Causal Learning and Inference: A Practical Guide — Part I

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Malaria Project: Leveraging Causal AI in Malaria Research

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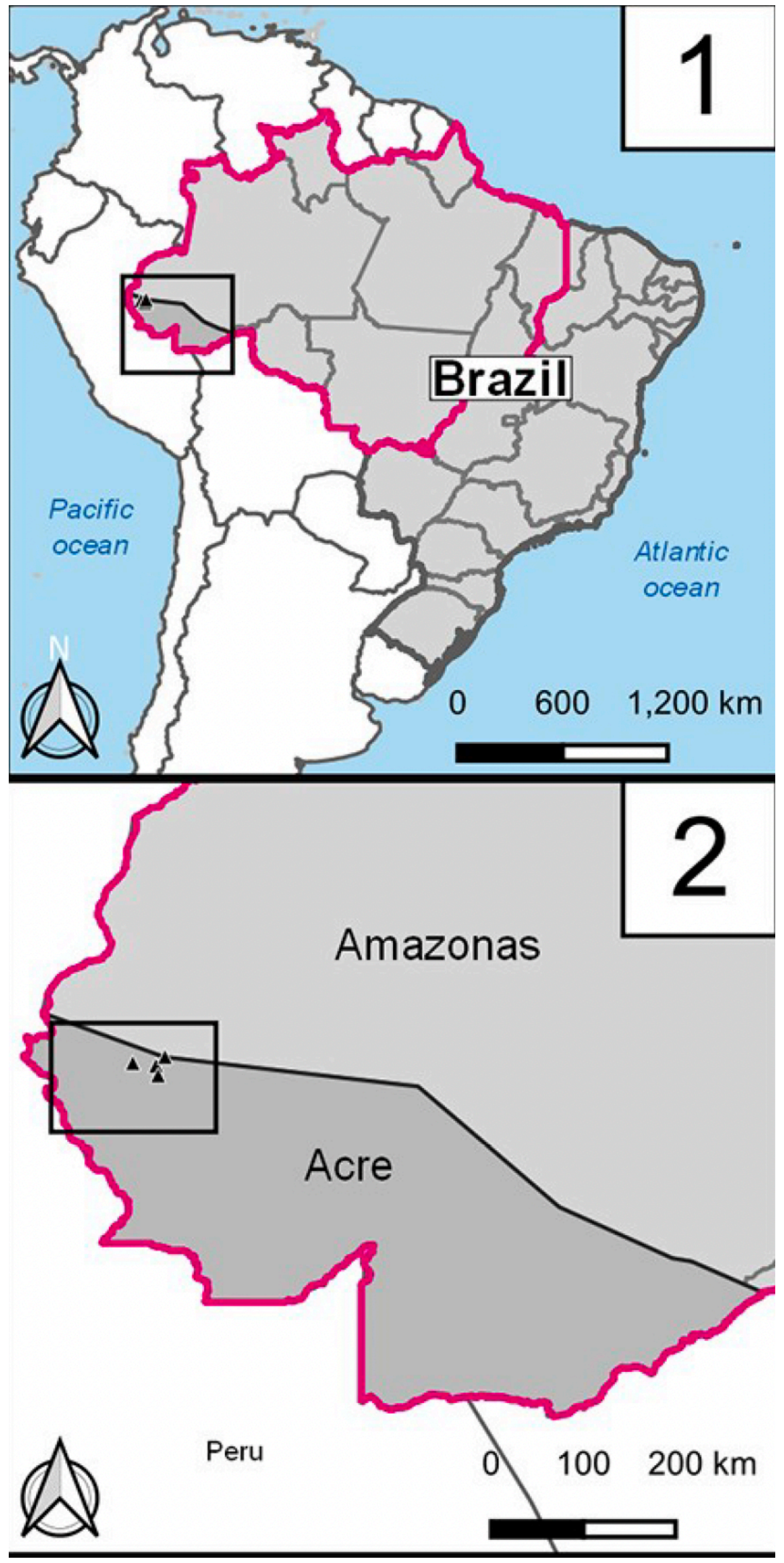
Malaria Project: Leveraging Causal AI in Malaria Research



Source: https://doi.org/10.1371/journal.pone.0242357.g001

Goal: To apply **causal discovery and inference tools** to identify and understand the factors influencing individual malaria risk in Brazil's Amazon.

The Mâncio Lima Cohort: An **observational longitudinal** study integrating genetic and epidemiological data.

- Municipality of Mâncio Lima, Acre, the highest annual parasite incidence in Brazil, with an estimated average of 422.8 cases per 1,000 people each year.
- Variables, including demographic and socioeconomic factors, geographic location (zone type), housing quality, lifestyle, occupation and travel histories.
  - **Longitudinal:** 7 cross-sectional surveys, from April 2018 to October 2021
  - **Household:** Continuously tracked 2,690 participants (594 households ~ 20% of households).
  - **Genetic:** High-throughput SNP genotyping for 1608 individuals (808 pseudo-independent observations).

Johansen, I. C., Rodrigues, P. T., and Ferreira, M. U. (2020). Human mobility and urban malaria risk in the main transmission hotspot of Amazonian Brazil. *PLOS ONE* 15 (11), e0242357. doi:10.1371/journal.pone.0242357

Malaria Project: Causal AI for Malaria Research

frontiers | Frontiers in Genetics

From bites to bytes: understanding how and why individual malaria risk varies using artificial intelligence and causal inference

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OPINION article
Front. Genet., 16 May 2025, Sec. Computational Genomics
Volume 16 - 2025 | https://doi.org/10.3389/fgene.2025.1599826

Key challenges:

Biological Complexity

- Interactions among humans, mosquitoes, and parasites
- Influences from environmental, genetic, and lifestyle factors

Analytical Complexity

- Heterogeneous data types and multiple sources of variation
- Complex dependencies (e.g., genetic, household, longitudinal)
- Incomplete and missing data

How can we infer causal effects in such complex, real-world scenarios?

Outline

Day 1:

Causal modeling: Structural Causal Models and Causal Bayesian Networks / d-Separation

Effect identification given a **causal diagram:** Backdoor Adjustment & ID Algorithm

Coding Exercises

Day 2:

Causal discovery: from **heterogenous** data types; **independent** and **dependent** observations

Effect identification given a **partial ancestral graph (PAG)**: Generalized Adjustment & PAG-ID

Coding Exercises

Causal Inference

In many areas of science we are often interested in questions of the form:

What is the effect of a specific intervention on a particular outcome?

For example, we might ask:

Does a medical treatment reduce the risk of developing a disease among individuals with specific risk factors or pre-existing conditions?

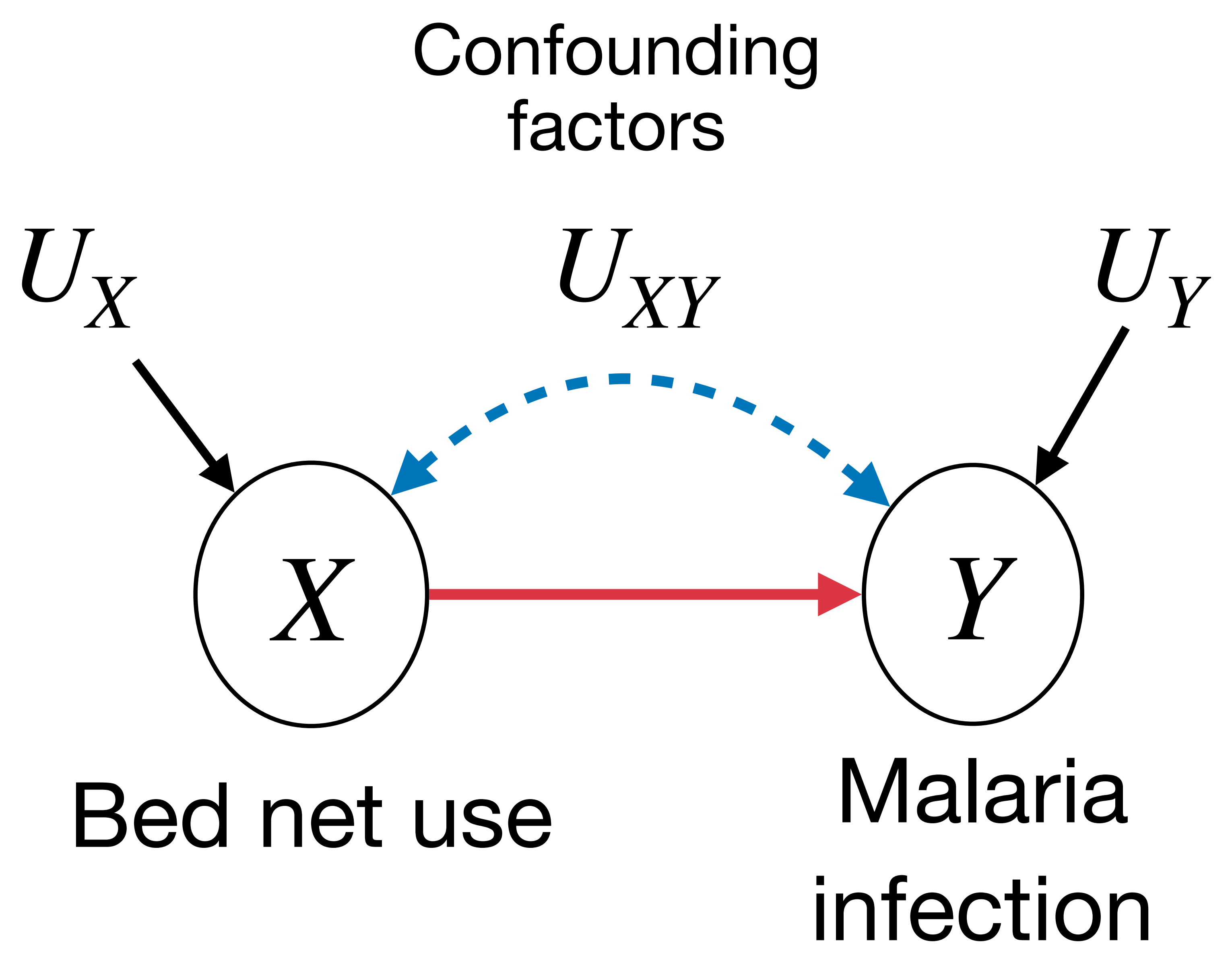
Is a malaria control intervention, such as the distribution of bed net, effective in reducing the incidence of malaria among high-risk populations?

Does poor housing quality increase the susceptibility to malaria for households in peri-urban areas?

Causal Inference

Insight: We observe that many individuals who use bed nets have lower rates of malaria infection.

Research question: Does bed net use causes a reduction in malaria, or are these individuals simply less likely to get malaria for other reasons?



Why these individuals use bed net?

$\{U_X, U_{XY}\}$:

- Availability of free or subsidized nets
- Health-seeking behavior
- Socioeconomic status (access to bed nets)

Which factors influence malaria incidence?

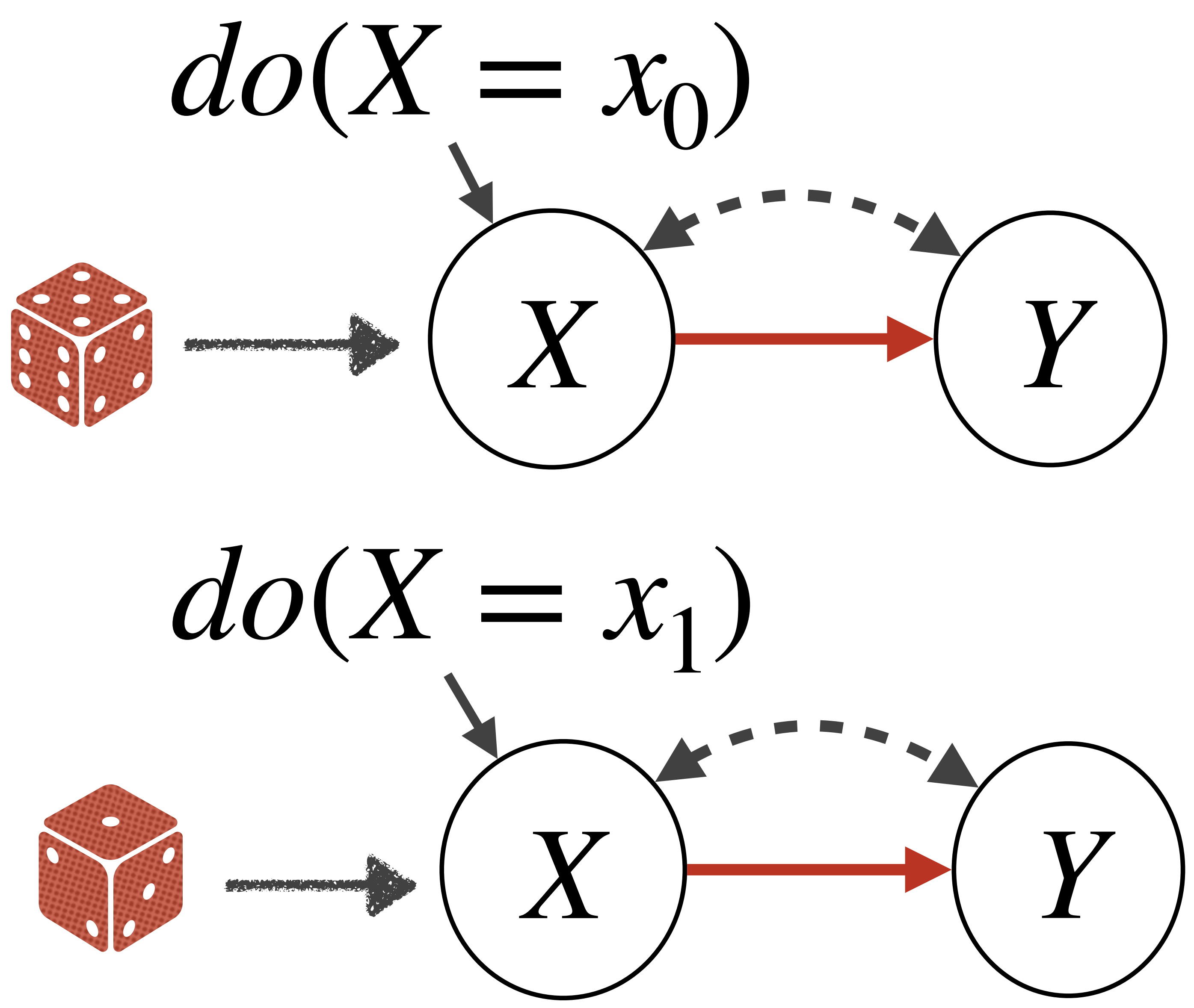
$\{U_Y, U_{XY}\}$:

- Biological / Ancestry susceptibility
- Previous exposure history
- Health-seeking behavior
- Socioeconomic status (housing quality)

Randomized Controlled Trials

A well accepted way to access the effect of an intervention is through a *perfectly realized* Randomized Controlled Trials (e.g. RCT):

Randomization of the X's assignment



$\mathbb{E}[Y \mid do(X = x_0)]$

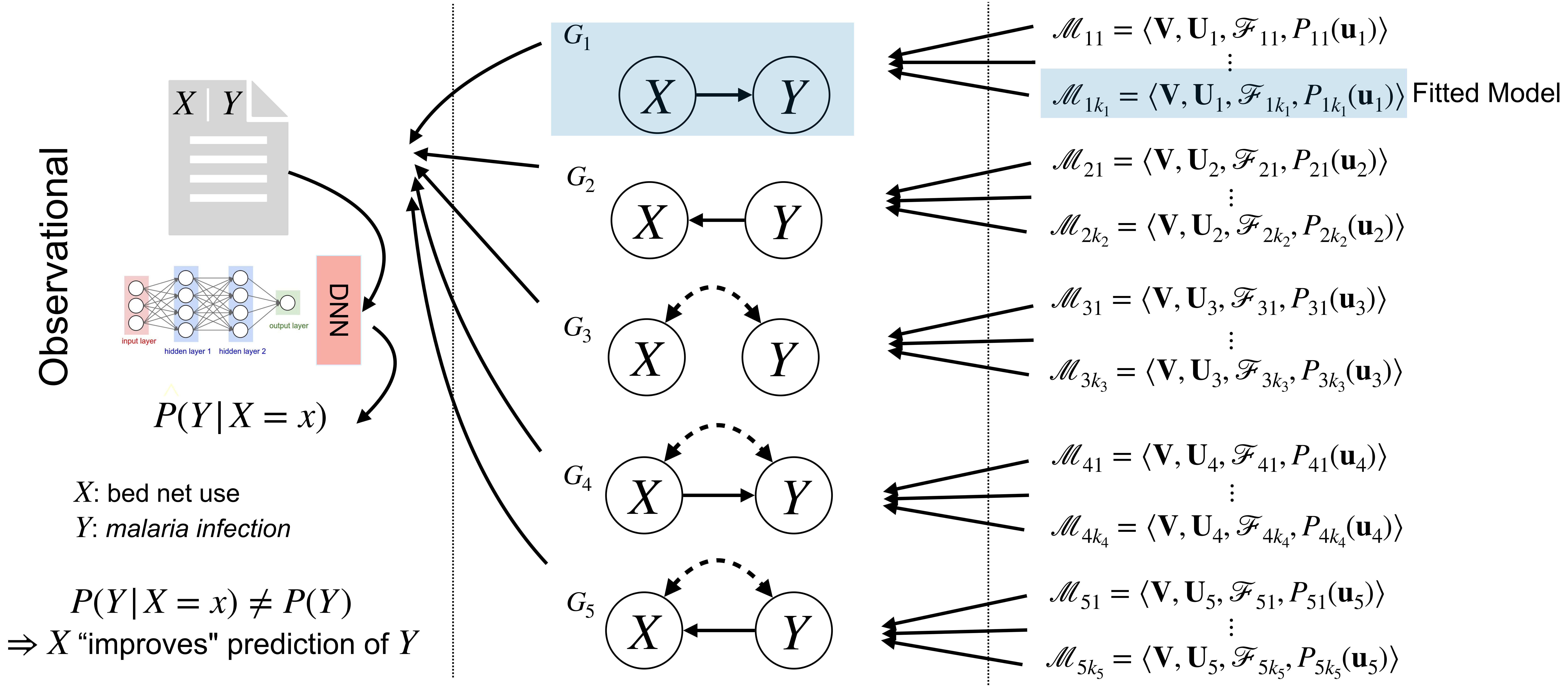
$\mathbb{E}[Y \mid do(X = x_1)]$

Average Causal Effect: $\mathbb{E}[Y \mid do(X = x_0)] - \mathbb{E}[Y \mid do(X = x_1)]$

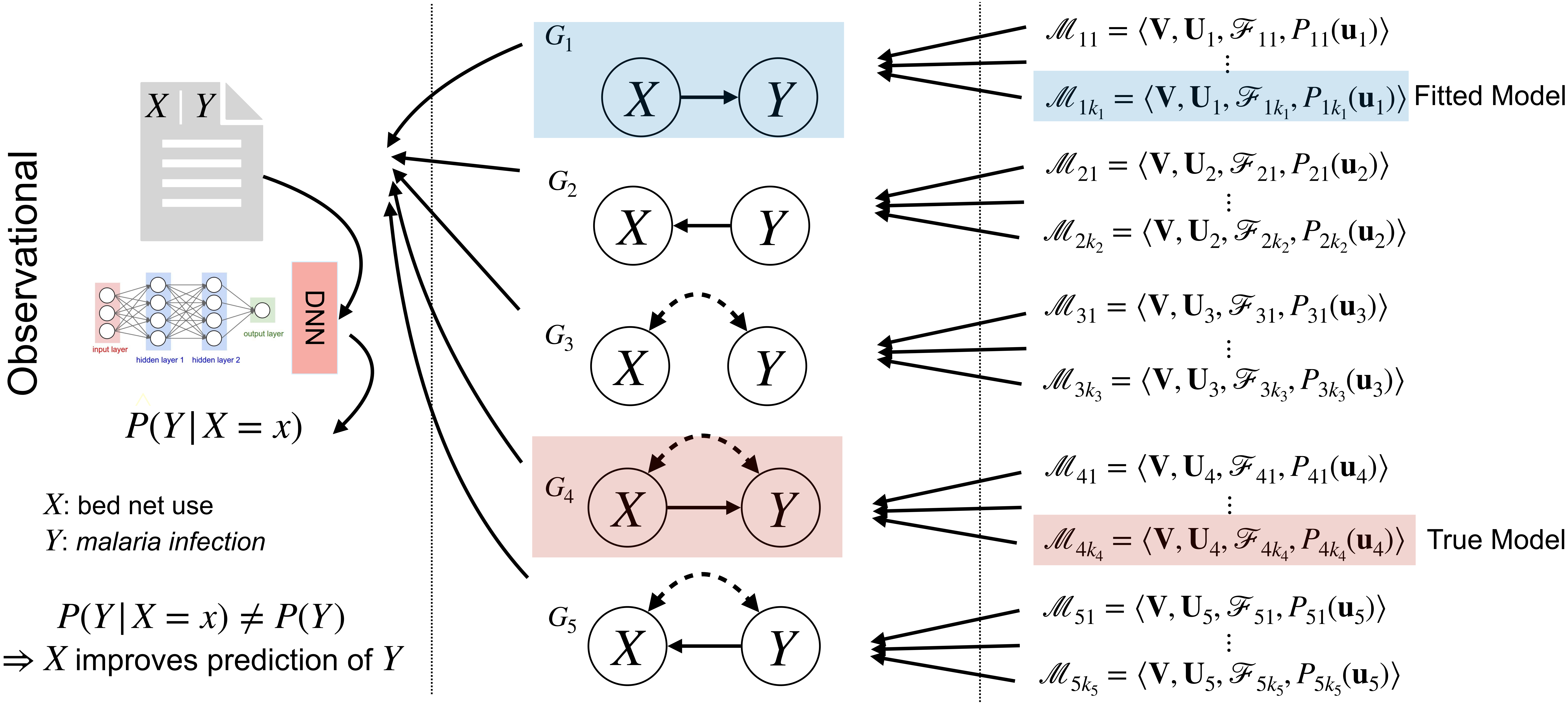
Can we still determine causality when RCTs are not an option?

