Univariate Investment Strategies

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Calculating Asset Returns

Given a time series of asset prices p_i, the dollar returns r_i^d, the percentage returns r_i^p, and the log returns r_i^l are defined as:

$$r_i^d = p_i - p_{i-1} \quad r_i^p = \frac{p_i - p_{i-1}}{p_{i-1}} \quad r_i^l = \log(\frac{p_i}{p_{i-1}})$$

The initial returns are all equal to zero.

If the log returns are small $r^l \ll 1$, then they are approximately equal to the percentage returns: $r^l \approx r^p$.

```
> library(rutils)
> # Extract the ETF prices from rutils::etfenv$prices
> pricev <- rutils::etfenv$prices
> pricev <- zoo::na.locf(pricev, na.rm=FALSE)
> pricev <- zoo::na.locf(pricev, fromLast=TRUE)
> datev <- zoo::index(pricev)
> # Calculate the dollar returns
> retd <- rutils::diffit(pricev)
> # Or
> # retd <- lapply(pricev, rutils::diffit)
> # retd <- rutils::do_call(cbind, retd)
> # Calculate the percentage returns
> retp <- retd/rutils::lagit(pricev, lagg=1, pad_zeros=FALSE)
> # Calculate the log returns
> retl <- rutils::diffit(log(pricev))
```

Compounding Asset Returns

The sum of the dollar returns: $\sum_{i=1}^{n} r_i^d$ represents the wealth path from owning a *fixed number of shares*.

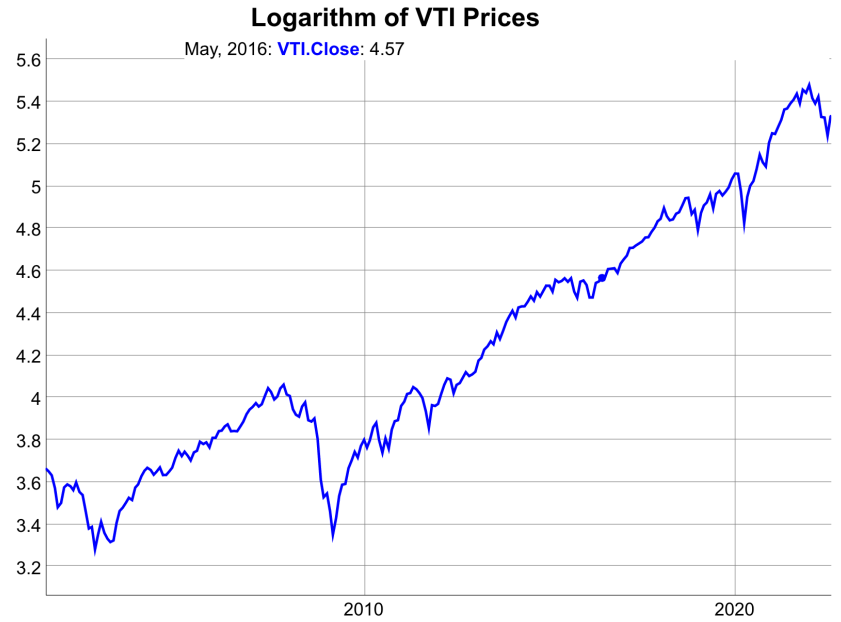
The compounded percentage returns: $\prod_{i=1}^{n}(1 + r_i^p)$ also represent the wealth path from owning a *fixed number of shares*, initially equal to \$1 dollar.

The sum of the percentage returns (without compounding): $\sum_{i=1}^{n} r_i^p$ represents the wealth path from owning a *fixed dollar amount* of stock.

Maintaining a *fixed dollar amount* of stock requires periodic *rebalancing* - selling shares when their price goes up, and vice versa.

This *rebalancing* therefore acts as a mean reverting strategy.

