Constraining Curvature Density Parameter by Combining Time-Delay Lenses with Other Probes: a Forecast for Next-Generation Surveys

Project Report

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1 Introduction

The curvature density parameter, Ω_k, is an important cosmological parameter that determines the geometry of our Universe. There are many probes that enable estimates on Ω_k of late universe. The constraint placed on Ω_k by probes such as Type Ia supernovae (SNe Ia), strong gravitational lenses and Baryon Acoustic Oscillation (BAO) have been widely analysed and discussed in past literature (cite). A promising study on constraining Ω_k is to look at a large number of time-delay distance measurements from strong gravitational lenses and combine it with complimentary probes, hoping that the combined data can overcome the deficiency of limited lenses events in previous analysis such as that by *H0LiCOW* on 6 lenses (cite) and degeneracy among cosmological parameters in single-probe analysis (cite). In light of the next-generation survey from the Rubin Observatory's legacy survey of space and time (*LSST*), it is expected that hundreds of strong lensing events will be observed during its 10-year survey baseline. This will provide a large dataset for studying the ability of strong lenses to constrain Ω_k.

In this project, we generate *LSST*-like data and use the simulated dataset to explore 3 tasks. Firstly, we combine it with a past survey, *Pantheon*, of apparent magnitude measurement of SNe Ia to see the improvement made on constraining Ω_k by adding lenses into the analysis. Different models of dark energy in a non-flat universe, i.e. oLCDM and owCDM, are used in the process of generating mock values of "measurements" for each next-generation survey. Then, we generate *Roman*-like SNe Ia data and *DESI*-like BAO data, both being the next-generation survey dataset, and combine them with *LSST*-like data to see their ability on constraining Ω_k. Finally, we use Gaussian Process to fit Hubble parameter H_z and look at a model-independent method to analyse the constraint that lenses and complimentary probes place on Ω_k. We hope that this project provides a forecast on using next-generation surveys to constrain Ω_k.

2 Theory and setups

The three late-universe probes used in this project are strong gravitational lenses, SNe Ia and BAO, and the method to get constraint on parameters is Markov Chain Monte Carlo (MCMC). Here, relevant formulae will be listed without explanation for reference purpose.

For lenses, we are interested in measuring time-delay distances $D_{\Delta t}$ of a lens at redshift z_l and a source at z_s:

$$D_{\Delta t} = \frac{(1+z_s)D_{A,l}D_{A,s}}{D_{A,ls}}, \tag{1}$$

where $D_{A,l}$ and $D_{A,s}$ are angular diameter distances at lens and source respectively, and $D_{A,ls}$ is the angular diameter distance between lens and source. D_A at a given redshift can be calculated by $D_L/(1+z)$ where D_L is the luminosity distance at the same redshift with which the formula will be given below.

For SNe Ia, what we measure is the apparent magnitude m_B given by:

$$m_B = 5\lg(D_L) + 25 + M_B, \tag{2}$$

where M_B is the absolute magnitude and D_L is the luminosity distances at z. D_L is given by:

$$D_L = \frac{c(1+z)}{H_0\sqrt{|\Omega_k|}} F(\sqrt{|\Omega_k|}\int_0^z \frac{\mathrm{d}z'}{E(z')}), \tag{3}$$

where function F is sin for $\Omega_k \leq 0$ and sinh for $\Omega_k > 0$. For BAO, we measure H_z, the Hubble parameter.

Throughout this project, for MCMC, we use uniform piror distributions for each cosmological parameter marginalised. The likelihood function is set to return a χ^2 term for lenses and BAO in this form:

$$\chi^2 = \sum_{\text{all probes used}} \sum_i \frac{(x_i - x_{m,i})^2}{\sigma_{x_i}^2}. \tag{4}$$

x_i, $x_{m,i}$ and σ_{x_i} indicate measured value, modelled value and percentage uncertainty in measurement x_i respectively. For example, for lenses, x_i is supposed to be a measured value of $D_{\Delta t}$; for BAO, x_i is a measured value of H_z. For SNe Ia, the χ^2 term takes this form instead:

$$\chi^2 = \Delta \boldsymbol{m_B}^T \cdot \boldsymbol{C}^{-1} \cdot \Delta \boldsymbol{m_B}, \tag{5}$$

