MODELLING AND NONLINEAR CONTROL OF A QUADCOPTER FOR STABILIZATION AND TRAJECTORY TRACKING

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OUTLINE

- INTRODUCTION
- AIM & OBJECTIVES OF THE STUDY
- SYSTEM MODELLING
- CONTROL ARCHITECTURE AND DEVELOPMENT
- PID STABILIZATION RESULTS
- TRAJECTORY TRACKING RESULTS
- ACHIEVEMENT AND CONCLUSION

INTRODUCTION

A quadcopter, also known as a quadrotor, is an unmanned aerial vehicle propelled by four rotors in cross-configuration. A quadcopter's dynamics are extremely nonlinear, it is an underactuated system with six degrees of freedom and four control inputs which are the rotor velocities.

The number and complexity of applications for quadcopter systems continues to grow on a daily basis, the control techniques that are employed must also improve in order to enhance performance and adaptability to new situations. This thesis would be studying the modelling of the quadcopter system and nonlinear control methods that can be implemented on the system for stability and trajectory tracking.

AIM AND OBJECTIVES OF THE STUDY

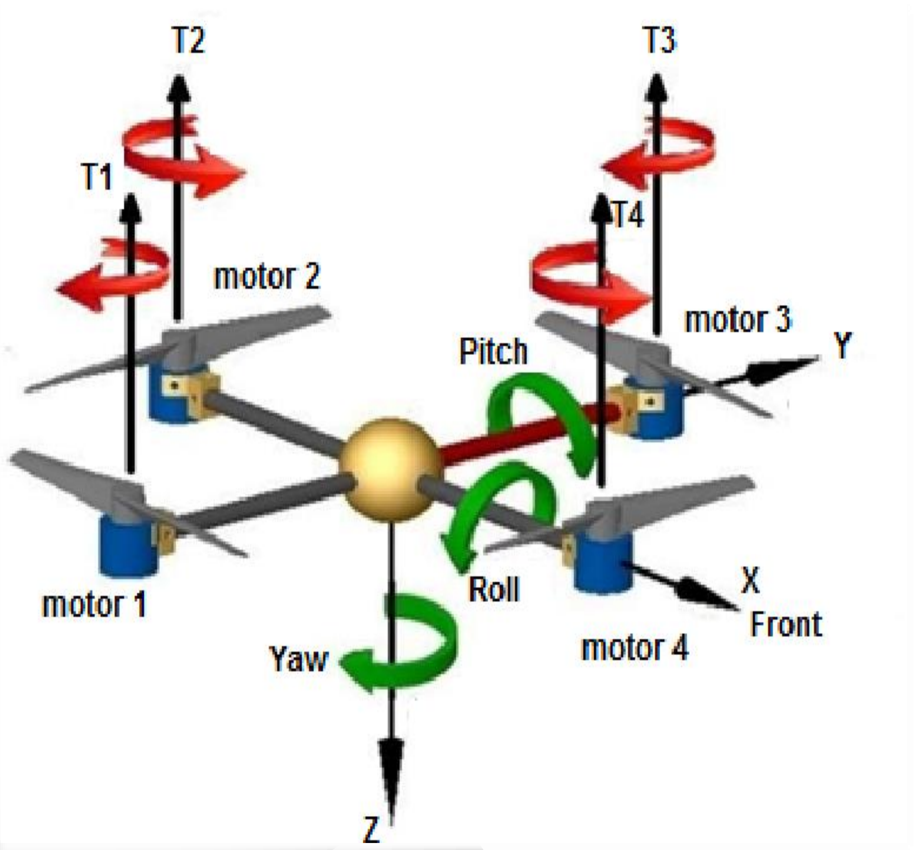
The project aims to develop a mathematical model of a quadcopter system and implement nonlinear control techniques on the derived model for stabilization and trajectory tracking of a quadcopter. The objectives of this project are to:

- Develop the mathematical model of a quadcopter system dynamics.
- Develop a PID control algorithm for the derived nonlinear quadcopter system dynamics.
- Derive the full state Feedback Linearized system of the derived nonlinear quadcopter system dynamics.
- Simulate and perform a comparative analysis of the implemented control techniques on the quadcopter system for stabilization and trajectory tracking.

SYSTEM MODELLING

The quadcopter system is complex and in order to control it, the quadcopter is modelled on the following assumptions:

- The structure is rigid
- The structure is axis symmetrical
- The Centre of Gravity and the body fixed frame origin coincide
- The propellers are rigid
- Thrust and drag are proportional to the square of the propeller’s speed



SYSTEM MODELLING

ROTATIONAL DYNAMICS

Using Euler Equation for Rigid bodies, the dynamics in the body frame is given as:

$$\boldsymbol{I\dot{\omega}_B + [\omega_B \times (I\omega_B)] + \Gamma = \tau_B}$$

$$\Gamma = J_r \omega_B \omega_r$$

$$where\ \omega_r = -\omega_1 + \omega_2 - \omega_3 + \omega_4$$

$$\tau_B = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} lk_t(-\omega_2^2 + \omega_4^2) \\ lk_t(-\omega_1^2 + \omega_3^2) \\ k_b(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2) \end{bmatrix}$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{(I_{yy}-I_{zz})qr}{I_{xx}} \\ \frac{(I_{zz}-I_{xx})pr}{I_{yy}} \\ \frac{(I_{xx}-I_{yy})pq}{I_{zz}} \end{bmatrix} - J_r \begin{bmatrix} \frac{q}{I_{xx}} \\ \frac{-p}{I_{yy}} \\ 0 \end{bmatrix} \omega_r + \begin{bmatrix} \frac{\tau_\phi}{I_{xx}} \\ \frac{\tau_\theta}{I_{yy}} \\ \frac{\tau_\psi}{I_{zz}} \end{bmatrix}$$

