Topology

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November 18, 2021

This is a short description of the course. It should give a little flavour of what the course is about, and what will be roughly covered in the notes.

This article constitutes my notes for the 'Non-Existent' course, held in Hillary 2052 at Cambridge. These notes are *not a transcription of the lectures*, and differ significantly in quite a few areas. Still, all lectured material should be covered.

1 Topological Spaces

The idea of a topological space is to provide just enough structure to a set so that the notion of continuity makes sense. Let's first think about what continuity looks like in a metric space.

Let $f : M \to M'$ be a function between metric spaces. Then the definition of continuity for f looks like this:

We say that f is continuous at $x \in M$ if given $\varepsilon > 0$,
there exists some $\delta > 0$ such that $d_M(x, y) < \delta$
implies that $d_{M'}(f(x), f(y)) < \varepsilon$.

This definition really centers around an idea of 'the functions values being arbitrarily close for sufficiently close points'. Here, the notion of 'closeness' is specified using the metric. But this isn't the only way to specify closeness.

Definition 1.1.

We care a significant amount about X random object.

Definition 1.2 (Random Object)**.** We say that an object X is a **random object** if we literally do not care about what it actually is.

It is trivial to check that all objects you will meet in this course are random objects.