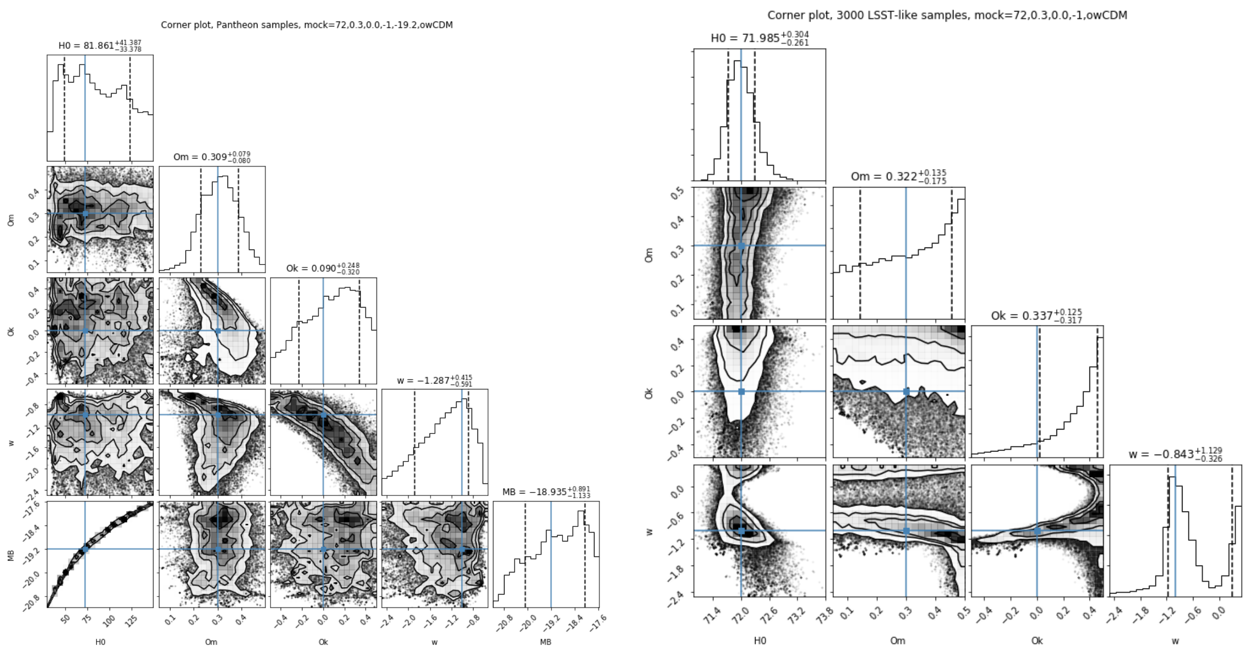
where $\Delta \boldsymbol{m_B} = \boldsymbol{m_B} - \boldsymbol{m_B}_{\text{model}}$ is a vector of difference between measured and modelled apparent magnitudes and $\boldsymbol{C} = \boldsymbol{D}_{\text{stat}} + \boldsymbol{C}_{\text{sys}}$ is the covariance matrix that takes into account both statistical and systematic uncertainty.

Data analysis in this project is done with *Python*. The key package used to calculate cosmological quantities in many cases is *astropy.cosmology*. The package used to run MCMC is *emcee*, and the package to do Gaussian Process is *george*. Note that in most cases in this project, we are doing a forecast for next-generation surveys, and the "measured" dataset is a simulated dataset generated from a mock universe of either Flat$w$CDM, $o\Lambda$CDM or owCDM model using methods in *astropy.cosmology*.

3 Combining Lenses with a past survey

It is well-known that for parametric analysis on SNe Ia, there exists a degeneracy between Hubble's constant H_0 and M_B. This is because both of them enter equation (2) as additive terms. Similar degeneracies exist between w in equation of state of dark energy and density parameters Ω_k and Ω_M. These set a limit on the ability of single-probe data to constrain Ω_k. To see these, we use the redshifts in binned *Pantheon* dataset and generate m_B measurements in a mock owCDM universe of $(H_0, \Omega_M, \Omega_k, w, M_B) = (72, 0.3, 0, -1, -19.2)$. MCMC Analysis with $(n_{\text{walkers}}, n_{\text{samples}}) = (32, 20000)$ clear shows the degeneracies and limitations on using SNe Ia alone to constrain Ω_k: $\Omega_k = 0.090^{+0.248}_{-0.320}$, a weak constraint (fig. 1 (a)). We will refer to such a mock owCDM universe and $(n_{\text{walkers}}, n_{\text{samples}}) = (32, 20000)$ as the "standard setting".

