$x := 0$

$J_{5\_100}(x) := 2 - 2\cos(\pi \cdot x)$ $\quad J_{5\_110}(x) := 2 - 2 \cdot \cos\left(\frac{\pi}{\sqrt{2}} \cdot x\right)^2$ $\quad J_{5\_111}(x) := 2 - 2\cos\left(\frac{\pi}{\sqrt{3}} \cdot x\right)^3$

$J_{1\_100}(x) := \sin(\pi \cdot x)^2$ $\quad J_{1\_110}(x) := 2\sin\left(\frac{\pi}{\sqrt{2}} \cdot x\right)^2$ $\quad J_{1\_111}(x) := 3\sin\left(\frac{\pi}{\sqrt{3}} \cdot x\right)^2$

$J_{11\_100}(x) := 2 - 2\cos(2 \cdot \pi \cdot x)$ $\quad J_{11\_110}(x) := 3 - \cos\left(\frac{2 \cdot \pi}{\sqrt{2}} x\right)^2 - 2\cos\left(\frac{2 \cdot \pi}{\sqrt{2}} x\right)$

$J_{11\_111}(x) := 3 - 3\cos\left(\frac{2 \cdot \pi}{\sqrt{3}} x\right)^2$

$J_{3\_100}(x) := 2 \cdot (3 - 2 \cdot \cos(\pi\, x) - \cos(3 \cdot \pi\, x))$ $\quad J_{3\_110}(x) := 2 \cdot \left(3 - \cos\left(\frac{\pi}{\sqrt{2}} x\right)^2 - 2\cos\left(\frac{3 \cdot \pi}{\sqrt{2}} x\right) \cdot \cos\left(\frac{\pi}{\sqrt{2}} x\right)\right)$

$J_{3\_111}(x) := 6 \cdot \left(1 - \cos\left(\frac{3 \cdot \pi}{\sqrt{3}} x\right) \cdot \cos\left(\frac{\pi}{\sqrt{3}} x\right)^2\right)$

$J_{111\_100}(x) := 2 - 2\cos(2\pi \cdot x)$ $\quad J_{111\_110}(x) := 2 - \cos\left(\frac{2\pi}{\sqrt{2}} \cdot x\right)^2 \cdot 2$ $\quad J_{111\_111}(x) := 2 - 2\cos\left(\frac{2\pi}{\sqrt{3}} \cdot x\right)^3$

DR expansion in 100 direction:

$$M1 := \begin{bmatrix} \frac{1}{2} \cdot \frac{d^2}{dx^2} J_{5\_100}(x) & \frac{1}{2} \cdot \frac{d^2}{dx^2} J_{1\_100}(x) & \frac{1}{2} \cdot \frac{d^2}{dx^2} J_{11\_100}(x) \\ \frac{1}{4!}\left(\frac{d^4}{dx^4} J_{5\_100}(x)\right) & \frac{1}{4!} \cdot \frac{d^4}{dx^4} J_{1\_100}(x) & \frac{1}{4!} \cdot \frac{d^4}{dx^4} J_{11\_100}(x) \end{bmatrix} \rightarrow \begin{pmatrix} \pi^2 & \pi^2 & 4 \cdot \pi^2 \\ -\frac{\pi^4}{12} & -\frac{\pi^4}{3} & -\frac{4 \cdot \pi^4}{3} \end{pmatrix}$$

DR expansion in 110 direction:

$$M2 := \begin{bmatrix} \frac{1}{2} \cdot \frac{d^2}{dx^2} J_{5\_110}(x) & \frac{1}{2} \cdot \frac{d^2}{dx^2} J_{1\_110}(x) & \frac{1}{2} \cdot \frac{d^2}{dx^2} J_{11\_110}(x) \\ \frac{1}{4!} \cdot \left(\frac{d^4}{dx^4} J_{5\_110}(x)\right) & \frac{1}{4!} \cdot \frac{d^4}{dx^4} J_{1\_110}(x) & \frac{1}{4!} \cdot \frac{d^4}{dx^4} J_{11\_110}(x) \end{bmatrix} \rightarrow \begin{pmatrix} \pi^2 & \pi^2 & 4 \cdot \pi^2 \\ -\frac{\pi^4}{6} & -\frac{\pi^4}{6} & -\frac{5 \cdot \pi^4}{3} \end{pmatrix}$$