Can large observational datasets and powerful prediction models (e.g., deep neural networks and transformers) **replace randomization** in identifying causal effects?

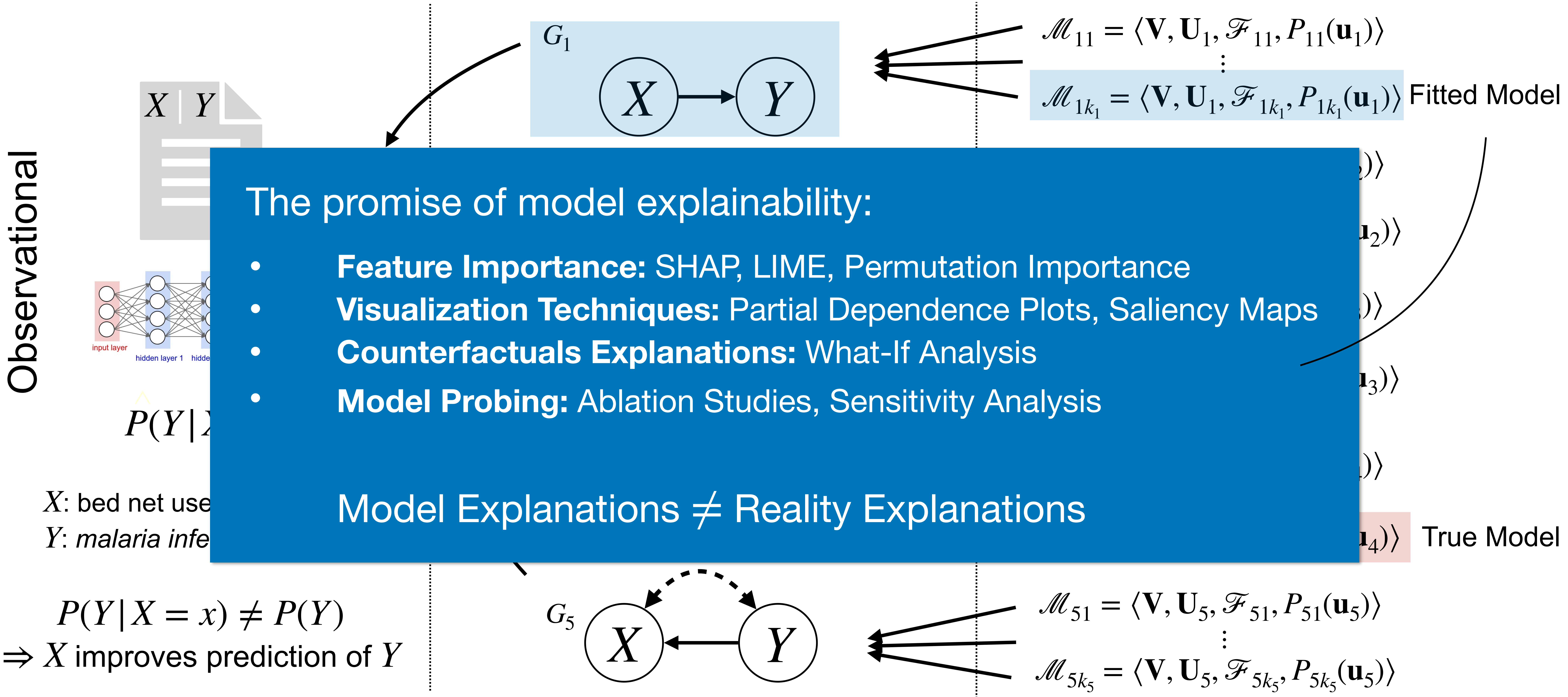
Why Doesn't Model Explainability Imply Causality?



Why Doesn't Model Explainability Imply Causality?



Why Doesn't Model Explainability Imply Causality?



The promise of model explainability:

- **Feature Importance:** SHAP, LIME, Permutation Importance
- **Visualization Techniques:** Partial Dependence Plots, Saliency Maps
- **Counterfactuals Explanations:** What-If Analysis
- **Model Probing:** Ablation Studies, Sensitivity Analysis

Model Explanations $\neq$ Reality Explanations

How do we move from prediction to true understanding of reality without randomized experiments?

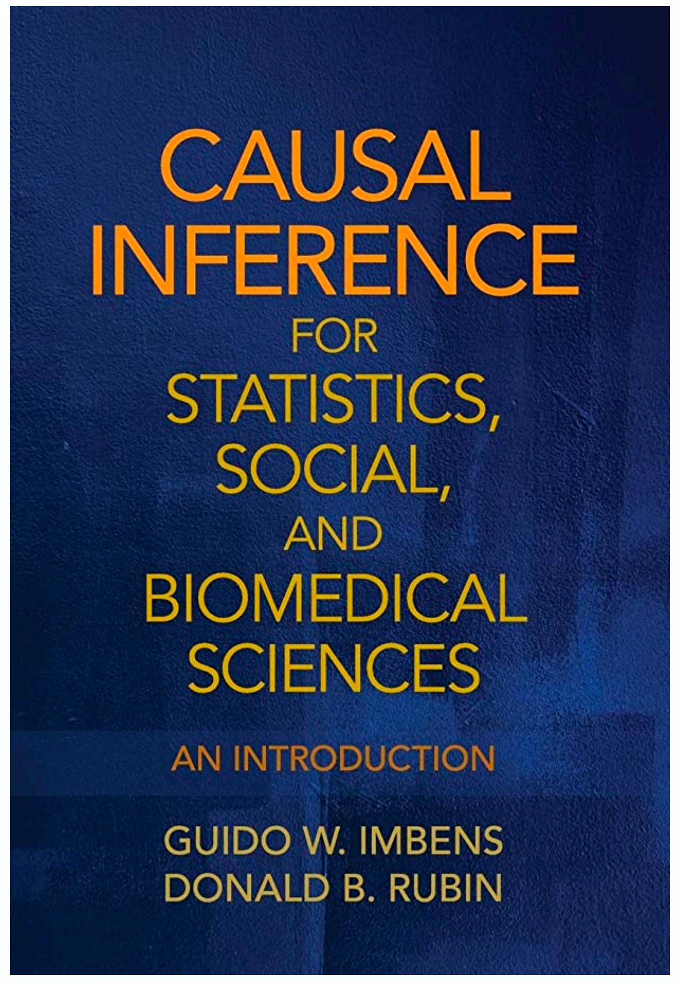
Donald B. Rubin, Guido W. Imbens & Joshua D. Angrist

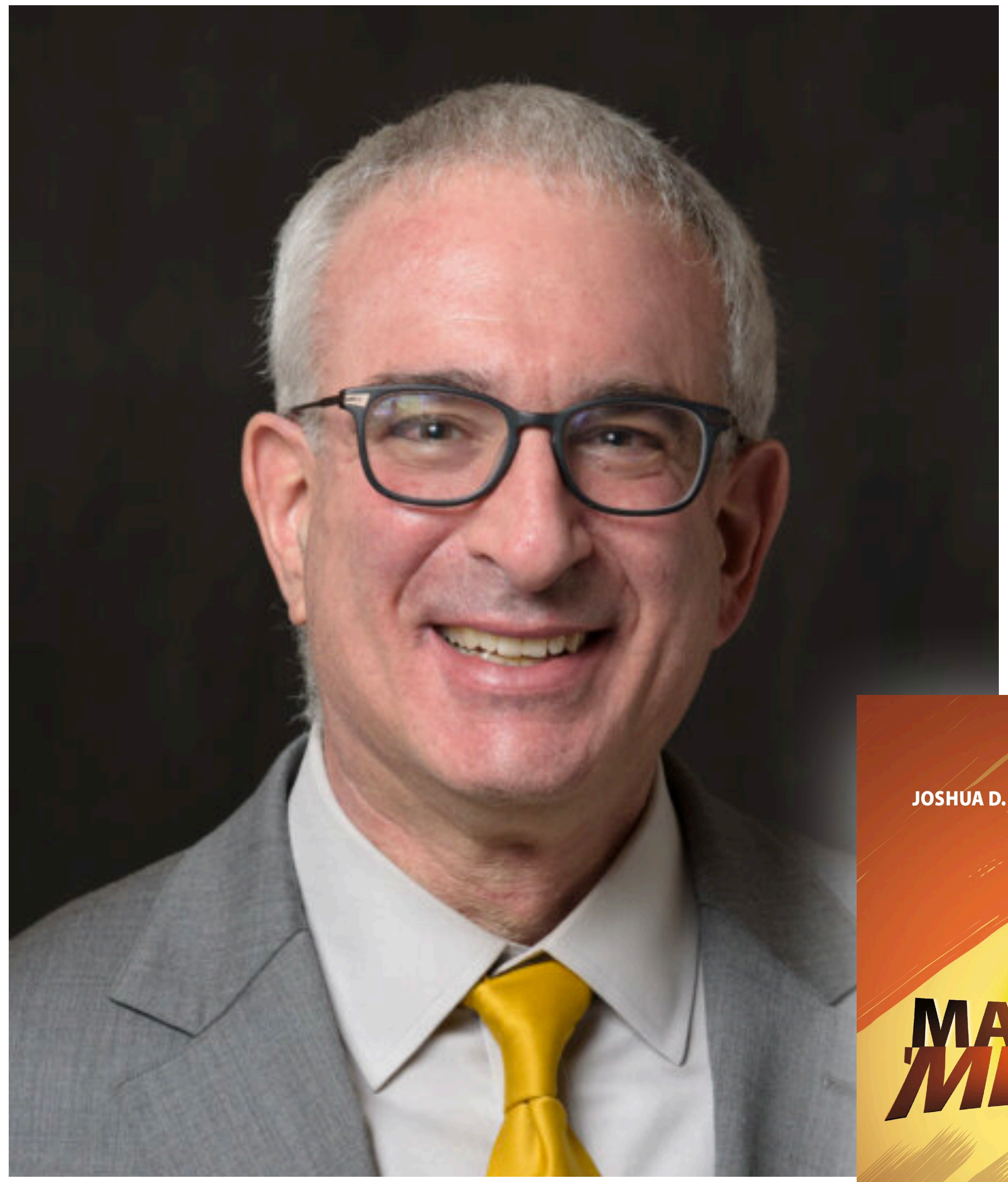
Donald B. Rubin

Professor of Statistics at Harvard University

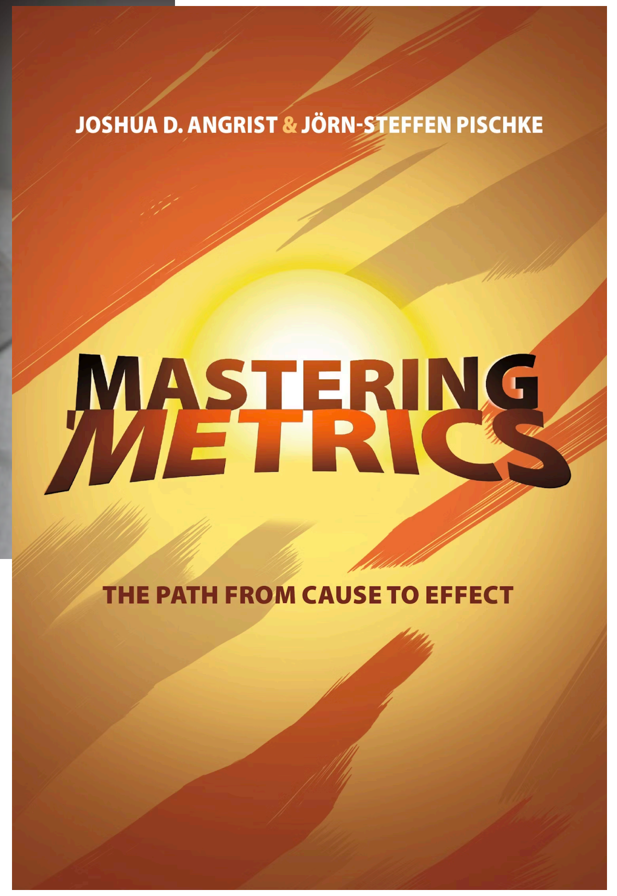
Guido W. Imbens

Professor of Applied Econometrics at Stanford University





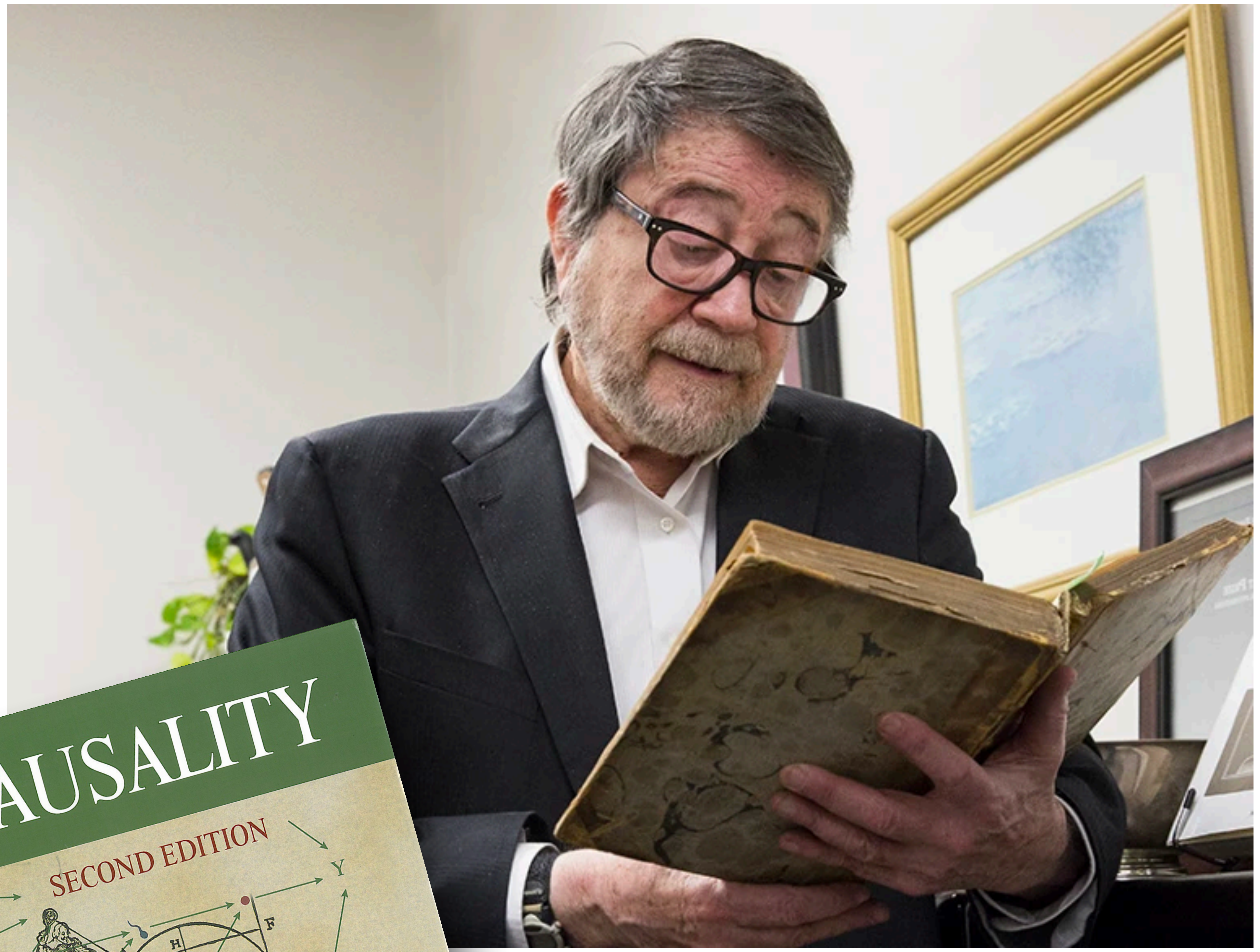
Joshua D. Angrist

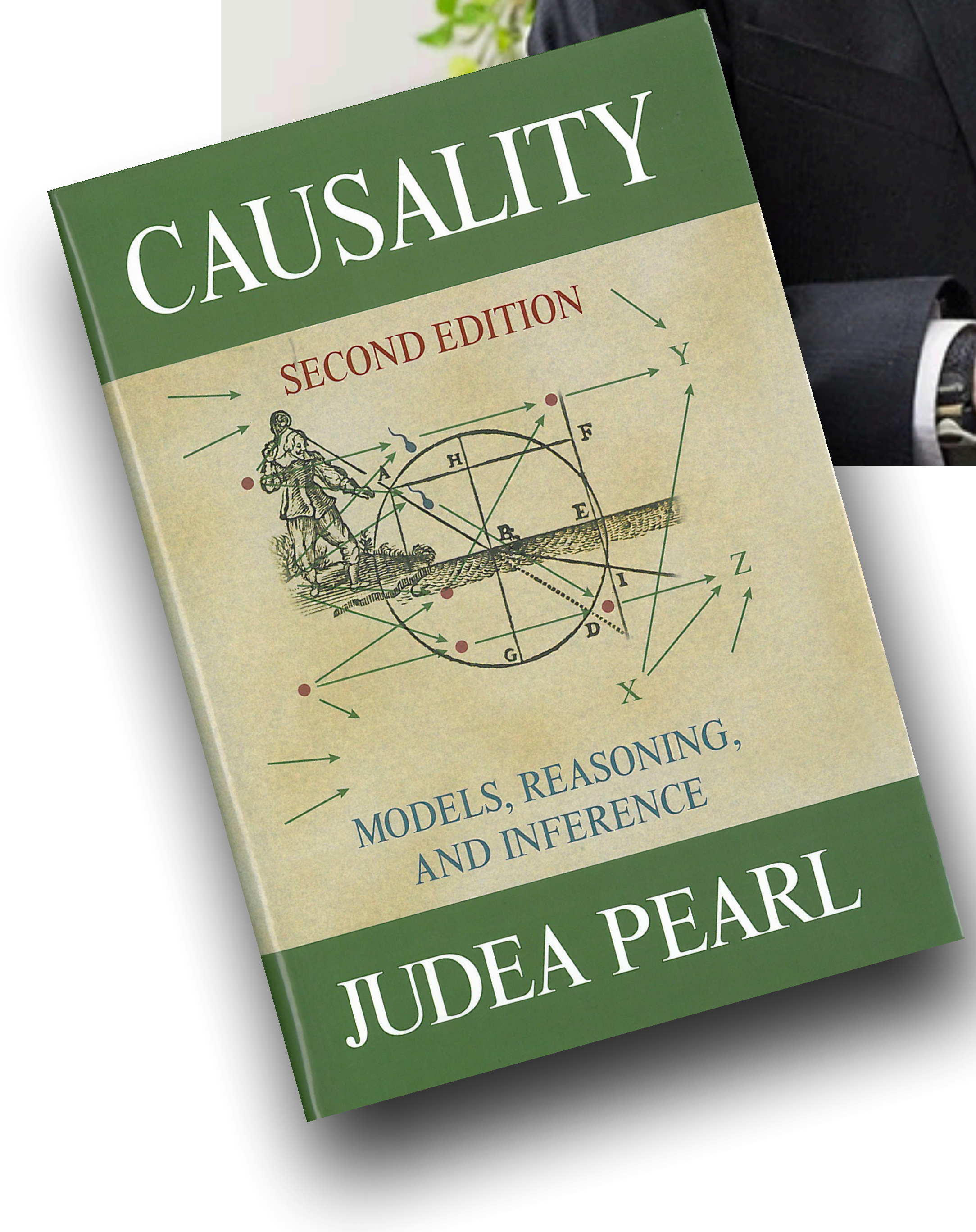


Professor of Economics at MIT

In 2021, Angrist & Imbens won the Nobel Prize in Economics "for their methodological contributions to the analysis of causal relationships"

Judea Pearl — Causal Artificial Intelligence





Director of the Cognitive Systems Laboratory at the University of California, Los Angeles.

