Quiz 1 - Good Luck Everyone! **Name:** ______________________________

1. Find the derivatives: $\boxed{\text{You do NOT need to simplify your answers}}$

   (a) $\dfrac{d}{dx}(\sin(x\ln x)) =$

   (b) $\dfrac{d}{dx}(\dfrac{x+\pi}{x^2+e^x}) =$

   (c) $\dfrac{d}{dx}(\sin^3(x)\cos(5x)) =$

2. Find the following anti-derivatives. Don't forget the $+C$

   (a) $\int (3x+2x^2)(1-\dfrac{1}{x^4})\ dx =$

   (b) $\int \dfrac{1}{\sqrt{2x+3}}\ dx =$

   (c) $\int \dfrac{1}{x\ln x} dx =$

Please turn over the page for problem 3:

3. A student graphs a function $y = f(x)$ and draws the tangent line at the point $(2, 1)$ and gets the line

$$y - 1 = -3(x - 2).$$

(a) This means that $f'(2) =$ ______________.

(b) Set up (but do not evaluate) a definite integral that calculates the area in the first quadrant (that is, $x, y \geq 0$) bounded between the x-axis and the line given above. A picture will probably help!

Makeup Quiz 1 - Good Luck Everyone! **Name:** ______________________

1. Find the derivatives: $\boxed{\text{You do NOT need to simplify your answers}}$

   (a) $\dfrac{d}{dx}(\sqrt{2x+5}) =$

   (b) $\dfrac{d}{dx}(\sin(\sqrt{x^2+7})) =$

   (c) $\dfrac{d}{dx}(\tan^5(x)\cos(x)) =$

   (d) $\dfrac{d}{dx}(x^{-5/2}) =$

2. Find the following anti-derivatives. Don't forget the $+C$

   (a) $\displaystyle\int x^2\left(\frac{3}{x^4}+\frac{2}{x^3}\right)\,dx =$

   (b) $\displaystyle\int \frac{1}{\sqrt{2x+3}}\,dx =$

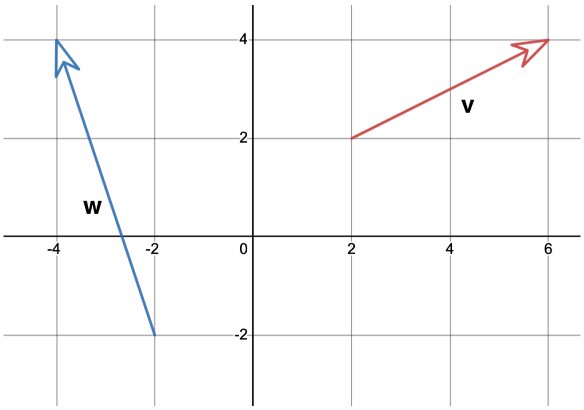
   (c) $\displaystyle\int xe^{4x^2}\,dx =$

Quiz 2 - Good Luck Everyone! **Name:** ________________________________

1. (1 point) In class, we learned that one of the simplest things to parametrize is the graph of a function. Fill in the blanks to write down a parametrization of the curve given by $f(x) = e^x + x^2$

$$\text{Answer: } c(t) = (\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_) \text{ with } -\infty < t < \infty.$$

2. (4 points) Consider the two vectors $\mathbf{v}, \mathbf{w}$ in the picture below.



(a) Find the position vectors (with the $\langle \ , \ \rangle$ brackets) for these vectors and write them down.

