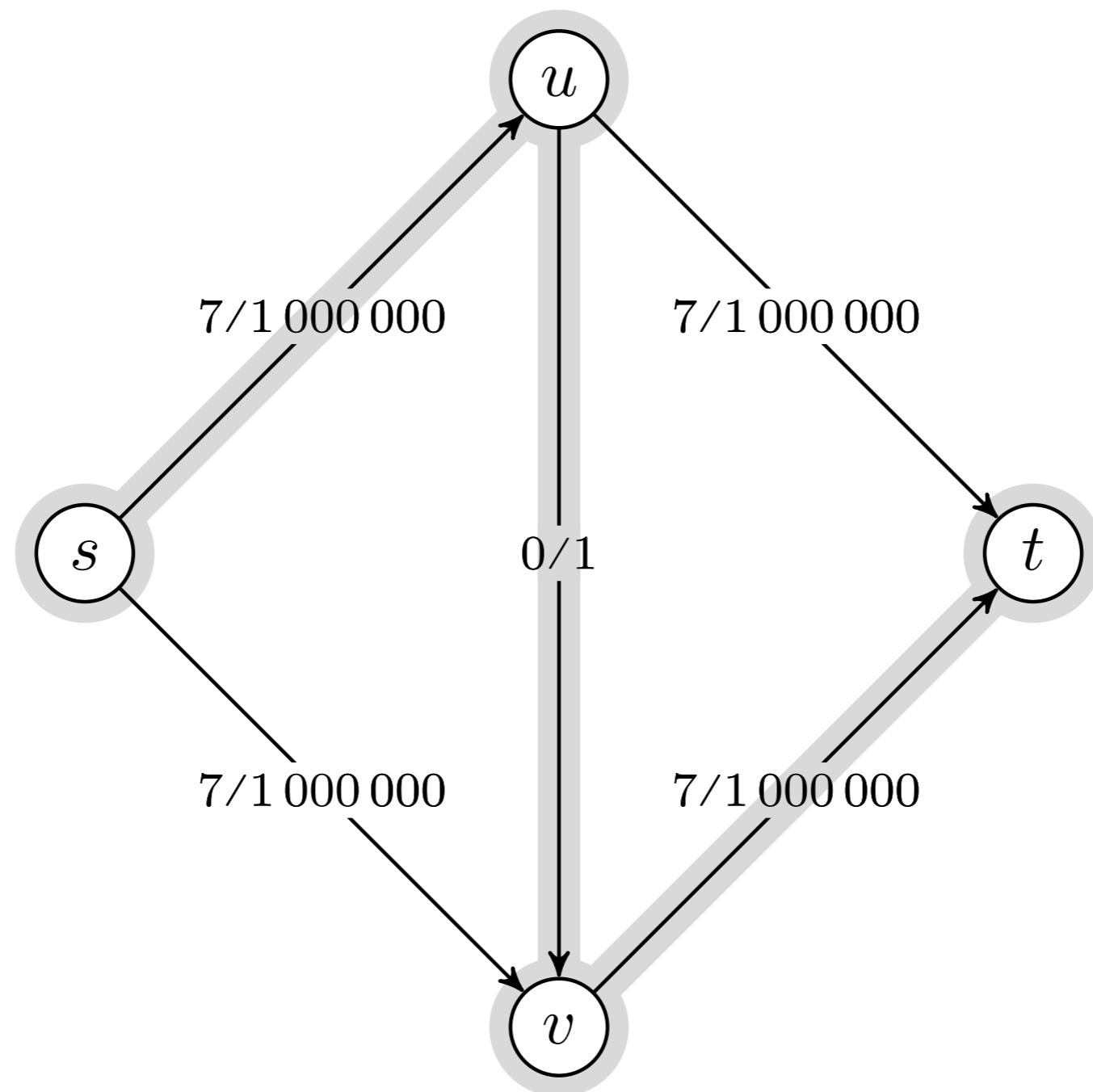


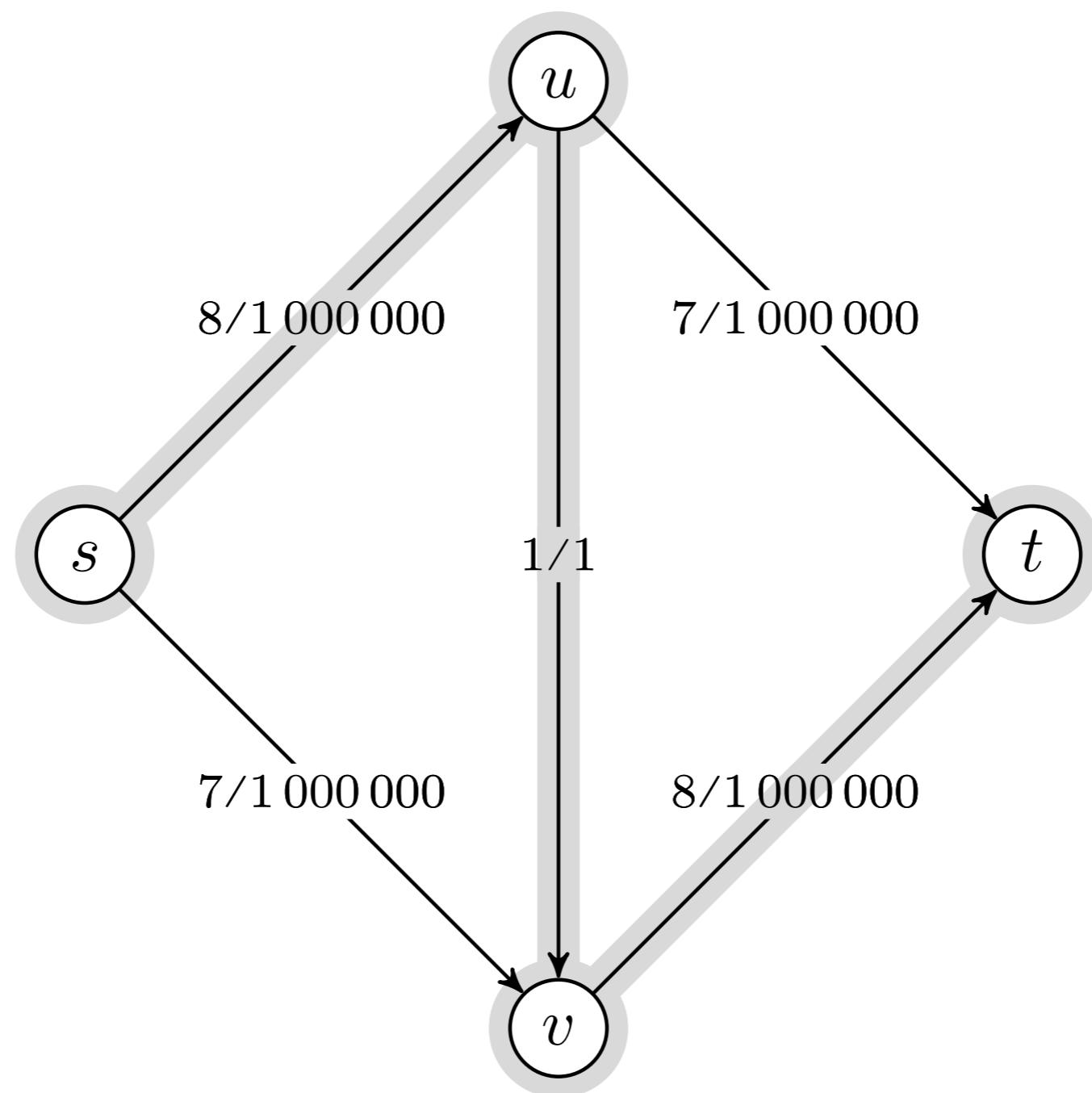


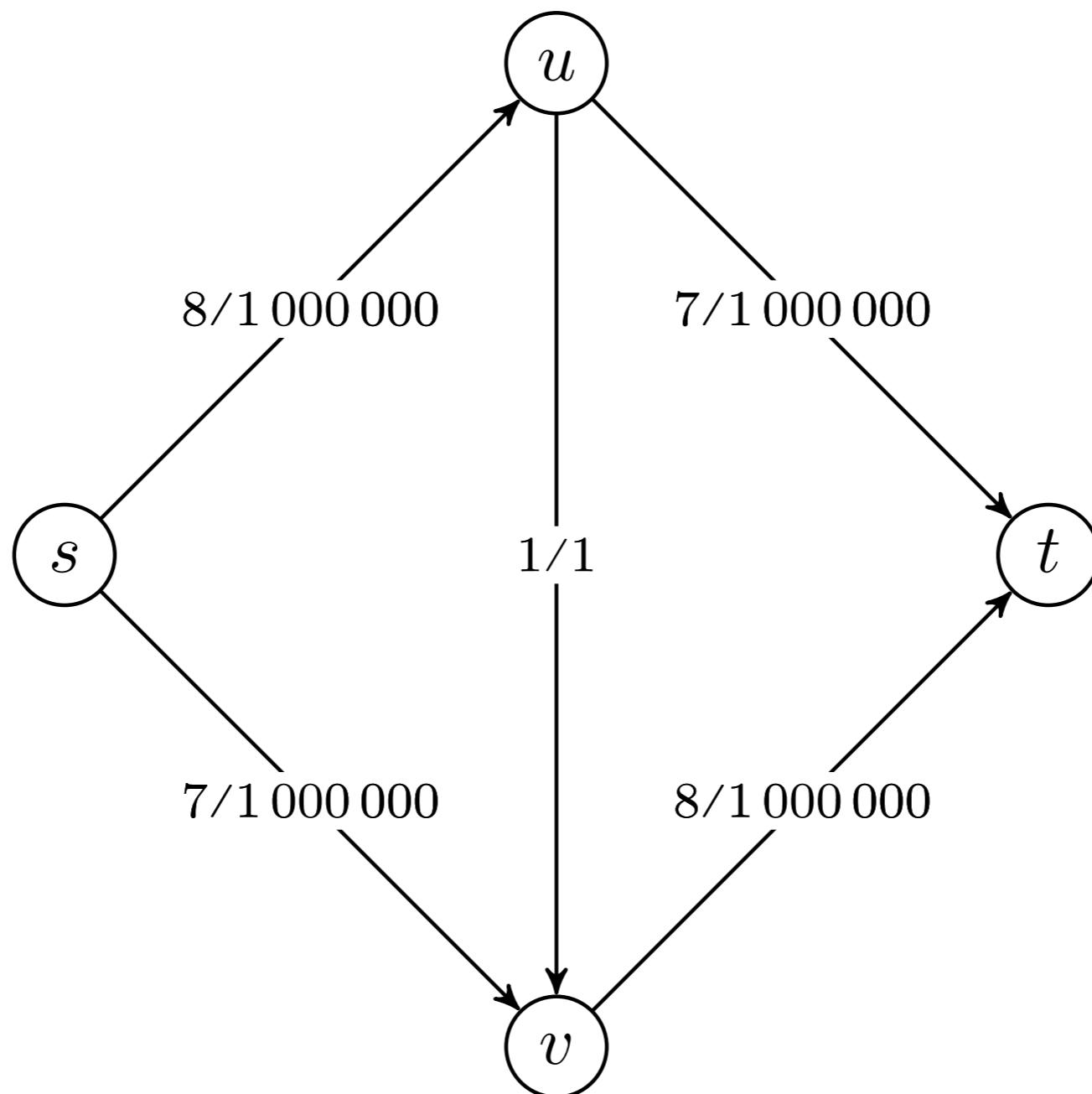


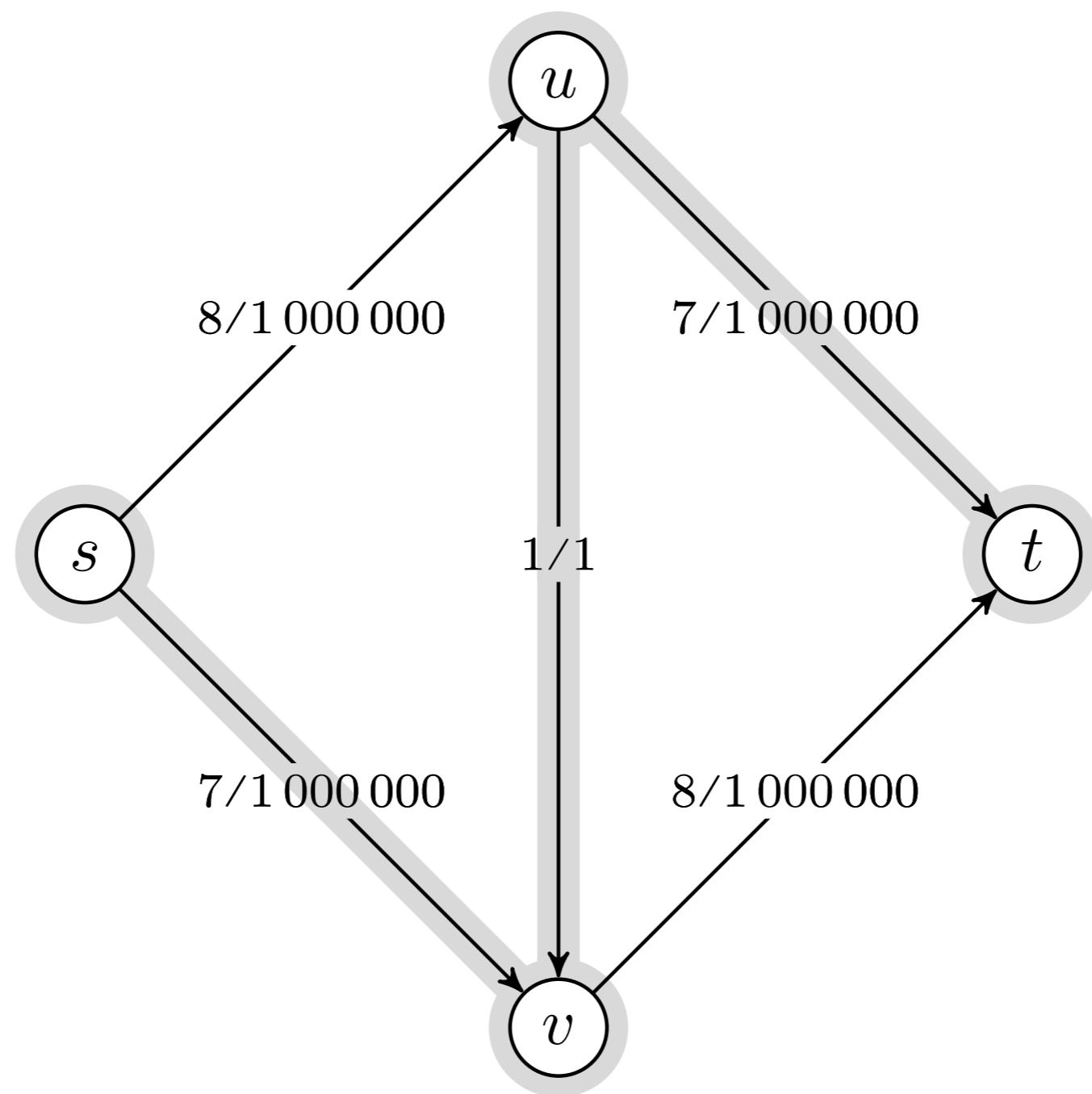


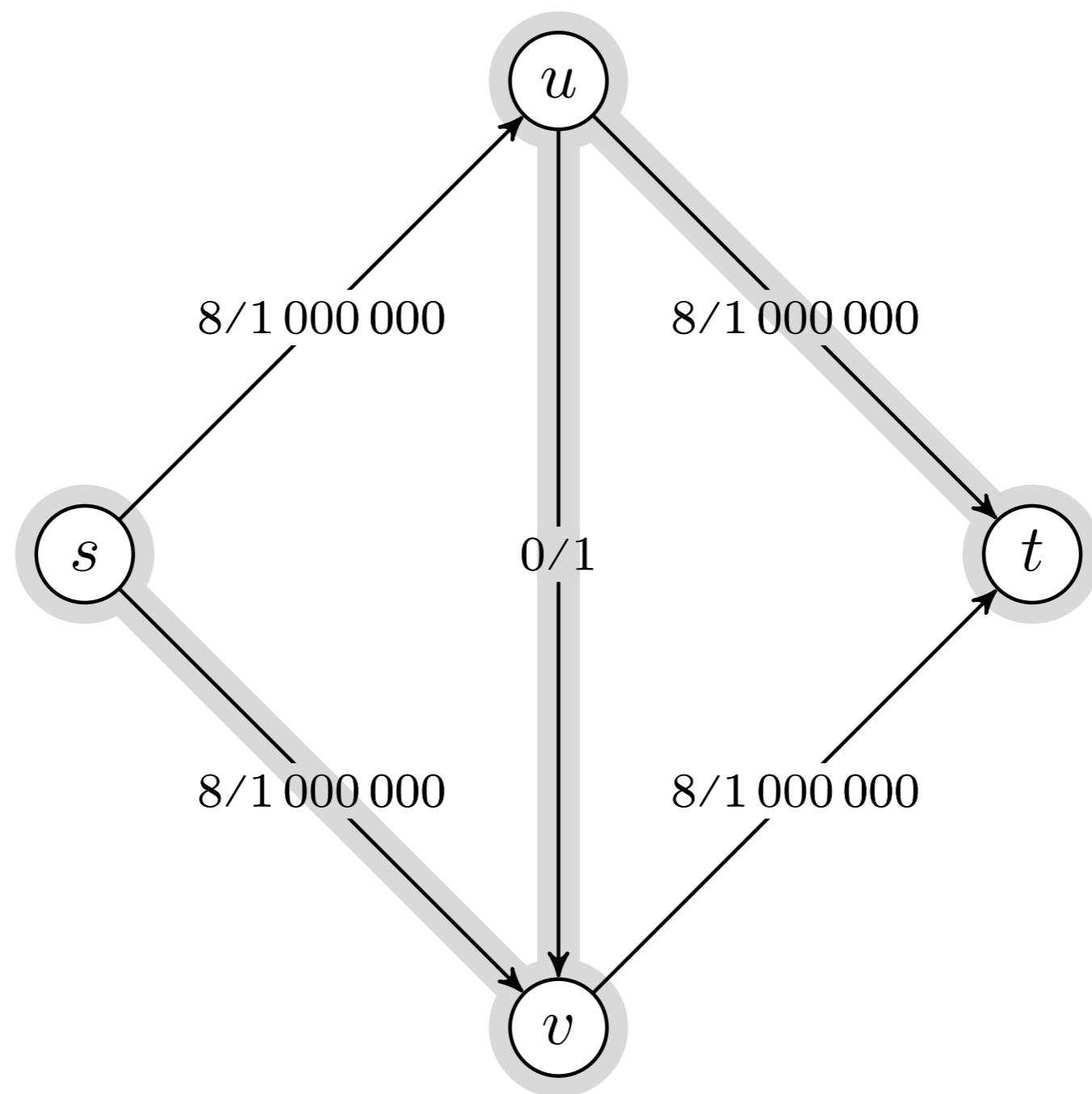


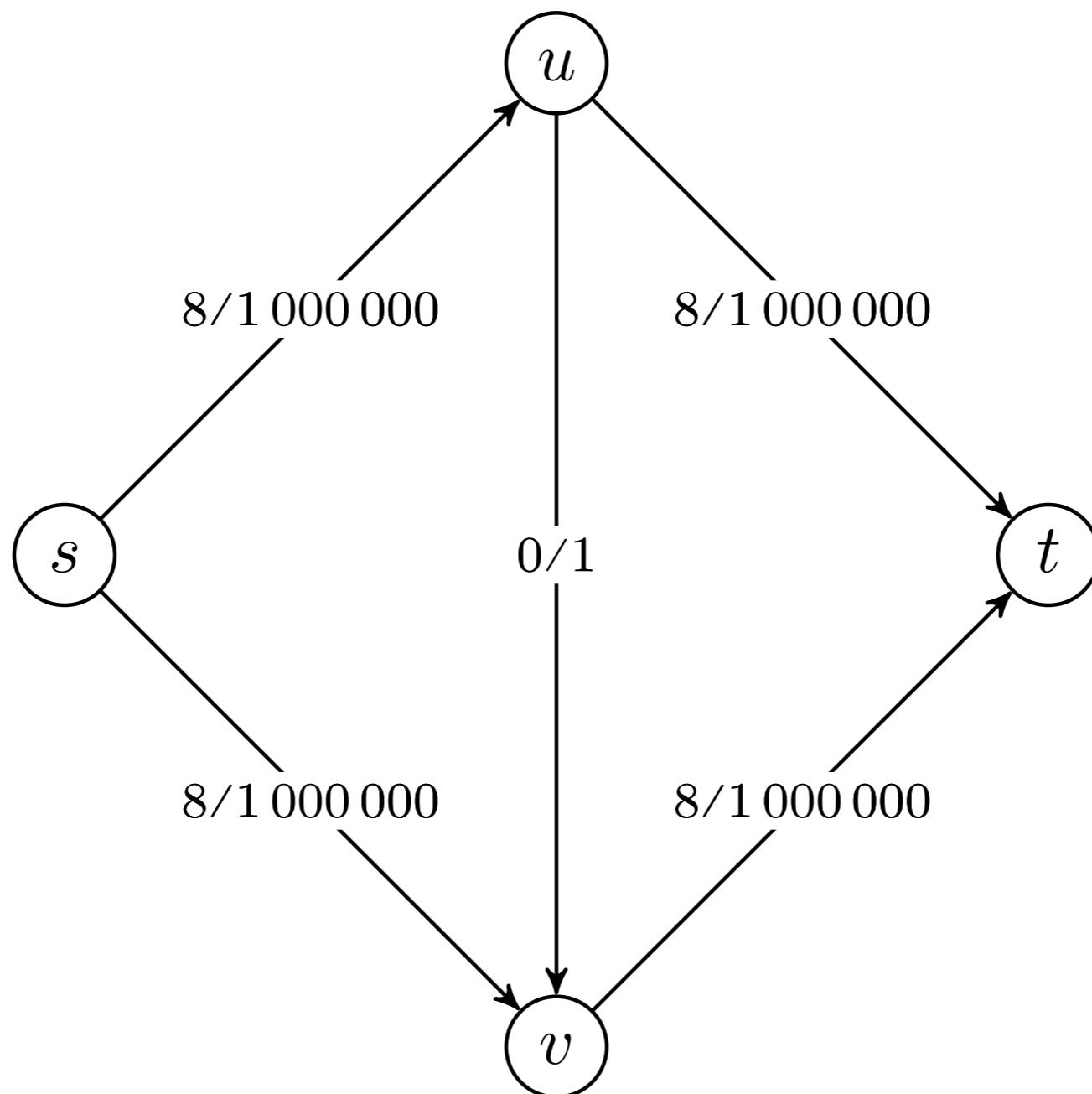


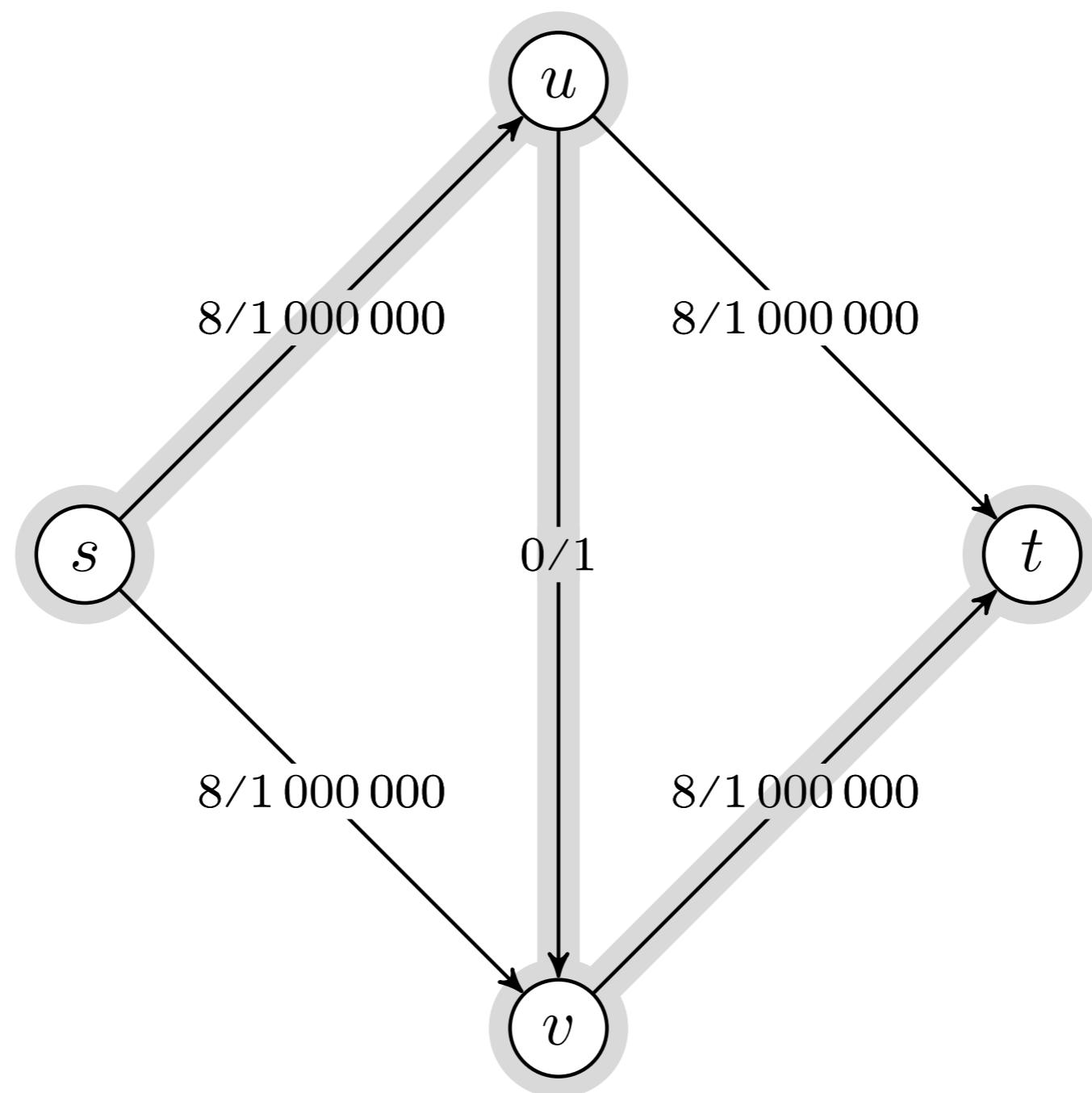


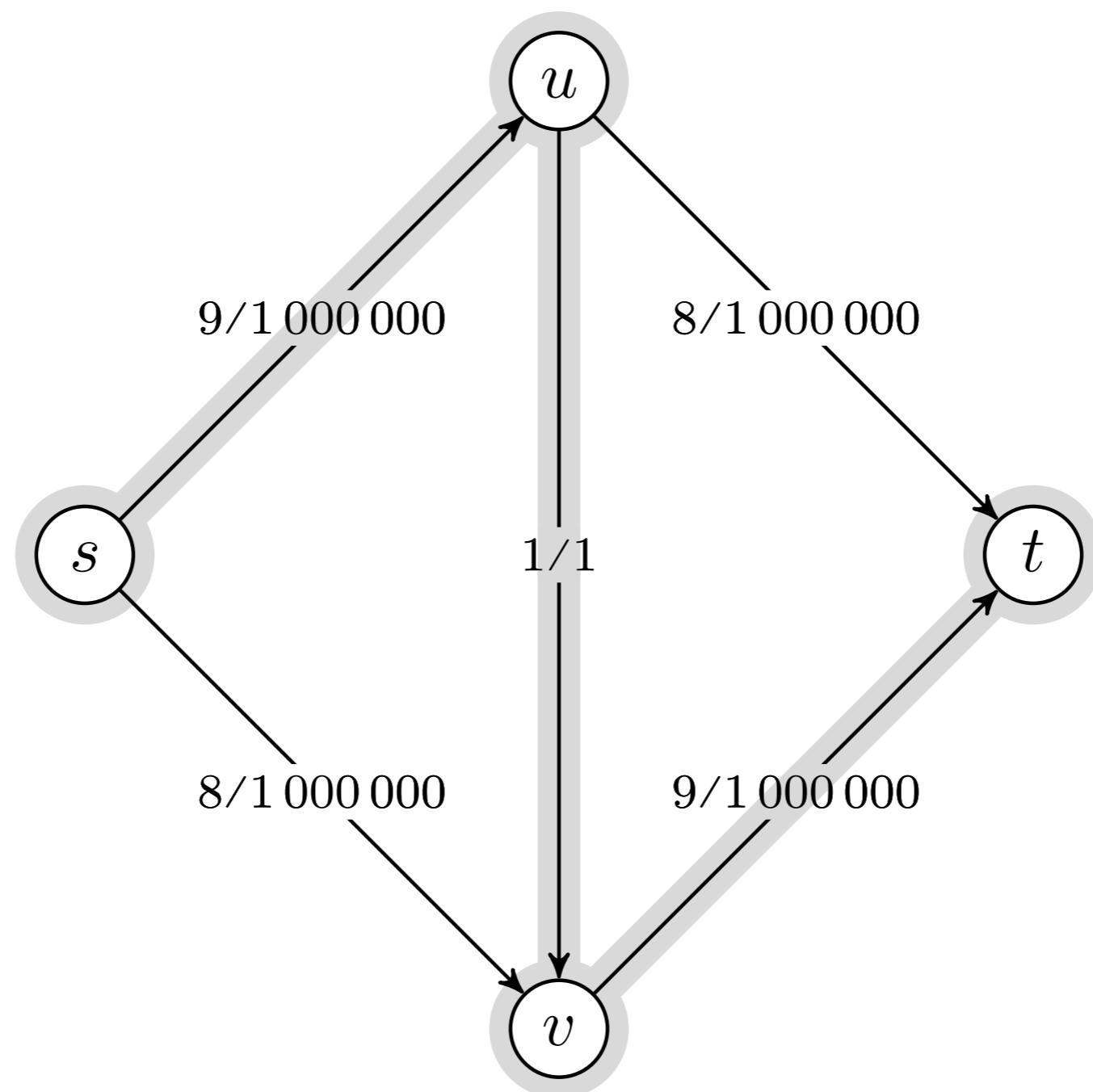


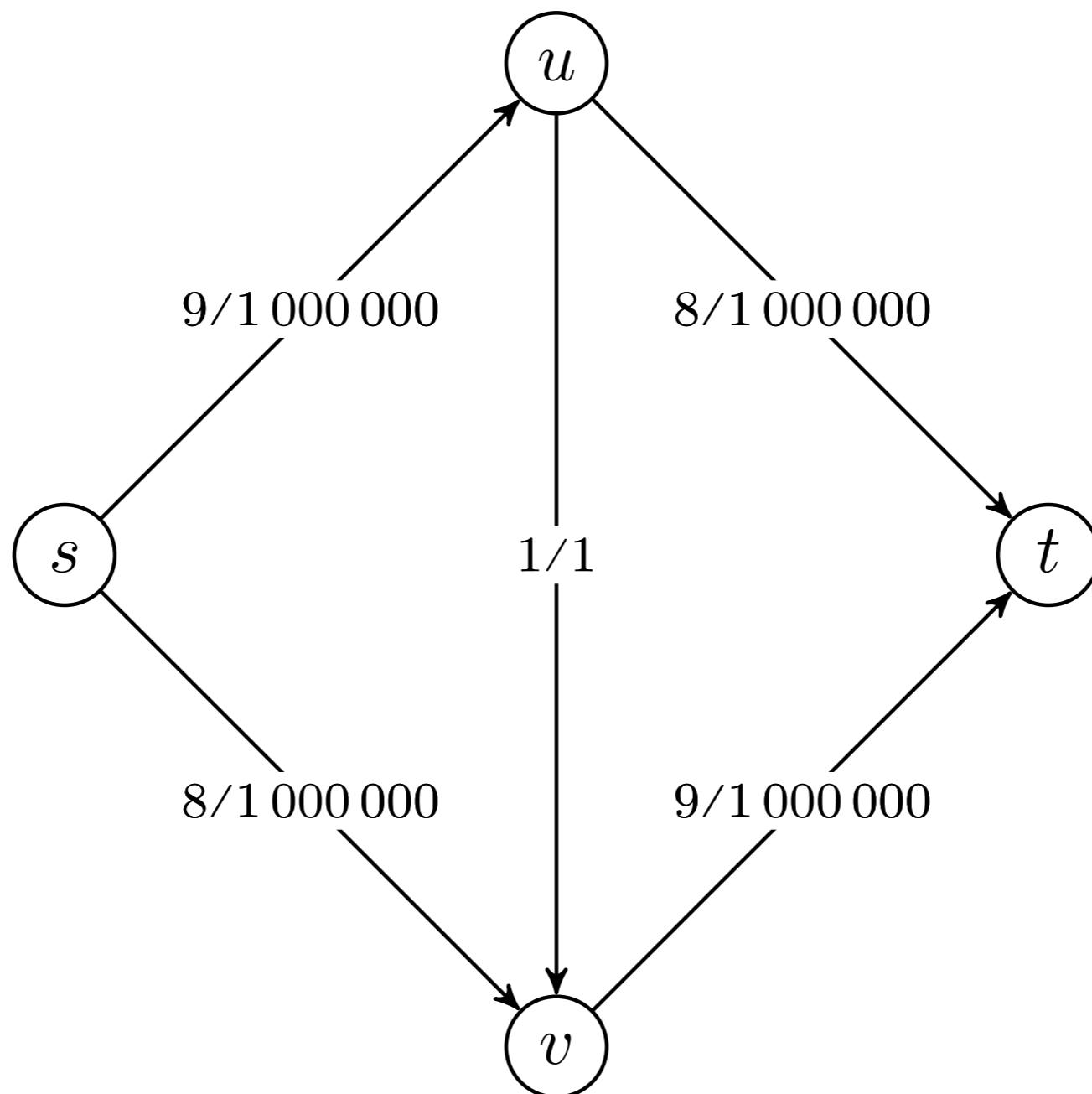


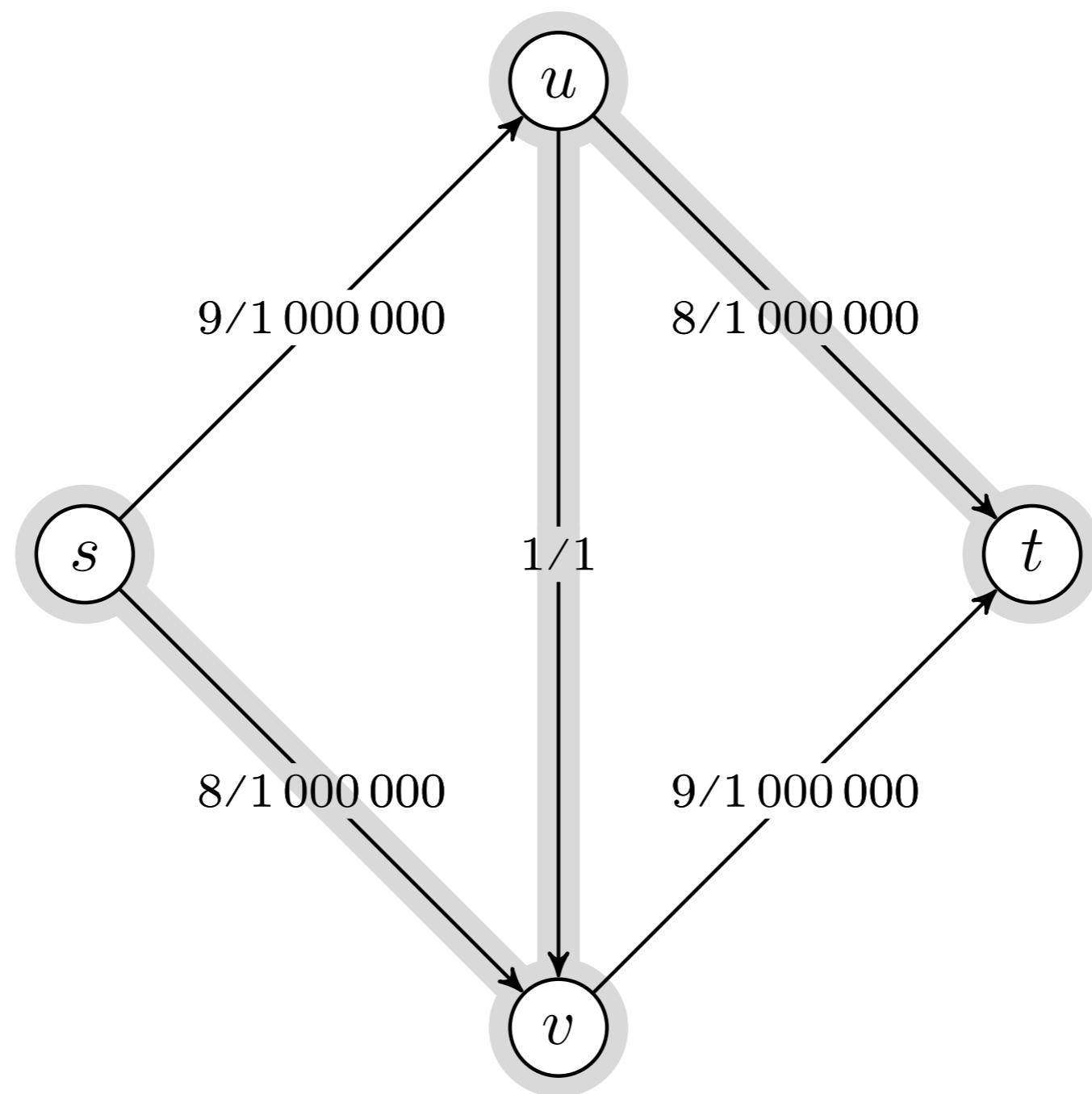


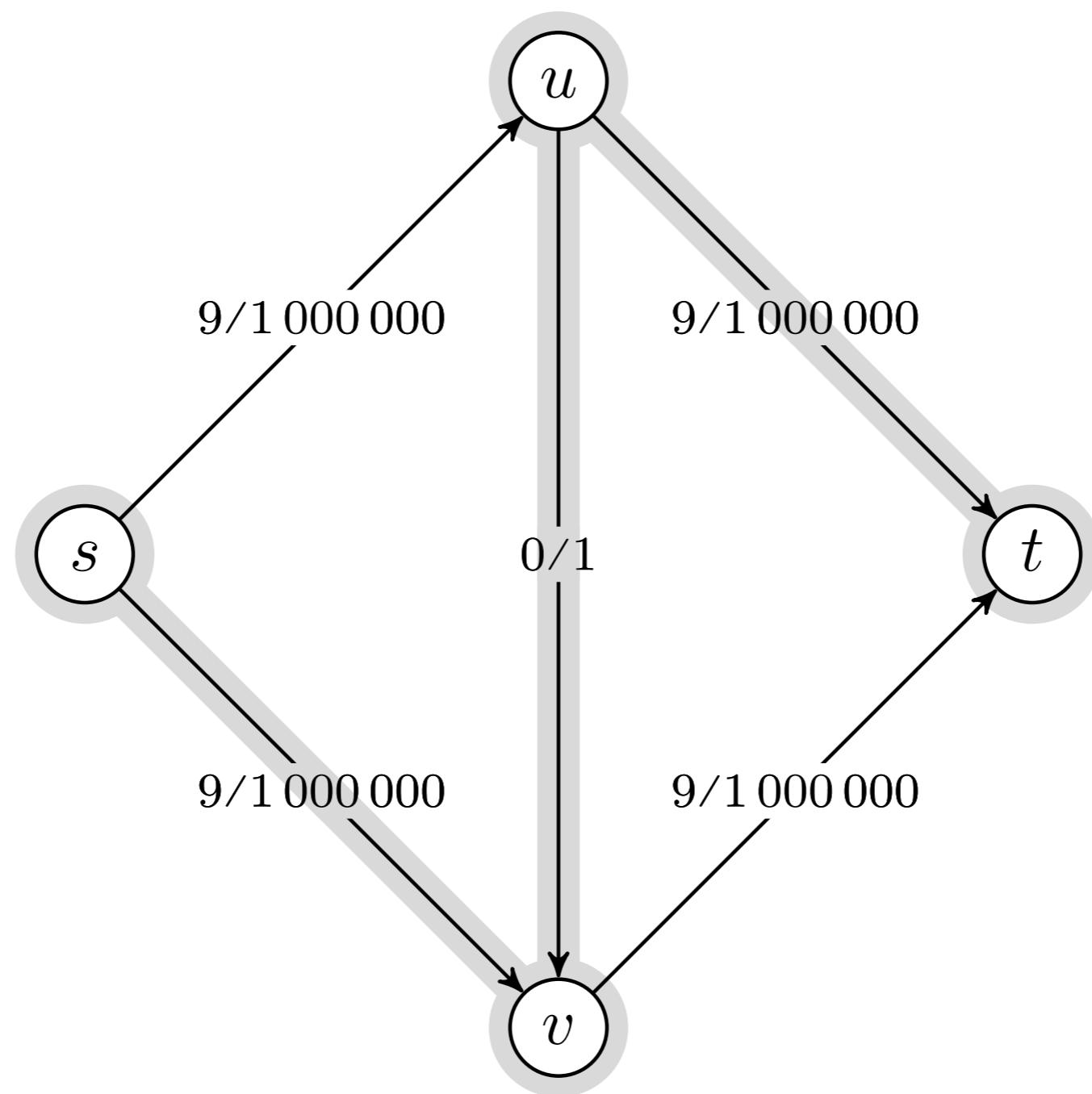


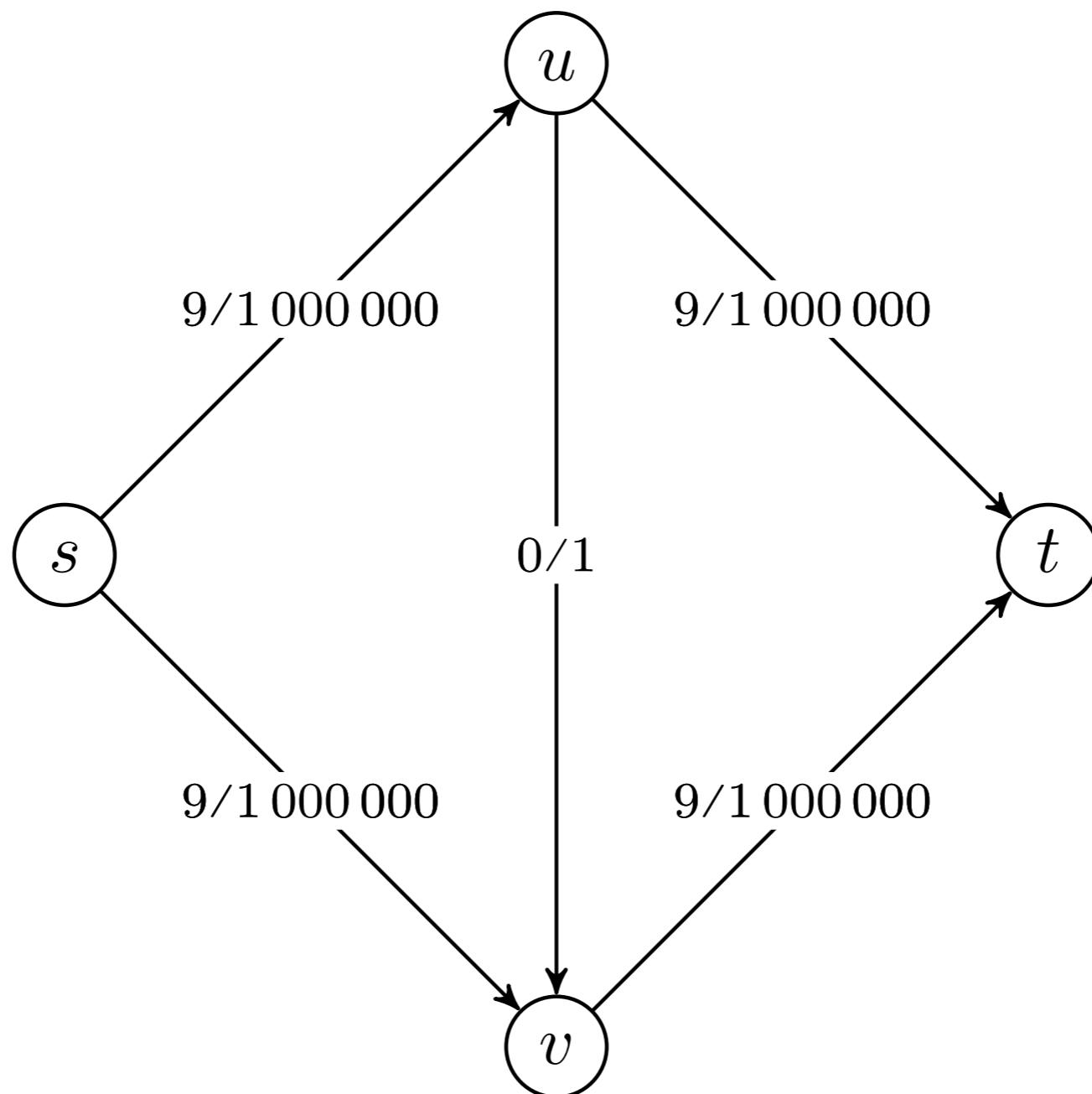


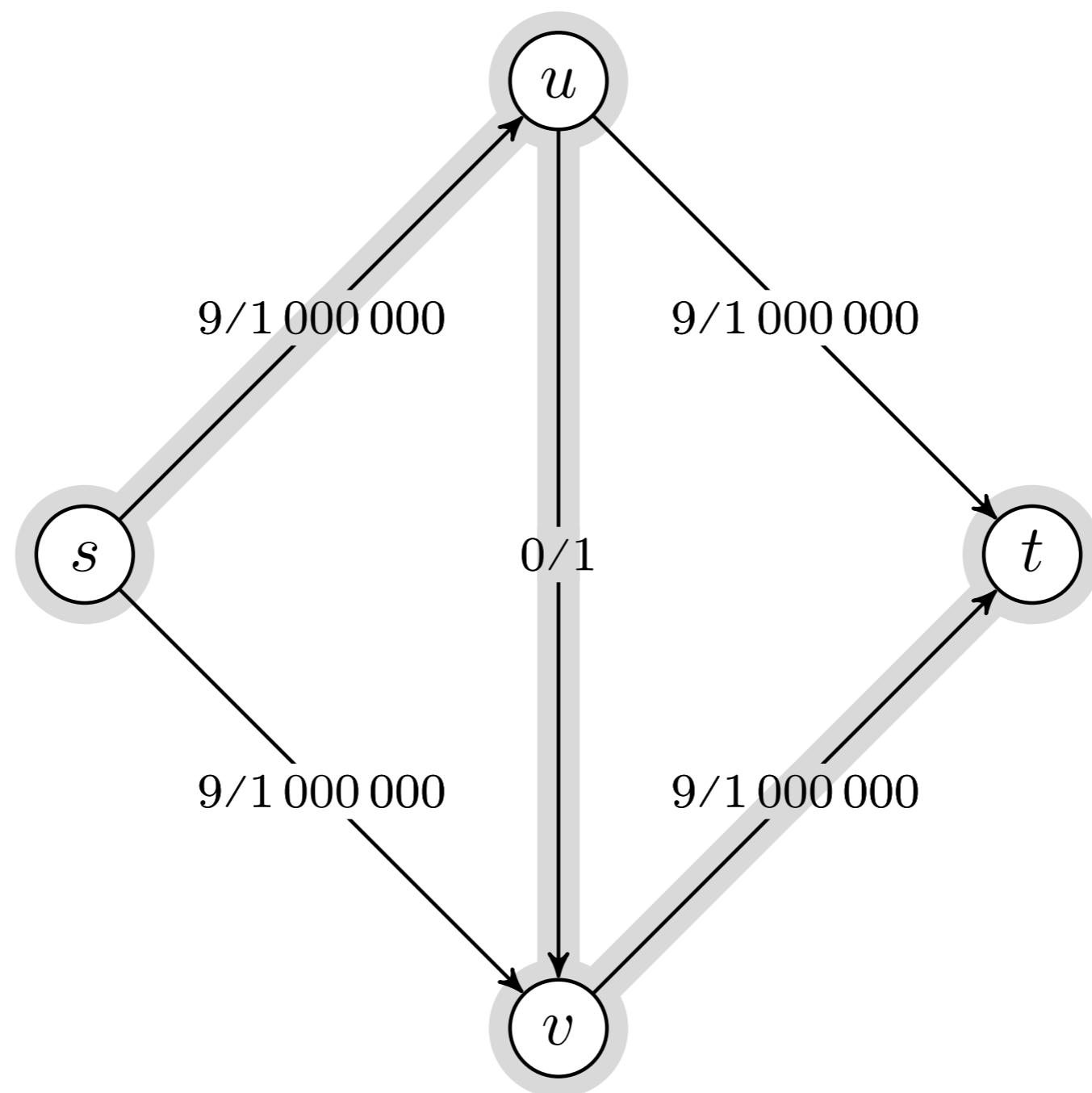


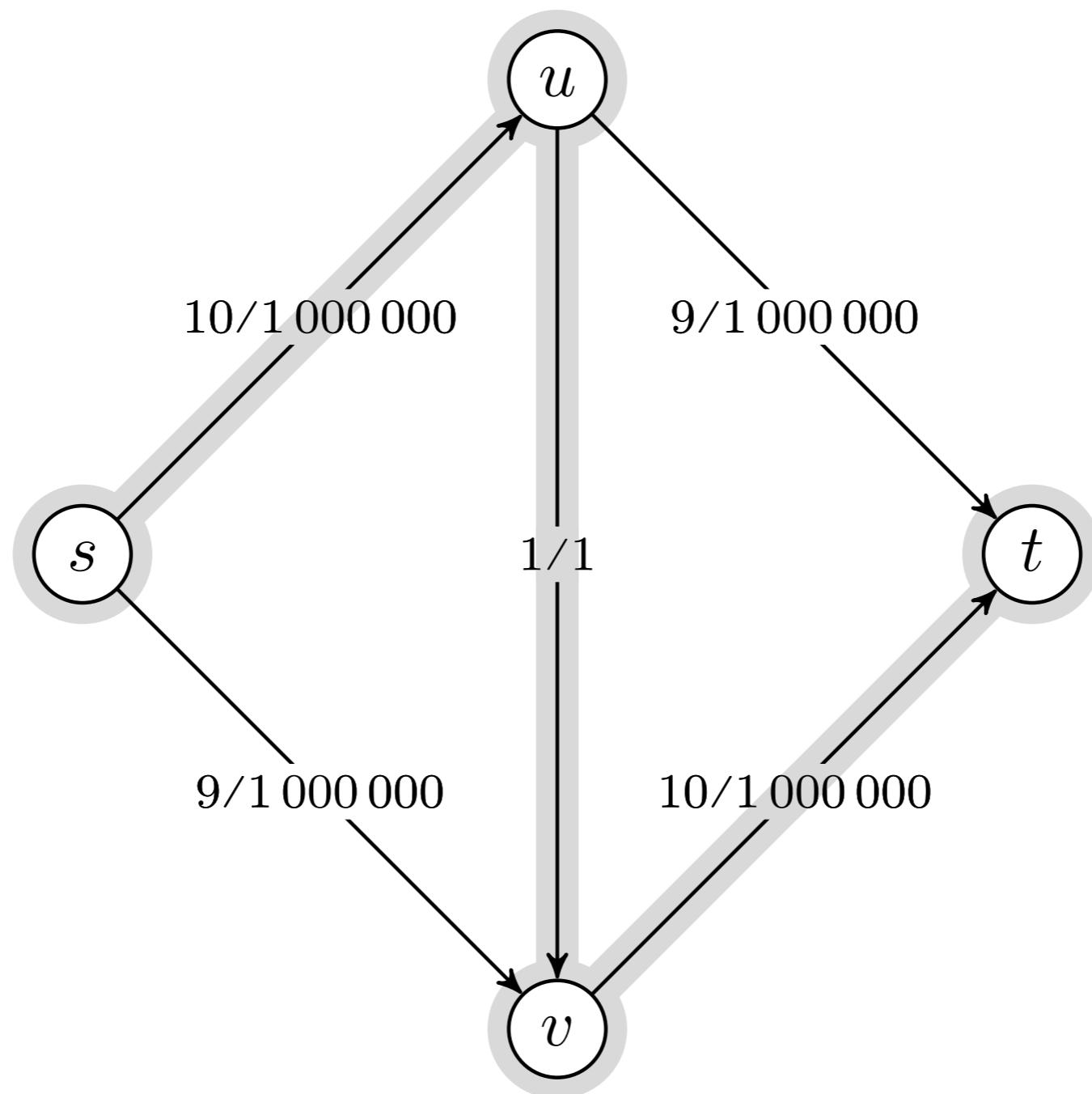


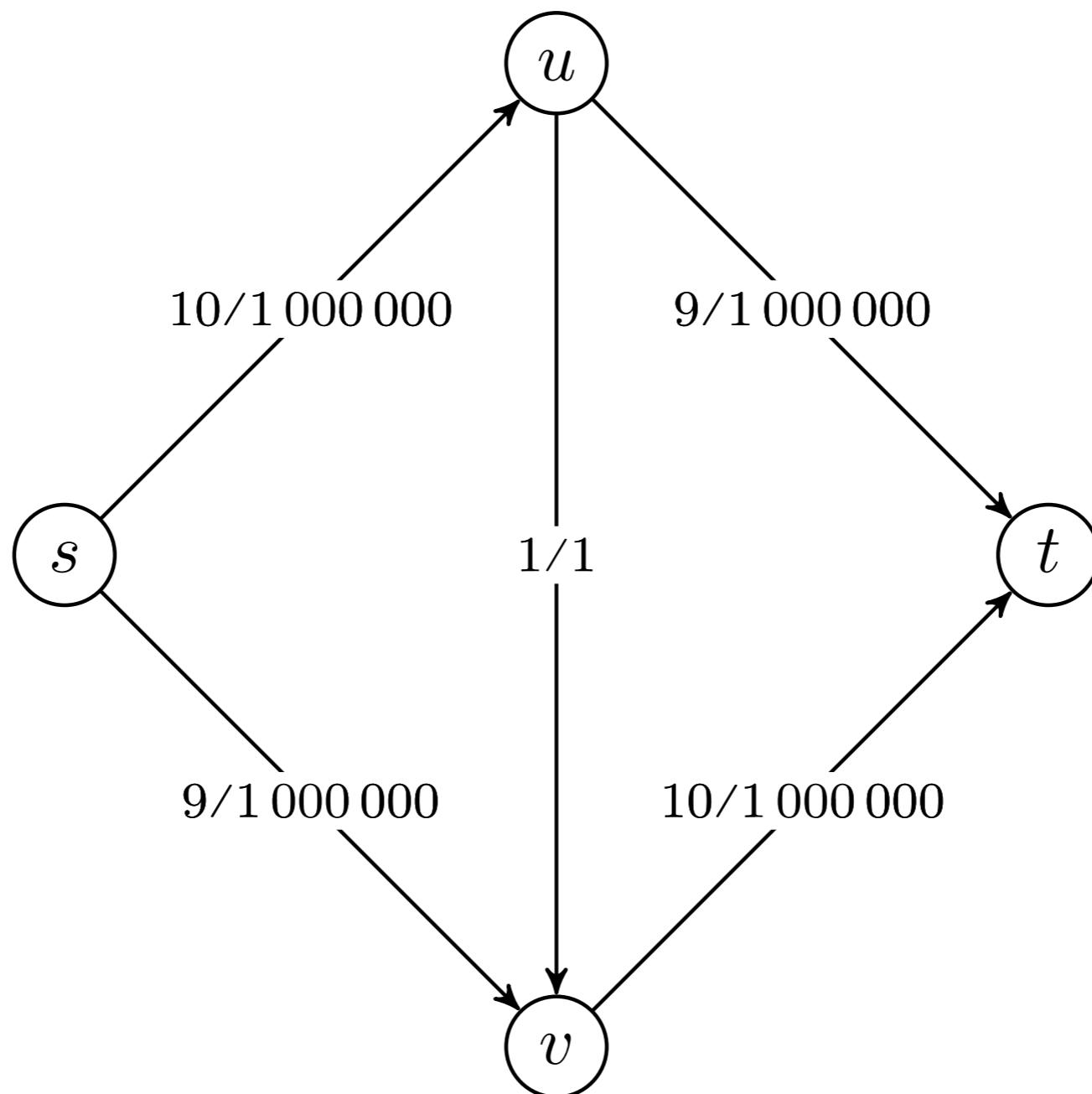


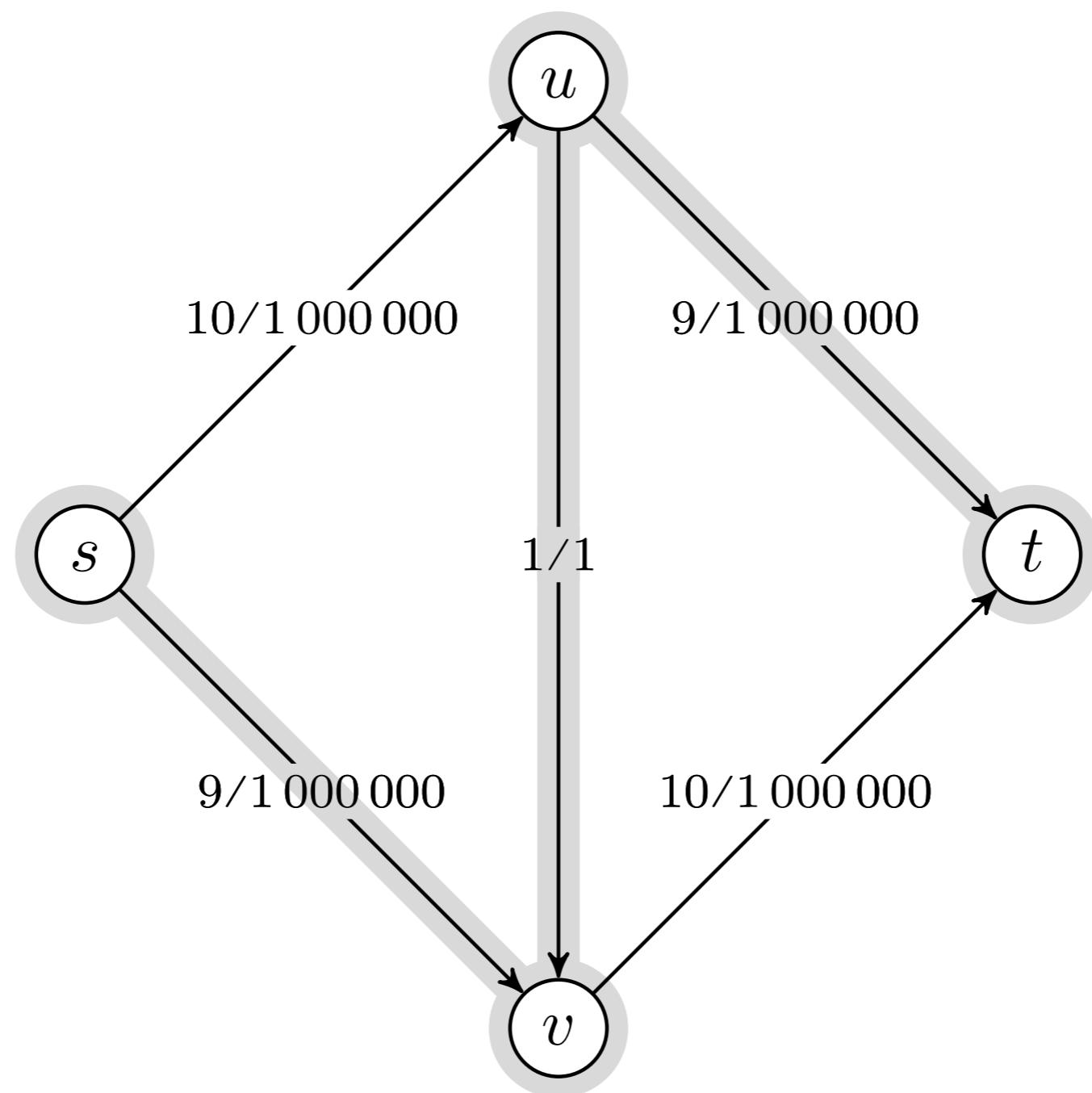


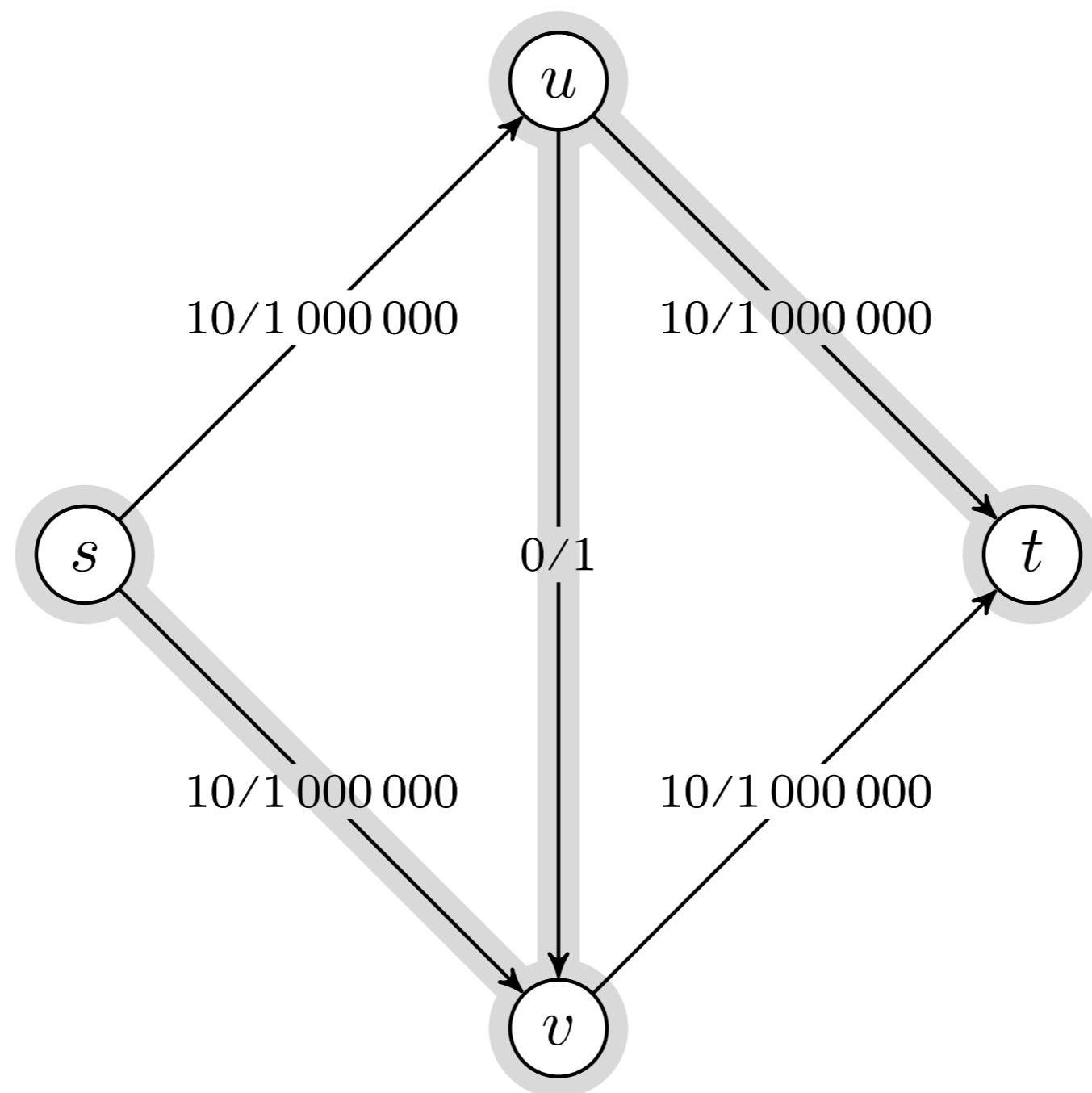


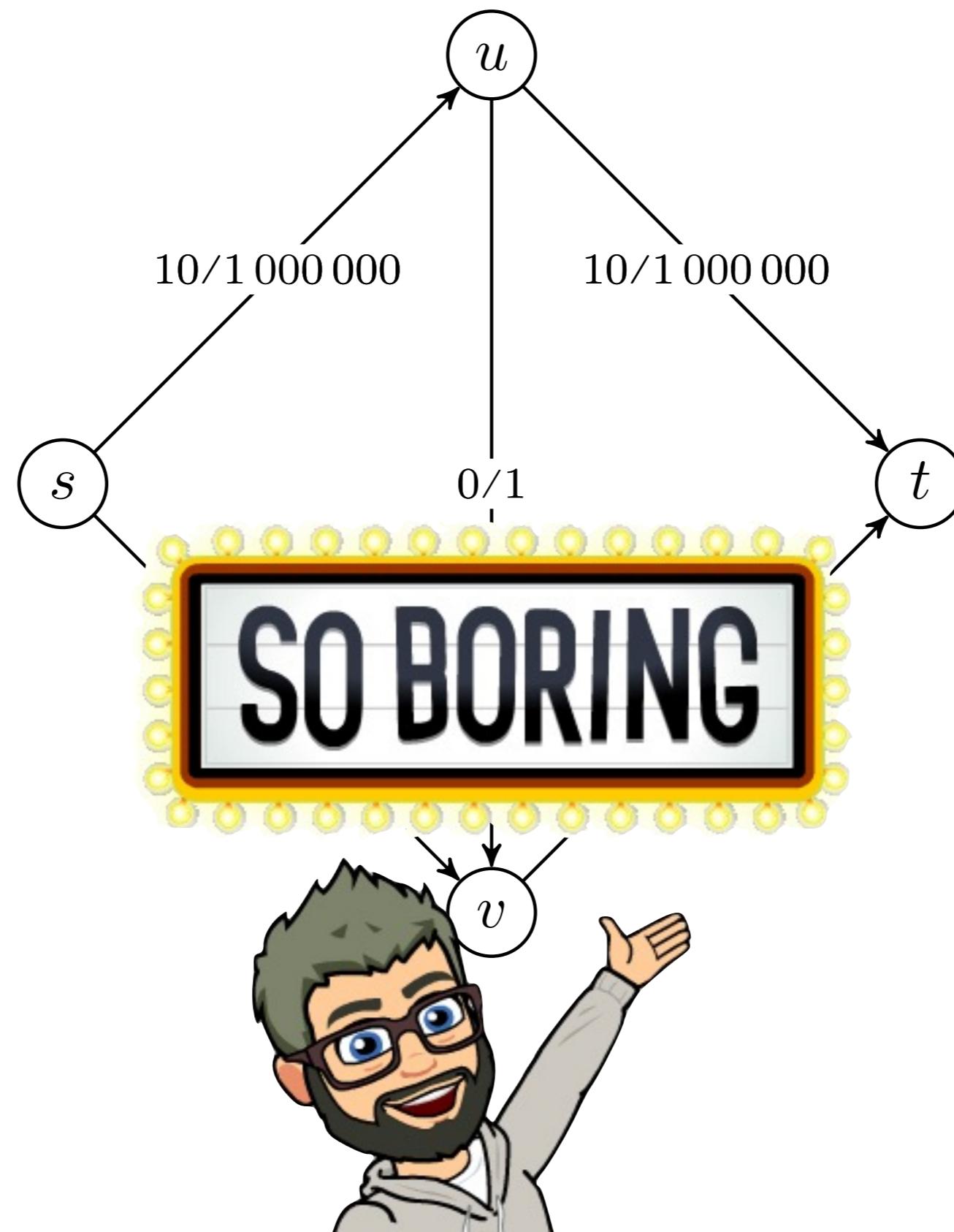












Operasjon	Antall	Kjøretid
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Operasjon	Antall	Kjøretid
Finn forøkende sti		

Med uspesifisert traversering i restnett

Operasjon	Antall	Kjøretid
Finn forøkende sti		$O(E)$

Alle nås fra  $s$ , så  $V + E = \Theta(E)$ . Kan stanses tidlig; derfor  $O$

Operasjon	Antall	Kjøretid
Finn forøkende sti	$O( f^* )$	$O(E)$

Heltallskapasiteter: Flyt økes med heltall  $\geq 1$  (Teorem 26.10)

Operasjon	Antall	Kjøretid
Finn forøkende sti	$O( f^* )$	$O(E)$

Totalt:  $O(E|f^*|)$

Operasjon	Antall	Kjøretid
Finn forøkende sti	$O( f^* )$	$O(E)$

Totalt:  $O(E|f^*|)$

