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Problem 1:

$$h_v^{(t+1)} = h_v^{(t)} + \sum_{u \in N(v)} h_u^{(t)}$$

$k = 2$:

$$h_{red}^{(2)} = h_{red}^{(1)} + h_5^{(1)} = (h_{red}^{(0)} + h_5^{(0)}) + (h_5^{(0)} + h_4^{(0)} + h_6^{(0)} + h_{red}^{(0)})$$

→ similar for both graphs

$k = 3$:

left →

$$h_{red}^{(3)} = h_{red}^{(2)} + h_5^{(2)} = (h_{red}^{(1)} + h_5^{(1)}) + (h_5^{(1)} + h_4^{(1)} + h_6^{(1)} + h_{red}^{(1)})$$

$$= [(h_{red}^{(0)} + h_5^{(0)}) + (h_5^{(0)} + h_4^{(0)} + h_6^{(0)} + h_{red}^{(0)})]$$

$$+ [(h_5^{(0)} + h_4^{(0)} + h_6^{(0)} + h_{red}^{(0)}) + (h_4^{(0)} + h_5^{(0)} + h_3^{(0)}) + (h_6^{(0)} + h_5^{(0)} + h_7^{(0)})] = [16]$$

right →

$$h_{red}^{(3)} = [(h_{red}^{(0)} + h_5^{(0)}) + (h_5^{(0)} + h_4^{(0)} + h_6^{(0)} + h_{red}^{(0)})]$$

$$+ [(h_5^{(0)} + h_4^{(0)} + h_6^{(0)} + h_{red}^{(0)}) + (h_4^{(0)} + h_5^{(0)} + h_3^{(0)}) + (h_6^{(0)} + h_5^{(0)} + h_7^{(0)} + h_8^{(0)})] = [17]$$

left and right graphs don't have similar embeddings after 3 message passings.

Problem 2:

$$H^{(l)} = (D^{-1}A)\, H^{(l-1)} = (D^{-1}A)^l H^{(0)}$$ → effects of nodes becomes less as $l \to \infty$

if $H_i^{(k)} = C \Rightarrow H_i^{(k+1)} = \frac{1}{|N(i)|} \times C\,|N(i)| = C \quad \forall i$

With respect to the hint, we can consider this as a Markov chain, transition matrix being $D^{-1}A$. $D^{-1}A$ is irreducible and can take us from any state to any other state. It is also aperiodic because starting from state i, we don't know when we will return to the same state after some transition.