During the expected 10-year *LSST* survey, approximately 310 lensing events are expected to be observed (cite). The number of effective lensing events can increase if quasars and other similar probes are taken into account. We generate pairs of (z_l, z_s) that satisfies a LSST-like distribution (I don't know the detail; help needed) and use the same mock universe above to generate $D_{\Delta t}$ measurements. The uncertainty of each simulated measurement is also generated as a random number between 6%-10%. It is also widely understood that the combination of angular diameter distances in equation (1) makes $D_{\Delta t}$ approximately inversly proportional to H_0, placing a strong constraint on H_0. However, even with 3000 lenses, the single-probe situation on constraining Ω_k is not optimistic, either. MCMC Analysis with standard setting shows that curvature density parameter is weakly constrained by lenses: $\Omega_k = 0.337^{+0.125}_{-0.317}$ (fig. 1 (b)).



(a) *Pantheon* (b) *LSST*

Figure 1: Constraints placed on cosmological parameters by 2 single-probe datasets

While lenses are decent at constraining H_0 and SNe Ia are not, it is promising to study how the combined data can compliment each other in constraining cosmological parameters, especially that of the Ω_k which both individual probe analysis fail to place a strong constraint alone. Looking at the shapes of the w-Ω_k contours in fig. 1, we see that *Pantheon* produces a "banana" shaped graph indicating degeneracy, while *LSST* has a more horizontal graph, indicating strong constraint on w and weak constraint on Ω_k. By adding equation (4) and (5) together as the output of log-likelihood function and run MCMC with standard setting, we obtain the constraint by *Pantheon+LSST*: $\Omega_k = -0.058^{+0.122}_{-0.113}$. A more detailed table of results of constraints placed on various parameters in non standard settings can be found in Appendix A.

We can see that the addition of lenses is helping to break the degeneracy between w and Ω_k in SNe Ia analysis, as seen from fig. 2, hence complimenting the ability of the past survey *Pantheon* on constraining Ω_k. However, due to the limitation in precision of measurement in past surveys, the systematic uncertainty of m_B measurements in *Pantheon* is relatively large. The constraint achieved on Ω_k is still not desirable.

