Probability

Wednesday, May 19, 2021 7:02 PM

Probability is study of Uncertainty.

P = Fav. Outcome / Total No of Outcomes

Random Experiment : It is the process for which outcome cannot be predicted with certainty.
For Ex. Tossing of Coin, Throwing of Dice etc.

Sample Space : Set of all possibilities of the Random Experiment.

Event : A subset of Sample Space.

R.E.	:	Tossing two coins
Sample Space	:	{ HT , HH, TH, TT}
Event	:	{ HH, HT }

P(E) = 2 / 4

<u>Axioms / Rules of Probability</u> :

0 <= P(E) <= 1

P(E) = n(E) / n(Sample Space)

P(SS) = 1

For any sequence of events that are mutually exclusive :

i.e. $E1 \cap E2 \cap E3 \ldots \cap En = \varphi$

P(E1 U E2 U E3…. U En) = P(E1) + P(E2) + P(E3)……+P(En)

Conditional Probability : Conditional probability is the probability of one event occurring with some relationship to one or more other events.

$P(A \mid B) = P(A \cap B) / P(B)$

<u>Random Variable</u> :

A **random variable** is a **variable** whose value is unknown or a function that assigns values to each of an experiment's outcomes. A **random variable** can be either discrete (having specific values) or continuous (any value in a continuous range).

For Example,

1. Random Exp. : Toss two coins
2. Sample Space : { HH, HT, TH, TT }
3. Random Variable (X) = Count the number of heads = **{ 2, 1, 0 }** (Set must contain

unique values)

4. Probability Distribution (for X) :

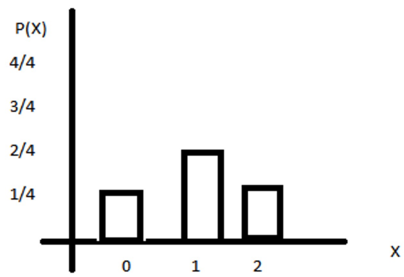
   P(X = 2) = { HH } = 1/4

   P(X = 1) = { HT, TH } = 2/4

   P(X = 0) = { TT } = 1/4

   (This Probability Distribution is used to plot Y-axis in Distplot of Seaborn Library)

5. Plot for Probability Distribution :



Types Of Random Variable :

- Discrete : Countable Random Variables (Countable Infinite Set)
- Continuous : Not Countable (Range of Values) Random Variables (Decimal Values)

Discrete R.V. :

A **discrete random variable** has a countable number of possible values. The probability of each value of a **discrete random variable** is between 0 and 1, and the sum of all the probabilities is equal to 1

- Bernoulli Random Variable : is the simplest kind of random variable. It can take on two values, 1 and 0. It takes on a 1 if an experiment with probability p resulted in success and a 0 otherwise. Some example uses include a coin flip, a random binary digit, whether a disk drive crashed, and whether someone likes a Netflix movie.

  If X is a Bernoulli random variable, denoted X ∼ Ber(p):

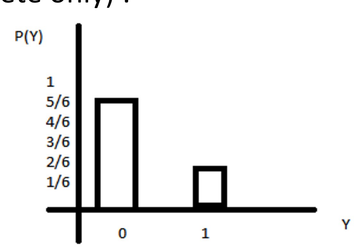
  Probability mass function: P(X = 1) = p

  P(X = 0) = (1 − p)

  Ex. Probability of getting 6 in a dice

1. Random Experiment : Rolling a dice
2. Sample Space : {1,2,3,4,5,6}
3. Random Experiment : {0,1}
4. Probability Distribution of Random Experiment : P(X = 0) = {1,2,3,4,5} = 5/6

   P(X = 1) = {6} = 1/6
5. Probability Mass Function (In Discrete only) :



- Binomial Random Variable : A binomial random variable is random variable that represents the number of successes in n successive independent trials of a Bernoulli experiment. Some example uses include the number of heads in n coin flips, the number of disk drives that crashed in a cluster of 1000 computers, and the number of advertisements that are clicked when 40,000 are served. If X is a Binomial random variable, we denote this $X \sim Bin(n, p)$, where p is the probability of success in a given trial. A binomial random variable has the following properties:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \qquad \text{if } k \in \mathbb{N}, 0 \le k \le n \text{ (0 otherwise)}$$

Binomial R.V. is also called the collection of Bernouli Random Variable. Because it has individual elements which are either success or failure.

Let X = number of heads after a coin is flipped three times. $X \sim Bin(3, 0.5)$. What is the probability of each of the different values of X?

$$P(X = 0) = \binom{3}{0} p^0 (1-p)^3 = \frac{1}{8}$$
$$P(X = 1) = \binom{3}{1} p^1 (1-p)^2 = \frac{3}{8}$$
$$P(X = 2) = \binom{3}{2} p^2 (1-p)^1 = \frac{3}{8}$$
$$P(X = 3) = \binom{3}{3} p^3 (1-p)^0 = \frac{1}{8}$$

Note :

- P = Probability of head in single coin

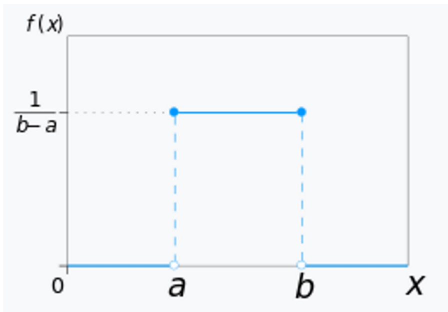
- Continuous Random Variable :

A **continuous random variable** is a **random variable** where the data can take infinitely many values. For example, a **random variable** measuring the time taken for something to be done is **continuous** since there are an infinite number of possible times that can be taken.

- Uniform Random Variable : In this Random Variable, the data is uniformly distributed over the graph and the probability of data is :

Probability Density Function (f(x)) =

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b, \\ 0 & \text{for } x < a \text{ or } x > b \end{cases}$$



Where b : max value, a: min value

- Normal Random Variable :

A **Normal** (or **Gaussian** or **Gauss** or **Laplace–Gauss**) **distribution** is a type of [continuous probability distribution] for a [real-valued] [random variable].

Probability Density Function (f(x)) =

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \text{ for all } x \in \square$$