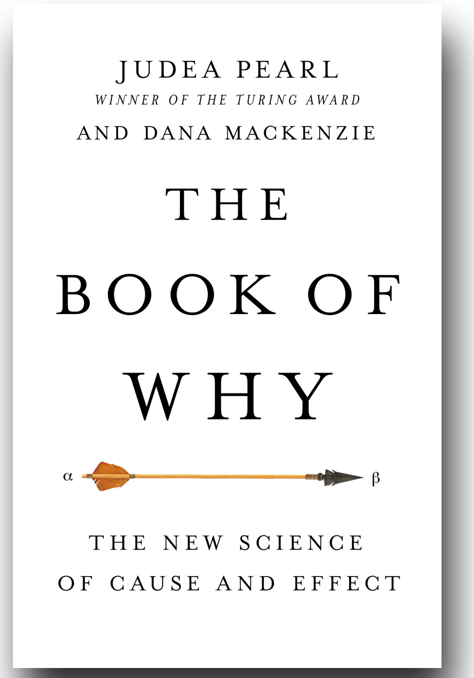
In 2011, he won the A. M. Turing Award (the highest distinction in computer science and a $250,000 prize)

"for fundamental contributions to artificial intelligence through the development of a calculus for probabilistic and causal reasoning."

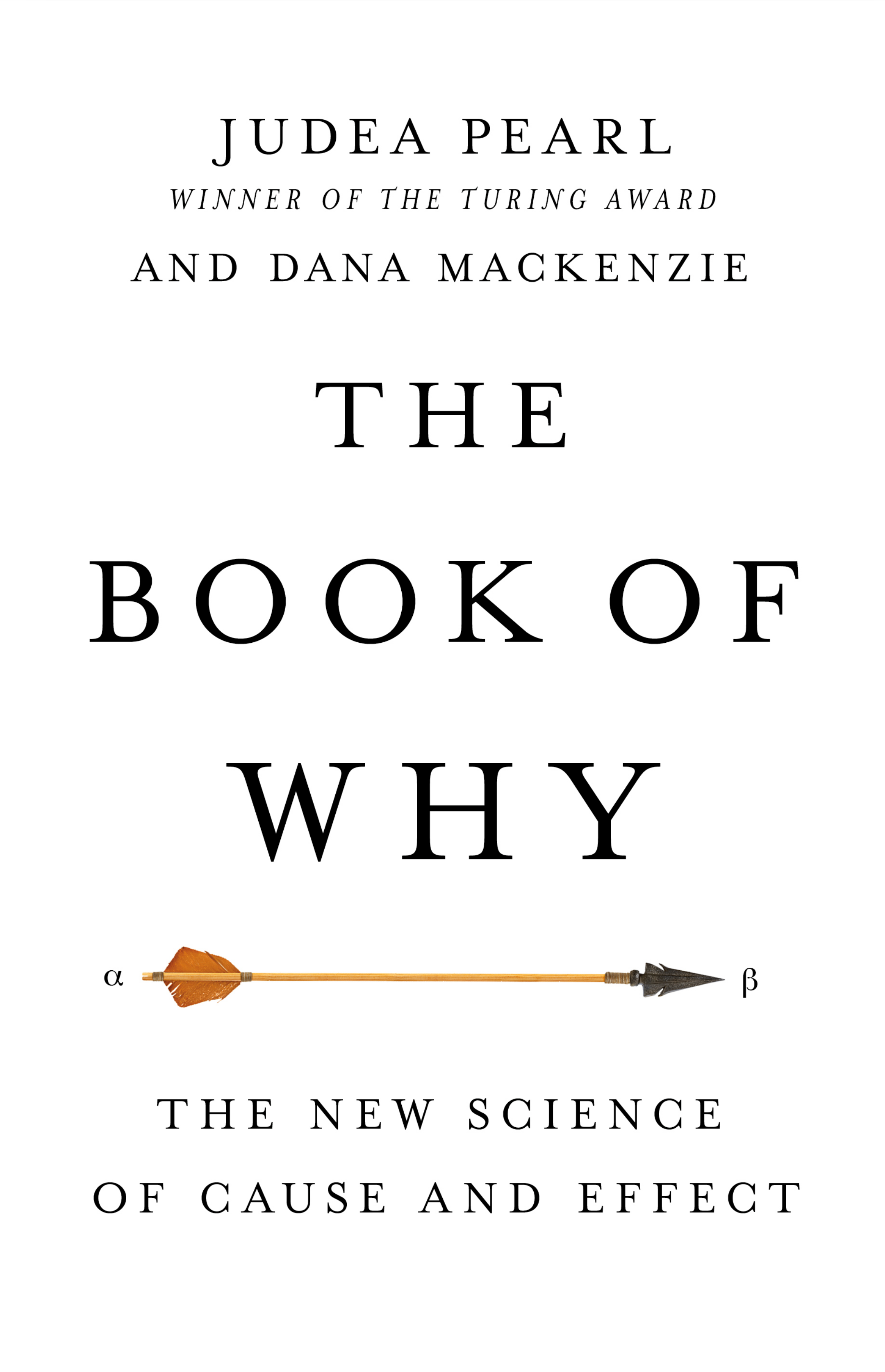
— Association for Computing Machinery (ACM)

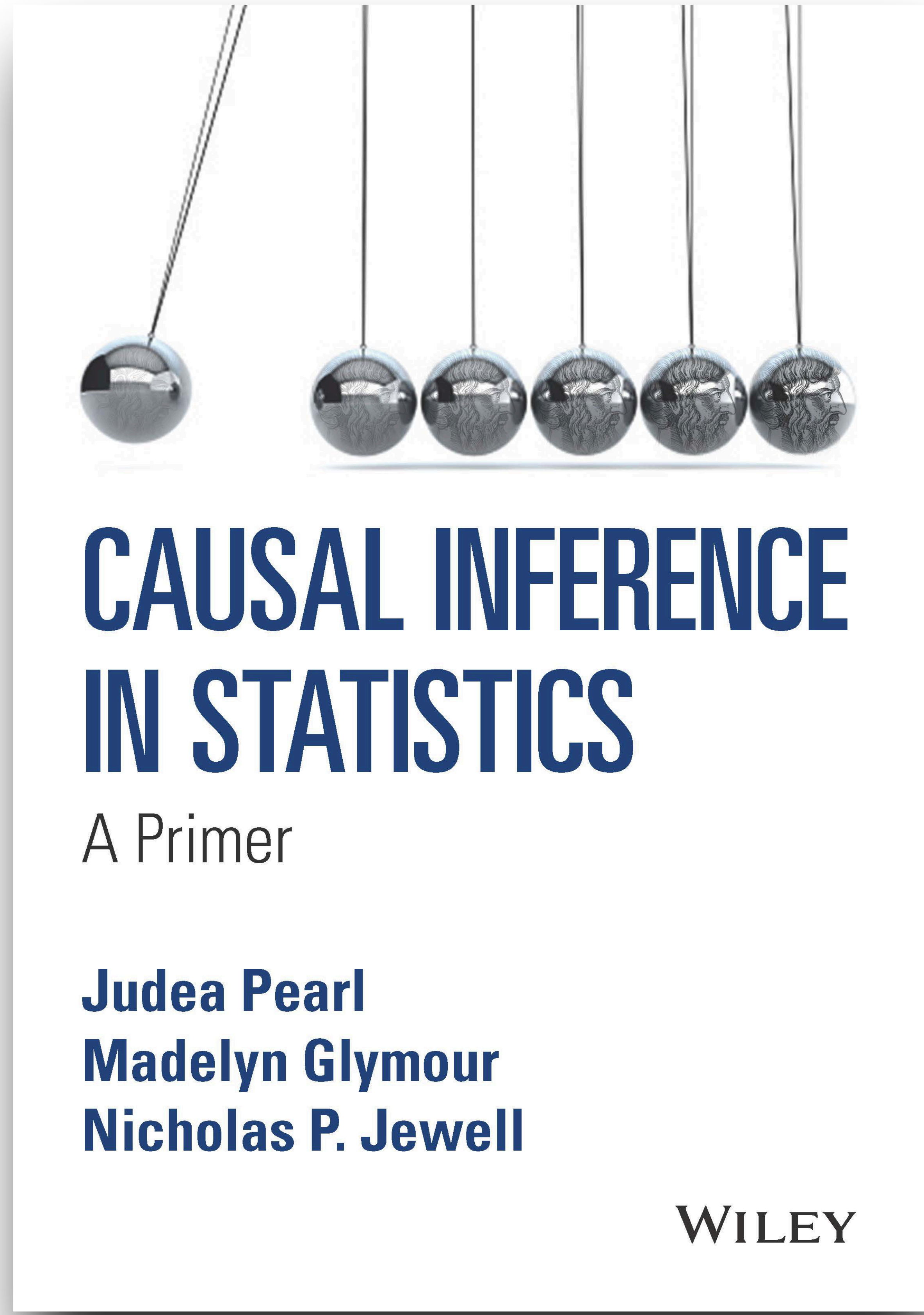
"Deep learning has instead given us machines with truly impressive abilities but no intelligence. The difference is profound and lies in the absence of a model of reality."

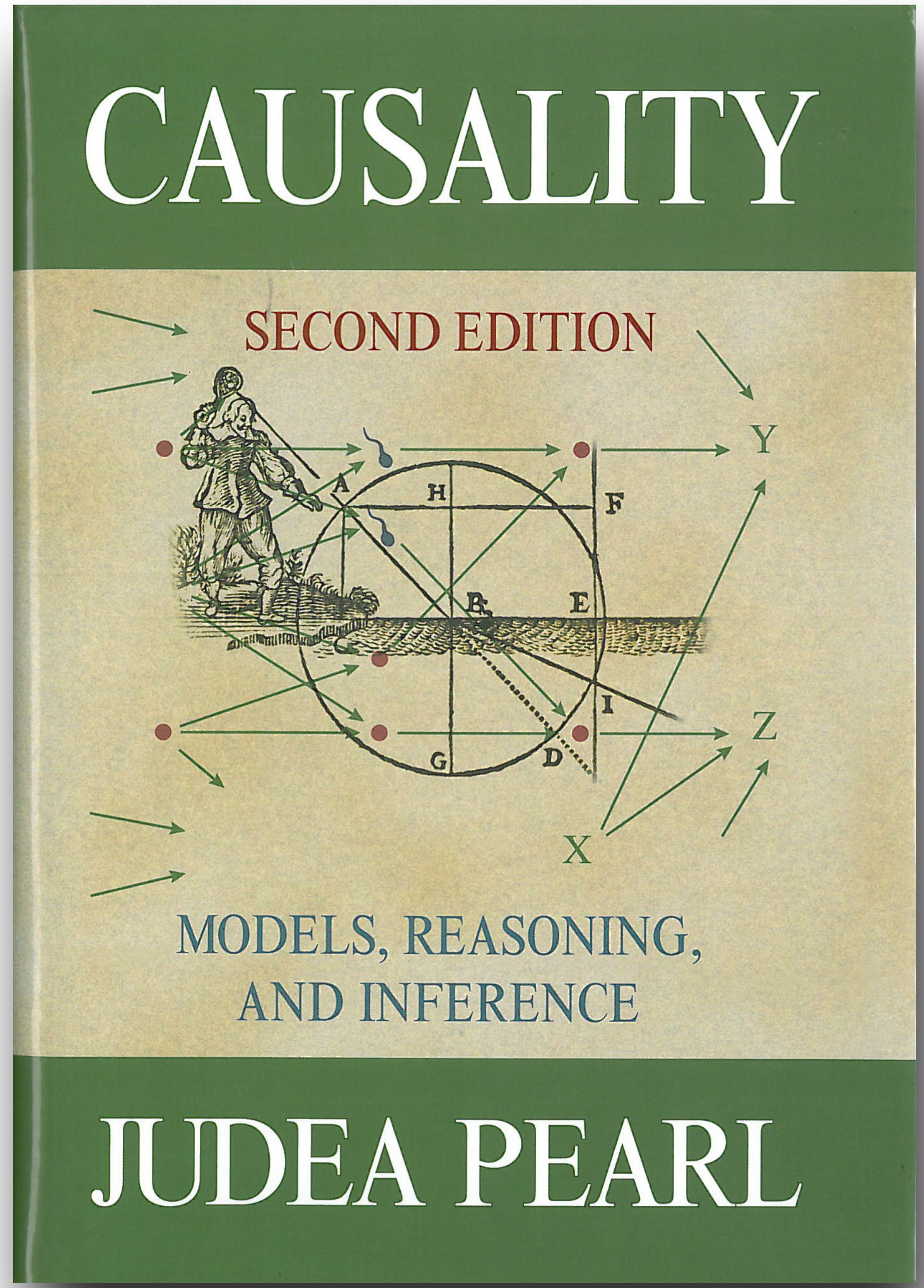
— The Book of Why: The New Science of Cause and Effect