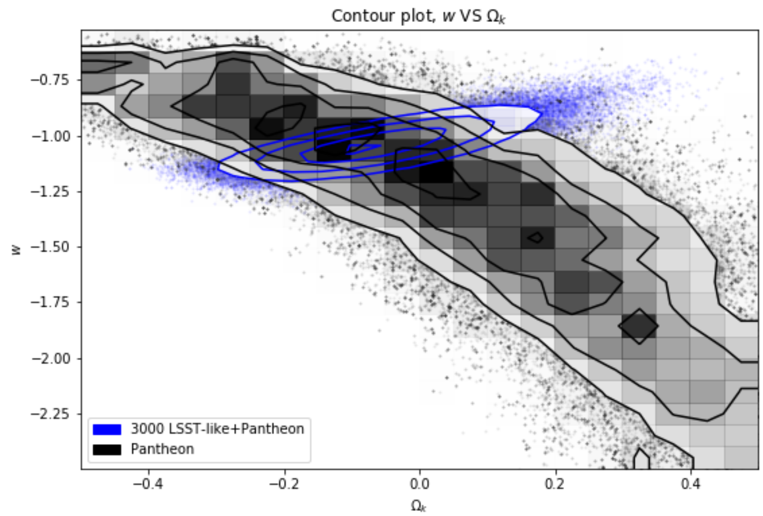


Figure 2: Comparison of w-Ω_k contour with and without lenses

4 Combining Lenses with next-generation surveys

To make fairer comparison and forecast for future analysis, we combine *LSST* lenses with next-generation surveys. The next-generation SNe Ia survey we used is *Roman* which has a systematic uncertainty matrix of approximately 10^1 magnitude smaller at every entry compared to that of *Pantheon*. We only take simulated redshifts and generate "measured" m_B from those redshifts using equation (2). An intrinsic magnitude scattering of a Gaussian distribution of $\mu = -19.2$mag and $\sigma = 0.02$mag is added to M_B for the binned *Roman* data to account for the statistical distribution of absolute magnitude of SNe Ia. Note that by adding the intrinsic scattering, the mock universe is close to but not exactly under standard setting anymore. This potentially causes a bias in the cosmological parameters obtained by MCMC from that of the mock values, and the magnitude and sign of bias depend on which random seed is used to generate the intrinsic scattering when applying *numpy.random.seed()* method. For BAO, we take simulated redshifts and directly compute H_z using a method in *astropy.cosmology*.

By running MCMC under standard setting with intrinsic scattering on single-probe and multi-probe datasets, we obtain the w-Ω_k contour plot shown in fig. 3. Notice how the the inclusion of lenses and BAO dataset help to break the w-Ω_k degeneracy in SNe Ia dataset. The introduction of intrinsic scattering generated by random seeds causes a noticeable bias in the w-Ω_k contour plot from the mock values $w = -1, \Omega_k = 0$, and fig. 4 shows such biases for seed 20, 21 and 22. Take note that if no seed number is mentioned in a plot, it is using seed 20 by default.

The constraint obtained by combining 3 probes and averaging for seed 20, 21 and 22 on curvature is: $\Omega_k = -0.037^{+0.014}_{-0.014}$. The 1σ interval has almost reduced by an order of magnitude as compard to the analysis with *Pantheon*, as the simulated *Roman* dataset has a much smaller systematic uncertainty in its covariance matrix, reducing the width of the banana-shaped contour in fig. 2, and the inclusion of the horizontal lenses contour helps to break the degeneracy of SNe Ia and BAO datasets, placing a strong contraint on Ω_k.

A more detailed record of results under other non-standard setting for the constraint by next-generation surveys on all relevant cosmological parameters can be found in Appendix B.