Rasjonale kapasiteter kan skaleres til heltall

Operasjon	Antall	Kjøretid
Finn forøkende sti	$O( f^* )$	$O(E)$

Totalt:  $O(E|f^*|)$

Kapasiteter kan være (eksponentielt) store!

Operasjon	Antall	Kjøretid
Finn forøkende sti	$O( f^* )$	$O(E)$

Totalt:  $O(E|f^*|)$

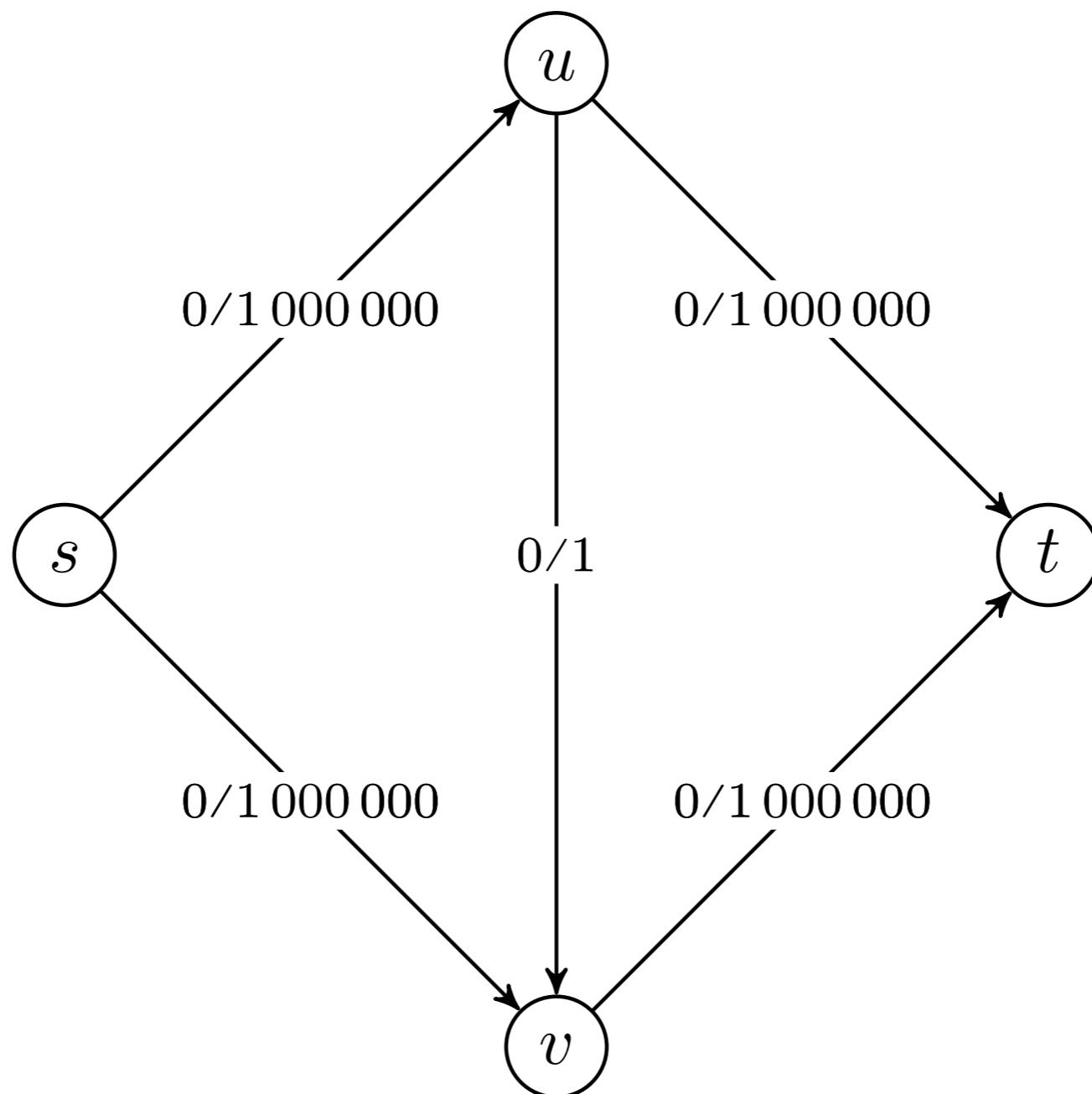
Irrasjonale kapasiteter: FORD-FULKERSON terminerer kanskje ikke!

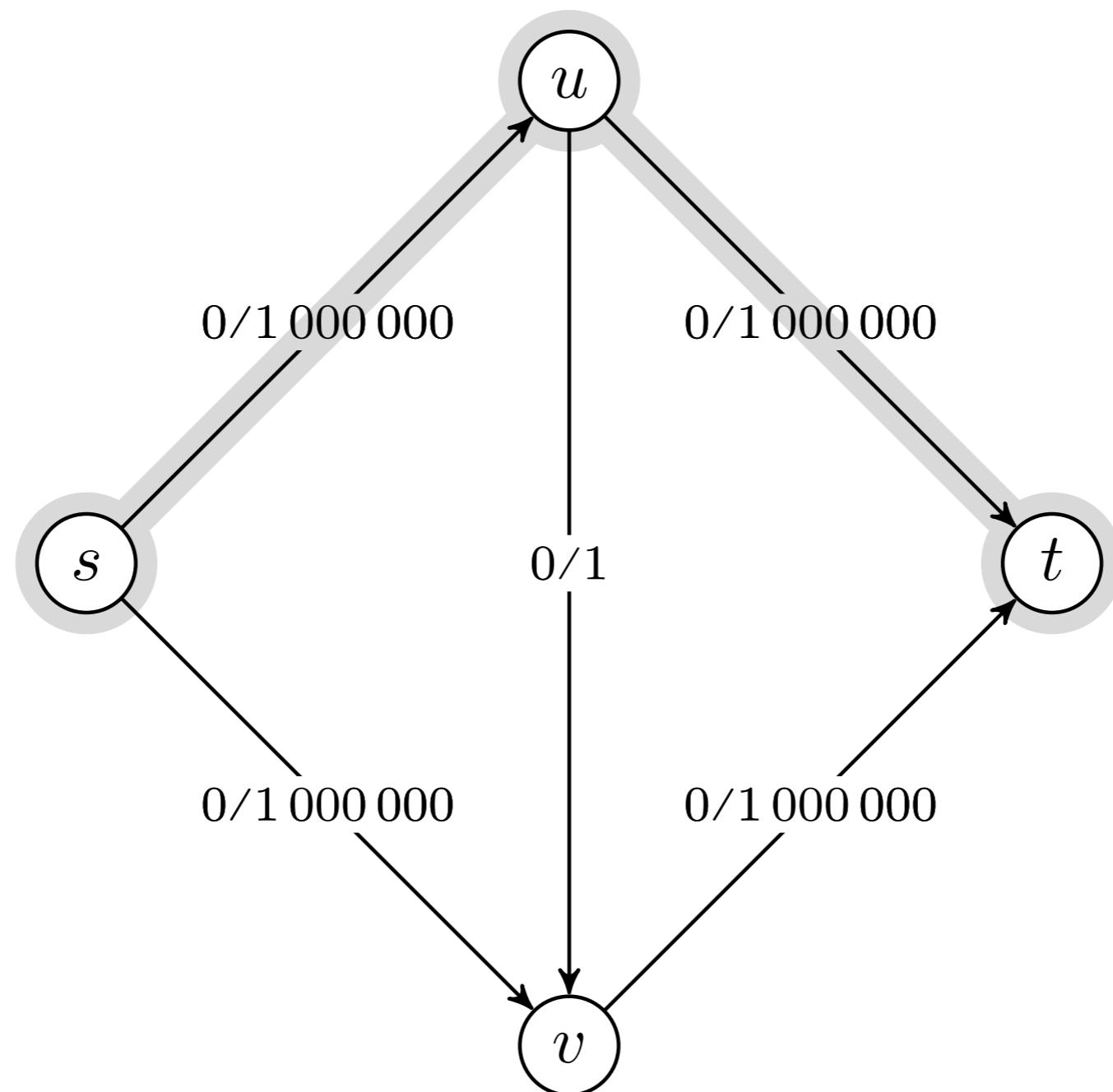
Pseudopolynomisk.

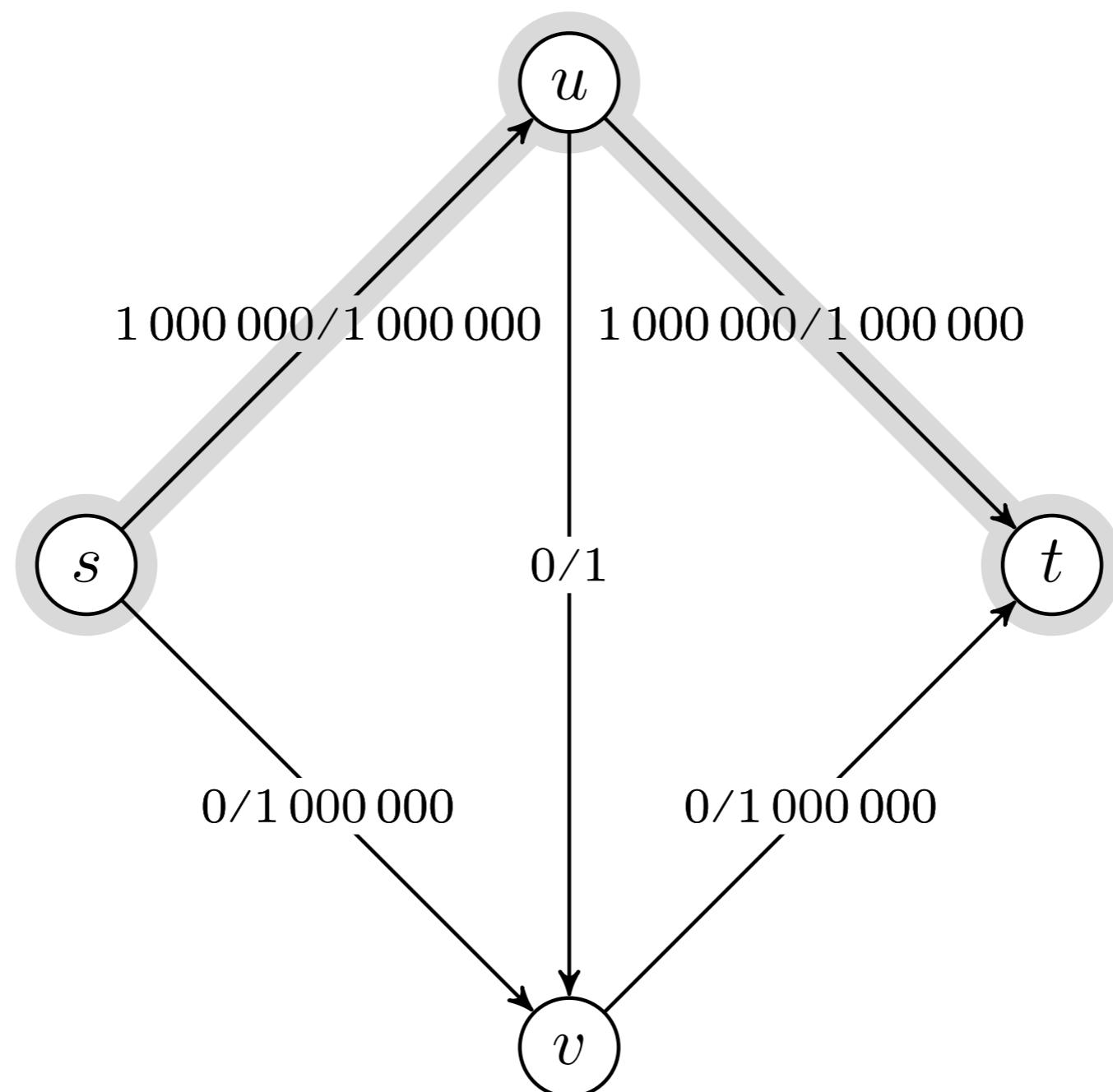
# Eksponentielt!

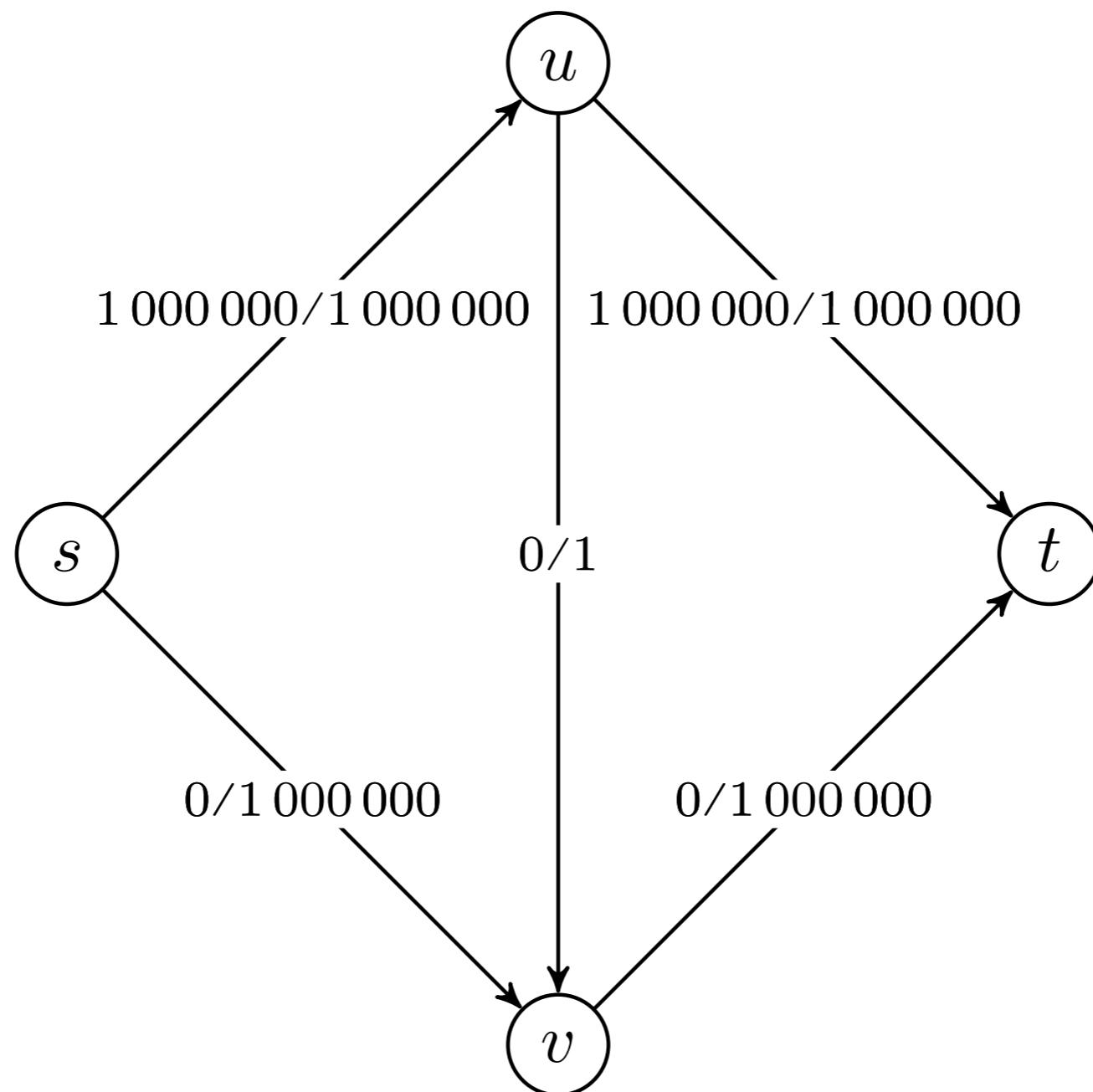
Dette blir som for 0-1-knapsack:  
Kjøretiden er en polynomisk  
funksjon av bl.a. ett av tallene i  
input; dermed er den  
eksponentiell som en funksjon  
av problemstørrelsen.

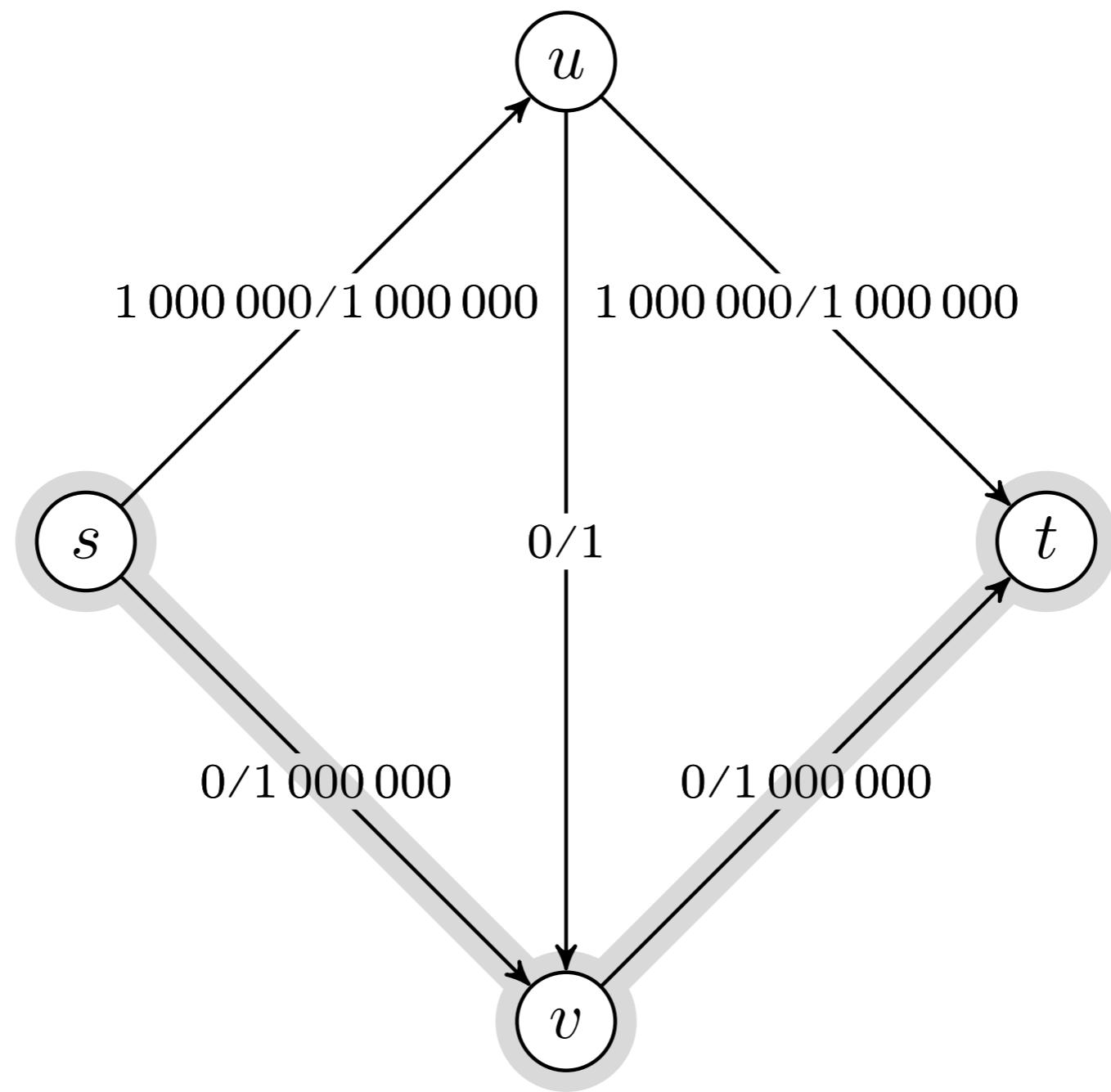
# **Bruk BFS!**

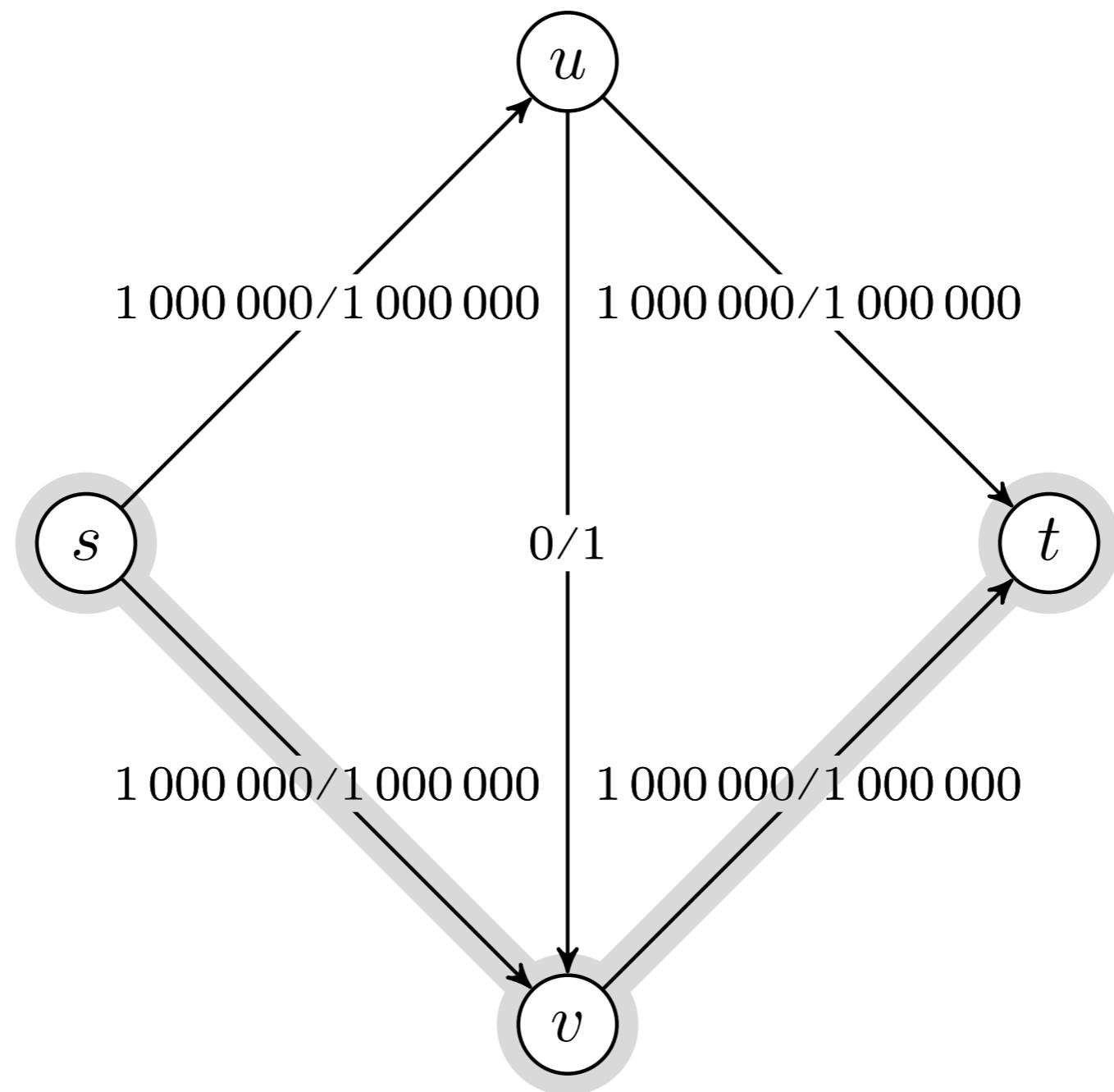


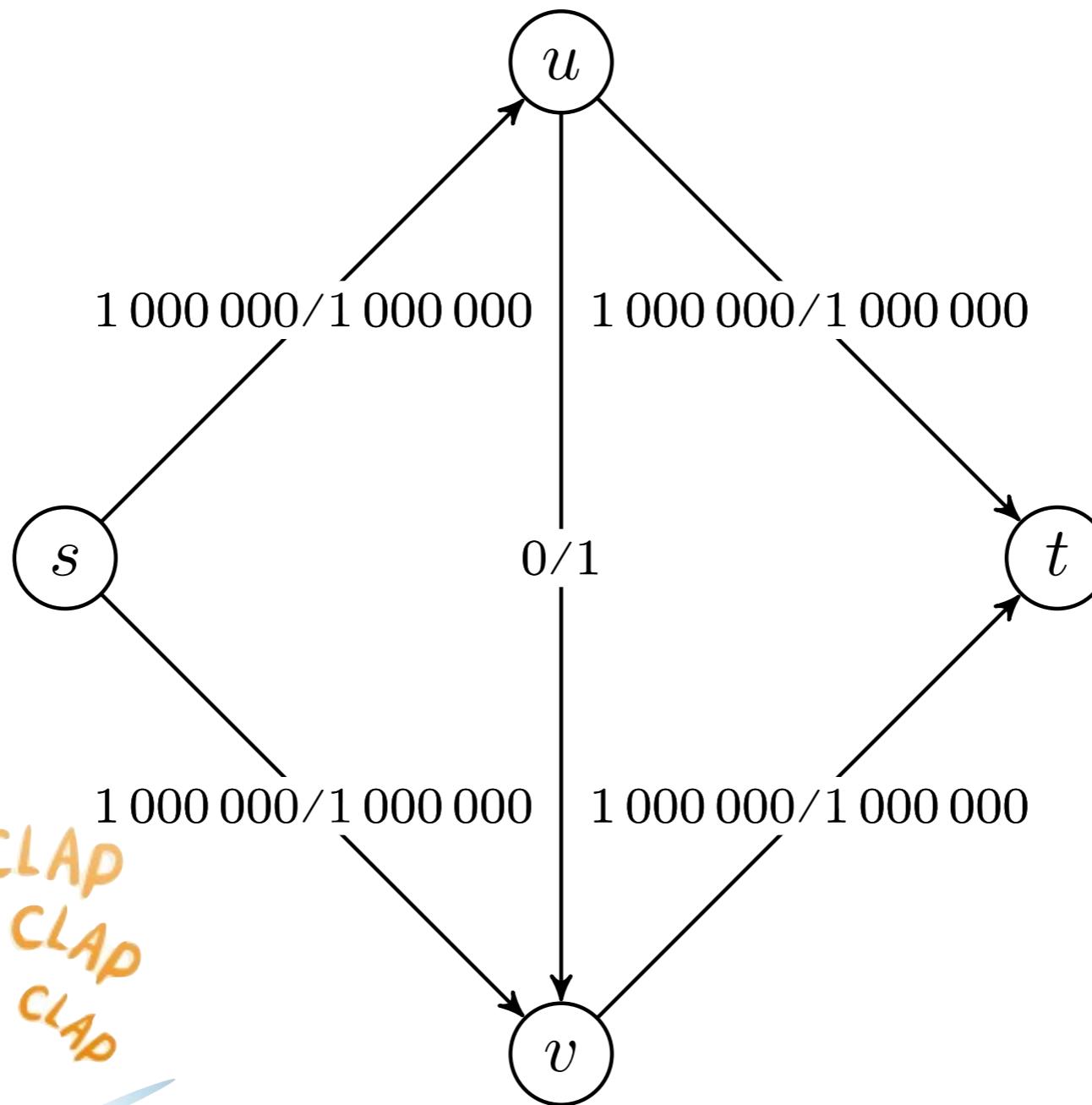












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Operasjon	Antall	Kjøretid
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Operasjon	Antall	Kjøretid
Finn forøkende sti		

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Med bredde-først-søk i restnett

Operasjon	Antall	Kjøretid
Finn forøkende sti		$O(E)$

Alle nås fra  $s$ , så  $V + E = \Theta(E)$ . Kan stanses tidlig; derfor  $O$

Operasjon	Antall	Kjøretid
Finn forøkende sti	$O(VE)$	$O(E)$

Det er her forskjellen mellom BFS og f.eks. DFS kommer inn!