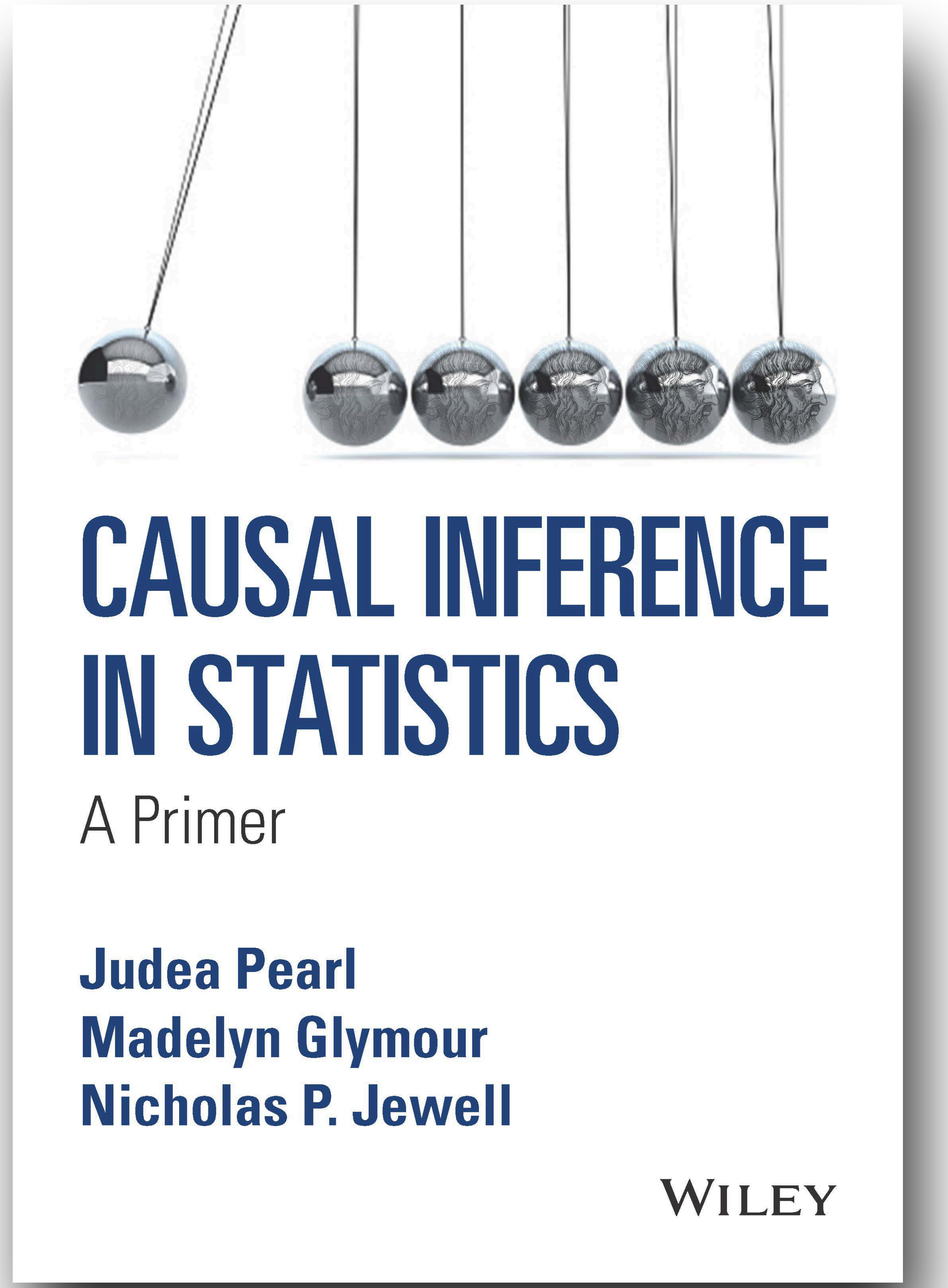
Causality Theory by Judea Pearl



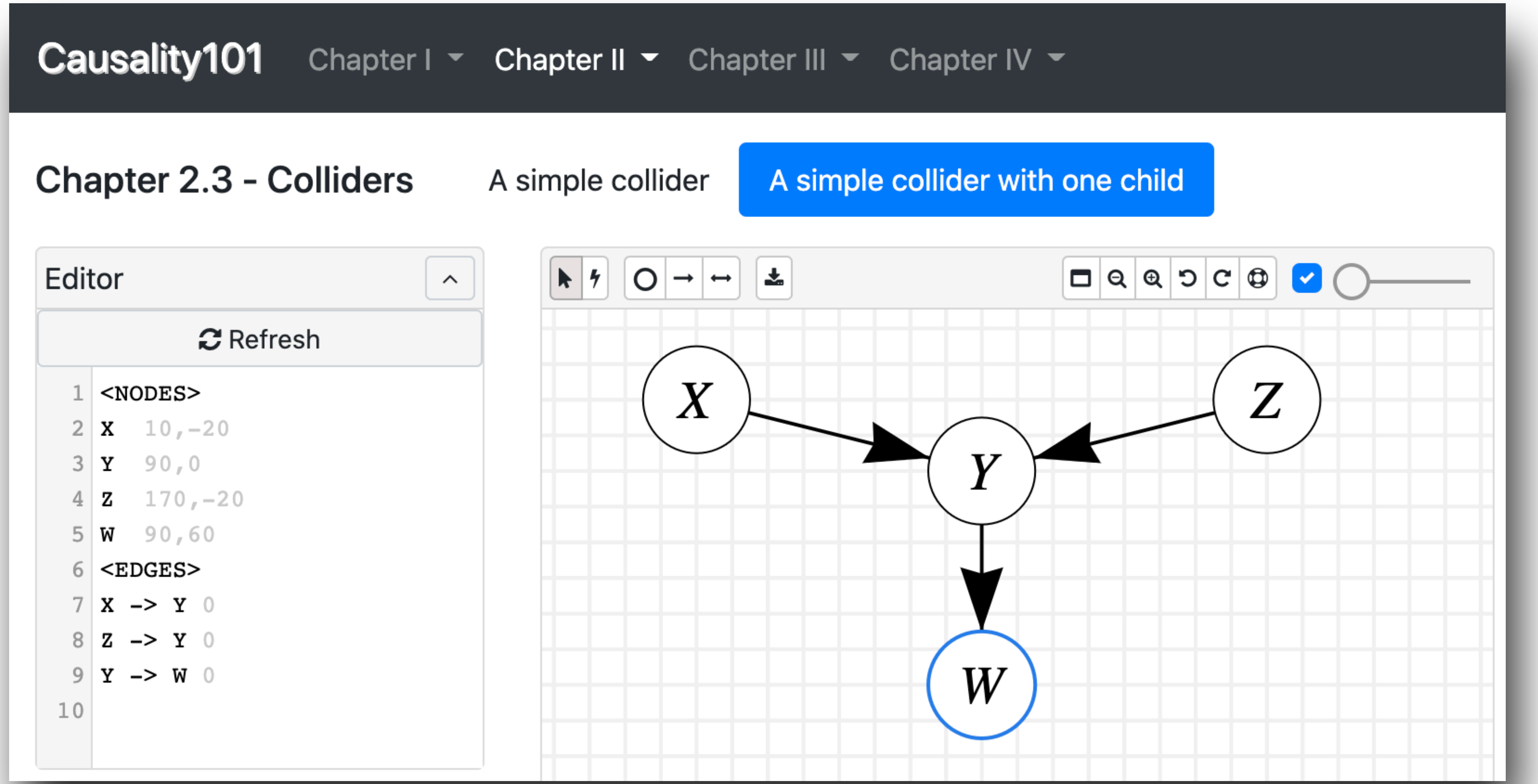




Causality Theory by Judea Pearl

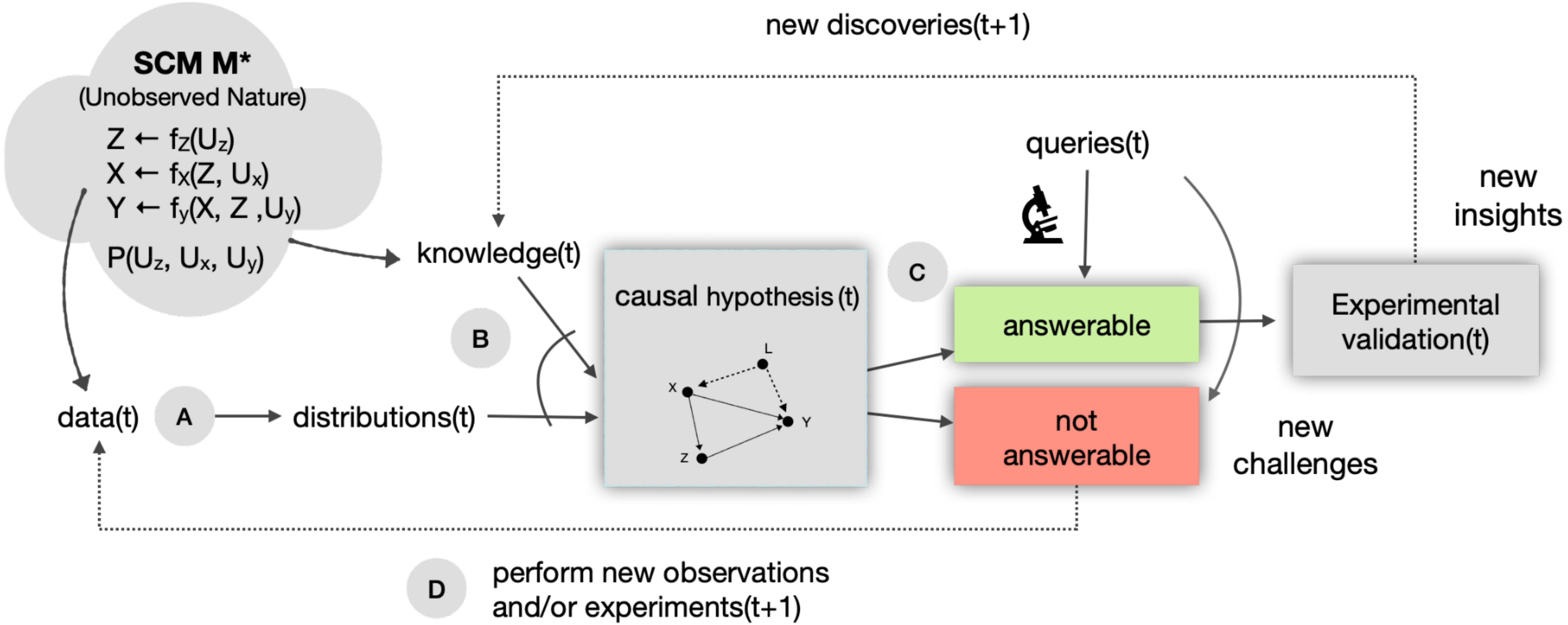


https://causality101.net/



Causal Inference Workflow

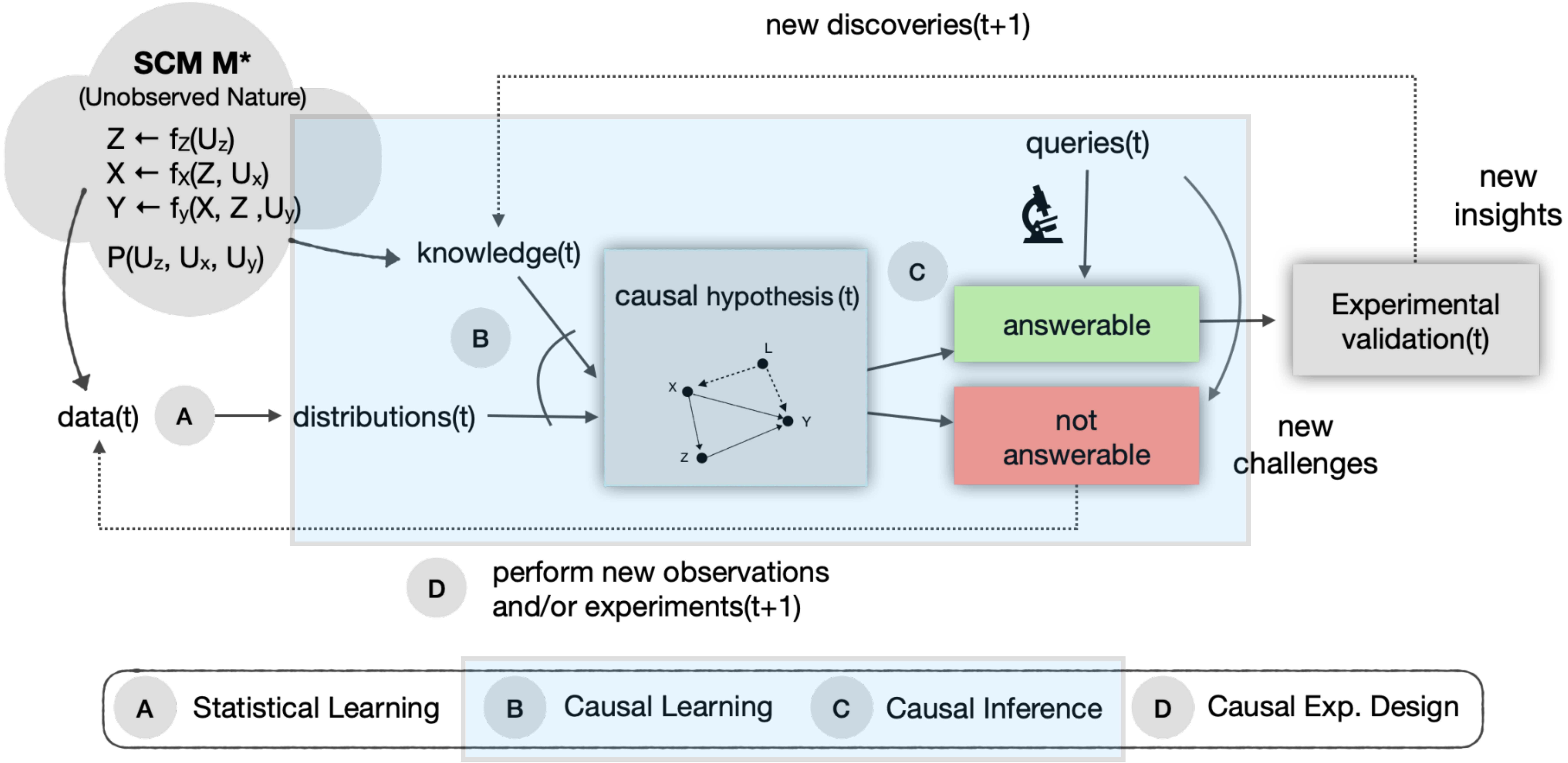
Continuous Process of Scientific Discovery and Causal Hypothesis Refinement



A Statistical Learning · B Causal Learning · C Causal Inference · D Causal Exp. Design

Causal Inference Workflow

Continuous Process of Scientific Discovery and Causal Hypothesis Refinement



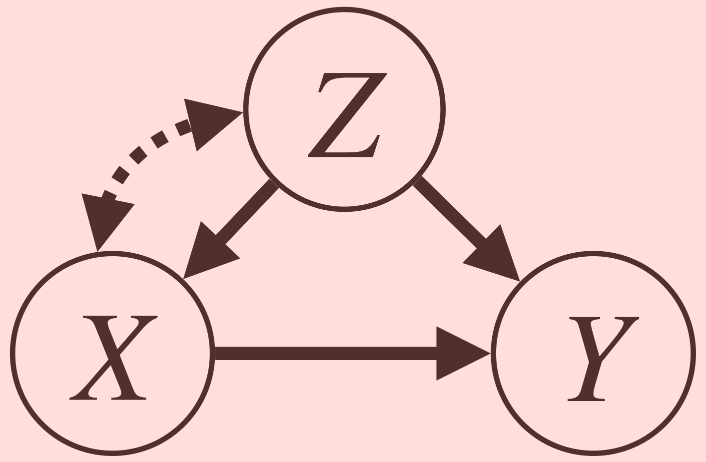
Classical Causal Pipeline - Judea Pearl's framework

Requires structural causal knowledge for **every pair** of variables!

1 **Query**

$P(y \mid do(x))$

2 **Causal Contraints**



3 $P(x, m, y)$

Observational Distribution

Inference Engine

Solution

yes / no

$$P(y \mid do(x)) = \sum_z P(y \mid x, z) P(z)$$

$$\mathbb{E}(y \mid do(x)) = \sum_z \mathbb{E}(y \mid x, z) P(z) \longrightarrow \hat{\mathbb{E}}(y \mid do(x)) = \sum_z \hat{\mathbb{E}}((y \mid x, z) \hat{P}(z)$$

Observational Data

Interventional Distribution ⟵ Available Distributions

👍

Causal Modeling → Causal Effect Identification → Causal Effect Estimation

Pearl, J. 1995. Causal diagrams for empirical research. Biometrika, 82(4): 669–688.

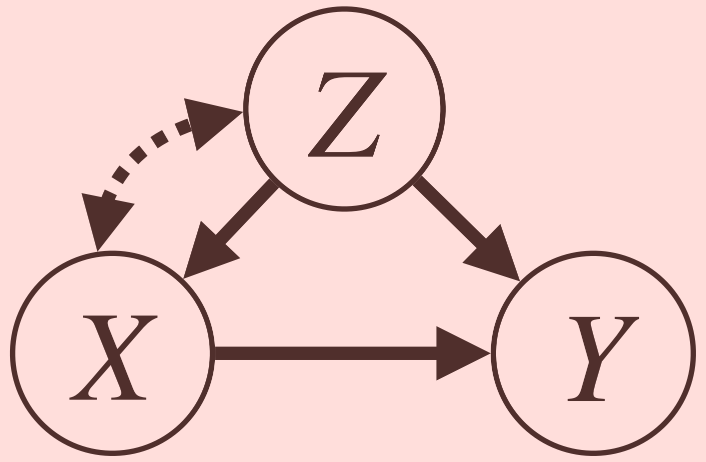
Classical Causal Pipeline - Judea Pearl's framework

Usually hard to be fully specified.
Can we relax some constraints?

1 **Query**

$P(y \mid do(x))$

2 **Causal Contraints**



3 $P(x, m, y)$

Observational Distribution

Inference Engine

Solution

yes / no

$$P(y \mid do(x)) = \sum_z P(y \mid x, z) P(z)$$

$$\mathbb{E}(y \mid do(x)) = \sum_z \mathbb{E}(y \mid x, z) P(z)$$

Interventional Distribution ← Available Distributions

Observational Data

$$\rightarrow \hat{\mathbb{E}}(y \mid do(x)) = \sum_z \hat{\mathbb{E}}((y \mid x, z) \hat{P}(z)$$

Causal Modeling → Causal Effect Identification → Causal Effect Estimation

Pearl, J. 1995. Causal diagrams for empirical research. Biometrika, 82(4): 669–688.

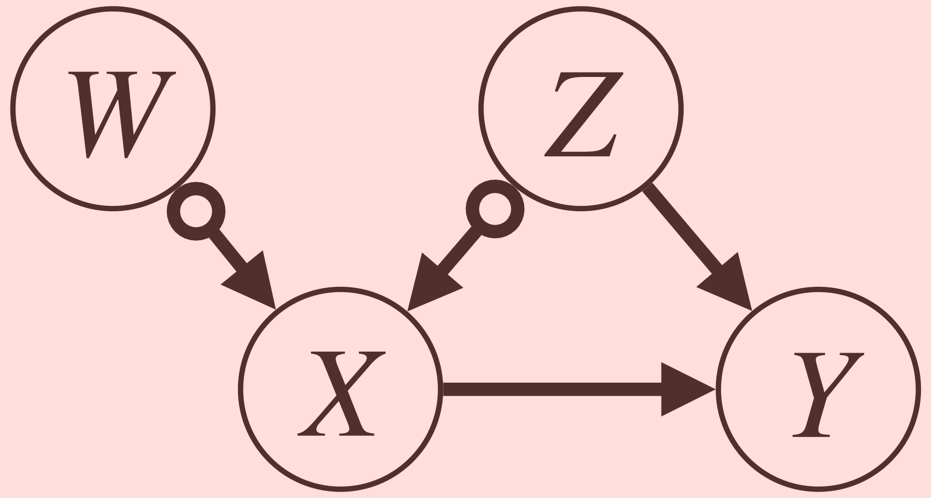
Classical Causal Pipeline - Judea Pearl's framework

Can be constructed in a fully data-driven way!
(e.g. using FCI algorithm)

1 **Query**

$$P(y \mid do(x))$$

2 **PAG**



3 $$P(x, m, y)$$

Observational Distribution

Causal Discovery

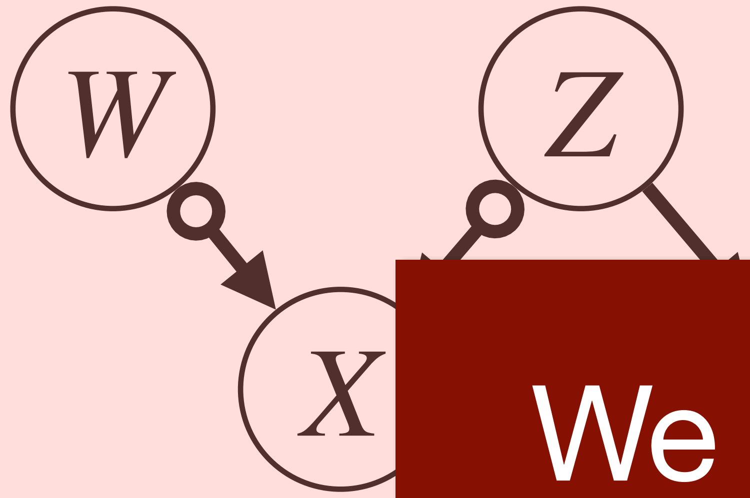
Zhang, J. (2008). On the completeness of orientation rules for causal discovery in the presence of latent confounders and selection bias. *Artificial Intelligence*, 172(16):1873–1896. (Link)

Classical Causal Pipeline - Judea Pearl's framework

1 **Query**

$P(y \mid do(x))$

2 **PAG**



3 $P(x, m, y)$

Observational Distribution

Inference Engine

Generalized Adjustment Criterion, IDP, CIPD Algorithms

PAGId R package: https://github.com/adele/PAGId

Solution

yes / no

$P(y \mid do(x)) = \sum P(y \mid x, z)P(z)$

Observational

$= \sum_{z} \hat{\mathbb{E}}((y \mid x, z)\hat{P}(z)$

Interventional Distribution ← Available Distributions

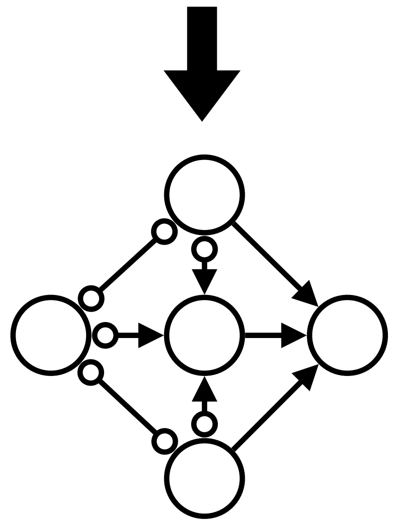
👍

We enable a complete, fully data-driven pipeline for causal inference, even with latent confounding!

Causal Discovery → Causal Effect Identification → Causal Effect Estimation

Jaber, A.*, **Ribeiro, A. H.***, Zhang, J., Bareinboim, E. (2022). Causal Identification under Markov equivalence: Calculus, Algorithm, and Completeness. *In Adv. Neural Inf Process Systems, 35, 3679-3690. (NeurIPS-22).* (Link)

Causality: A Key to Overcoming AI's Greatest Challenges

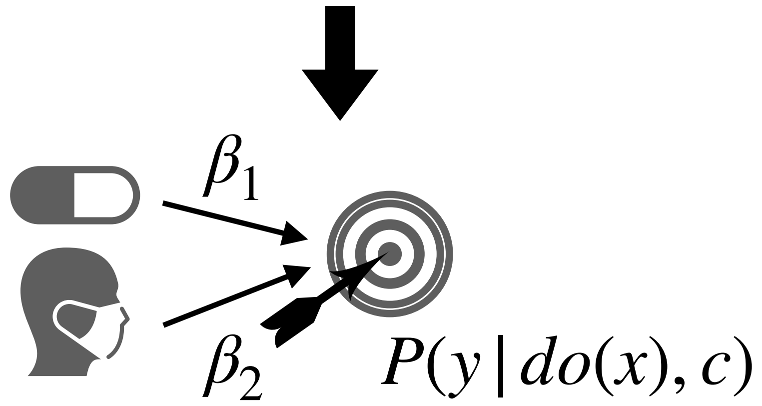


Data Fusion: Provides language and inferential machinery to cohesively combine prior knowledge and data from multiple and heterogeneous studies.

