Percy PreCal's Missing Treasure

Attached is a linear project (in 3 separate parts) that is suitable for students in a first-semester calculus course. The project is included in its original form, including my specific instructions to the students. This project was first given to students during the Fall 2018 semester at the University of San Diego. This project is part of a broader collection of project-based assessment being developed at the University of San Diego.

The mathematical concepts in this project involve optimization, and tangent lines to curves defined implicitly. Components of this project were later developed further in "The Mysterious Orb Incident," which is distinct in that a solution to later portions of the project requires a successful solution to the earlier portions.

Adam Boocher
University of San Diego

Math 150 Exam 3 - Project

We have reached a point in this course where we are exploring the applications of derivatives. Many real life problems involve many steps and often involve teams of people working on different pieces. In all of your future careers, problem solving, writing and communication will be very important.

In my view, what's important with applications is being able to correctly set-up, solve and communicate our solutions to the problems. A timed in-class exam isn't the best way to assess this. So I've designed this "treasure hunt" project. There are three problems below, and each requires solving problems similar to those that we have solved in class and on the homework. What's different with these is that you will write your responses as if you were writing to Percy PreCal, an explorer who doesn't know calculus but knows some precalculus. For each of the three problems, I would like you to write up your answers as if you were writing to our treasure seeker. Simply writing $x = 5/3$ won't be much help to Percy, who might want instructions from the nearest tree, or coconut. You may begin each problem by saying "Dear Percy," and feel free to tell a story with your answers - draw pictures, use full sentences, if you are breaking the problem into parts, say what those parts are.

- Each problem is worth 35 points, with the following breakdown:

    1. (10 Writing and Communication) Your solution should include lots of sentences and words. Grammar and style are important! I won't be super picky, but write as if you were writing a response to a colleague, explaining how you have set up each problem and why you are setting it up that way. You may want to type your answers and then leave space for the mathematics if you want to hand-write it in. (See the 4 videos on this site for ideas about how to write mathematics `https://instruct.math.lsa.umich.edu/support/teamhomework/characteristic1.html`)
    2. (10 points) Mathematical Setup - Is the problem properly set up? If there are letters being used, is it clear what they mean? Are there labels and carefully drawn images? If we set a derivative equal to zero, why are we doing that? How does that help?
    3. (10 points) Correct Solution - Given the setup can you correctly solve the problem and answer Percy's questions. Make sure the final answer is given as a set of instructions for Percy to follow and one that answers all the questions asked.
    4. (5 points) Presentation - is your work neat and clear? If you have graphs are they clear and well-drawn? (Or better yet, use Desmos and take screenshots to include.)

- Regarding length, none of these need to be excessively long (roughly a page or two for each problem seems appropriate) but the more insight you can give into the problem, the better.

- Can I use a graphing calculator? Yes - you totally can use a calculator - and you may want to use Desmos to draw pictures, find where lines intersect etc. Whatever you do, just make sure you explain what you did, and how you did it.

- **This project is due on Thursday November 29.**

- You are encouraged to work with others on this project and are definitely encouraged to come to my office hours. However, I request that everyone write up their answers themselves.

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- My hope is that this can be an opportunity to have some fun and gain a sense of ownership of a problem, talk through ideas with your friends, draw pictures, and explore calculus without the pressure of a timed exam. As you'll see, Percy writes in a funny, fanciful way - you might enjoy writing back to Percy in a similar way. Or perhaps you might want to write back the way that a technical engineer might, or like a teacher - what do you want to bring to the world and how can you incorporate that into your writing?

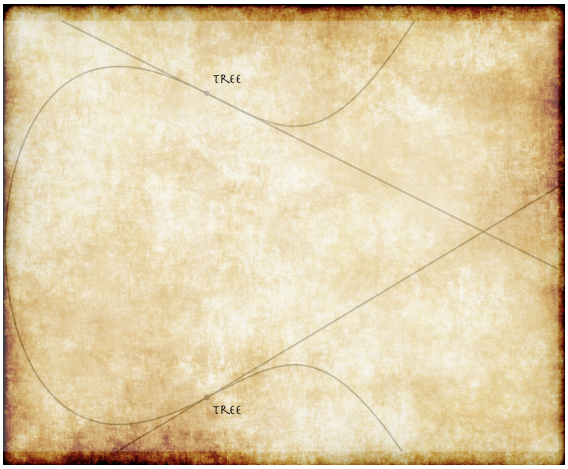
- You've totally got this - and I want to help as much as I can. If you want to submit drafts to me for feedback, come by my office.