TRANSLATIONAL DYNAMICS

Using Newtonian equation to model the linear dynamics:

$$\dot{m}\boldsymbol{v_1} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \boldsymbol{RT_B + F_D}$$

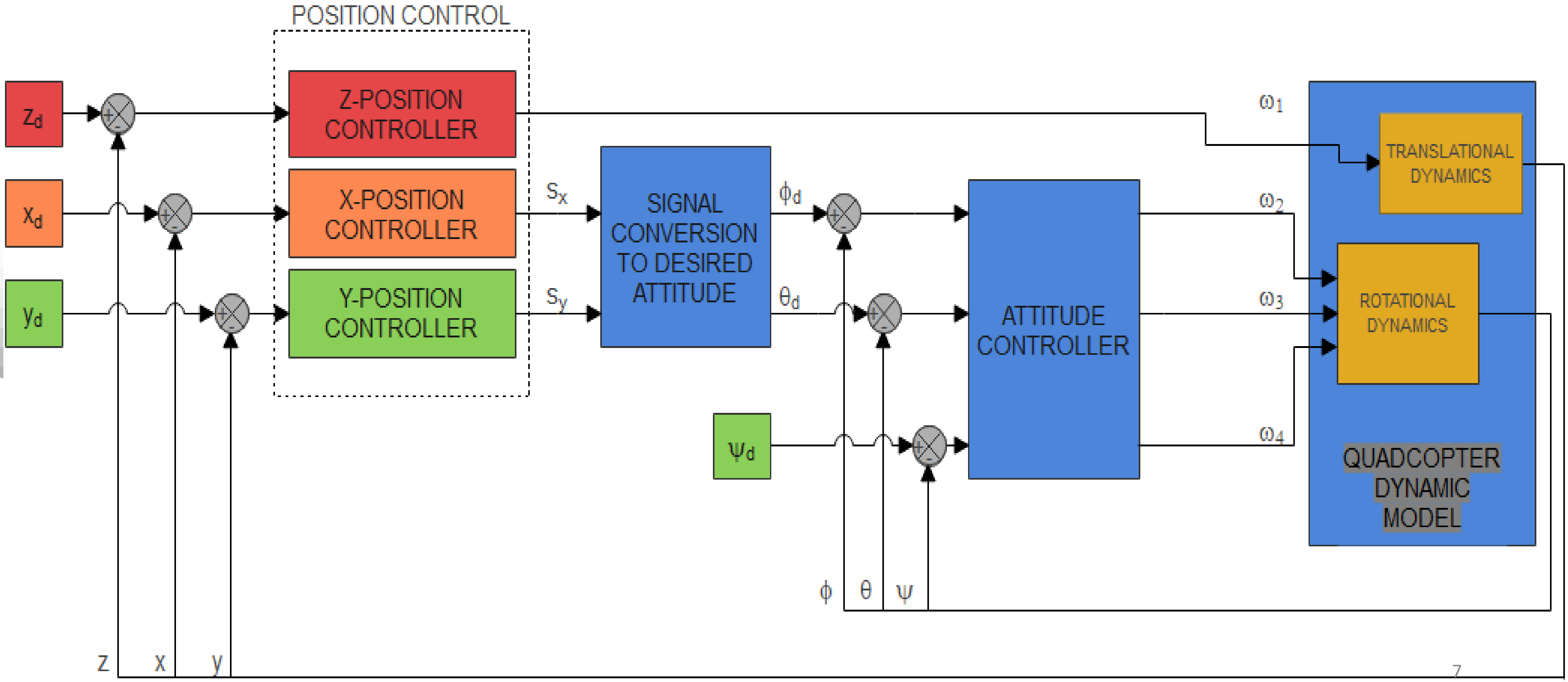
$$F_D = -k_d v_I$$

$$T = \sum_{i=1}^{4} F_i = k_t \sum_{i=1}^{4} \omega_i^2$$

$$T_B = \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix} = k_t \begin{bmatrix} 0 \\ 0 \\ \Sigma_{i=1}^4 \omega_i^2 \end{bmatrix}$$

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{T}{m} \begin{bmatrix} c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi s_\theta c_\phi + c_\psi s_\phi \\ c_\theta c_\phi \end{bmatrix} - \frac{1}{m} \begin{bmatrix} k_{dx} & 0 & 0 \\ 0 & k_{dy} & 0 \\ 0 & 0 & k_{dz} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