DR expansion in 111 direction:

$$M3 := \begin{bmatrix} \frac{1}{2} \cdot \frac{d^2}{dx^2} J_{5\_111}(x) & \frac{1}{2} \cdot \frac{d^2}{dx^2} J_{1\_111}(x) & \frac{1}{2} \cdot \frac{d^2}{dx^2} J_{11\_111}(x) \\ \frac{1}{4!} \cdot \left(\frac{d^4}{dx^4} J_{5\_111}(x)\right) & \frac{1}{4!} \cdot \frac{d^4}{dx^4} J_{1\_111}(x) & \frac{1}{4!} \cdot \frac{d^4}{dx^4} J_{11\_111}(x) \end{bmatrix} \rightarrow \begin{pmatrix} \pi^2 & \pi^2 & 4 \cdot \pi^2 \\ -\frac{7 \cdot \pi^4}{36} & -\frac{\pi^4}{9} & -\frac{16 \cdot \pi^4}{9} \end{pmatrix}$$

rank(M1) = 2 rank(M2) = 2 rank(M3) = 2

Matrix to get j0,j1,j2 from power 4 polynomial fitting:

$$MS := \begin{bmatrix} \frac{1}{2}\cdot\frac{d^2}{dx^2}J_{5\_100}(x) & \frac{1}{2}\cdot\frac{d^2}{dx^2}J_{1\_100}(x) & \frac{1}{2}\cdot\frac{d^2}{dx^2}J_{11\_100}(x) \\ \frac{1}{4!}\left(\frac{d^4}{dx^4}J_{5\_100}(x)\right) & \frac{1}{4!}\cdot\frac{d^4}{dx^4}J_{1\_100}(x) & \frac{1}{4!}\cdot\frac{d^4}{dx^4}J_{11\_100}(x) \\ \frac{1}{2}\cdot\frac{d^2}{dx^2}J_{5\_110}(x) & \frac{1}{2}\cdot\frac{d^2}{dx^2}J_{1\_110}(x) & \frac{1}{2}\cdot\frac{d^2}{dx^2}J_{11\_110}(x) \\ \frac{1}{4!}\left(\frac{d^4}{dx^4}J_{5\_110}(x)\right) & \frac{1}{4!}\cdot\frac{d^4}{dx^4}J_{1\_110}(x) & \frac{1}{4!}\cdot\frac{d^4}{dx^4}J_{11\_110}(x) \\ \frac{1}{2}\cdot\frac{d^2}{dx^2}J_{5\_111}(x) & \frac{1}{2}\cdot\frac{d^2}{dx^2}J_{1\_111}(x) & \frac{1}{2}\cdot\frac{d^2}{dx^2}J_{11\_111}(x) \\ \frac{1}{4!}\cdot\frac{d^4}{dx^4}J_{5\_111}(x) & \frac{1}{4!}\cdot\frac{d^4}{dx^4}J_{1\_111}(x) & \frac{1}{4!}\cdot\frac{d^4}{dx^4}J_{11\_111}(x) \end{bmatrix} \rightarrow \begin{pmatrix} \pi^2 & \pi^2 & 4\cdot\pi^2 \\ -\frac{\pi^4}{12} & -\frac{\pi^4}{3} & -\frac{4\cdot\pi^4}{3} \\ \pi^2 & \pi^2 & 4\cdot\pi^2 \\ -\frac{\pi^4}{6} & -\frac{\pi^4}{6} & -\frac{5\cdot\pi^4}{3} \\ \pi^2 & \pi^2 & 4\cdot\pi^2 \\ -\frac{7\cdot\pi^4}{36} & -\frac{\pi^4}{9} & -\frac{16\cdot\pi^4}{9} \end{pmatrix}$$

