Diskrete Signale u. Filter

elementare diskrete Signale

Einheitsimpuls

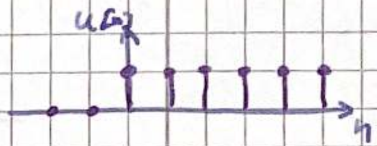
$$\delta[n] = \begin{cases} 1 & \text{für } n = 0 \\ 0 & \text{für } n \neq 0 \end{cases}$$

→ $h[n]$ ist Antwort des Systems auf $\delta[n]$

Einheitssprung

„unit step"

$$u[n] = \begin{cases} 1 & \text{für } n \geq 0 \\ 0 & \text{für } n < 0 \end{cases}$$



Signale zeichnen

$$s\left[\pm \frac{n - n_0}{N}\right]$$

Spiegeln an der y-Achse

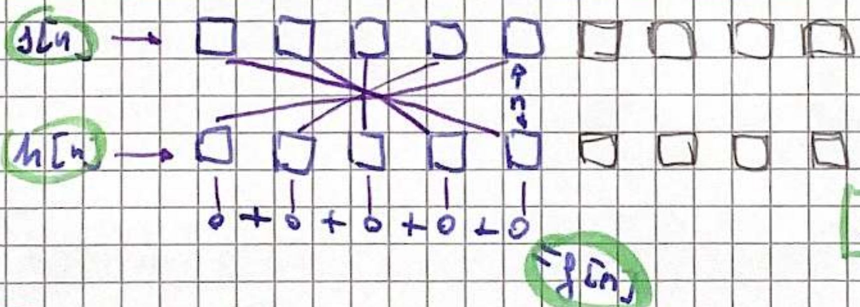
Dehnen $N > 1$ — $N < 1$

Verschieben $n_0 > 0$ — $n_0 < 0$

diskrete Faltung

$$g[n] = s[n] * h[n] = \sum_{k=-\infty}^{\infty} s[k] \cdot h[n-k]$$

- kommutativ
- assoziativ
- distributiv



$$s[n] * \delta[n-T] = s[n-T]$$

Bsp. $s[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

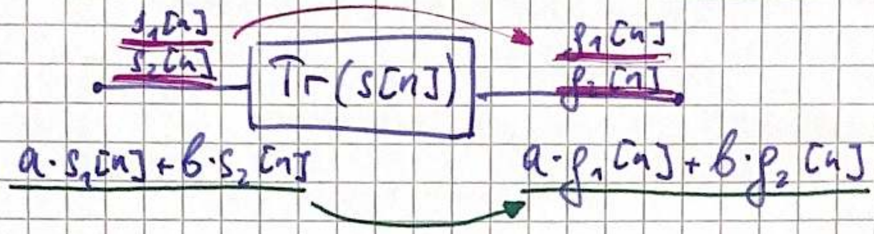
$h[n] = \delta[n-1] + 2\delta[n-2]$

n	<0	0	1	2	3	4	5	6
$s[n]$	0	1	2	1	0	0	0	0
$h[n]$	0	0	1	2	0	0	0	0
$s[0] \cdot h[n]$	0	0	1	2	0	0	0	0
$s[1] \cdot h[n-1]$	0	0	0	2	4	0	0	0
$s[2] \cdot h[n-2]$	0	0	0	0	1	2	0	0
$g[n]$	0	0	1	4	5	2	0	0

LSI-Systeme

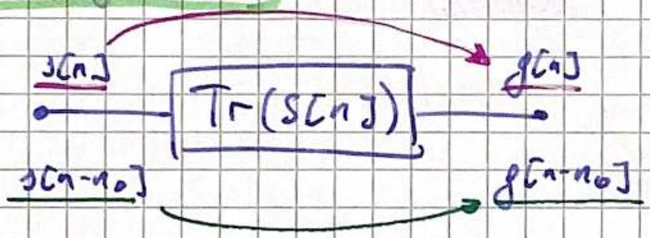
linear

$$Tr\left\{\sum_i s_i[n]\right\} = \sum_i Tr\{s_i[n]\} = \sum_i g_i[n]$$



Verschiebungsinvariant / Shift-invariant

$$s[n-n_0] = g[n-n_0]$$



Kausal

$h[n] = 0$ für $n<0$

z.B. gleitende Mittelwert- u. Differenzfilter

FIR-Filter (finite impulse response): $h[n]$ endlich

Übertragung von Signalen

- $H(\hat{\omega}) \rightarrow |H(\hat{\omega})| \cdot e^{j\vartheta(\hat{\omega})}$

