Constraining Curvature Density Parameter with Time-Delay Strong Lenses and Complimentary Probes: a Forecast for Next-Generation Surveys

Project Report

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Abstract

It is of great interest among cosmologists to obtain a high-fidelity constraint on curvature density parameter Ω_k of late universe that is independent of the early universe Cosmic Microwave Background measurement, and this report presents a framework to do so with the combined data of time-delay distances from strong gravitational lenses and complimentary probes. Simulated data of next-generation surveys are used in model-dependent and non-parametric analysis where Markov chain Monte Carlo sampling is used to constrain Ω_k. By assuming a non-flat wCDM model, the addition of strong lensing data into the analysis with data of Type Ia supernovae and Baryon Acoustic Oscillation breaks the w-Ω_k degeneracy, placing a strong constraint on Ω_k. Similar order of magnitude of constraint on Ω_k was obtained in non-parametric analysis. This serves as a promising forecast for next-generation surveys on constraining Ω_k.

1 Introduction

The curvature density parameter, Ω_k, is an important cosmological parameter that determines the geometry of our Universe. Probes such as Type Ia supernovae (SNe Ia), strong gravitational lenses and baryon acoustic oscillations (BAO) give estimates on Ω_k of the late universe, leading to possible ways of obtaining a high-fidelity constraint on Ω_k independent of the early-universe Cosmic Microwave Background (CMB) measurement. The constraint placed on Ω_k by SNe Ia, strong gravitational lenses and BAO individually have been widely analysed and discussed in past literature. A promising study on constraining Ω_k is to look at a large number of time-delay distance measurements from strong gravitational lenses and combine it with complimentary probes, hoping that the combined data can overcome the deficiency of limited lenses events in previous analysis such as that by H0LiCOW on 6 lenses [1] and degeneracies among cosmological parameters in single-probe analysis.

In light of the next-generation survey from the Rubin Observatory's legacy survey of space and time (LSST), it is expected that hundreds of strong lensing events will be observed during its 10-year survey baseline [2]. This will provide a large dataset for studying the ability of strong lenses to constrain Ω_k. In this project, we use simulated LSST-like data [3] to explore 3 tasks. Firstly, we combine it with a past survey, Pantheon [4], of apparent magnitude measurement of SNe Ia to see the improvement made on constraining Ω_k by adding lenses into the analysis. Different models of dark energy in a non-flat universe, i.e. $o\Lambda$CDM and owCDM, are used in the process of generating mock values of "measurements" for each next-generation survey. Then, we use simulated next-generation Roman-like SNe Ia data [5] and DESI-like BAO data [6] and combine them with LSST-like data to see their ability on constraining Ω_k. Finally, we use Gaussian Process to fit Hubble parameter H_z and look at a non-parametric method to analyse the constraint that lenses and complimentary probes place on Ω_k. We hope that this project provides an elementary but instructive forecast on using next-generation surveys to constrain Ω_k.

2 Methodology

2.1 Formulae in cosmology

The three late-universe probes used in this project are strong gravitational lenses, SNe Ia and BAO. Each of them has a measurable quantity that is linked to the curvature density parameter Ω_k via some formulae. For the purpose of having a reference section for this brief report, relevant formulae will be listed below without derivations.

For lenses, we are interested in measuring time-delay distances $D_{\Delta t}$ of a lens at redshift z_l and a source at z_s:

$$D_{\Delta t} = \frac{(1+z_s)D_{A,l}D_{A,s}}{D_{A,ls}}, \tag{1}$$

where $D_{A,l}$ and $D_{A,s}$ are angular diameter distances at lens and source respectively, and $D_{A,ls}$ is the angular diameter distance between lens and source. D_A at a given redshift can be calculated by $D_L/(1+z)$ where D_L is the luminosity distance at the same redshift with which the formula will be given below. Ω_k appears in D_L.

For SNe Ia, what we measure is the apparent magnitude m_B given by:

$$m_B = 5\lg(D_L) + 25 + M_B, \tag{2}$$

where M_B is the absolute magnitude and D_L is the luminosity distances at z. D_L is given by:

$$D_L = \frac{c(1+z)}{H_0\sqrt{|\Omega_k|}} F(\sqrt{|\Omega_k|}\int_0^z \frac{\mathrm{d}z'}{E(z')}), \tag{3}$$

where function F is sin for $\Omega_k \leq 0$ and sinh for $\Omega_k > 0$.

For BAO, we measure H_z, the Hubble parameter. Its relation to H_0, Ω_k and other density parameters is trivially derived from the Friedman equations.