The logarithm of the wealth of a *fixed number of shares* is approximately equal to the sum of the percentage returns.




```
> # Plot log VTI prices
> endd <- rutils::calc_endpoints(rutils::etfenv$VTI, interval="weeks
> dygraphs::dygraph(log(quantmod::Cl(rutils::etfenv$VTI)[endd]),
+   main="Logarithm of VTI Prices") %>%
+   dyOptions(colors="blue", strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

```
> # Set the initial dollar returns
> retd[1, ] <- pricev[1, ]
> # Calculate the prices from dollar returns
> pricen <- cumsum(retd)
> all.equal(pricen, pricev)
> # Compound the percentage returns
> pricen <- cumprod(1 + retp)
> # Set the initial prices
> prici <- as.numeric(pricev[1, ])
> pricen <- lapply(1:NCOL(pricen), function (i) prici[i]*pricen[, i])
> pricen <- rutils::do_call(cbind, pricen)
> # pricen <- t(t(pricen)*prici)
> all.equal(pricen, pricev, check.attributes=FALSE)
```

Funding Costs of Single Asset Rebalancing

The rebalancing of stock requires borrowing from a *margin account*, and it also incurs trading costs.

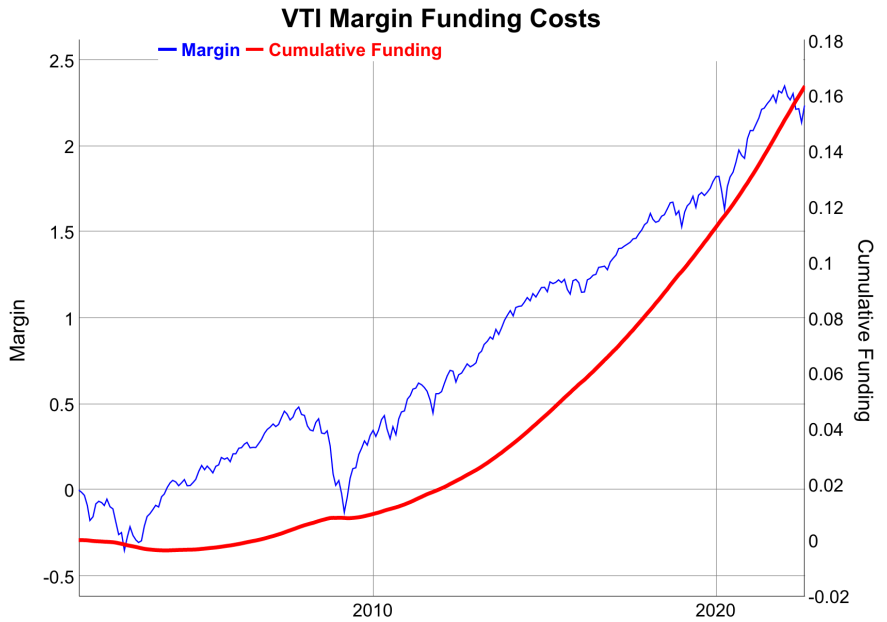
The wealth accumulated from owning a *fixed dollar amount* of stock is equal to the cash earned from rebalancing, which is proportional to the sum of the percentage returns, and it's kept in a *margin account*: $m_t = \sum_{i=1}^{t} r_i^p$.

The cash in the *margin account* can be positive (accumulated profits) or negative (losses).

The *funding costs* c_t^f are approximately equal to the *margin account* m_t times the *funding rate* f: $c_t^f = f\, m_t = f \sum_{i=1}^{t} r_i^p$.

Positive *funding costs* represent interest profits earned on the *margin account*, while negative costs represent the interest paid for funding stock purchases.

The *cumulative funding costs* $\sum_{i=1}^{t} c_t^f$ must be added to the *margin account*: $m_t + \sum_{i=1}^{t} c_t^f$.




```
> # Calculate the percentage VTI returns
> pricev <- rutils::etfenv$prices$VTI
> pricev <- na.omit(pricev)
> retp <- rutils::diffit(pricev)/rutils::lagit(pricev, lagg=1, pad
```