① Ausschreiben
② Symmetrie
③ Euler

$$\boxed{\begin{aligned} H(\hat{\omega}) &= e^{-j\hat{\omega}} + 2e^{-j3\hat{\omega}} + \overset{\text{Mitte}}{9e^{-j4\hat{\omega}}} + 2e^{-j5\hat{\omega}} + e^{-j7\hat{\omega}} \\ &= e^{-j4\hat{\omega}}\left(9 + \left(e^{j3\hat{\omega}} + e^{-j3\hat{\omega}}\right) + 2\left(e^{j\hat{\omega}} + e^{-j\hat{\omega}}\right)\right) \\ &= \underbrace{e^{-j4\hat{\omega}}}_{\vartheta(\hat{\omega}) = -4\hat{\omega}} \underbrace{\left(9 + 2\cos(3\hat{\omega}) + 4\cos(\hat{\omega})\right)}_{|H(\hat{\omega})| = 9 + 2\cos(3\hat{\omega}) + 4\cos(\hat{\omega})} \end{aligned}}$$

⚠ $H(\hat{\omega}) < 0 \rightarrow \boxed{\vartheta(\hat{\omega}) + \pi}$

- Übertragung
  - Gleichanteil
    $$\boxed{\begin{aligned} s[n] &= A = A \cdot e^{j0} \\ \Rightarrow g[n] &= H(0) \cdot A \end{aligned}}$$
  - Dirac-Impuls
    $$\boxed{\begin{aligned} s[n] &= \delta[n-n_0] \\ \Rightarrow g[n] &= \delta[n-n_0] * h[n] = h[n-n_0] \end{aligned}}$$
  - cos-Signal
    $$\boxed{\begin{aligned} s[n] &= A \cdot \cos(\hat{\omega}n + \varphi) \\ \Rightarrow g[n] &= |H(\hat{\omega})| \cdot A \cdot \cos(\hat{\omega}n + \varphi + \vartheta(\hat{\omega})) \end{aligned}}$$
  - Linearität
    $$\boxed{\begin{aligned} s[n] &= s_1[n] + s_2[n] \\ \Rightarrow g[n] &= g_1[n] + g_2[n] \end{aligned}}$$

Übertragungsfunktionen-Übung

$$\boxed{H(\hat{\omega}) = \sum_{n=-\infty}^{\infty} h[n] \cdot e^{-j\hat{\omega}n}}$$

$$\boxed{H(\hat{\omega}) = |H(\hat{\omega})| \cdot e^{j\varphi(\hat{\omega})}}$$

"Übertragungsfkt"

→ periodisch mit Periode 2π

$$\boxed{S(\hat{\omega}) = \sum_{n=-\infty}^{\infty} s[n] \cdot e^{-j\hat{\omega}n}}$$

→ Spektrum eines zeitdiskreten Signals

$$\boxed{s[n] = \frac{1}{2\pi} \cdot \int_{-\pi}^{\pi} S(\hat{\omega}) \cdot e^{j\hat{\omega}n} \, d\hat{\omega}}$$

$\hat{\omega} = 2\pi f$

$$s[n] \xrightarrow{\text{LSI},\ h[n]} g[n] = s[n] * h[n]$$ — Zeit

$$S(\hat{\omega}) \xrightarrow{H(\hat{\omega})} G(\hat{\omega}) = S(\hat{\omega}) \cdot H(\hat{\omega})$$ — Frequenz/Spektrum

Kaskade:

$$s[n] \rightarrow \underset{H_1(\hat{\omega})}{h_1[n]} \rightarrow \underset{H_2(\hat{\omega})}{h_2[n]} \rightarrow \dots \rightarrow \underset{H_N(\hat{\omega})}{h_N[n]} \rightarrow g[n]$$

$$h_{ges}[n] = h_1[n] * h_2[n] * \dots * h_N[n]$$

$$H_{ges}(\hat{\omega}) = H_1(\hat{\omega}) \cdot H_2(\hat{\omega}) \cdot \dots \cdot H_N(\hat{\omega})$$

reelles System $\Longleftrightarrow$ gerade Übertragungsfkt

$h[n]$ reell

$$\mathrm{Re}\{H(\hat{\omega})\} = \mathrm{Re}\{H(-\hat{\omega})\}$$

$$\mathrm{Im}\{H(\hat{\omega})\} = -\mathrm{Im}\{H(-\hat{\omega})\}$$

$$\boxed{H(\hat{\omega}) = H^*(-\hat{\omega})}$$

Amplitudengang $\boxed{|H(\hat{\omega})| = |H(-\hat{\omega})|}$ → gerade

Phasengang $\boxed{\varphi(\hat{\omega}) = -\varphi(-\hat{\omega})}$ → ungerade

FIR-Filter u. Übertragungsfkt

Finite Impulse Response Filter (→ sind LSI-Systeme)

