Orbits

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Central Forces

If a force is given by a potential V, depending only on the distance from the origin, then

$$\mathbf{F}(\mathbf{x}) = -\nabla V(|\mathbf{x}|) = -\frac{dV}{dr}\hat{\mathbf{r}},$$

where $\hat{\mathbf{r}} = \mathbf{x}/|\mathbf{x}|$. Such forces are known as *central forces*.

Angular Momentum

An important result when dealing with central forces is the existence of another conserved quantity, *angular momentum*. We define angular momentum to be

$$\mathbf{L} = \mathbf{x} \times \mathbf{p} = m\mathbf{x} \times \dot{\mathbf{x}}.$$

With a general force $\mathbf{F}$, we have $d\mathbf{L}/dt = \mathbf{x} \times \mathbf{F} = \boldsymbol{\tau}$, a quantity named *torque*. If a force is central, then $\mathbf{F} \parallel \mathbf{x}$, and thus $\boldsymbol{\tau} = \mathbf{0}$, and $\mathbf{L}$ is conserved.