Finite State Machines: Moore and Mealy

Abyan Majid

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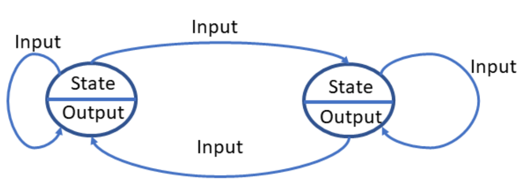
1 Finite state machines

A "Finite State Machine" (FSM) is a model used to provide a diagrammatic representation of systems that (1) has a finite number of states, and (2) and undergo transitions between states in response to inputs. Finite state machines are typically made up of the following elements:

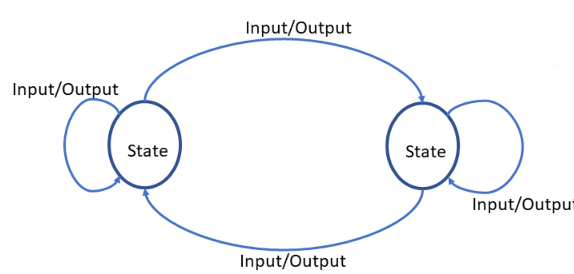
- **States** (denoted as a node/circle $\circ$): The finite set of states the machine can be in. Each state is typically given a binary or symbol that uniquely defines it.
- **Transitions** (denoted as arrows $\rightarrow$): A possible transition from one state to another if a condition (input) is True.
- **Inputs** (denoted as symbols, often binary): It's any signal that allows either or both a transition from one state to another OR the output (in the context of a Mealy machine)
- **Outputs** (denoted as symbols, often binary): The set of symbols produced by the machine during the state transitions.

There are several types of finite state machines, but we only learn of two in ELEC1601:

- **Moore FSM**: An FSM where state transition depends on the current state and the input values, while the outputs are ONLY dependent on the current state.
- **Mealy FSM**: An FSM where both the state transition and outputs are dependent on the current state and input values.



Generic Moore model



Generic Mealy model

2 Deriving truth tables from FSMs

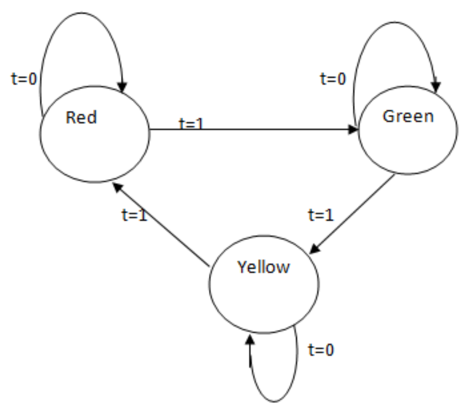
Given an FSM, you could derive its truth table where the "Result" or "Output" column depends on the type of the FSM. Typically, for a:

- Moore FSM, the output is whatever each state implies.
- Mealy FSM, the output is whatever could (but not always) happen in each state.

2.1 Example 1 with Moore FSM

Suppose that we have the following Moore FSM, which models the states of a traffic light. Btw, we can tell that it is a Moore FSM because:

- there is no output label beside the inputs, therefore indicating that the outputs are independent of the inputs.
- in a traffic light, red represents "stop", green represents "go", and yellow represents "caution". In other words, the outputs (ie. stop, go, caution) are dependent on the colour of the traffic lights, all of which are the states of the FSM.



Suppose that t is the input and it denotes time, where $t = 1$ is a time for state transition, and $t = 0$ is time to stay in the current state. Since we have the input and the current states, we can easily derive the truth table of our FSM like so:

t	Current State	Next State	Output
0	Red	Red	Stop
1	Red	Green	Go
0	Green	Green	Go
1	Green	Yellow	Caution
0	Yellow	Yellow	Caution
1	Yellow	Red	Stop

We can also encode all symbols in binary such that the truth table is comprised of binary values only.

State symbols:

- Red: 00
- Green: 01
- Yellow: 10

Outputs:

- Stop: 00
- Go: 01
- Cauation: 10

t	S_0S_1	$S_0^+S_1^+$	**Output**
0	0 0	0 0	00
1	0 0	0 1	01
0	0 1	0 1	01
1	0 1	1 0	10
0	1 0	1 0	10
1	1 0	0 0	00

It is also the convention that if the states are represented in binary notation, the column heading for the:

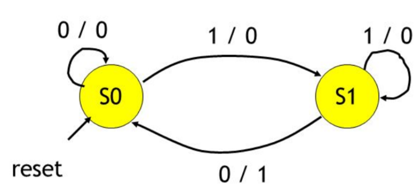
- "current states" be expressed as $S_0S_1S_2...S_{n-1}$, where S denotes a bit in the binary.
- "next states" be expressed as $S_0^+S_1^+S_2^+...S_{n-1}^+$, where S^+ denotes a single bit in the binary.

As you can see, the outputs are perfectly dependent on the state. If you're:

- in a state "red" (00), the ouput will always be "stop" (00).
- If you're in a state "green" (01), the output will always be "go" (01).
- If you're in a state "yellow" (10), the output will always be "caution" (10).

2.2 Example 2 with Mealy FSM

Suppose we have the following FSM, for which we are given no context whatsoever. But we can definitely tell that this is a Mealy FSM because the outputs are labeled beside the inputs, ie. (input / output), therefore indicating that the output (besides depending on the current state) also depends on the input



Here's how our truth table would look like:

Input	S	S^+	**Output**
0	0	0	0
1	0	1	0
0	1	0	1
1	1	1	0

As you can see, the outputs of a Mealy FSM also depends on the inputs, unlike Moore FSMs. For example, being in state "S0" can get you an output 0 or 1 depending on whether you stayed in S0 or had transitioned from S1 to S0. If this was a Moore FSM, the output for being in S0 would either always be 0, or always be 1. It cannot be different under any circumstances.