2.2 Statistical setup

The method to get constraints on parameters in this project is Markov chain Monte Carlo (MCMC), a sampling technique in Bayesian inference. Using MCMC, we drawn samples for cosmological parameters to get distributions of them based on measured data provided. The mean and the standard deviation of the distributions become the constraints placed on cosmological parameters.

In Bayesian inference, a prior distribution and a likelihood function are required. Throughout the project, we use conservative uniform prior distributions given by the table below for each cosmological parameter marginalised:

Parameter	Prior
H_0	U(0, 150)
Ω_m	U(0.05, 0.5)
Ω_k	U(-0.5, 0.5)
w	U(-2.5, 0.5)
M_B	U(-38.4, 0)

The likelihood function is set to return a χ^2 term for lenses and BAO in this form:

$$\chi^2 = \sum_{\text{all probes used}} \sum_i \frac{(x_i - x_{m,i})^2}{\sigma_{x_i}^2}. \tag{4}$$

x_i, $x_{m,i}$ and σ_{x_i} indicate measured value, modelled value and percentage uncertainty in measurement x_i respectively. For example, for lenses, x_i is supposed to be a measured value of $D_{\Delta t}$; for BAO, x_i is a measured value of H_z. For SNe Ia, the χ^2 term takes this form instead:

$$\chi^2 = \Delta \boldsymbol{m_B}^T \cdot \boldsymbol{C}^{-1} \cdot \Delta \boldsymbol{m_B}, \tag{5}$$

where $\Delta\boldsymbol{m_B} = \boldsymbol{m_B} - \boldsymbol{m_B}_{\text{model}}$ is a vector of difference between measured and modelled apparent magnitudes and $\boldsymbol{C} = \boldsymbol{D}_{\text{stat}} + \boldsymbol{C}_{\text{sys}}$ is the covariance matrix that takes into account both statistical and systematic uncertainty.

Data analysis in this project is done with *Python*. The key package used to calculate cosmological quantities in many cases is *astropy.cosmology*. The package used to run MCMC is *emcee*, and the package to do Gaussian Process is *george*. Note that in most cases in this project, we are doing a forecast for next-generation surveys, and the "measured" dataset is a simulated dataset generated from a mock universe of either Flat$w$CDM, $o\Lambda$CDM or owCDM model using methods in *astropy.cosmology*.

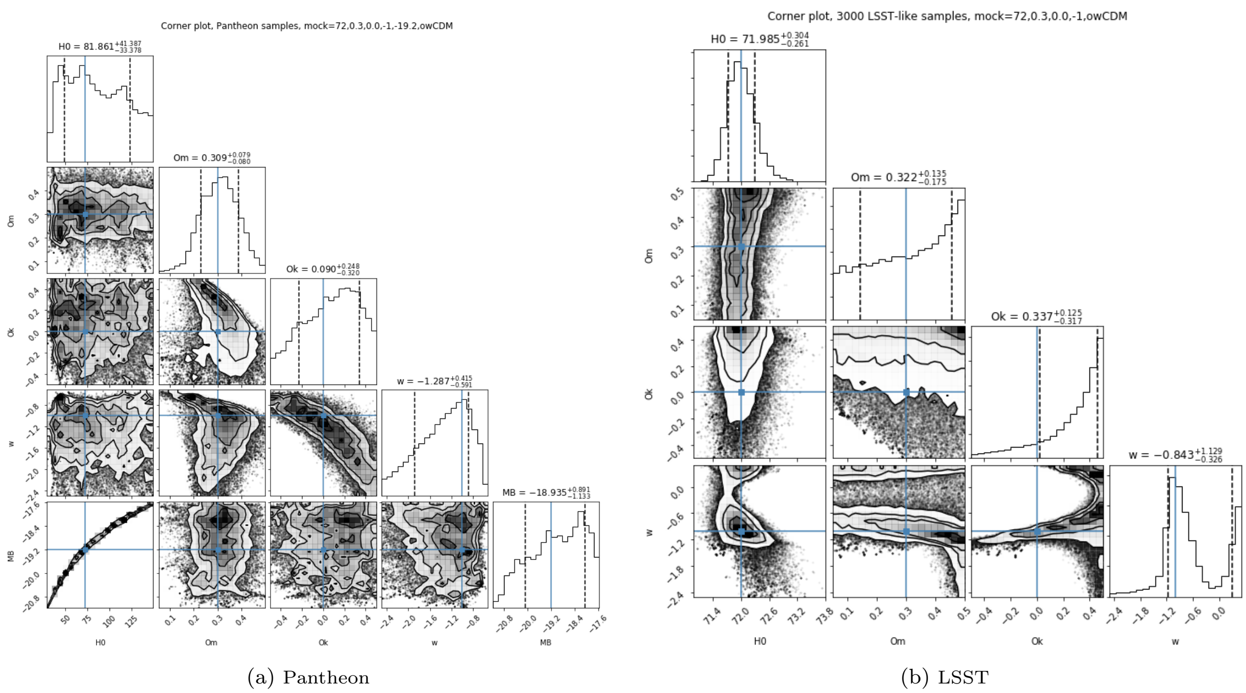
In most cases, we work in a mock owCDM universe with cosmological parameters set to be $(H_0, \Omega_M, \Omega_k, w, M_B) = (72, 0.3, 0, -1, -19.2)$ and run MCMC analysis with $(n_{\text{walkers}}, n_{\text{samples}}) = (32, 20000)$. We will refer to such a statistical setting and default values of cosmological parameters as the "standard setting".