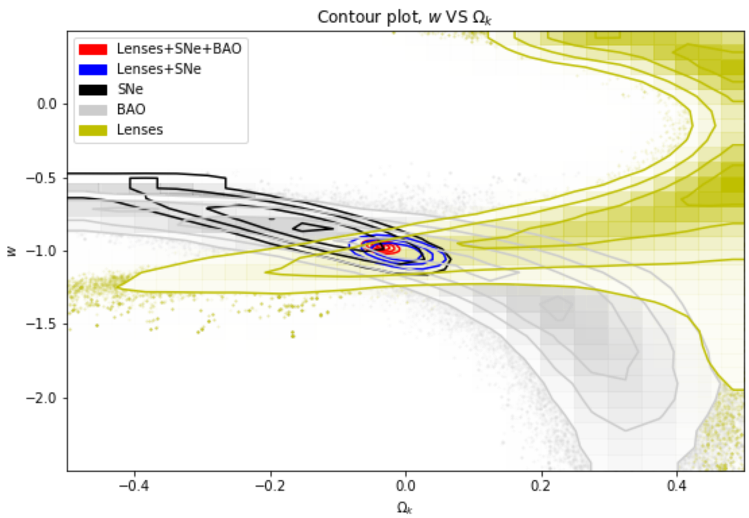


Figure 3: Comparison of w-Ω_k contour with different probes

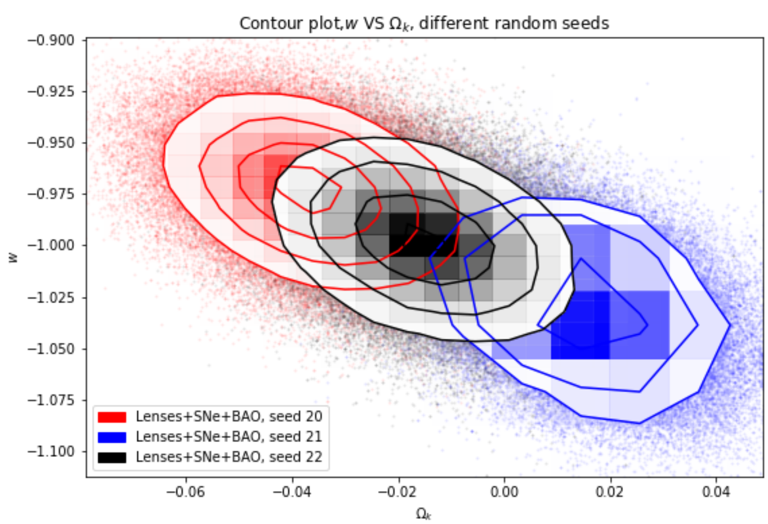


Figure 4: Comparison of w-Ω_k contour with different seeds

5 Model-indepentent inference

It is also worthwhile to consider the effect of adding lenses into model-independent analysis on constraining Ω_k. To carry out model-independent analysis, we no longer relying on assuming a mock universe with a set of truth values of $w = -1$, $\Omega_k = 0$, etc. What we do is interpolating values of $E(z) = H_z/H_0$, which appears in equation (3), using Gaussian Process (GP). $E(z)$, the normalised Hubble parameter, appears in all cosmological distances involved in this project. It is a function of density parameters and w, and the form of this function depends on the model chosen. GP does not require an assumed model of the universe, but rather it is interpolating values statistically using a few given data points. We mentioned that H_z will be measured in BAO survey, so, we use 19 simulated BAO H_z "measurement" to run GP using package *george* with exponential-squared kernel and optimal hyper-parameters. The optimal hyper-parameter for this kernel, A (amplitude) and l (length scale), are determined by MCMC.

The outcome of GP is an interpolated curve for H_z between redshift 0 and 1.85. This curve of H_z is then used to calculate $E(z)$ which appears in equation (1)&(3) for $D_{\Delta t}$ and D_L. This enables the previously modelled values in equation (4)&(5) to be generated in a model-independent way. The rest of the steps is the same as those in section 4. We run MCMC with 310 lenses cases and $(n_{\text{walkers}}, n_{\text{samples}}) = (32, 20000)$. Figure 5 shows the constraints obtained with *Roman* and 310 *LSST*-like lenses. This again corroborates with our previous discussion that SNe Ia gives poor constraint on H_0 and lenses give poor constraint on Ω_k, and they are complimenting each other to give strong constraints on both H_0 and ω_k.

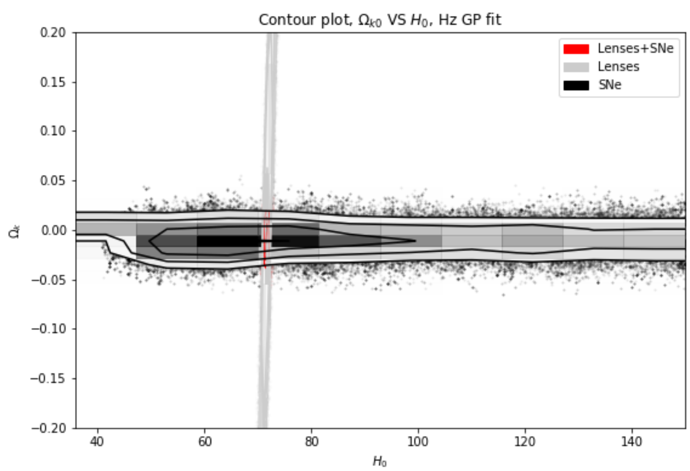


Figure 5: Comparison of Ω_k-H_0 contour in model-independent analysis

Figure 6 shows a summary of model-independent analysis with 310 lenses on constraining parameters. The curvature density parameter is constrained to be: $\Omega_k = -0.009^{+0.015}_{-0.015}$. This is comparable to the constraint obtained in model-dependent analysis with 3000 lenses! The histograms shows the improvements made by adding lenses into the analysis. A detailed record of result can be found in Appendix C.

