

11/01/2024  
Tuesday

- \* **Longest common subsequence**  
→ is a subsequence that is common to both strings.  
Given two strings  $\text{text1}$  and  $\text{text2}$ , return the length of their longest common subsequence.  
If there is no common subsequence, return 0.

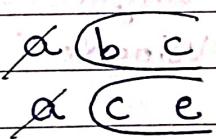
I/P -  $\text{text1} = \text{"abcde"}$ ,  $\text{text2} = \text{"ace"}$

O/P - 3

Explanation - Here, LCS is "ace" and its length is 3.

Approach - Base case → return 0 when there is no character in string.

If character matches -



$$\text{ans} = 1 + \text{recursionKAAns}$$

→ adding this one because 1 char match ho gya.

If character doesn't matches -

Case A → a b c  
x y z

a b c ← Case B  
x y z

$$\text{ans} = 0 + \max(\text{caseA}, \text{caseB})$$

## • Code using recursion

```
class Solution {
public:
    int solveUsingRec(string str1, string str2, int i, int j) {
        // base case
        if (i >= str1.length()) return 0;
        if (j >= str2.length()) return 0;

        int ans = 0;
        // characters matched
        if (str1[i] == str2[j]) {
            ans = 1 + solveUsingRec(str1, str2, i+1, j+1);
        } else {
            // characters not matched
            ans = max(solveUsingRec(str1, str2, i, j+1),
                      solveUsingRec(str1, str2, i+1, j));
        }
        return ans;
    }

    int lcs(string text1, string text2) {
        int i = 0;
        int j = 0;
        return solveUsingRec(text1, text2, i, j);
    }
};
```

## • Code using recursion + memoization

class Solution {

public:

```
int solveUsingMemo(string& str1, string& str2, int i,
                   int j, vector<vector<int>>& dp) {
    // base case
    if (i >= str1.length()) return 0;
    if (j >= str2.length()) return 0;

    // if ans already exists
    if (dp[i][j] != -1) {
        return dp[i][j];
    }

    int ans = 0;
    if (str1[i] == str2[j]) {
        ans = 1 + solveUsingRec(str1, str2, i+1, j+1, dp);
    }
    else {
        ans = 0 + max(solveUsingRec(str1, str2, i, j+1, dp),
                      solveUsingRec(str1, str2, i+1, j, dp));
    }

    // store & return using dp array
    dp[i][j] = ans;
    return dp[i][j];
}
```

```
int lcs(string text1, string text2) {
    int i = 0;
```

```
    int j = 0;
```

```
    vector<vector<int>> dp(text1.length() + 1,
```

```
                           vector<int>(text2.length() + 1, -1))
```

```
    return solveUsingMemo(text1, text2, i, j, dp);
```

```
}
```

## • Code using tabulation

```
class Solution {
```

```
public:
```

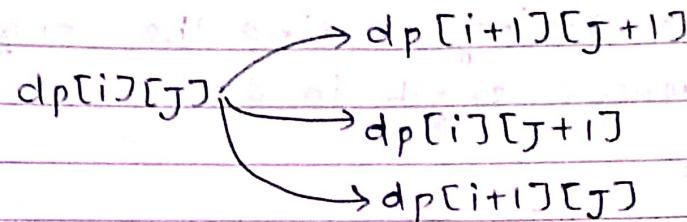
```
int solveUsingTabu(string& str1, string& str2) {
    vector<vector<int>> dp(str1.length() + 1, vector<int>(str2.length() + 1, 0));
    for (int i = str1.length() - 1; i >= 0; i--) {
        for (int j = str2.length() - 1; j >= 0; j--) {
            int ans = 0;
            if (str1[i] == str2[j]) {
                ans = 1 + dp[i + 1][j + 1];
            } else {
                ans = 0 + max(dp[i][j + 1], dp[i + 1][j]);
            }
            dp[i][j] = ans;
        }
    }
    return dp[0][0];
}
```

```
int LCS(string text1, string text2) {
```

```
    return solveUsingTabu(text1, text2);
```

```
}
```

from tabulation approach, we can see that



The final ans  
would form here ↪

curr ↪				
	(i, j)	(i, j+1)		
next ↪	(i+1, j)	(i+1, j+1)		

So, we don't need the whole 2-D array every time. We just need is 2 1-D arrays and we can build the answer either row-wise or column-wise.

- Optimising the code space-wise

```
int solveUsingSO(string& str1, string& str2){  
    vector<int> next(str2.length() + 1, 0);  
    vector<int> curr(str2.length() + 1, 0);  
  
    for(int i = str1.length() - 1; i >= 0; i--) {  
        for(int j = str2.length() - 1; j >= 0; j--) {  
            int ans = 0;  
            if(str1[i] == str2[j]) ans = 1 + next[j + 1];  
            else ans = 0 + max(curr[j + 1], next[j]);  
  
            curr[j] = ans;  
        }  
        next = curr; // shifting  
    }  
    return curr[0];
```

## \* Longest Palindromic Subsequence

Given a string  $s$ , find the longest palindromic subsequence's length in  $s$ .

