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Discrete Random Variables

Bernoulli(p)

$\wp(X=1)=p, \quad \wp(X=0)=1-p.$
$\mathsf{E}[X]=p, \quad \mathsf{var}(X)=p(1-p), \quad G_X(z)=(1-p)+pz.$

$E[X^2] = VAR[X] + E[X]^2$
$= p(1-p) + p^2 = p - p^2 + p^2$
$= p$

binomial(n, p)

$\wp(X=k)=\binom{n}{k}p^k(1-p)^{n-k}, \quad k=0,\ldots,n.$

$\mathsf{E}[X]=np, \quad \mathsf{var}(X)=np(1-p), \quad G_X(z)=[(1-p)+pz]^n.$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

geometric$_0$(p)

$\wp(X=k)=(1-p)p^k, \quad k=0,1,2,\ldots.$

$\mathsf{E}[X]=\frac{p}{1-p}, \quad \mathsf{var}(X)=\frac{p}{(1-p)^2}, \quad G_X(z)=\frac{1-p}{1-pz}.$

geometric$_1$(p)

$\wp(X=k)=(1-p)p^{k-1}, \quad k=1,2,3,\ldots.$

$\mathsf{E}[X]=\frac{1}{1-p}, \quad \mathsf{var}(X)=\frac{p}{(1-p)^2}, \quad G_X(z)=\frac{(1-p)z}{1-pz}.$

negative binomial or Pascal(m, p)

$\wp(X=k)=\binom{k-1}{m-1}(1-p)^m p^{k-m}, \ k=m,m+1,\ldots.$

$\mathsf{E}[X]=\frac{m}{1-p}, \quad \mathsf{var}(X)=\frac{mp}{(1-p)^2}, \quad G_X(z)=\left[\frac{(1-p)z}{1-pz}\right]^m.$

Note that Pascal($1,p$) is the same as geometric$_1$(p).

Poisson(λ)

$\wp(X=k)=\frac{\lambda^k e^{-\lambda}}{k!}, \quad k=0,1,\ldots.$

$\mathsf{E}[X]=\lambda, \quad \mathsf{var}(X)=\lambda, \quad G_X(z)=e^{\lambda(z-1)}.$