Darstellung	Formel	Bsp.
Filterkoeffizienten	$\{b_k\}$	$\{\overset{0}{3}, \overset{1}{0}, \overset{2}{7}\}$
Impulsantwort	$h[n]=\sum_{k=-\infty}^{+\infty} b_k \cdot \delta[n-k]$	$3\cdot\delta[n-0]+7\cdot\delta[n-2]$
Differenzengleichung	$y[n]=\sum_{k=-\infty}^{+\infty} b_k \cdot x[n-k]$	$3\cdot x[n-0]+7\cdot \delta[n-2]$
Übertragungsfkt	$H(\hat{\omega})=\sum_{k=-\infty}^{+\infty} b_k \cdot e^{-j\hat{\omega}k}$	$3+7\cdot e^{-j\hat{\omega}2}$
Systemfkt	$H_z(z)=\sum_{k=-\infty}^{\infty} b_k \cdot z^{-k}$	$3+7\cdot z^{-2}$

Übertragungsfkt

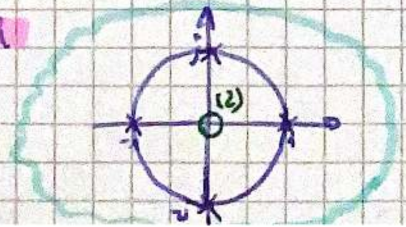
- $\boxed{h[n] \circ\!\!-\!\!\bullet\, H(\hat{\omega})=\sum_{n=-\infty}^{\infty} h[n]\cdot e^{-j\hat{\omega}n} = |H(\hat{\omega})|\cdot e^{j\vartheta(\hat{\omega})}}$
- $|H(\hat{\omega})|\geq 0$, gerade
- $-\pi<\vartheta(\hat{\omega})\leq\pi$, ungerade
- komplex konj. Symmetrie $H(\hat{\omega})=H^*(-\hat{\omega})$
- inverse Fouriertransf. $\boxed{H(\hat{\omega}) \bullet\!\!-\!\!\circ\, h[n]=\frac{1}{2\pi}\cdot\int_{-\pi}^{\pi}H(\hat{\omega})\cdot e^{j\hat{\omega}n}\,d\hat{\omega}}$

Pol-Nullstellen-Diagramm

1. $H(z)$ zu einem Bruch zusammenfassen
2. alle Potenzen von z positiv machen
3. N-/P-Stellen bestimmen

$$\boxed{H_z(z)=\frac{z^2}{z^4-1}} \quad \begin{matrix}\leftarrow \text{Nullstellen}\\ \leftarrow \text{Polstellen}\end{matrix}$$

4. zeichnen



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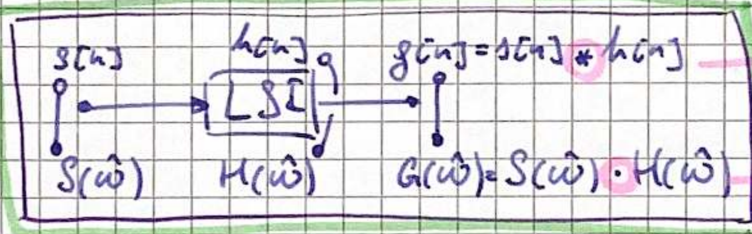
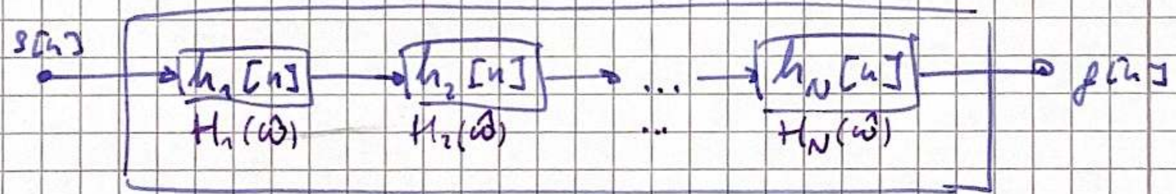


— Zeit

— Frequenz/Spektrum

Kaskade:



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