I/P -  $s = "c b b d"$

O/P - 2

Explanation - Here, LPS is "bb" whose length is 2.

Approach - Given is string  $s$ . Let the reverse of string  $s$  be  $str$ .  
Longest common subsequence of  $s$  and  $str$  is the longest common subsequence of  $s$  and  $s$ .

Find LCS of  $(s)$  and  $(str)$ .

Code -

```
class Solution {
public:
    int findLCS(string& str1, string& str2) {
        // this function will find LCS of two strings
        vector<int> next(str1.length() + 1, 0);
        vector<int> curr(str1.length() + 1, 0);
```

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```
for (int i = str1.length() - 1; i >= 0; i--) {  
    for (int j = str2.length() - 1; j >= 0; j--) {  
        int ans = 0;  
        if (str1[i] == str2[j]) {  
            ans = 1 + next[j + 1];  
        } else {  
            ans = 0 + max(curr[j + 1], next[j]);  
        }  
        curr[j] = ans;  
    }  
    next = curr; // signed assignment  
} // end of main function  
return curr[0];  
}
```

int LPS(string s) {

```
    string str = s; // reverse the string  
    reverse(str.begin(), str.end());  
    return findLCS(s, str);  
}
```

}; // end of main(( ))

\*

## Edit distance

Given two strings word1 and word2, return min no. of operations required to convert word1 to word2.

Permitted operations - replace, insert, delete.

I/P - word1 = "horse", word2 = "course"

O/P - 2

Explanation - horse → corse (replace 'h' with 'c')  
corse → course (insert 'c' b/w 'o' and 'r').

Approach → Base Case

(1)

word1 = h o r s e  
word2 = h i o r s e  
                                    ↑  
                                    J

if ( $i == \text{word1.length()}$ ). Here the only option, we have is to insert the remaining words.  
No. of words to be inserted is  $\rightarrow (\text{word2.length()} - J)$

(2)

word1 = h o r s e  
word2 = h o r s e  
                                    ↑  
                                    J

if ( $j == \text{word2.length()}$ ). Here, the only option, we have is to delete the remaining words.  
No. of words to be deleted is  $\rightarrow (\text{word1.length()} - i)$ .

characters matched -

call recursion for  $(i+1, j+1)$

Characters not matched -

replace  $\rightarrow l + (i+1, j+1)$  } minimum of these  
insert  $\rightarrow l + (i, j+1)$  } 3 operations  
remove  $\rightarrow l + (i+1, j)$

### Code using recursion

```
class Solution {
public:
    int solveUsingRec(string& word1, string& word2, int i,
                      int j) {
        if (i == word1.length()) {
            return word2.length() - j;
        }
        if (j == word2.length()) {
            return word1.length() - i;
        }
        int ans = 0;
        if (word1[i] == word2[j]) {
            ans = solveUsingRec(word1, word2, i+1, j+1) + 0;
        } else {
            int insert = 1 + solveUsingRec(word1, word2, i, j+1);
            int remove = 1 + solveUsingRec(word1, word2, i+1, j);
            int replace = 1 + solveUsingRec(word1, word2, i+1, j+1);
            ans = min(insert, min(remove, replace));
        }
        return ans;
    }
}
```

int minDistance(string word1, string word2){  
 return solveUsingRec(word1, word2, 0, 0);  
}

### Code using recursion + memoization

```
class Solution{  
public:  
    int solveUsingMemo(string& word1, string& word2, int i,  
                      int j, vector<vector<int>>& dp){  
        if (i == word1.length()) {  
            return word2.length() - j;  
        }  
        if (j == word2.length()) {  
            return word1.length() - i;  
        }  
        // if ans already exists,  
        if (dp[i][j] != -1) {  
            return dp[i][j];  
        }  
        int ans = 0;  
        if (word1[i] == word2[j]) {  
            ans = solveUsingMemo(word1, word2, i+1, j+1, dp);  
        }  
        else {  
            int insert = 1 + solveUsingMemo(word1, word2, i, j+1, dp);  
            int remove = 1 + solveUsingMemo(word1, word2, i+1, j, dp);  
            int replace = 1 + solveUsingMemo(word1, word2, i+1, j+1, dp);  
            ans = min(insert, min(remove, replace));  
        }  
        dp[i][j] = ans;  
        return ans;  
    }  
};
```

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```

dp[i][j] = ans;
return dp[i][j];
}
int minDistance(string word1, string word2) {
    vector<vector<int>> dp(word1.length() + 1,
                           vector<int>(word2.length() + 1, -1));
    return solveUsingMemo(word1, word2, i, j, dp);
}