- **Causal Modeling, Causal Representation Learning and Causal Abstraction**

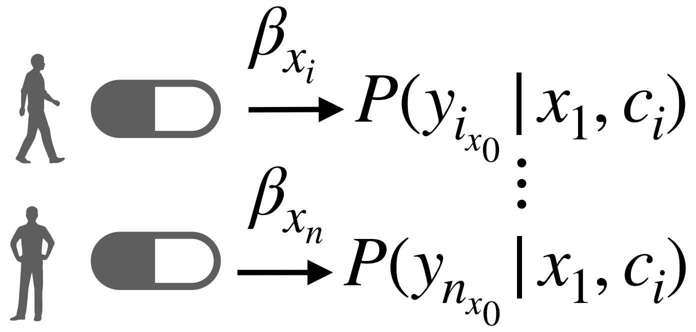
Explainability: Provides a better understanding of the true underlying mechanisms

- **Causal Discovery**

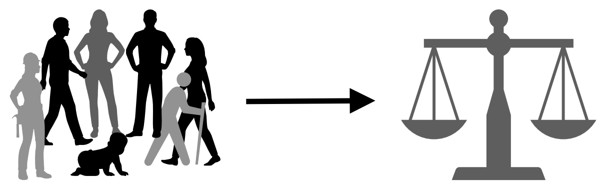


Optimal Decision Making: Can determine the *unbiased* effect of *unrealized* interventions, distinguishing between association and causation, rather than just predicting outcomes.

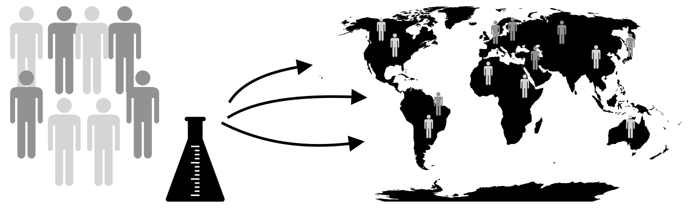
- **Causal Effect Identification and Estimation**



Personalized Inferences: Enables ***counterfactual reasoning*** by considering alternate scenarios and individual variability.



Fairness: Identifies and disentangles any mechanisms of discrimination, whether direct or indirect (potentially mediated or confounded).



Generalizability: Enables effect *transportability* across different populations.

Structural Causal Model (SCM)

The true model behind the data

Full explainability for all layers of the causal hierarchy

Structural Causal Model (SCM)

Definition: A structural causal model $\mathscr{M}$ (or, data generating model) is a tuple $\langle \mathbf{V}, \mathbf{U}, \mathscr{F}, P(\mathbf{u}) \rangle$, where

- $\mathbf{V} = \{V_1, \ldots, V_n\}$: are endogenous variables
- $\mathbf{U} = \{U_1, \ldots, U_m\}$: are exogenous variables
- $\mathscr{F} = \{f_1, \ldots, f_n\}$: are functions determining $\mathbf{V}$, i.e., $v_i \leftarrow f_i(pa_i, u_i)$, where
  - $Pa_i \subseteq \mathbf{V}$ are endogenous causes (parents) of V_i
  - $U_i \subseteq \mathbf{U}$ are exogenous causes of V_i.
- $P(\mathbf{U})$ is the probability distribution over $\mathbf{U}$.

Assumption: $\mathscr{M}$ is recursive, i.e., there are no feedback (cyclic) mechanisms.

Causal (Intervention) Effects from SCMs

Pre-Interventional/ Observational SCM

$$\mathcal{M} = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_{XY}, U_X, U_Y\} \\ \mathcal{F} = \begin{cases} X = f_X(U_X, U_{XY}) \\ Y = f_Y(X, U_Y, U_{XY}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

Observational Distribution

$$P(\mathbf{V}) \doteq P_{\mathcal{M}}(\mathbf{V})$$

Can we ***predict*** better the value of Y after ***observing*** that $X = x$?

$P(Y = y \mid X = x) \neq P(Y = y) \implies X$ is ***correlated*** to Y

$$do(X = x)$$

$$\neq$$

Post-Interventional / Interventional SCM

$$\mathcal{M}_x = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_{XY}, U_X, U_Y\} \\ \mathcal{F} = \begin{cases} X = x \\ Y = f_Y(x, U_Y, U_{XY}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

Interventional Distribution

$$P(\mathbf{V} \mid do(X = x)) \doteq P_{\mathcal{M}_x}(\mathbf{V})$$

Can we ***predict*** better the value of Y after ***making an intervention*** $do(X = x)$?

$\exists x$ s.t. $P_{\mathcal{M}_x}(Y = y) \neq P(Y = y) \implies X$ is ***a cause*** of Y

Structural Equation Model (SEM)

$$
\mathscr{M} = \begin{cases} \mathbf{V} = \{X, Y, Z\} \\ \mathbf{U} = \{\epsilon_X, \epsilon_Y, \epsilon_Z\} \\ \mathscr{F} = \begin{cases} Z = \beta_{Z0} + \epsilon_Z \\ X = \beta_{X0} + \beta_{XZ} Z + \epsilon_X \\ Y = \beta_{Y0} + \beta_{YZ} Z + \beta_{YX} X + \epsilon_Y) \end{cases} \\ \mathbf{U} \sim \mathscr{N}\left(\mathbf{0}, \Sigma = \begin{bmatrix} \sigma_X & 0 & 0 \\ 0 & \sigma_Y & 0 \\ 0 & 0 & \sigma_Z \end{bmatrix} \right) \end{cases}
$$

- **Linear functions**
- **Normal distribution**
- **Markovianity / Causal Sufficiency**: Error terms in $\mathbf{U}$ are independent of each other (diagonal covariance matrix).

Full specification of an SCM <u>requires</u> parametric and distributional assumptions.

Estimation of such models usually requires strong assumptions (e.g., Markovianity).

SCM: Encoder of Functional Knowledge

The knowledge required to fully specify an SCM is usually *unavailable* in practice.

Is it possible to identify the effect of interventions from *observational* data without fully specifying the SCM (i.e., in a non-parametric fashion)?

Yes, with structural knowledge encoded as a causal diagram!

Causal Bayesian Network

Directed
Acyclic Graph

Acyclic Directed
Mixed Graph

A DAG, or, more generally, an ADMG for latent confounders
that encodes both the **conditional independencies** of
a Bayesian network and the **causal structure** implied by an SCM.

CBN: Encoder of Structural Causal Knowledge

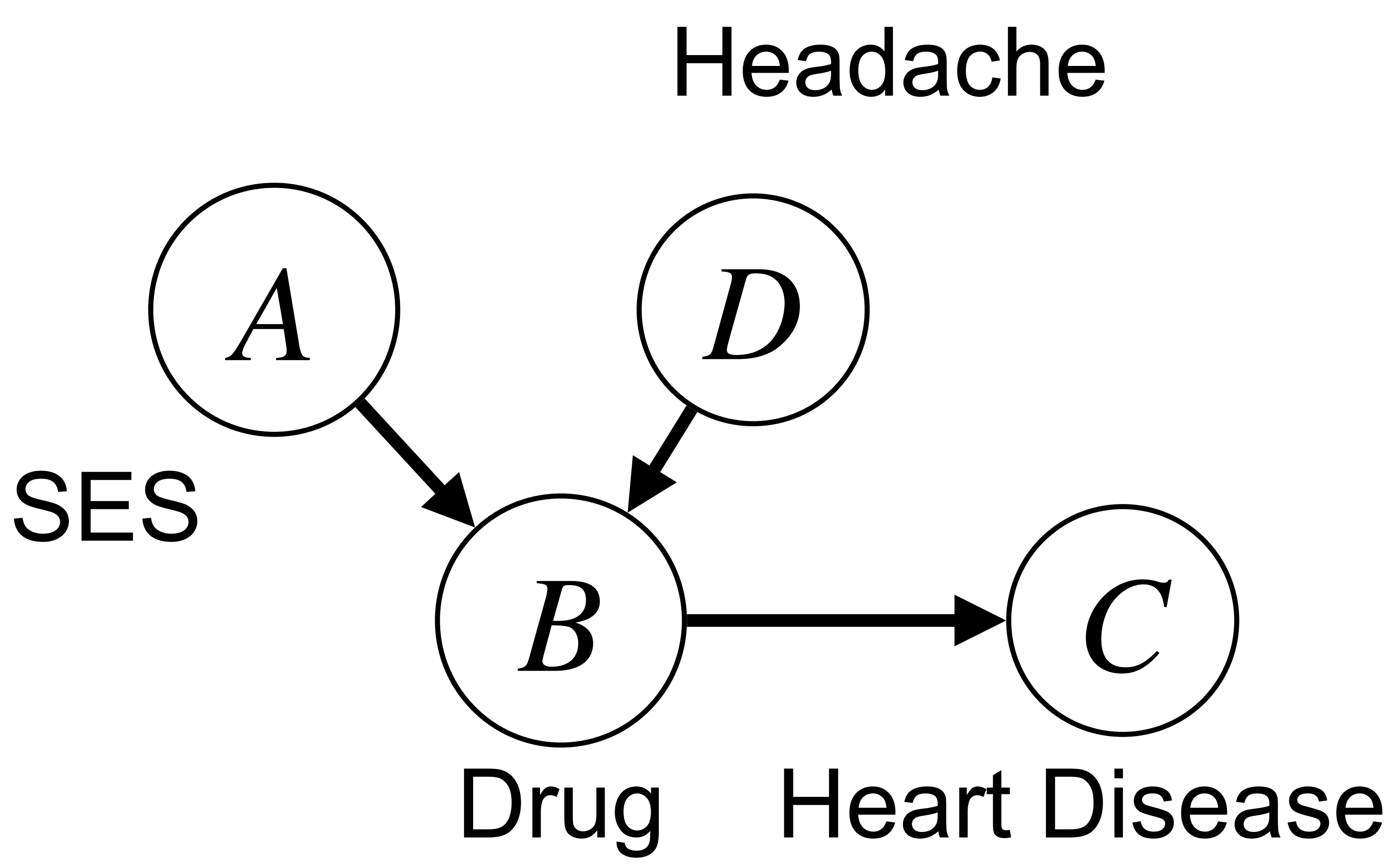
Structural Causal Model (SCM)

$\mathscr{M} = \langle \mathbf{V}, \mathbf{U}, \mathscr{F}, P(\mathbf{u}) \rangle$

$$
\mathscr{M} = \begin{cases} \mathbf{V} = \{A, B, C, D\} \\ \mathbf{U} = \{U_A, U_B, U_C, U_D, U_{CD}\} \\ \mathscr{F} = \begin{cases} A \leftarrow f_A(U_A) \\ B \leftarrow f_B(A, D, U_B) \\ D \leftarrow f_Z(U_D, U_{CD}) \\ C \leftarrow f_X(B, U_C, U_{CD}) \end{cases} \\ P(\mathbf{U}) \end{cases}
$$

Induced Causal Bayesian Network (CBN)

Causal Diagram



An SCM $\mathscr{M} = \langle \mathbf{V}, \mathbf{U}, \mathscr{F}, P(\mathbf{u}) \rangle$ induces a causal diagram such that, **for every** $V_i, V_j \in \mathbf{V}$:

$V_i \rightarrow V_j$, if V_i appears as argument of $f_j \in \mathscr{F}$.

CBN: Encoder of Structural Causal Knowledge

Structural Causal Model (SCM)

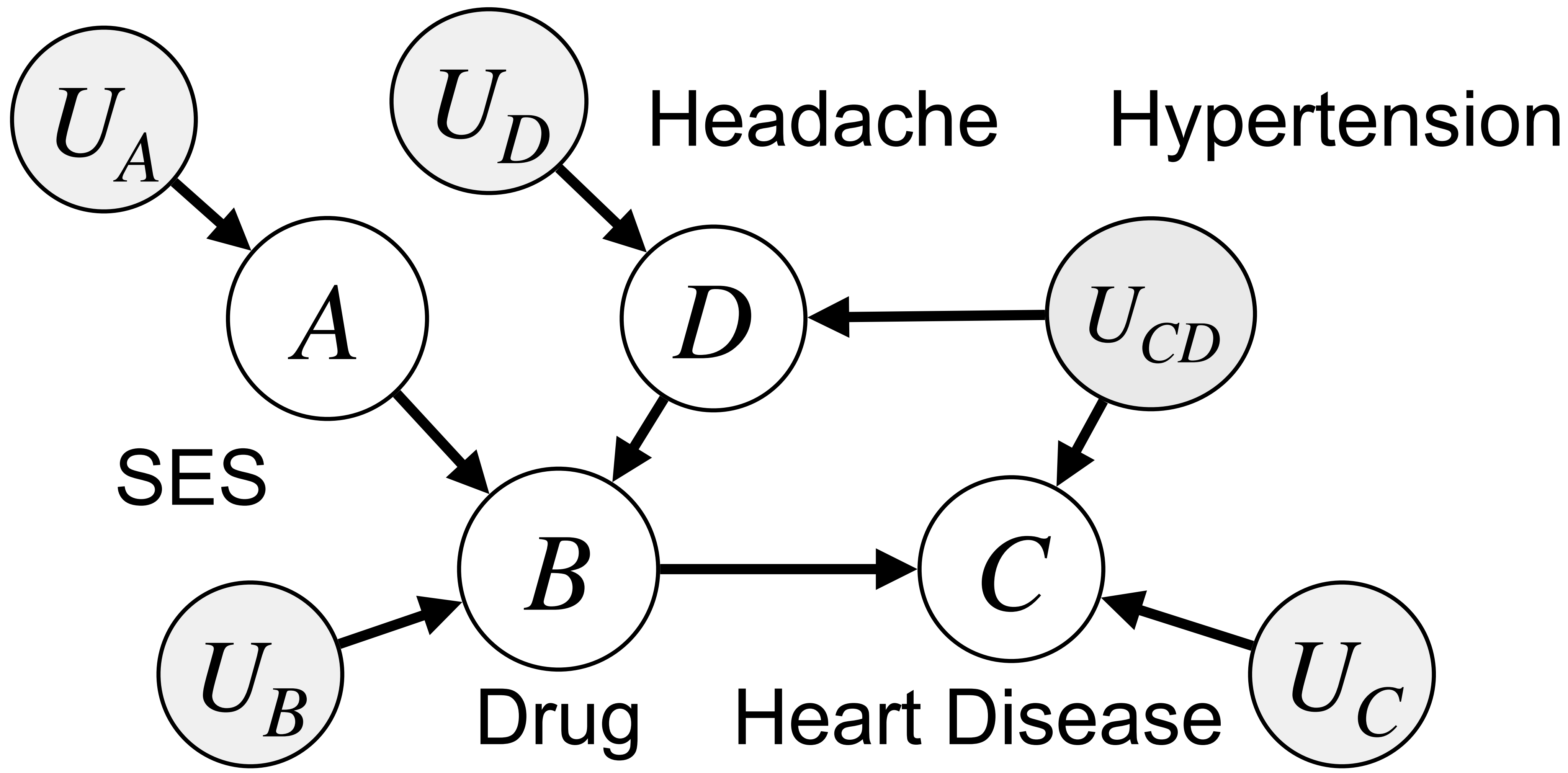
$$\mathcal{M} = \langle \mathbf{V}, \mathbf{U}, \mathcal{F}, P(\mathbf{u}) \rangle$$

$$\mathcal{M} = \begin{cases} \mathbf{V} = \{A, B, C, D\} \\ \mathbf{U} = \{U_A, U_B, U_C, U_D, U_{CD}\} \\ \mathcal{F} = \begin{cases} A \leftarrow f_A(U_A) \\ B \leftarrow f_B(A, D, U_B) \\ D \leftarrow f_Z(U_D, U_{CD}) \\ C \leftarrow f_X(B, U_C, U_{CD}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

$\longrightarrow$

Induced Causal Bayesian Network (CBN)

Causal Diagram



An SCM $\mathcal{M} = \langle \mathbf{V}, \mathbf{U}, \mathcal{F}, P(\mathbf{u}) \rangle$ induces a causal diagram such that, **for every** $V_i, V_j \in \mathbf{V}$:

$V_i \rightarrow V_j$, if V_i appears as argument of $f_j \in \mathcal{F}$.

CBN: Encoder of Structural Causal Knowledge

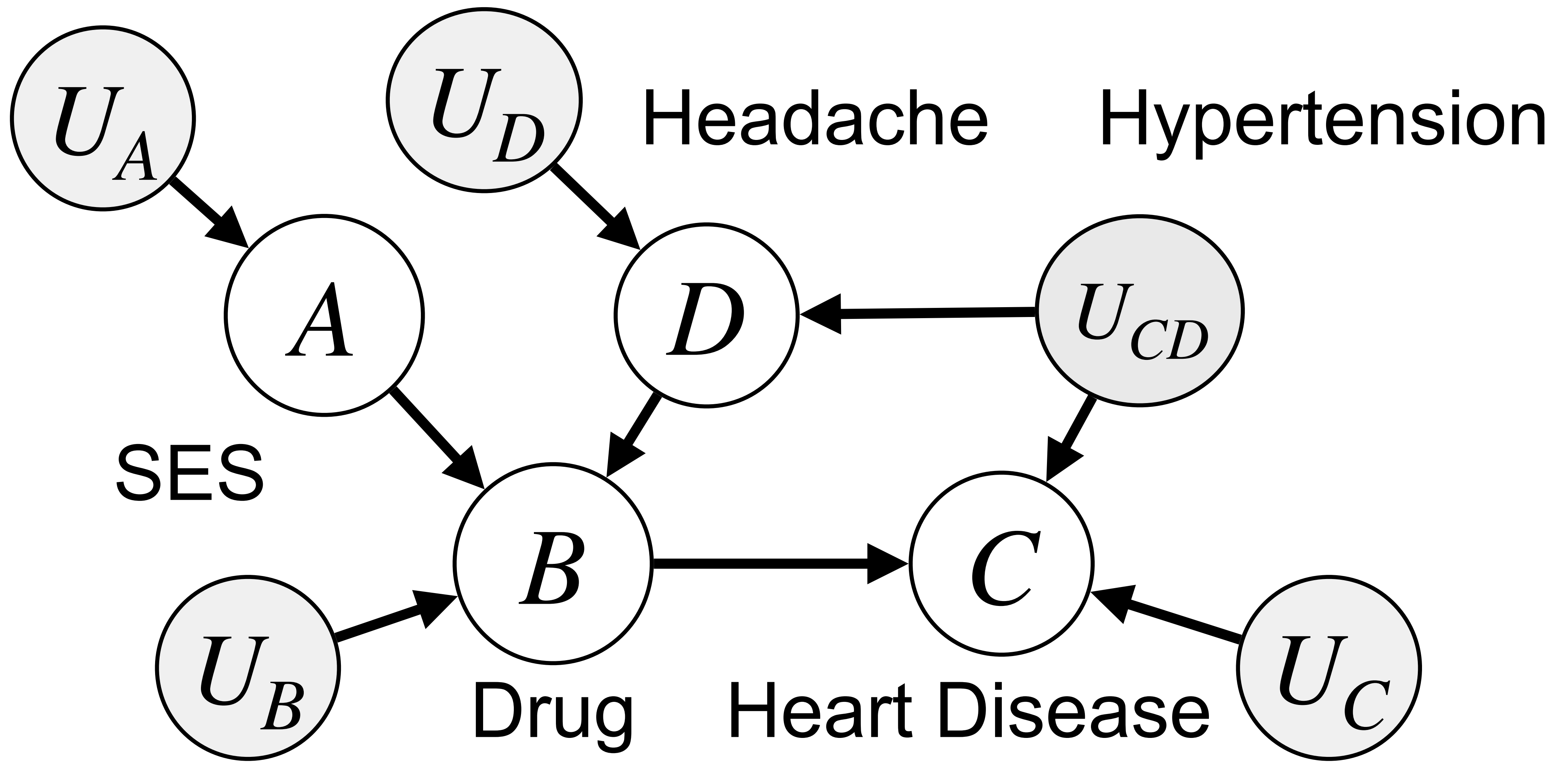
Structural Causal Model (SCM)

$\mathscr{M} = \langle \mathbf{V}, \mathbf{U}, \mathscr{F}, P(\mathbf{u}) \rangle$

$$\mathscr{M} = \begin{cases} \mathbf{V} = \{A, B, C, D\} \\ \mathbf{U} = \{U_A, U_B, U_C, U_D, U_{CD}\} \\ \mathscr{F} = \begin{cases} A \leftarrow f_A(U_A) \\ B \leftarrow f_B(A, D, U_B) \\ D \leftarrow f_Z(U_D, U_{CD}) \\ C \leftarrow f_X(B, U_C, U_{CD}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

Induced Causal Bayesian Network (CBN)

Causal Diagram



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$V_i \leftarrow\!\!\dashrightarrow V_j$ if the corresponding $U_i, U_j \in \mathbf{U}$ are correlated or f_i , f_j share some argument $U \in \mathbf{U}$.