$\mathbf{v} = \langle \qquad , \qquad \rangle$

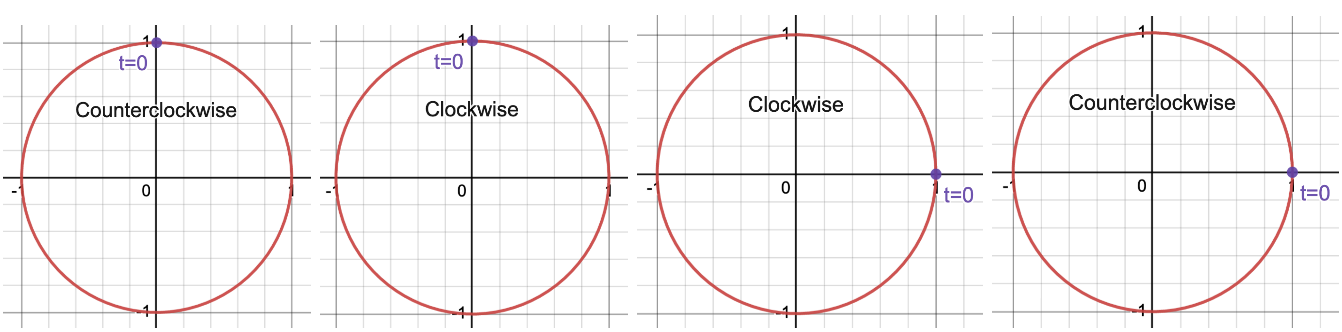
$\mathbf{w} = \langle \qquad , \qquad \rangle$

(b) Find $\mathbf{v} \cdot \mathbf{w}$.

3. (2 points) Let $\mathbf{v} = \langle 3, 4 \rangle$. If $\mathbf{w}$ is a vector of length 3 such that the angle between $\mathbf{w}$ and $\mathbf{v}$ is 60 degrees, what is $\mathbf{w} \cdot \mathbf{v}$?

4. (1 point) A student wants to use the formula $P + t\mathbf{v}$ to parametrize the line through the point $(1, 3, 2)$ that is parallel to the line through the points $(3, 1, 4)$ and $(5, 7, 8)$. What would $\mathbf{v}$ be in this problem? (There are multiple correct answers)

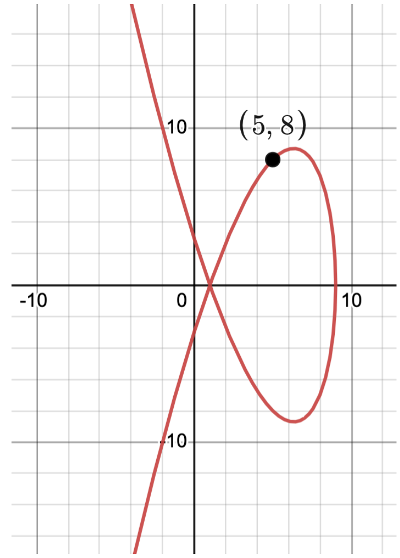
5. (1 point) The parametric curve $(\sin t, \cos t)$ corresponds to which of the curves below? (The word in the center describes the way the curve is traversed as $0 \leq t \leq 2\pi$.



6. Consider the parametric curve described by

$$c(t) = \left(9 - t^2, 8t - t^3\right).$$

A graph is given at the right



(a) (4 points) Find the equation of the tangent line at the point $(5, 8)$. **Show your work clearly.**

(b) (1 point) On the graph, if we were to animate this graph, say starting with $t = -1000$ and going to $t = 1000$ in which direction would this curve be traced? (Please draw an arrow to indicate your answer).

7. (Extra Credit) Determine (with explanation) whether or not the line parametrized by $(1 - t, 2 - t, 8 - 2t)$ intersects the x-axis.

Quiz 3 - Good Luck Everyone! **Name:** ______________________________

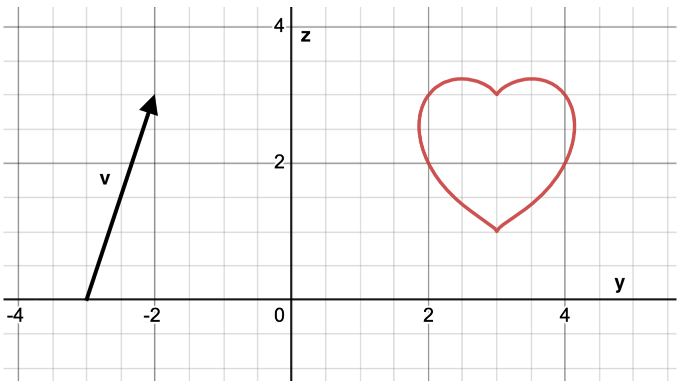
Extra Credit:

This quiz has 16 possible points but will be graded out of a total of 15.