Operasjon	Antall	Kjøretid
Finn forøkende sti	$O(VE)$	$O(E)$

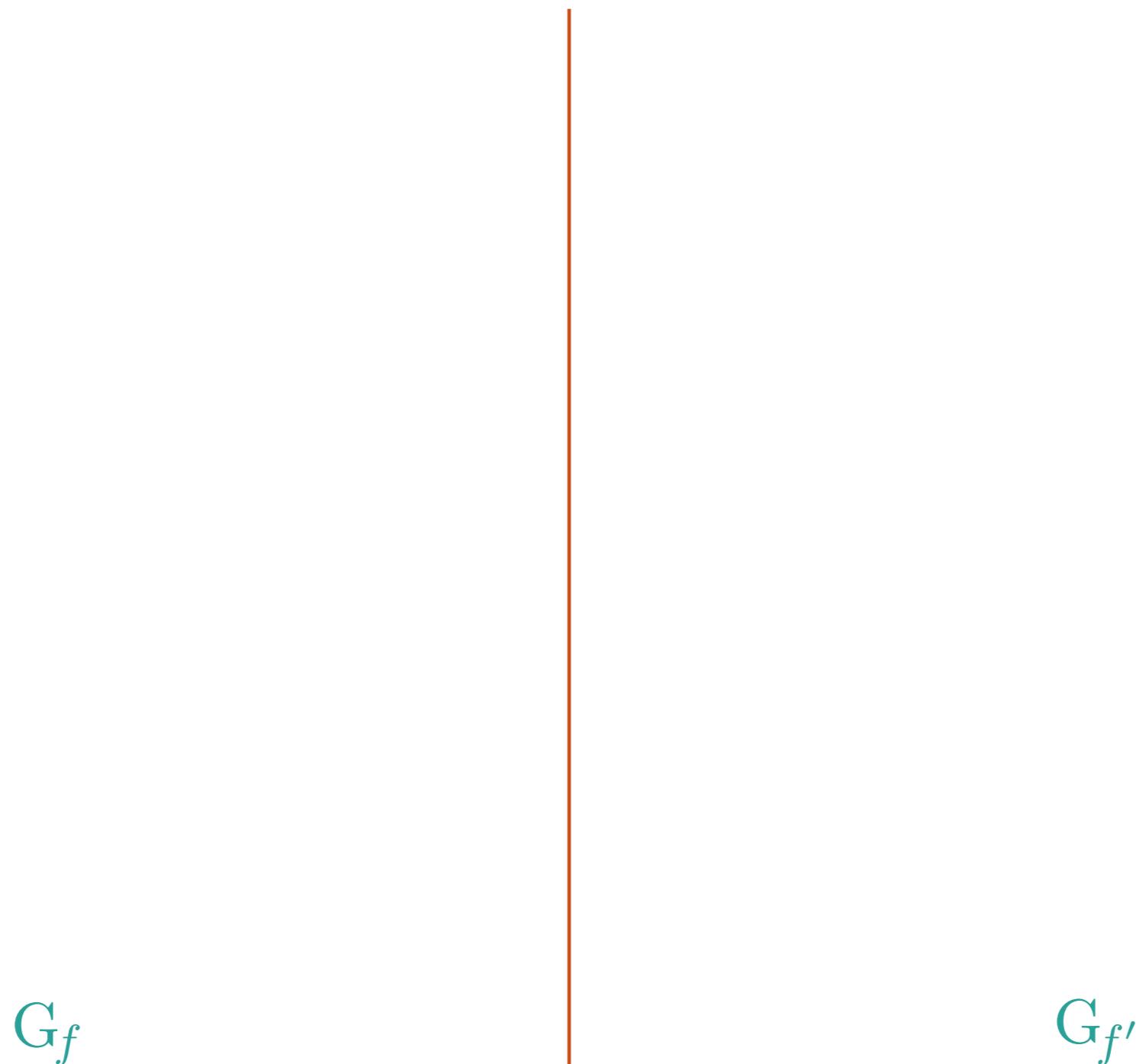
Totalt:  $O(VE^2)$

Operasjon	Antall	Kjøretid
Finn forøkende sti	$O(VE)$	$O(E)$

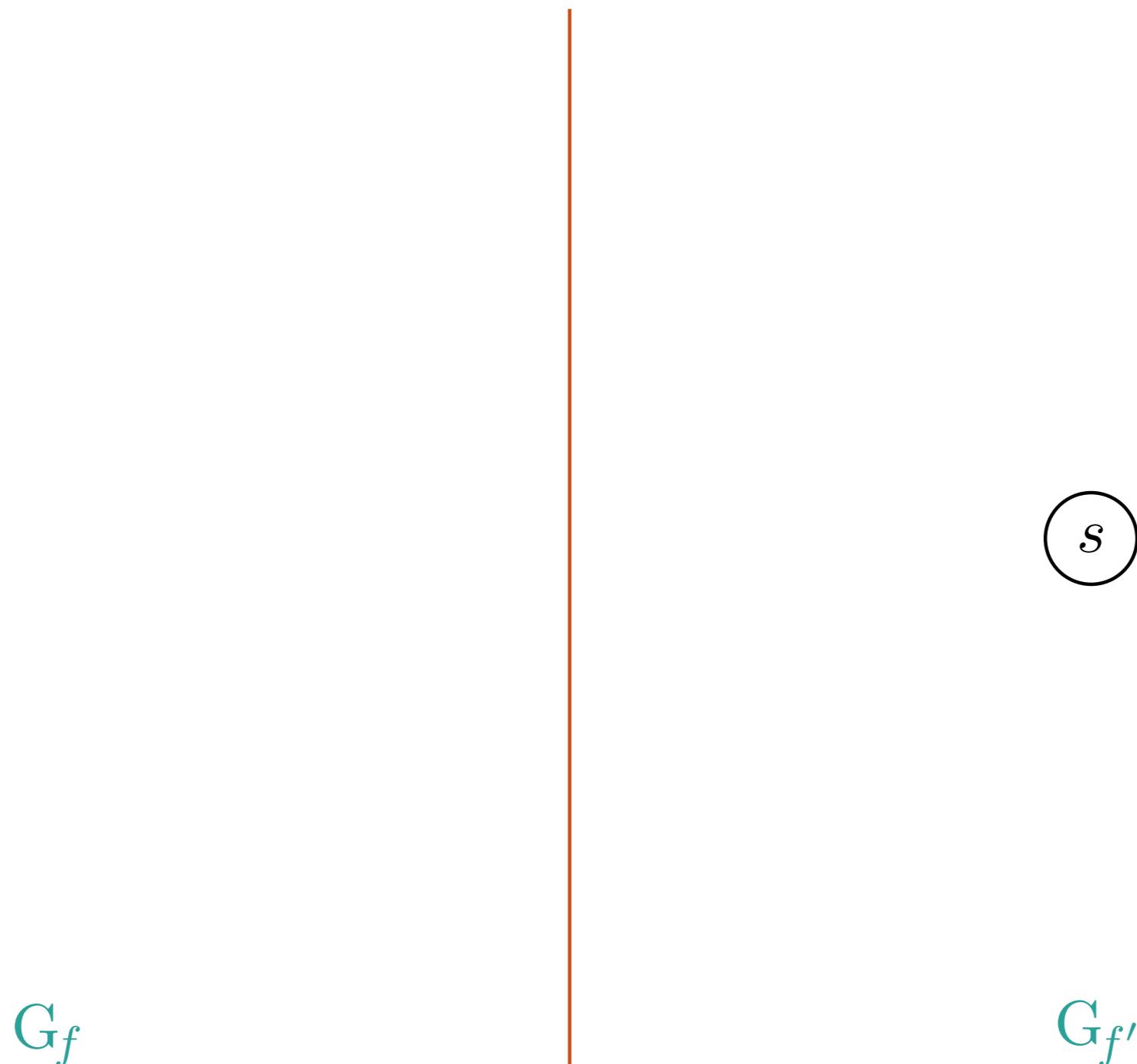
Totalt:  $O(VE^2)$

Men ... hvorfor  $O(VE)$  iterasjoner?

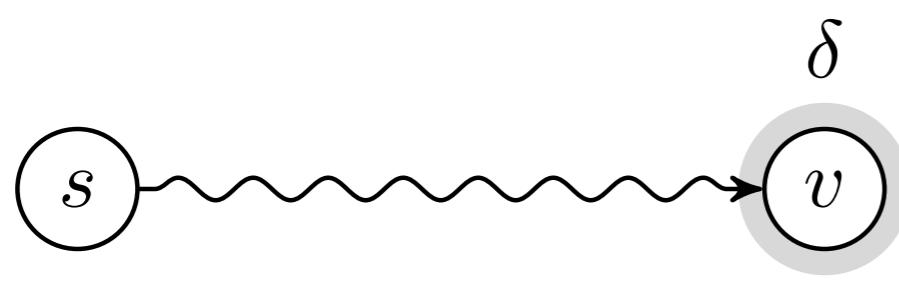
**Avstander synker ikke  
I restnettet**



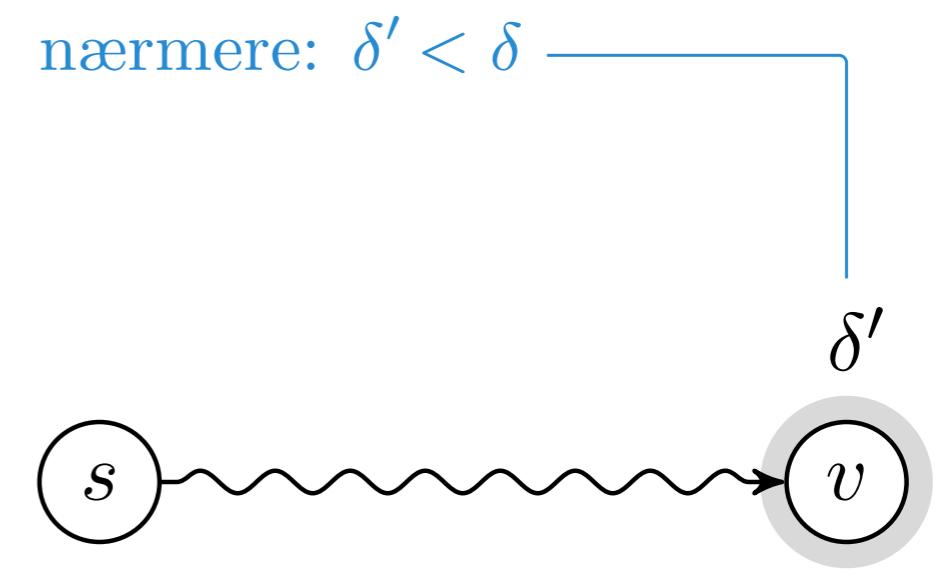
Restnett før og etter en flytøkning



Kan noen noder ha kommet nærmere *s*?

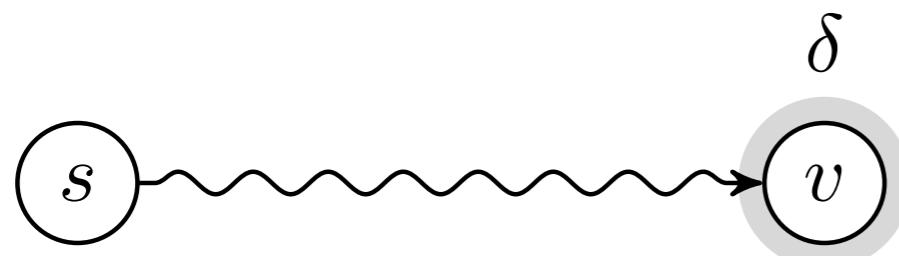


$G_f$

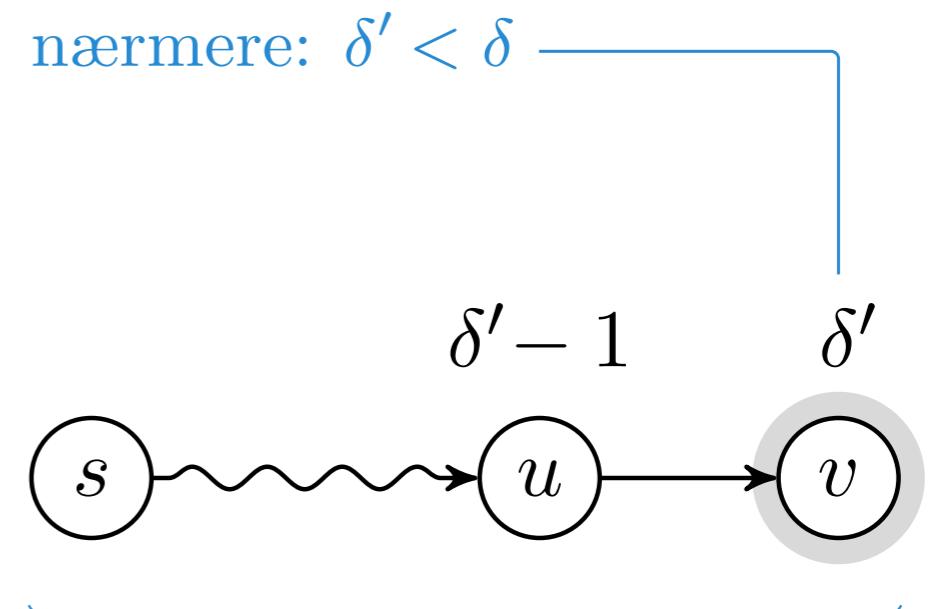


$G_{f'}$

La  $v$  være den som kom nærmere og havnet nærmest

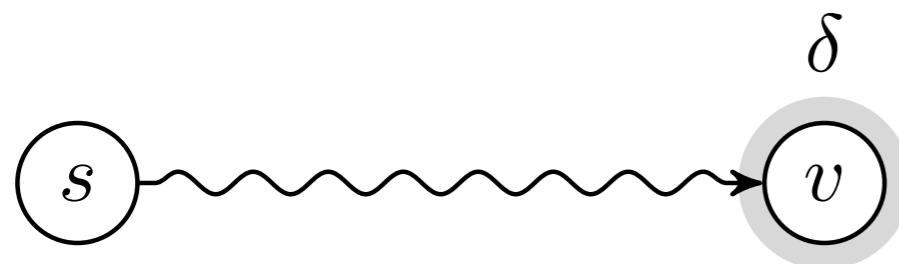


$G_f$

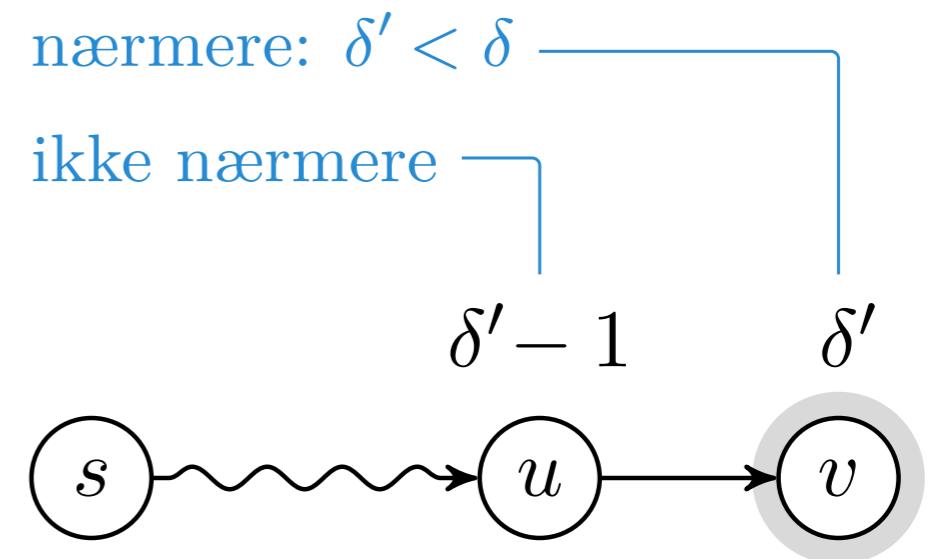


$G_{f'}$

La  $u$  være forrige node langs korteste vei etterpå



$G_f$

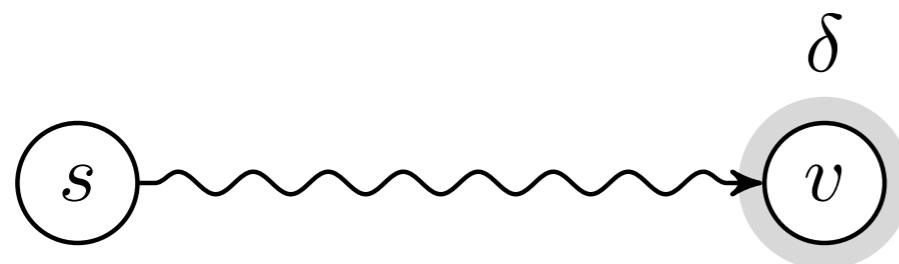


$G_{f'}$

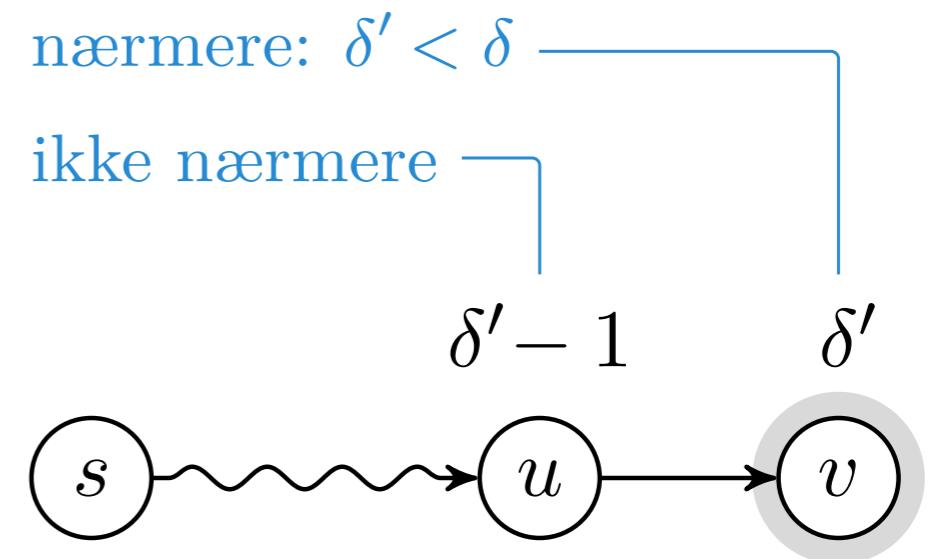
nærmere:  $\delta' < \delta$

ikke nærmere

$v$  var den nærmeste som kom nærmere, så  $u$  kom ikke nærmere

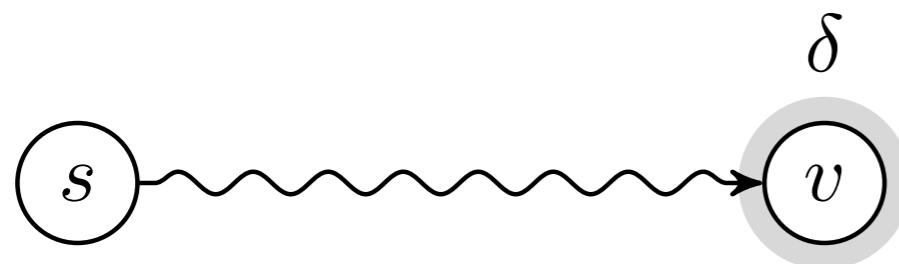


$G_f$



$G_{f'}$

Om vi hadde  $(u, v)$  fra før, måtte  $u$  også ha kommet nærmere

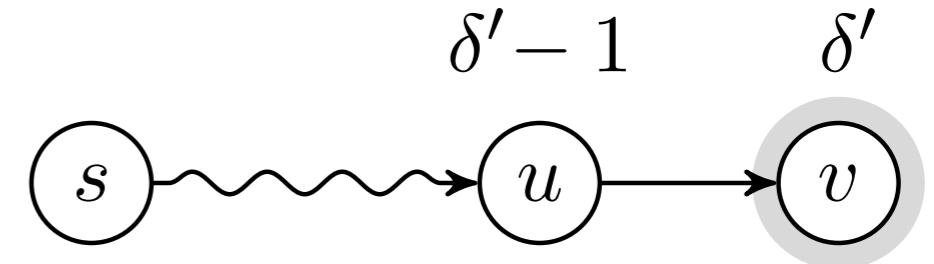


$(u, v)$  finnes ikke

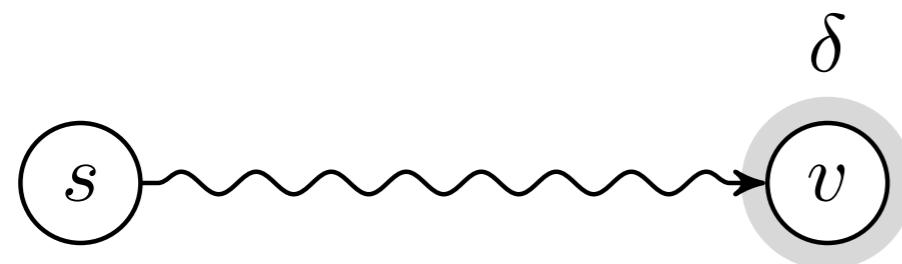
$G_f$

nærmere:  $\delta' < \delta$

ikke nærmere



Om vi hadde  $(u, v)$  fra før, måtte  $u$  også ha kommet nærmere

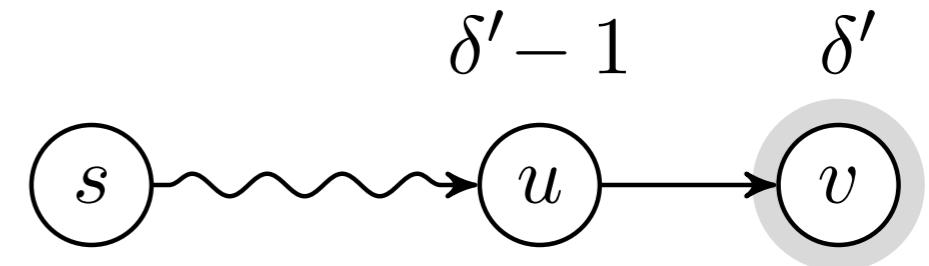


$(u, v)$  finnes ikke

$G_f$

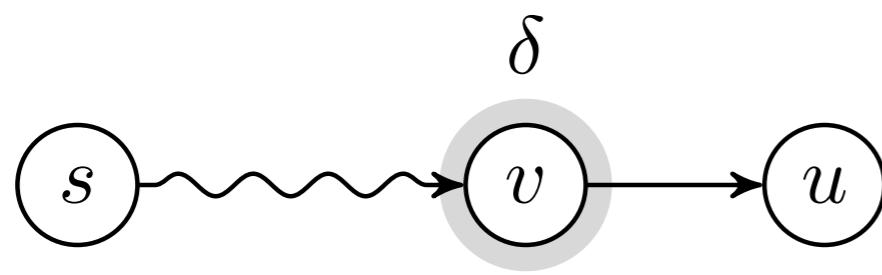
nærmere:  $\delta' < \delta$

ikke nærmere



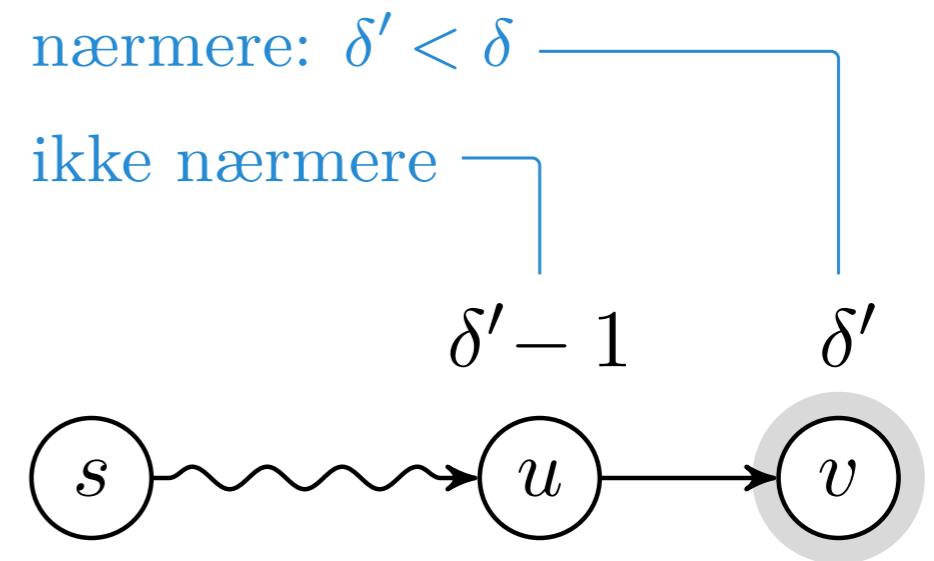
$G_{f'}$

Siden  $(u, v)$  dukket opp, må vi ha økt flyt fra  $v$  til  $u$



$(u, v)$  finnes ikke

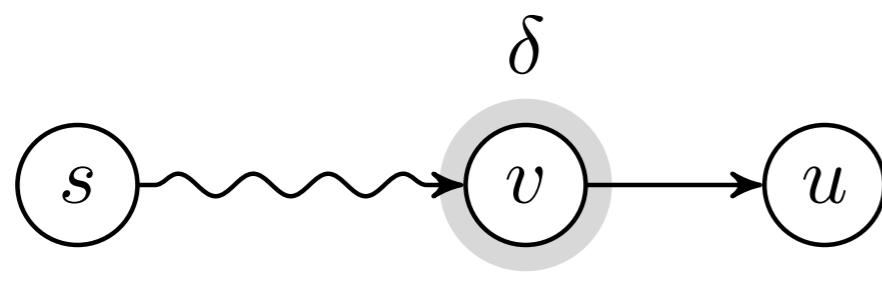
$G_f$



nærmere:  $\delta' < \delta$

ikke nærmere

Siden  $(u, v)$  dukket opp, må vi ha økt flyt fra  $v$  til  $u$



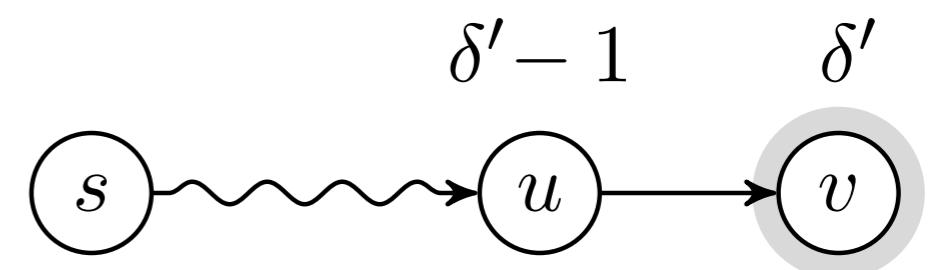
korteste vei

$(u, v)$  finnes ikke

$G_f$

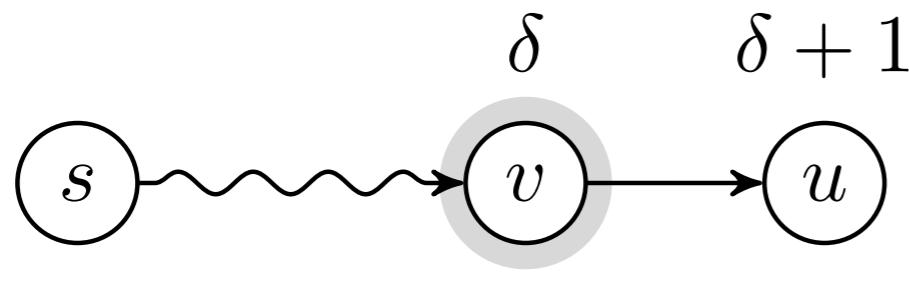
nærmere:  $\delta' < \delta$

ikke nærmere

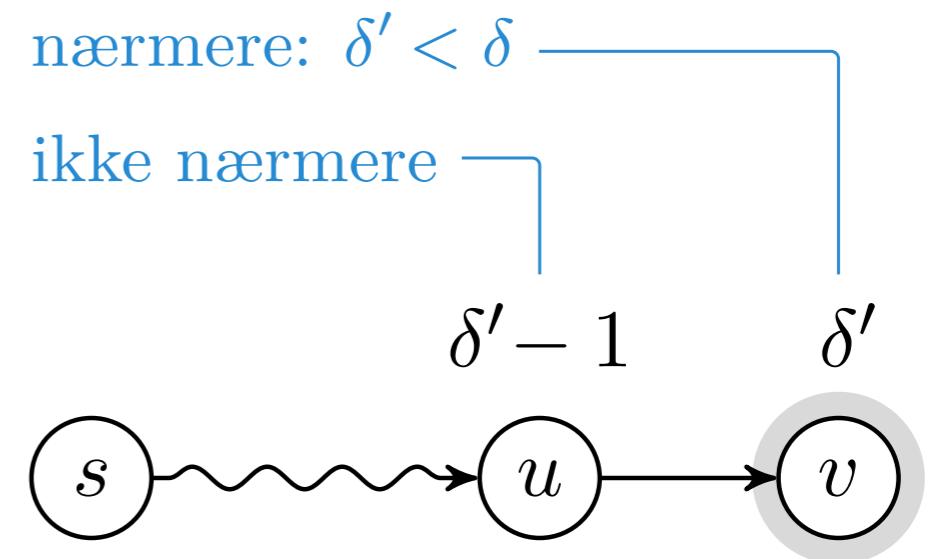


$G_{f'}$

Vi velger korteste forøkende stier, her med  $v$  før  $u$

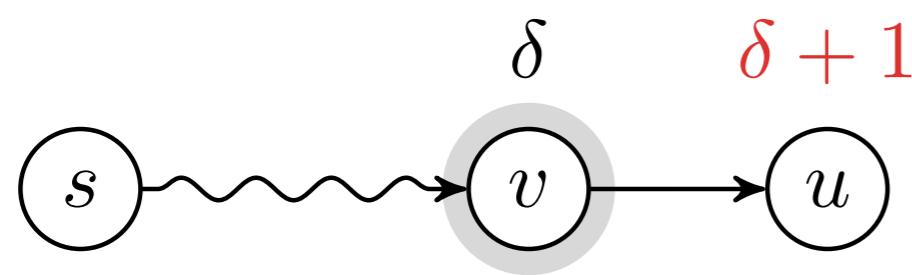


$G_f$

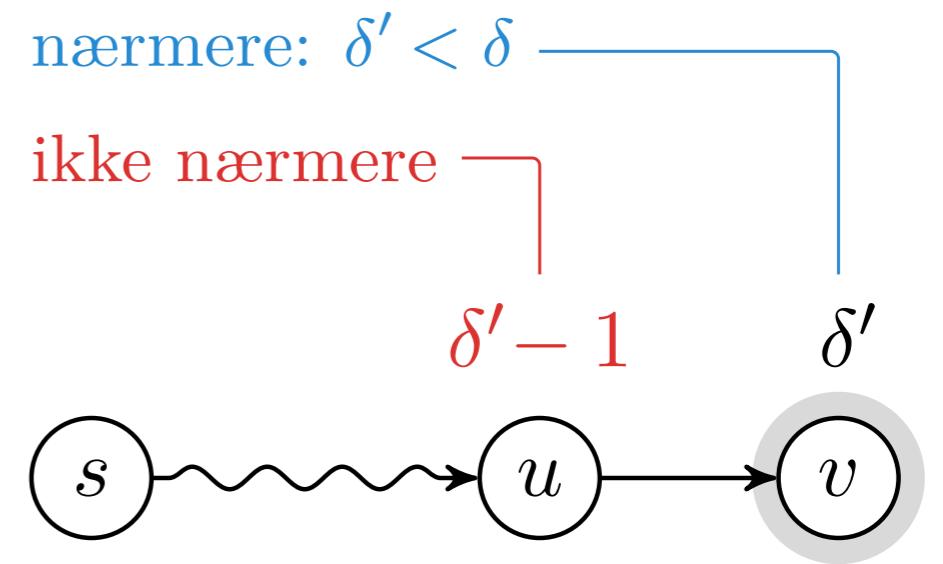


$G_{f'}$

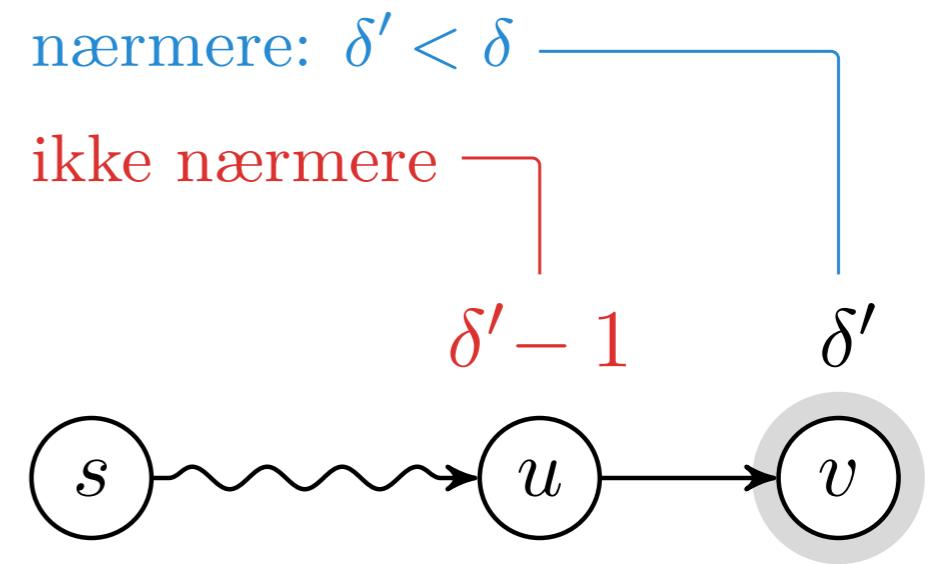
$v$  må ha beveget seg forbi  $u$  i feil retning!



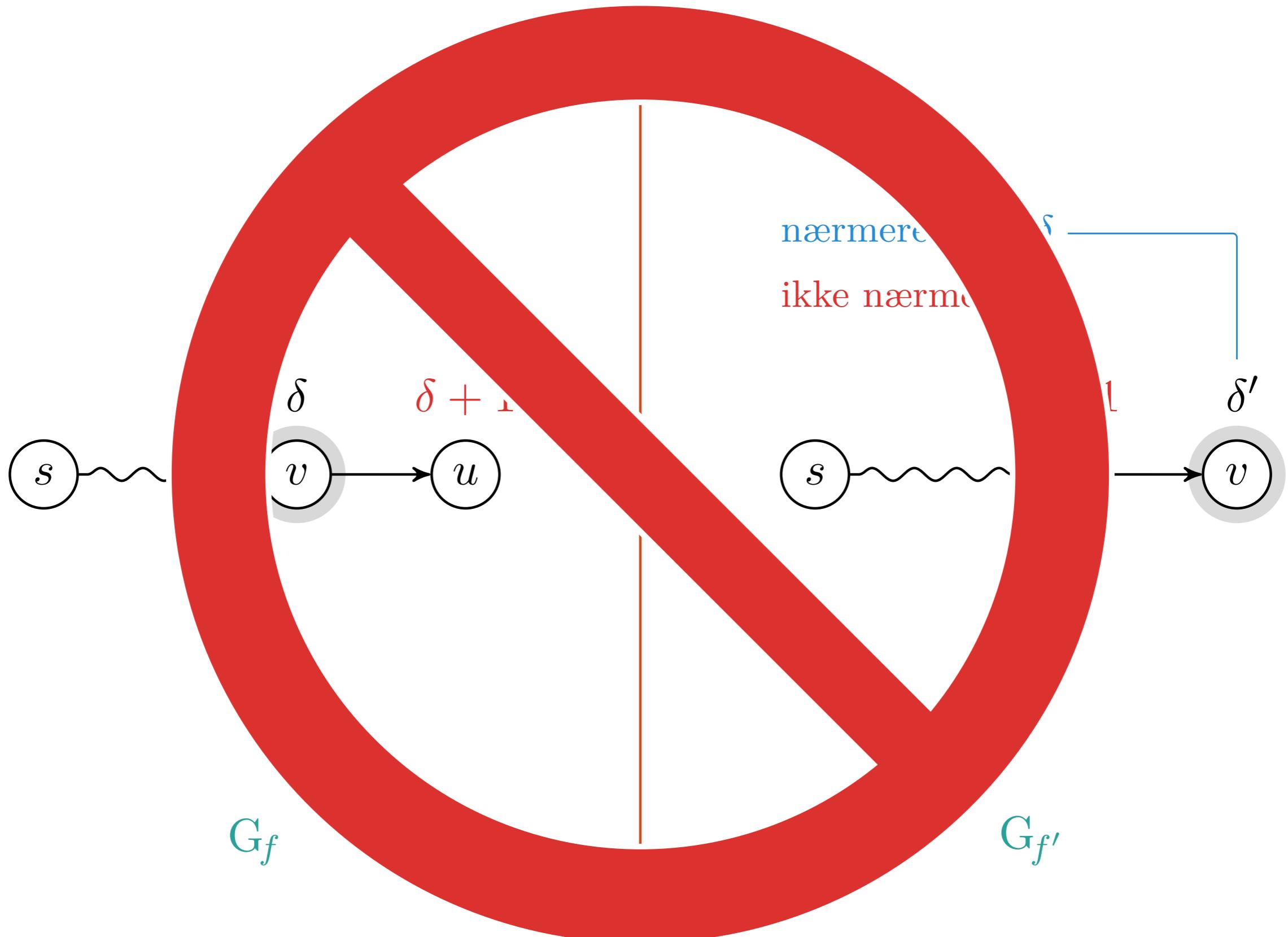
$G_f$



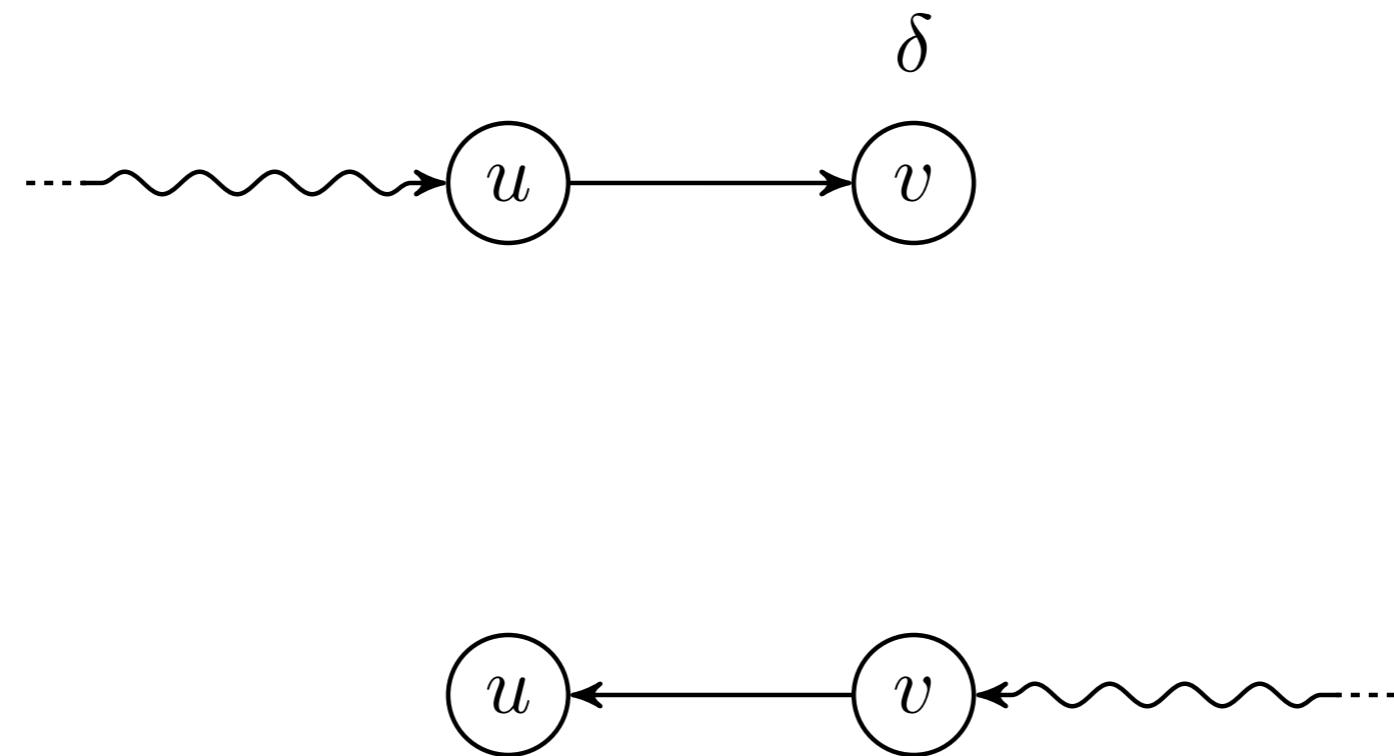
$G_{f'}$



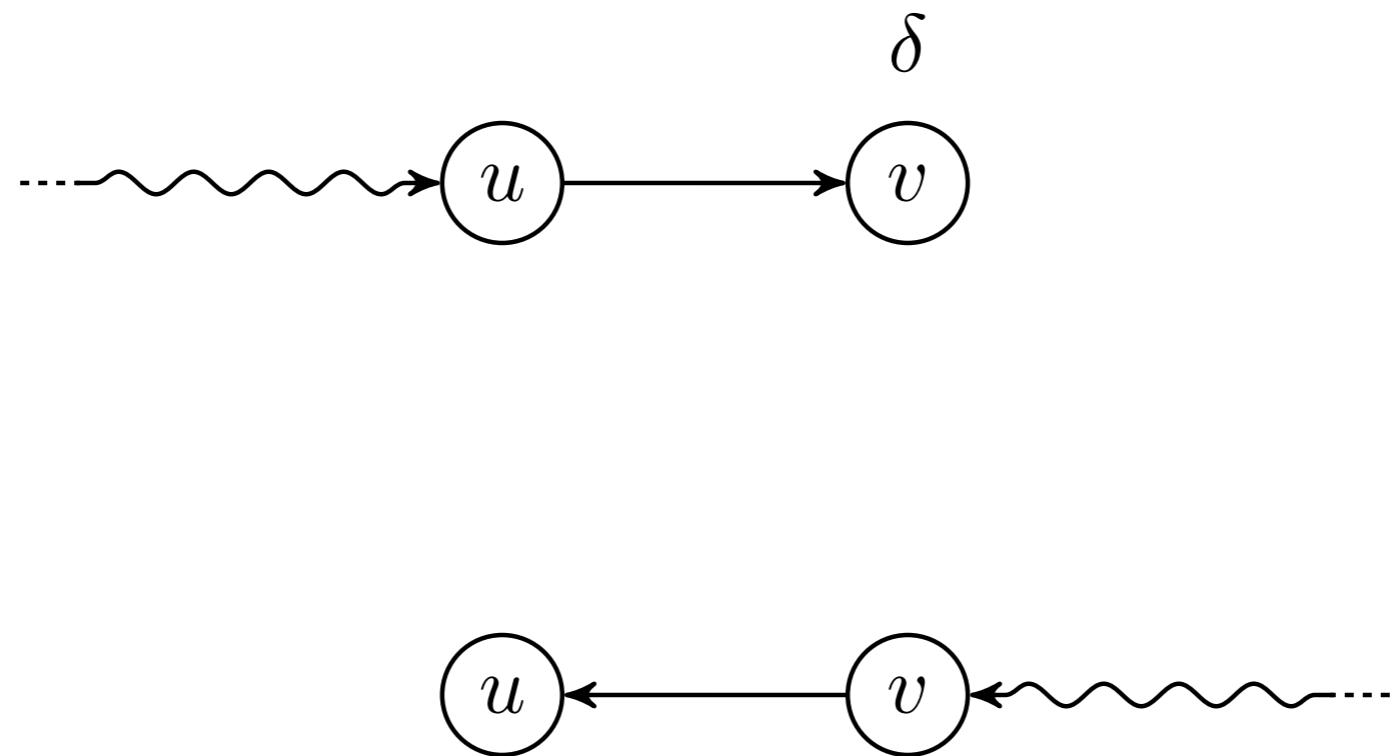
$$\delta < \delta + 1 \leq \delta' - 1 < \delta' \implies \delta < \delta'; \text{ vi antok } \delta' < \delta!$$



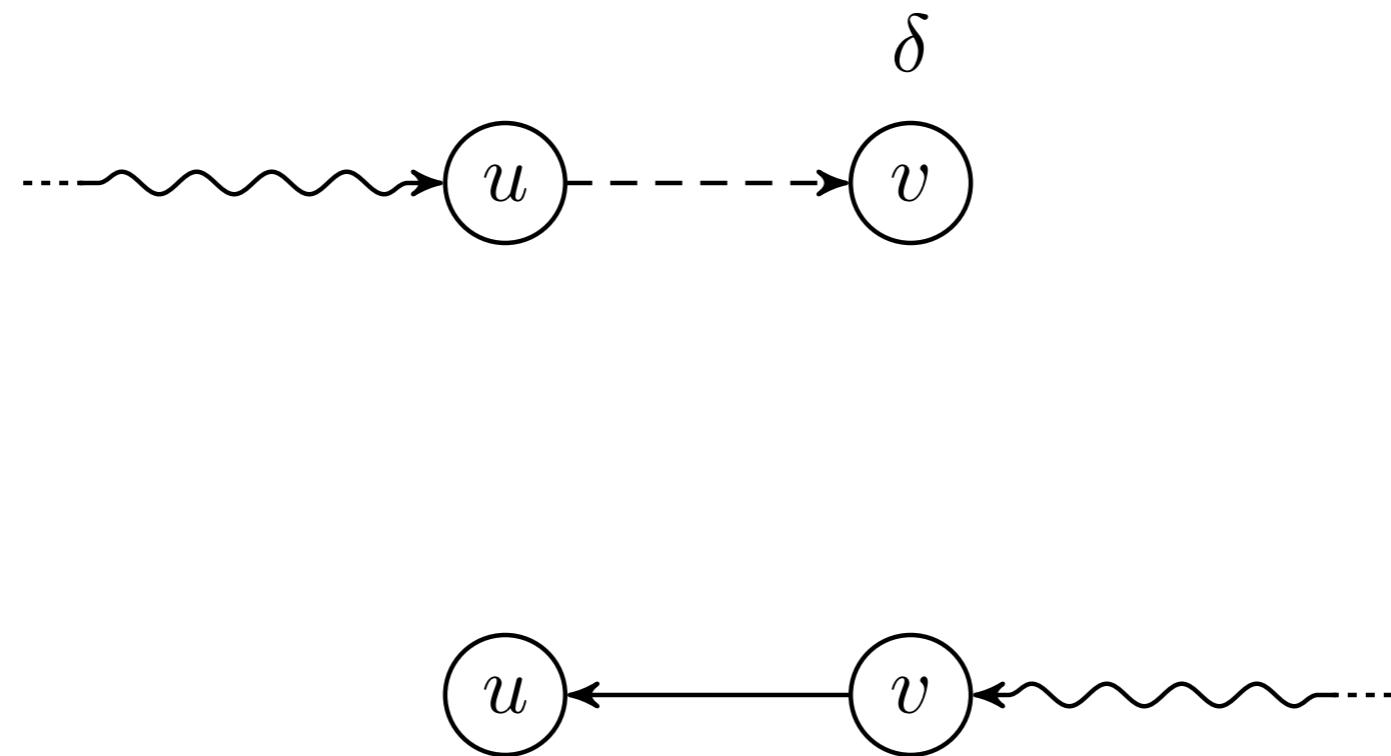
# Begrenset gjenbruk av flaskehalser



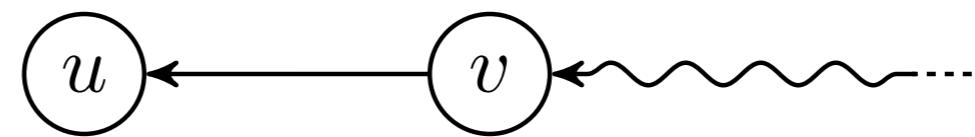
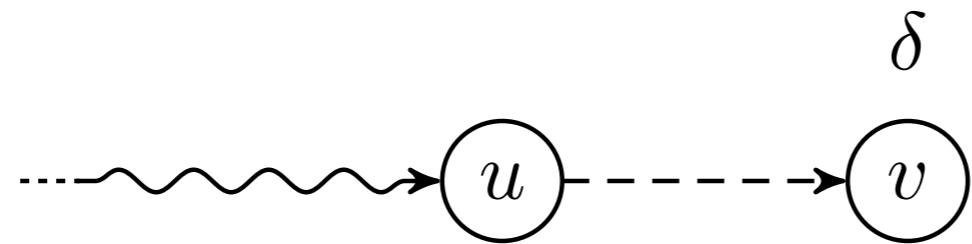
La  $(u, v)$  være flaskehals i en flytøkning