CONTROL ARCHITECTURE AND DEVELOPMENT



CONTROL ARCHITECTURE AND DEVELOPMENT

PID CONTROL

$$\boldsymbol{e(t) = r(t) - y(t)}$$

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{d}{dt} e(t)$$

$$\tau = I \times u(t)$$

$$\begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} I_{xx}\left(k_{P\phi}(\phi_{des} - \phi) + k_{D\phi}(\dot{\phi}_{des} - \dot{\phi})\right) \\ I_{yy}\left(k_{P\theta}(\theta_{des} - \theta) + k_{D\theta}(\dot{\theta}_{des} - \dot{\theta})\right) \\ I_{zz}\left(k_{P\psi}(\psi_{des} - \psi) + k_{D\psi}(\dot{\psi}_{des} - \dot{\psi})\right) \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{w_1^2} \\ \boldsymbol{w_2^2} \\ \boldsymbol{w_3^2} \\ \boldsymbol{w_4^2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{k_t} & \boldsymbol{k_t} & \boldsymbol{k_t} & \boldsymbol{k_t} \\ \boldsymbol{0} & \boldsymbol{-lk_t} & \boldsymbol{0} & \boldsymbol{lk_t} \\ \boldsymbol{-lk_t} & \boldsymbol{0} & \boldsymbol{lk_t} & \boldsymbol{0} \\ \boldsymbol{-k_b} & \boldsymbol{k_b} & \boldsymbol{-k_b} & \boldsymbol{k_b} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{T} \\ \boldsymbol{\tau_\phi} \\ \boldsymbol{\tau_\theta} \\ \boldsymbol{\tau_\psi} \end{bmatrix}$$

FEEDBACK LINEARIZATION

$$\boldsymbol{\frac{T}{m} \begin{bmatrix} c(\psi)s(\theta)c(\phi) + s(\psi)s(\phi) \\ s(\psi)s(\theta)c(\phi) + c(\psi)s(\phi) \\ c(\theta)c(\phi) \end{bmatrix} = \begin{bmatrix} k_{ax}(\dot{x} - \dot{x}_d) \\ k_{ay}(\dot{y} - \dot{y}_d) \\ k_{az}(\dot{z} - \dot{z}_d) \end{bmatrix} + g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{m} \begin{bmatrix} k_{dx}k_x(x - x_d) \\ k_{dy}k_y(y - y_d) \\ k_{dz}k_z(z - z_d) \end{bmatrix}}$$

$$F_x = \frac{\left(mk_{ax}(\dot{x} - \dot{x}_d) + k_{dx}k_x(x - x_d)\right)}{T}$$

$$F_y = \frac{\left(mk_{ay}(\dot{y} - \dot{y}_d) + k_{dy}k_y(y - y_d)\right)}{T}$$

$$F_z = \frac{\left(mk_{az}(\dot{z} - \dot{z}_d) + mg + k_{dz}k_z(z - z_d)\right)}{T}$$

$$\begin{bmatrix} p_d \\ q_d \\ r_d \end{bmatrix} = \begin{bmatrix} 1 & s(\phi_d)t(\theta_d) & c(\phi_d)t(\theta_d) \\ 0 & c(\phi_d) & -s(\phi_d) \\ 0 & \frac{s(\phi_d)}{c(\theta_d)} & \frac{c(\phi_d)}{c(\theta_d)} \end{bmatrix}^{-1} \begin{bmatrix} k_\phi(\phi - \phi_d) \\ k_\theta(\theta - \theta_d) \\ k_\psi(\psi - \psi_d) \end{bmatrix}$$

$$\boldsymbol{\begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} I_{xx}k_{P\phi}(p - p_d) \\ I_{yy}k_{P\theta}(q - q_d) \\ I_{zz}k_{P\psi}(r - r_d) \end{bmatrix} - \begin{bmatrix} (I_{yy} - I_{zz})q_d r_d + J_r q_d \omega_r \\ (I_{zz} - I_{xx})p_d r_d + J_r p_d \omega_r \\ (I_{xx} - I_{yy})p_d q_d \end{bmatrix}}$$

CONTROL ARCHITECTURE AND DEVELOPMENT

TRAJECTORY TRACKING WITH PD CONTROL

The goal of trajectory control is to take the quadcopter system from its current position to the desired position by regulating the quadcopter's rotor angular velocities. To achieve this with a PD controller, we make two assumptions:

- Small angle approximation such that $\sin(x) \approx x$ and $\cos(x) \approx 1$
- The desired ψ angle is zero.

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -g\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{T}{m}\begin{bmatrix} c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi s_\theta c_\phi + c_\psi s_\phi \\ c_\theta c_\phi \end{bmatrix} - \frac{1}{m}\begin{bmatrix} k_{dx} & 0 & 0 \\ 0 & k_{dy} & 0 \\ 0 & 0 & k_{dz} \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

$$\ddot{x}_{des} = \frac{T}{m}\theta_{des} - \frac{k_{dx}}{m}\dot{x}_{des}$$

$$\ddot{y}_{des} = \frac{T}{m}\phi_{des} - \frac{k_{dy}}{m}\dot{y}_{des}$$

$$\ddot{z}_{des} = -g + \frac{T}{m} - \frac{k_{dz}}{m}\dot{z}_{des}$$

Mapping Equation for Stabilization:

$$\theta_{des} = \frac{1}{T}\left[m\ddot{x}_{des} + k_{dx}\dot{x}_{des} + k_{Pxy}(x_{des} - x)\right]$$

$$\phi_{des} = -\frac{1}{T}\left[m\ddot{y}_{des} + k_{dy}\dot{y}_{des} + k_{Pxy}(y_{des} - y)\right]$$

$$T_{des} = m(\ddot{z}_{des} + g) + k_{dz}\dot{z}_{des}$$

CONTROL ARCHITECTURE AND DEVELOPMENT

TRAJECTORY TRACKING WITH FBL CONTROL

To relate the desired position x and y which are not controllable to the desired angles of ϕ and θ that are controllable using FBL control technique:

- No small angle approximation.
- The desired ψ angle is zero.

