

The Mysterious Orb Incident

Attached is a non-linear project (in 3 parts) that is suitable for students in a first-semester calculus course. The project is included in its original form, including my specific instructions to the students. This project was first given to students during the Spring 2019 semester at the University of San Diego. A modified version of the project was given in Fall 2020. This project is part of a broader collection of project-based assessment being developed at the University of San Diego.

The mathematical concepts in this project all involve optimization of some sort, though each is slightly different. A correct solution requires students to analyze what is being asked (e.g. is the length of day being optimized or is it the rate of change of the length? Is it the distance from the beach to the boat that is being minimized, or is it the time it takes to run / swim?) Additionally the project requires students to read (a rather fanciful) story and collect clues in order to answer the questions.

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Hi Students - it's me Dr. Boocher! As you know we won't be having a third "in-class" midterm. Instead, we will work on a project that uses Calculus to help solve "applied" problems. As you'll see, I've concocted a zany mystery puzzle for you to solve by using the methods of Calculus. Everything takes place on the island of Calculia where all the laws of calculus and physics apply, except the constants are maybe a little different (different number of days, acceleration due to gravity etc). All of the problems you'll have to solve are like problems we've seen in class (we'll work on a few in the coming days), but what's new in this project is that **you have to figure out what the problem is asking and you have to write up your solutions in a way that offers an explanation using concepts from class as well as any relevant figures, graphs, formulas, etc.**

Here's how everything will go down: Because this is a mystery problem, at each stage the number of suspects will be reduced (at the end of this document there is a list of 200 scholars who have visited Calculia. The culprit is one of these!) Your first task will be to *greatly* reduce that down. Then you'll narrow it down further. And then finally you will solve the mystery in the final stage by figuring out what happens on the beach as one of our heroes watches in terror from a boat! By the end of the project, you should be able to determine who the mystery suspect is. By that point you will also have written many pages of well-written mathematics that explains the concepts we've been learning in the course.

Since I don't want you to get too far off track, I'm going to ask you to turn in this project in stages. This is for several reasons:

- It gives you a chance to get feedback before you turn in the remaining parts.
- If you made a mistake and narrowed it down to the wrong suspects, then you'll be able to correct and adjust for the subsequent parts of the project.

You are welcome to turn in any part of the project early.

What follows might seem like a lot - (there are admittedly a lot of pages) but most of it is letters between our two main characters. What I recommend doing is reading everything (estimated time: 10-15 minutes) and trying to plot out how you're going to tackle each of the tasks below. The rubric for this project is:

Each Task will be worth 25 points:

- 10 points will be for the mathematical correctness of your solution. Did you correctly solve what the task asked for and clearly indicate your steps in the solution?
- 5 points will be for mathematical correctness of explanation. Did you include labels, did you specify what your functions were, your variables, and did you correctly use mathematical terminology.
- 5 points will be for your exposition. Did you explain your terms in a way that a fellow Calculus student could understand your reasoning and how you came to your conclusions?
- 5 points is for presentation and creativity. Are your equations clearly separated from the text. Is your work clear and easy to read. Do you refer to the appropriate parts of the document when making your case? Have you personalized this project - feel free to create a conversational tone. Maybe you want this to read like a note to your friend Detective P. Or maybe you want to practice writing formal business or scientific writing. This is all up to you. What's important is that you write clearly.

Task 1: (Due Tuesday April 16th at the beginning of class)

When is Festival Day?

You'll notice that on Calculia's list of visitors the dates are just written using the numbers 1-100. That's right, there are 100 days in Calculia's calendar. **Your first task is to figure out which of those days is Festival Day.** The clues you need are in the letters. Your solutions should include the following:

- It should be addressed to Detective P. and provide an introduction and explain your methods. For instance, you might want to say something like "I know that festival day is X because I looked at these graphs and charts and did these calculations."
- Your solution should include a calculation of a derivative. You may want to include relevant graphs to support your answer. Screenshots from Desmos are strongly encouraged. Be particularly careful about your explanations of *why* your answer is correct.
- If you are making any assumptions make them clear - this is a "real life" problem, so you may need to make some assumptions.
- At the end of your letter to Detective P. you should know when Festival Day is, and I'd like you to thus determine possible suspects based off of who was on the island at that time. You can trust the data in the table at the end of this document. You should have it narrowed down to at most 5 people.