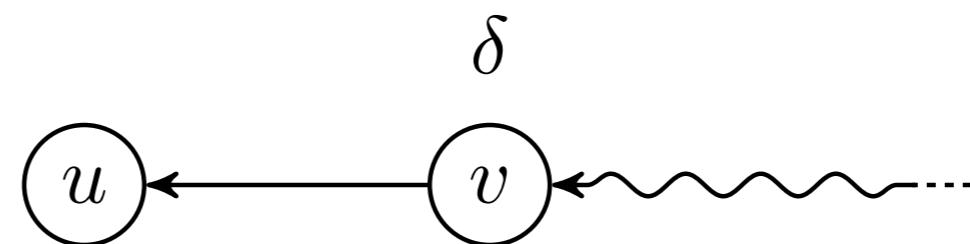
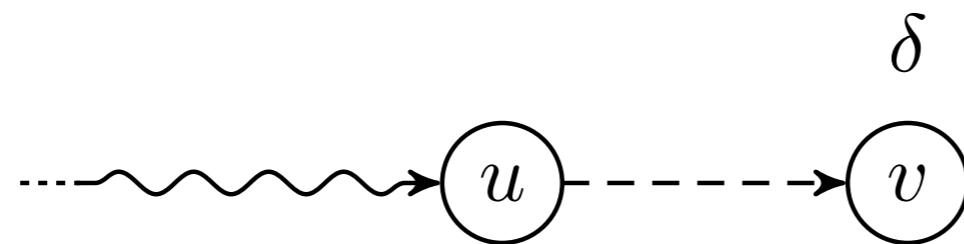
Etter økningen forsvinner  $(u, v)$  fra  $G_f$



Etter økningen forsvinner  $(u, v)$  fra  $G_f$

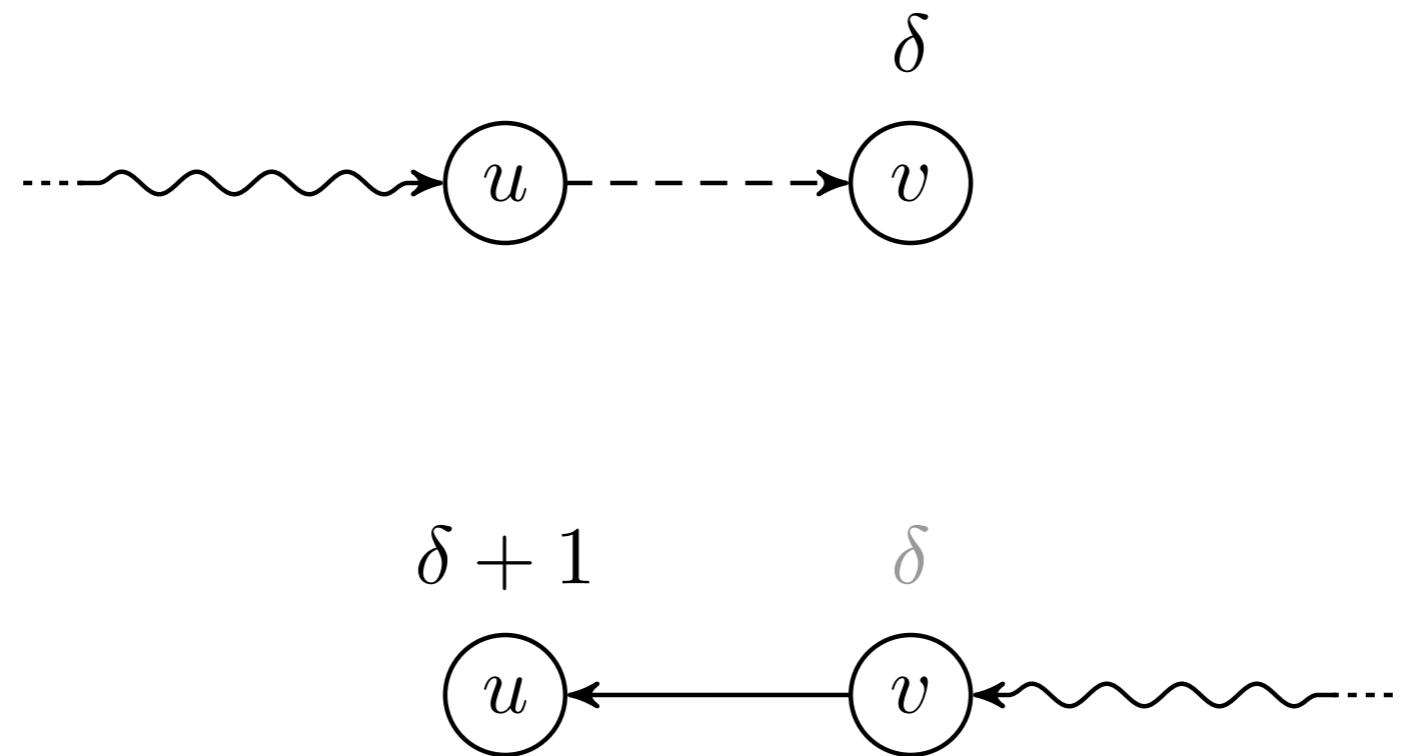


For å gjeninnføre  $(u, v)$  må vi øke flyt fra  $v$  til  $u$

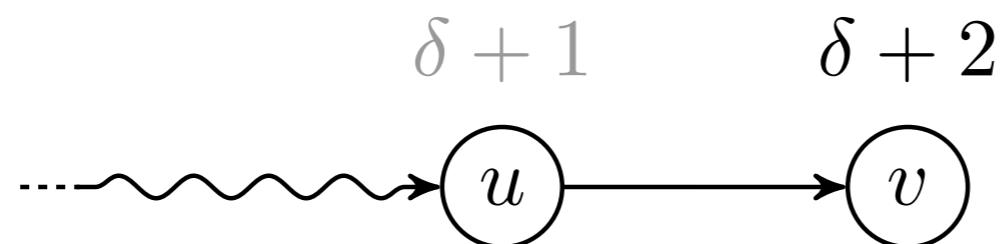
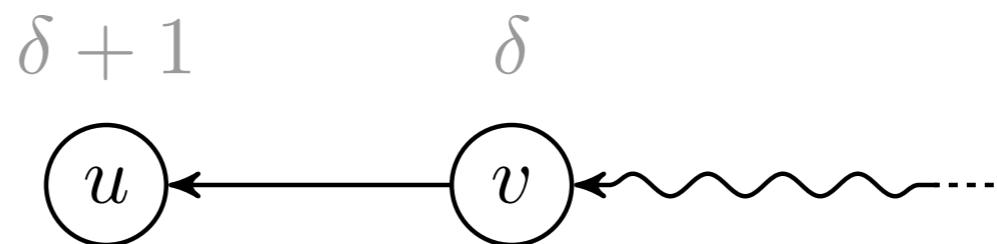
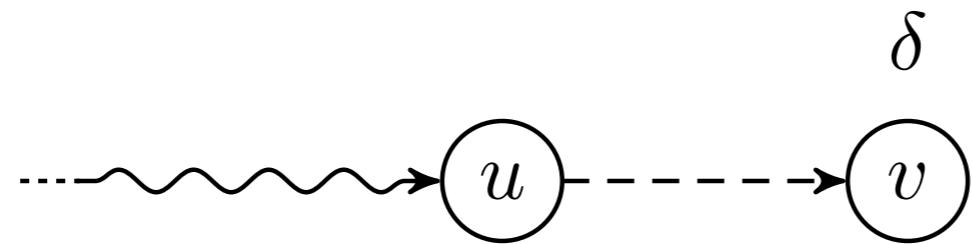


Har kan altså avstandene være enda større; dette er bare minimum.

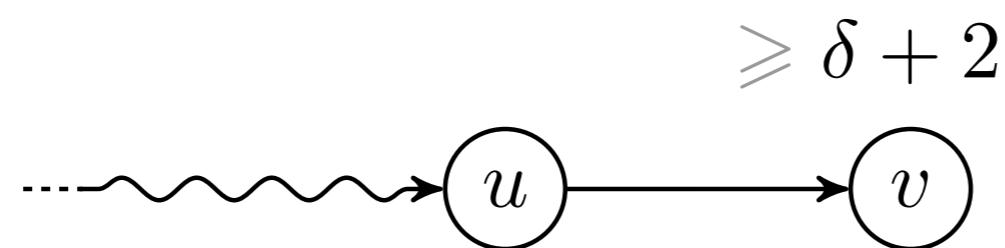
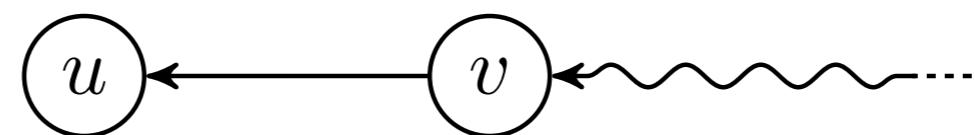
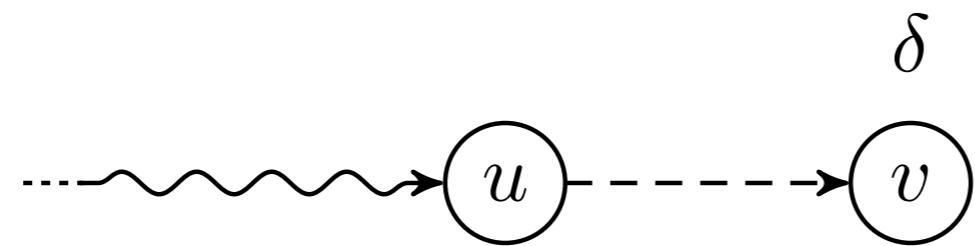
Husk: Avstander synker ikke. (De kan naturligvis øke)



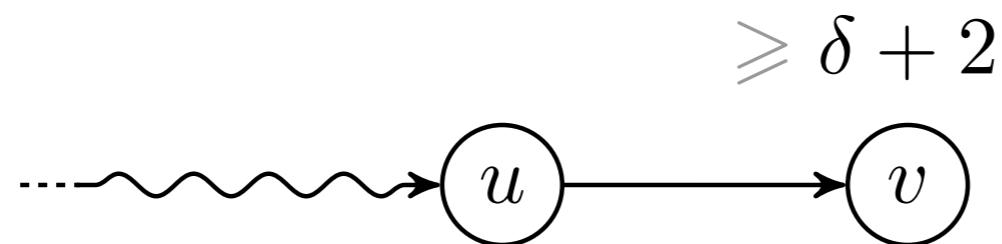
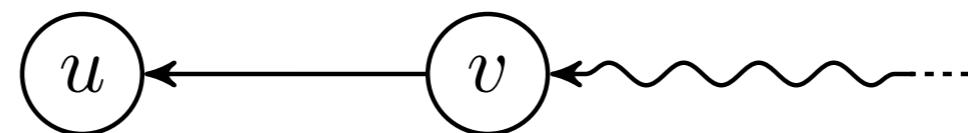
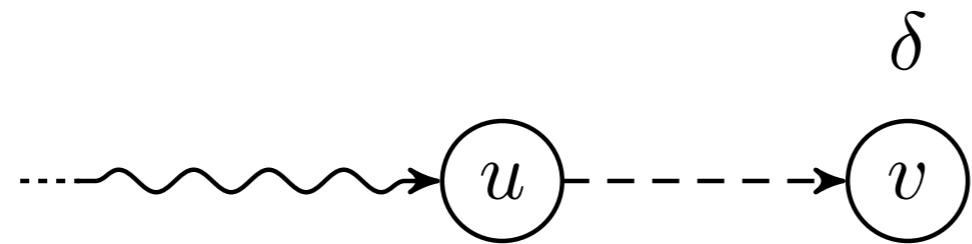
Vi øker flyt langs korteste vei



Så  $(u, v)$  er flaskehals maks annenhver gang . . .



... og  $\delta(s, v)$  øker da med minst 2



O( $E$ ) nodepar er flaskehals O( $V$ ) ganger: Kjøretid O( $VE$ )

# Korrekthet, da?

Vi ser på det såkalt duale problemet, der vi forsøker å finne flaskehalsen i nettet – det minimale snittet. Det vil hjelpe oss med å vise korrekthet.

Min. snitt og maks. flyt er såkalt duale problemer: Det ene er minimering, det andre er maksimering, og de har samme optimum.

# 4.5

## Minimalt snitt

### MAXIMAL FLOW THROUGH A NETWORK

L. R. FORD, JR. AND D. R. FULKERSON

**Introduction.** The problem discussed in this paper was formulated by T. Harris as follows:

"Consider a rail network connecting two cities by way of a number of intermediate cities, where each link of the network has a number representing its capacity. Assuming a steady flow from one given

Om du er nysgjerrig på dualitet generelt, så finner du mer om det i delkapittel 29.4.

# MAXIMAL FLOW THROUGH A NETWORK

L. R. FORD, JR. AND D. R. FULKERSON

**Introduction.** The problem discussed in this paper was formulated by T. Harris as follows:

"Consider a rail network connecting two cities by way of a number of intermediate cities, where each link of the network has a number assigned to it representing its capacity. Assuming a steady state condition, find a maximal flow from one given city to the other."

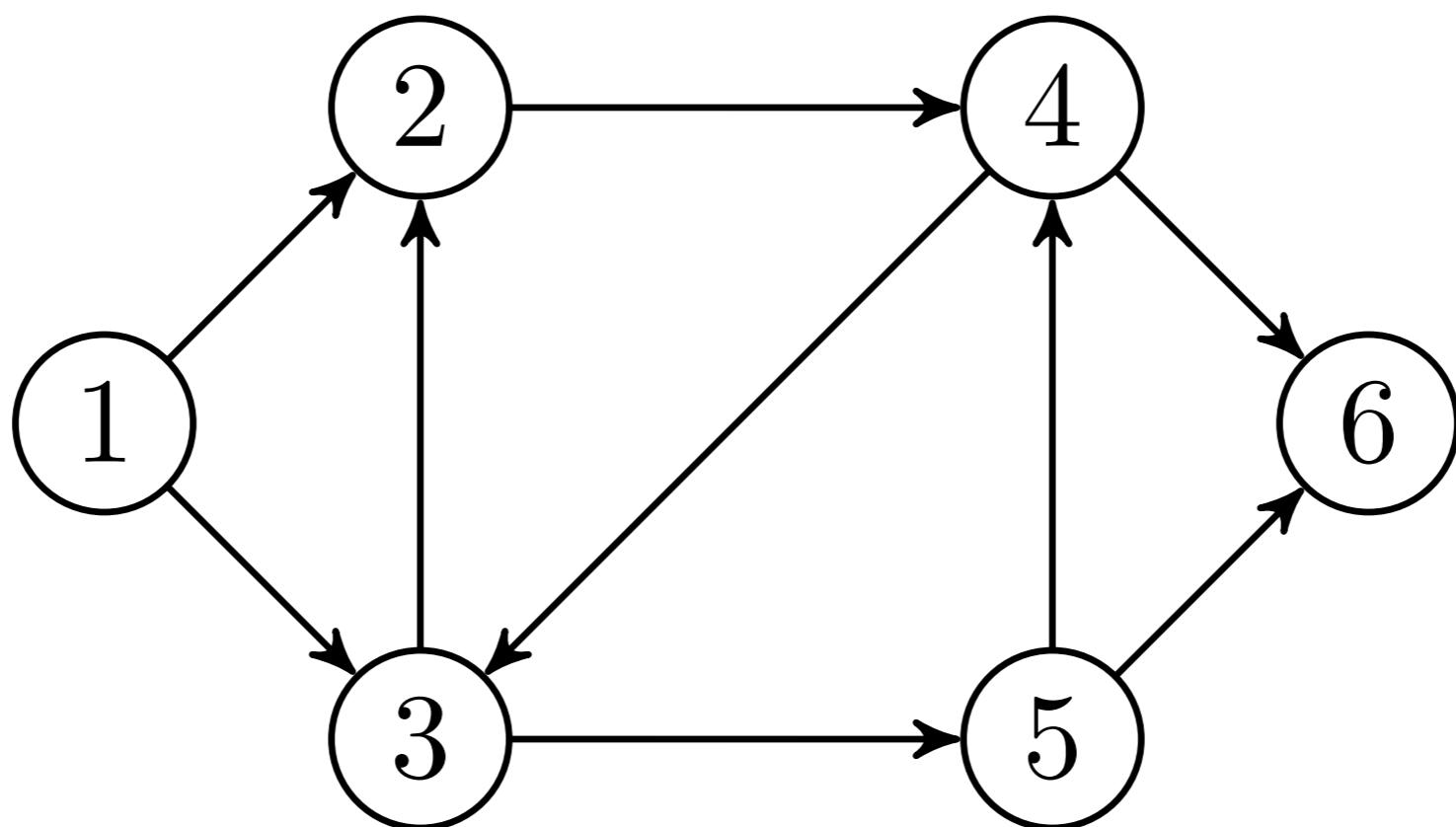
While this can be set up as a linear programming problem with as many equations as there are cities in the network, and hence can be solved by the simplex method (1), it turns out that in the cases of most practical interest, where the network is planar in a certain restricted sense, a much simpler and more efficient hand computing procedure can be described.

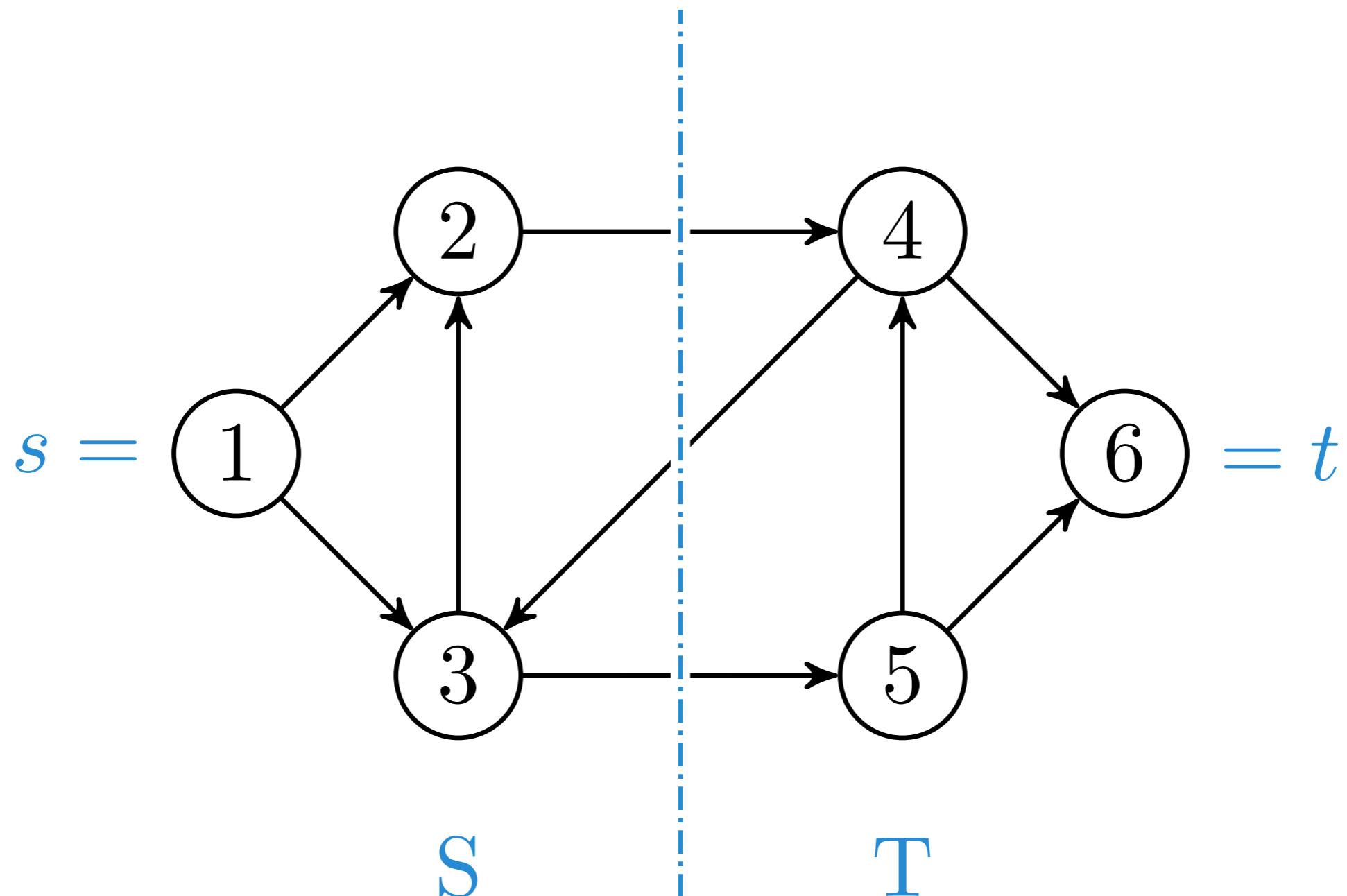
In §1 we prove the minimal cut theorem, which establishes that an obvious upper bound for flows over an arbitrary network can always be achieved. The proof is non-constructive. However, by specializing the network (§2), we obtain as a consequence of the minimal cut theorem an effective computational scheme. Finally, we observe in §3 the duality between the capacity problem and that of finding the shortest path, via a network, between two given points.

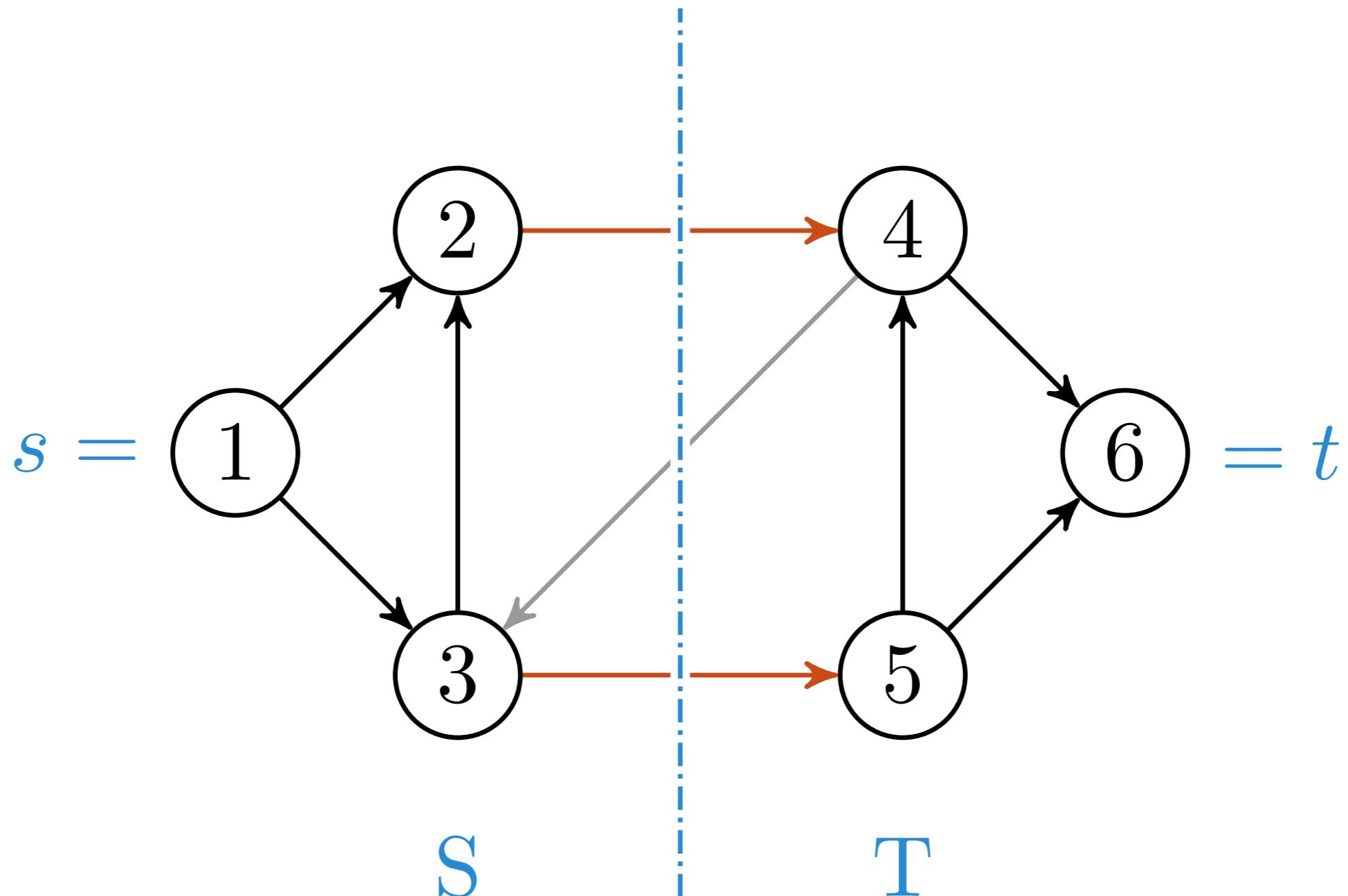
**1. The minimal cut theorem.** A *graph*  $G$  is a finite, 1-dimensional complex, composed of *vertices*  $a, b, c, \dots, e$ , and *arcs*  $\alpha(ab), \beta(ac), \dots, \delta(ce)$ . An arc  $\alpha(ab)$  *joins* its end vertices  $a, b$ ; it passes through no other vertices of  $G$  and intersects other arcs only in vertices. A *chain* is a set of distinct arcs of  $G$  which can be arranged as  $\alpha(ab), \beta(bc), \gamma(cd), \dots, \delta(gh)$ , where the vertices  $a, b, c, \dots, h$  are distinct, i.e., a chain does not intersect itself; a chain *joins* its end vertices  $a$  and  $h$ .

We distinguish two vertices of  $G$ :  $a$ , the *source*, and  $b$ , the *sink*.<sup>1</sup> A *chain flow* from  $a$  to  $b$  is a couple  $(C; k)$  composed of a chain  $C$  joining  $a$  and  $b$ , and a non-negative number  $k$  representing the flow along  $C$  from source to sink.

Each arc in  $G$  has associated with it a positive number called its *capacity*. We call the graph  $G$ , together with the capacities of its individual arcs, a







Kapasitet gitt av kanter  $S \rightarrow T$

## Snitt i flytnett: Partisjon $(S, T)$ av $V$

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- ›  $s \in S$  og  $t \in T$

## Snitt i flytnett: Partisjon $(S, T)$ av $V$

- ›  $s \in S$  og  $t \in T$
- › Netto-flyt:

$$f(S, T) = \sum_{u \in S} \sum_{v \in T} f(u, v) - \sum_{u \in S} \sum_{v \in T} f(v, u)$$

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› Kapasitet:

$$c(S, T) = \sum_{u \in S} \sum_{v \in T} c(u, v)$$

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$$c(S, T) = \sum_{u \in S} \sum_{v \in T} c(u, v)$$

**Lemma 26.5:**  $f(S, T) = |f|$

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› Kapasitet:

$$c(S, T) = \sum_{u \in S} \sum_{v \in T} c(u, v)$$

**Lemma 26.5:**  $f(S, T) = |f|$

Beviset (s.722) krever en del utregning, men er ganske rett frem

## Snitt i flytnett: Partisjon $(S, T)$ av $V$

›  $s \in S$  og  $t \in T$

› Netto-flyt:

$$f(S, T) = \sum_{u \in S} \sum_{v \in T} f(u, v) - \sum_{u \in S} \sum_{v \in T} f(v, u)$$

› Kapasitet:

$$c(S, T) = \sum_{u \in S} \sum_{v \in T} c(u, v)$$

### Lemma 26.5: $f(S, T) = |f|$

› Korollar 26.5:  $|f| \leq c(S, T)$

Beviset (s.722) krever en del utregning, men er ganske rett frem

---

**Input:** Et flytnett  $G = (V, E)$  med kilde  $s$  og sluk  $t$ .

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---

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**Output:** Et snitt  $(S, T)$  med minst mulig kapasitet, dvs., der  $c(S, T)$  er minimal.

---

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**Output:** Et snitt  $(S, T)$  med minst mulig kapasitet, dvs., der  $c(S, T)$  er minimal.

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For anvendelser vil  $c$  her ofte være en form for *kostnad*

**Maks. flyt = min. snitt**

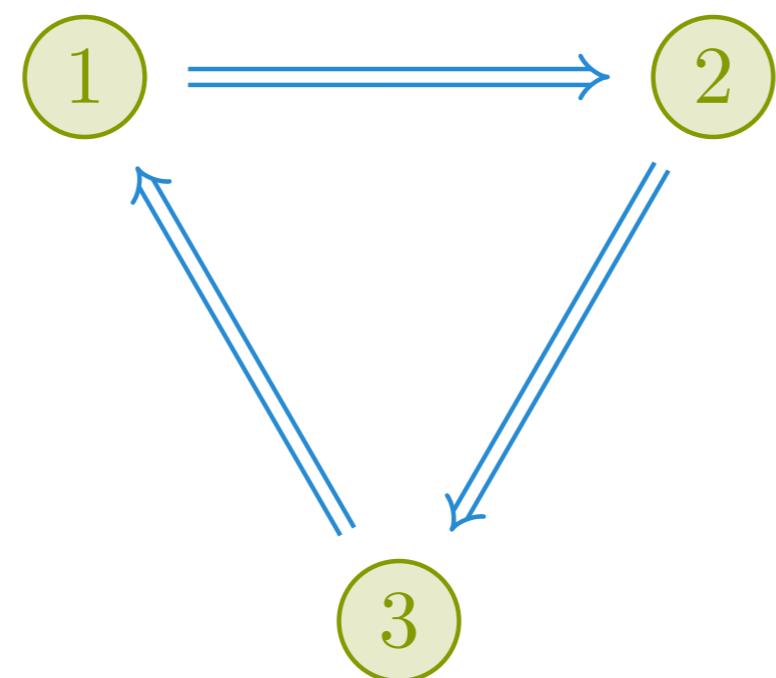
$f$  er maks-flyt for  $G$

$G_f$  har ingen forøkende sti



$f$  er maks-flyt for  $G$

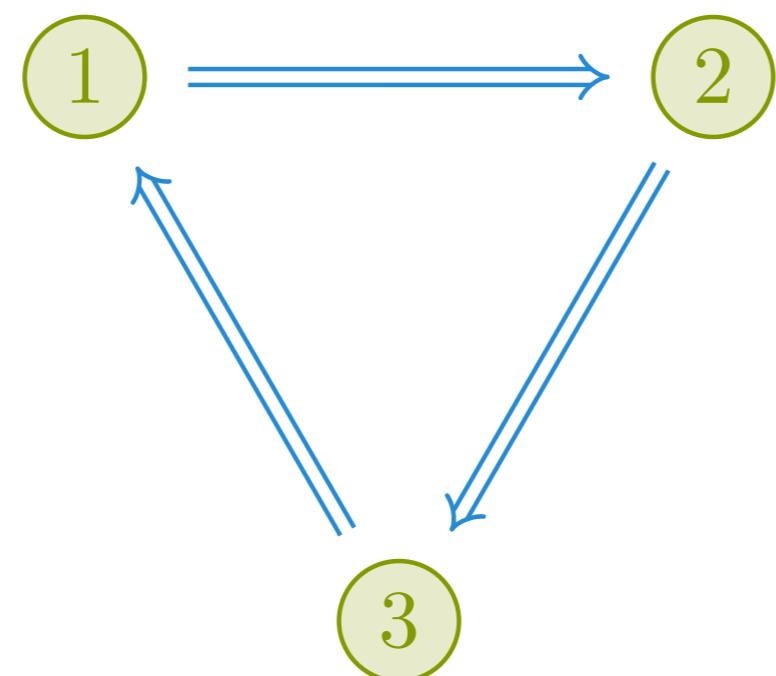
$G_f$  har ingen forøkende sti



$|f| = c(S, T)$  for et snitt  $(S, T)$

$f$  er maks-flyt for  $G$

$G_f$  har ingen forøkende sti



$|f| = c(S, T)$  for et snitt  $(S, T)$

Eksempel på såkalt *dualitet*

$f$  er maks-flyt for  $G$

$G_f$  har ingen forøkende sti

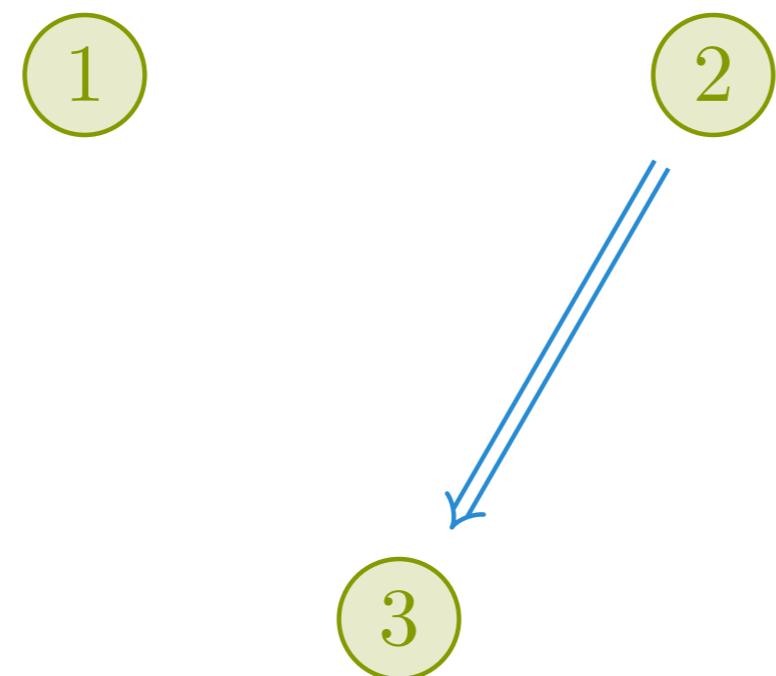


$|f| = c(S, T)$  for et snitt  $(S, T)$

Ved selvmotsigelse: En slik sti ville kunne øke  $f$

$f$  er maks-flyt for  $G$

$G_f$  har ingen forøkende sti

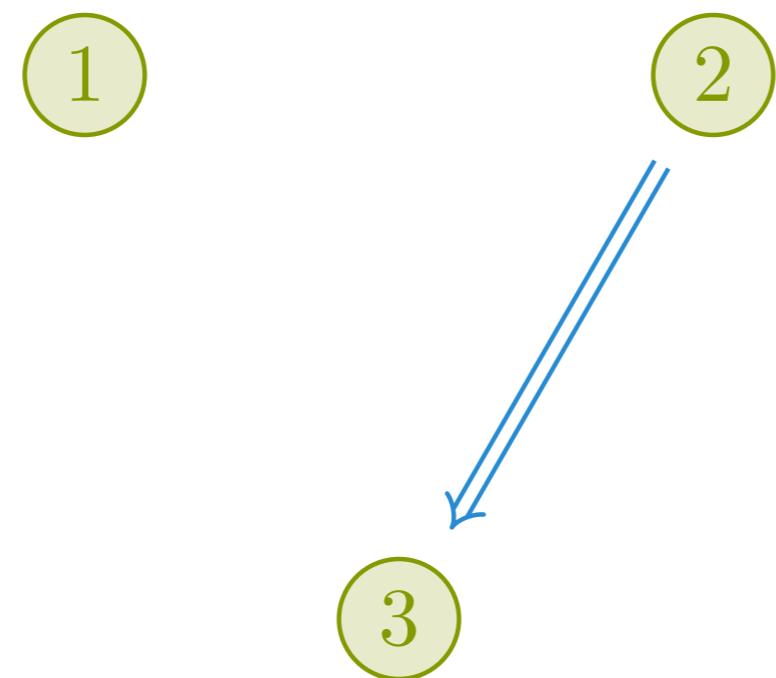


$|f| = c(S, T)$  for et snitt  $(S, T)$

La  $S$  være noder som kan nås i  $G_f$ , og la  $T = V - S$

$f$  er maks-flyt for  $G$

$G_f$  har ingen forøkende sti

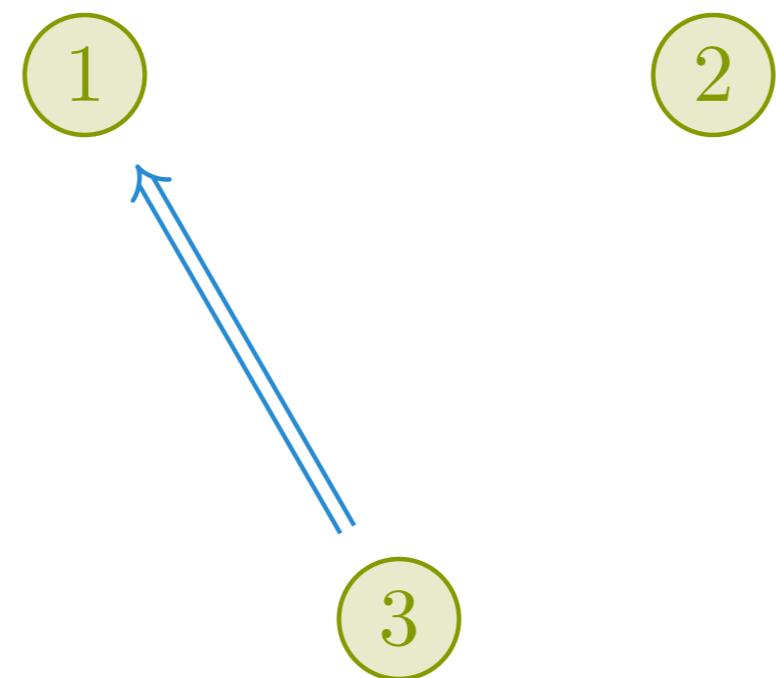


$|f| = c(S, T)$  for et snitt  $(S, T)$

Snittet blokkerer: Kanter er fulle ( $S \rightarrow T$ ) eller tomme ( $S \leftarrow T$ )

$f$  er maks-flyt for  $G$

$G_f$  har ingen forøkende sti

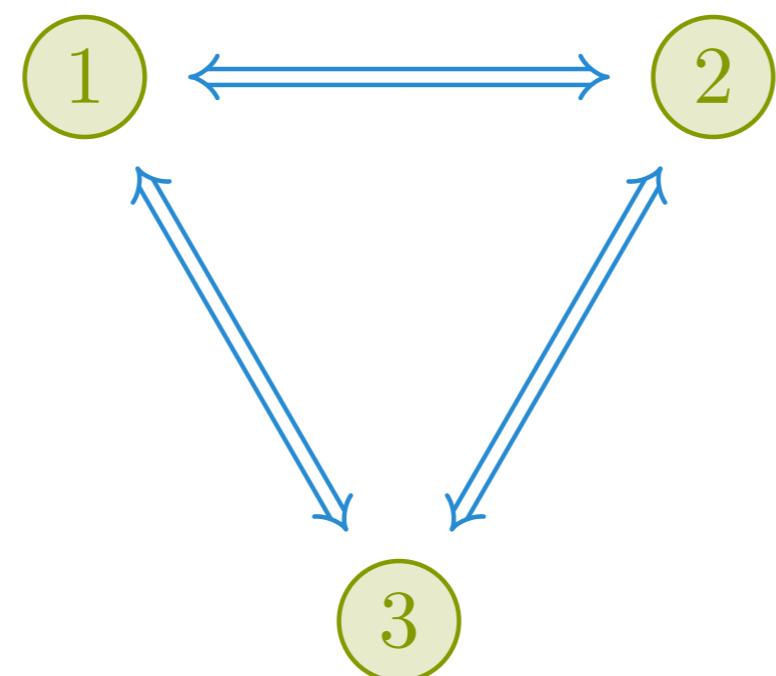


$$|f| = c(S, T) \text{ for et snitt } (S, T)$$

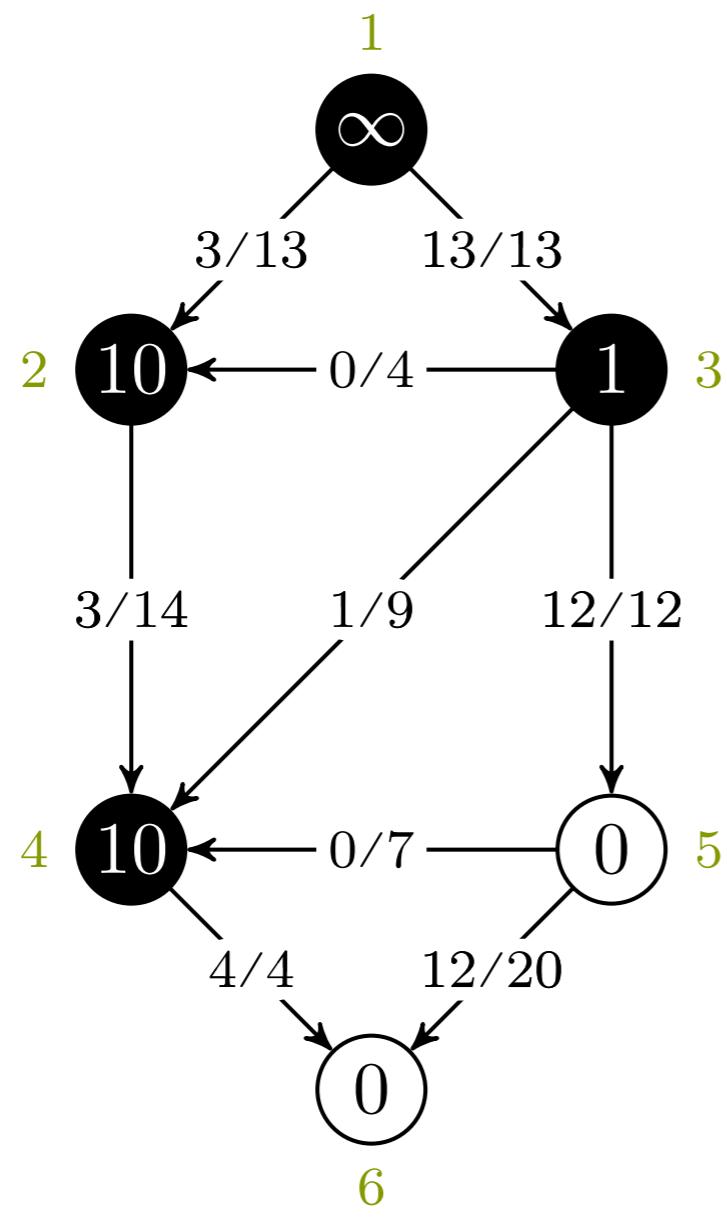
For enhver flyt har vi  $|f| \leq c(S, T)$ ; siden  $|f| = c(S, T)$  er  $f$  maksimal

$f$  er maks-flyt for  $G$

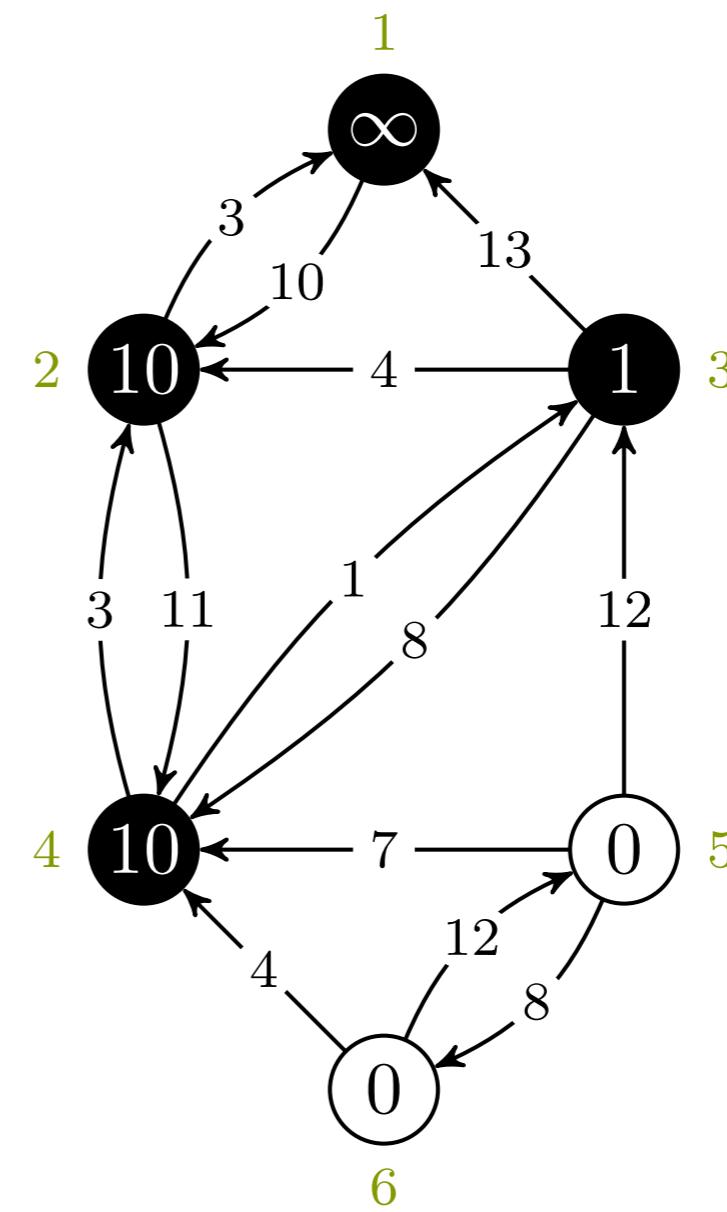
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$|f| = c(S, T)$  for et snitt  $(S, T)$

