Quantum Mechanics

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October 18, 2021

Quantum mechanics is the mathematical framework used to describe nature at the scale of subatomic particles. This course will primarily focus on building up this mathematical framework, and not so much on the actual applications or physical background to it.

This article constitutes my notes for the 'Quantum Mechanics' course, held in Michaelmas 2021 at Cambridge. These notes are *not a transcription of the lectures*, and differ significantly in quite a few areas. Still, all lectured material should be covered.

Contents

§1 Foundations of Quantum Mechanics

§1.1 Wavefunctions and Probabilistic Interpretation

How do we describe a point particle? In classical mechanics, we had a vector $\mathbf{x}$ and a vector $\mathbf{p}$ which gave the position and momentum of the particle.

The corresponding description in quantum mechanics is the state ψ, described[1] of by the complex valued *wavefunction* $\psi(\mathbf{x}, t)$.

Definition 1.1 (Wavefunction)
A **wavefunction** $\psi(\mathbf{x}, t)$ is a complex valued function $\mathbb{R}^3 \times \mathbb{R} \to \mathbb{C}$ that satisfies the mathematical properties dictated by its physical interpretation.

To see how the wavefunction related to the underlying physical object, we need to have some sort of interpretation of it. This is given by the *Born rule*.

Law 1.2 (The Born Rule)
The probability density for a particle to be at $\mathbf{x}$ at a time t is given by

$$\rho(\mathbf{x}, t) = |\psi(\mathbf{x}, t)|^2.$$

[1] The state ψ should be thought of as an abstract entity, with the wavefunction $\psi(\mathbf{x}, t)$ being the complex coefficients of ψ in a continuous basis.

The Born rule immediately imposes some restrictions on the wavefunction. Because the particle has to be *somewhere* with probability 1, the wavefunction has normalized such that

$$\int_{\mathbb{R}^3} |\psi(\mathbf{x}, t)|^2 \mathrm{d}V = 1.$$

It's sometimes useful to work with unnormalized wavefunctions (where we just impose the restriction that this integral is finite), but it must be remembered that the underlying physical interpretation requires this to be normalized.