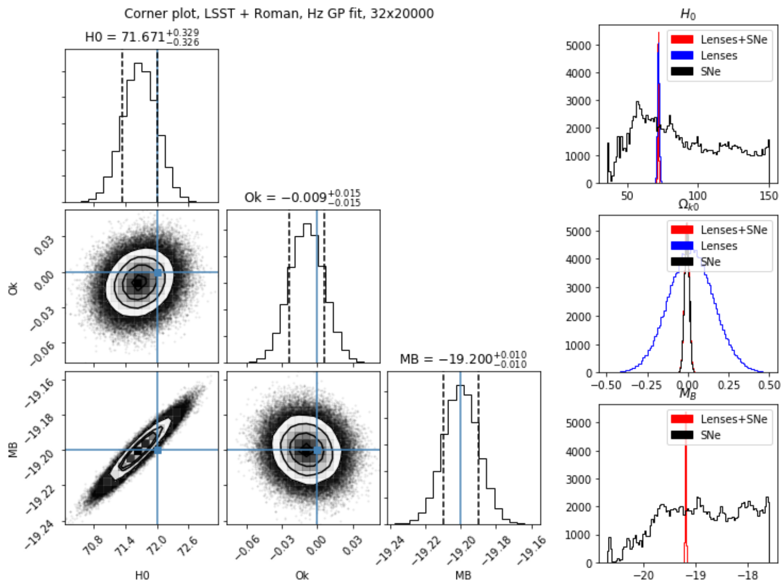


Figure 6: Summary of constraints with *LSST* and *Roman* datasets in model-independent analysis

Finally, we do the same model-independent analysis with 3000 lenses. It takes extremely long computing time to run MCMC with large number of walkers and samples, so we run that with $(n_{\rm walkers}, n_{\rm samples}) = (32, 8000)$ instead. The constraint placed on Ω_k by 3000 *LSST* lenses and *Roman* with model-independent analysis is the strongest achieved in this project: $\Omega_k = 0.002^{+0.010}_{-0.010}$. The bias of H_0 in the corner plot below still remains a puzzle to us. It is not due to choice of seed or MCMC uncertainty, and we will update the report when it is solved.

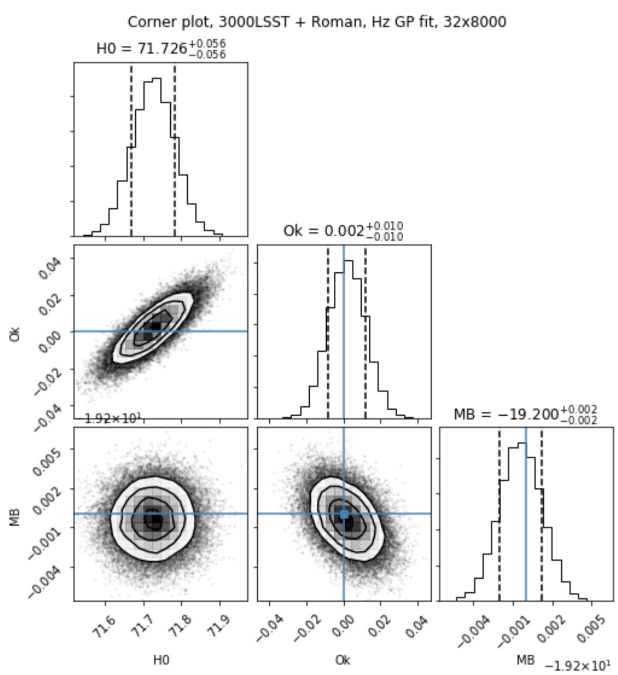


Figure 7: Constraints placed on cosmological parameters by *LSST* and *Roman*, model-independent analysis

6 Conclusion

In this project, we explored the ability of lenses to constrain Ω_k and combine it to a past survey and future surveys to see the improvements made on constraining Ω_k in both model-dependent and independent analysis. Result shows optimistic forecast on adding lenses into analysis to get strong constraint on Ω_k. When adding lenses to the past *Pantheon* survey, the constraint obtained is $\Omega_k = -0.058^{+0.122}_{-0.113}$. When combining lenses with simulated next-generation surveys, the constraint is $\Omega_k = -0.037^{+0.014}_{-0.014}$. In model-independent analysis, the constraints achieved are $\Omega_k = -0.009^{+0.015}_{-0.015}$ and $\Omega_k = 0.002^{+0.010}_{-0.010}$ for 310 and 3000 lenses cases respectively.