3 Results and analysis

3.1 Combining LSST lenses with a past survey

It is well-known that for parametric analysis on SNe Ia, there exists a degeneracy between Hubble's constant H_0 and M_B. This is because both of them enter equation (2) as additive terms. Similar degeneracies exist between w in equation of state of dark energy and density parameters Ω_k and Ω_M. These set a limit on the ability of single-probe data to constrain Ω_k. To see these, we use the redshifts in binned Pantheon dataset and generate m_B measurements in the standard setting. Figure 1 (a) clear shows the degeneracies and limitations on using SNe Ia alone to constrain Ω_k: $\Omega_k = 0.090^{+0.248}_{-0.320}$, a weak constraint.

On the other hand, during the 10-year LSST survey, approximately 310 lensing events are expected to be observed. We call the 310 pairs of (z_l, z_s) in the source file given by LSST *DESC data products*[3] data that satisfies a LSST-like distribution, or LSST-like data. For model-dependent analysis, we assumed the standard setting. LSST-like data was used to generate $D_{\Delta t}$ measurements using equation (1). The number of effective lensing events can increase if quasars and other similar probes are taken into account, and at some instances in this project we increased the number to 3000 by drawing samples from the original source file and appended to it while maintaining the LSST-like distribution. The uncertainty of each simulated measurement was generated as a random number between 6%-10%. It is also widely understood that the combination of angular diameter distances in equation (1) makes $D_{\Delta t}$ approximately inversely proportional to H_0, placing a strong constraint on H_0. However, even with 3000 lenses, the single-probe situation on constraining Ω_k is not optimistic, either. MCMC analysis with standard setting shows that curvature density parameter is weakly constrained by lenses: $\Omega_k = 0.337^{+0.125}_{-0.317}$ (fig. 1 (b)).



(a) Pantheon

(b) LSST

Figure 1: Constraints placed on cosmological parameters by 2 single-probe datasets

While lenses are decent at constraining H_0 and SNe Ia are not, it is promising to study how the combined data can compliment each other in constraining cosmological parameters, especially that of the Ω_k which both individual probe analysis fail to place a strong constraint alone. Looking at the shapes of the w-Ω_k contours in fig. 1, we see that Pantheon produces a “banana” shaped graph indicating degeneracy, while LSST has a more horizontal graph, indicating strong constraint on w and weak constraint on Ω_k. By adding equation (4) and (5) together as the output of log-likelihood function and run MCMC with standard setting, we obtain the constraint by Pantheon+LSST: $\Omega_k = -0.058^{+0.122}_{-0.113}$. A more detailed table of results of constraints placed on various parameters in non-standard settings can be found in Appendix A.

We can see that the addition of lenses is helping to break the degeneracy between w and Ω_k in SNe Ia analysis, as seen from fig. 2, hence complimenting the ability of the past survey Pantheon on constraining Ω_k. However, due to the limitation in precision of measurement in past surveys, the systematic uncertainty of m_B measurements in Pantheon is relatively large. The constraint achieved on Ω_k is still not desirable.