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = -g\begin{bmatrix}0\\0\\1\end{bmatrix} + \frac{T}{m}\begin{bmatrix}c_\psi s_\theta c_\phi + s_\psi s_\phi\\ s_\psi s_\theta c_\phi + c_\psi s_\phi\\ c_\theta c_\phi\end{bmatrix} - \frac{1}{m}\begin{bmatrix}k_{dx} & 0 & 0\\ 0 & k_{dy} & 0\\ 0 & 0 & k_{dz}\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix}$$

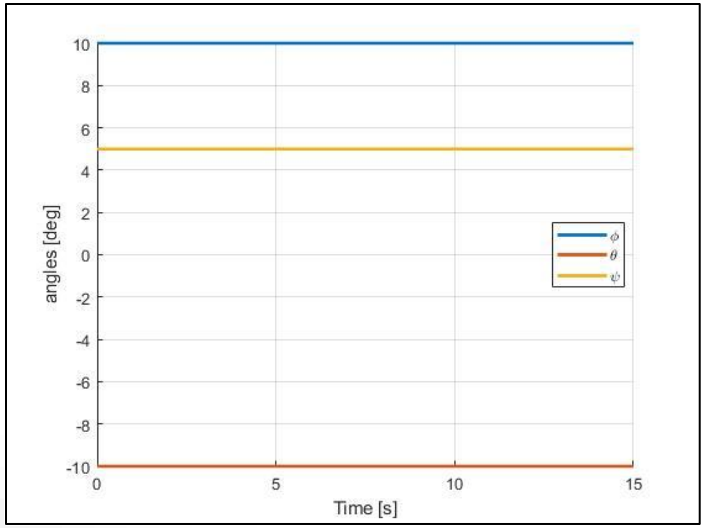
$$\ddot{x}_{des} = \frac{T}{m}\theta_{des} - \frac{k_{dx}}{m}\dot{x}_{des}$$

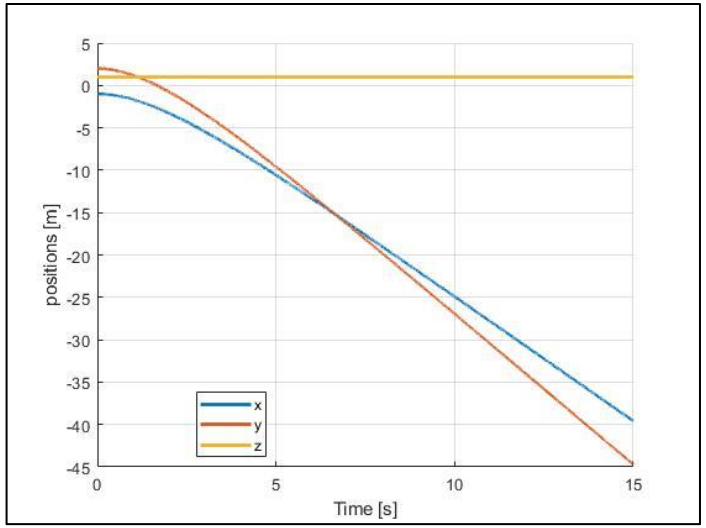
$$\ddot{y}_{des} = \frac{T}{m}\phi_{des} - \frac{k_{dy}}{m}\dot{y}_{des}$$

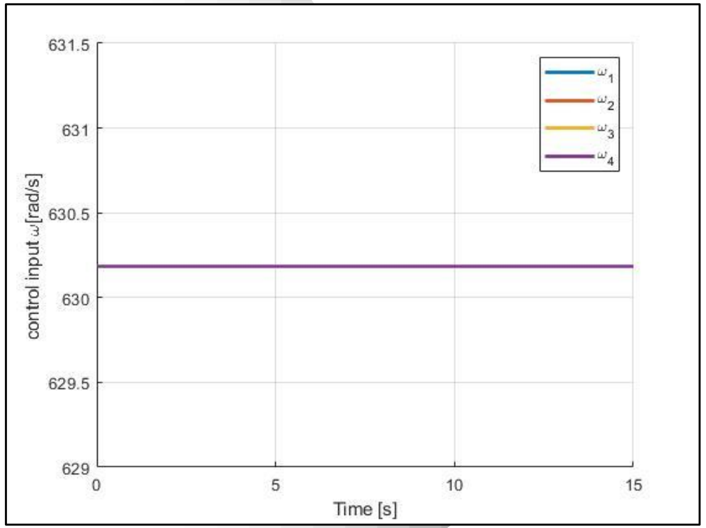
$$\ddot{z}_{des} = -g + \frac{T}{m} - \frac{k_{dz}}{m}\dot{z}_{des}$$

