Geometry

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1 Surfaces

1.1 Topological Surfaces

Definition 1.1. A **topological surface** is a topological space Σ such that

(i) for all points $p \in \Sigma$, there is an open neighborhood $p \in U \subset \Sigma$ such that U is homeomorphic to $\mathbb{R}^2$ with its usual Euclidean topology.

(ii) Σ is Hausdorff and second countable.

1.2 Examples of Topological Surfaces

- The graph of a function
- The sphere
- The real projective plane
- Torus
- General polygons with identified edges
- Connect sum

The graph of a function

1.3 Subdivisions and Triangulations

Definition 1.2. A **subdivision** of a compact topological surface Σ comprises of a finite subset $V \subseteq \Sigma$ of **vertices**, and a finite collection of continuous embeddings $\{e_i : [0,1] \to \Sigma\}$ called **edges**, each of which has endpoints in C and any two of which are disjoint except perhaps at endpoints. Both of these are such that each connected component of the complement of $V \cup E$ in Σ is homeomorphic to an open disk. We call each component a **face**.

A subdivision is a **triangulation** if the closure of each face contains exactly three edges, and two closed faces meet each-other at exactly one edge, or they don't meet.

Definition 1.3. The **Euler characteristic** of a subdivision is $V - E + F$.

Theorem 1.4. *Every compact topological surface has a subdivision, and the Euler characteristic is invariant under choice of subdivision, and is topologically invariant.*

1.4 Smooth Surfaces

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