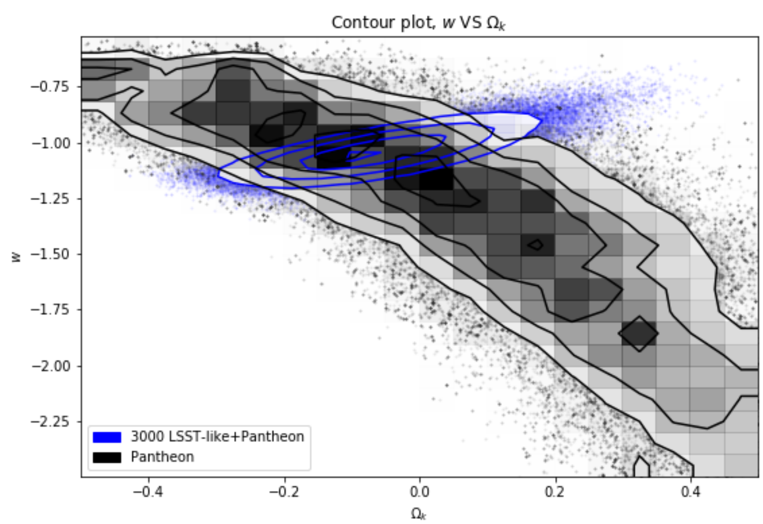


Figure 2: Comparison of w-Ω_k contour with and without lenses

3.2 Combining LSST lenses with next-generation surveys

To make fairer comparison and forecast for future analysis, we combine LSST lenses with next-generation surveys. The next-generation SNe Ia survey we used is Roman which has a systematic uncertainty matrix of approximately 10^1 magnitude smaller at every entry compared to that of *Pantheon*. We take simulated redshifts from *Hounsell et al 2018* [5] and generate m_B measurement from those redshifts under the same mock universe using equation (2). An intrinsic magnitude scattering of a Gaussian distribution of $\mu = -19.2$mag and $\sigma = 0.02$mag is added to M_B for the binned Roman data to account for statistical distribution of absolute magnitude of SNe Ia. Note that by adding the intrinsic scattering, the mock universe is close to but not exactly under standard setting. This potentially causes a bias in the cosmological parameters obtained by MCMC from that of the mock values, and the magnitude and sign of bias depend on which random seed is used to generate the intrinsic scattering when applying *numpy.random.seed()* method.

By running MCMC under standard setting with intrinsic scattering on single-probe and multi-probe datasets, we obtain the w-Ω_k contour plot shown in fig. 3. Notice how the the inclusion of lenses and BAO dataset help to break the w-Ω_k degeneracy in SNe Ia dataset. The introduction of intrinsic scattering generated by random seeds causes a noticeable bias in the w-Ω_k contour plot from the mock values $w = -1, \Omega_k = 0$, and fig. 4 shows such biases for seed 20, 21 and 22. Take note that if no seed number is mentioned in a plot, it is using seed 20 by default.

The constraint obtained by combining 3 probes and averaging for seed 20, 21 and 22 on curvature is: $\Omega_k = -0.037^{+0.014}_{-0.014}$. The 1σ interval has almost reduced by an order of magnitude as compared to the analysis with Pantheon, as the simulated Roman dataset has a much smaller systematic uncertainty in its covariance matrix, reducing the width of the banana-shaped contour in fig. 2, and the inclusion of the horizontal lenses contour helps to break the degeneracy of SNe Ia and BAO datasets, placing a strong contraint on Ω_k.

A more detailed record of results under other non-standard setting for the constraint by next-generation surveys on all relevant cosmological parameters can be found in Appendix B.