CBN: Encoder of <u>Structural</u> <u>Causal</u> Knowledge

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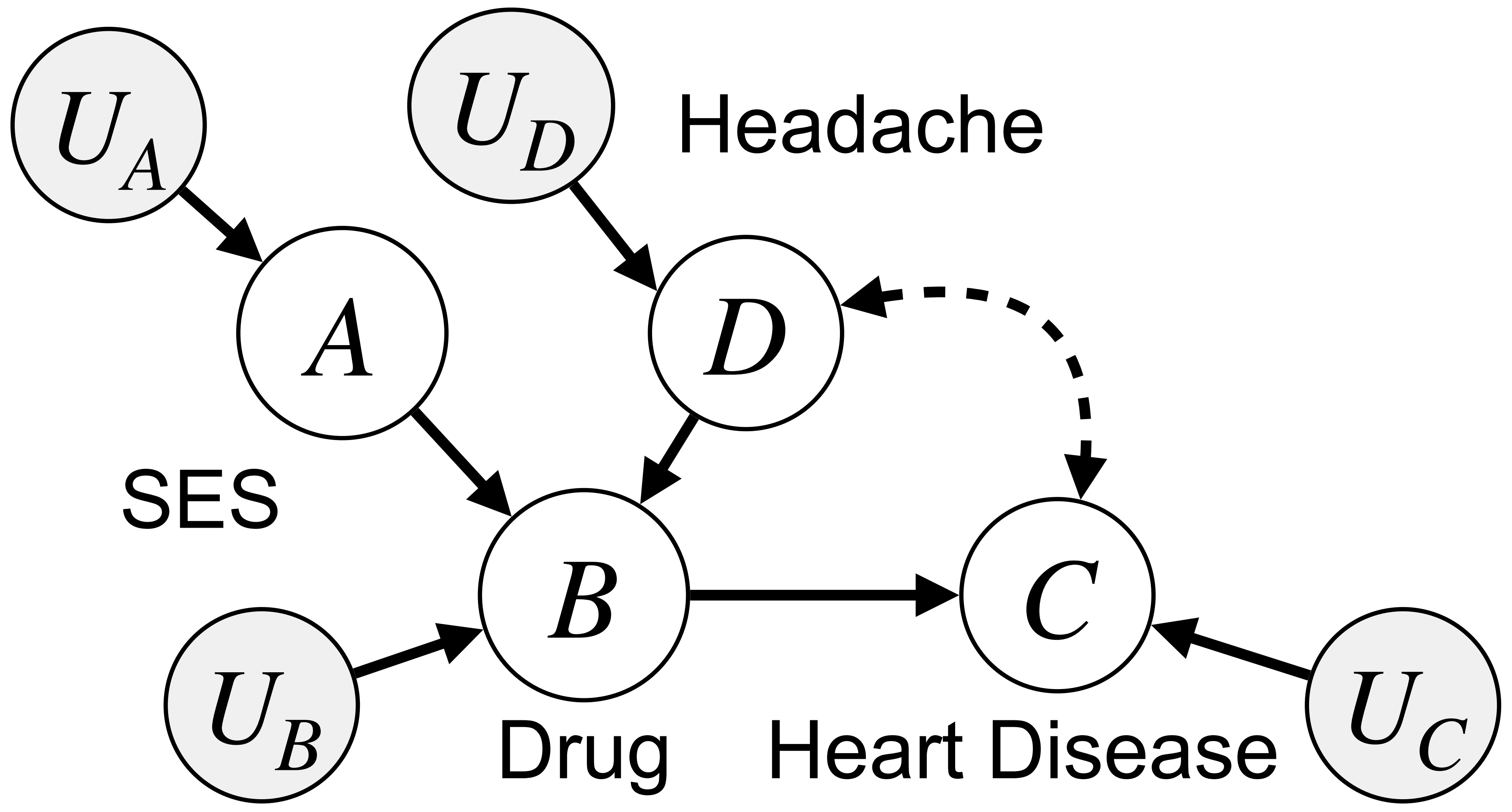
$\longrightarrow$

Induced Causal Bayesian Network (CBN)

Causal Diagram



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CBN: Encoder of Structural Causal Knowledge

Structural Causal Model (SCM)

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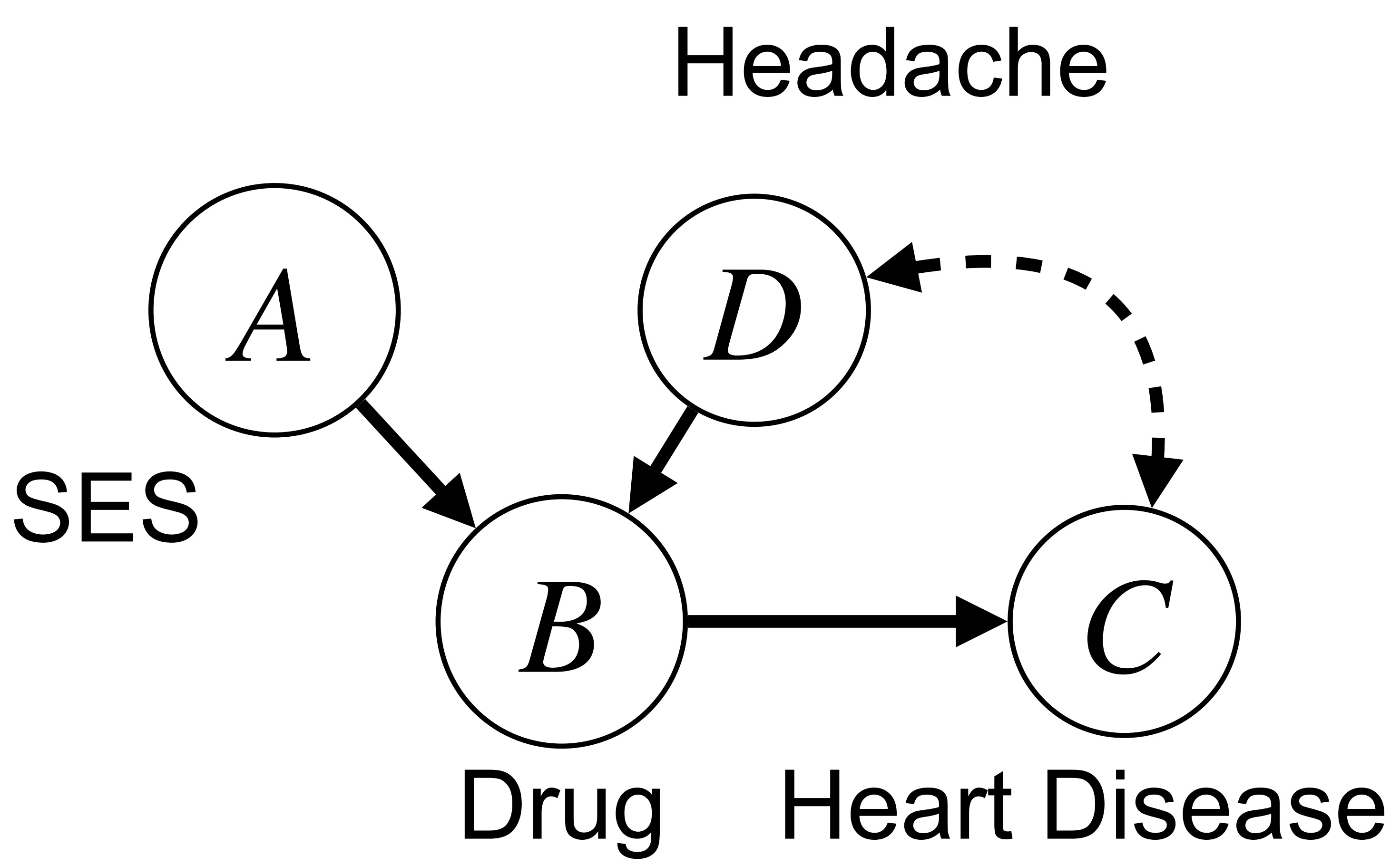
Induced Causal Bayesian Network (CBN)

Causal Diagram



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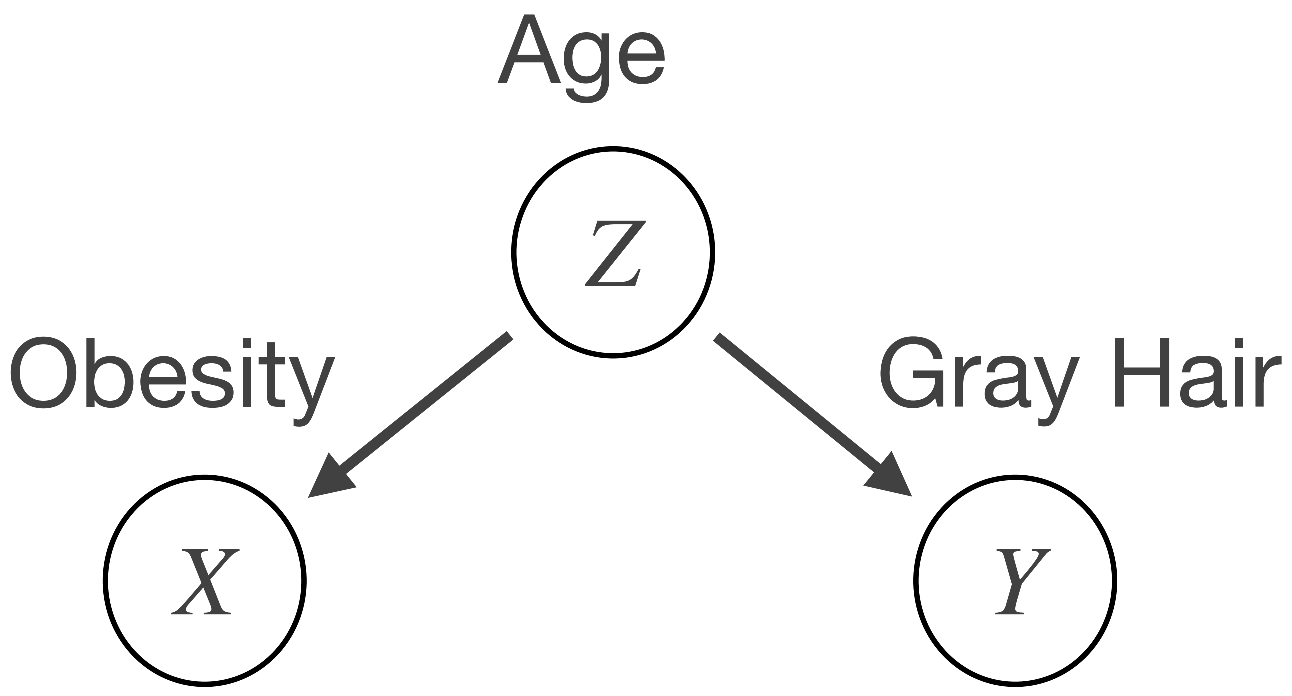
D-Separation

Graphical Tool for Identifying Conditional Independencies implied by Bayesian Networks

Implied Conditional independencies - Examples

Fork

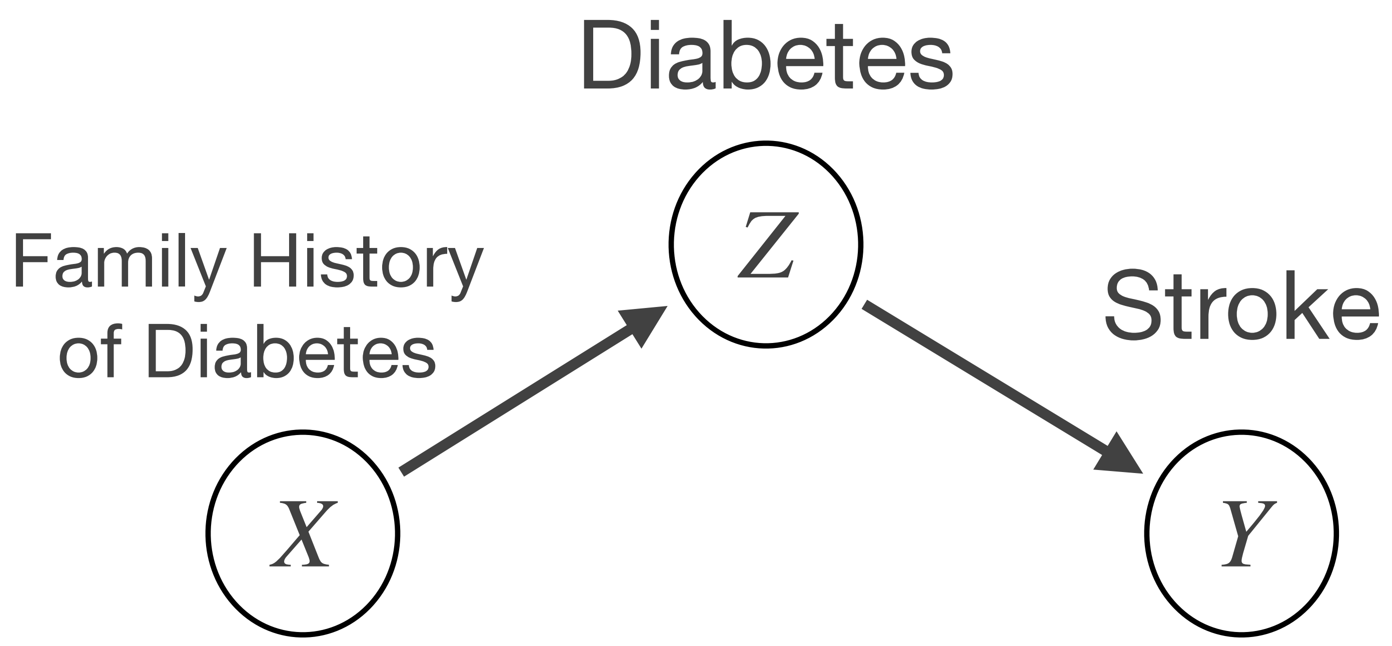
Z **as a common cause**



$$X \not\perp\!\!\!\perp Y$$
$$X \perp\!\!\!\perp Y | Z$$

Chain

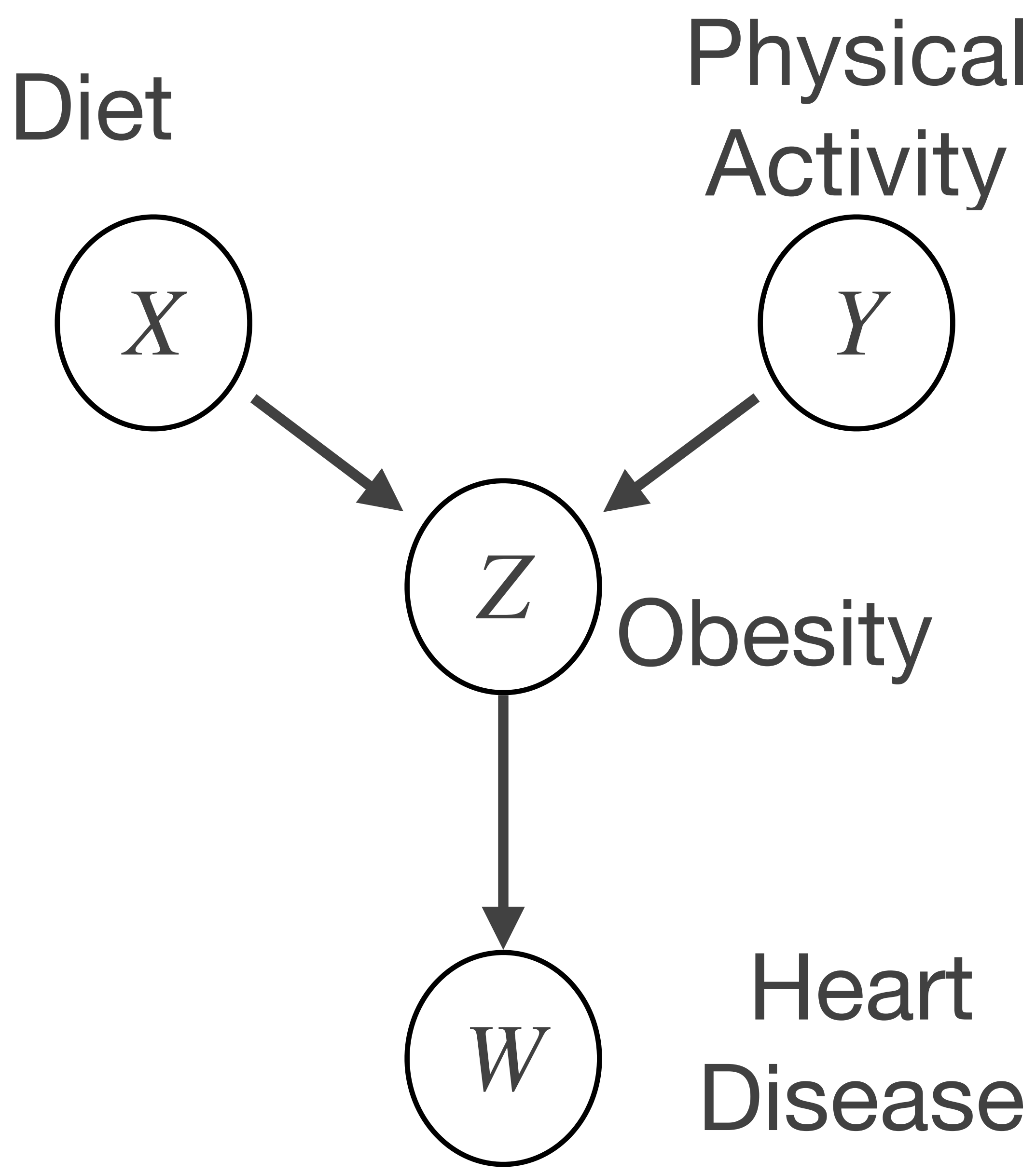
Z **as a mediator**



$$X \not\perp\!\!\!\perp Y$$
$$X \perp\!\!\!\perp Y | Z$$

V-Structure

Z **as a collider or common effect**



$$X \perp\!\!\!\perp Y$$
$$X \not\perp\!\!\!\perp Y | Z$$
$$X \not\perp\!\!\!\perp Y | W$$

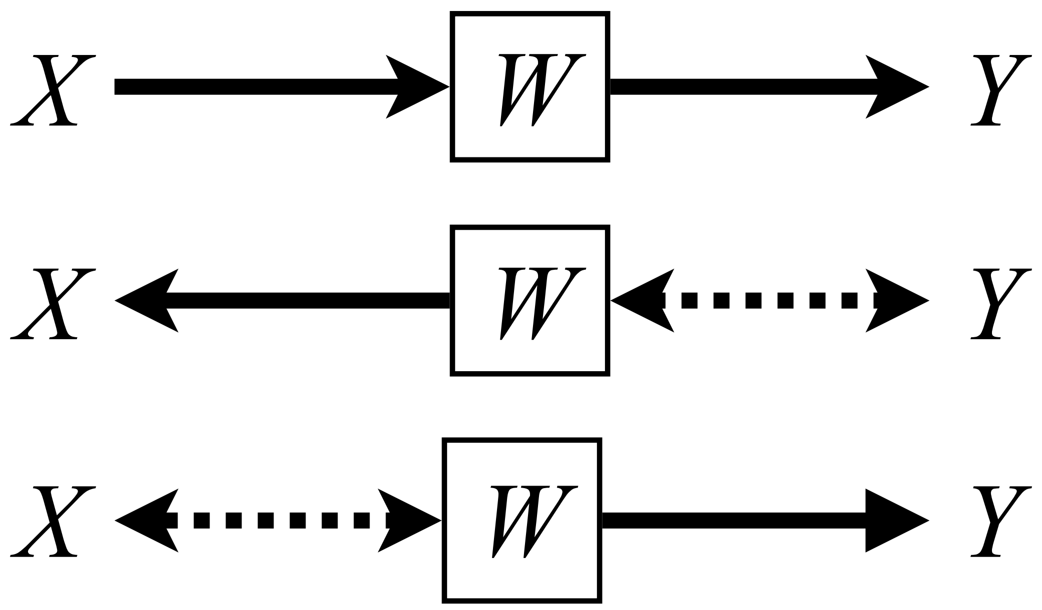
Markov-equivalent models: imply the same set of conditional independencies.