Task 2: Due Tuesday April 30th

Whose Alibi Doesn't Hold Up - Who Dropped the Orb?

- By this point you should narrowed down the list of suspects (and I will send out the correct list in case there were any errors. According to the story, all of these people have an alibi at 11:59, and the orb hit the ground at 12:00. What gives? Someone's alibi doesn't hold up.
- To further reduce this list you must determine "Who Could Have Thrown The Orb and still had time to get to the vestibule by 11:59am" based on the information in the puzzle. Assume there is no air resistance, and that physics in Calculia behaves as it does here, just with perhaps different constants.
- Your answer should exonerate some people as possible suspects, but there might still be ambiguity as to who the true culprit is. Give an updated list of possible suspects.
- Please address everything to Detective P. as before and explain your mathematics! I want you to include details about how you know the formula for the height of the ball, from the acceleration, initial velocity, etc.

Task 3: Due Thursday May 9th

Who Was Running on the Beach?

- We are now ready to determine the final answer to this problem - who stole the orb, shattered it and is now on their way to escape! Something happens on the beach. You can assume that the criminal is trying to get to the boat as fast as possible. This should be reminiscent of a problem we've solved in class. (In fact, most of this project should feel that way).

- Use the information in this story to figure out who it was that was heading for the boat. Make sure your explain is clear with proper figures and you explain who your final choice is for the suspect.
- I'm looking for a good setup and explanation for this problem.

You are absolutely welcome to turn things in early! In fact there will be a prize for whoever is the first to catch the thief! You may work with your classmates but please do not ask tutors or other professors for help with this project. This should be treated like an exam, but one where you are encouraged to collaborate, brainstorm and of course ask me lots of questions. My goal with this project is to give you an alternative to timed in-class exams and to give you a chance to express yourself in writing. I want everyone to succeed on this project, so that's why I've scaffolded everything in this way.

Good Luck and I hope this is fun!

Hint: You might want to learn how to use sliders / functions on desmos. It will really help for this project - just give it a try, type $f(x) = ax^2$ and it should prompt you to make a slider for the letter a. Then slide away!

Attention: Calculus Students
The Orb Incident

April 2019
The Desk of Detective P.
California, USA

Dear Students,

Allow me to introduce myself. My name is Detective P. (private eye. The eye stands for investigator, but it's also a pun because you see.. oh you don't care about puns bah, I'm writing this on a typewriter and don't have time to go back and make corrections. . . this is urgent!)

I've been working on this case for about 3 months now and I'm no closer to a solution than ere I began. I had given up hope, but then I noticed that since this all takes place at a mathematical institute maybe I should consult some mathematical experts. Your professor recommended you for the job! I'm sending you everything that I have, but as you'll see it's quite the terrible tale. One with many characters, magical orbs, a spectacular tower, love, pirates, and this blasted Festival day, about which I've been able to find no information! Let me catch you up to speed, because this is no usual case....

A few months ago I was walking along the beach in Pacific Beach and a bottle washed ashore. Inside was something unlike anything I'd ever seen. Fantastical drawings of a tower - love letters - a police report - and discussion of gravity, sunlight, and treachery at the Festival! I sat down at Coffee Cycle (truly the best cup of coffee in San Diego) and got to work reading these notes. I should say that I have no idea who sent me this bottle, and indeed it seems almost like it comes from another planet where the sun and earth behave under different rules. As I read the documents I became so caught up in the story that I had to figure out what happened. Who stole the orb and who could have dropped it from the tower? Clearly it must be the same person who ran from the police and escaped by swimming to the boat. But who was it? And when did it even happen? Yes, on Festival day, I get it, but what day was that? We have lists of every guest who set foot on the Island, but only in terms of their silly calendar. When is Festival day? Is it the 10th day of the year or the 100th? If only I knew that I could narrow down the search and have some culprits, but at the moment I don't know where to start.

And what happened to poor Wesley and Buttercup? Alas we may never be able to figure any of this out. What I'm certain of is that the same person who stole the orb is the one who dropped it from the tower. And this is the same masked person who ran away from the police that festival evening and swam out to poor Wesley's ship.