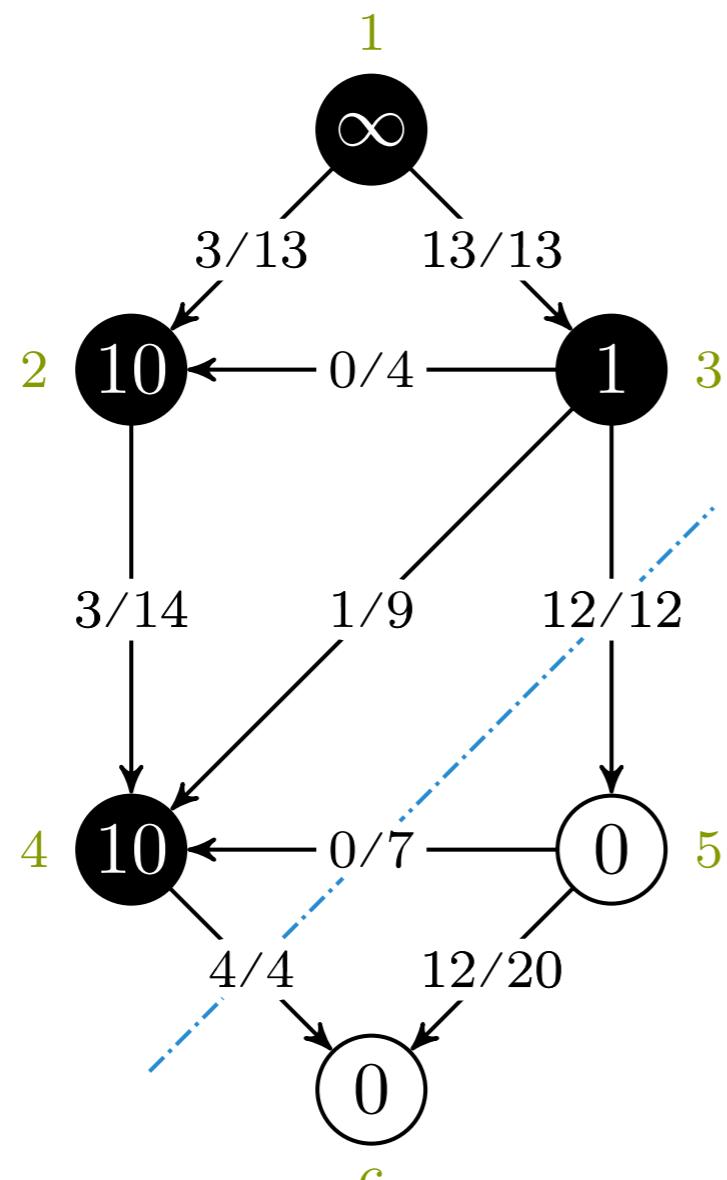


Flyt

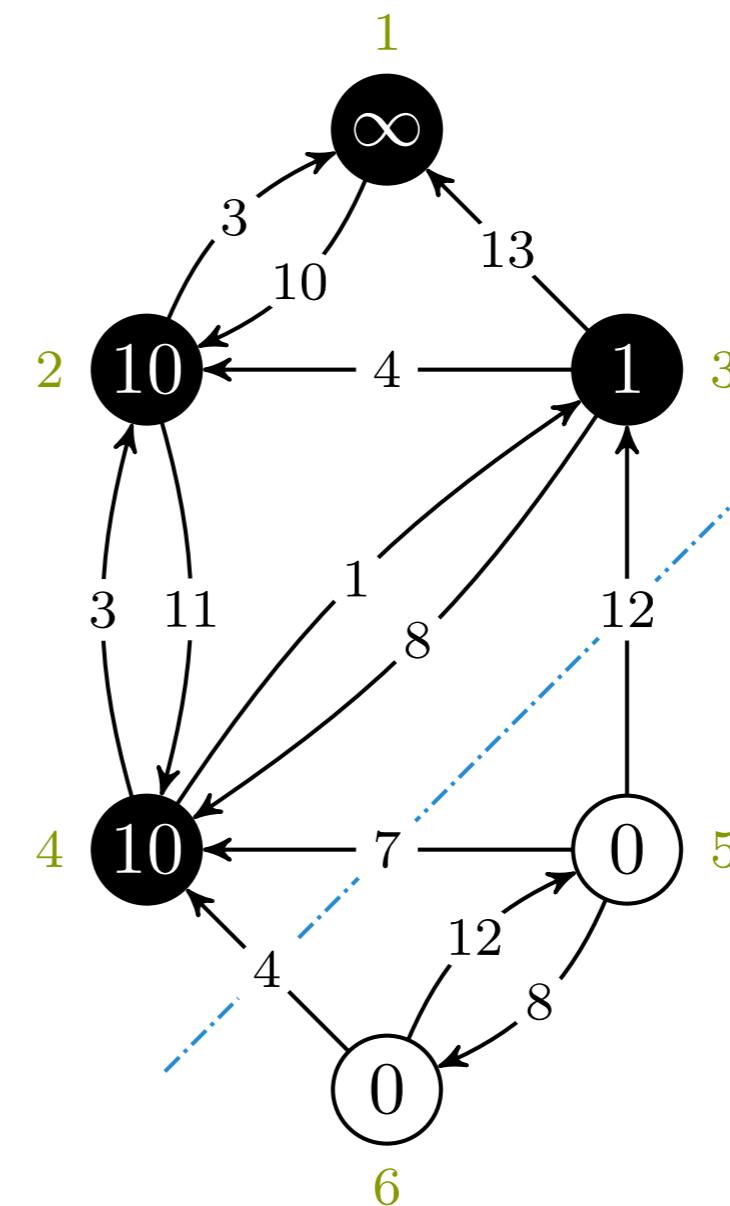


Rest

Etter kjøringen vår av EDMONDS-KARP

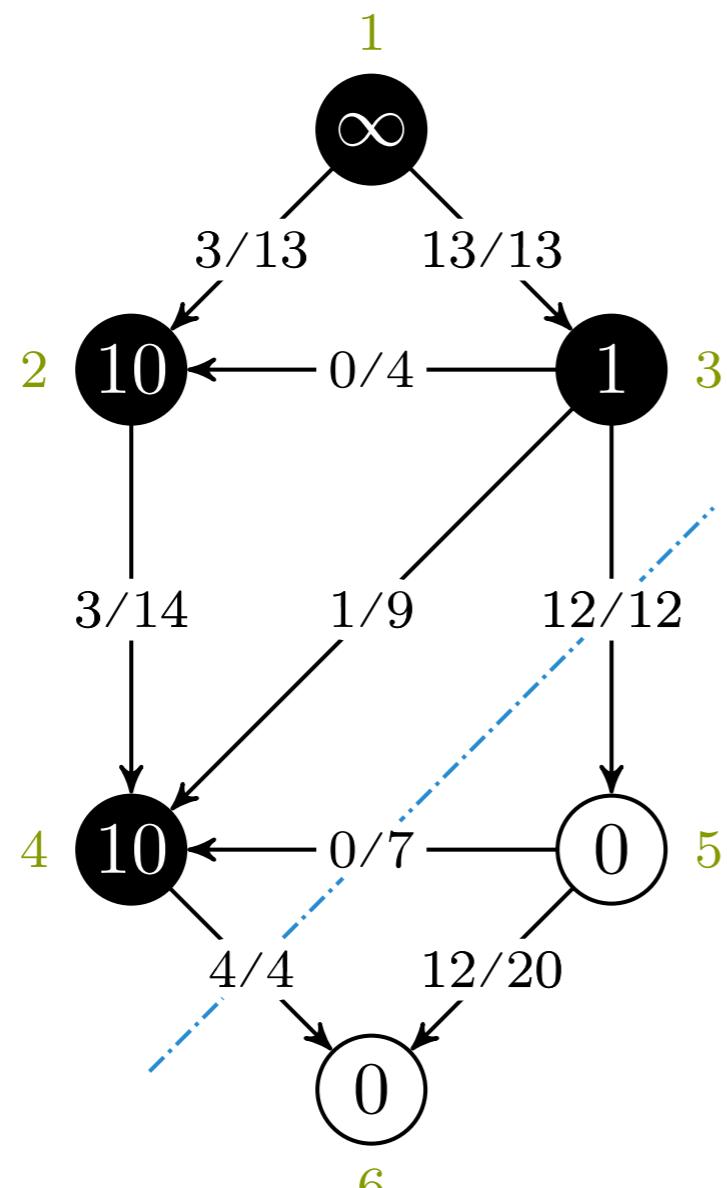


Flyt

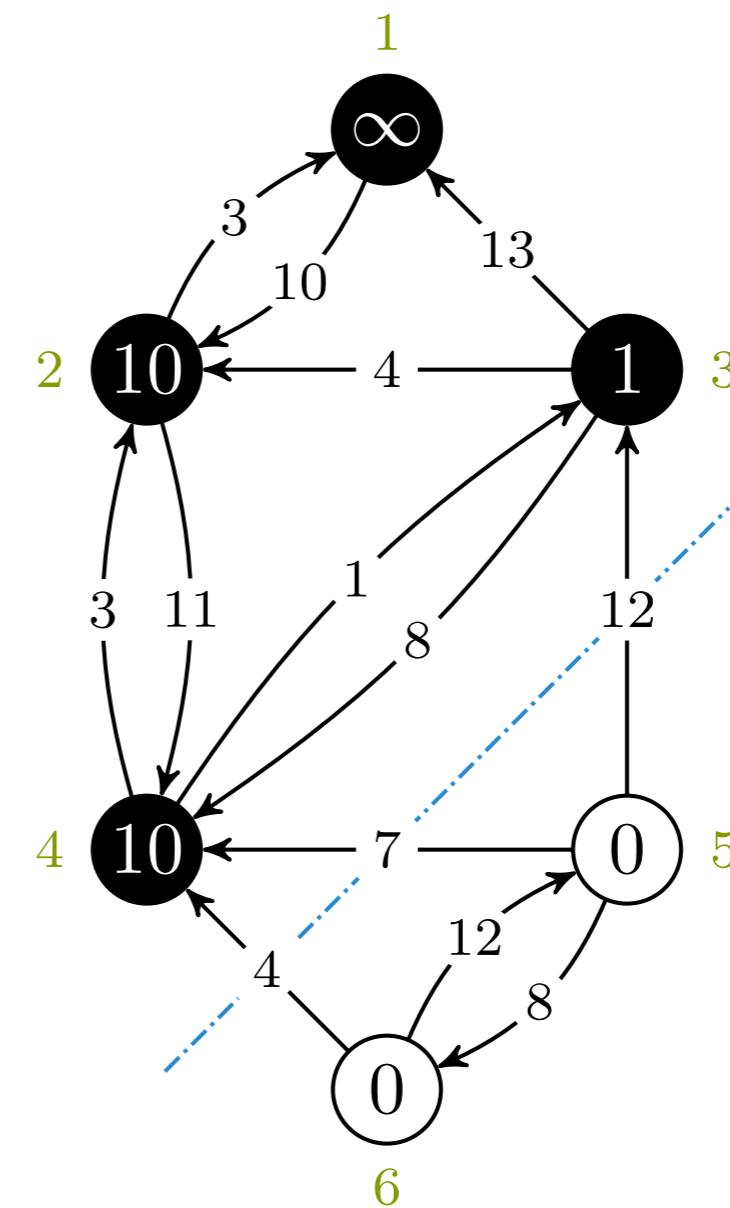


Rest

Min. snitt: Mellom svart og hvit.  $c(S, T) = f(S, T) = 16$



Flyt



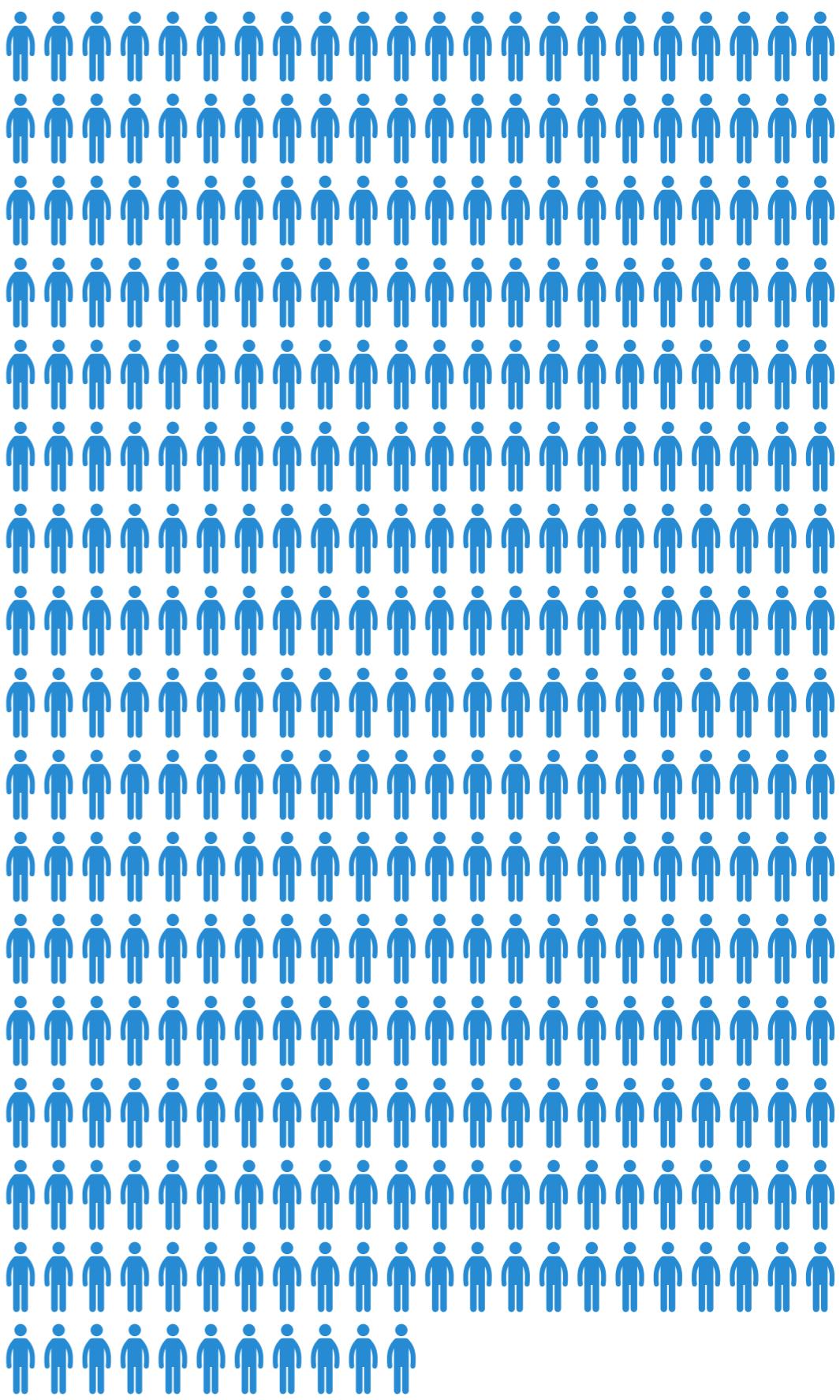
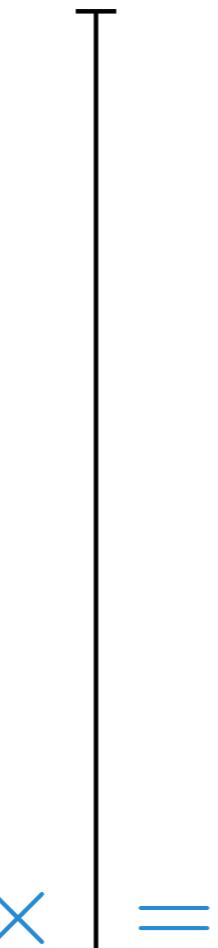
Rest

$S \rightarrow T$ : Fullt.  $S \leftarrow T$ : Tomt. Ingen sti  $S \rightsquigarrow T$  i restnetverk!

Så, endelig:  
Hvordan kommer vi fra ...

13.8 Ga

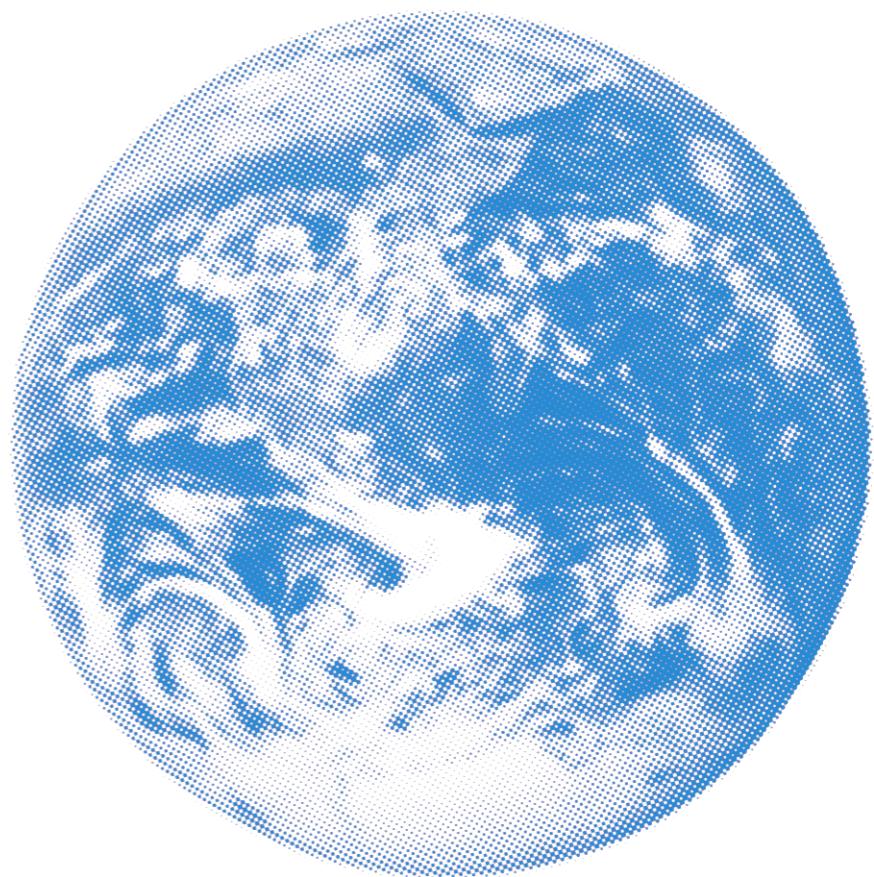
402



... til ...

$\frac{1}{2}h$ 

=



$\frac{1}{2}h?$



Vi bruker riktignok en litt langsommere algoritme enn det jeg brukte i utregningen, men likevel...

Dette er på mange vis et \*eksempel\* på bruk av flyt – eller \*reduksjon til flyt\*. Det går an å tenke seg at reduksjonen foregår i to trinn:

1. Reduser til flyt med mange kilder og sluk – hver donor er en kilde og hver mottaker er et sluk.
2. Reduser dette videre til å bruke én kilde og ett sluk, på den vanlige måten (med superkilde og supersluk).

5:5

Matching

**Matching:** Delmengde  $M \subseteq E$  for en urettet graf  $G = (V, E)$

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- › Ingen av kantene i  $M$  deler noder
- › Bipartitt matching:  $M$  matcher partisjonene

---

**Input:** En bipartitt urettet graf  $G = (V, E)$ .

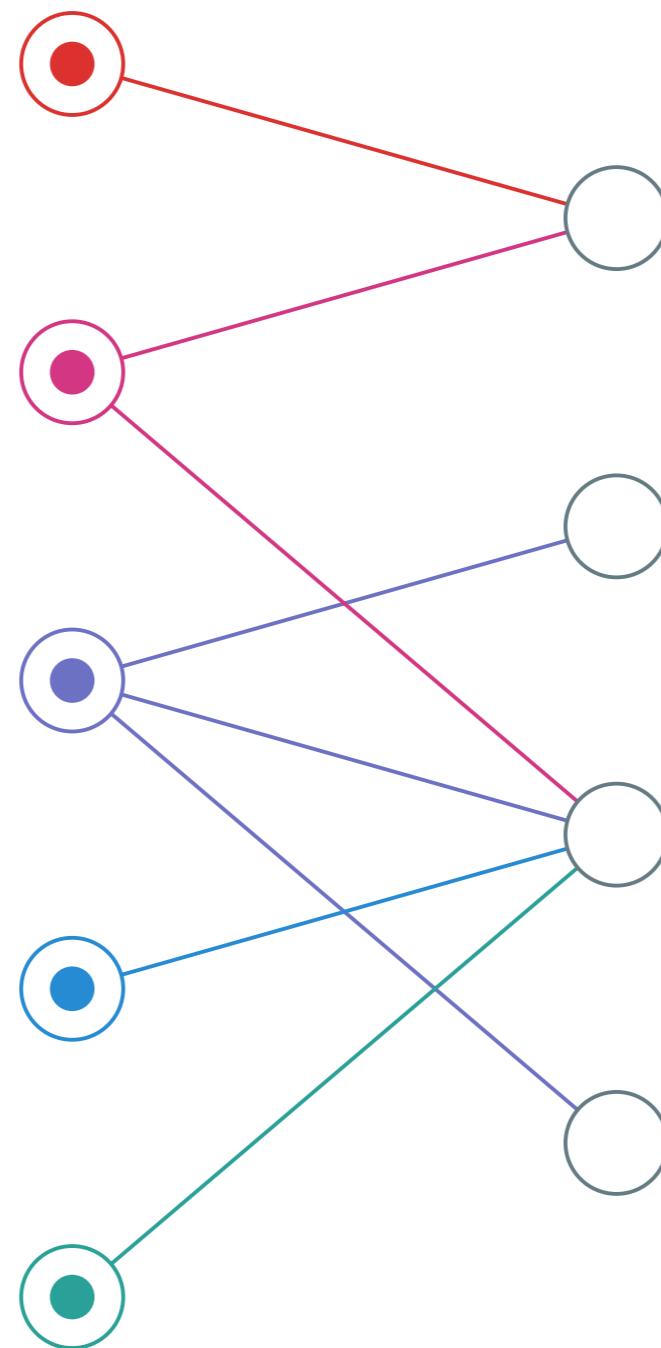
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**Input:** En bipartitt urettet graf  $G = (V, E)$ .

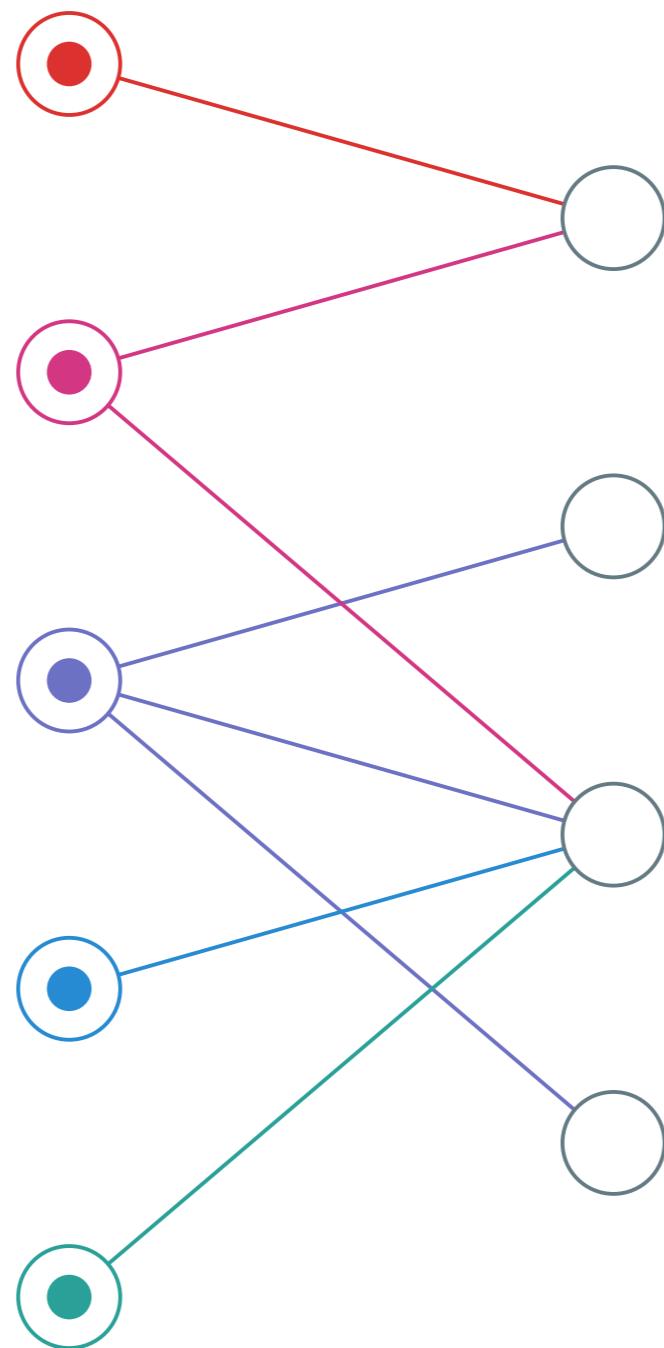
**Output:** En matching  $M \subseteq E$  med flest mulig kanter, dvs., der  $|M|$  er maksimal.

---

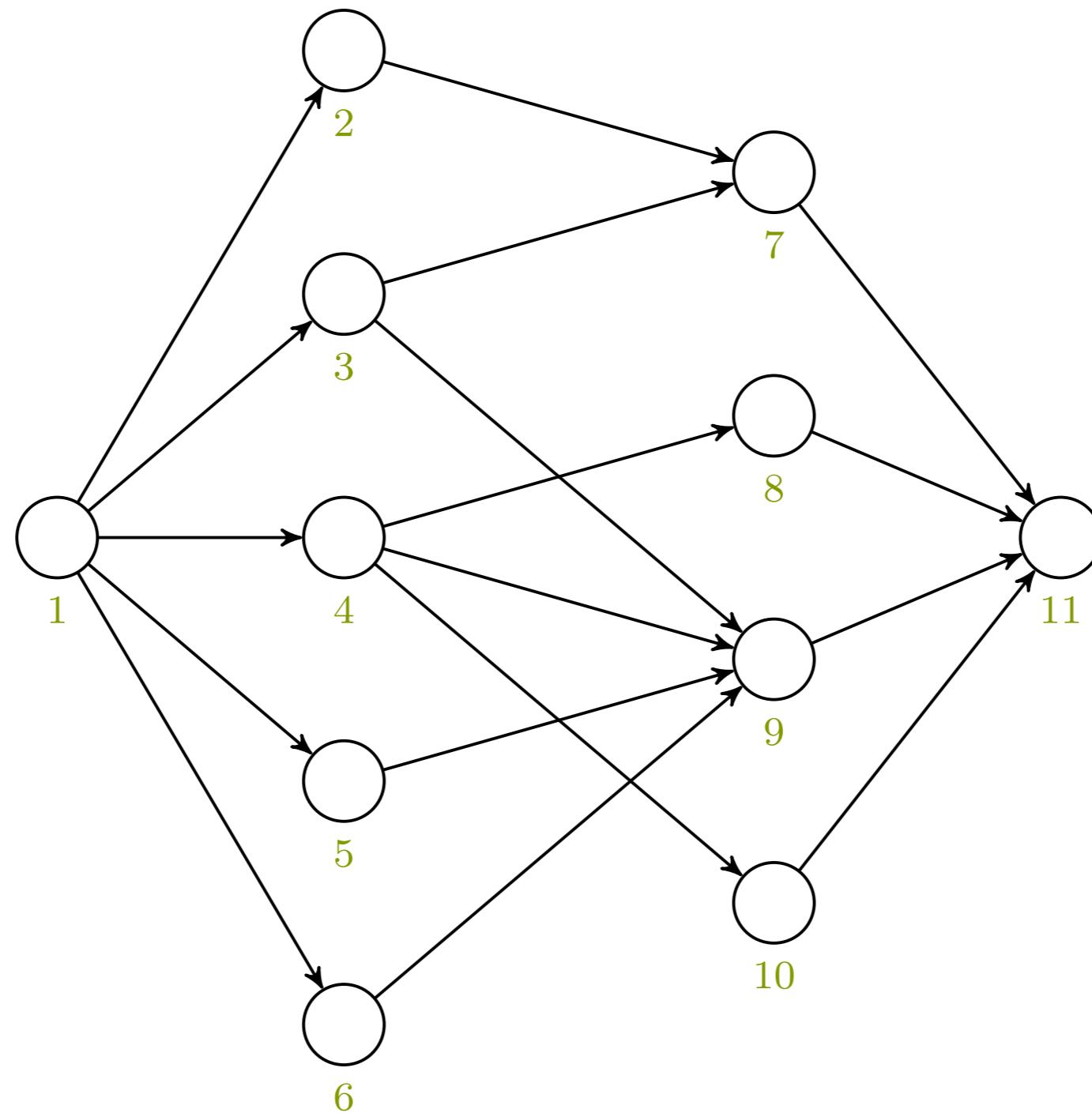


Hvordan får vi matchet flest mulig?

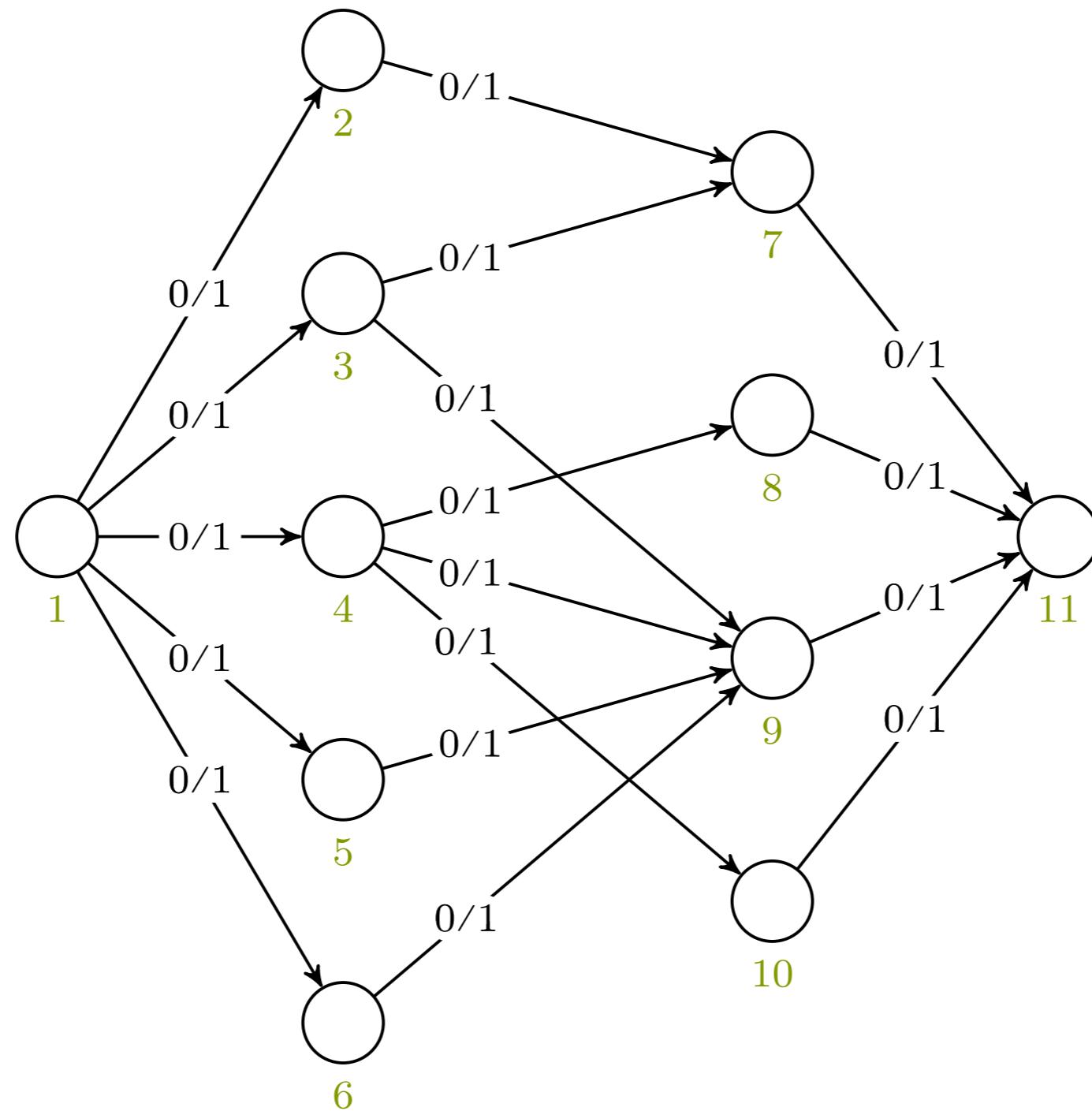
**Heltallsteoremet (26.10):**  
For heltallskapasiteter gir Ford-Fulkerson  
heltallsflyt



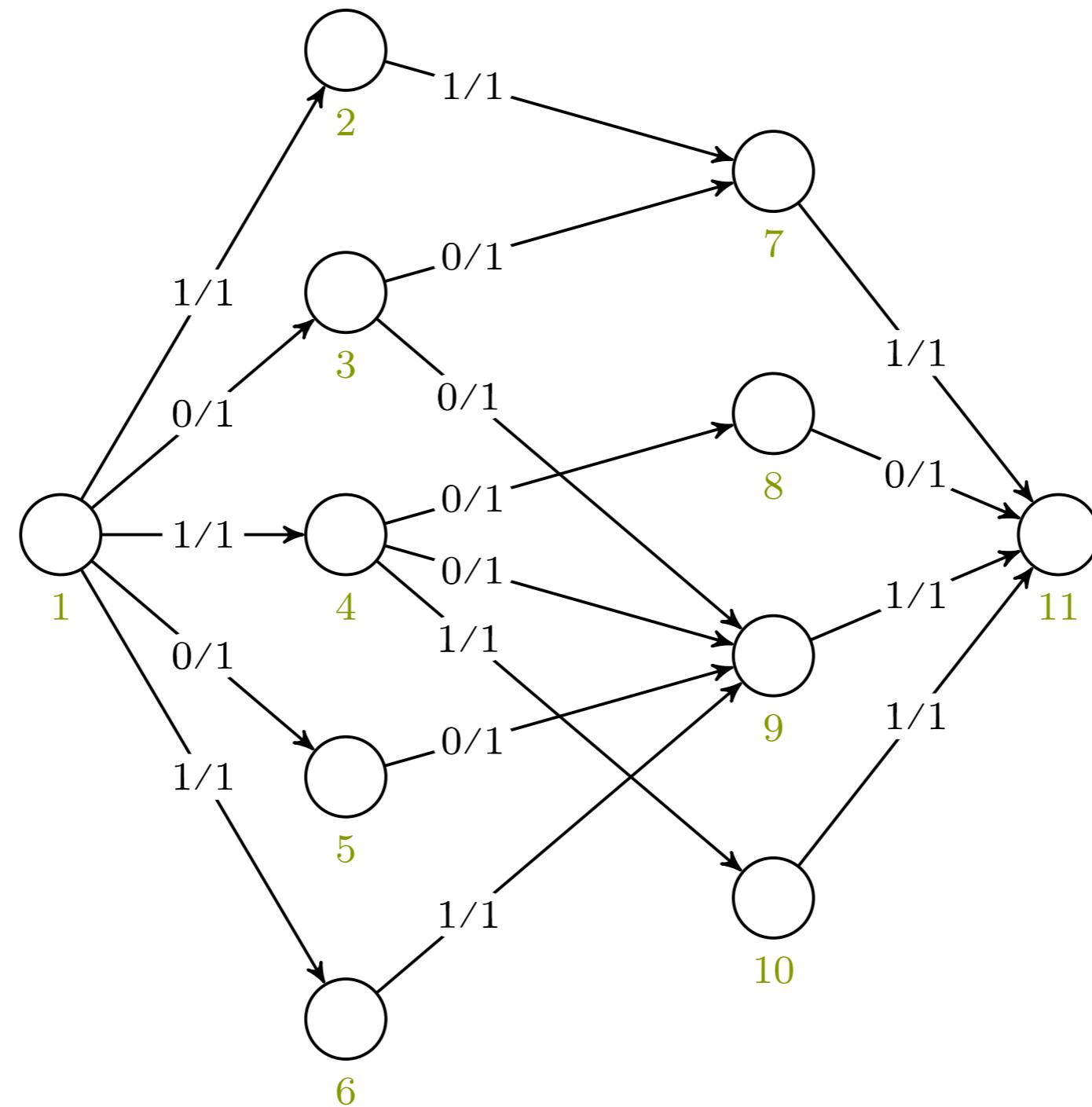
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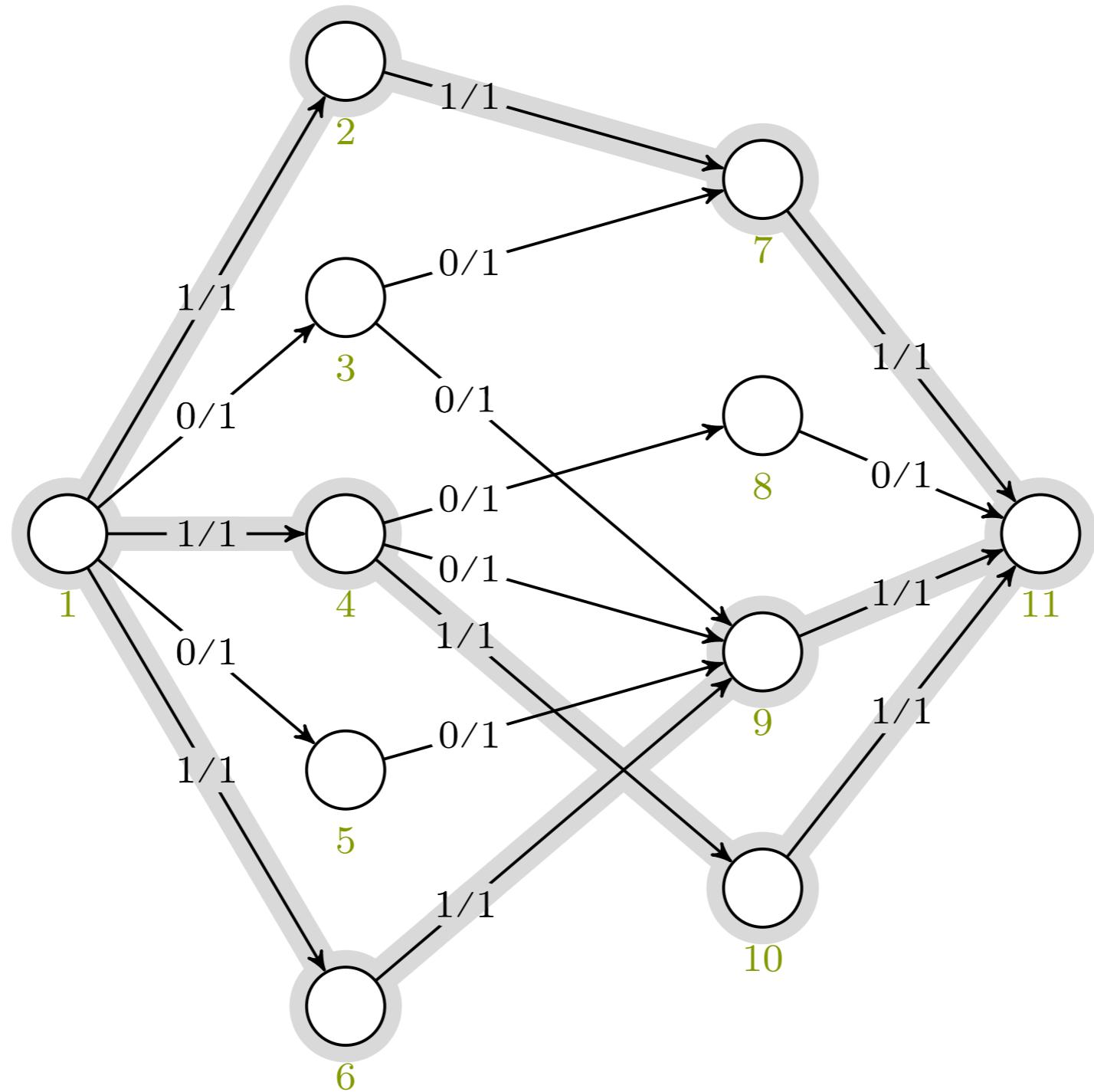
Legg til kilde og sluk



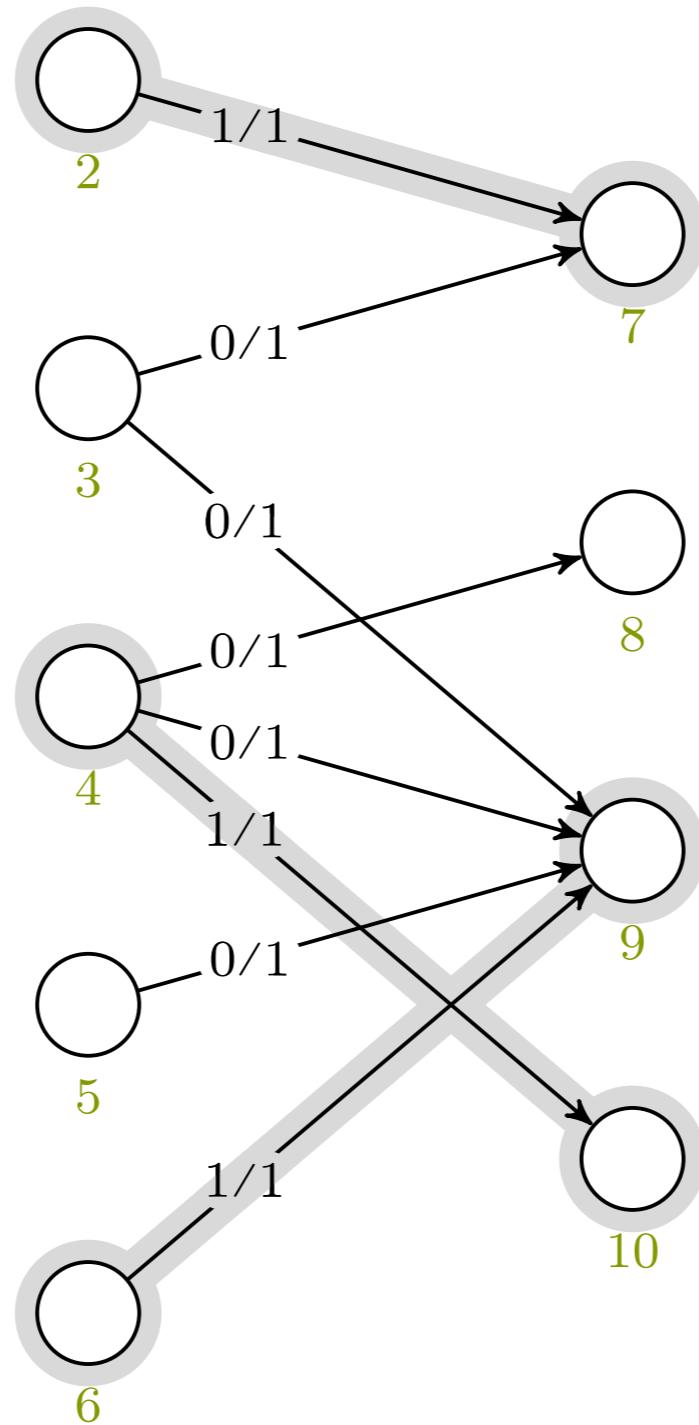
Hver kant og hver node kan inngå i maks ett par



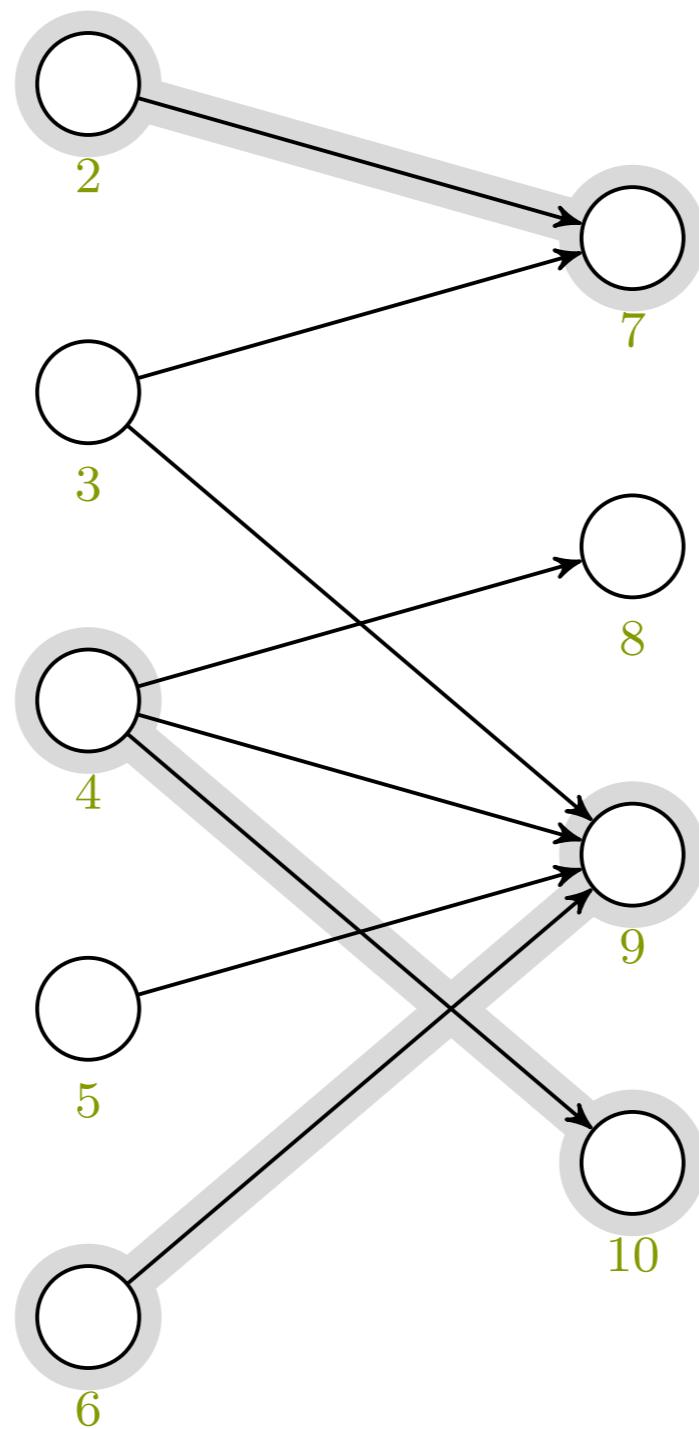
Kanter med flyt inngår i matchingen



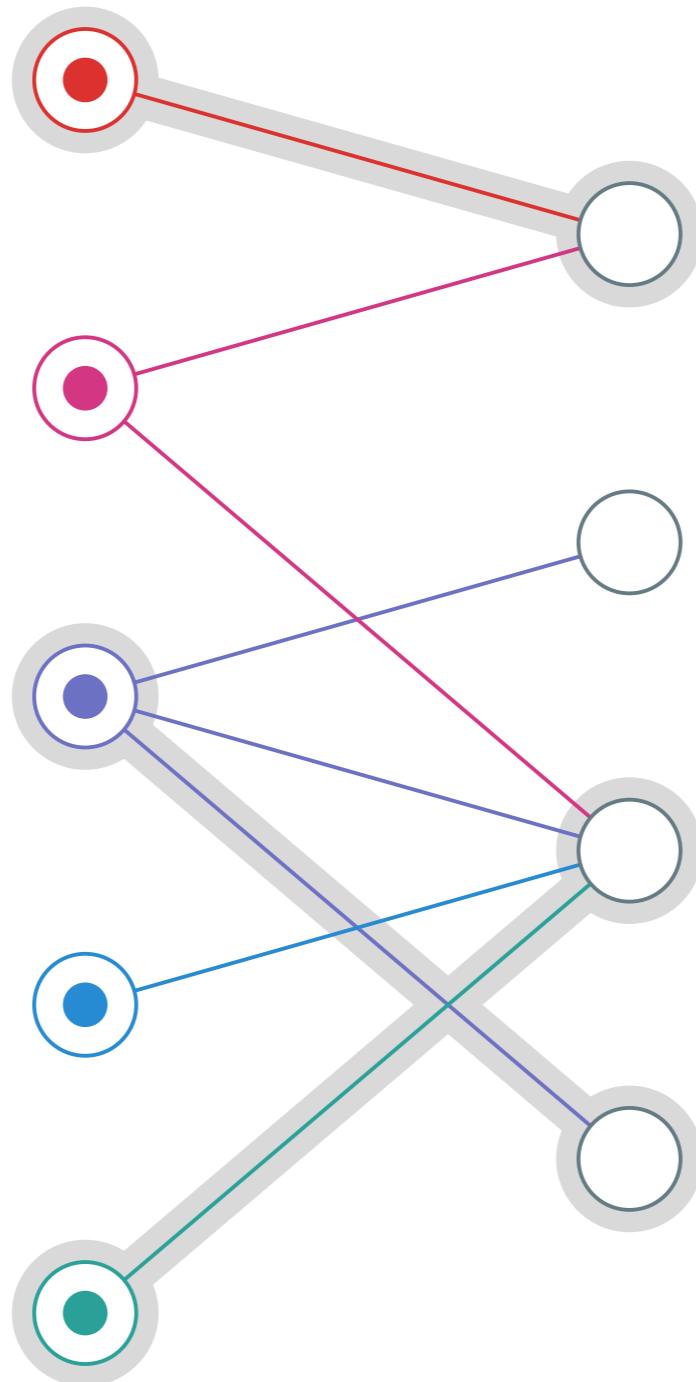
Kanter med flyt inngår i matchingen



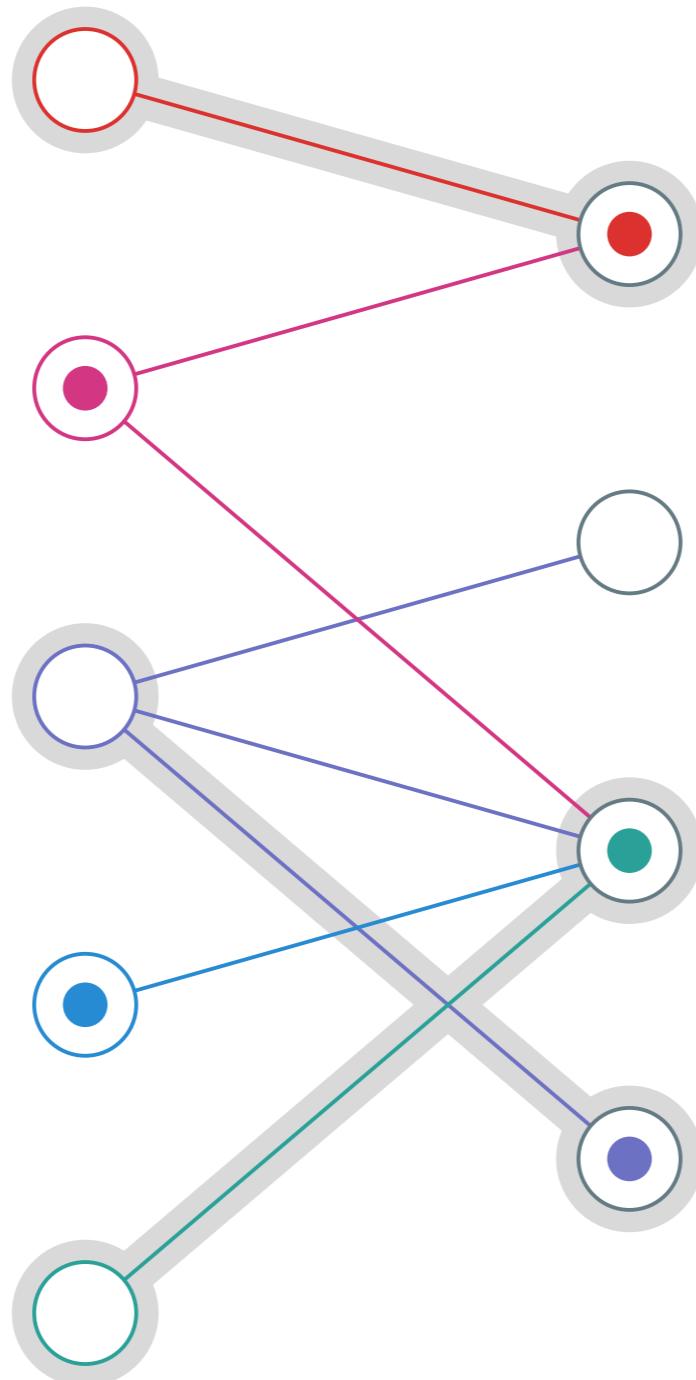
Flytnettverket har gjort jobben sin



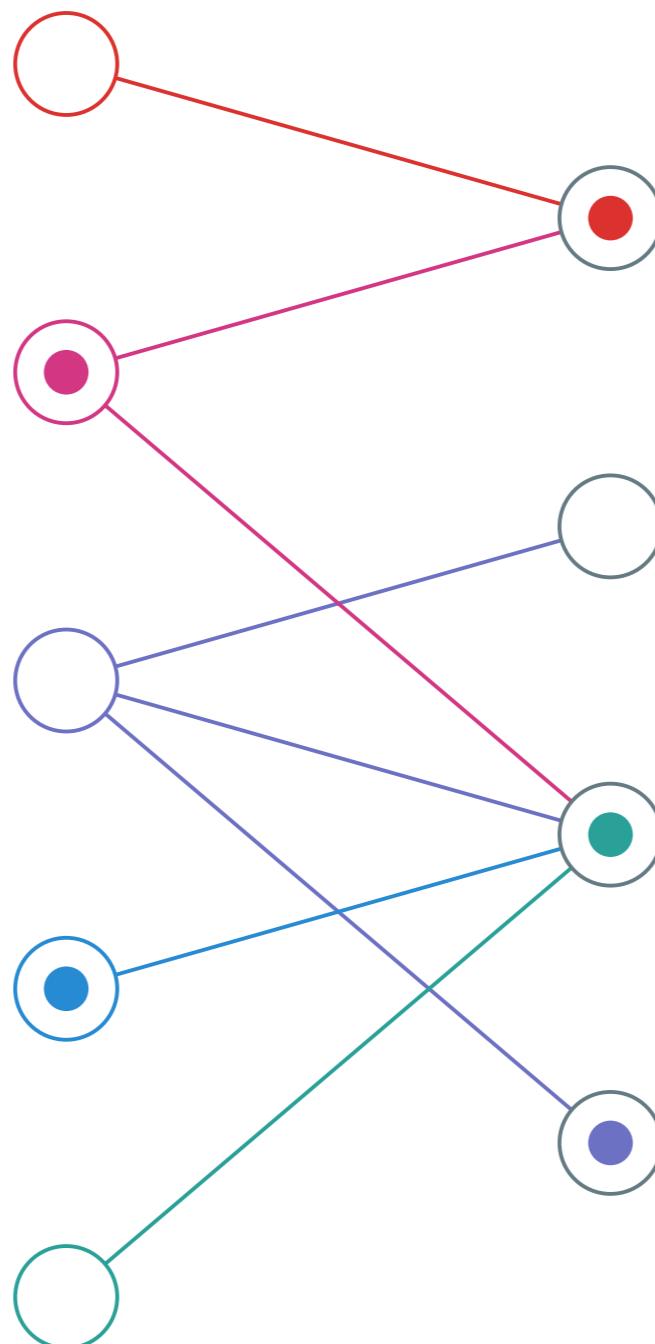
Flytnettverket har gjort jobben sin



Og her har vi endelig løsningen!