Although failed to place a strong constraint on Ω_k alone, lenses contributes positively at constraining Ω_k when combined with other probes explored in this project, as lenses compliments the weakly constrained parameters of other probes (e.g. H_0), hence improving the constraint achieved on all parameters overall. It will be promising to study what constraint on Ω_k can be achieved when combining lenses with other consistent and complimentary probes, as well as analysis with more complicated model of the universe such as ow_0w_aCDM and slow-roll model. Hopefully, this project can serve as an elementary but instructive forecast on using next-generation surveys to constrain Ω_k.

Appendix A

Constraints on parameters with simulated lenses and *Pantheon*

Cosmological model		H_0	Ω_m	Ω_k	w	M_B
1000 *Strong lenses + SNe Ia*						
$o\Lambda$CDM	Mock	72	0.3	0.00	-1	-19.2
	MCMC	$71.906^{+0.324}_{-0.327}$	$0.312^{+0.047}_{-0.048}$	$-0.039^{+0.118}_{-0.110}$	$\equiv -1$	$-19.207^{+0.018}_{-0.018}$
2000 *Strong lenses + SNe Ia*						
$o\Lambda$CDM	Mock	72	0.3	0.00	-1	-19.2
	MCMC	$71.939^{+0.254}_{-0.255}$	$0.307^{+0.039}_{-0.040}$	$-0.024^{+0.093}_{-0.090}$	$\equiv -1$	$-19.205^{+0.016}_{-0.016}$
3000 *Strong lenses + SNe Ia*						
$o\Lambda$CDM	Mock	72	0.3	0.00	-1	-19.2
	MCMC	$71.959^{+0.212}_{-0.215}$	$0.304^{+0.035}_{-0.035}$	$-0.017^{+0.080}_{-0.076}$	$\equiv -1$	$-19.205^{+0.015}_{-0.015}$
$o\Lambda$CDM	Mock	72	0.3	0.03	-1	-19.2
	MCMC	$71.952^{+0.210}_{-0.211}$	$0.304^{+0.035}_{-0.035}$	$0.010^{+0.081}_{-0.078}$	$\equiv -1$	$-19.205^{+0.015}_{-0.015}$
$o\Lambda$CDM	Mock	72	0.3	0.05	-1	-19.2
	MCMC	$71.953^{+0.211}_{-0.211}$	$0.304^{+0.035}_{-0.036}$	$0.030^{+0.084}_{-0.080}$	$\equiv -1$	$-19.205^{+0.015}_{-0.015}$
$o\Lambda$CDM	Mock	72	0.3	-0.03	-1	-19.2
	MCMC	$71.954^{+0.212}_{-0.214}$	$0.304^{+0.034}_{-0.035}$	$-0.048^{+0.078}_{-0.075}$	$\equiv -1$	$-19.205^{+0.015}_{-0.015}$
$o\Lambda$CDM	Mock	72	0.3	-0.05	-1	-19.2
	MCMC	$71.954^{+0.213}_{-0.213}$	$0.304^{+0.034}_{-0.034}$	$-0.066^{+0.076}_{-0.072}$	$\equiv -1$	$-19.205^{+0.015}_{-0.015}$
owCDM	Mock	72	0.3	0	-1	-19.2
	MCMC	$72.059^{+0.285}_{-0.275}$	$0.337^{+0.071}_{-0.076}$	$-0.058^{+0.122}_{-0.113}$	$-1.043^{+0.087}_{-0.078}$	$-19.205^{+0.015}_{-0.015}$
3000 *Strong lenses*						
FwCDM	Mock	72	0.3	0	-1	-
	MCMC	$71.997^{+0.319}_{-0.287}$	$0.313^{+0.131}_{-0.162}$	$\equiv 0$	$-1.029^{+0.063}_{-0.081}$	-