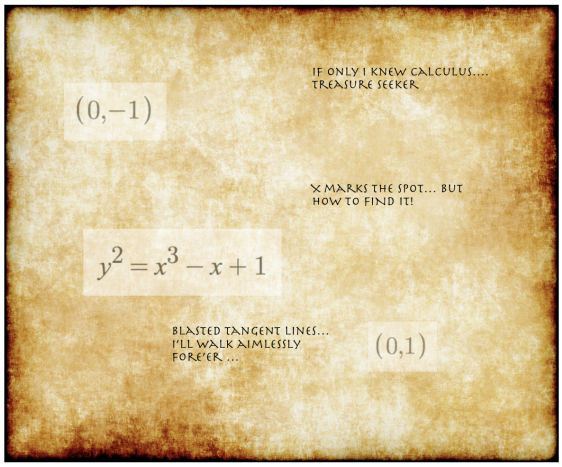
- Get started *early* on this project. Already, you should be able to solve the first two problems, and we'll have seen enough math to solve all the problems by Nov. 9th.

Part 1: Math on The Beach

Ahoy! Dear Calculus Students, if you can read this letter then my missive has reached you... I only hope that it reaches you in time! I have been stranded on an island for what seems like years.... The sun bears down on me as I search for the treasure that seems so close... Allow me to introduce myself. My name is Percy PreCal, and I've never taken calculus, but I know some precalculus. It turns out to find this treasure, I need more than just Precal - can you help me?

It started off as a treasure hunting trip. While exploring the local archives at the USD library I found a hidden room (revealed by pulling the right book out of a bookcase). Inside the room were dozens of manuscripts, mathematical tomes, and magical potions! (Future adventures perhaps?) But what captured my attention was the treasure map I've included in this bottle. That X on the map - that must be it.. It must be where the treasure lies. X marks the spot... oh if only it were that easy!





On 1st November I set off on my voyage from San Diego to the mysterious island. Oh mercifully, the waves carried me safely and 30 days later I arrived on a beautiful island, exactly as the map had predicted. As I scouted out the island, I found the two "TREES" mentioned on the map - of this I am certain. I know this because there is a very faint trail in the shape of the curve on the map. Some explorer has written the equation of this curve on the back of the map. I have no idea how they did this, but I am grateful for their work. And they're written some points - maybe the trees? Alas, I have no graphing calculator to get insight, but I think that must be so...

As soon as I found the trees, I thought that I would be able to easily find the treasure. My plan was simple: Go along the path, and then cut off tangent at the tree and make a mark in the sand. Then do the same again at the other tree and see where the paths cross. But curses, the sand on this island is unlike any I've ever seen. As soon as I lift my feet, the sand moves and leaves no trace that I've been there. I cannot make any marks in the ground to help me. All I can hope for now is to calculate how many paces I need to go along the tangent...

Oh how many times have I walked along that trail, and how many times have I come across those trees. And how many times have I wondered off tangent to the curve. If only I knew how far to walk along my tangent path... That must be the key right? The map says to go off tangent to the PATH but for how many paces?

Curses to the gods ... I thought it would be so easy - the map even included the information:

$$y^2 = x^3 - x + 1, \quad (0,1), \quad (0,-1)$$

But what good does that do me? I don't have access to Desmos here, and I've never taken calculus before. Can you help me find the treasure, students?

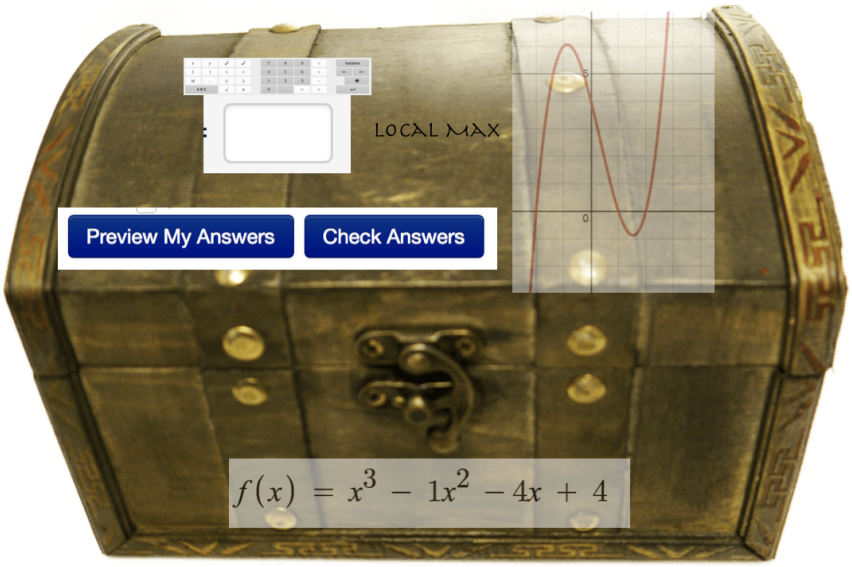
Here is what I can offer:

- The trees are 300 paces apart
- I think your Professor will offer hints and help during office hours!
- In your write up, please show your work and write your instructions for me in complete sentences. And remember that your answers have to be in terms of walking directions - if you just say "Go to point $(53, 124)$" that won't be helpful to me - I have no grid!