Og her har vi endelig løsningen!



Det finnes mer effektive løsninger enn Edmonds-Karp for bipartitt matching. F.esk. Hopcroft-Karp, som har en liten vri: I stedet for å finne én forkende sti, så finner den så mange som mulig som ikke krysser hverandre. Kjøretiden går fra  $O(VE^2)$  til  $O(E \cdot \sqrt{V})$ . Det finnes mange andre algoritmer som virker på andre måter (og som har bedre kjøretider) for både flyt generelt, og matching mer spesifikt.

Og her har vi endelig løsningen!





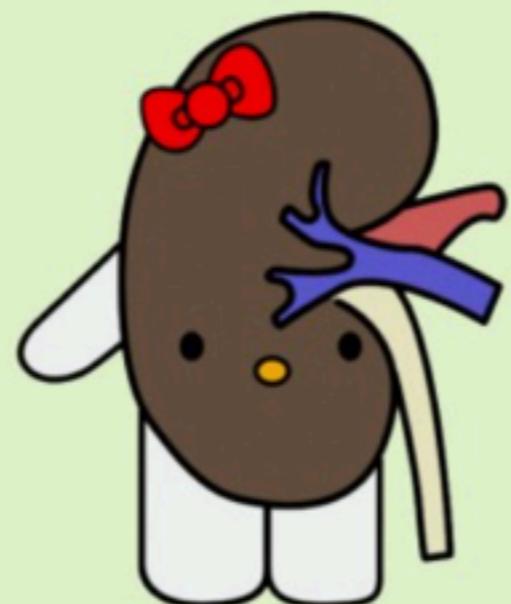
SCIENTIFIC METHOD —

# The math of organ donation: Kidneys are an NP-hard problem

Matching donors and recipients is a bit like the traveling salesman problem.

JOHN TIMMER - 1/6/2015, 3:18 PM

Dette gjelder naturligvis en litt annen variant enn det vi har sett på. Akkurat hva dette innebærer betyr kommer vi tilbake til i neste forelesning.



Hello Kidney

THAT  
ESCALATED  
QUICKLY



We're bilaterally symmetric organisms—we've got matching bits on our left and right side. But many critical organs are present in only a single copy (hello heart) or we need both to function optimally (see: lungs). The kidneys are rare exceptions, as your body gets by just fine with only a single one. That has enabled people to become living kidney donors, with both the donor and recipient continuing life with one kidney.



ars TECHNICA

BIZ & IT TECH SCIENCE POLICY CARS GAMING & CULTURE FORUMS

SIGN IN



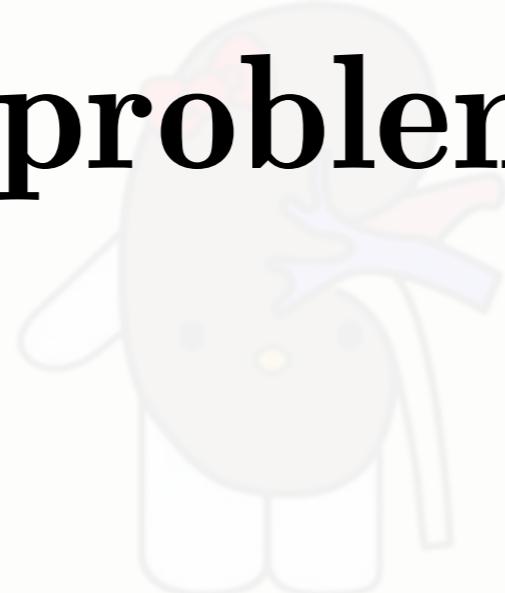
SCIENTIFIC METHOD —

## The math of organ donation: Kidneys are an NP-hard problem

Matching donors and recipients is a bit like the traveling salesman problem.

JOHN TIMMER - 1/15/2015

Merk: Det gjelder et litt annet problem, ikke matching



Hello Kidney

We're bilaterally symmetric organisms—we've got matching bits on our left and right side. But many critical organs are present in only a single copy (hello heart) or we need both to function optimally (see: lungs). The kidneys are rare exceptions, as your body gets by just fine with only a single one. That has enabled people to become living kidney donors, with both the donor and recipient continuing life with one kidney.

1. Problemet

2. Ideer

3. Ford-Fulkerson

4. Minimalt snitt

5. Matching

For den nysgjerrige:  
[http://www.idi.ntnu.no/~mlh/  
algkon/flow.pdf](http://www.idi.ntnu.no/~mlh/algkon/flow.pdf)

# Bonusmateriale

Altså ting det er helt frivillig å se på – bare ting som har blitt til overs eller kanskje kan være nyttig.

# **Eksempel:**

# **Bildesegmentering**

maks-flyt › min. snitt › eksempel



- Hver piksel, én node
- Kap. fra kilde:  
«Forgrunnsaktighet»
- Kap. til sluk:  
«Bakgrunnsaktighet»
- Kap. mellom naboer:  
Hvor like de er

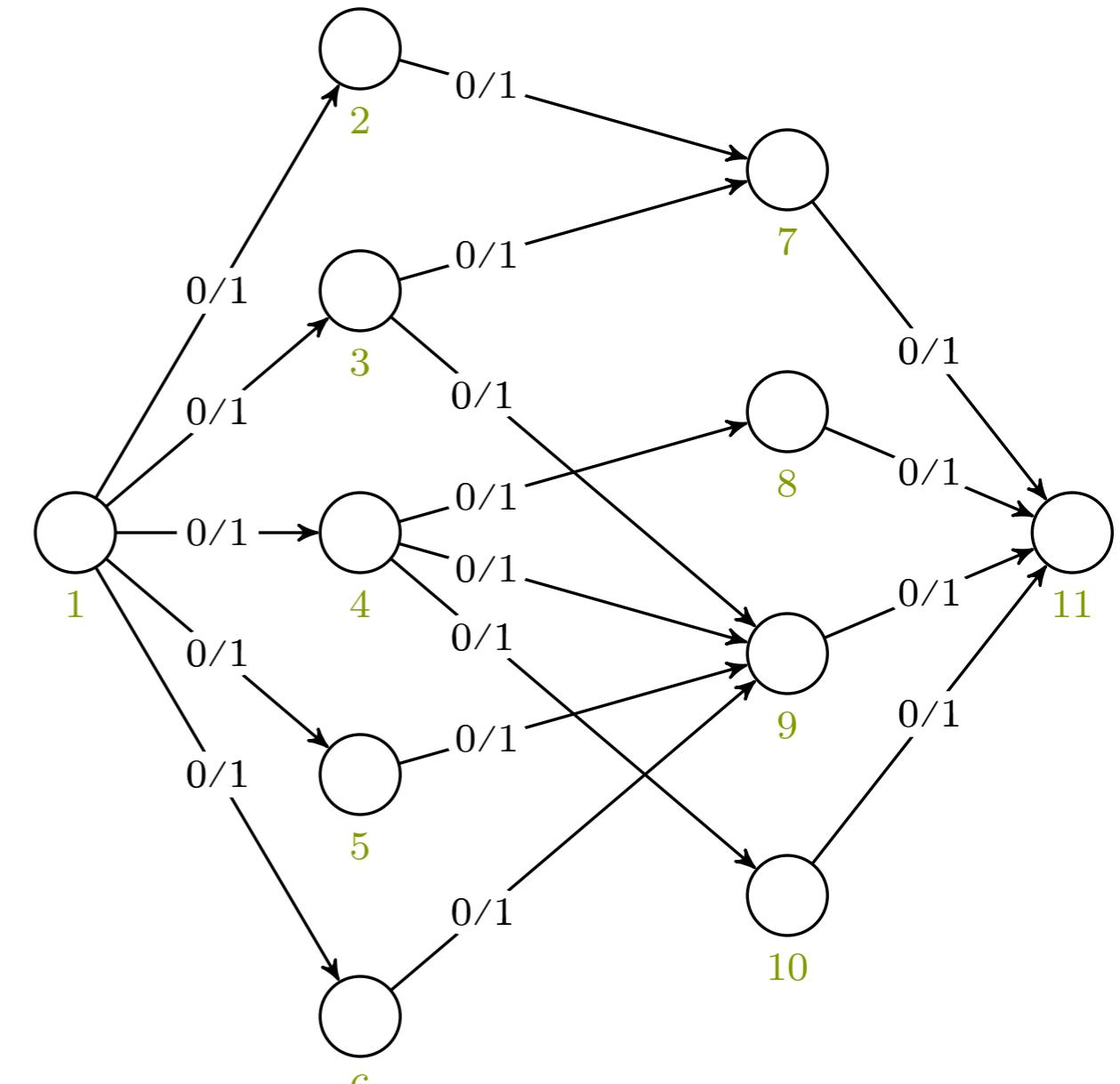


Minimalt snitt skiller forgrunn fra bakgrunn.

EDMONDS-KARP( $G, s, t$ )

```

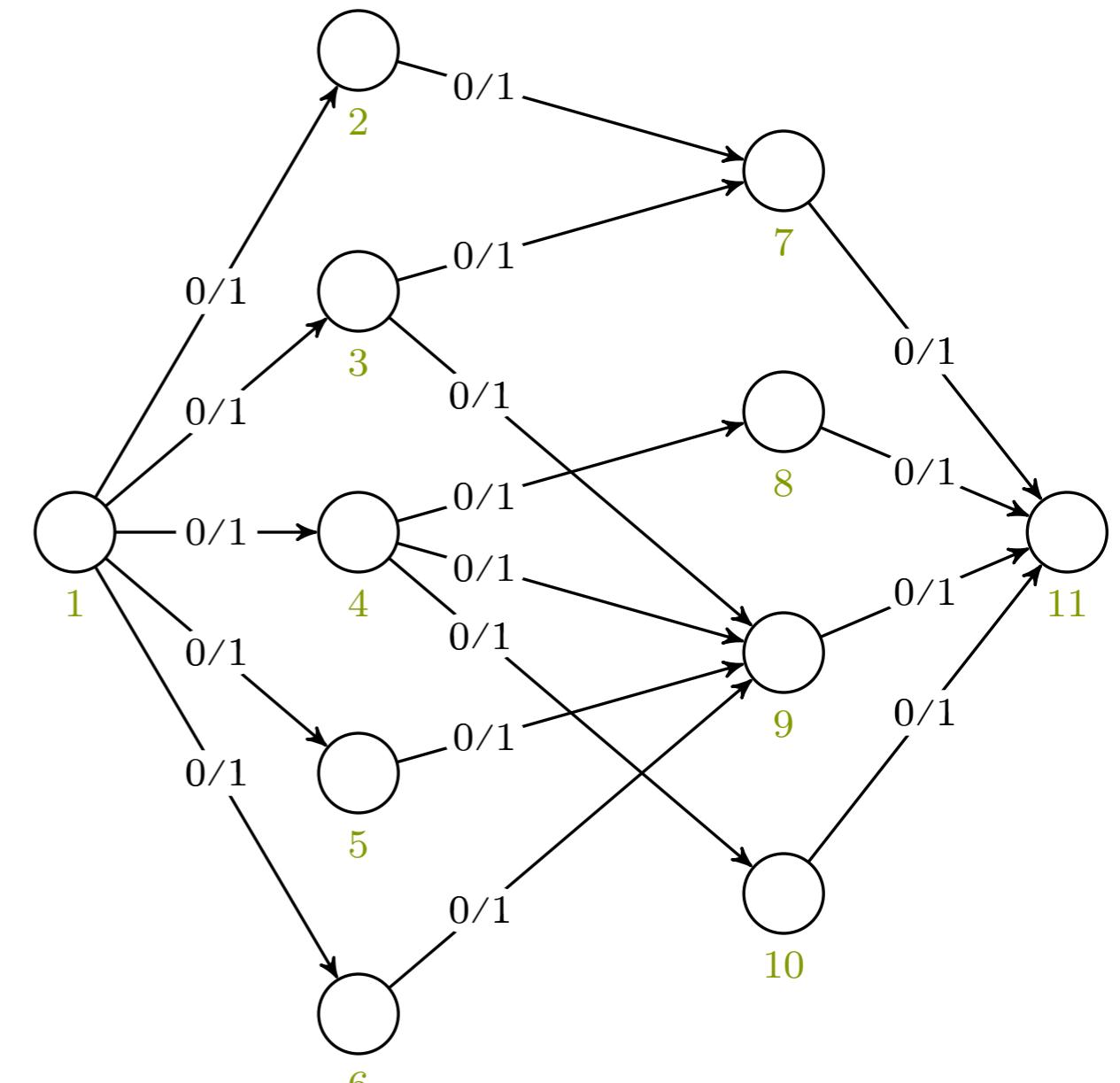
1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
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26 until  $t.a == 0$ 
```

 $u, v = -, -$ 

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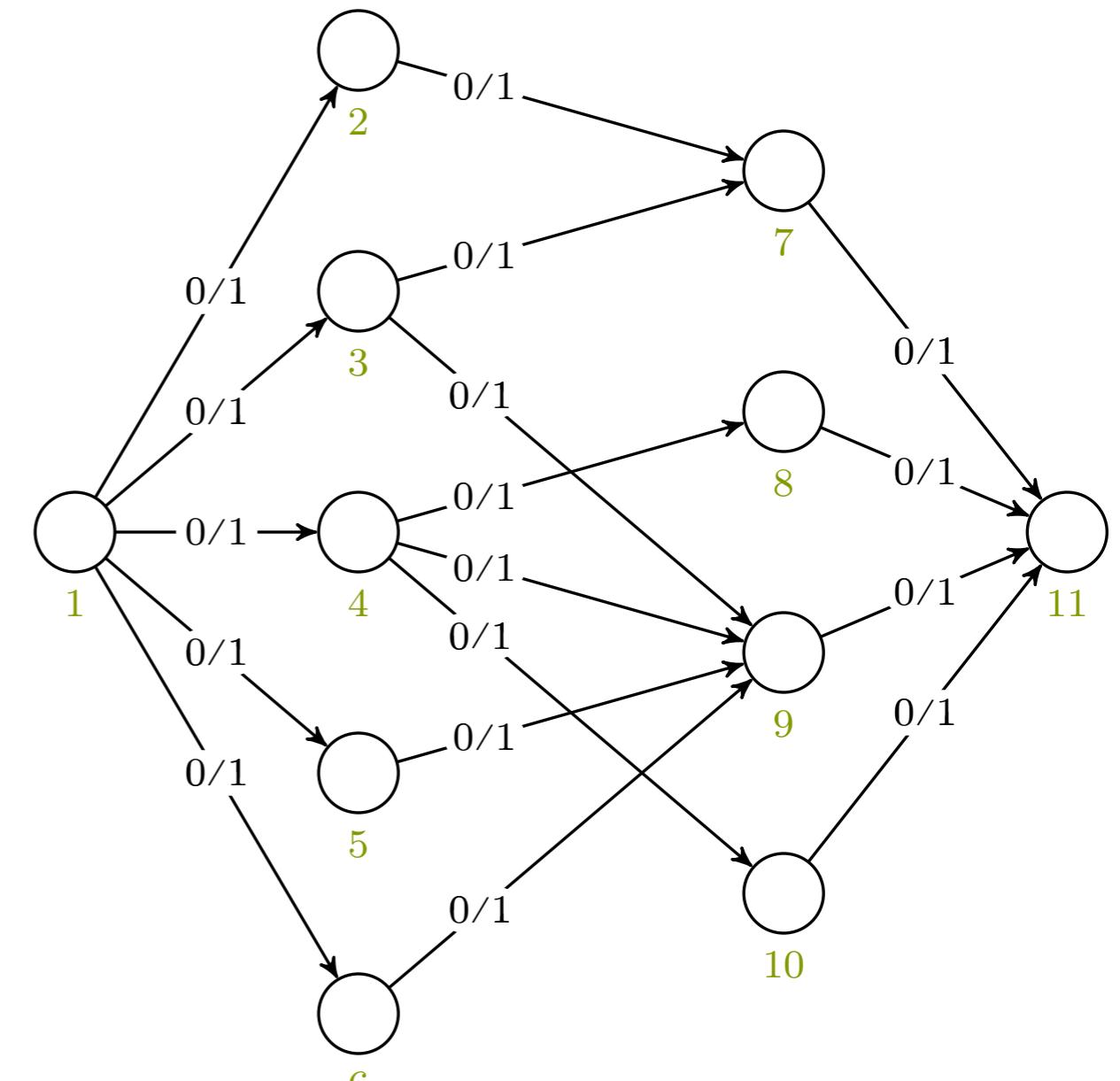
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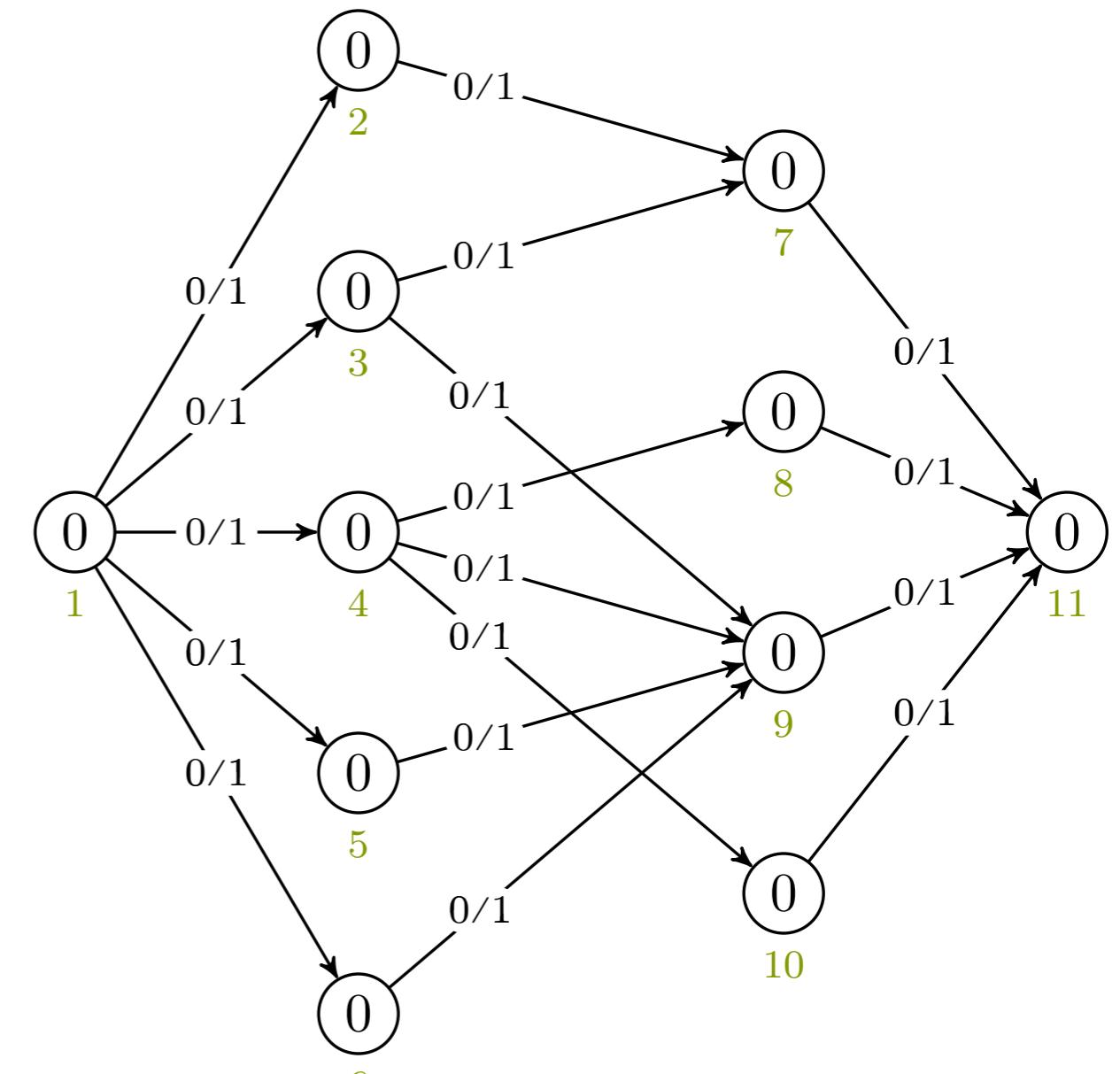
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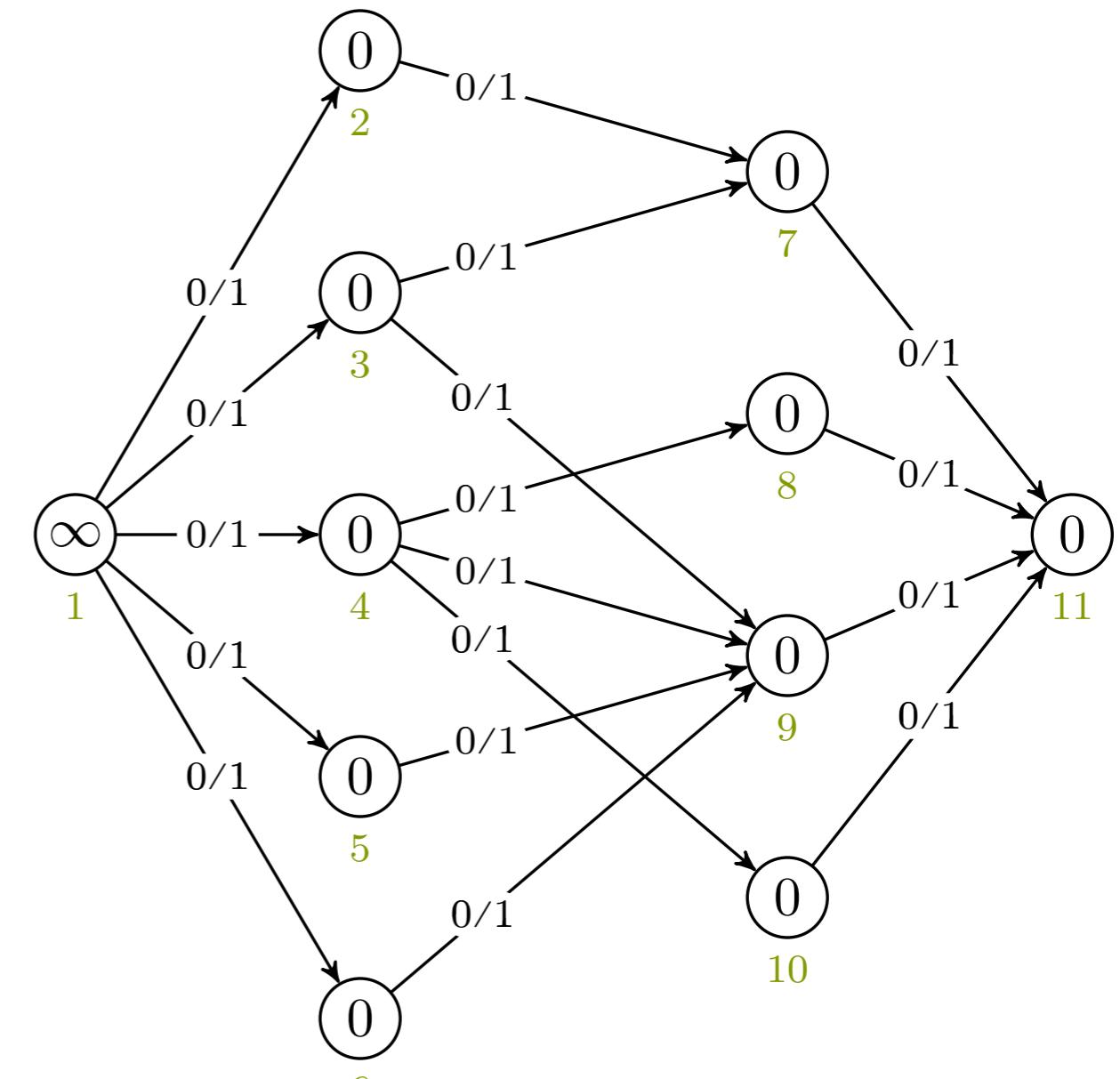
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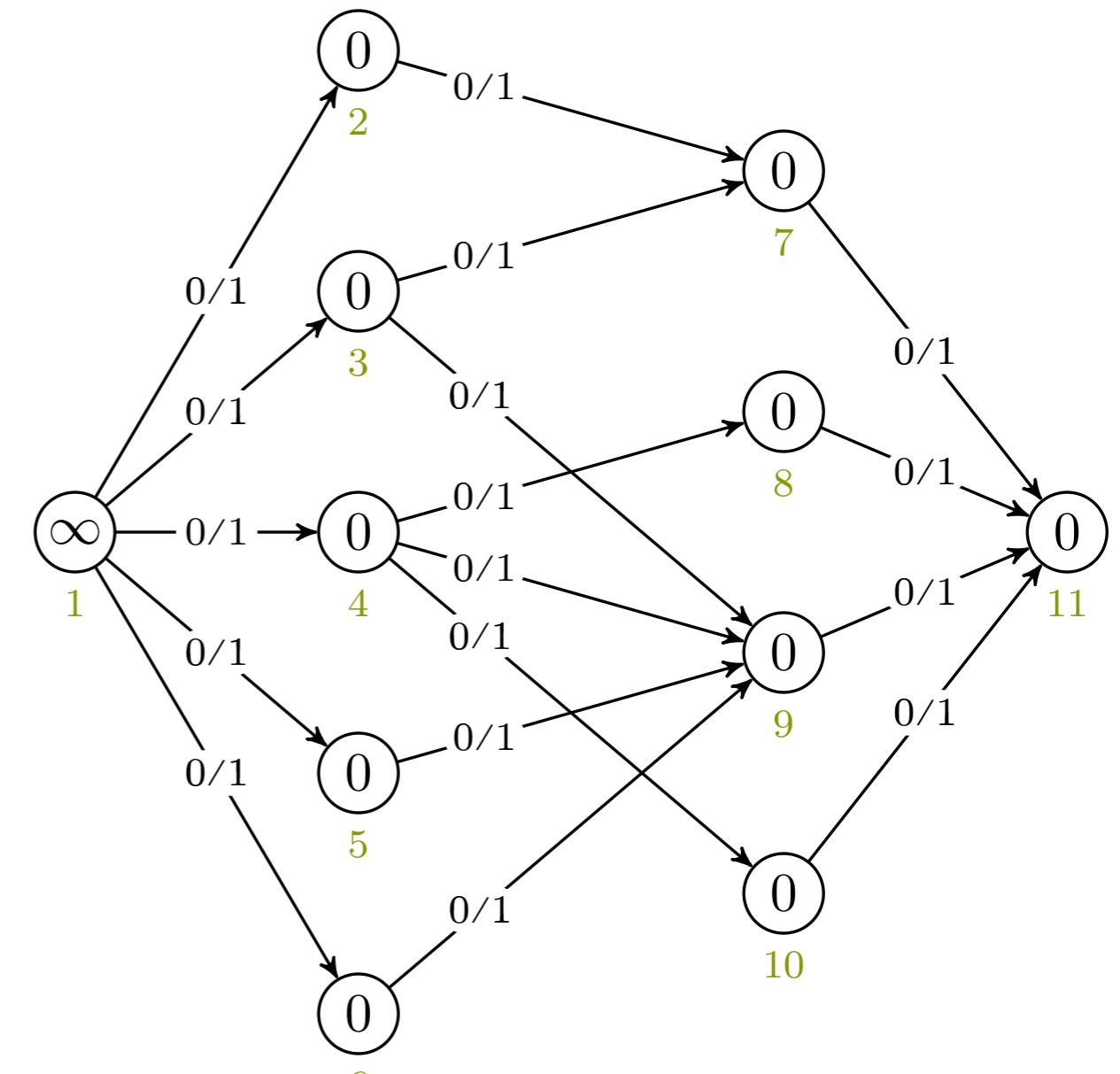
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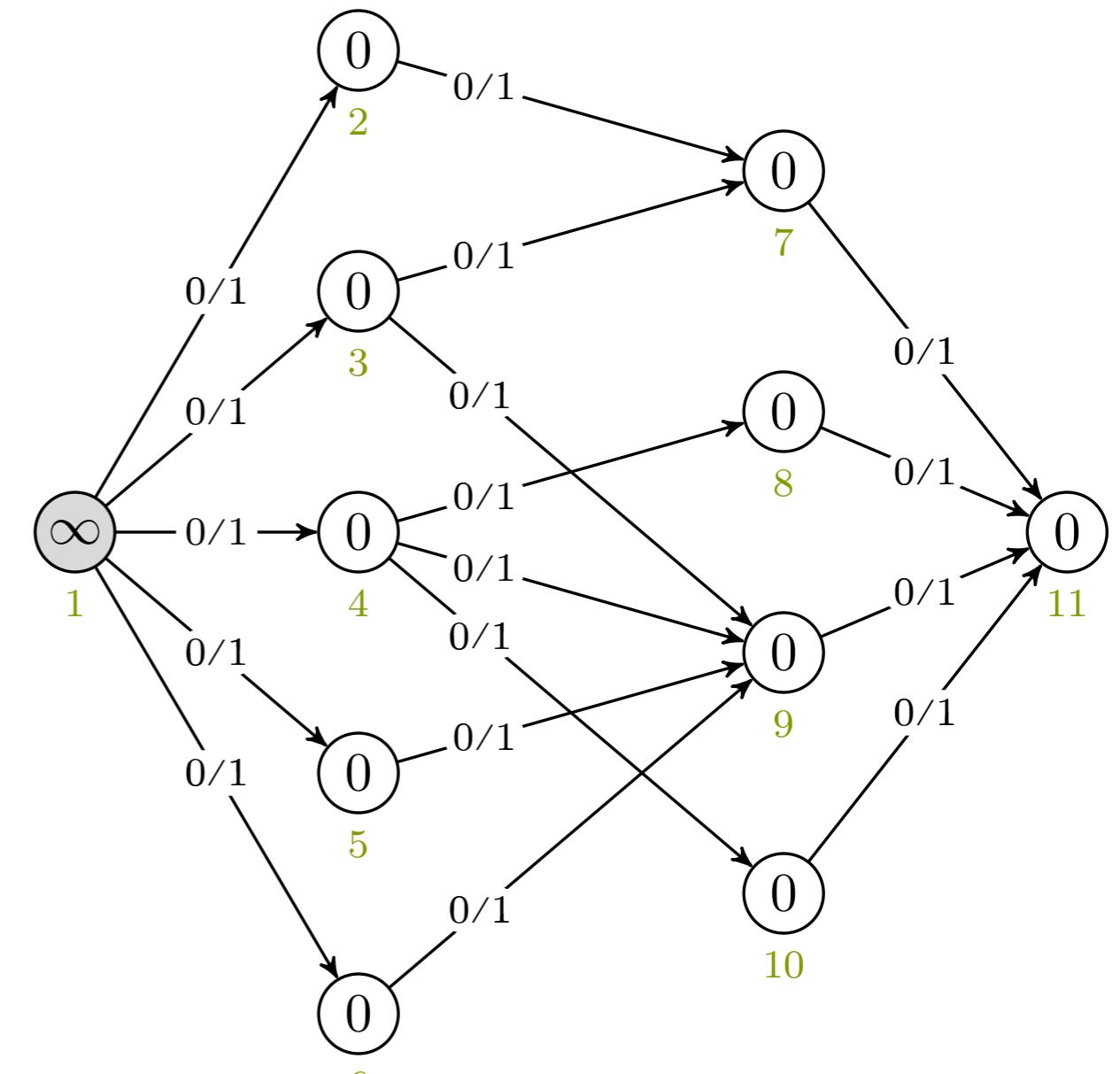
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EDMONDS-KARP( $G, s, t$ )

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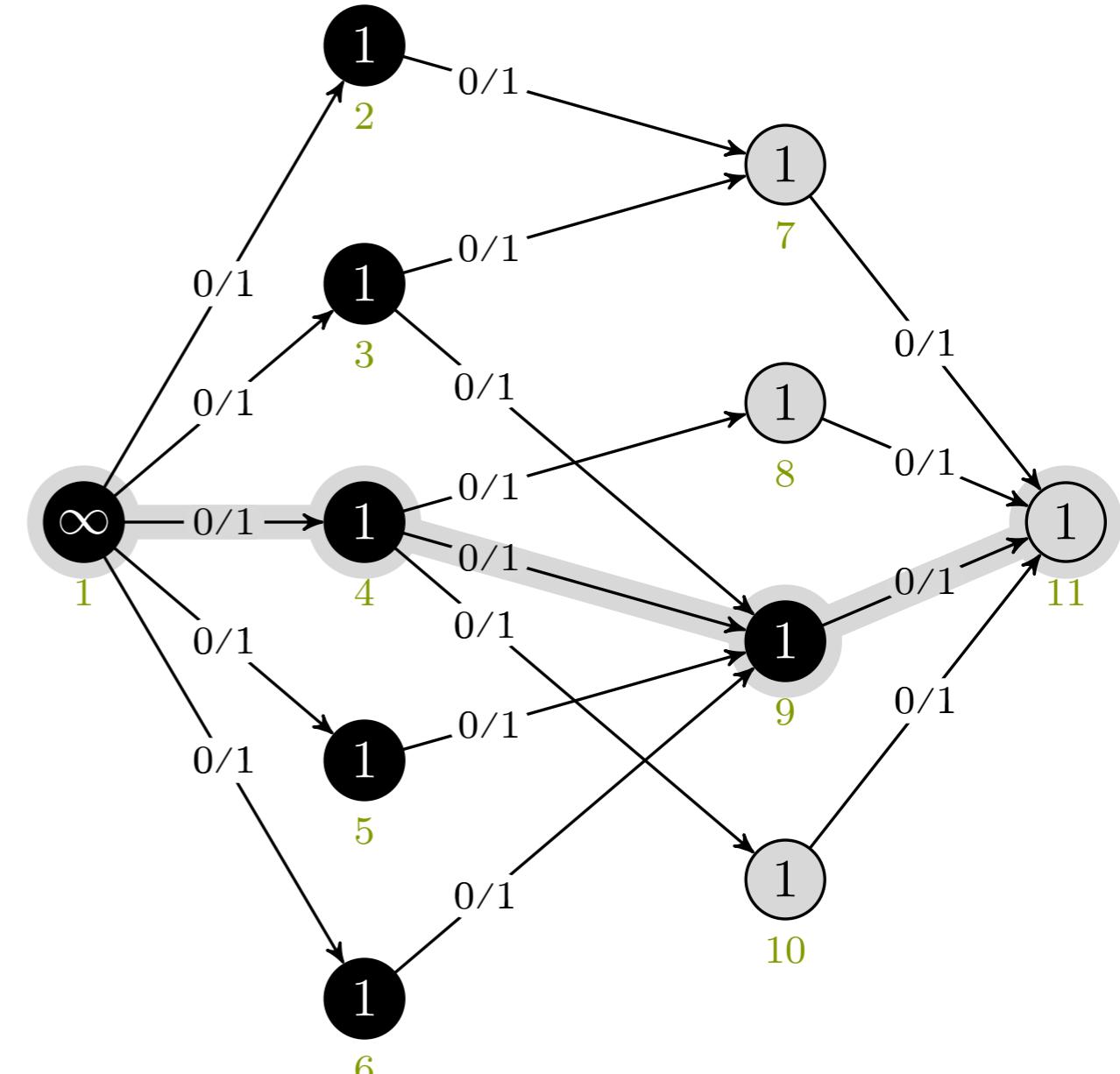
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EDMONDS-KARP( $G, s, t$ )

```

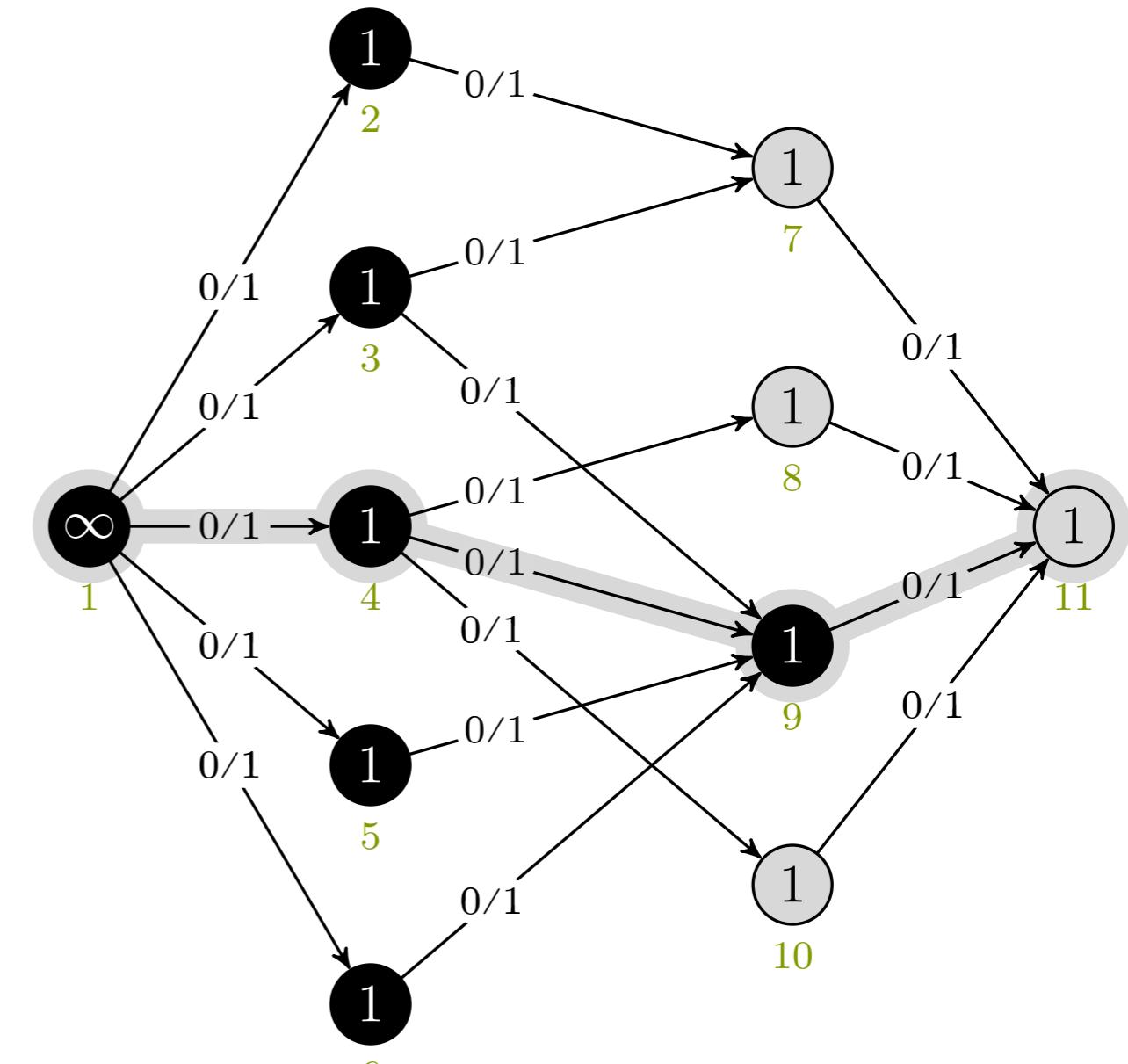
1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = -, -$ 

EDMONDS-KARP( $G, s, t$ )

```

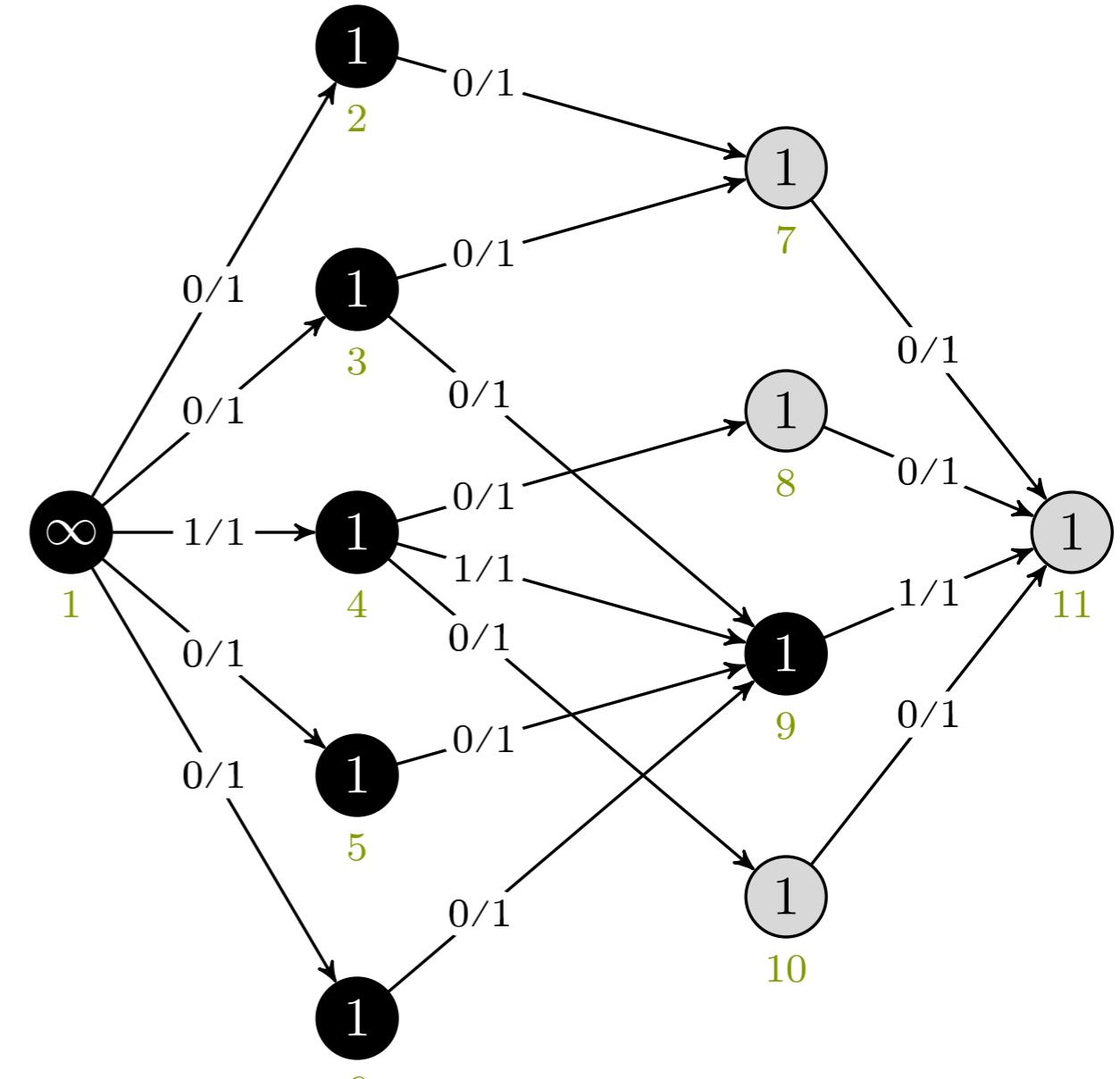
1  for each edge  $(u, v) \in G.E$ 
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3  repeat
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24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = 9, 11$ 