No. of strong gravitational lensing events = 3000

No. of binned SNe Ia events = 40

No. of walkers in MCMC = 32

No. of samples in MCMC = 20000

numpy.random.seed() used in all random processes: 20

Appendix B

Constraints on parameters with simulated lenses and other probes

Cosmological model		H_0	Ω_m	Ω_k	w	M_B
Lenses						
owCDM	Mock	72	0.3	0	-1	-
	MCMC*	$71.985^{+0.304}_{-0.261}$	$0.322^{+0.135}_{-0.175}$	$0.337^{+0.125}_{-0.317}$	$-0.843^{+1.129}_{-0.326}$	-
SNe Ia						
owCDM	Mock	72	0.3	0	-1	-19.2
	MCMC	$74.121^{+28.898}_{-20.841}$	$0.302^{+0.033}_{-0.074}$	$-0.156^{+0.121}_{-0.150}$	$-0.829^{+0.155}_{-0.167}$	$-19.111^{+0.713}_{-0.718}$
BAO						
owCDM	Mock	72	0.3	0	-1	-
	MCMC	$72.738^{+5.279}_{-3.941}$	$0.246^{+0.096}_{-0.078}$	$0.180^{+0.166}_{-0.351}$	$-1.215^{+0.382}_{-0.658}$	-
Lenses + SNe Ia						
$o\Lambda$CDM	Mock	72	0.3	0	-1	-19.2
	MCMC	$71.957^{+0.112}_{-0.112}$	$0.317^{+0.005}_{-0.005}$	$-0.017^{+0.018}_{-0.017}$	$\equiv -1$	$-19.192^{+0.007}_{-0.007}$
owCDM	Mock	72	0.3	0	-1	-19.2
	MCMC	$72.004^{+0.259}_{-0.261}$	$0.318^{+0.007}_{-0.008}$	$-0.012^{+0.028}_{-0.028}$	$-1.008^{+0.042}_{-0.044}$	$-19.191^{+0.009}_{-0.009}$
Lenses + SNe Ia + BAO						
$o\Lambda$CDM	Mock	72	0.3	0	-1	-19.2
	MCMC	$71.905^{+0.100}_{-0.101}$	$0.315^{+0.005}_{-0.005}$	$-0.031^{+0.013}_{-0.013}$	$\equiv -1$	$-19.201^{+0.004}_{-0.004}$
owCDM	Mock	72	0.3	0	-1	-19.2
	MCMC1*	$71.787^{+0.148}_{-0.148}$	$0.313^{+0.006}_{-0.006}$	$-0.036^{+0.014}_{-0.014}$	$-0.974^{+0.024}_{-0.024}$	$-19.199^{+0.004}_{-0.004}$
owCDM	Mock	72	0.3	0	-1	-19.2
	MCMC2*	$72.170^{+0.155}_{-0.164}$	$0.296^{+0.006}_{-0.006}$	$0.014^{+0.014}_{-0.015}$	$-1.028^{+0.027}_{-0.026}$	$-19.201^{+0.004}_{-0.004}$
owCDM	Mock	72	0.3	0	-1	-19.2
	MCMC3*	$71.954^{+0.145}_{-0.150}$	$0.306^{+0.006}_{-0.006}$	$-0.015^{+0.014}_{-0.014}$	$-0.997^{+0.024}_{-0.024}$	$-19.202^{+0.004}_{-0.004}$

No. of strong gravitational lensing events = 3000

No. of binned SNe Ia events = 40

No. of BAO measurements = 18

No. of walkers in MCMC = 32

No. of samples in MCMC = 20000

* MCMC & MCMC1 indicate seed=20, MCMC2 indicates seed=21, MCMC3 indicates seed=22

Appendix C

Constraint on parameters in model-independent analysis

Probe		H_0	Ω_k	M_B
SNe Ia	Mock	72	0	-19.2
	MCMC	$68.619^{+14.652}_{-11.738}$	$-0.011^{+0.016}_{-0.016}$	$-19.295^{+0.419}_{-0.406}$
Strong lenses	Mock	72	0	-
	MCMC	$71.183^{+1.335}_{-1.328}$	$0.070^{+0.116}_{-0.115}$	-
Strong lenses+SNe Ia	Mock	72	0	-19.2
	MCMC	$70.440^{+0.723}_{-0.702}$	$-0.008^{+0.015}_{-0.014}$	$-19.238^{+0.022}_{-0.021}$

No. of strong gravitational lensing events = 310

No. of binned SNe Ia events = 40

No. of walkers in MCMC = 32

No. of samples in MCMC = 20000