Part 2: The Locked Chest

Dear Students, thank you so much for your help finding the treasure! But now I fear that I must ask you again for your help. I'm beginning to think that this is all some elaborate mathematical joke....

Let me recap: Your instructions were perfect - you told me where to dig and I found this chest buried beneath the surface. This LOCKED chest...



Yeah, I know what you're thinking. This is ridiculous. It looks like the webwork I did in my Precalculus class. What sort of person buries a webwork-locked chest??

It brought back some good memories - I was really good at precalculus. I didn't always know the right answer at the start, but after talking with my friends and my teachers, I gained confidence that I could do mathematics! Oh those were the days... I knew how to find the vertex of any parabola. Yep, $-b/2a$ I was a pro!

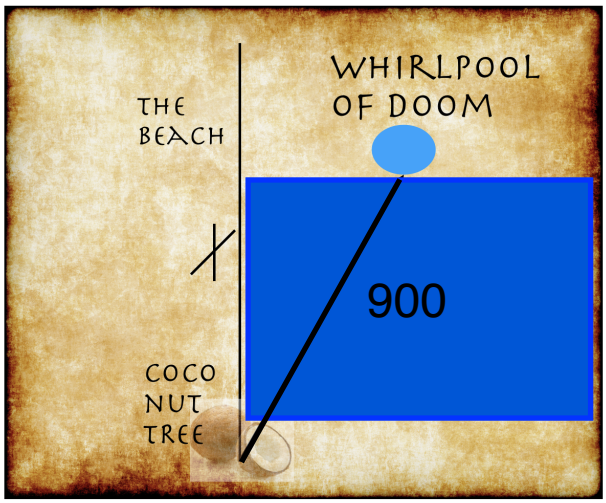
But I never learned how to find those max and min points of a cubic equation - which is what I think the chest is asking for... What sort of magic do you need to solve this? That point where the function goes from increasing to decreasing... I mean I can tell you that the "answer" should be something like -0.869, I've spent days looking closely at the graph and crunching numbers. But I have no idea what the EXACT answer should be. And to make matters worse, every few hours the problem changes and I have new numbers to deal with. When you write me back, can you tell me what the answer is for this problem? But also, can you show me your work and explain how I might be able to use these for whatever cubic equation I have by the time I get your response.

- The chest says "Local Max" what is a local max?
- And how do I find it? Please tell me what the exact x-coordinate of the local maximum is for this function $f(x) = x^3 - 1x^2 - 4x + 4$.
- But also describe a method for how to find the $x-$coordinate of the local maximum for a different cubic equation. For instance, something like $8x^3 + 8x^2 + 8x$? Or $x^3 - x$? What steps could I use? Remember I probably don't know any of the terms from your calculus class, so if you want to use them, you'll have to explain them first. But I know what slope is!
- (Instructor's Note) - this problem is intentionally a little bit vague: Think about how you might best answer Percy's question. Maybe you want to explain it by doing a few examples and through examples show how to find these points. You've already done

problems like this on the webwork, so maybe you can use those in your explanation. You can include graphs from Desmos as well as the steps to find the local maximum. Maybe you want to do things generally with $ax^3 + bx^2 + cx + d$ and find a "formula"? These are up to you!

Part 3: Elvis???

Ok, you're never going to believe what was in the chest.... There were some old photos and another map.



On the back of the map was written:

"3 feet per second to swim the sea;
10 feet per second to run the beach;
From coconut to whirlpool as fast as can be;
Ere you dive into water, the treasure you'll reach!"

Ok, let me rant for a second here.

- I came for treasure, not webwork problems...
- What's with Elvis? And these dogs? (See Blackboard Readings: Do Dogs Know Calculus?)
- And that guy, is he a mathematician - I wonder if he does research with students and his dogs...
- More pressingly there's a Whirlpool of Doom???? That would have been helpful to know before sailing out here. Seriously...

So you're really going to have to help me here. Here's what I'm thinking - I seems like if my goal were to get from the coconut tree to the whirlpool, I'd want to do some combination of running along the beach and then diving into the water. I need to know when to cut into the water. It looks like the whirlpool is 900 feet from the coconut, and I measure that the angle it makes with the beach is 30 degrees. But that's about as much as I know. Can you tell me how many feet up the beach I should go from the coconut tree before digging for whatever "treasure" might be there?