Week 6 - Chance

Created	September 11, 2023 8:56 PM
Date	Empty
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0) Summary of important things

0.1) Mutually exclusive vs. Independence

Term	Definition	Example
Mutually exclusive	The occurrence of Event1 prevents Event2 occurring	Getting a sum of 12 from 2 dice throws and throwing a 1
Independence	The occurrence of Event1 does not change the chance of Event2	Getting a head from a coin toss and throwing a 3 from a die

0.2) Addition rule vs. Multiplication rule

What	When	Formula	Condition
Addition Rule	P(At least 1 of 2 events occurs)	P(Event1) + P(Event2)	if mutually exclusive
Multiplication Rule	P(Both events occur)	P(Event1) x P(Event2)	if independent
		P(Event1) x P(Event2| Event 1)	if dependent

6.1) Chance

6.1.1) Prosecutor's fallacy

Prosecutor's fallacy states that:

$$P(A|B) = P(B|A)$$

which is impossible, as it is factual that:

$$P(A|B) \neq P(B|A)$$

6.1.2) Definition of chance

Chance (or probability) is the the percentage of time an event is expected to happen, if the same process is repeated in the long term.

6.1.3) Properties of chance

- Chances are between 0% (impossible) and 100% (certain)
- The chance of some event happening is 100% minus its complement

$$P(X) = 1 - P(\overline{X})$$

- Randomly drawing from a set of elements means each element has the same chance of being picked.

6.1.4) Conditional probability

Conditional probability is the chance that an event X occurs, given that another event Y has occured.

$$P(X|Y)$$

6.1.5) Independence & dependence

6.1.5.1) Independence

Independence: 2 events are independent if the chance of the 2nd event E_2, given the 1st event E_1 has occurred, is the same as E_2.

$$P(E_2|E_1) = P(E_2)$$

- Drawing randomly *WITH* **replacement ensures independence.**
- **MULTIPLICATION PRINCIPLE (Special case for independence)**: If 2 events E_1 and E_2 are independent, the chance of both occurring is the product of their unconditional probabilities

$$P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$$

6.1.5.2) Dependence

Dependence: 2 events are dependent if the chance of the 2nd event E_2, given the 1st event E_1 has occurred, is **NOT** the same as E_2.

$$P(E_2|E_1) \neq P(E_2)$$

- Drawing randomly *WITHOUT* **replacement ensures dependence.**
- **MULTIPLICATION PRINCIPLE:** The probability that 2 **DEPENDENT** events E_1 and E_2 occur is the chance that E_1 occurs multiplied by the chance that E_2 occurs, given that E_1 has occurred.

$$P(E_1 \wedge E_2) = P(E_1) \times P(E_2|E_1)$$

6.2) More chance

6.2.1) Solving problems by making lists

6.2.1.1) METHOD 1: Writing lists of outcomes

For simple chance problems, a good way to start is by:

1. Write a list of all outcomes
2. Count which outcomes belong to the event of interest

6.2.1.2) METHOD 2: Summarize in a tree diagram

1. Create a tree diagram branching from "Start"
2. Keep branching respectively until we get all events of interest

6.2.1.3) METHOD 3: Simulate

Simulate the problem and record the findings for each trial. (You can use R for this)

Chance simulation in R template:

```
# Template
set.seed(1)
X = sample(<possible outcomes>, <number of trials>, <repetition: TRUE/FALSE>)
```

Example: Simulating coin toss in R

```
# Example: Simulating coin toss in R
set.seed(1)
totals = sample(0:1, 1000, rep = TRUE)
```

```
OUT:
0 1 0 0 1 0 0 0 1 1 0 0 0 0 0 1 1 1 1 0
0 0 0 0 0 1 0 0 1 1 1 0 1 0 0 1 0 1 1 1
...
0 1 0 1 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0
```

```
# Computing probability
(table(totals))/1000
```

```
OUT:
    0      1
0.492  0.508
```

6.2.2) Mutually exclusive events

2 events E_1 and E_2 are **mutually exclusive** when the occurrence of one event prevents the other.

Example: The event of getting a sum of 12 (E_1) and the event of throwing a (E_2) are mutually exclusive because E_2 prevents E_1 from happening.

6.2.3) Addition rule

If 2 events E_1 and E_2 are mutually exclusive, then the chance of **AT LEAST** 1 of them occurring is the sum of the individual chances.

$$P(\text{At least one of } E_1 \text{ or } E_2) = P(E_1) + P(E_2)$$

6.3) Binomial formulas

6.3.1) Factorial

The number of ways of rearranging n distinct objects in a row is $n!$

$$n! = n \times (n-1) \times ... \times 2 \times 1$$

To note, $0!$ is `1`.

$$0! = 1$$

Factorial in R:

```
factorial(6)
```

```
OUT:
720
```

6.3.2) Binomial coefficient

If we have n objects in a row, made up of 2 types:

- x, and
- $n - x$

The number of ways of rearranging the n objects is given by the **binomial coefficient:**

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

with special case:

$$\binom{n}{0} = 1$$

getting binomial coefficient in R:

```
choose(n, x)
```

6.3.3) Binomial model

6.3.3.1) Binary trials

A binary trial is at trial of one of two possibilities: either an event E occurred or not. Therefore we have the following properties in a binary trial for some event E:

- $P(E) = p$
- $P(\overline{E}) = 1 - p$

6.3.3.2) Binomial theorem

If we have fixed n number of independent binary trials for some event E with $P(E) = p$ at every trial, the chance that **EXACTLY** x events occurs is given by:

$$\binom{n}{x} = p^x(1-p)^{n-x}$$

Binomial theorem in R:

```
dbinom(x, n, p)
```