Quantum Mechanics

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This article constitutes my notes for the 'Quantum Mechanics' course, held in Michaelmas 2021 at Cambridge. These notes are *not a transcription of the lectures*, and differ significantly in quite a few areas. Still, all lectured material should be covered.

Contents

§1 Historical Introduction

1. *1801-03.* Interference/diffraction experiments (by Young), found that light was a wave.
2. *1862-64.* The development of electromagnetism (by Maxwell), found that light was an electromagnetic wave.
3. *1897.* The discovery of the electron (by Thomson).
4. *1900.* The Plack law, explaining black body radiation.
5. *1905.* The photo-electric effect (by Einstein).
6. *1909.* Wave-light interference patterns with one photon recorded at a time (by G. I. Taylor).
7. *1911.* Rutherford's experiment giving an atomic model.
8. *1913.* Bohr model of the atom.
9. *1923.* The Compton experiment of x-ray scattering off electrons.
10. *1923-24.* Wave-particle duality.
11. *1925-30.* The emergence of Quantum Mechanics (by Heisenberg, Bohr, Jordan, Dirac, Pauli, Schrödinger, . . .).
12. *1927-28.* Diffraction experiment with electrons (by Davisson, Germer, and Thomson).

§1.1 Particles and Waves in Classical Mechanics

We will begin by describing the physical background to particles and waves, as studied in classical mechanics.

A **particle** is an object carrying energy and momentum in an point-like position in space. Particles are determined by two vectors, $\mathbf{x}$ (position) and $\mathbf{v} = \dot{\mathbf{x}}$ (velocity). Newton's second law gave us that

$$m\ddot{\mathbf{x}} = \mathbf{F}(\mathbf{x}(t), \dot{\mathbf{x}}(t)),$$

and solving this equation would then describe the position totally, so long as the initial conditions are known.

A **wave** is any real or complex valued function with periodicity in time and/or space. If the period of the wave is T, then it has frequency $1/T$, and we define the **angular frequency** as $\omega = 2\pi/T$. If we have a function of x in one dimension, $f(x+\lambda) = f(x)$, we call λ the **wavelength** and $k = 2\pi/\lambda$ the **wavenumber**. In three dimensions, we have $f(x) = \exp(i\mathbf{k}\cdot\mathbf{x})$, which we call **plane waves**. We call $\mathbf{k}$ the **wavevector**, and $\lambda = 2\pi/|\mathbf{k}|$.

One dimensional waves obey the wave equation

$$\frac{\partial^2 f(x,t)}{\partial t^2} - c^2\frac{\partial^2 f(x,t)}{\partial x^2} = 0,$$

with $c \in \mathbb{R}$. The solutions are given by

$$f(x,t) = A\exp(\pm ikx - i\omega t),$$

with $\omega = ck$ and $\lambda = cT$ (the dispersion relations). We call A the amplitude.

The three dimensional wave equation is given by

$$\frac{\partial^2 f(\mathbf{x},t)}{\partial t^2} - c^2\nabla^2 f(\mathbf{x},t) = 0,$$

and the solutions are given by

$$f(\mathbf{x},t) = A\exp(i\mathbf{k}\cdot\mathbf{x} - i\omega t),$$

with $\omega = c|\mathbf{k}|$ and $\lambda = cT$.

Other kinds of waves arise as solutions to other governing wave equations, provided other dispersion relations are given. Also for any governing equation that is linear in f, we always have the superposition principle, saying that if f_1, f_2 are solutions to the equation, then so is $f = f_1 + f_2$.

§1.2 Particle-Like Behavior of Light-Waves

If you heat a body to a temperature T, it will emit radiation. The simplest system which this behavior can be studied in is a *black-body*, a totally absorbing surface. In studying such a system, Kirchoff looked at the intensity of light emitted by the black body as a function of the radiation frequency. He observed a