Complex Analysis

Adam Kelly (ak2316@cam.ac.uk)

April 13, 2022

This article constitutes my notes for the 'Complex Analysis' course, held in Lent 2022 at Cambridge. These notes are *not a transcription of the lectures*, and differ significantly in quite a few areas. Still, all lectured material should be covered.

1 Analytic Functions

1.1 Complex Differentiability

This course is about complex valued functions of a (single) complex variable, that is, functions

$$f : A \to \mathbb{C}, \quad \text{where } A \subset \mathbb{C}.$$

In this course we will be particularly interested in functions that are *complex differentiable*.

Definition 1.1. Let $f : U \to \mathbb{C}$ where $U \subseteq \mathbb{C}$ is open. We say that f is **differentiable** at $z \in U$ if the limit

$$f'(z) = \lim_{h \to 0} \frac{f(z+h) - f(z)}{h}$$

exists.

- *Analytic functions.* Complex differentiation and Cauchy-Riemann equations. Examples. Conformal mappings. Informal discussion of branch points, examples of $\log z$ and z^c.
- *Contour Integration and Cauchy's theorem.* Contour integration (for piecewise continuously differentiable curves). Statement and proof of Cauchy's theorem for star domains. Cauchy's integral formula. Maximum modulus theorem. Louiville's theorem. Fundamental theorem of algebra. Morera's theorem.
- *Expansions and singularities.* Uniform convergence of analytic functions. Local uniform convergence. Differentiability of power series. Taylor and Laurent expansions. Principle of isolated zeros. Residue at an isolated singularity. Classification of isolated singularities.
- *Residue Theorem.* Winding numbers. Residue theorem. Jordan's lemma. Evaluation of definite integrals by contour integration. Rouche's theorem, principle of the argument. Open mapping theorem.