1. (1 points)

   The normal vector to the plane $3x - 2y + 5z = 1$ is $\langle \qquad\qquad\qquad \rangle$.

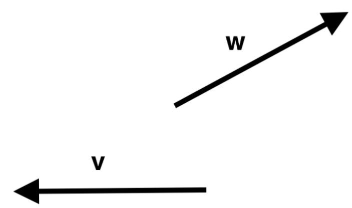
2. (3 points)



Cupid shoots an arrow in the direction of the vector **v** which lies in the yz plane. Find the equation of the plane through the the **origin** that is perpendicular to the vector **v**. No need to simplify your answer, just fill in the blanks below:

$$\quad(x- \quad) + \quad (y- \quad) + \quad (z- \quad) =$$

3. (6 points) Find the equation of the plane that contains the point $(1, 4, 5)$ and the line given by $\mathbf{r}(t) = \langle 3t, 4t, 5t \rangle$. Hint: Draw a picture.



4. (6 points) Suppose that $\mathbf{v}, \mathbf{w}$ are the two vectors in the picture above.

   Then $\mathbf{v} \times \mathbf{w}$ points [out of the page / into the page] (Circle one)

   How many vectors are perpendicular to both $\mathbf{v}, \mathbf{w}$? [One/Two/ Three/More than Three] (Circle one)

   How many **unit** vectors are perpendicular to both $\mathbf{v}, \mathbf{w}$? [One/Two/ Three/More than Three] (Circle one)

   Which of the following would necessarily be a unit vector perpendicular to both $\mathbf{v}, \mathbf{w}$: (Circle all that apply):

   (a) $\dfrac{\mathbf{w} \times \mathbf{v}}{||\mathbf{v}||||\mathbf{w}||}$

   (b) $-\dfrac{\mathbf{w} \times \mathbf{v}}{||\mathbf{v}||||\mathbf{w}||}$

   (c) $\dfrac{\mathbf{w} \times \mathbf{v}}{||\mathbf{w} \times \mathbf{v}||}$

   (d) $-\dfrac{\mathbf{w} \times \mathbf{v}}{||\mathbf{w} \times \mathbf{v}||}$

   (e) $\dfrac{\mathbf{v} \times \mathbf{w}}{||\mathbf{v} \times \mathbf{w}||}$

   (f) $\langle 0, 0, 0 \rangle$.

Quiz 4 - Good Luck Everyone! **Name:** ______________________________

1. A particle is moving along the path with parametrization:

$$\mathbf{r}(t) = \langle 3 - t^2 + 4t, 3t - 12, t^3 - 18t \rangle, \qquad -\infty < x < \infty$$

   (a) (3 points) At what time(s) is the particle at rest? If none, explain why not.

   (b) (3 points) At what point(s) does the particle cross the plane xz plane?