rank(MS) = 3

$$4\cdot MS = \begin{pmatrix} 39.478 & 39.478 & 157.914 \\ -32.47 & -129.879 & -519.515 \\ 39.478 & 39.478 & 157.914 \\ -64.939 & -64.939 & -649.394 \\ 39.478 & 39.478 & 157.914 \\ -75.763 & -43.293 & -692.687 \end{pmatrix}$$

rank(MS) = 3

$$Rh := \begin{pmatrix} -142.5352 \\ 1.8760e{+}04 \\ -80.6464 \\ 1.7088 \cdot 10^4 \\ -60.8472 \\ 1.6527 \cdot 10^4 \end{pmatrix} = \begin{pmatrix} -142.535 \\ 1.876 \times 10^4 \\ -80.646 \\ 1.709 \times 10^4 \\ -60.847 \\ 1.653 \times 10^4 \end{pmatrix}$$

dir 100
DH: -0.249392; + 1994.875876*x^2 + -925.847152*x^4
ERR: -0.000000*x + 0.000000*x^3
Delta: 0.39164
dir 110
DH: -0.581097; + 2027.377915*x^2 + -2121.094392*x^4
ERR: -0.000000*x + 0.000000*x^3
Delta: 0.10225
dir 111
DH: -0.165590; + 1990.125056*x^2 + -1785.202625*x^4
ERR: -0.000000*x + 0.000000*x^3

$$Rh := \begin{pmatrix} 1994.875876 \\ -925.847152 \\ 2027.377915 \\ -2121.094392 \\ 1990.125056 \\ -1785.202625 \end{pmatrix} = \begin{pmatrix} 1.995 \times 10^3 \\ -925.847 \\ 2.027 \times 10^3 \\ -2.121 \times 10^3 \\ 1.99 \times 10^3 \\ -1.785 \times 10^3 \end{pmatrix}$$

$$\frac{\text{lsolve}(MS, Rh)}{4} = \begin{pmatrix} 57.312 \\ 9 \\ -3.887 \end{pmatrix}$$

3

$$\left\| \begin{pmatrix} 9.87 & 19.739 \\ -97.409 & -779.273 \\ 961.389 & 3.076 \times 10^{4} \end{pmatrix} \right\|$$

$$\mathrm{svd2}\left(\begin{pmatrix} \frac{\pi^{2}}{2} & - \\ -\frac{\pi^{4}}{24} & - \\ \frac{\pi^{6}}{720} & \frac{2}{-} \end{pmatrix} \right.$$

$$\mathrm{aa} := \mathrm{svd2}\left(\right.$$

$$aa_0 = \begin{pmatrix} 1.269 \times 10^5 \\ 73.624 \\ 0.04 \end{pmatrix}$$

$$\left|\begin{pmatrix} 78.957 \\ -3.117 \times 10^{3} \\ 1.231 \times 10^{5} \end{pmatrix}\right| = -3.746 \times 10^{5}$$

$$\left|\left|\begin{pmatrix} \pi^{2} & 4\cdot\pi^{2} \\ -\frac{\pi^{4}}{3} & -\frac{4\cdot\pi^{4}}{3} \\ \frac{2\cdot\pi^{6}}{45} & \frac{8\cdot\pi^{6}}{45} \end{pmatrix}\right|\right| = \begin{pmatrix} \{3,1\} \\ \{3,3\} \\ \{3,3\} \end{pmatrix}$$

$$\left|\left|\begin{pmatrix} 9.87 & 19.739 & 78.957 \\ -97.409 & -779.273 & -3.117 \times 10^{3} \\ 961.389 & 3.076 \times 10^{4} & 1.231 \times 10^{5} \end{pmatrix}\right|\right| = \begin{pmatrix} \{3,1\} \\ \{3,3\} \\ \{3,3\} \end{pmatrix}$$