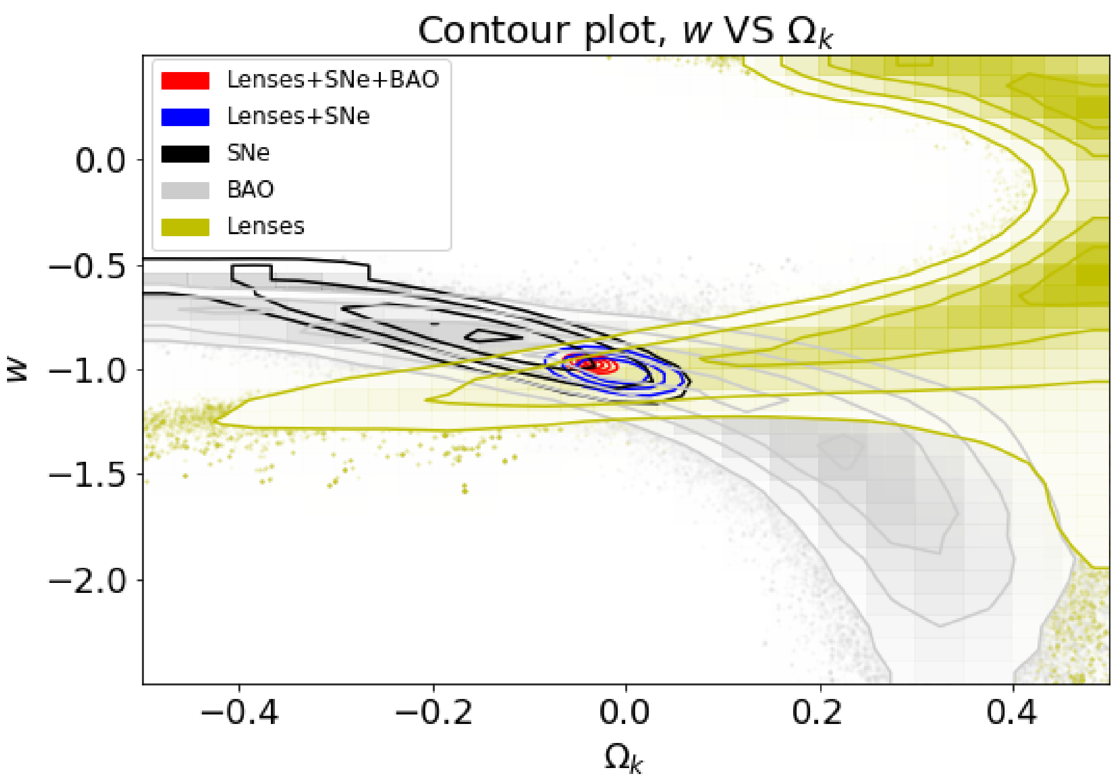


Figure 3: Comparison of w-Ω_k contour with different probes

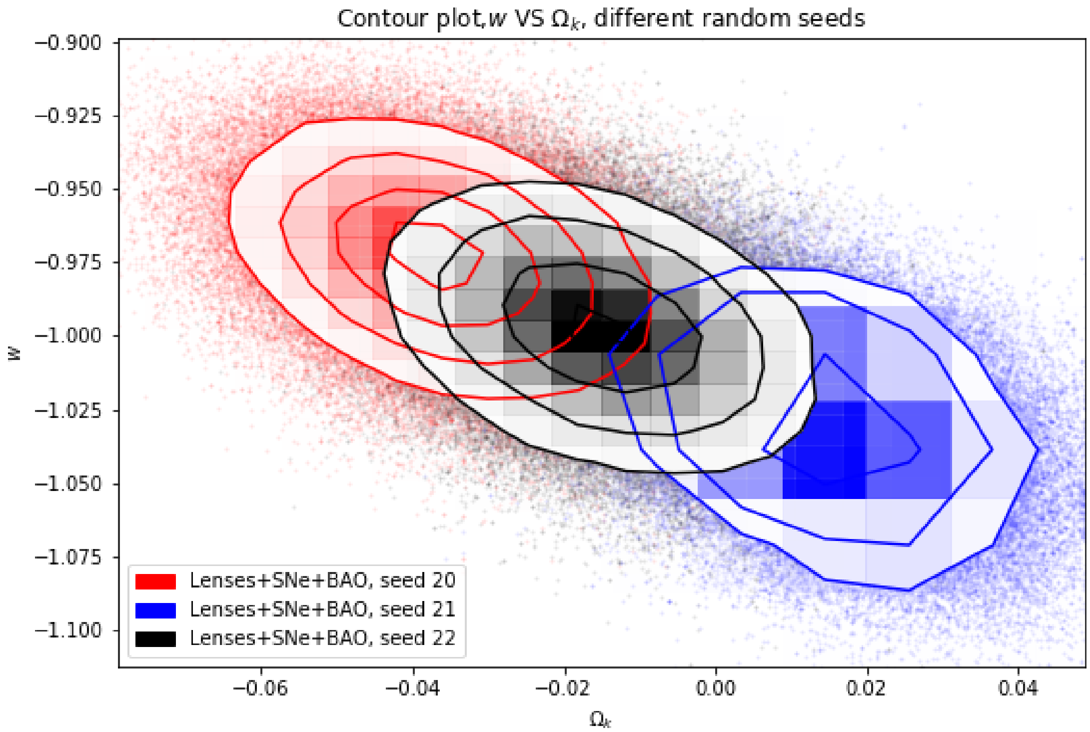


Figure 4: Comparison of w-Ω_k contour with different seeds

3.3 Non-parametric inference

It is also worthwhile to consider the effect of adding lenses into non-parametric analysis on constraining Ω_k. To carry out non-parametric analysis, we no longer relying on assuming a mock universe with a set of truth values of $w = -1$, $\Omega_k = 0$, etc. What we do is interpolating values of $E(z) = H_z/H_0$, which appears in equation (3), using Gaussian Process (GP). $E(z)$, the normalised Hubble parameter, appears in all cosmological distances involved in this project. It is a function of density parameters and w, and the form of this function depends on the model chosen. GP does not require an assumed model of the universe, but rather it is interpolating values statistically using a few given data points. We mentioned that H_z will be measured in BAO survey, so, we use 19 simulated BAO H_z "measurement" to run GP using package *george* with exponential-squared kernel and optimal hyper-parameters. The optimal hyper-parameter for this kernel, A (amplitude) and l (length scale), are determined by MCMC.