Students, I'm stuck and I confess I think I may need to know some calculus to solve these problems.

When is Festival Day? Who could have been able to drop the Orb from the tower? and who could have made it to Wesley's ship? I ask that if you can help me, please send your answers to your professor. He will hopefully be able to help and give you some deadlines for working on different pieces. I hear he might also give you some hints and help you along the way. He'll even shift his office hours around to accommodate your schedule. Yes *your* schedule. He wants you to succeed and wants to help!

Some things you should know:

::: The record-keeping on Calculia seems impeccable. I have every
::: reason to believe that the records are accurate. You'll see
::: that they kept track of so many different things.

::: I trust the records of the local officers. Ack it sounds like
::: they nearly caught the suspect. It is to bad that their letters
::: were damaged.

Sincerely,

Detective P.

Enclosures:

- 1) Local Report (page 2 missing)
- 2) Letters between B. and W.
- 3) Brochure for Calculia including ~~artist's~~ rendering of island.
- 4) Guest Ledger and Data

... An equation so simple yet it captures
light and dark, day and night...

Dear Wesley,

The days are getting shorter now ... that time of the year now approaches. As the sunset gets earlier and earlier, as I look from the Room of Learning it reminds me of our sunset walks down at Eagle's Point as we'd watch the ships sail off. And to think - you're now on one of those ships! Finished with the naval training and now off to see the world. How excited am I for you! I was reminiscing the other day in my calculus class about how we met - at the Room of The Math Learning Center. You were learning about the physics of motion and me about concavity. I think it was just around our second midterm. I still remember our breakthrough when we realized the subtle difference between "the slope is positive" and "the slope is increasing". The tutor pointed across the room to your downward facing parabola and said "see - on the left the slope is positive, but the slope is decreasing as it flattens out." And that's when I first saw you. And after that our friendship began.

Buttercup

Dear Buttercup,

How could I forget that day? The equations of motion - that acceleration due to gravity is given by

$$a = -0.72 \text{ ft/sec}^2$$

I've often wondered if this number would be different on another planet or perhaps whether it's due to the porous rock underfoot. I like that word, underfoot. And this simple acceleration yields parabolic motion for objects. You're the one who showed me that you could derive the equations using calculus! In the Room of Physics we just used the parabolic equation. I remember you practicing your mathematical writing (for your class project) and asking if you could practice by teaching me. "As you wish", I said. Living on a ship is different - I feel a bit isolated from the world. Only via these carrier pigeons can we send and receive letters. Whenever I see a letter in the post bag I hope it's something from Calculia.

Buttercup - I hope my officer will have gotten bored of all that mathematics above (though how could math be boring!) I fear that there's trouble afoot (heh I like that word too). Top secret business they say. Nothing to worry about they say. But all I know is that they've been censoring our messages and they've been increasing our nightly vigils. Every week there are new scholars going to the institute and security is increasing. I've heard word that not everyone can be trusted. Please be careful!

Wesley

Dear Wesley,

Things are wonderful at the Tower - scholars are joining every day. How I long to be a scholar some day! To be able to go into the Room of Scholars at the top of The Tower. Wesley, they say that if one stretches out their arm from the Room of Scholars it is exactly 314 feet above the earth. Clearly built by a mathematician, the circular tower is "pi in all dimensions" they say. But security is impossible - only a scholar can enter that top room.

Professor Plum taught us today about how our planetary motions are such that the length of "daylight" each day is governed by the formula

$$L(t) = 3\sin(\pi/50*(t+39.4))+5$$

I think there's something beautiful about this fact. ... An equation so simple yet it captures light and dark; day and night.

Speaking of the sun, the days are getting longer again - I usually love this time of the year. A time of renewal. Though something feels off - I cannot place it. I'm sure it's nothing. I'm looking forward to your return. Will you be back before Festival Day?

Buttercup

PS As I drop this off at the post, I see that you've just sent me a letter.