Active and Inactive Triplets

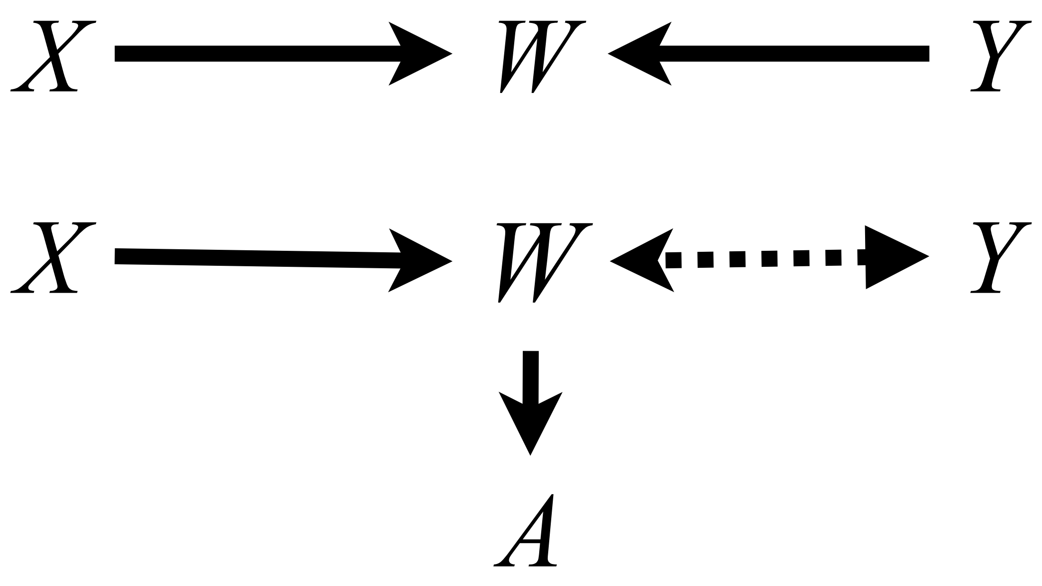
Definition (inactive): A triplet $\langle V_i, V_m, V_j \rangle$ is said to be ***inactive*** relative to a set $\mathbf{Z}$ if the middle node V_m:

1. Is a non-collider and is in $\mathbf{Z}$; or
2. Is a collider and neither it nor any of its descendants in $\mathbf{Z}$.

W is non-collider and $W \in \mathbf{Z}$



W is (descendant of) a collider and $W, A \notin \mathbf{Z}$



D-Separation

Definition (d-separation): A path p in an ADMG G is said to be **blocked (inactive)** by a set of variables $\mathbf{Z}$ if and only if p contains an inactive triplet in it.

A set $\mathbf{Z}$ d-separates $\mathbf{X}$ and $\mathbf{Y}$ if and only if $\mathbf{Z}$ blocks every path between a node in $\mathbf{X}$ and a node in $\mathbf{Y}$. We denote that by $(\mathbf{X} \perp\!\!\!\perp \mathbf{Y} \,|\, \mathbf{Z})_G$.

Does $\mathbf{Z}$ d-separate X and Y ?

$X \leftarrow B \rightarrow W \rightarrow Y$ **Z:** ✗ $\{\}$ ✓ $\{B\}$ ✓ $\{W\}$ ✓ $\{B, W\}$

$X \rightarrow B \leftrightarrow W \leftarrow Y$ **Z:** ✓ $\{\}$ ✓ $\{B\}$ ✓ $\{W\}$ ✗ $\{B, W\}$

$X \rightarrow B \rightarrow W \rightarrow Y$, $B \leftrightarrow Y$ **Z:** ✗ $\{\}$ ✗ $\{B\}$ ✗ $\{W\}$ ✗ $\{B, W\}$

$$(\mathbf{X} \perp\!\!\!\perp \mathbf{Y} \,|\, \mathbf{Z})_G \Rightarrow (\mathbf{X} \perp\!\!\!\perp \mathbf{Y} \,|\, \mathbf{Z})_P$$

D-separations in G correspond to conditional independencies in P

Causal Effect Identification Given a Causal Diagram

Graphical Criteria, Do-Calculus, and ID-Algorithm

Tools for Causal Identification

1. Truncated Factorization / G-computation formula

Markovian Models

2. Graphical criteria
   i. Backdoor Adjustment
   ii. Front-door Adjustment

A few interesting (albeit still constrained) scenarios

3. Do-Calculus (a.k.a Causal Calculus)
4. Identify Algorithm (a.k.a. ID algorithm)

General Semi-Markovian Scenarios

Pearl, J. (2000). Causality: Models, Reasoning, and Inference. Cambridge University Press, New York. http://dx.doi.org/10.1017/CBO9780511803161

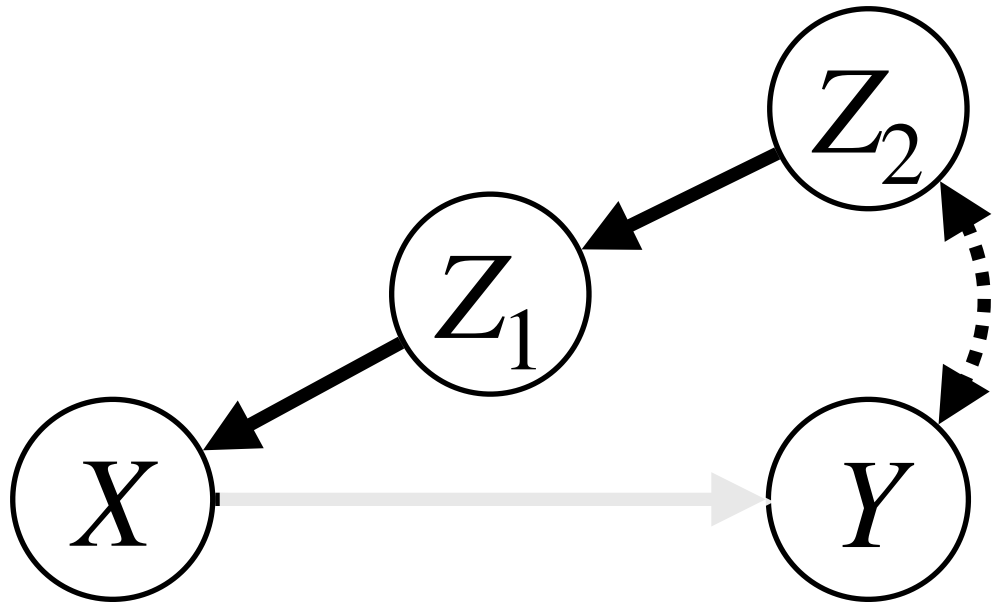
Jin Tian. Studies in causal reasoning and learning. PhD thesis, University of California, Los Angeles, 2002.

Identification via Backdoor Criterion

$$P(\mathbf{y} \mid do(\mathbf{x})) = ?$$

$\mathbf{X} = \{X\}$

$\mathbf{Y} = \{Y\}$



$\mathbf{Z} = \{Z_1\}$

$\mathbf{Z} = \{Z_1, Z_2\}$

Let $\mathbf{X}$ be a set of treatment variables and $\mathbf{Y}$ a set of outcome variables in the causal graph G.

If there exists a set $\mathbf{Z}$ such that:

In $G_{\underline{\mathbf{X}}}$, all non-backdoor paths are severed

1. $\mathbf{Z}$ d-separates $\mathbf{X}$ and $\mathbf{Y}$ in the graph $G_{\underline{\mathbf{X}}}$, i.e., the graph resulting from cutting the arrows out of $\mathbf{X}$

2. no node in $\mathbf{Z}$ is a descendant of a variable $X \in \mathbf{X}$ in G (all variables in $\mathbf{Z}$ are pre-treatment)

Then, $\mathbf{Z}$ satisfies the ***backdoor criterion*** for $(\mathbf{X}, \mathbf{Y})$ and, then the effect of $\mathbf{X}$ on $\mathbf{Y}$ is given by:

$$P(\mathbf{y} \mid do(\mathbf{x})) = \sum_{\mathbf{z}} P\left(\mathbf{y} \mid \mathbf{x}, \mathbf{z}\right) P\left(\mathbf{z}\right)$$

$\mathbf{Z}$, a set of covariates, admissible for backdoor adjustment

Judea Pearl. Comment: Graphical models, causality and intervention. Stat. Sci., 8:266–269, 1993.

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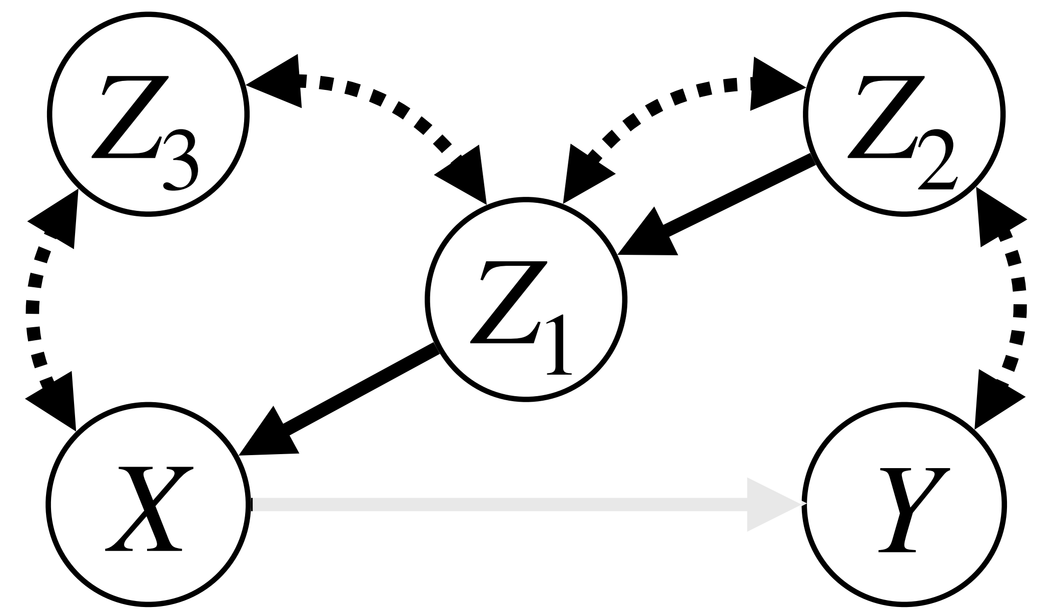

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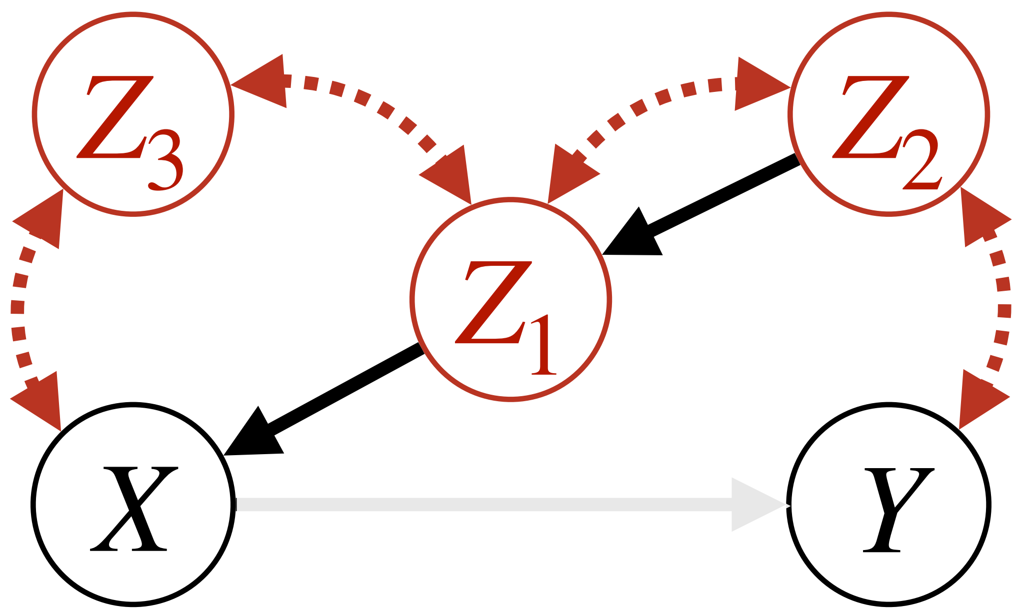

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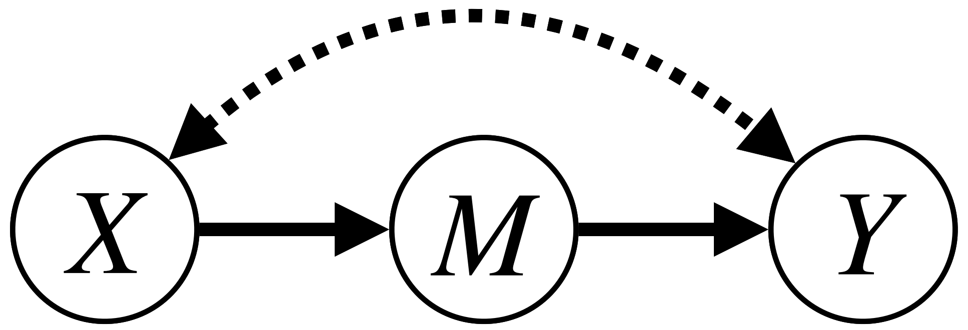
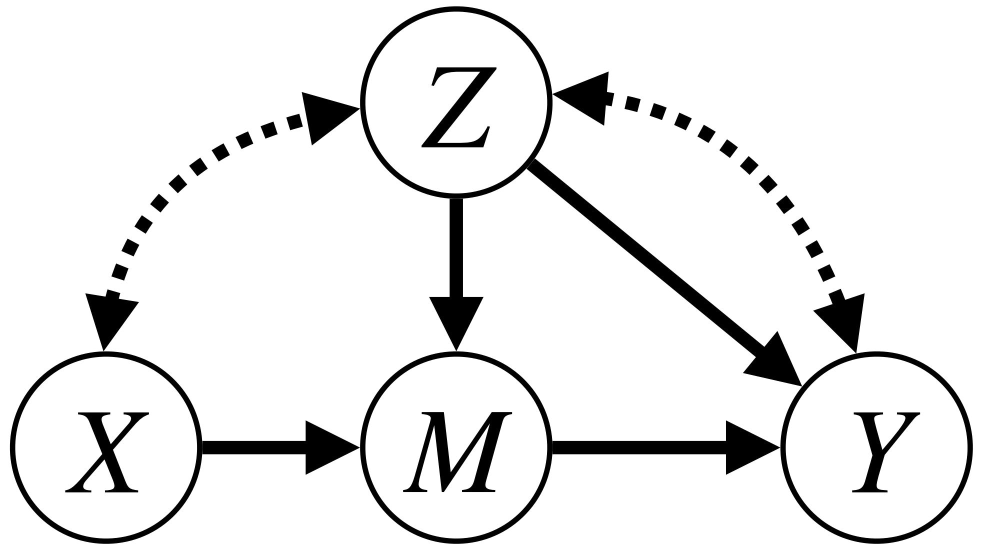
Many Scenarios Beyond Backdoor …



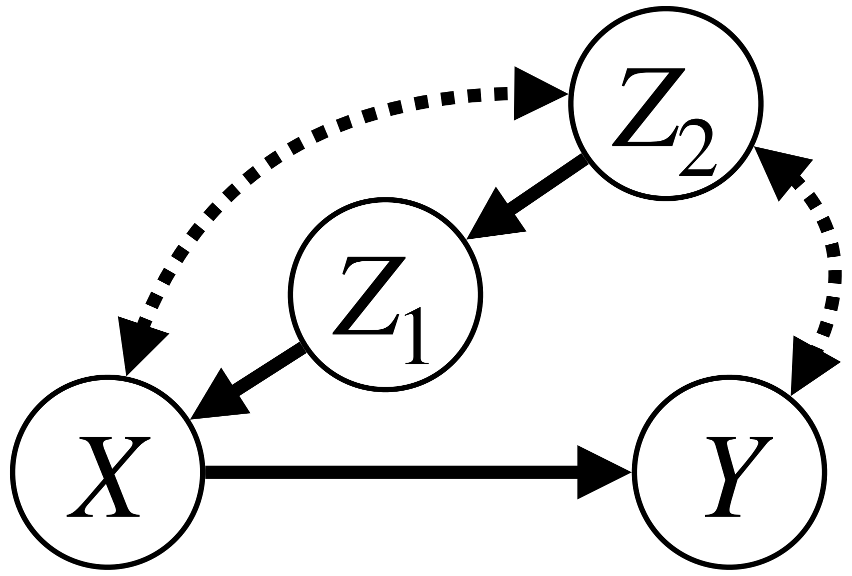
Front-Door

$$P(y \mid do(x)) = \sum_{m} P(m \mid x) \sum_{x'} P(y \mid m, x')P(x')$$



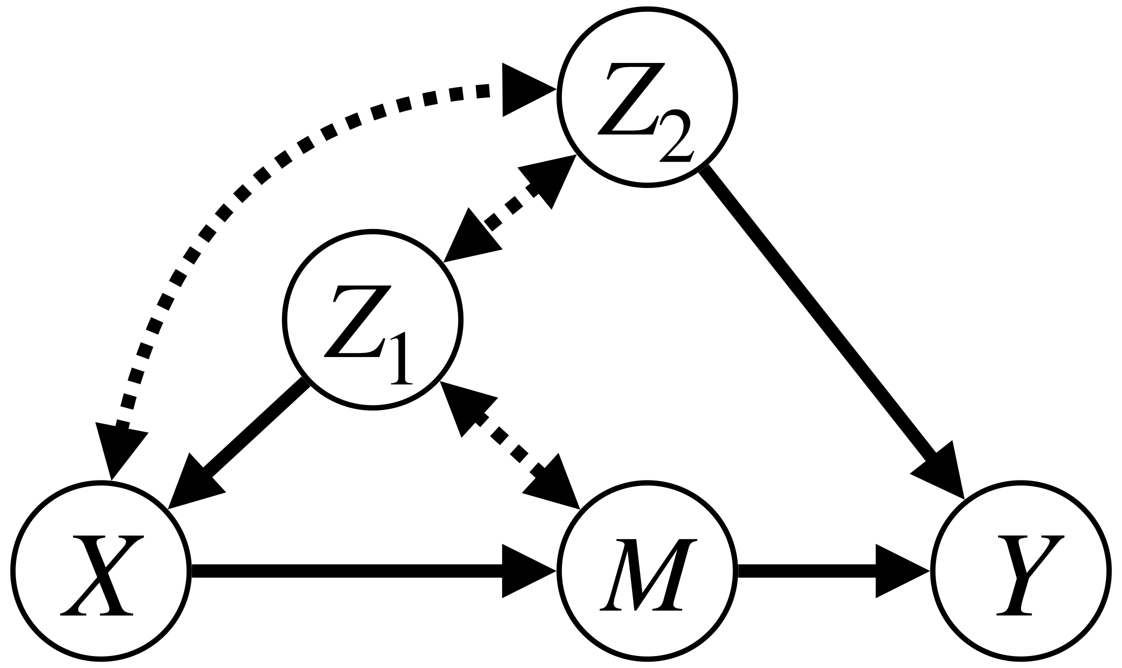
Conditional Front-Door

$$P(y \mid do(x)) = \sum_{m,z} P(m \mid x, z) \sum_{x'} P(y \mid m, x', z)P(x', z)$$



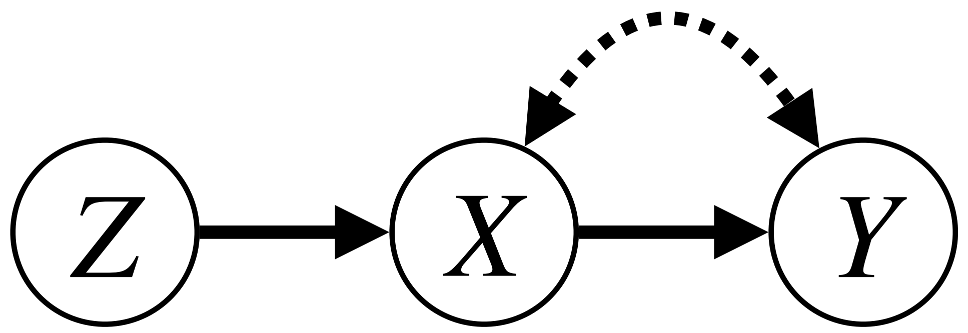
Napkin

$$P(y \mid do(x)) = \frac{\sum_{z_2} P(x, y \mid z_1, z_2)P(z_2)}{\sum_{z_2} P(x \mid z_1, z_2)P(z_2)}$$



Unnamed

$$P(y \mid do(x)) = \sum_{z_2,z_3} P(y \mid x, z_1, z_2, z_3)P(z_2) \sum_{z_1} P(z_3 \mid x, z_1)P(z_1)$$



Linear IV Graph

$$\frac{d}{dx}\mathbb{E}[Y \mid \mathrm{do}(X = x)] = \beta = \frac{Cov(Z, Y)}{Cov(Z, X)}$$

And many others….