The outcome of GP is an interpolated curve for H_z between redshift 0 and 1.85. This curve of H_z is then used to calculate $E(z)$ which appears in equation (1)&(3) for $D_{\Delta t}$ and D_L. This enables the previously modelled values in equation (4)&(5) to be generated in a non-parametric way. The rest of the steps is the same as those in section 4. We run MCMC with 310 lenses cases and $(n_{\text{walkers}}, n_{\text{samples}}) = (32, 20000)$. Figure 5 shows the constraints obtained with Roman and 310 LSST-like lenses. This again corroborates with our previous discussion that SNe Ia gives poor constraint on H_0 and lenses give poor constraint on Ω_k, and they are complimenting each other to give strong constraints on both H_0 and ω_k.

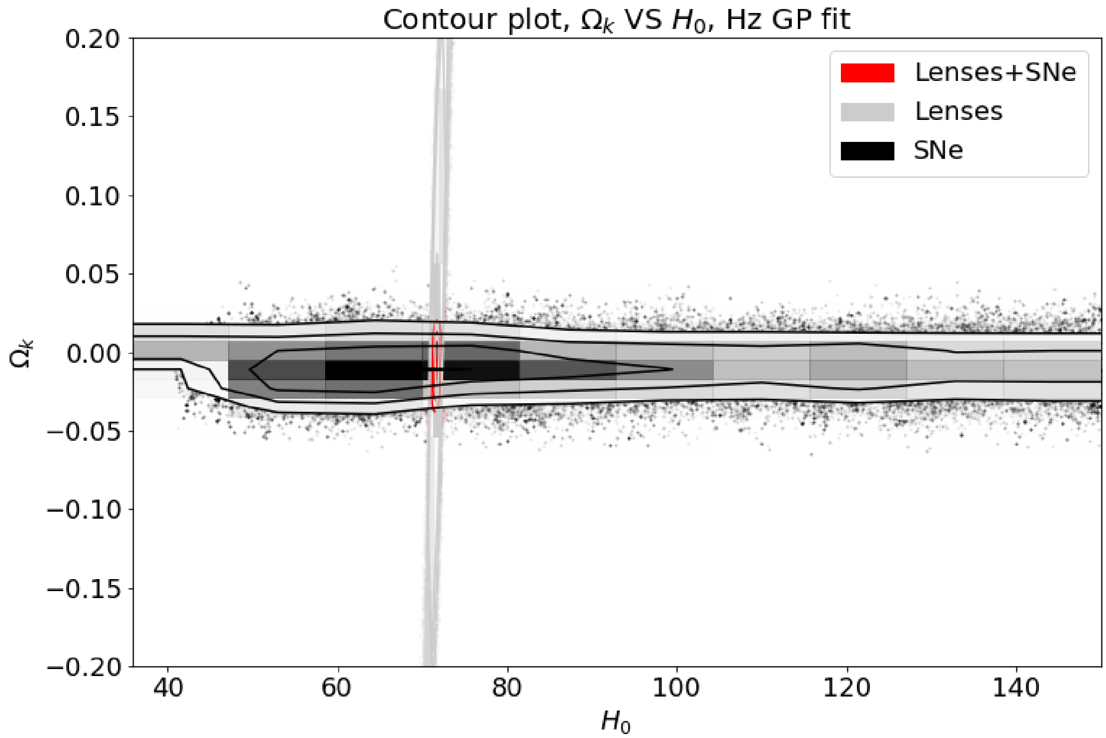


Figure 5: Comparison of Ω_k-H_0 contour in non-parametric analysis

Figure 6 shows a summary of non-parametric analysis with 310 lenses on constraining parameters. The curvature density parameter is constrained to be: $\Omega_k = -0.009^{+0.015}_{-0.015}$. This is comparable to the constraint obtained in model-dependent analysis with 3000 lenses! The histograms shows the improvements made by adding lenses into the analysis. A detailed record of result can be found in Appendix C.

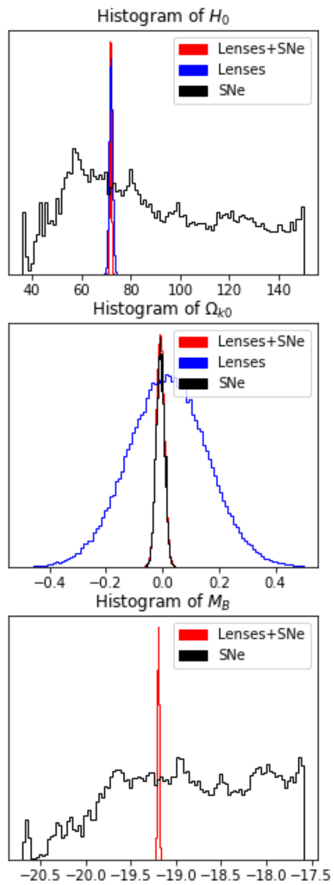


Figure 6: Summary of constraints with LSST and Roman datasets in non-parametric analysis