```
> # Funding rate per day
> ratef <- 0.01/252
> # Margin account value
> marginv <- cumsum(retp)
> # Cumulative funding costs
> costf <- cumsum(ratef*marginv)
> # Add funding costs to margin account
> marginv <- (marginv + costf)
> # dygraph plot of margin and funding costs
> datav <- cbind(marginv, costf)
> colnamev <- c("Margin", "Cumulative Funding")
> colnames(datav) <- colnamev
> endd <- rutils::calc_endpoints(datav, interval="weeks")
> dygraphs::dygraph(datav[endd], main="VTI Margin Funding Costs") %>
+   dyAxis("y", label=colnamev[1], independentTicks=TRUE) %>%
+   dyAxis("y2", label=colnamev[2], independentTicks=TRUE) %>%
+   dySeries(name=colnamev[1], axis="y", col="blue") %>%
+   dySeries(name=colnamev[2], axis="y2", col="red", strokeWidth=3)
```

Transaction Costs of Trading

The total *transaction costs* are the sum of the *broker commissions*, the *bid-ask spread* (for market orders), *lost trades* (for limit orders), and *market impact*.

Broker commissions depend on the broker, the size of the trades, and on the type of investors, with institutional investors usually enjoying smaller commissions.

The *bid-ask spread* is the percentage difference between the *ask* (offer) minus the *bid* prices, divided by the *mid* price.

Market impact is the effect of large trades pushing the market prices (the limit order book) against the trades, making the filled price worse.

Limit orders are not subject to the bid-ask spread but they are exposed to *lost trades*.

Lost trades are limit orders that don't get executed, resulting in lost potential profits.

Limit orders may receive rebates from some exchanges, which may reduce transaction costs.

The bid-ask spread for many liquid ETFs is about 1 basis point. For example the *XLK ETF*

In reality the *bid-ask spread* is not static and depends on many factors, such as market liquidity (trading volume), volatility, and the time of day.

The *transaction costs* due to the *bid-ask spread* are equal to the number of traded shares times their price, times half the *bid-ask spread*.

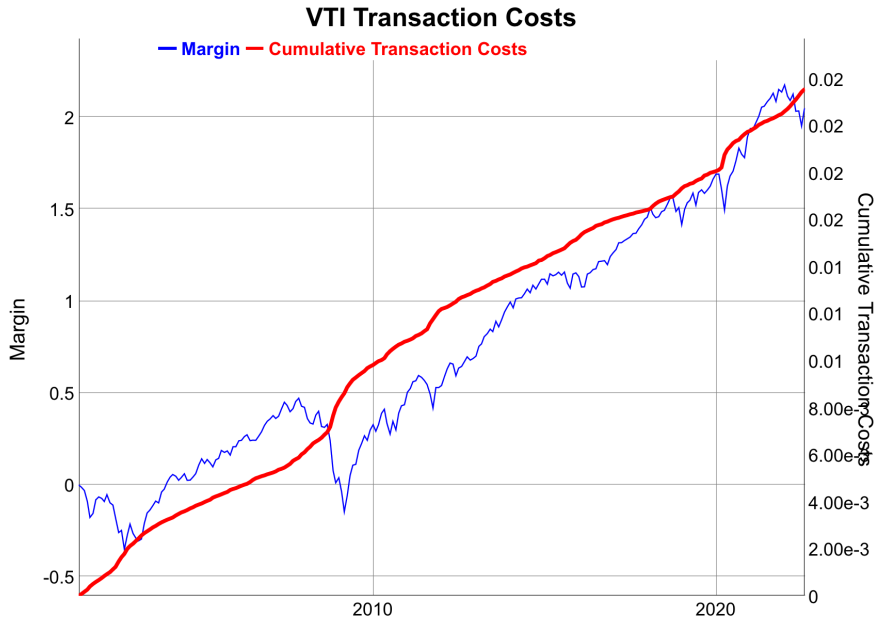
Transaction Costs of Single Asset Rebalancing

Maintaining a *fixed dollar amount* of stock requires periodic *rebalancing*, selling shares when their price goes up, and vice versa.

The dollar amount of stock that must be traded in a given period is equal to the absolute of the percentage returns: $|r_t|$.

The *transaction costs* c_t^r due to rebalancing are equal to half the *bid-ask spread* δ times the dollar amount of the traded stock: $c_t^r = \frac{\delta}{2} |r_t|$.

The *cumulative transaction costs* $\sum_{i=1}^{t} c_t^r$ must be subtracted from the *margin account* m_t: $m_t - \sum_{i=1}^{t} c_t^r$.