PID STABILIZATION RESULTS

A

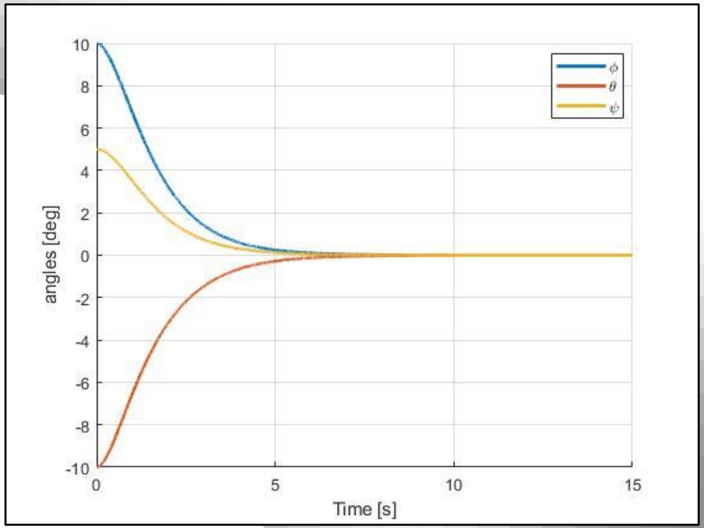


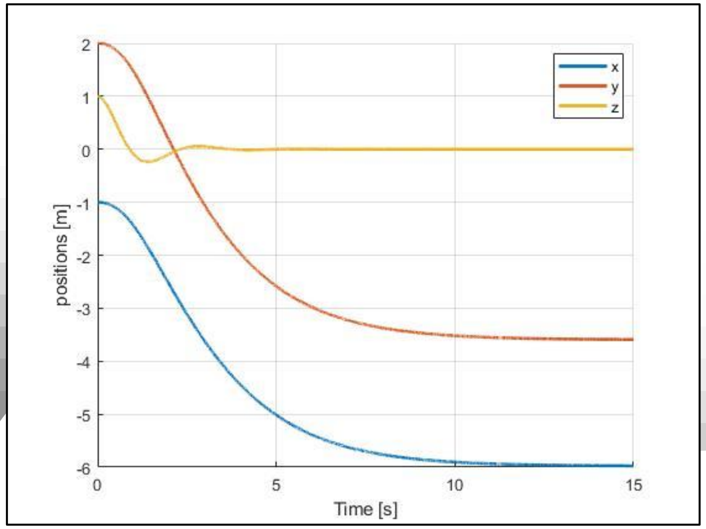


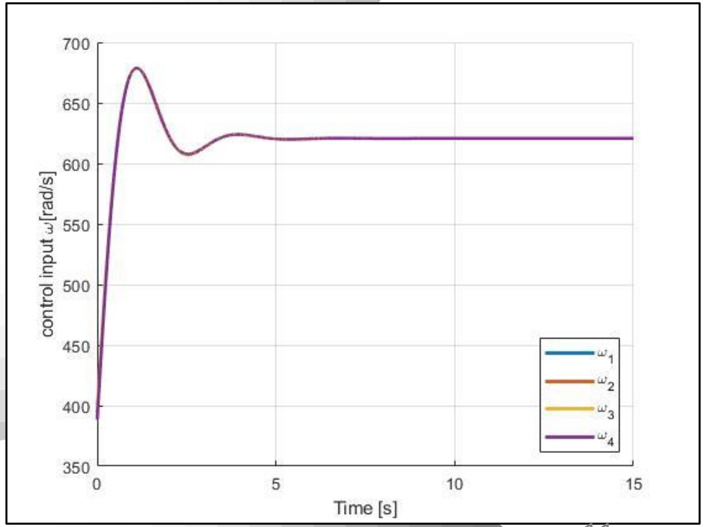


Simulation without PD Control

B



