Dynamics and Relativity

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This set of notes is a work-in-progress account of the course 'Dynamics and Relativity', originally lectured by Prof Peter Haynes in Lent 2020 at Cambridge. These notes are not a transcription of the lectures, but they do roughly follow what was lectured (in content and in structure).

These notes are my own view of what was taught, and should be somewhat of a superset of what was actually taught. I frequently provide different explanations, proofs, examples, and so on in areas where I feel they are helpful. Because of this, this work is likely to contain errors, which you may assume are my own. If you spot any or have any other feedback, I can be contacted at ak2316@cam.ac.uk.

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1 Newtonian Dynamics – Basic Concepts

A central aspect of this course is Newtonian dynamics. In this chapter we will develop some of the ideas and definitions needed to discuss this in detail.

§1.1 Particles

When dealing with Newtonian dynamics, we will often use and refer to *particles*, as a way of describing phenomina.

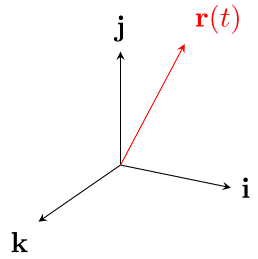
Definition 1.1.1 (Particle)

A **particle** is an object of negligible size. It has some mass $m > 0$, and can also have other properties such as (perhaps) an electric charge q.

A particle is completely described by a **position vector**, usually denoted $\mathbf{r}(t)$ or $\mathbf{x}(t)$, with respect to some origin O. The cartesian coordinates of $\mathbf{r}$ are (x, y, z), where

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k},$$

with $\mathbf{i}, \mathbf{j}, \mathbf{k}$ being orthonormal basis vectors.



The choice of coordinate axes defines a **frame of reference** S.

Of course, we will be considering particles that are moving, so we will define the velocity, momentum and acceleration of the particle.

Definition 1.1.2 (Velocity)

The **velocity** of a particle is $\mathbf{u}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{r}(t) = \dot{\mathbf{r}}$, and is tangent to the path (or trajectory) of the particle.

Definition 1.1.3 (Momentum)

The **momentum** of a particle is $\mathbf{p} = m\mathbf{u} = m\dot{\mathbf{r}}$, where m is the mass of the particle.

Definition 1.1.4 (Acceleration)

The **acceleration** of the particle is $\dot{\mathbf{u}} = \ddot{\mathbf{r}} = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{r}(t)$.

§1.2 Newton's Laws of Motion

We can now write down Newton's three laws of motion, which govern the motion of particles. All of these statements about particles can be extended to finite bodies (which are composed of many particles).

Law (Newton's First Law/Galileo's Law of Inertia)

There exist inertial frames of reference in which a particle remains at rest or moves at constant velocity unless it is acted on by a force.

Law (Newton's Second Law)

In an inertial frame the rate of change of momentum of a particle is equal to the force acting on it.

Law (Newton's Third Law)

To every action there is an equal and opposite reaction. That is, forces excreted between two particles are equal in magnitude and opposite in direction.