Wesley,

I haven't received a letter in awhile. I hope everything is all right on the open sea. It makes me think of a joke I learned in my math seminar. It was about the real numbers, denoted \mathbb{R} and the complex numbers denoted \mathbb{C} . The joke is "What's a pirate's favorite number system?" We all said "ARRRR!" but then Professor Plum said "Nay, that be the \mathbb{C} ". What a character, that Professor Plum!

Did I tell you that one of Professor Plum's colleagues was in town last week and delivered a mysterious package? I was in her office hours when the visitor arrived. Professor Plum peeked in the box but then quickly put it away. I wonder what it could be?

Also, did you ever know why we celebrate Festival Day on the day that we do? It's because it's the day that the length of the day is increasing at the fastest rate in the whole year! They say that Festival Day will be a full 11 minutes longer than the day before it. Professor Plum will be making a presentation at the Festival - she says that she will be revealing something new. I hope it's a new theorem. Did you know that Mathematicians are very active today proving new theorems? I might do some undergraduate research this summer with Professor Plum.

Buttercup

Buttercup,

I found the remains of my last three letters in the Room of the Garbage aboard the ship. Shredded!! I don't know what is going on, but someone is trying to cut off communication and I don't think that it's Calculia. I'm sending this on the pigeon myself (I won't say how I've arranged this, but it wasn't easy).

I've detected two spies aboard our ship and I think they've been taking my letters. Yours are still getting through just fine, but alas I know that mine make it naut. (at least I can still make puns, Buttercup!) But seriously, these spies, they use a very peculiar word - they call each other "Bro". I've researched this and in the land of California there is a word "Brother" and this is a common word for them to say. However, these fools don't realize that this isn't in our language. I'm pretty sure that bro is coded language for "Orb", but I have no idea what this could mean. Buttercup, this might be my last letter to you. I should return on the evening of the festival. It's just a few weeks away but I'm afraid of what will come...

Wesley

Wesley,

Professor Plum has showed me what was in the secret box. She calls it "The Orb" I've never seen anything like it! It's a ball of glass with the ability to do incredible things. It can graph any function you ask! You really won't believe it. Exponentials, Sine and Cosine ... it even has the ability to add parameters and adjust them using sliders. How awesome this would have been during my Calculus Project. I could used it to measure slope! It will even calculate derivatives, give values and has this feature called "Orbshot" which captures an image of the Orb and prints it on parchment. Professor Plum has been developing this for years and will unveil it at the Festival. This could mean great things for Calculia - more scholars might come to visit and we might be able to test conjectures and learn! Oh, I'm so happy to be here.

The days are getting longer and longer by the day. Can you believe that Festival Day is just three days away? I've been volunteering at our local Moonball event. Of course you know that our scholars are very active (there's a pun for you, Wesley) and while they're not doing mathematics, they go for runs, swims, and even practice throwing balls in the air. (To the moon they say!) I've been in charge of recording all of this data. It's a lot of work writing on paper. Hopefully some day there will be an Orb for that. I hope you arrive before Festival Day! No letters in awhile from you Wesley, but I'm not going to think of the worst.. I can't...

Buttercup

Wesley,

I got your letter only yesterday and a chill went down my spine. How did the people on your boat know about the Orb? Gosh, I'm not even sure I should be writing this. Wesley - the Orb is real. I must try to warn Professor Plum, but I haven't been able to find her. I think she's answering questions up in the Room of Scholars.

Buttercup

Wesley,

It's Festival Day and it's horrible - just horrible! The Orb has been stolen!!! Professor Plum was in the Room of Auditoriums this morning and had an audience of 500 people waiting for the great unveiling and when she opened the box it was empty. EMPTY! I know it's got to be one of the new scholars that recently arrived. If only we could figure it out.

You'll be back tonight though... I can't wait to see you. Wesley, it's been hard these past few months, but being able to write to you has been invaluable. Thank you for being a friend - but don't do anything to put yourself in danger.

Buttercup

Wesley,

SMASHED! The orb has been smashed - I think dropped from the Room of Scholars. I was with Professor Plum in her office for an hour and then all of a sudden we looked out the window and saw the Orb fall past her window. We ran up to the Room of Lunch and all of the scholars were there. Who could have dropped it. It *HAD* to have been one of them, but which one? I know there's got to be a way to catch them. If only real life was as simple as those equations back in the Room of The Math Learning Center. But sadly, life isn't the same as..... Golly! I have an idea - I need to round up some other math students, but this just might work.

