DATA1001 Topic 6 Flashcards

Understanding Chance

Abyan Majid

Question	My answer
What is the prosecutor's fallacy?	Prosecutor's fallacy is the mistake of thinking that the probability of a DNA match given innocence is the same as the probability of innocence given a DNA match, where in fact $P(DNA\ Match \mid Innocence) \neq P(Innocence \mid DNA\ Match)$ A more generalized and succinct definition: It is the mistake of thinking $P(A\mid B) = P(B\mid A)$ where in $P(A\mid B) \neq P(B\mid A)$
What is the chance?	By the frequentist definition, chance is the expected percentage of time an event/outcome is expected to occur if we were to repeatedly draw from the sample space over a long period of time
Are the following statements true or false? (1) Chances are between 0 and 1 (2) The chance of an event E is $P(E) = 1 - \overline{E}$ (3) Drawing at random does not necessarily mean that all outcomes in the sample space have an equal chance of being drawn.	(1) True (2) True (3) False; drawing at random ALWAYS implies that all outcomes have an equal chance of being drawn.
What is conditional probability?	The probability than an event A occurs given that B occurred. $P(A\mid B)$
Recite the multiplication principle (the probability that two events E_1 and E_2 occur)!	The probability that two events E_1 and E_2 both occur is given by $P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2 \mid E_1)$
When are two events independent?	Two events are independent if and only if the probability of the second event occurring given the first occurred $P(E_2\mid E_1)$ is equal to the probability of the second event $P(E_2)$, So: E_1 and E_2 are independent if and only if $P(E_2\mid E_1) = P(E_2)$

How do you draw such that independence is ensured?	Draw randomly with replacement.
Assuming we draw without replacement, in a deck of 52 cards: (1) What is the chance of drawing 3 aces? (2) What is the chance of drawing 1 king and 1 other than isn't a king?	Let A be the event of drawing an ace, K the event of drawing a king (1) $P(A_1 \text{ and } A_2 \text{ and } A_3) = \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \approx 0.000181$ (2) $P(K \text{ and } \overline{K}) = \frac{4}{52} \times \frac{48}{51} \approx 0.0724$
What is the chance of both events E_1 and E_2 occurring, given that they are independent?	$P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$
A fair coin is tossed twice. If the coin lands heads on the 2nd toss, you win $1. (1) If the 1st coin is a head, what is the chance of winning $1? (2) If the 1st coin is a tail, what is the chance of winning $1? (3) Are the tosses independent? (4) What is the chance of winning $1?	Let F be first roll, and S be second roll (1) Sample space for F_H : $\{(H, H),\ (H, T)\}$, so $P(S_H \| F_H) = \frac{1}{2}$ (2) Sample space for F_T : $\{(T,\ H),\ (T,\ T)\}$ so $P(S_H \| F_T) = \frac{1}{2}$ (3) Yes, because we draw randomly (since the coin is "fair") with replacement. (4) Sample space: $\{(H,\ H),\ (H,\ T),\ (T,\ H),\ (T,\ T)\}$ so $P(S_H) = \frac{2}{4} = \frac{1}{2}$
Two dice are thrown. What is the chance the dice sum to 6? How would you simulate this in R?	There are $6^2 = 36$ possible outcomes for the sum of two dies. The combinations $\{(1,\ 5),\ (2,\ 4),\ (3,\ 3),\ (4,\ 2),\ (5,\ 1))\}$ result in a sum of 6. So $P(D_1 + D_2 = 6) = \frac{5}{36}$ To simulate in R, (1) \`totals=sample(1:6, 1000, rep = T)+sample(1:6, 1000, rep = T)\` (2) \`table(totals)\` \`1000\` can be replaced by another number of trials where theoretically the greater the number of trials, the more reliable the simulation becomes.
Define mutual exclusivity!	Two events are mutually exclusive if the occurrence of one event prevents the occurrence of another.
Recite the addition rule!	If two events E_1 and E_2 are mutually exclusive, then the probability that at least one of them occurs is given by $P(E_1) + P(E_2)$
A die is rolled 6 times and a deck of cards is shuffled. What is chance that:	(1) $\frac{1}{6} + \frac{1}{6} - (\frac{1}{6} \times \frac{1}{6}) \approx 0.306$ (2) $\frac{1}{6} \times \frac{1}{6} \approx 0.0278$

(1) the 1st roll is a 1 or the 6th roll is a 1, or both? (2) both the 1st and 6th rolls are 1s? (3) the top card is the ace of spades or the bottom card is the ace of spades? (4) both the top card and the bottom card are the ace of spades?	(3) $\frac{1}{52} + \frac{1}{52} - (\frac{1}{52} \times \frac{1}{52}) \approx 0.0381$ (4) 0
(1) What is the number of ways of rearranging n distinct objects IN A ROW (order matters)? (2) What is the number of ways of rearranging n distinct objects (order DOESN'T matter)?	(1) $n!$ (2) $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ Note that $0! = 1$, $\binom{n}{0} = 1$
What is a binary trial?	A trial where only 2 things can occur; either E occurs or it does not ($\overline{E}$).
Recite the binomial theorem! How would you use it in R?	Given n binary trials, the chance that exactly x events occur is given by $\binom{n}{x} p^x (1 - p)^{n-x}$ To use it via R, `binom(x, n, p)`
(1) A fair coin is tossed 5 times. What is the probability of getting 3 heads? (2) A fair coin is tossed 500 times. What is the probability of getting 300 heads? (3) Suppose 100 babies are born at RPA hospital today with P(boy) = 0.51. What is the probability that 55 boys were born?	(1) $\binom{5}{3} 0.5^3 (1 - 0.5)^{5-3} = 0.3125$ (2) $\binom{500}{300} 0.5^{300} (1 - 0.5)^{500-300} \approx 0.000000154$ (3) $\binom{100}{55} 0.51^{55} (1 - 0.51)^{100-55} \approx 0.0580$