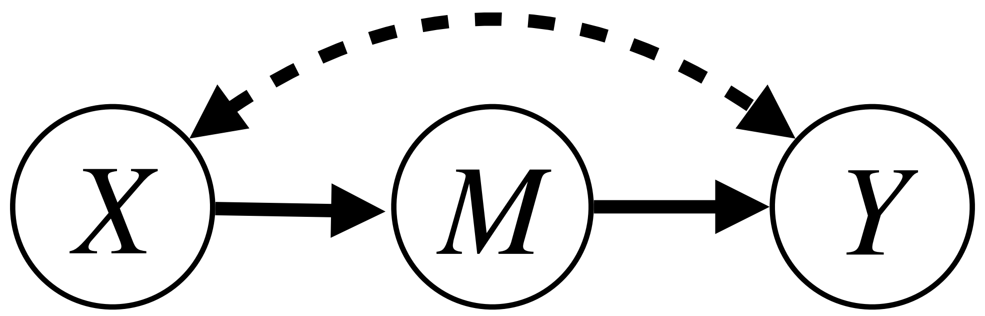
Causal Effect Identification (ID) Algorithm

ID-Algorithm and many recent generalizations.

1. **Query**

   $P(y \mid do(x))$

2. **Causal Contraints**



3. **Probability Distributions**

   $P(x, m, y)$ **Observational Distribution**

Inference Engine

Solution

yes / no

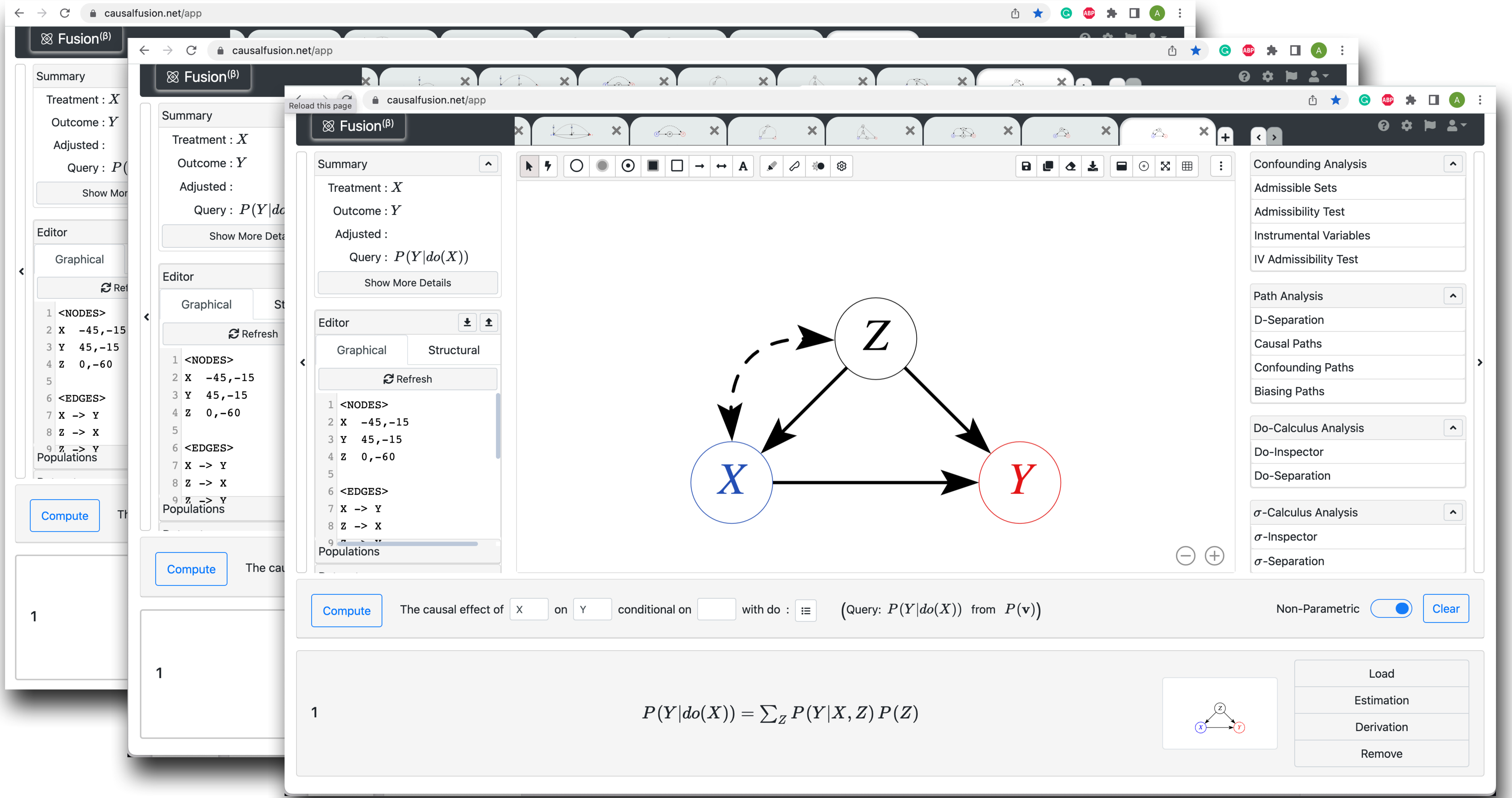
$$P(y \mid do(x)) = \sum_{m} P(m \mid x) \sum_{x'} P(y \mid m, x') P(x')$$

Interventional Distribution ← Available Distributions 👍

- Tian, J. and Pearl, J. A General Identification Condition for Causal Effects. In Proceedings of the Eighteenth National Conference on Artificial Intelligence (AAAI 2002), pp. 567–573, Menlo Park, CA, 2002. AAAI Press/MIT Press.

http://causalfusion.net

By Elias Bareinboim - Causal AI Lab (causalai.net)



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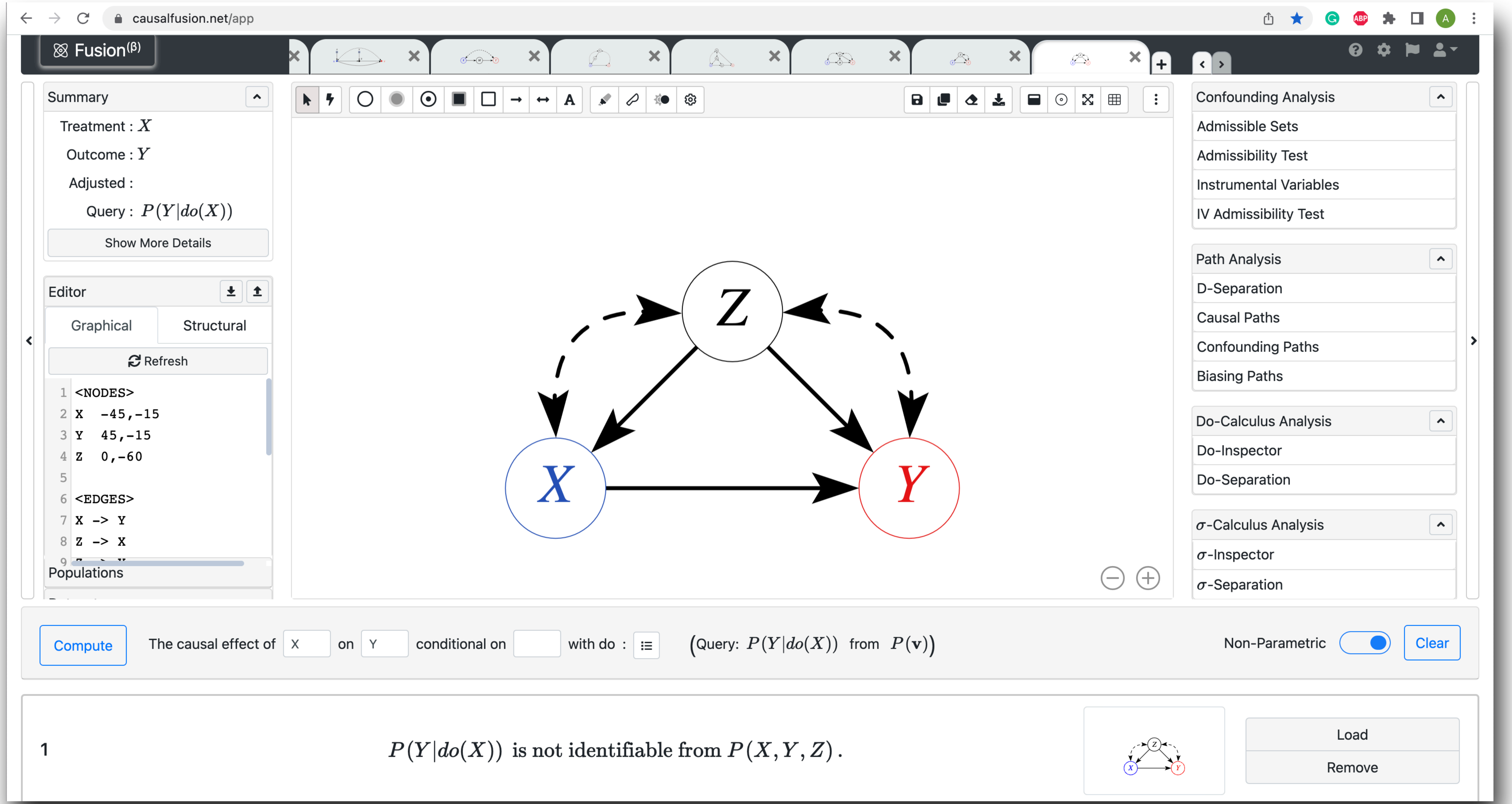

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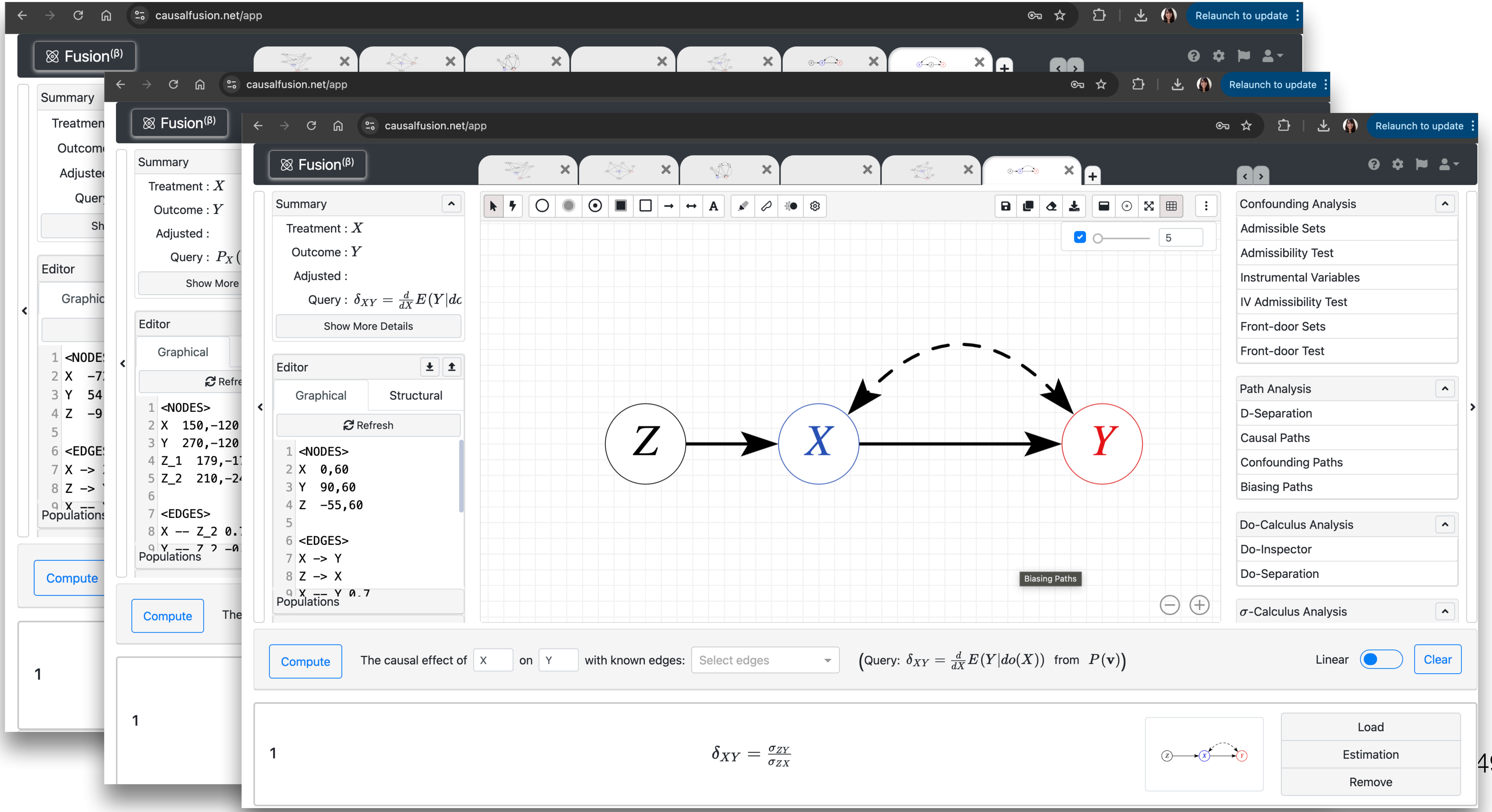

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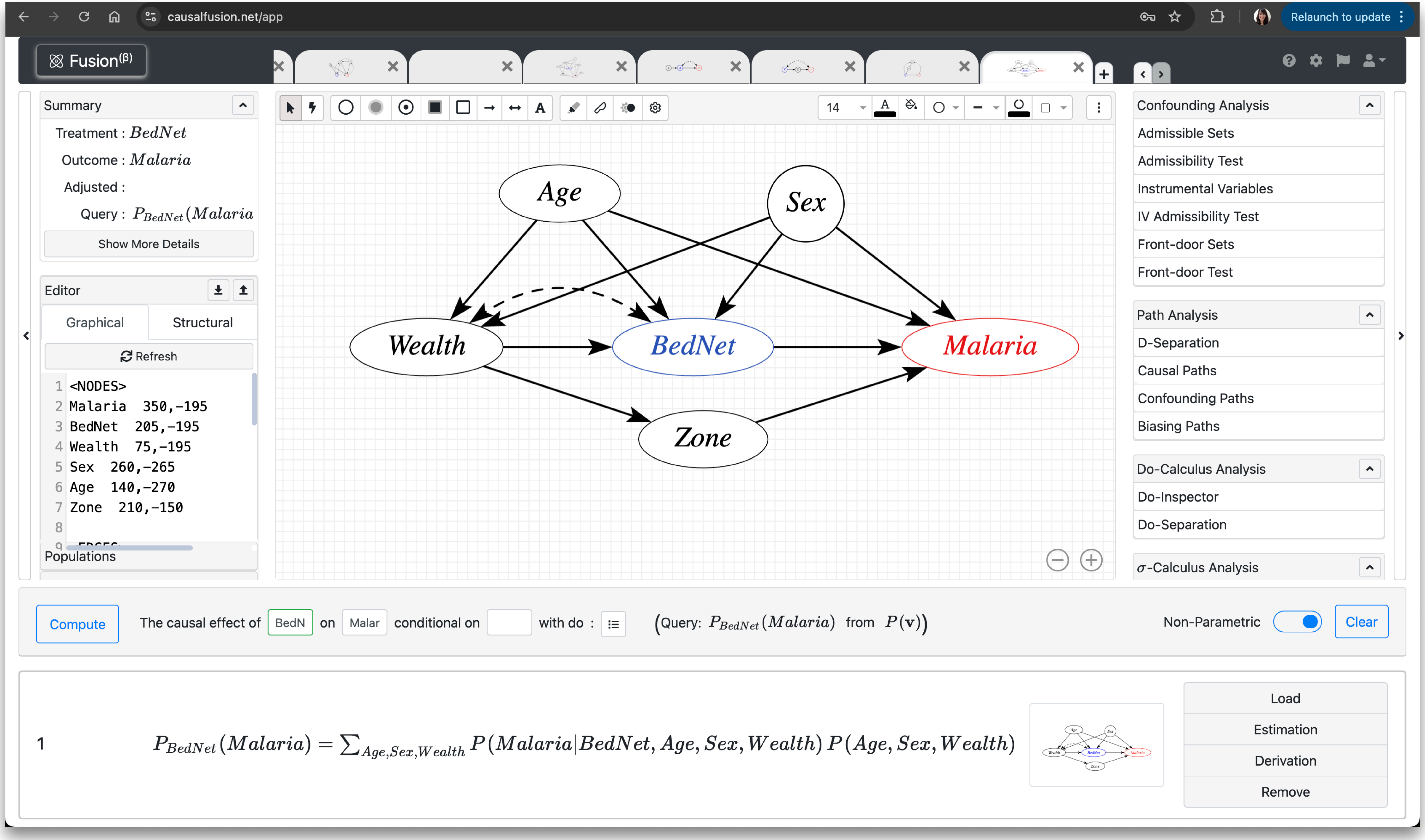

Coding Exercises

Causality Tutorial:

- **Google Colab Notebook:** (Link)

  Please, open and make a local copy: File → Save a copy in Drive

- Slides and Google Colab link also available on GitHub → RBras 2025

Setup: Download the required packages — it takes a few minutes.

Check Part I: Causal Modeling

Check Part II: Causal Effect Identification from Causal Diagrams

1. Backdoor / Adjustment Criterion -- pcalg R package
2. ID Algorithm -- causaleffect R package

What if domain knowledge does not allow you construct the true causal diagram?

We'll explore this tomorrow!!