4 Conclusion

In this project, we explored the ability of lenses to constrain Ω_k and combine it to a past survey and future surveys to see the improvements made on constraining Ω_k in both model-dependent and independent analysis. Result shows optimistic forecast on adding lenses into analysis to get strong constraint on Ω_k. When adding lenses to the past Pantheon survey, the constraint obtained is $\Omega_k = -0.058^{+0.122}_{-0.113}$. When combining lenses with simulated next-generation surveys, the constraint is $\Omega_k = -0.037^{+0.014}_{-0.014}$. In non-parametric analysis, the constraint achieved is $\Omega_k = -0.009^{+0.015}_{-0.015}$ for 310 lenses.

Although failed to place a strong constraint on Ω_k alone, lenses contributes positively at constraining Ω_k when combined with other probes explored in this project, as lenses compliments the weakly constrained parameters of other probes (e.g. H_0), hence improving the constraint achieved on all parameters overall. It will be promising to study what constraint on Ω_k can be achieved when combining lenses with other consistent and complimentary probes, as well as analysis with more complicated model of the universe such as ow_0w_aCDM and slow-roll model. Hopefully, this project can serve as an elementary but instructive forecast on using next-generation surveys to constrain Ω_k.

References

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[5] R Hounsell et al. "Simulations of the WFIRST supernova survey and forecasts of cosmological constraints". In: *The Astrophysical Journal* 867.1 (2018), p. 23.

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Appendix A

Constraints on parameters with simulated lenses and Pantheon

Cosmological model		H_0	Ω_m	Ω_k	w	M_B
1000 *Strong lenses + SNe Ia*						
$o\Lambda$CDM	Mock	72	0.3	0.00	-1	-19.2
	MCMC	$71.906^{+0.324}_{-0.327}$	$0.312^{+0.047}_{-0.048}$	$-0.039^{+0.118}_{-0.110}$	$\equiv -1$	$-19.207^{+0.018}_{-0.018}$
2000 *Strong lenses + SNe Ia*						
$o\Lambda$CDM	Mock	72	0.3	0.00	-1	-19.2
	MCMC	$71.939^{+0.254}_{-0.255}$	$0.307^{+0.039}_{-0.040}$	$-0.024^{+0.093}_{-0.090}$	$\equiv -1$	$-19.205^{+0.016}_{-0.016}$
3000 *Strong lenses + SNe Ia*						
$o\Lambda$CDM	Mock	72	0.3	0.00	-1	-19.2
	MCMC	$71.959^{+0.212}_{-0.215}$	$0.304^{+0.035}_{-0.035}$	$-0.017^{+0.080}_{-0.076}$	$\equiv -1$	$-19.205^{+0.015}_{-0.015}$
$o\Lambda$CDM	Mock	72	0.3	0.03	-1	-19.2
	MCMC	$71.952^{+0.210}_{-0.211}$	$0.304^{+0.035}_{-0.035}$	$0.010^{+0.081}_{-0.078}$	$\equiv -1$	$-19.205^{+0.015}_{-0.015}$
$o\Lambda$CDM	Mock	72	0.3	0.05	-1	-19.2
	MCMC	$71.953^{+0.211}_{-0.211}$	$0.304^{+0.035}_{-0.036}$	$0.030^{+0.084}_{-0.080}$	$\equiv -1$	$-19.205^{+0.015}_{-0.015}$
$o\Lambda$CDM	Mock	72	0.3	-0.03	-1	-19.2
	MCMC	$71.954^{+0.212}_{-0.214}$	$0.304^{+0.034}_{-0.035}$	$-0.048^{+0.078}_{-0.075}$	$\equiv -1$	$-19.205^{+0.015}_{-0.015}$
$o\Lambda$CDM	Mock	72	0.3	-0.05	-1	-19.2
	MCMC	$71.954^{+0.213}_{-0.213}$	$0.304^{+0.034}_{-0.034}$	$-0.066^{+0.076}_{-0.072}$	$\equiv -1$	$-19.205^{+0.015}_{-0.015}$
owCDM	Mock	72	0.3	0	-1	-19.2
	MCMC	$72.059^{+0.285}_{-0.275}$	$0.337^{+0.071}_{-0.076}$	$-0.058^{+0.122}_{-0.113}$	$-1.043^{+0.087}_{-0.078}$	$-19.205^{+0.015}_{-0.015}$
3000 *Strong lenses*						
FwCDM	Mock	72	0.3	0	-1	-
	MCMC	$71.997^{+0.319}_{-0.287}$	$0.313^{+0.131}_{-0.162}$	$\equiv 0$	$-1.029^{+0.063}_{-0.081}$	-