```

### Code using Tabulation

```

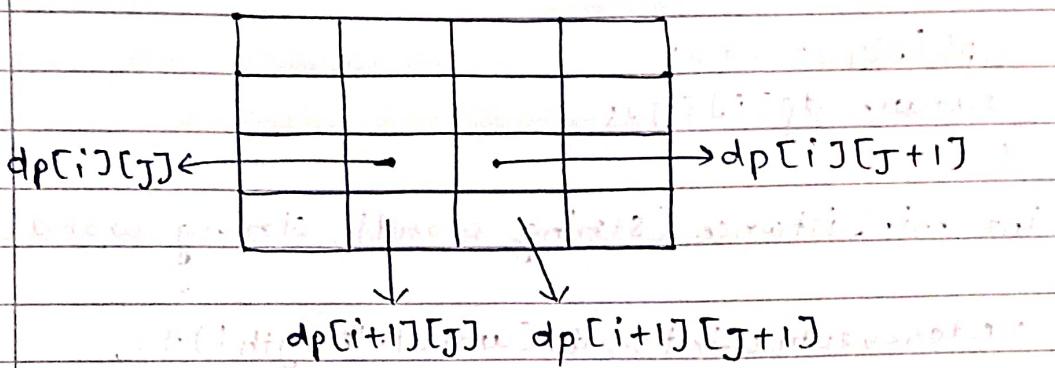
int solveUsingTabu(string &a, string &b) {
    int m = a.length(); int n = b.length();
    vector<vector<int>> dp(m + 1, vector<int>(n + 1, -1));

    for (int j = 0; j <= n; j++) dp[m][j] = n - j;
    for (int i = 0; i <= m; i++) dp[i][n] = m - i;

    for (int i = m - 1; i >= 0; i--) {
        for (int j = n - 1; j >= 0; j--) {
            int ans = 0;
            if (a[i] == b[j]) {
                ans = a[i] + dp[i + 1][j + 1];
            } else {
                int insert = 1 + dp[i][j + 1];
                int remove = 1 + dp[i + 1][j];
                int replace = 1 + dp[i + 1][j + 1];
                ans = min(insert, min(replace, remove));
            }
            dp[i][j] = ans;
        }
    }
    return dp[0][0];
}

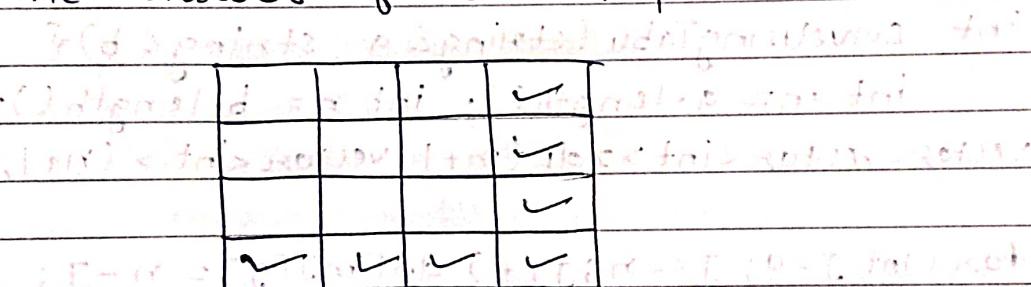
```

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for calculating answer of  $dp[i][j]$ , we need to know the value of 3 cells i.e.  $dp[i][j+1]$ ,  $dp[i+1][j]$  and  $dp[i+1][j+1]$ .

Overall, we need not the whole 2-D array, we need only 2 1-D arrays to calculate the answer of current position.



In tabulation approach, we are filling the last row and last column by analysing base cases to maintain initial states.

In space optimised, we are using 2 1-D arrays only let them be next and curr. So, we will fill the next array beforehand and everytime, when row is changed, will fill the last block of curr array.

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- Space optimisation

```
int solveUsingSO ( string& word1, string& word2 ) {  
    int m = word1.length();  
    int n = word2.length();  
  
    vector<int> next (n+1, -1);  
    vector<int> curr (n+1, -1);  
  
    for (int j=0; j<=n; j++) {  
        next[j] = n-j;  
    }  
  
    for (int i=m-1; i>=0; i--) {  
        // har nyi row ke last block ko fill krna he  
        curr[n] = m-i; // most important line  
        for (int j=n-1; j>=0; j--) {  
            int ans = 0;  
            if (word1[i] == word2[j]) {  
                ans = 0 + next[j+1];  
            }  
            else {  
                int insert = 1 + curr[j+1];  
                int remove = 1 + next[j];  
                int replace = 1 + next[j+1];  
                ans = min(insert, min(remove, replace));  
            }  
            curr[j] = ans;  
        }  
        next = curr; // shifting  
    }  
    return next[0];  
}
```