EDMONDS-KARP( $G, s, t$ )

```

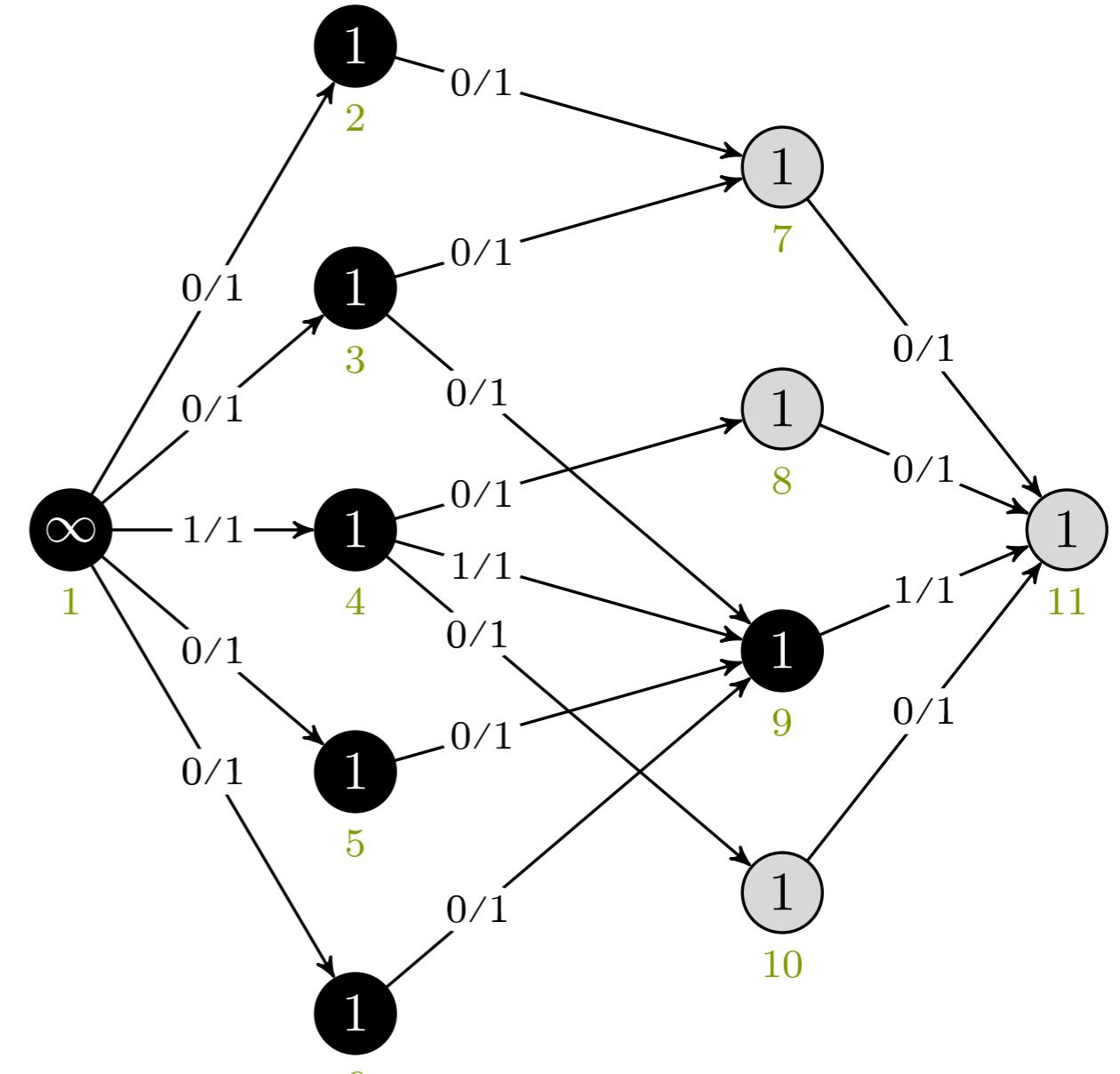
1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
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25         $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

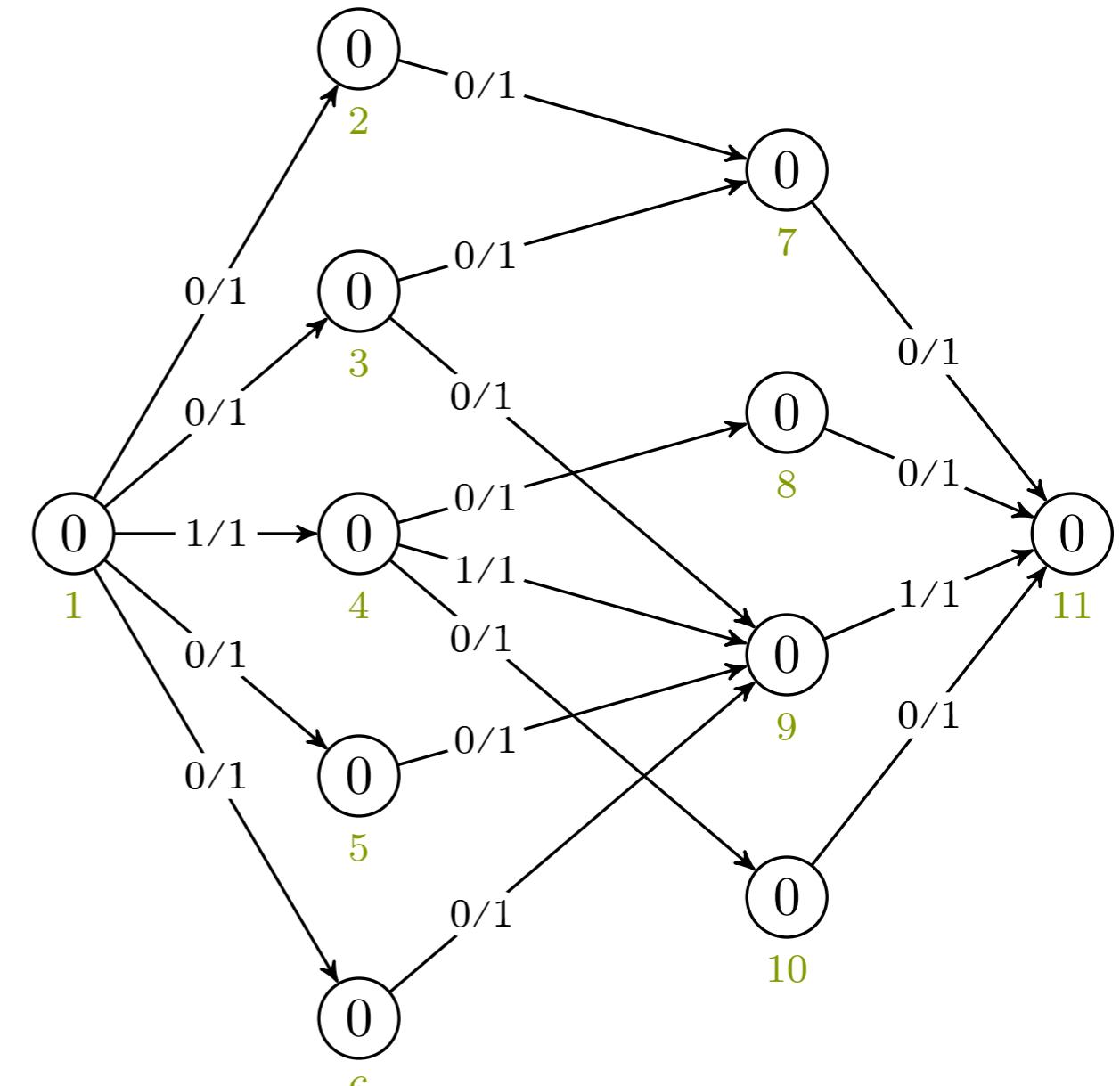
1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
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 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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26 until  $t.a == 0$ 
```

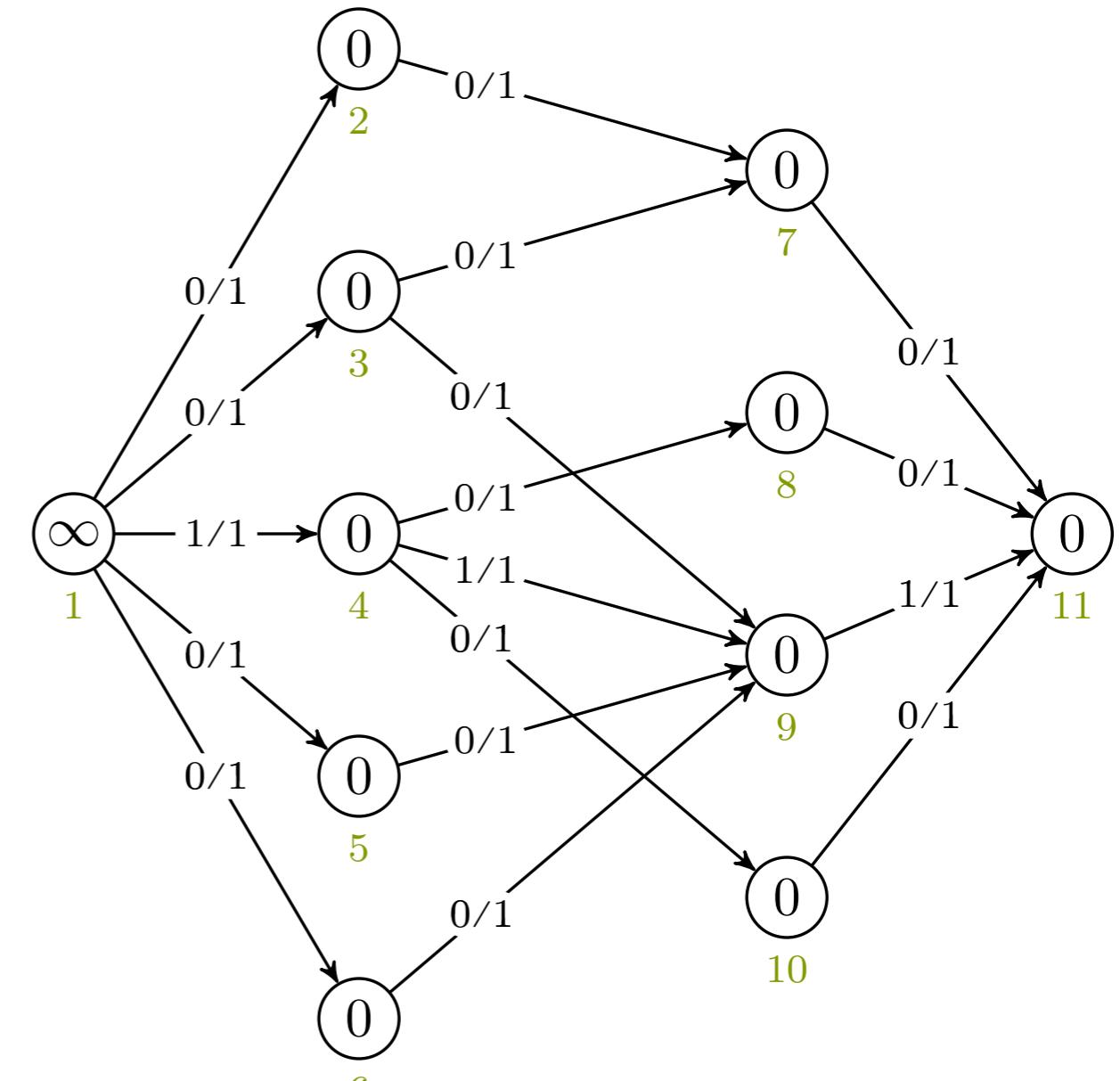
 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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2     $(u, v).f = 0$ 
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26 until  $t.a == 0$ 

```

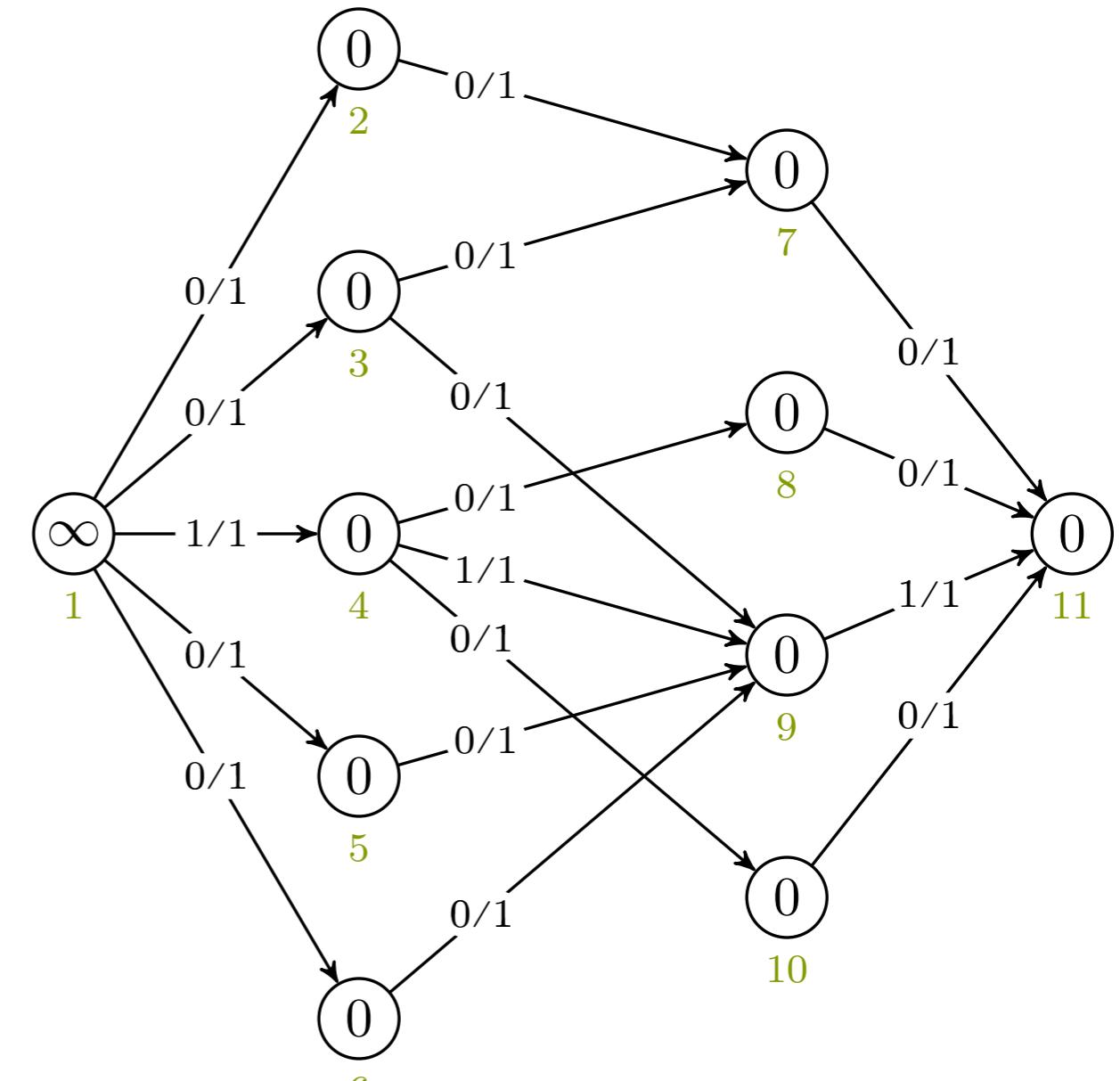
 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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```

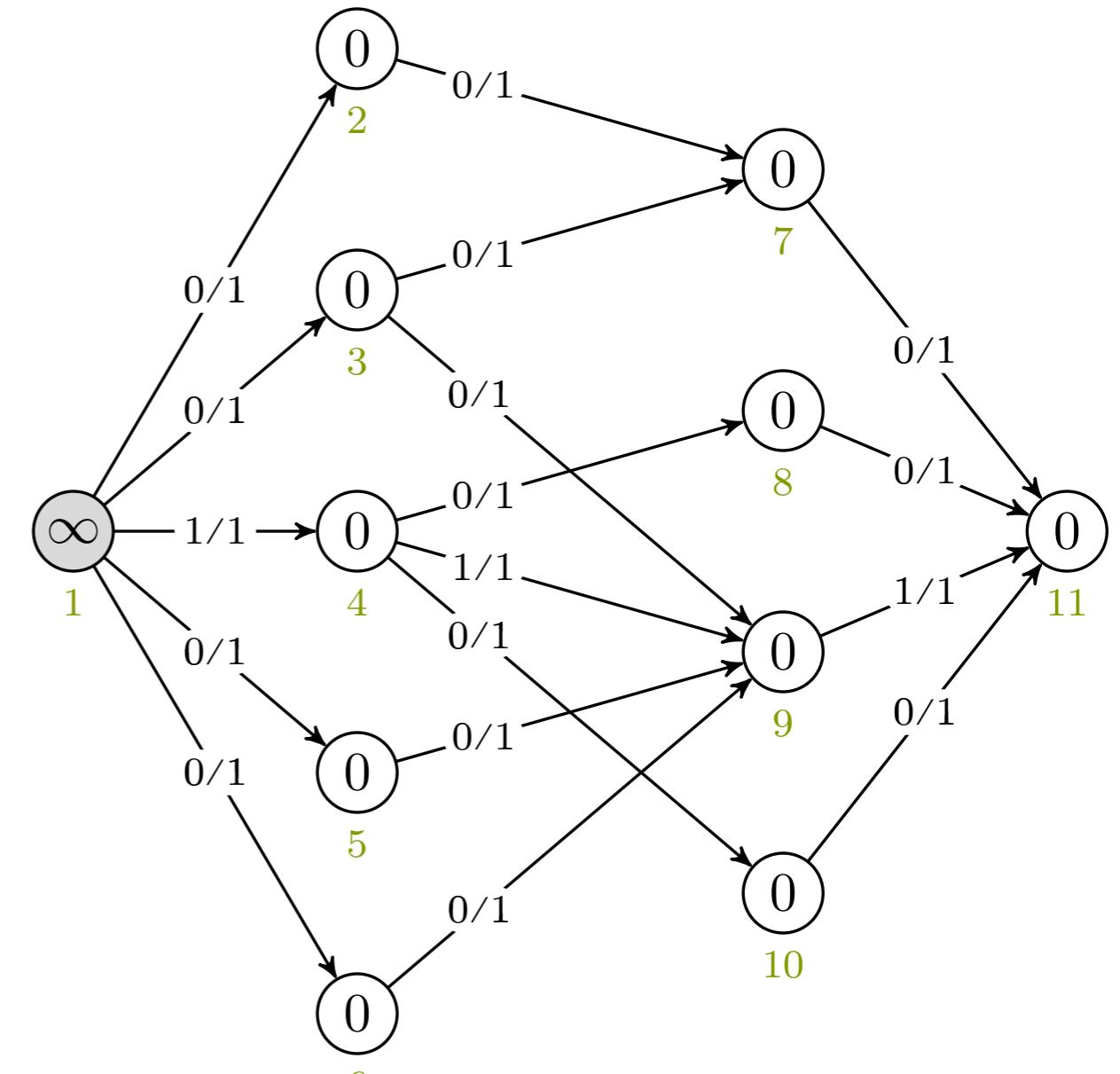
 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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25        $u, v = u.\pi, u$ 
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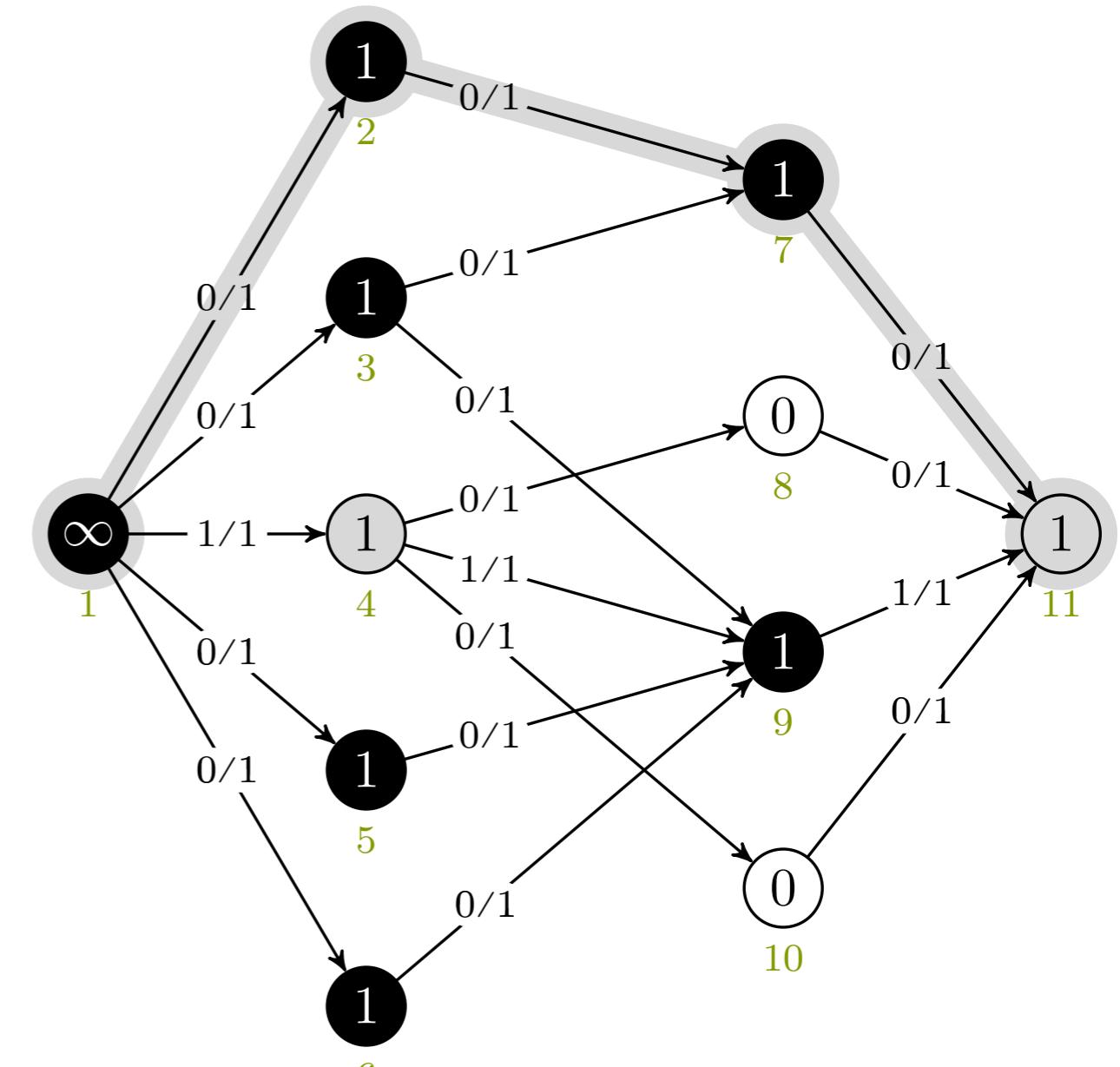
```

 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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```

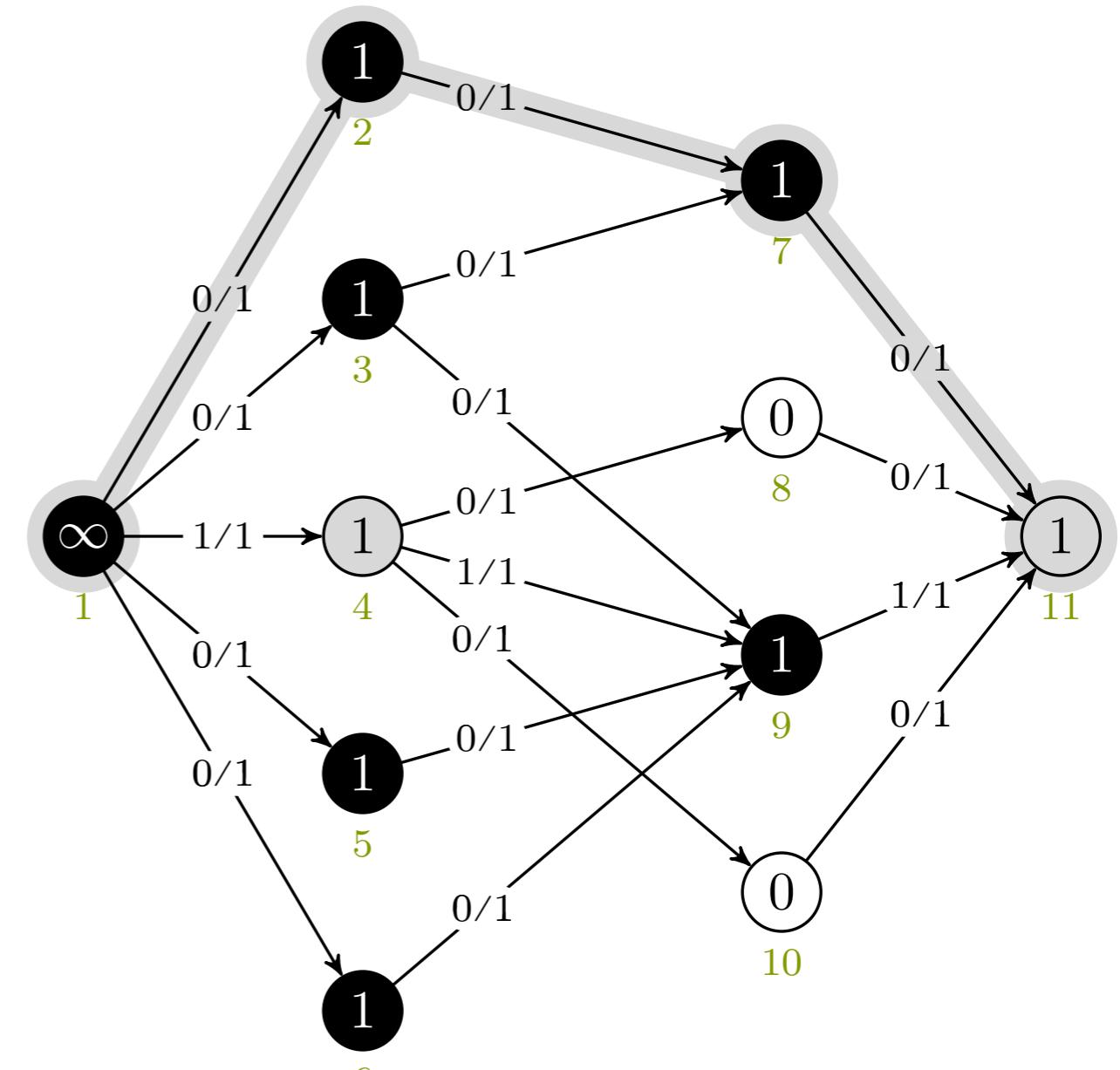
 $u, v = -, -$ 

EDMONDS-KARP( $G, s, t$ )

```

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```

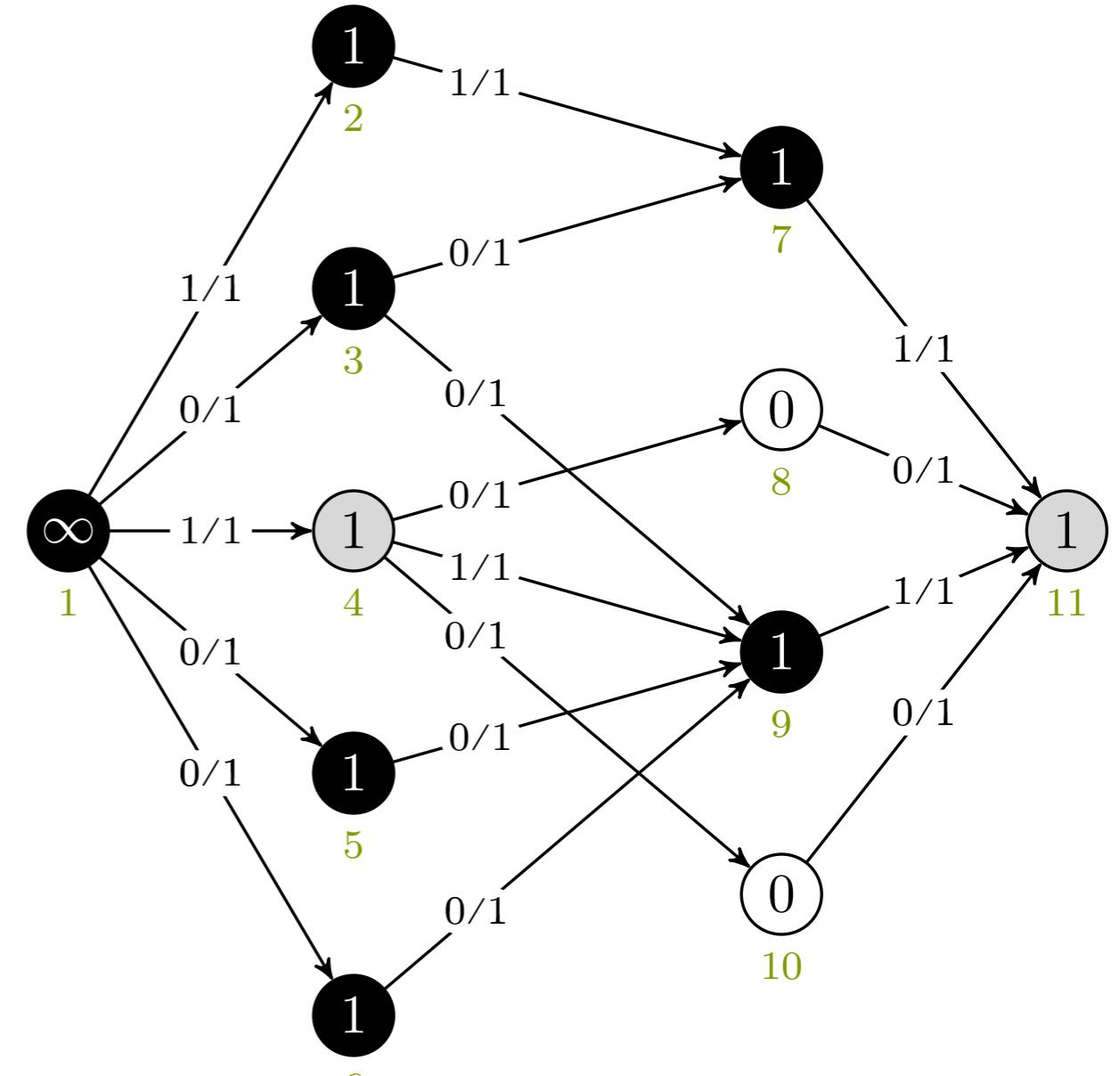
$u, v = 7, 11$



EDMONDS-KARP( $G, s, t$ )

```

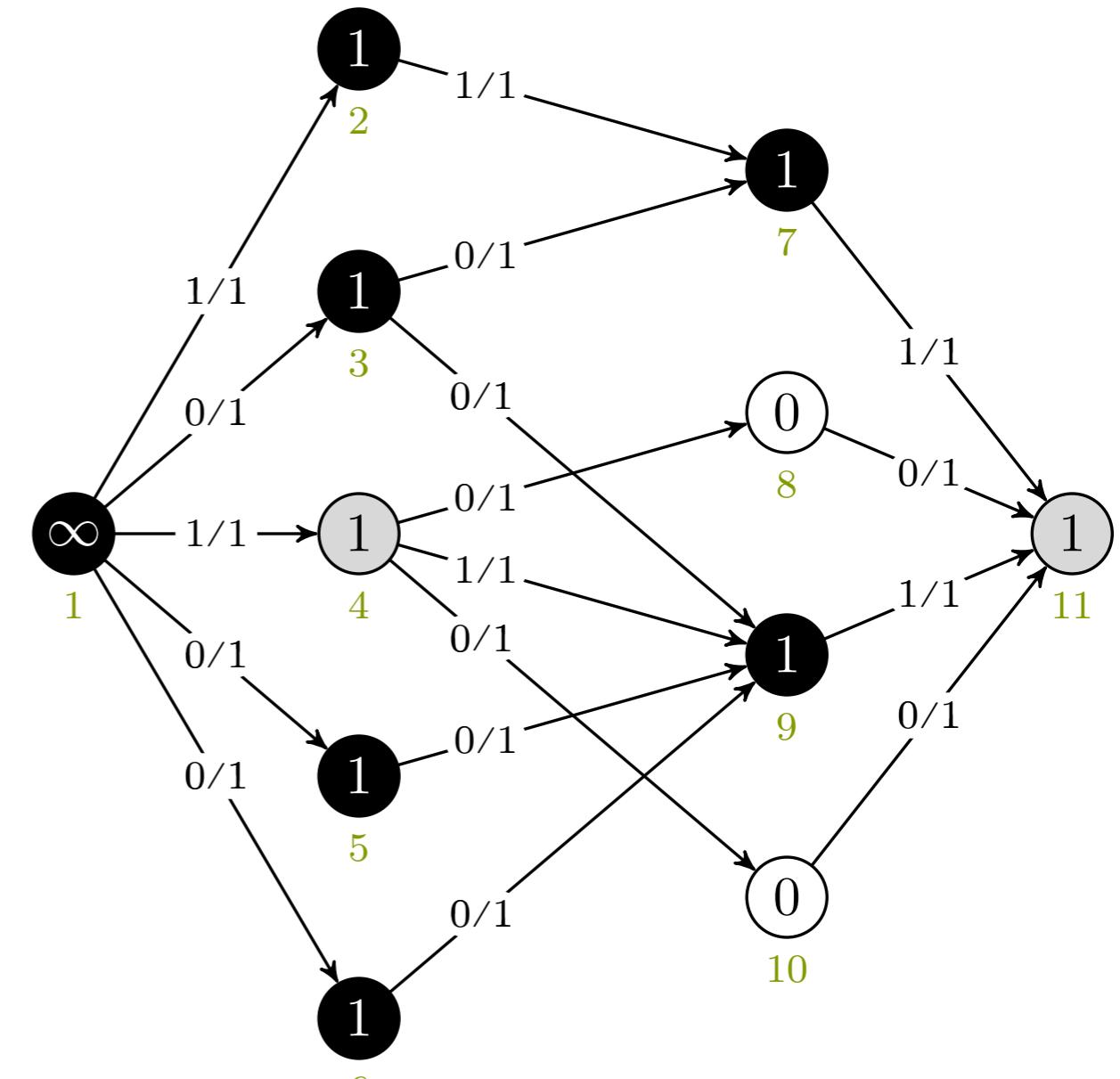
1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
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```

 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

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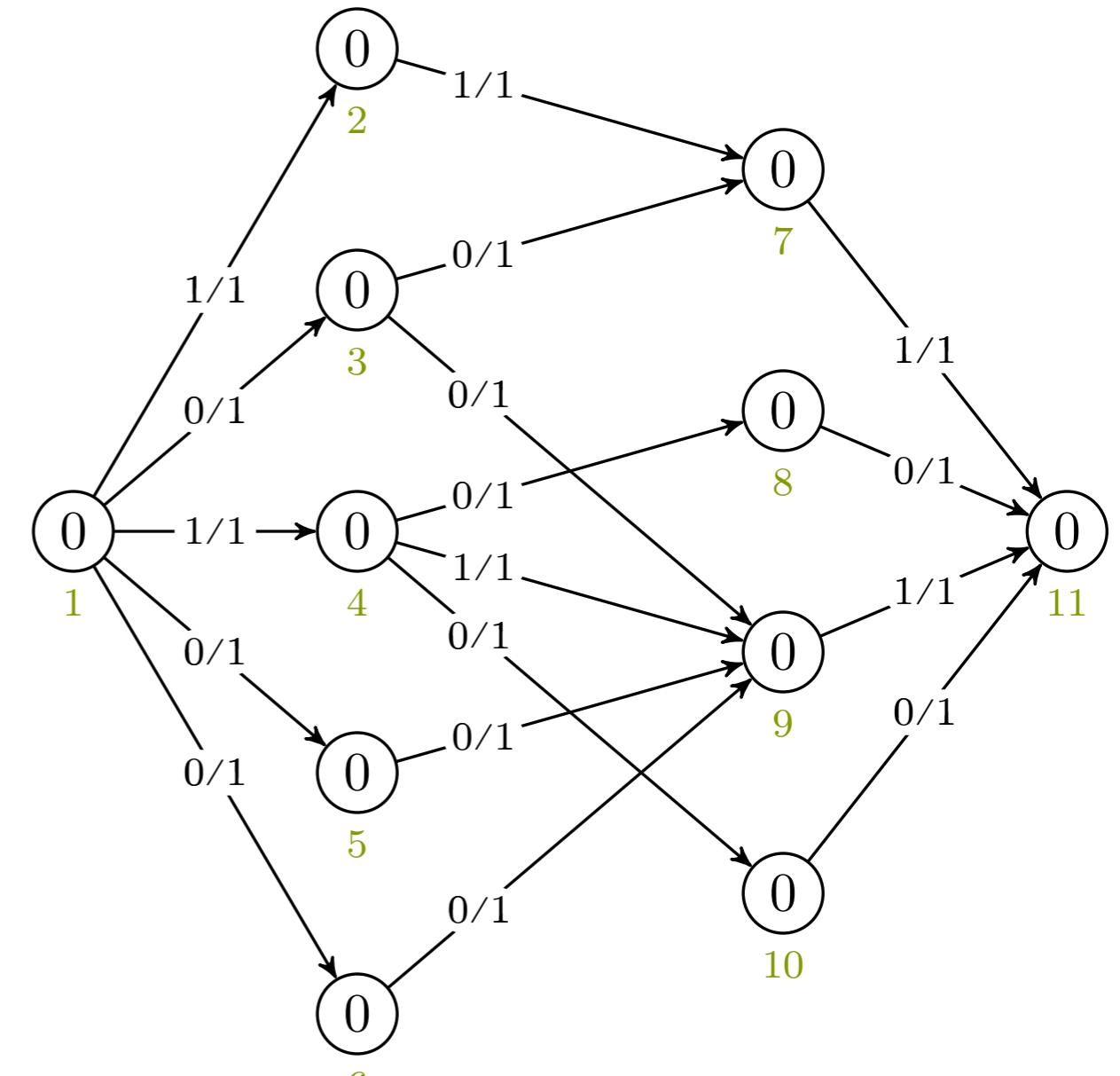
 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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```

$u, v = \text{NIL}, 1$

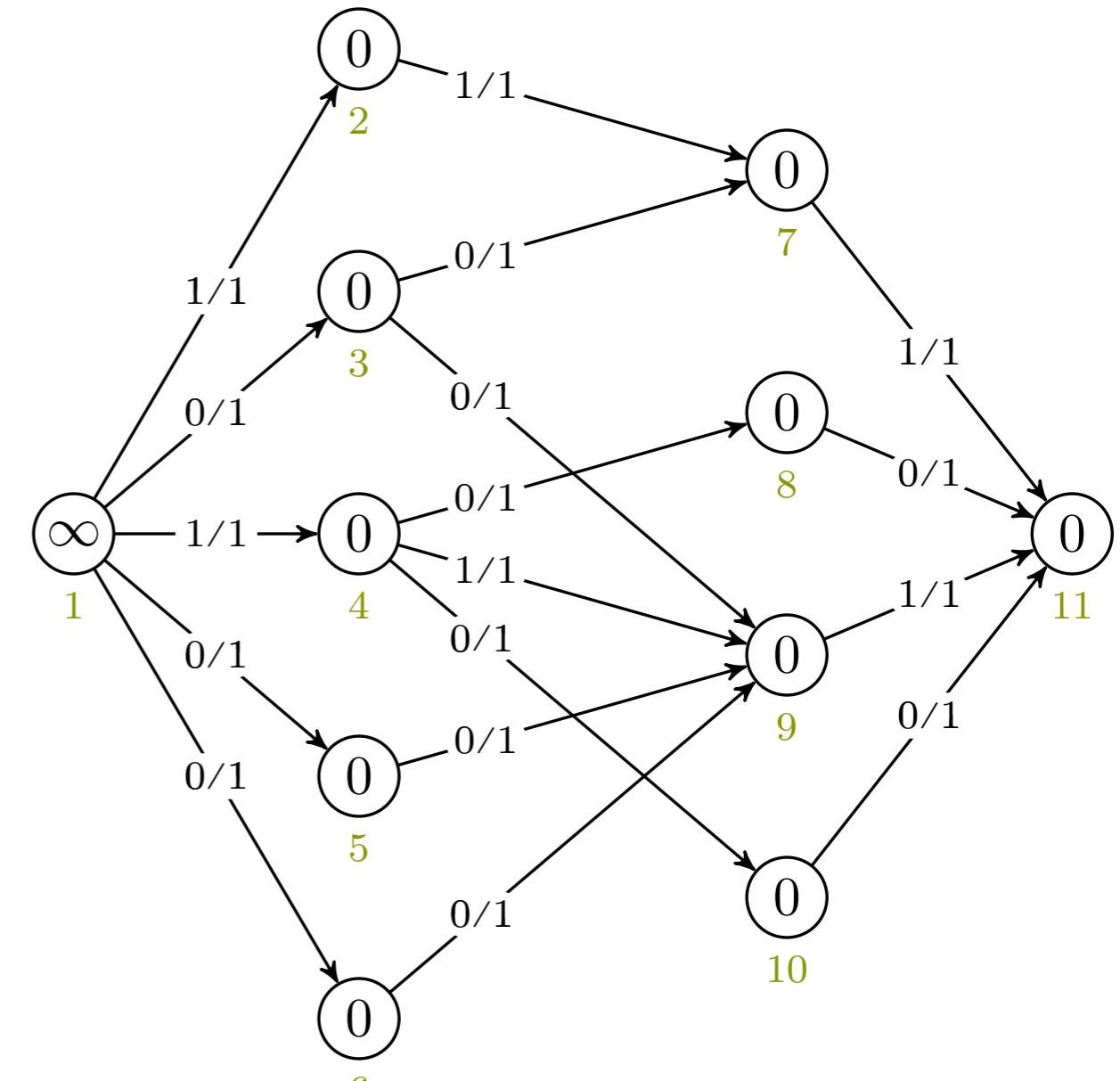


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19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

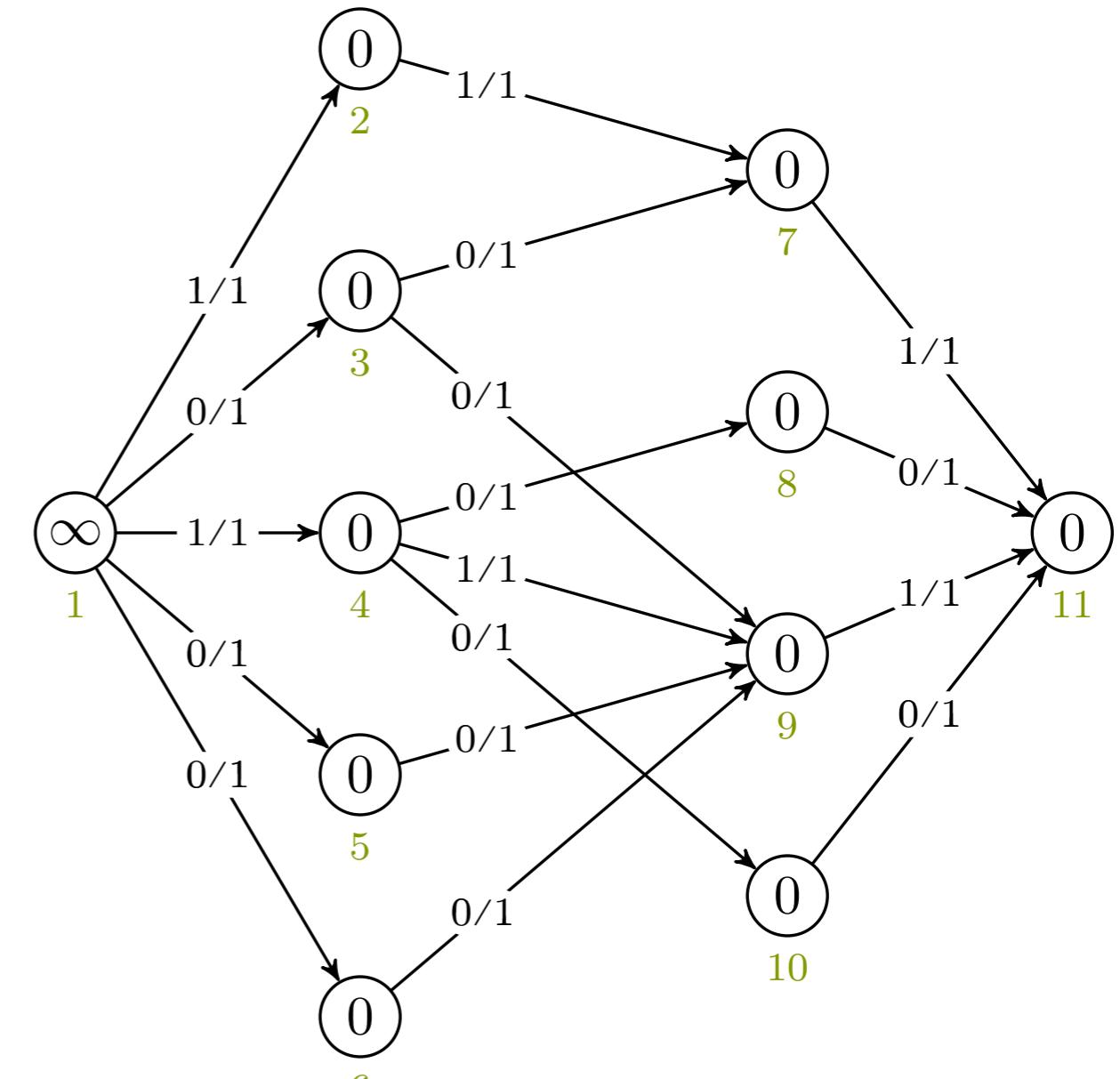
 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
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23            $(u, v).f = (u, v).f + t.a$ 
24         else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

```

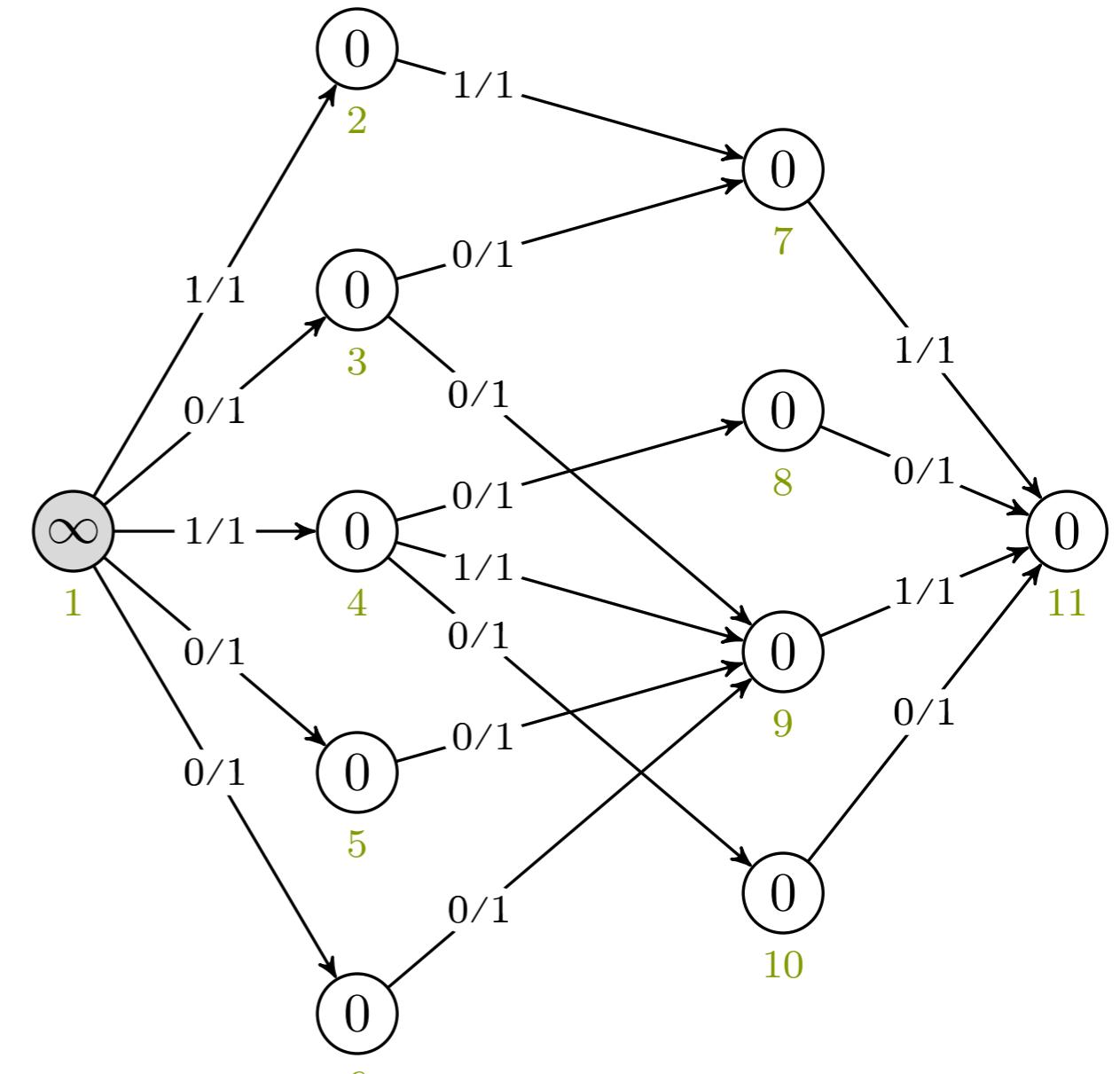
 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
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19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21       while  $u \neq \text{NIL}$ 
22         if  $(u, v) \in G.E$ 
23            $(u, v).f = (u, v).f + t.a$ 
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25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 

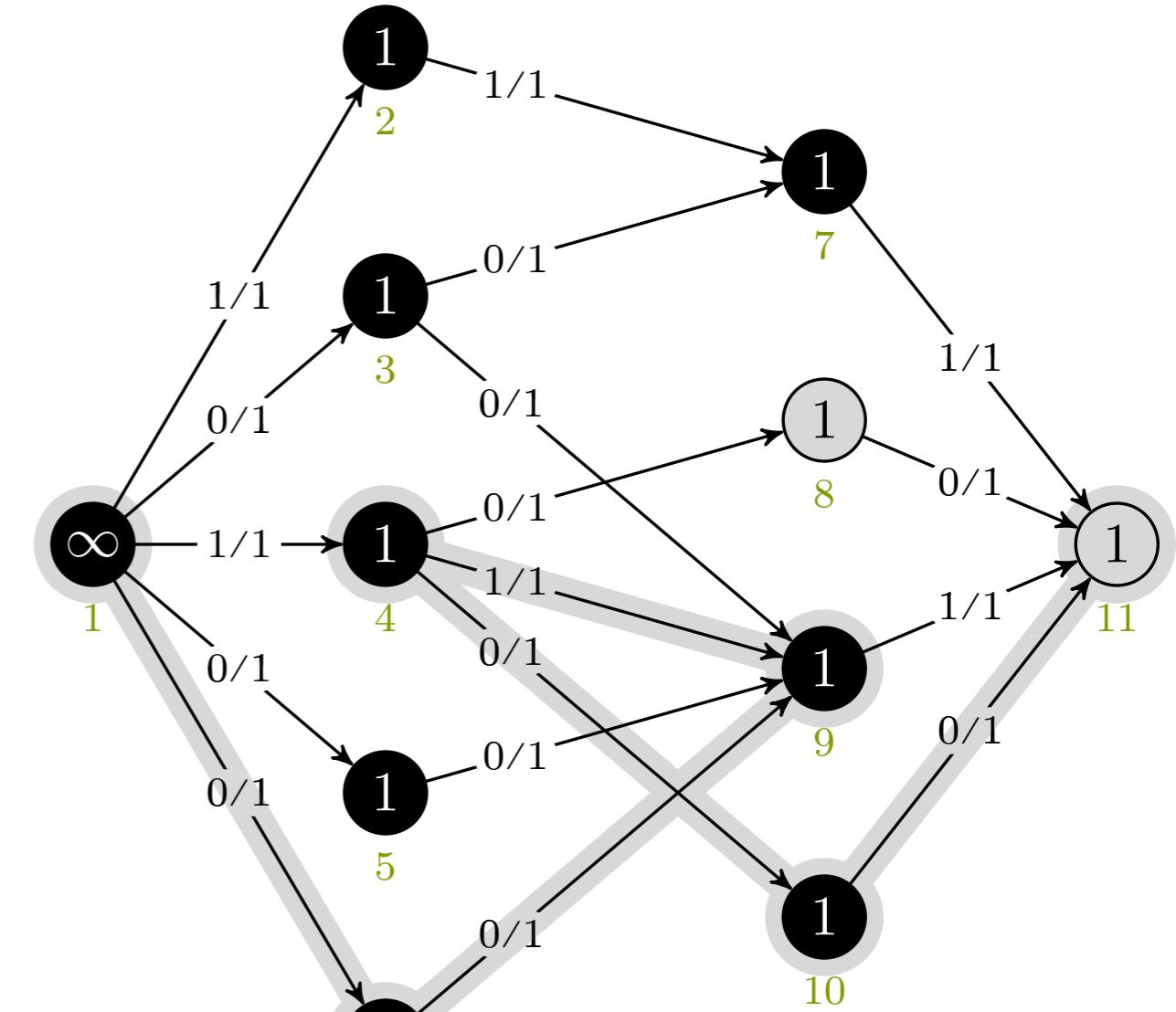
```

