Topics in Combinatorics

Adam Kelly (ak2316@cam.ac.uk)

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It is relatively hard to pin down exactly what Combinatorics is. Many different types of problems are studied, usually with some recurring characters: graphs, hypergraphs, sets of numbers, sets of points, subsets of $\{0,1\}^n$, subsets of a group, and so on. These objects all have a 'subset of a structured object' flavour, and it is this type of object that combinatorics usually deals with. In this article, we will be more concerned with various *techniques* used in combinatorics.

This article constitutes my notes for the 'Topics in Combinatorics' course, held in Lent 2021 at Cambridge and lectured by Prof. Tim Gowers. These notes are *not a transcription of the lectures*, and differ significantly in quite a few areas. Still, all lectured material should be covered.

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§1 Averaging Arguments

A seemingly obvious fact[1] in probability is that for a random variable X we have

$$\mathbb{P}[X \geq \mathbb{E}X] > 0.$$

Despite being basic, this fact is incredibly useful in combinatorics. In the discrete case, the way we use it is to say that it is possible for X to be at least its mean. This sort of argument is known as an *averaging argument*, and in this section we are going to look at a few ways to apply it.

[1] For discrete random variables, this is actually trivial, but needs a little work for continuous random variables.