2. Find parametrizations for the following:

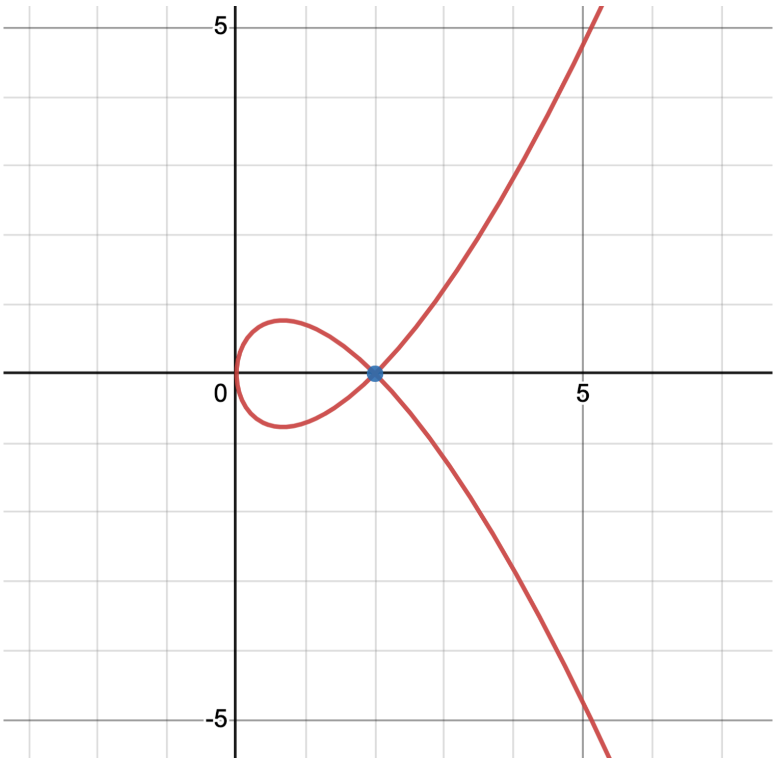
   (a) (3 points) The circle that is parallel to the yz plane that has radius 4 and center at the point $(1, 1, 1)$? (Include your range for t)

   (b) (3 points) The intersection of the cylinder $x^2 + y^2 = 25$ with the plane $x + y + z = 4$. (Include your range for t.)

3. (6 points) Below is the graph of the curve with parametrization

$$\mathbf{r}(t) = (2t^2, 2t^3 - 2t), \quad -\infty < x < \infty$$

Showing your steps carefully, find the tangent vectors at the point $(2, 0)$ and draw them in as vectors starting at $(2, 0)$. Hint: there are two tangent vectors that happen at two different times.



Quiz 5 - Good Luck Everyone! **Name:** ______________________________

This is a group quiz!

1. A student is trying to classify the critical points of a function $f(x,y)$. The student works out that

$$\frac{\partial f}{\partial x} = 3x^2 - 3 + 3y^2, \qquad \frac{\partial f}{\partial y} = 6xy.$$

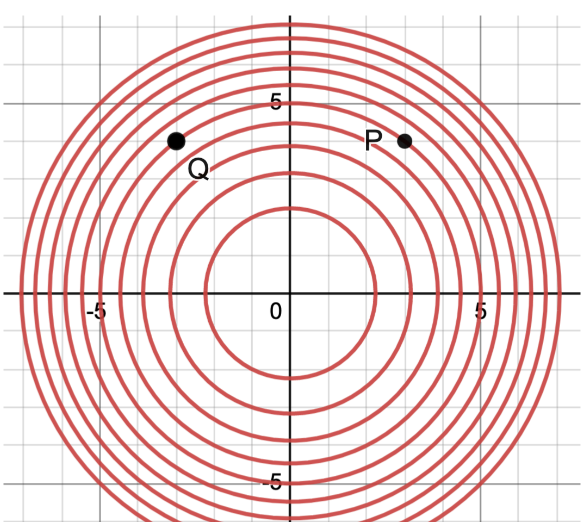
And finds four critical points, $(0,1), (0,-1), (1,0), (-1,0)$.

(a) (3 points) Write down the Hessian matrix H in terms of x and y.

(b) (2 points) Determine whether the point $(-1,0)$ is a local max, local min, saddle point. Explain your answer briefly.

(c) (2 points) Determine whether the point $(0,-1)$ is a local max, local min, saddle point. Explain your answer briefly.

(d) (2 points) What is ∇f at the point $(1,1)$?



