Functions Basics

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1 Definition

A function in mathematics is some sort of relationship that can be established between two variables as input and output.

Say that you have a variable y, of which value depends on another variable x. You write $y = f(x)$ to denote that "y is a function of x" - and so $f(x)$ establishes that relationship that the value of y depends on the value of x.

2 Vertical line test - is that a function or not?

To determine if a mathematical construct is a function, you can do the vertical line test - which states that if every vertical line $x = a$, where $a \in \mathbb{R}$, intersects the figure ONLY once, then this "figure" is a function. If it intersects more the once, it is not a function.
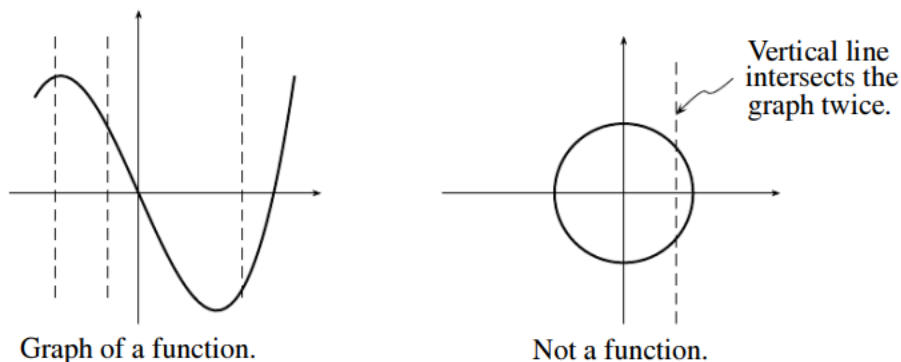

Graph of a function.   Not a function.

Figure 1: Vertical line test

3 Domain and range

- A "**domain**" is the set of all possible INPUT values (x) for a function. To specify, the domain of a function $f(x)$, you write $\{x \mid x_1 \leq x \leq x_2\}$, which is read as "all x values, such that $x^1 \leq x \leq x_2$"

- A "**range**" is the set of all possible OUTPUT values (y) for a function. To specify the range of a function $f(x)$, you write $\{y \mid y_1 \leq y \leq y_2\}$, which is read as "all y values, such that $y_1 \leq y \leq y_2$"

Of course, you may replace the inequality sign with $<$, and you may also exclude one of the bounds. Or you may even say something like $x \in \mathbb{Z}$. It's all just set notation.

4   Combining functions arithmetically

If you have a function $f(x)$ and another $g(x)$, both of which have a common domain, you can combine the two by:

- $f(x) + g(x)$

- $f(x) - g(x)$

- $f(x) \times g(x)$

- $f(x) \div g(x)$

Just combine respective terms as you would algebraically with any expression.

5   Composite function

A "composite function", denoted as $(f \circ g)(x)$ or $f(g(x))$, is essentially a function that takes another function as an input.

So if we have $f(x) = 2x^2 + 4x - 1$ and $g(x) = 2x + 5$, our composite function would be:

$$(f \circ g) = f(g(x)) = 2(2x + 5)^2 + 4(2x + 5) - 1$$

You can choose to simplify it further, like so:

$$= 8x^2 + 48x + 69$$

6   Inverse function

The "inverse" of the function $f(x)$, denoted as $f^{-1}(x)$, is the reflection of $f(x)$ about the line $y = x$ in the cartesian plane. To find the inverse of a function, you do as demonstrated in the following example:

Find the inverse of the function $f(x) = -\frac{1}{3}x + 1$

1. Rewrite $f(x)$ as y or any other variable besides the input.

$$y = -\frac{1}{3}x + 1$$

2. Switch y and x.

$$x = -\frac{1}{3}y + 1$$

3. Solve for y.

$$x = -\frac{1}{3}y + 1$$
$$3x = -y + 3$$
$$3x - 3 = -y$$
$$y = -3x + 3$$

4. y is now the inverse of $f(x)$, therefore rewrite y as $f^{-1}(x)$.

$$f^{-1}(x) = -3x + 3$$