 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

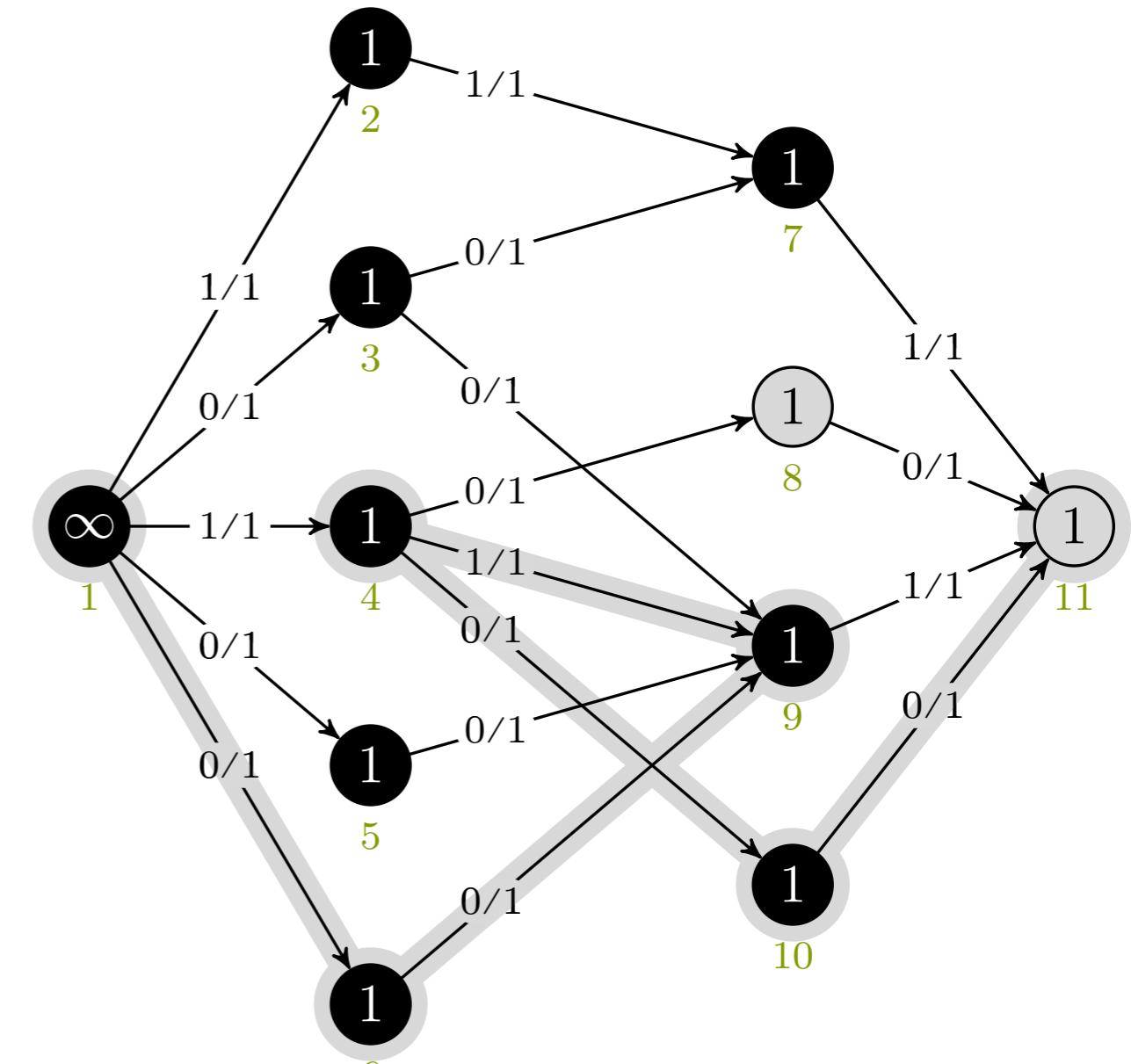
1  for each edge  $(u, v) \in G.E$ 
2     $(u, v).f = 0$ 
3  repeat
4    for each vertex  $u \in G.V$ 
5       $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6       $u.\pi = \text{NIL}$ 
7       $s.a = \infty$ 
8       $Q = \emptyset$ 
9      ENQUEUE( $Q, s$ )
10     while  $t.a == 0$  and  $Q \neq \emptyset$ 
11        $u = \text{DEQUEUE}(Q)$ 
12       for all edges  $(u, v), (v, u) \in G.E$ 
13         if  $(u, v) \in G.E$ 
14            $c_f(u, v) = c(u, v) - (u, v).f$ 
15         else  $c_f(u, v) = (v, u).f$ 
16         if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17            $v.a = \min(u.a, c_f(u, v))$ 
18            $v.\pi = u$ 
19           ENQUEUE( $Q, v$ )
20        $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21     while  $u \neq \text{NIL}$ 
22       if  $(u, v) \in G.E$ 
23          $(u, v).f = (u, v).f + t.a$ 
24       else  $(v, u).f = (v, u).f - t.a$ 
25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = -, -$ 

EDMONDS-KARP( $G, s, t$ )

```

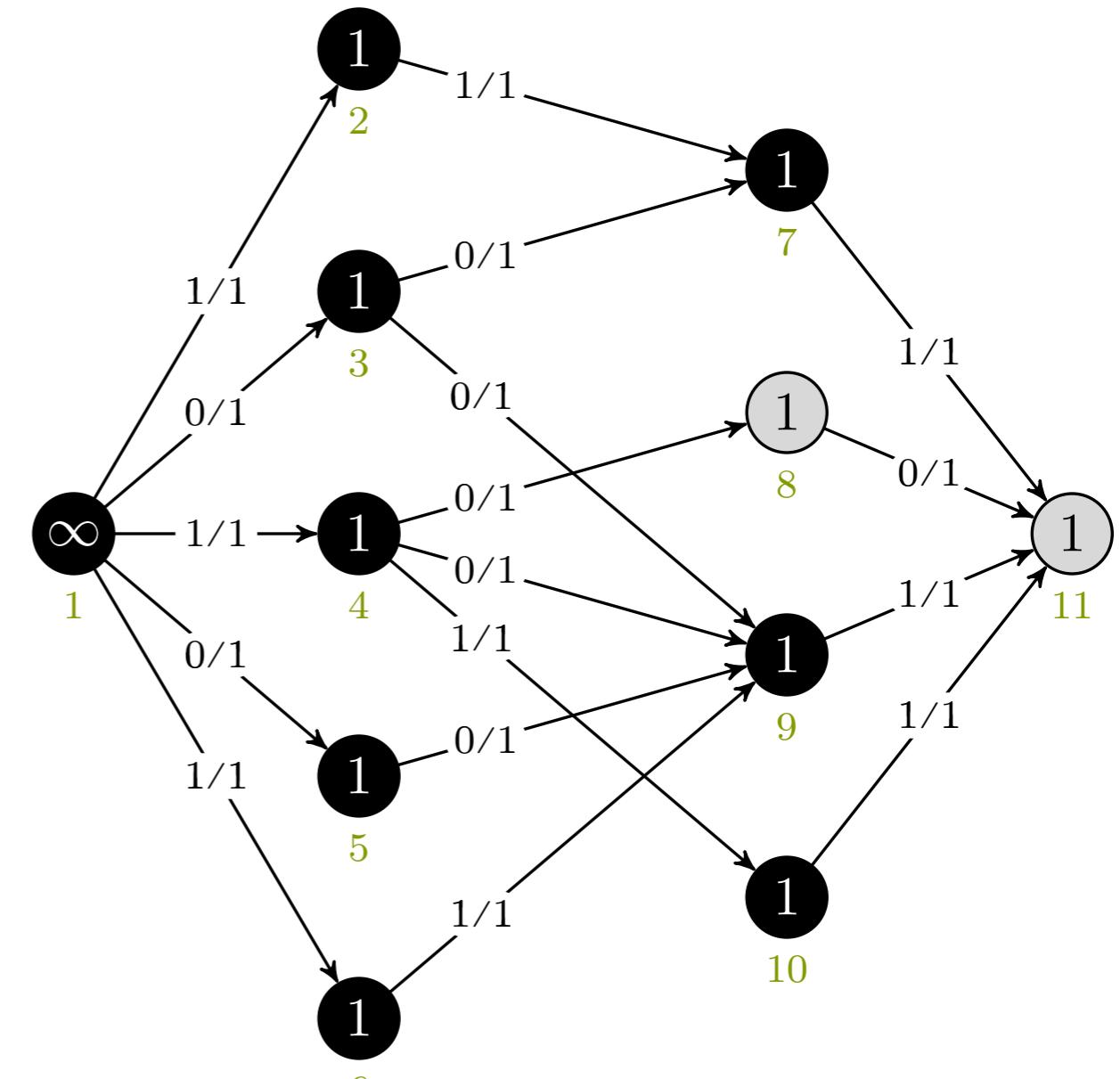
1 for each edge  $(u, v) \in G.E$ 
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23         $(u, v).f = (u, v).f + t.a$ 
24      else  $(v, u).f = (v, u).f - t.a$ 
25       $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = 10, 11$ 

EDMONDS-KARP( $G, s, t$ )

```

1 for each edge  $(u, v) \in G.E$ 
2    $(u, v).f = 0$ 
3 repeat
4   for each vertex  $u \in G.V$ 
5      $u.a = 0 \rightarrow$  flow reaching  $u$  in  $G_f$ 
6      $u.\pi = \text{NIL}$ 
7      $s.a = \infty$ 
8      $Q = \emptyset$ 
9     ENQUEUE( $Q, s$ )
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11       $u = \text{DEQUEUE}(Q)$ 
12      for all edges  $(u, v), (v, u) \in G.E$ 
13        if  $(u, v) \in G.E$ 
14           $c_f(u, v) = c(u, v) - (u, v).f$ 
15        else  $c_f(u, v) = (v, u).f$ 
16        if  $c_f(u, v) > 0$  and  $v.a == 0$ 
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20         $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21        while  $u \neq \text{NIL}$ 
22          if  $(u, v) \in G.E$ 
23             $(u, v).f = (u, v).f + t.a$ 
24          else  $(v, u).f = (v, u).f - t.a$ 
25         $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

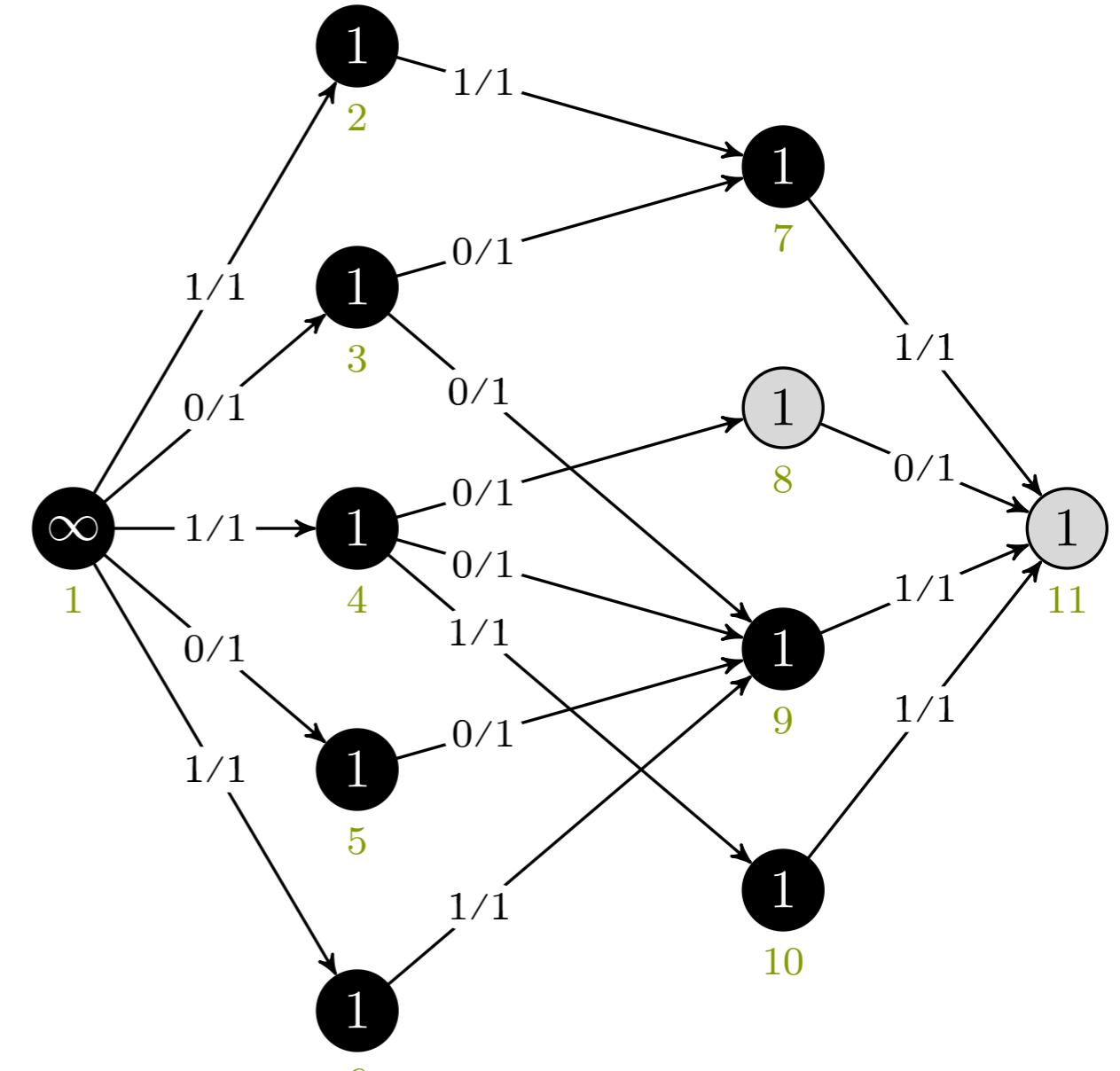
 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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2    $(u, v).f = 0$ 
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4   for each vertex  $u \in G.V$ 
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7      $s.a = \infty$ 
8      $Q = \emptyset$ 
9     ENQUEUE( $Q, s$ )
10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11       $u = \text{DEQUEUE}(Q)$ 
12      for all edges  $(u, v), (v, u) \in G.E$ 
13        if  $(u, v) \in G.E$ 
14           $c_f(u, v) = c(u, v) - (u, v).f$ 
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19          ENQUEUE( $Q, v$ )
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26 until  $t.a == 0$ 

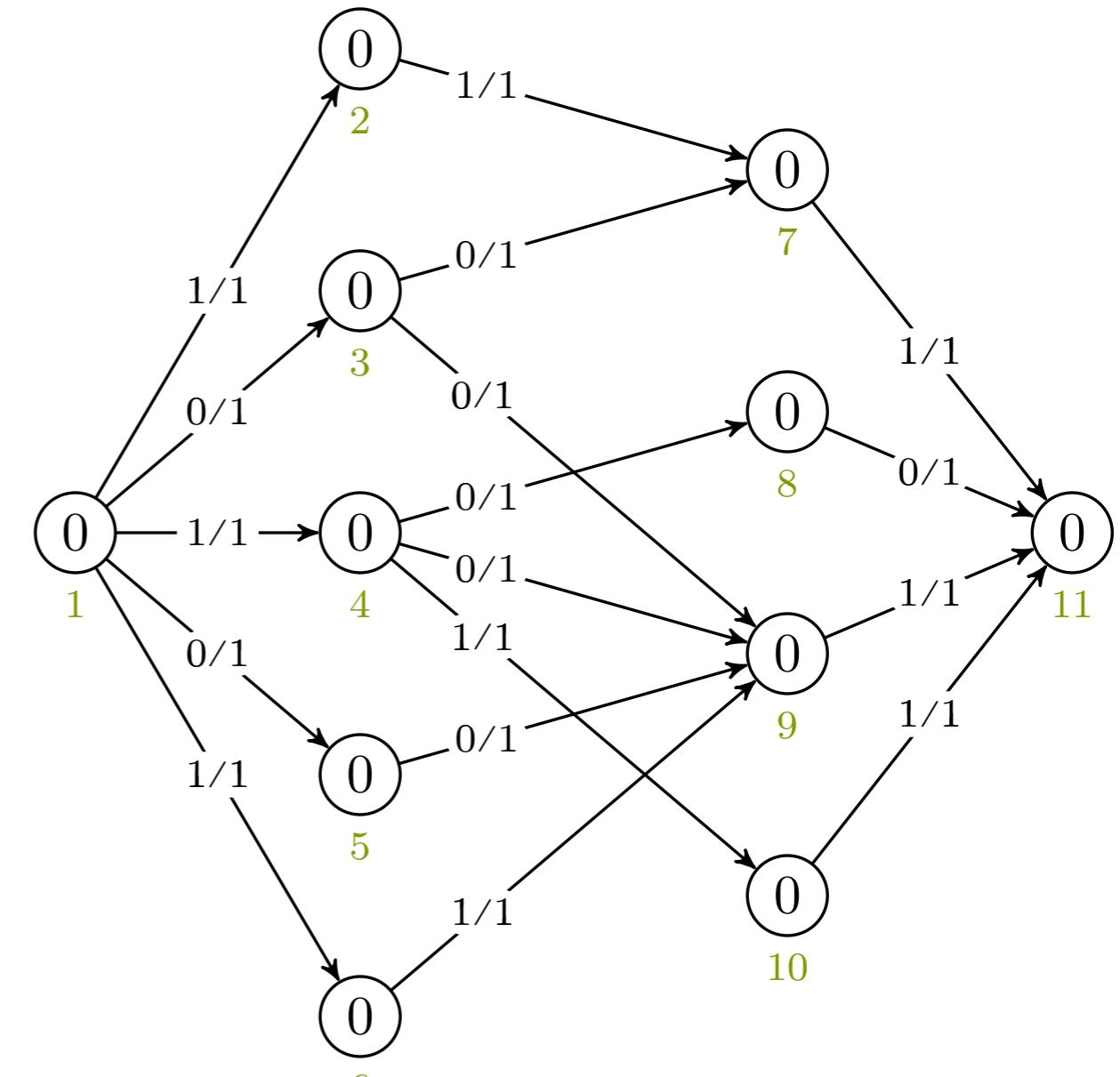
```

 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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25      $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

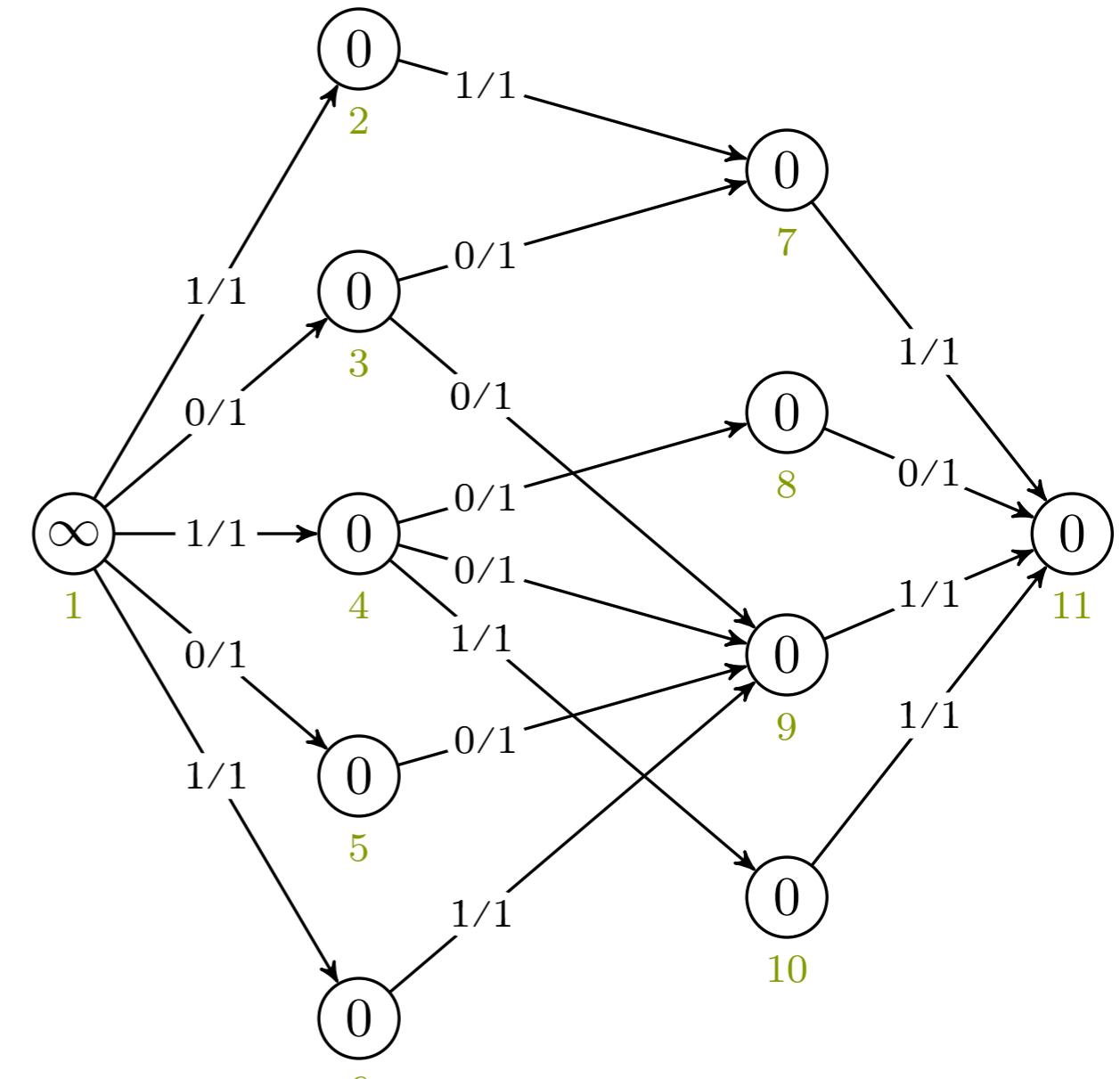
 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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```

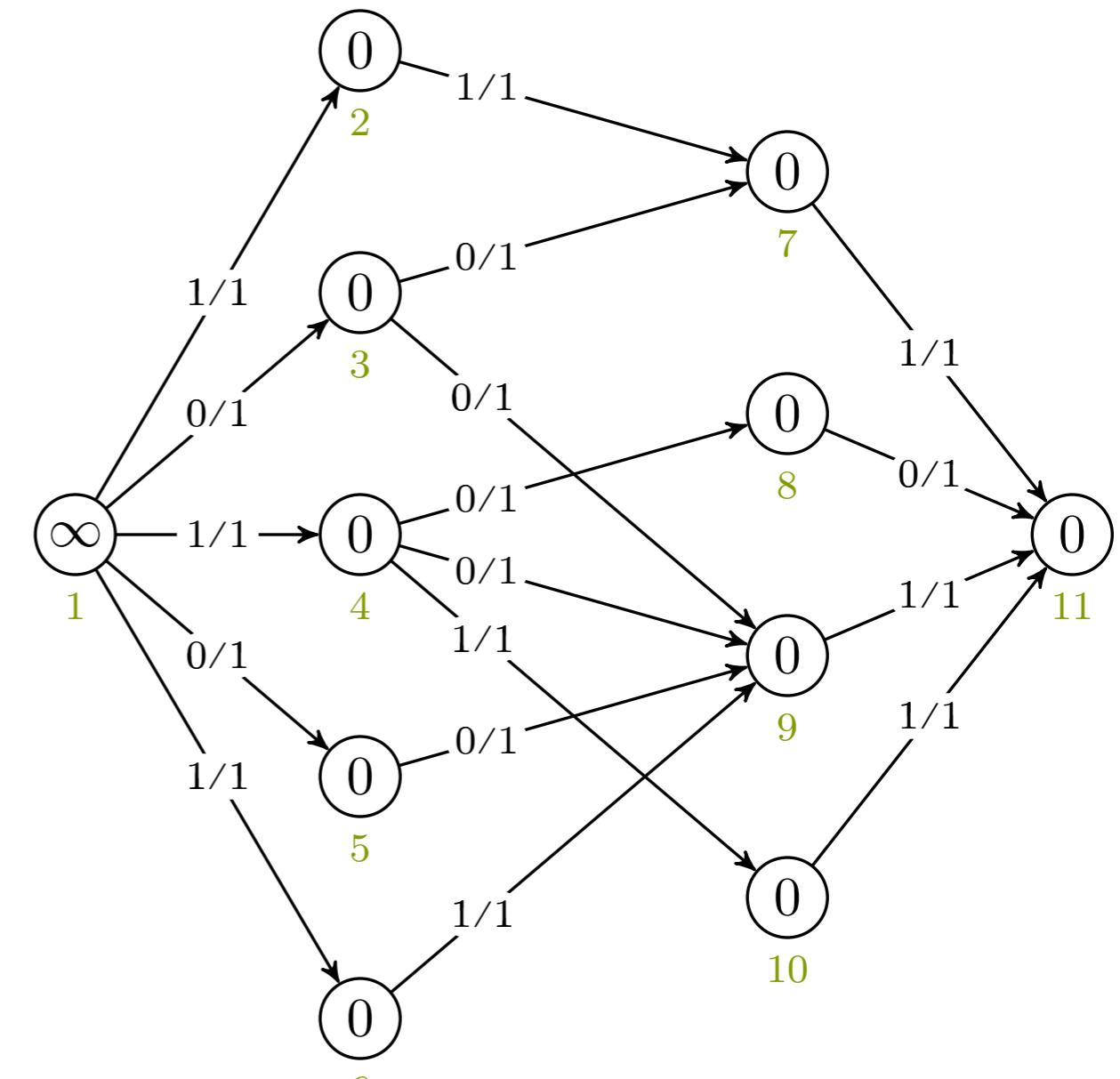
 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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26 until  $t.a == 0$ 

```

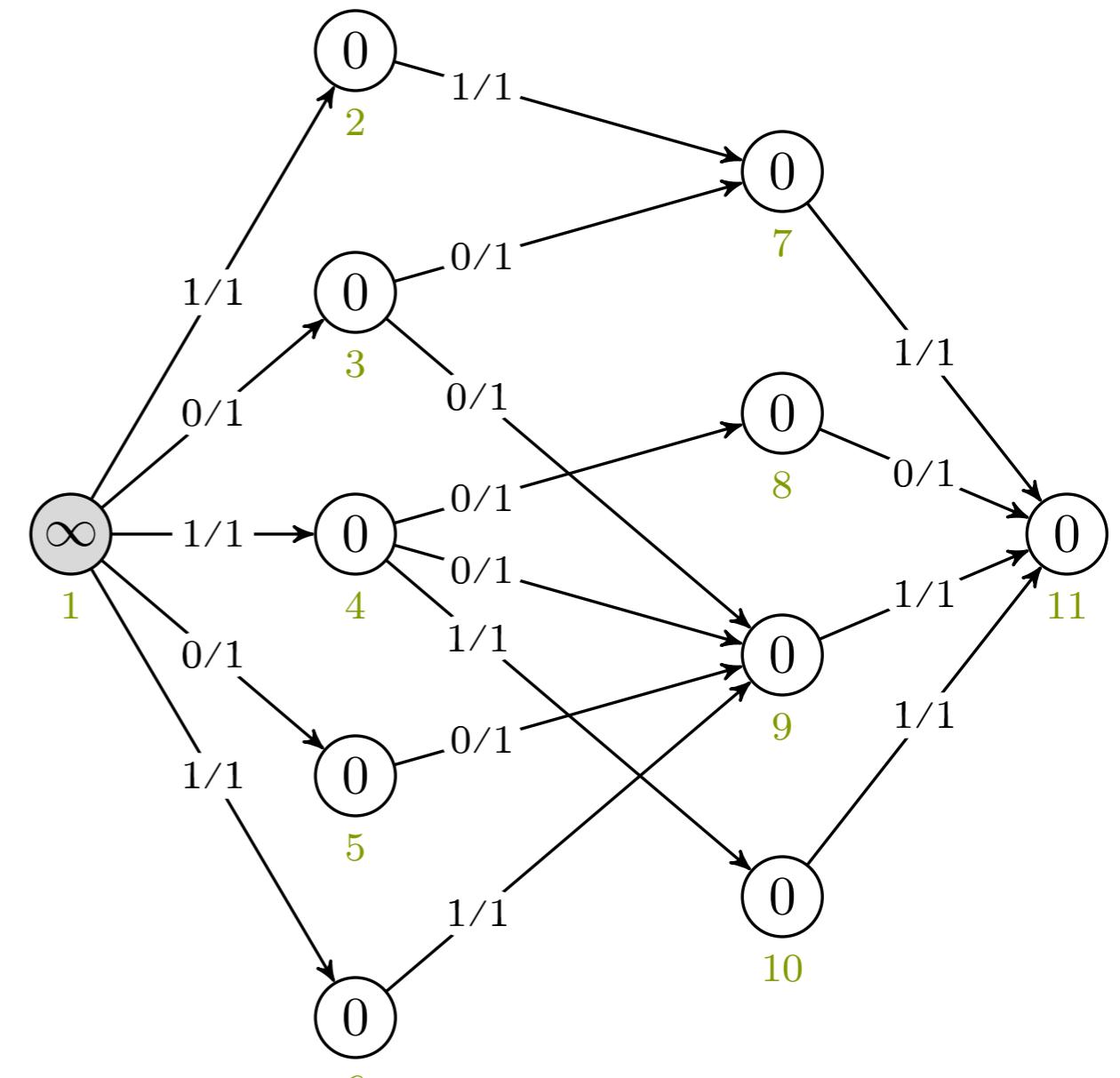
 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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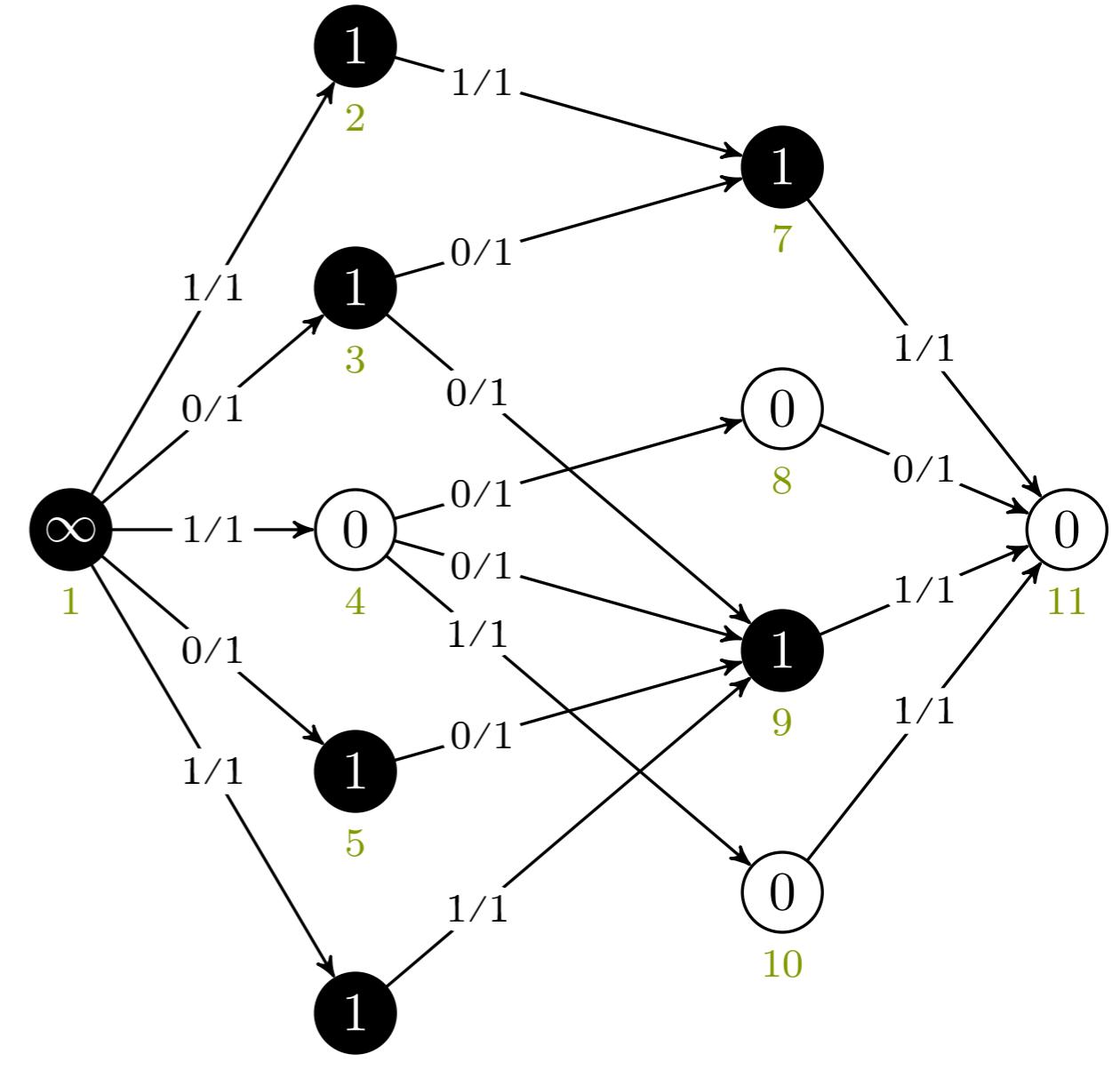
```

 $u, v = \text{NIL}, 1$ 

EDMONDS-KARP( $G, s, t$ )

```

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25        $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = -, -$ 

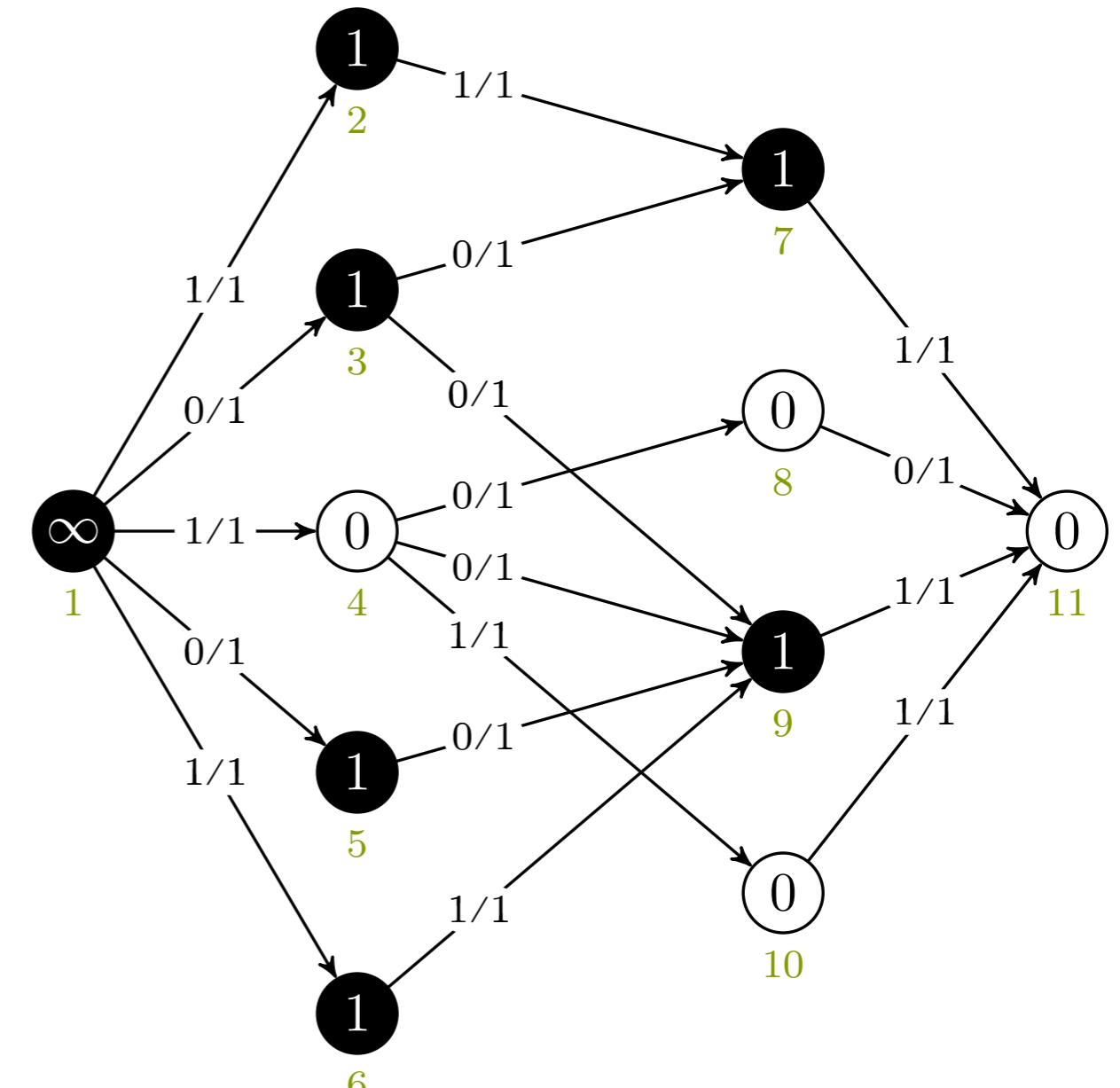
## EDMONDS-KARP(G, $s, t$ )

```

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9          ENQUEUE( $Q, s$ )
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11              $u = DEQUEUE(Q)$ 
12             for all edges  $(u, v), (v, u) \in G.E$ 
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24                 else  $(v, u).f = (v, u).f - t.a$ 
25                  $u, v = u.\pi, u$ 
26             until  $t.a == 0$ 

```

*u, v* = NIL, 11

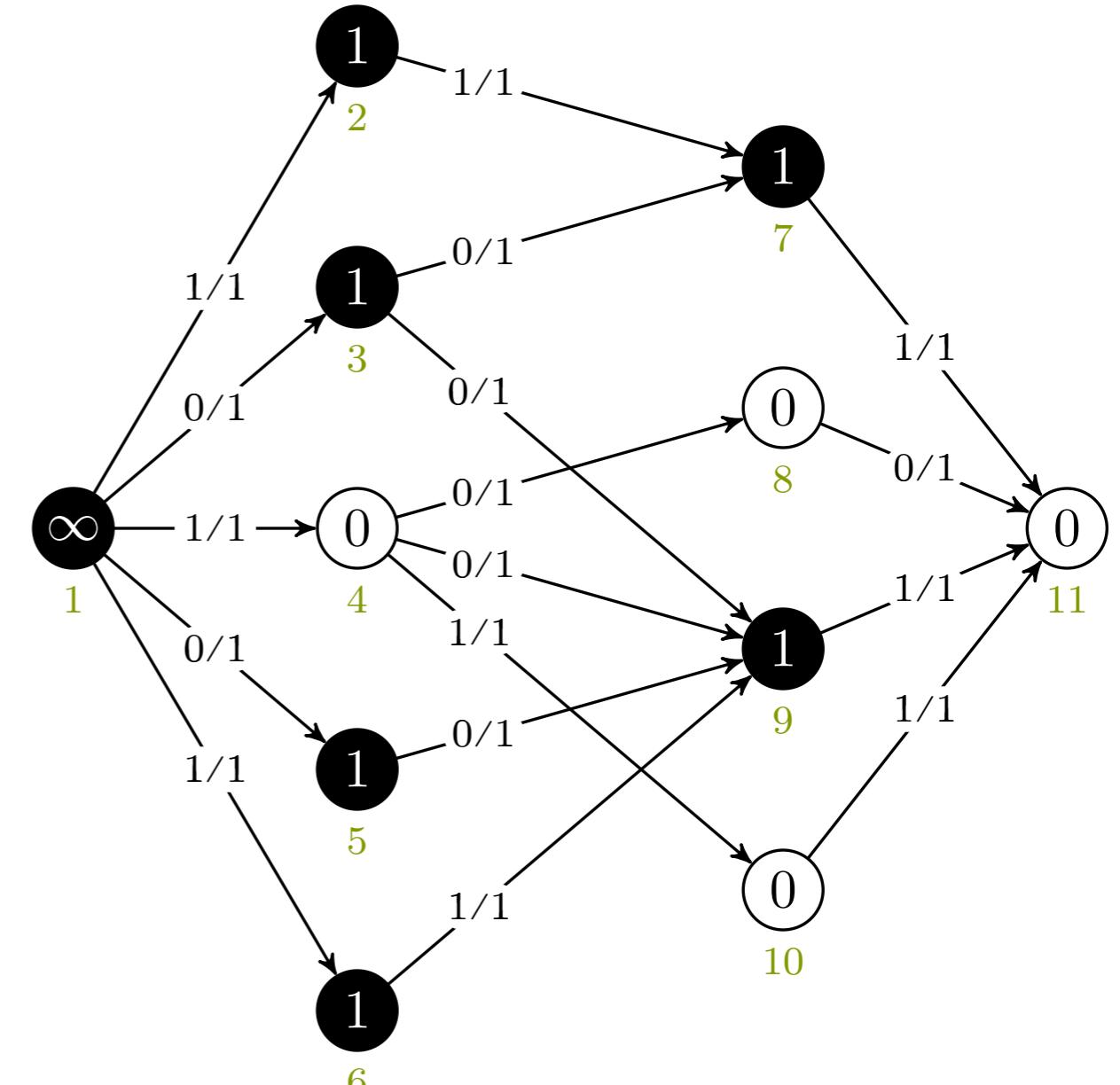


EDMONDS-KARP( $G, s, t$ )

```

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```

$u, v = \text{NIL}, 11$



EDMONDS-KARP( $G, s, t$ )

```

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10    while  $t.a == 0$  and  $Q \neq \emptyset$ 
11       $u = \text{DEQUEUE}(Q)$ 
12      for all edges  $(u, v), (v, u) \in G.E$ 
13        if  $(u, v) \in G.E$ 
14           $c_f(u, v) = c(u, v) - (u, v).f$ 
15        else  $c_f(u, v) = (v, u).f$ 
16        if  $c_f(u, v) > 0$  and  $v.a == 0$ 
17           $v.a = \min(u.a, c_f(u, v))$ 
18           $v.\pi = u$ 
19          ENQUEUE( $Q, v$ )
20       $u, v = t.\pi, t \rightarrow$  at this point,  $t.a == c_f(p)$ 
21      while  $u \neq \text{NIL}$ 
22        if  $(u, v) \in G.E$ 
23           $(u, v).f = (u, v).f + t.a$ 
24        else  $(v, u).f = (v, u).f - t.a$ 
25       $u, v = u.\pi, u$ 
26 until  $t.a == 0$ 
```

 $u, v = -, -$ 