n-dim Gaussian

$\gamma = \sqrt{n+\lambda}$

$\lambda = \alpha^2(n+\kappa) - n$

α, κ are scaling parameters

Algorithm UKF_localization($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, m$):

Generate augmented mean and covariance

$$M_t = \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2 \omega_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 \omega_t^2 \end{pmatrix}$$

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{pmatrix}$$

zero-mean motion noise

zero-mean measurement noise

$$\mu_{t-1}^a = (\overset{3\times 1}{\mu_{t-1}^T} \quad \overset{2\times 1}{(0\ 0)^T} \quad \overset{2\times 1}{(0\ 0)^T})^T$$ — augmented state (7×1) L = 7

$$\Sigma_{t-1}^a = \begin{pmatrix} \overset{3\times 3}{\Sigma_{t-1}} & \overset{3\times 2}{\mathbf{0}} & \overset{3\times 2}{\mathbf{0}} \\ \overset{2\times 3}{\mathbf{0}} & \overset{2\times 2}{M_t} & \overset{2\times 2}{\mathbf{0}} \\ \overset{2\times 3}{\mathbf{0}} & \overset{2\times 2}{\mathbf{0}} & \overset{2\times 2}{Q_t} \end{pmatrix}$$ — cov. of aug. state est. error (7×7)

Generate sigma points

7×15

$$\mathcal{X}_{t-1}^a = (\mu_{t-1}^a \quad \mu_{t-1}^a + \gamma\sqrt{\Sigma_{t-1}^a} \quad \mu_{t-1}^a - \gamma\sqrt{\Sigma_{t-1}^a})$$ 15 sigma points — 2L+1

Pass sigma points through motion model and compute Gaussian statistics

control noise comes in here

$$\bar{\mathcal{X}}_t^x = g(u_t + \mathcal{X}_t^u, \mathcal{X}_{t-1}^x)$$ propogate state sigma pts from (t-1) → t

$$\bar{\mu}_t = \sum_{i=0}^{2L} w_i^{(m)} \bar{\mathcal{X}}_{i,t}^x$$ — weighted mean of σ-pt state

$$\bar{\Sigma}_t = \sum_{i=0}^{2L} w_i^{(c)} (\bar{\mathcal{X}}_{i,t}^x - \bar{\mu}_t)(\bar{\mathcal{X}}_{i,t}^x - \bar{\mu}_t)^T$$ weighted cov of σ-pt state

Predict observations at sigma points and compute Gaussian statistics

sensor noise comes in here

$$\bar{\mathcal{Z}}_t = h(\bar{\mathcal{X}}_t^x) + \mathcal{X}_t^z$$ measurement σ-pts

model uncertainty effect on measurement uncertainty

$$\hat{z}_t = \sum_{i=0}^{2L} w_i^{(m)} \bar{\mathcal{Z}}_{i,t}$$ — mean of predicted meas.

$$S_t = \sum_{i=0}^{2L} w_i^{(c)} (\bar{\mathcal{Z}}_{i,t} - \hat{z}_t)(\bar{\mathcal{Z}}_{i,t} - \hat{z}_t)^T$$ — cov. of predicted meas.

$$\Sigma_t^{x,z} = \sum_{i=0}^{2L} w_i^{(c)} (\bar{\mathcal{X}}_{i,t}^x - \bar{\mu}_t)(\bar{\mathcal{Z}}_{i,t} - \hat{z}_t)^T$$ — cross cov. between state and observations (meas)

cross-correlation between uncertainty in x and z

Update mean and covariance

$$K_t = \Sigma_t^{x,z}\, S_t^{-1}$$ — Kalman gain

$$\mu_t = \bar{\mu}_t + K_t(z_t - \hat{z}_t)$$ — update location estimate — meas. update

$$\Sigma_t = \bar{\Sigma}_t - K_t\, S_t\, K_t^T$$ — update location cov. est.

$$p_{z_t} = \det(2\pi S_t)^{-\frac{1}{2}} \exp\left\{-\tfrac{1}{2}\,(z_t - \hat{z}_t)^T S_t^{-1}(z_t - \hat{z}_t)\right\}$$

return μ_t, Σ_t, p_{z_t}

Table 7.4 The unscented Kalman filter (UKF) localization algorithm, formulated here for a feature-based map and a robot equipped with sensors for measuring range and bearing. This version handles single feature observations only and assumes knowledge of the exact correspondence. L is the dimensionality of the augmented state vector, given by the sum of state, control, and measurement dimensions.