# Continuous Random Variables

**uniform$[a,b]$**

$f_X(x) = \dfrac{1}{b-a}$ and $F_X(x) = \dfrac{x-a}{b-a}$, $a \le x \le b$.

$\mathsf{E}[X] = \dfrac{a+b}{2}$, $\mathsf{var}(X) = \dfrac{(b-a)^2}{12}$, $M_X(s) = \dfrac{e^{sb}-e^{sa}}{s(b-a)}$.

**exponential, $\exp(\lambda)$**

$f_X(x) = \lambda e^{-\lambda x}$ and $F_X(x) = 1 - e^{-\lambda x}$, $x \ge 0$.

$\mathsf{E}[X] = 1/\lambda$, $\mathsf{var}(X) = 1/\lambda^2$, $\mathsf{E}[X^n] = n!/\lambda^n$.

$M_X(s) = \lambda/(\lambda - s)$, $\operatorname{Re} s < \lambda$.

**Laplace$(\lambda)$**

$f_X(x) = \frac{\lambda}{2} e^{-\lambda|x|}$.

$\mathsf{E}[X] = 0$, $\mathsf{var}(X) = 2/\lambda^2$. $M_X(s) = \lambda^2/(\lambda^2 - s^2)$, $-\lambda < \operatorname{Re} s < \lambda$.

**Gaussian or normal, $N(m, \sigma^2)$**

$f_X(x) = \dfrac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\dfrac{1}{2}\left(\dfrac{x-m}{\sigma}\right)^2\right]$. $F_X(x) = \texttt{normcdf(x, m, sigma)}$.

$\mathsf{E}[X] = m$, $\mathsf{var}(X) = \sigma^2$, $\mathsf{E}[(X-m)^{2n}] = 1 \cdot 3 \cdots (2n-3)(2n-1)\sigma^{2n}$,

$M_X(s) = e^{sm + s^2\sigma^2/2}$.

**gamma$(p, \lambda)$**

$f_X(x) = \lambda \dfrac{(\lambda x)^{p-1} e^{-\lambda x}}{\Gamma(p)}$, $x > 0$, where $\Gamma(p) := \int_0^\infty x^{p-1} e^{-x}\,dx$, $p > 0$.

Recall that $\Gamma(p) = (p-1) \cdot \Gamma(p-1)$, $p > 1$.

$F_X(x) = \texttt{gamcdf(x, p, 1/lambda)}$.

$\mathsf{E}[X^n] = \dfrac{\Gamma(n+p)}{\lambda^n \Gamma(p)}$. $M_X(s) = \left(\dfrac{\lambda}{\lambda - s}\right)^p$, $\operatorname{Re} s < \lambda$.

Note that gamma$(1, \lambda)$ is the same as $\exp(\lambda)$.

## Continuous Random Variables (continued)

---

**Erlang($m, \lambda$) := gamma($m, \lambda$), $m$ = integer**

Since $\Gamma(m) = (m-1)!$

$$f_X(x) = \lambda \frac{(\lambda x)^{m-1}\, e^{-\lambda x}}{(m-1)!} \quad \text{and} \quad F_X(x) = 1 - \sum_{k=0}^{m-1} \frac{(\lambda x)^k}{k!} e^{-\lambda x}, \quad x > 0.$$

Note that Erlang($1, \lambda$) is the same as exp($\lambda$).

---

**chi-squared($k$) := gamma($k/2, 1/2$)**

If $k$ is an even integer, then chi-squared($k$) is the same as Erlang($k/2, 1/2$).
Since $\Gamma(1/2) = \sqrt{\pi}$,

for $k = 1$, $f_X(x) = \dfrac{e^{-x/2}}{\sqrt{2\pi x}}, \quad x > 0.$

Since $\Gamma\left(\frac{2m+1}{2}\right) = \frac{(2m-1)\cdots 5\cdot 3\cdot 1}{2^m}\sqrt{\pi}$,

for $k = 2m+1$, $f_X(x) = \dfrac{x^{m-1/2} e^{-x/2}}{(2m-1)\cdots 5\cdot 3\cdot 1\sqrt{2\pi}}, \quad x > 0.$

$F_X(x) = \texttt{chi2cdf(x,k)}$.

Note that chi-squared(2) is the same as exp(1/2).

---

**Rayleigh($\lambda$)**

$f_X(x) = \dfrac{x}{\lambda^2} e^{-(x/\lambda)^2/2}$ and $F_X(x) = 1 - e^{-(x/\lambda)^2/2}, \quad x \geq 0.$

$\mathsf{E}[X] = \lambda\sqrt{\pi/2}, \quad \mathsf{E}[X^2] = 2\lambda^2, \quad \mathsf{var}(X) = \lambda^2(2 - \pi/2).$

$\mathsf{E}[X^n] = 2^{n/2}\lambda^n \Gamma(1 + n/2).$

---

**Weibull($p, \lambda$)**

$f_X(x) = \lambda p x^{p-1} e^{-\lambda x^p} \quad \text{and} \quad F_X(x) = 1 - e^{-\lambda x^p}, \quad x > 0.$

$\mathsf{E}[X^n] = \dfrac{\Gamma(1 + n/p)}{\lambda^{n/p}}.$

Note that Weibull($2, \lambda$) is the same as Rayleigh$\left(1/\sqrt{2\lambda}\right)$ and that Weibull($1, \lambda$) is the same as exp($\lambda$).

---

**Cauchy($\lambda$)**

$f_X(x) = \dfrac{\lambda/\pi}{\lambda^2 + x^2}, \quad F_X(x) = \dfrac{1}{\pi}\tan^{-1}\left(\dfrac{x}{\lambda}\right) + \dfrac{1}{2}.$

$\mathsf{E}[X] = \text{undefined}, \quad \mathsf{E}[X^2] = \infty, \quad \varphi_X(\nu) = e^{-\lambda|\nu|}.$

Odd moments are not defined; even moments are infinite. Since the first moment is not defined, central moments, including the variance, are not defined.

---