Raising a Complex Number to A Power Using De Moivre's Theorem

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De Moivre's Theorem gives us a way to raise any complex number z to any integer power n. Specifically, De Moivre's Theorem is defined as:

$$\boxed{z^n = r^n(\cos(n\theta) + i\sin(n\theta)), \text{ for } n \in \mathbb{Z}}$$

De Moivre's theorem actually also applies to real and complex powers, but such case is rare.

Example: Given that $z = 3(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$, find z^4

$$z^4 = 3^4[\cos(4)(\frac{\pi}{4}) + i\sin(4)(\frac{\pi}{4})]$$

$$z^4 = 81(\cos\pi + i\sin\pi)$$

$$z^4 = -81$$

You can of course apply De Moivre's Theorem in the exponential form, like so: $\boxed{z = r^n e^{in\theta}}$

(Btw, pronounce "De Moivre" as "De Moave" because it's french)