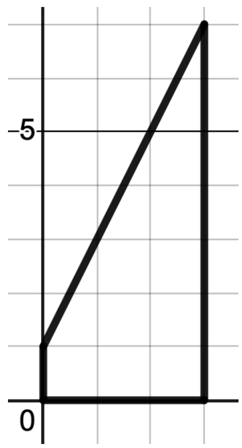
2. (4 points) The contour map of a function $f(x,y)$ is given in the picture to the left, you are given that the point $(0,0)$ is a local maximum. Then

$f_x(P)$ is positive/negative/zero

$f_y(P)$ is positive/negative/zero

$f_x(Q)$ is positive/negative/zero

$f_y(Q)$ is positive/negative/zero



3. (11 points) A student is trying to find the global maximum and global minimum temperature on the region indicated at the left.

   The equation for temperature is given by

$$T(x,y) = 12x - y^2$$

In order to find the global min and max of the temperature on this region,

you would start by finding the ____________________________ which are obtained by setting the partial derivatives equal to 0. You would then find the temperature at these points.

Then you would study the ____________________________ (8 letter word with a y in it) of this region, which has 4 pieces.

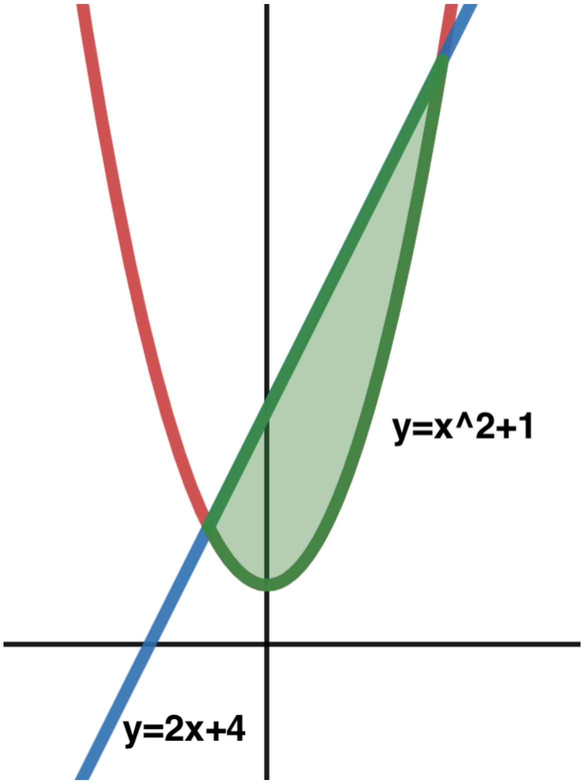
($\star$) On each piece you would parametrize the edge and then optimize the function, meaning you would note its min and max. You would record where these happen.

All together this problem would then have 5 steps, plus a conclusion.

On this problem I am only asking you to do step ($\star$) for the top left edge. To receive full credit you must include a sketch of a parabola like we did in class. Please write complete sentences including a conclusion: "On this edge the minimum of .... occurs at the point .... " (and similarly for the maximum). You might want to label where the min and max occur on the picture above, though that is not necessary.

Quiz 6 - Good Luck Everyone! **Name:** ______________________________

1. Consider the region graphed below:



Set up **but do not evaluate** a double integral of the function $f(x,y) = x + 2y$ over the region D. You may use $dy\ dx$ or $dx\ dy$ setup.

2. In the following integral, please

   A) sketch the domain of integration

   B) reverse the order of integration. By this I mean write an iterated integral of the form $dy\ dx$.

