μ_t = estimate of $(x_t\ y_t\ \theta_t)$

$u_t = (v_t\ \ \omega_t)^T$

$x_t = (x_t\ \ y_t\ \ \theta_t)^T$

scalar

vector

Algorithm EKF_localization_known_correspondences($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t, m$):

$\theta = \mu_{t-1,\theta}$

$$G_t = \begin{pmatrix} 1 & 0 & -\frac{v_t}{\omega_t}\cos\theta + \frac{v_t}{\omega_t}\cos(\theta+\omega_t\Delta t) \\ 0 & 1 & -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta+\omega_t\Delta t) \\ 0 & 0 & 1 \end{pmatrix}$$

Jacobian of $g(u_t, x_{t-1})$

$$V_t = \begin{pmatrix} \frac{-\sin\theta+\sin(\theta+\omega_t\Delta t)}{\omega_t} & \frac{v_t(\sin\theta-\sin(\theta+\omega_t\Delta t))}{\omega_t^2} + \frac{v_t\cos(\theta+\omega_t\Delta t)\Delta t}{\omega_t} \\ \frac{\cos\theta-\cos(\theta+\omega_t\Delta t)}{\omega_t} & -\frac{v_t(\cos\theta-\cos(\theta+\omega_t\Delta t))}{\omega_t^2} + \frac{v_t\sin(\theta+\omega_t\Delta t)\Delta t}{\omega_t} \\ 0 & \Delta t \end{pmatrix}$$

Jacobian to map noise in control (input) space to state space

$$M_t = \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2\omega_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4\omega_t^2 \end{pmatrix}$$

$$\bar\mu_t = \mu_{t-1} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta+\omega_t\Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta+\omega_t\Delta t) \\ \omega_t\Delta t \end{pmatrix}$$

$\bar\Sigma_t = G_t\,\Sigma_{t-1}\,G_t^T + V_t\,M_t\,V_t^T$

uncertainty due to from motion noise

from position uncertainty at time (t-1)

uncertainty due to measurement noise

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

we won't consider signature of landmark

for all observed features $z_t^i = (r_t^i\ \phi_t^i\ s_t^i)^T$ *do*

$j = c_t^i$ — ith feature at time t corresponds to jth landmark in map

$q = (m_{j,x} - \bar\mu_{t,x})^2 + (m_{j,y} - \bar\mu_{t,y})^2$

predicted measurement based on state estimate

$$\hat z_t^i = \begin{pmatrix} \sqrt{q} \\ \operatorname{atan2}(m_{j,y}-\bar\mu_{t,y}, m_{j,x}-\bar\mu_{t,x}) - \bar\mu_{t,\theta} \\ m_{j,s} \end{pmatrix}$$

range

bearing

$$H_t^i = \begin{pmatrix} -\frac{m_{j,x}-\bar\mu_{t,x}}{\sqrt{q}} & -\frac{m_{j,y}-\bar\mu_{t,y}}{\sqrt{q}} & 0 \\ \frac{m_{j,y}-\bar\mu_{t,y}}{q} & -\frac{m_{j,x}-\bar\mu_{t,x}}{q} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

Jacobian of h wrt x_t

uncertainty in measurement due to uncertainty in robot location

$S_t^i = H_t^i\,\bar\Sigma_t\,[H_t^i]^T + Q_t$ — uncertainty due to measurement noise

uncertainty in $\hat z_t$

$K_t^i = \bar\Sigma_t\,[H_t^i]^T[S_t^i]^{-1}$ — the lower the uncertainty, the higher the gain!

Kalman gain

$\bar\mu_t = \bar\mu_t + K_t^i(z_t^i - \hat z_t^i)$ — Kalman gain maps innovation (in measurement space) into state space.

$\bar\Sigma_t = (I - K_t^i\,H_t^i)\,\bar\Sigma_t$

update to uncertainty based on measurement update

endfor

$\mu_t = \bar\mu_t$

$\Sigma_t = \bar\Sigma_t$

$p_{z_t} = \prod_i \det\left(2\pi S_t^i\right)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\,(z_t^i-\hat z_t^i)^T[S_t^i]^{-1}(z_t^i-\hat z_t^i)\right\}$

return μ_t, Σ_t, p_{z_t}

Table 7.2 The extended Kalman filter (EKF) localization algorithm, formulated here for a feature-based map and a robot equipped with sensors for measuring range and bearing. This version assumes knowledge of the exact correspondences.