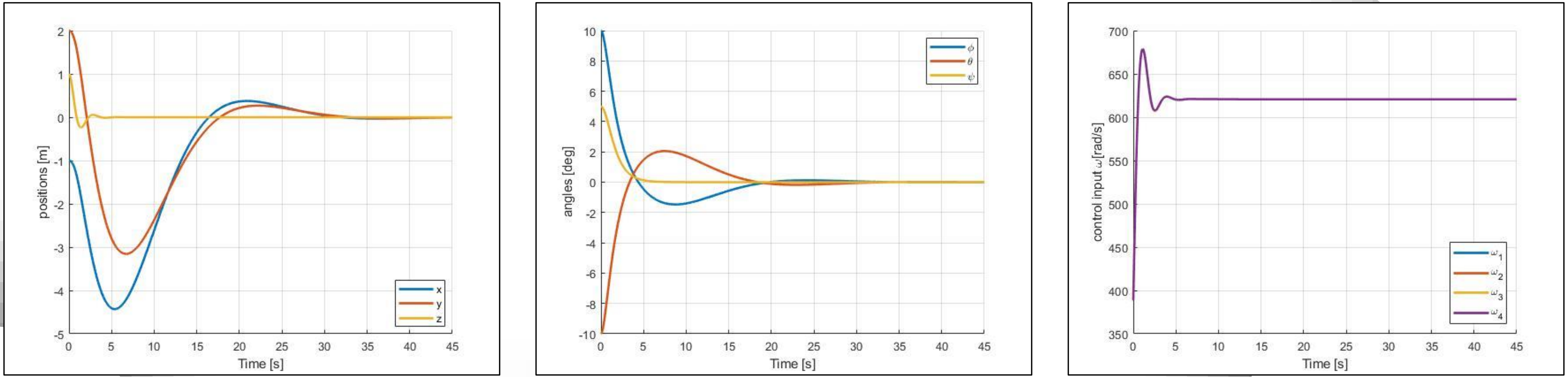




Simulation with PD Control

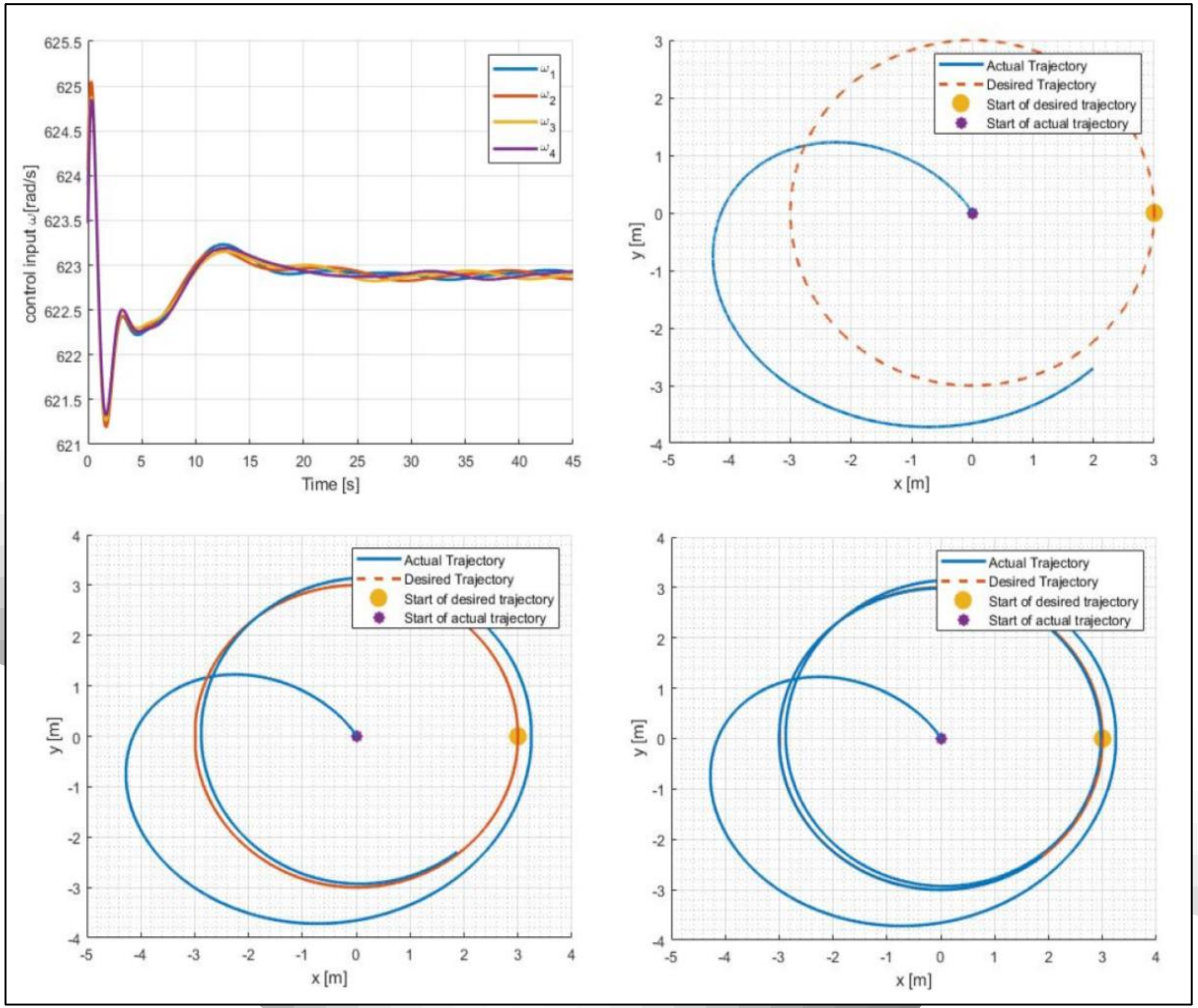
PID STABILIZATION RESULTS

C



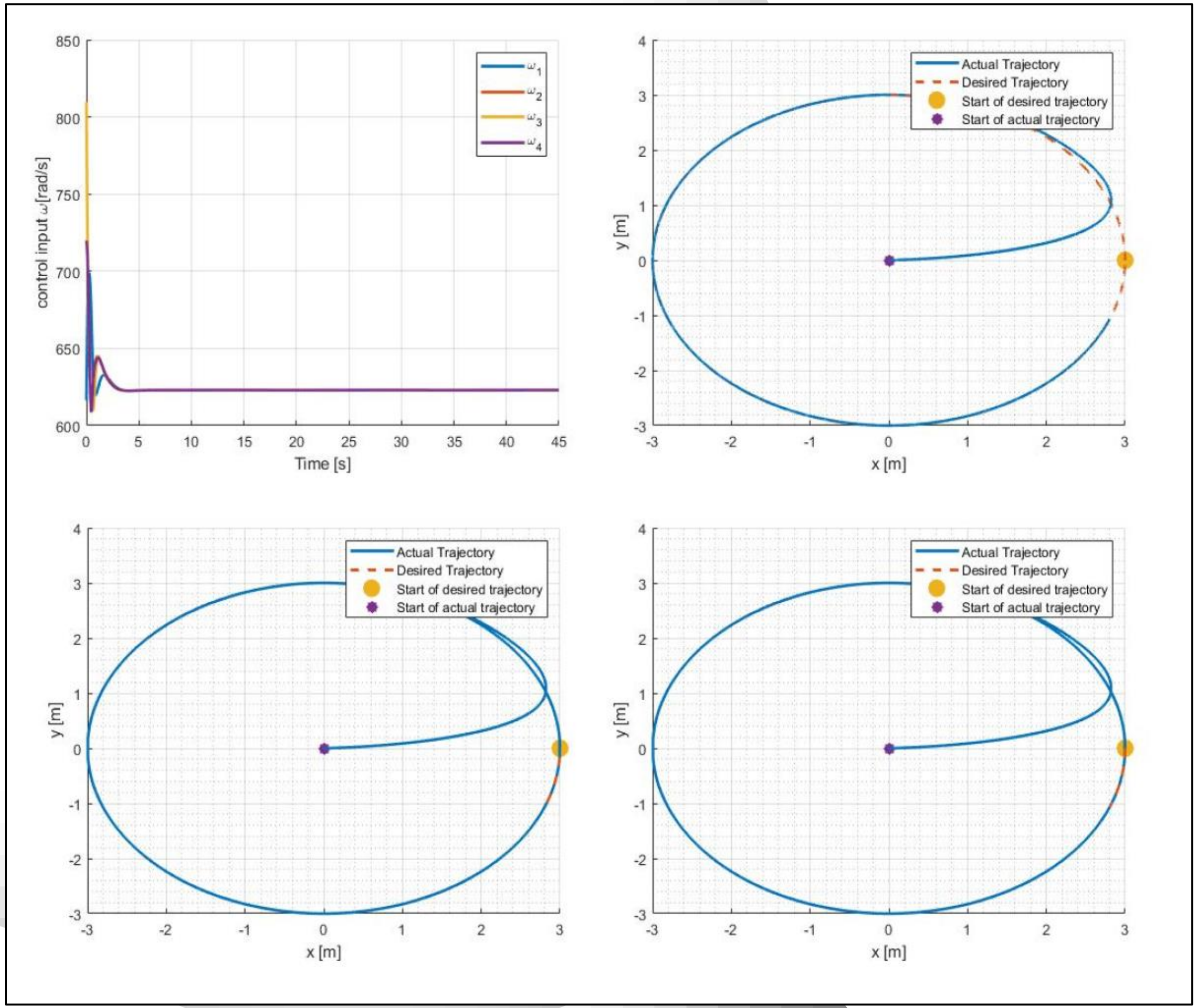
Stabilization with Zero Mapping of Position States