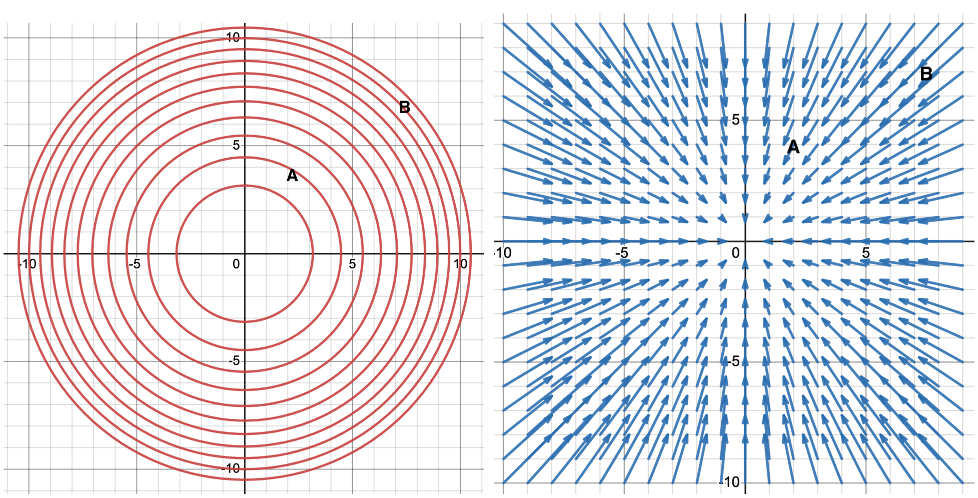
$$\int_1^4 \int_{1-y}^{y-1} f(x,y)\ dx\ dy.$$

Quiz 7 - Good Luck Everyone! **Name:** ____________________

1. (6 points) One of the following two vectors fields is conservative and the other is not. Determine which is which. For the conservative one, I want you to find the potential. For the other one, you should explain why you know it cannot be conservative like we did in class.

$$\mathbf{F} = \langle xy^2, x^2 \rangle, \qquad \mathbf{G} = \langle 16x + 4y, 4x + 6y \rangle$$

2. (2 points) Below are two pictures. One is the contour map of a function $T(x, y)$ that gives the temperature. The other is the vector field given by its gradient ∇T. Using complete sentences, explain whether the temperature at point A or point B must be greater (or whether we cannot tell).



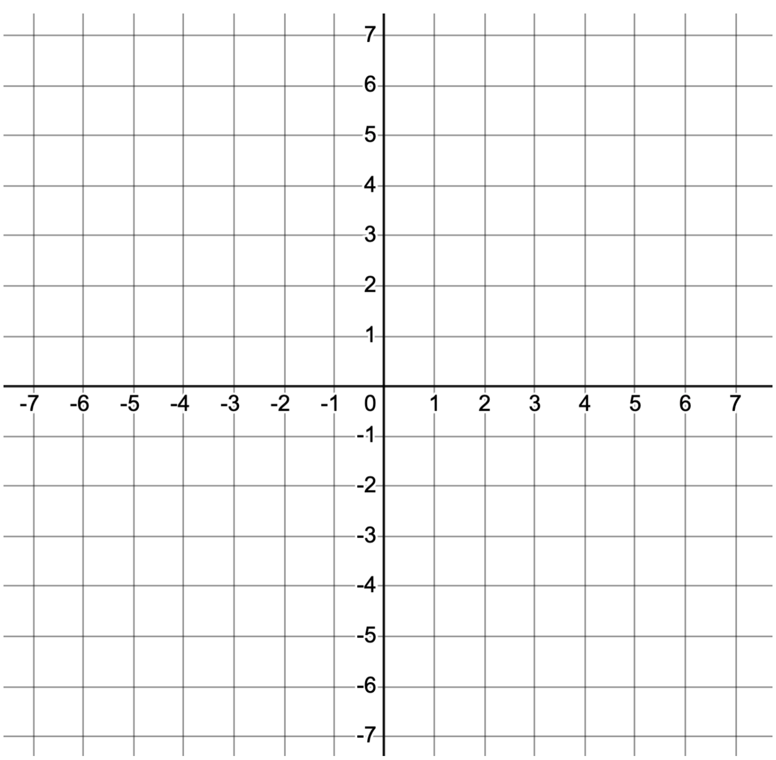
3. (10 points) A researcher is walking around the rainforest. At each point (x, y), she measures an oxygen density given by $f(x, y) = 2 + x^2 + y^2$ and a temperature $T(x, y) = 30 + \sqrt{x^2 + 2y^2}$ in degrees Celsius. She also measures the wind, and determines that it is given by the vector field $\mathbf{W} = \langle -y, x - 1 \rangle$. One day the researcher is walking along the path with parametrization $\mathbf{c}(t) = (t^2, 3t - t^3)$. Use the above information to answer the following:

(a) What is the wind vector at the point $(4, 2)$? ____________________________.