Calculus the Rescue. I just hope we're in time.

Buttercup

Buttercup,

My messages have likely been intercepted, but I'm sending this message along with all the letters I have from you in a glass bottle. I hope that it reaches the shore at some point. The "bros" on our ship have taken over command and have stopped the boat just short of Calculia. So close, but so far! We're but 150 feet from shore to the closest point, and then from there it's just 1000 feet to where the path meets the beach. I'm so close, but I dare not jump - the spies are dangerous and I fear them. I can only hope that you receive these messages and know that California is the culprit. Some place called "Hollywood".

What! Someone is running on the beach. They've just left the path and are running along the beach. And Calculia's defenders are chasing them along the sand! And now they're swimming in the water. They've just cut in at an angle. I think they're heading towards our boat! Clearly they're trying to minimize the time it takes to get from the path our ship. Eek I'd better send this letter before they arrive.

Wesley

From the Land of Calculia Report of the Great Orb Incident

Surely you've heard of the great orb incident, indeed, at this point there are books filling the great Library that describe it in detail. And the great story tellers tell of it near and far. And of the disappearance of the vessel of Wesley. For never was there a tale of more woe... dear Wesley where did he go?

I'll assume you know the basics, like that the date of its happening was



And you know that by using Calculus we had narrowed it down to who the suspect was shortly after the orb was dropped. Thanks to the meticulous record keeping we knew who was on the island and their relative arm strengths. I mean at this point we know who the suspect was. Their name is included on the next page of this report.

Here is what we know: The orb was dropped from the Room of Scholars and hit the ground at 12:00 noon exactly. Our record keeper, L. Simpson was walking to lunch on the day and was nearly hit by the crashing object. She noted that the orb struck at precisely 12:00:00 to the second. Clearly this was the work of a skilled athlete and cunning scholar of mathematics... For only the scholars could have had access to the Room of Scholars. No one else could have gained entry. Of that we were sure.

Here's what we know: Of the scholars who were at the Institute that week, all of them were present in the vestibule at 11:59 am. Our record keepers don't miss anything. So it seems like everyone had an alibi at the time of the orb-drop. Indeed it takes a whole 30 seconds to walk from the windows out to the vestibule. And how long could it possibly take for an Orb to hit the ground? A clear alibi for everyone... or was it? Not so fast!! Our resident students, led by Buttercup found out that this alibi might not be what it seems.... (see our analysis on page 2) you see, what we did was notice that we had some information about the arm strength of these guests, and you see that they could have thrown the orb at just the right speed so that when

and that's how we figured out all the mathematics and figured out who it was! I guess when we were chasing the suspect on the beach we hadn't completely narrowed it down, but once we measured the distance they ran on the beach by looking at their footprints (930 ft) we were certain! I suppose it makes sense in hindsight, who else could it be?

The rest is ... as they say ... history. Wesley's boat was soon taken captive and disappeared. Luckily Wesley had been able to drop all his letters from Buttercup into a bottle and we received them for our Archives. So that's how we solved it. We don't know what happened to Wesley, perhaps his story continued - perhaps Buttercup found him on her voyage. She set sail years ago and hasn't returned. Their letters tell a good tale. They are included in our Archives below as well as a manifest of all the scholars who visited that fateful year. As far as we knew these records are accurate.

A Brochure for
Calculia: A Mathematics Scholar Retreat Center

Do you long to study mathematics with Scholars from all over the world? Do you want to conduct research in a room called the Room of Scholars? Do you want to work with someone called Professor Plum and meet some of the world's sharpest mathematical minds?

A world-class resort and mathematical retreat center, Calculia was founded by Professor Plum in order to advance the mathematical sciences and serve as a center for research and education. Situated in a tower that is “Pi in all dimensions” - (circular and standing at a height of 314 feet from the Room of Scholars) our resort is on a small island complete with resort activities - running, swimming, throwing a ball into the air, Scrabble and more! We have visitors every day of the year - yes that's 100 days!

Direct all enquiries to ProfessorPlum@ThisIsProfessorPlumsEmail.gov.island.