TRAJECTORY TRACKING RESULTS



D

Trajectory Tracking with PD Control



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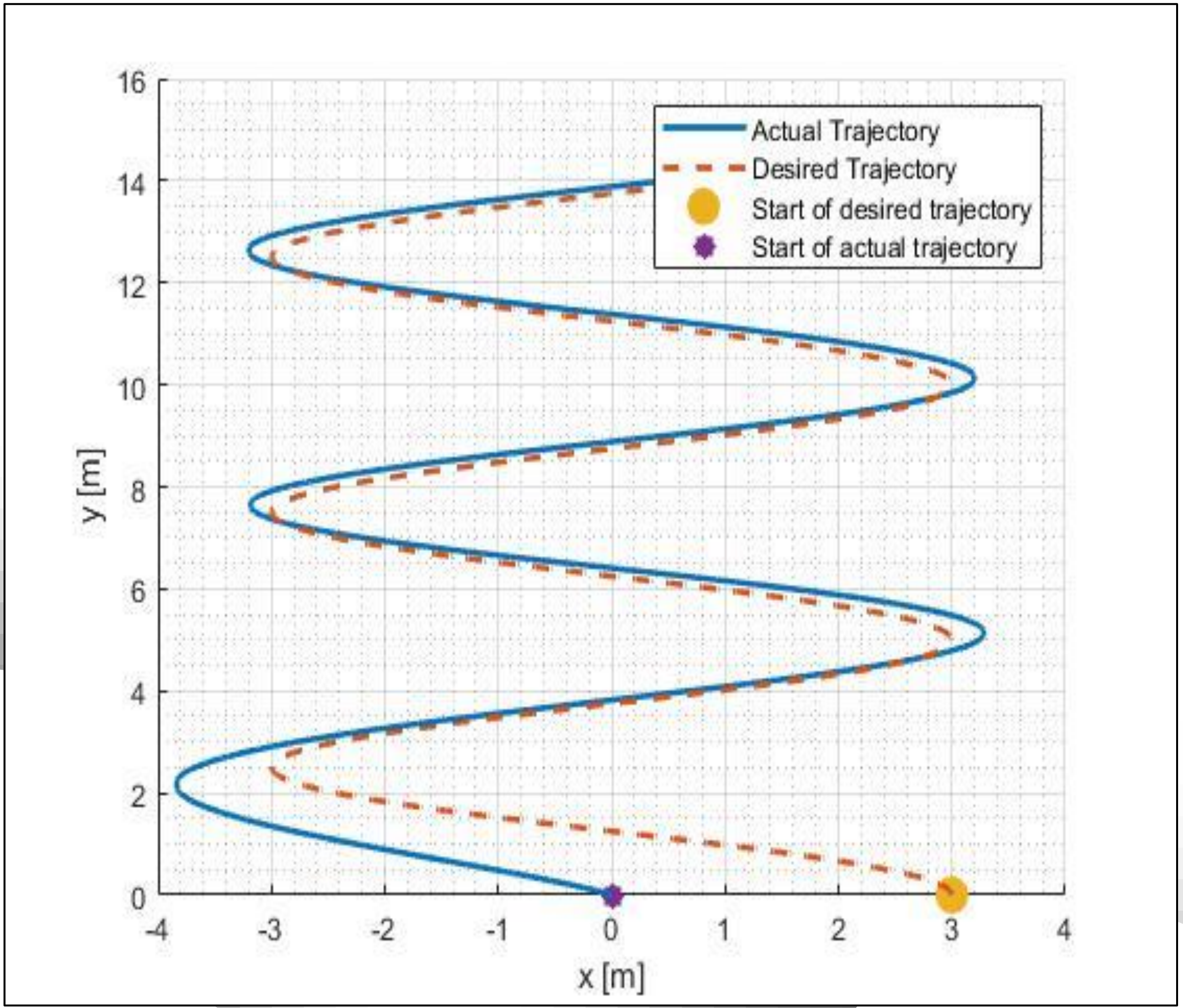
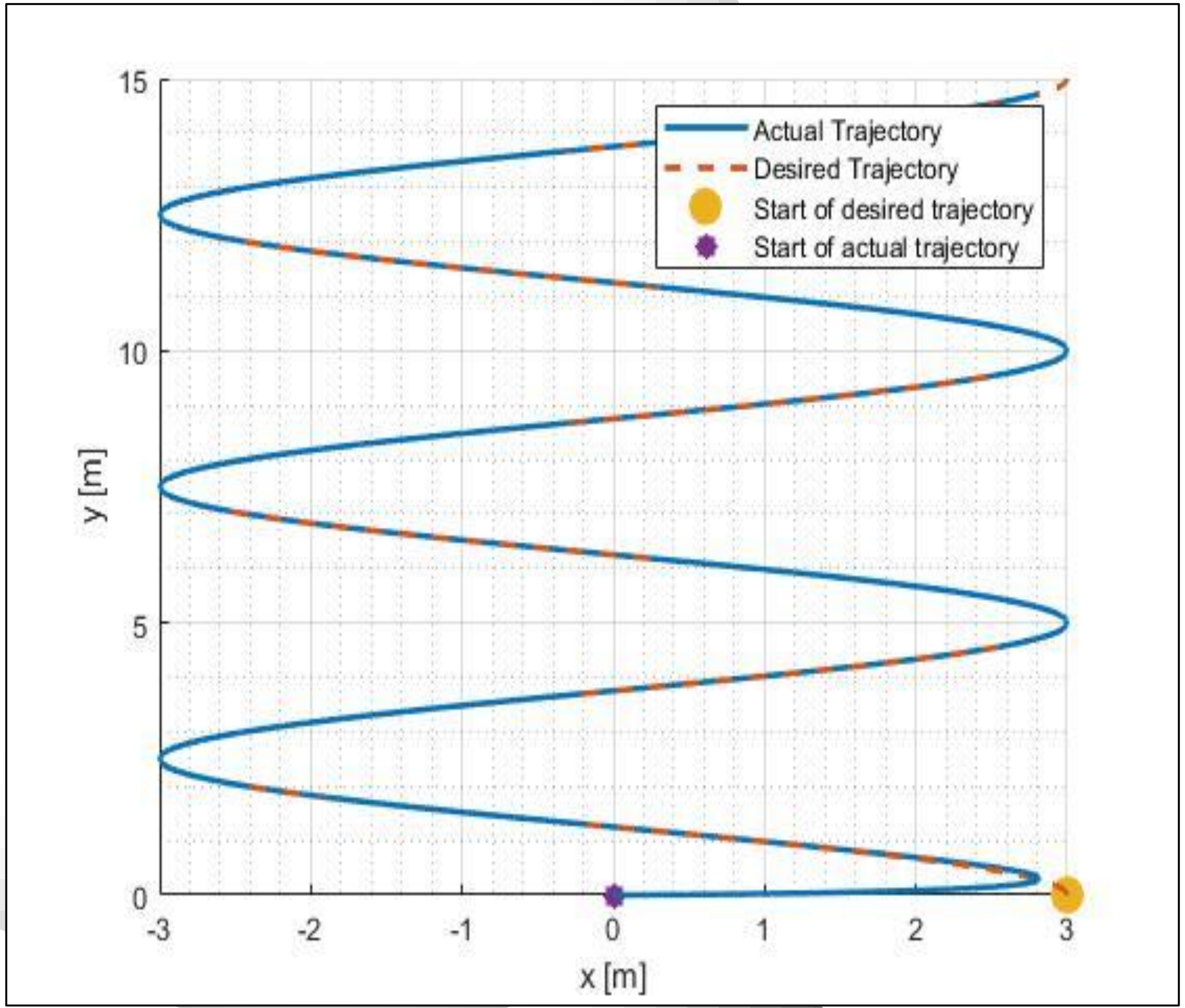
Trajectory Tracking with FBL Control

TRAJECTORY TRACKING RESULTS



F

Trajectory Tracking with PD Control



G

Trajectory Tracking with FBL Control

ACHIEVEMENT AND CONCLUSION

The aim and objectives stated at the beginning of the project have been achieved.

- This thesis presented a mathematical method of mapping the desired position variables to the desired angles to stabilize all the states including the position variables to zero
- It also provides the complete derivation of the feedback linearized system based on the full state Nonlinear Dynamic Inversion (NDI) technique.
- The FBL system was computed and simulated without small signal approximations.

The results from simulation show that the PD control produces satisfactory results for system stabilization but fails to do so for trajectory tracking of the quadcopter. Hence the need of a different control technique called the feedback linearization control which produces excellent results for trajectory tracking of the quadcopter system.

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Thank you!!! ...