(b) Draw this vector in the picture (start the vector at the point $(4, 2)$ and label it carefully as $\vec{W}$. You must use the vector notation to get full credit.

(c) Find the velocity vector for the researcher's path at the point $(4, 2)$. Circle your answer and add it to the picture with an appropriate label.

(d) At time $t = 1$ what is the temperature where the researcher is? (Indicate this by putting a point at the appropriate part of the graph below and labeling it with the "Temp ="



Quiz 8 - Good Luck Everyone! **Name:** ________________________

1. (2 points) Antoine the Ant is walking along the curve described below:

   $\mathbf{c}(t) = (t^2 + 5, t^3 - 6), \quad 1 \le t \le 2.$

   $\mathbf{c}'(t) = \langle 2t, 3t^2 \rangle$

   Find the coordinates of the start and end of Antoine's path. Write your answers on the blanks:

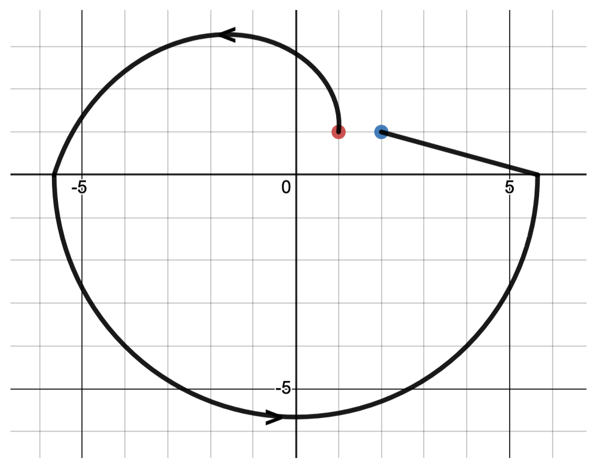
   START: END :

---

2. (5 points) Evaluate the following line integral by finding a potential and using the Fundamental Theorem for Gradients. Simplify all answers completely.

$$\int_C (2x + y + y^2)\ dx + (x + 2xy)\ dy$$

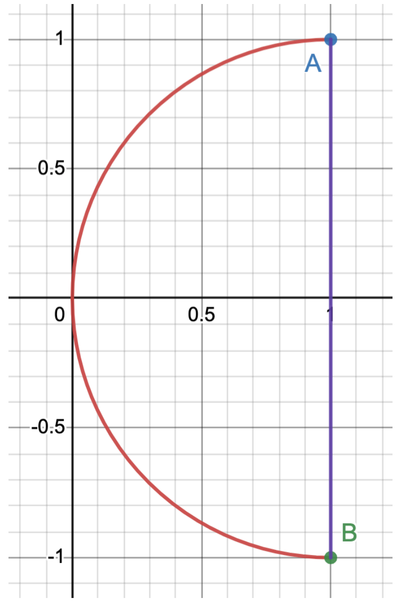
where C is the curve pictured at right:



3. John B. is walking in the wind, which is given by the vector field:

$$\mathbf{W} = \langle y, 2x \rangle.$$

He is walking along the closed path (walking counterclockwise) in the figure to the right. This path consists of two different curves, a straight line and an arc of a circle. Please set up and **fully evaluate** the line integral of $\mathbf{W}$ along the **straight line** with the orientation described.