No. of strong gravitational lensing events = 3000

No. of binned SNe Ia events = 40

No. of walkers in MCMC = 32

No. of samples in MCMC = 20000

numpy.random.seed() used in all random processes: 20

Appendix B

Constraints on parameters with simulated lenses and other probes

Cosmological model		H_0	Ω_m	Ω_k	w	M_B
Lenses						
owCDM	Mock	72	0.3	0	-1	-
	MCMC*	$71.985^{+0.304}_{-0.261}$	$0.322^{+0.135}_{-0.175}$	$0.337^{+0.125}_{-0.317}$	$-0.843^{+1.129}_{-0.326}$	-
SNe Ia						
owCDM	Mock	72	0.3	0	-1	-19.2
	MCMC	$74.121^{+28.898}_{-20.841}$	$0.302^{+0.033}_{-0.074}$	$-0.156^{+0.121}_{-0.150}$	$-0.829^{+0.155}_{-0.167}$	$-19.111^{+0.713}_{-0.718}$
BAO						
owCDM	Mock	72	0.3	0	-1	-
	MCMC	$72.738^{+5.279}_{-3.941}$	$0.246^{+0.096}_{-0.078}$	$0.180^{+0.166}_{-0.351}$	$-1.215^{+0.382}_{-0.658}$	-
Lenses + SNe Ia						
$o\Lambda$CDM	Mock	72	0.3	0	-1	-19.2
	MCMC	$71.957^{+0.112}_{-0.112}$	$0.317^{+0.005}_{-0.005}$	$-0.017^{+0.018}_{-0.017}$	$\equiv -1$	$-19.192^{+0.007}_{-0.007}$
owCDM	Mock	72	0.3	0	-1	-19.2
	MCMC	$72.004^{+0.259}_{-0.261}$	$0.318^{+0.007}_{-0.008}$	$-0.012^{+0.028}_{-0.028}$	$-1.008^{+0.042}_{-0.044}$	$-19.191^{+0.009}_{-0.009}$
Lenses + SNe Ia + BAO						
$o\Lambda$CDM	Mock	72	0.3	0	-1	-19.2
	MCMC	$71.905^{+0.100}_{-0.101}$	$0.315^{+0.005}_{-0.005}$	$-0.031^{+0.013}_{-0.013}$	$\equiv -1$	$-19.201^{+0.004}_{-0.004}$
owCDM	Mock	72	0.3	0	-1	-19.2
	MCMC1*	$71.787^{+0.148}_{-0.148}$	$0.313^{+0.006}_{-0.006}$	$-0.036^{+0.014}_{-0.014}$	$-0.974^{+0.024}_{-0.024}$	$-19.199^{+0.004}_{-0.004}$
owCDM	Mock	72	0.3	0	-1	-19.2
	MCMC2*	$72.170^{+0.155}_{-0.164}$	$0.296^{+0.006}_{-0.006}$	$0.014^{+0.014}_{-0.015}$	$-1.028^{+0.027}_{-0.026}$	$-19.201^{+0.004}_{-0.004}$
owCDM	Mock	72	0.3	0	-1	-19.2
	MCMC3*	$71.954^{+0.145}_{-0.150}$	$0.306^{+0.006}_{-0.006}$	$-0.015^{+0.014}_{-0.014}$	$-0.997^{+0.024}_{-0.024}$	$-19.202^{+0.004}_{-0.004}$

No. of strong gravitational lensing events = 3000

No. of binned SNe Ia events = 40

No. of BAO measurements = 18

No. of walkers in MCMC = 32

No. of samples in MCMC = 20000

* MCMC & MCMC1 indicate seed=20, MCMC2 indicates seed=21, MCMC3 indicates seed=22

Appendix C

Constraint on parameters in non-parametric analysis

Probe		H_0	Ω_k	M_B
SNe Ia	Mock	72	0	-19.2
	MCMC	$68.619^{+14.652}_{-11.738}$	$-0.011^{+0.016}_{-0.016}$	$-19.295^{+0.419}_{-0.406}$
Strong lenses	Mock	72	0	-
	MCMC	$71.183^{+1.335}_{-1.328}$	$0.070^{+0.116}_{-0.115}$	-
Strong lenses+SNe Ia	Mock	72	0	-19.2
	MCMC	$70.440^{+0.723}_{-0.702}$	$-0.008^{+0.015}_{-0.014}$	$-19.238^{+0.022}_{-0.021}$

No. of strong gravitational lensing events = 310

No. of binned SNe Ia events = 40

No. of walkers in MCMC = 32

No. of samples in MCMC = 20000