Quiz 9 - Good Luck Everyone! **Name:** ______________________________

1. (12 points) Let C be the boundary of the circle $x^2 + y^2 = 9$ oriented **clockwise.**

   - Use Green's Theorem to evaluate $\oint\limits_C (e^{\cos x} - 5y)\, dx + (3x - e^{y^3})dy$. You must simplify your answer completely. You may evaluate your integral or appeal to area / center of mass reasons, but you must show all your work and write in complete sentences. Any symbols you use should be clearly defined. Defining with a picture is ok.

2. (12 points) Short Answer

(a) If D is a region with area 6, then $\iint_D 2\, dA = 12$ [True/False]

(b) If D is a region with area 6, the $\iint_D 2x\, dA = 12x$ [True/ False]

(c) If D is a region with area 6 whose center of mass is at $(4,5)$ then $\iint_D x\, dA =$ ______________________________

(fill in a number)

(d) Suppose D is a region in the plane and $\iint_D x\, dA = 10$. Is this enough information to know the value of $\iint_D x^2\, dA$? [Yes / No]

(e) Suppose D is a region in the plane and $\iint_D x\, dA = 10$ and D has area 5. Is this enough information to know the value of $\iint_D x^2\, dA$? [Yes / No]

(f) Suppose that D is the unit circle and H is the right half of the unit circle. (I recommend drawing a quick picture to help!) Which of the following are true? Circle all that apply:

i. $\iint_D dA = 2\iint_H dA.$

ii. $\iint_D x dA = 2\iint_H x\, dA.$

iii. $\iint_D y dA = 2\iint_H y\, dA.$

iv. $\iint_D x^2 dA = 2\iint_H x^2\, dA.$

(g) Choose one of the items from the previous part, (part (f)) and explain why you made your choice. You can explain "I circled XXX because ..." or you can explain "I didn't circle XXX because."'

Quiz 10 - Good Luck Everyone!

This quiz is take home. It is due Friday May 12th in Class

All parts of this problem are about the same surface S. Since this is a take-home quiz, I will be paying more attention to your writeups and presentation, so please **write very neatly.**

This quiz score (if it is greater) will replace your lowest quiz score.

You **may** work with classmates on this quiz, but you must write up your own solutions.

I'm happy to help in office hours.

Asking for help from tutors, or anyone else outside of our class is forbidden. This includes posting these questions online asking for help.

For what it's worth, this question is something that could appear on the final, so I highly recommend everyone try it! Good luck!

Let S be the surface consisting of the boundary of the 3D region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$. Suppose that S is oriented outwards.

1. Draw a very careful picture of S.
2. Your picture should show you that your surface has two different pieces. Describe them in words, and parametrize each of them. For each picture, carefully say what your parameter domain D is. You will use these in the next problem.
3. Find the flux of the vector field $\mathbf{F} = \langle xz, z, 2\rangle$ through the surface S. You should have two different parts, since your surface has two different pieces. Please write your work very neatly. You can use a calculator/Desmos to calculate your double integrals, but I will pay careful attention to your writeup. In lowest terms your answer should be a fraction with 3 in the denominator and π in the numerator.

   Since your surface has 2 parts, you will need to calculate 2 integrals. Make sure your final answer makes it clear how you are combining your answers. This Quiz might take up more space than you're used to - that's ok, give yourself enough space to present everything clearly.
4. Find the divergence of the vector field from part 3. Calculate that the triple integral of $\operatorname{div}\mathbf{F}$ over the **solid region bounded by** S is the same answer you got in part a. This is a special case of the **divergence** theorem.

   (Optional) Which method was faster - the one in part 3 or in part 4?

   (Optional) This problem was mostly about setting up surface integrals and triple integrals. You might want to add some advice to your note card (which can be a full sheet of notes, front and back) for the final.

Math 250 Midterm 1

Your Name

- You have 55 minutes to do this exam.
- You may use a scientific calculator and a small note card.
- For justifications, please use complete sentences and make sure to explain any steps which are questionable.
- Good luck!

Problem	Total Points	Score
1	24	
2	17	
3	24	
4	8	
5	12	
EC	4	
Total	85	