```
> # The bid-ask spread is equal to 1 bp for liquid ETFs
> bidask <- 0.001
> # Cumulative transaction costs
> costv <- bidask*cumsum(abs(retp))/2
> # Subtract transaction costs from margin account
> marginv <- cumsum(retp)
> marginv <- (marginv - costv)
> # dygraph plot of margin and transaction costs
> datav <- cbind(marginv, costv)
> colnamev <- c("Margin", "Transaction Costs")
> colnames(datav) <- colnamev
> dygraphs::dygraph(datav[endd], main="VTI Transaction Costs") %>%
+   dyAxis("y", label=colnamev[1], independentTicks=TRUE) %>%
+   dyAxis("y2", label=colnamev[2], independentTicks=TRUE) %>%
+   dySeries(name=colnamev[1], axis="y", col="blue") %>%
+   dySeries(name=colnamev[2], axis="y2", col="red", strokeWidth=3)
+   dyLegend(show="always", width=300)
```

Combining the Returns of Multiple Assets

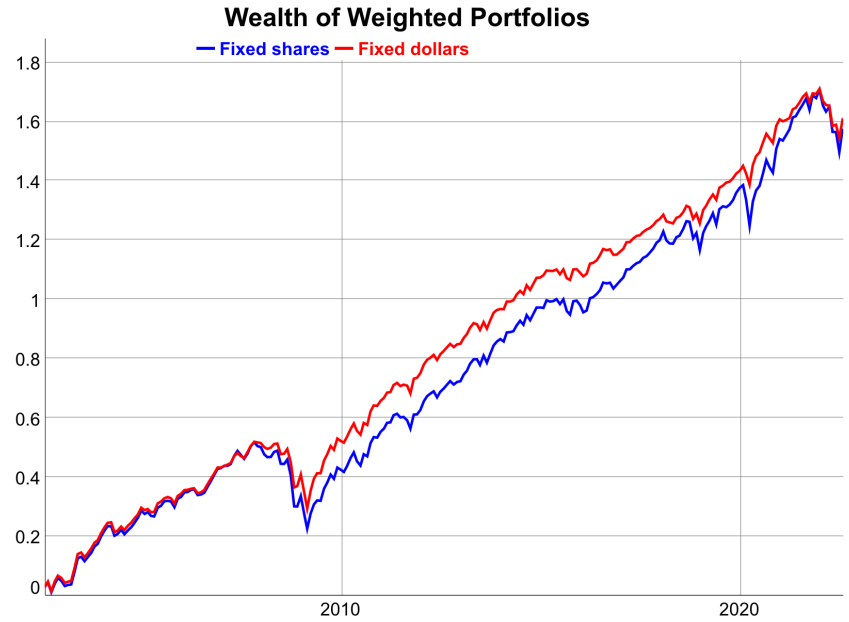
Multiplying the weights times the dollar returns is equivalent to buying a *fixed number of shares* proportional to the weights (aka *Fixed Share Allocation* or FSA).

Multiplying the weights times the percentage returns is equivalent to investing in *fixed dollar amounts of stock* proportional to the weights (aka *Fixed Dollar Allocation* or FDA).

The portfolio allocations must be periodically rebalanced to keep the dollar amounts of the stocks proportional to the weights.

This *rebalancing* acts as a mean reverting strategy - selling shares when their price goes up, and vice versa.

The *FDA* portfolio has a slightly higher Sharpe ratio than the *FSA* portfolio.




```
> # Calculate the VTI and IEF dollar returns
> pricev <- rutils::etfenv$prices[, c("VTI", "IEF")]
> pricev <- na.omit(pricev)
> retd <- rutils::diffit(pricev)
> datev <- zoo::index(pricev)
> # Calculate the VTI and IEF percentage returns
> retp <- retd/rutils::lagit(pricev, lagg=1, pad_zeros=FALSE)
> # Wealth of fixed shares equal to $0.5 each at start (without rel
> weightv <- c(0.5, 0.5)  # dollar weights
> wealthfs <- drop(cumprod(1 + retp) %*% weightv)
> # Or using the dollar returns
> prici <- as.numeric(pricev[1, ])
> retd[1, ] <- pricev[1, ]
> wealthfs2 <- cumsum(retd %*% (weightv/prici))
> all.equal(wealthfs, drop(wealthfs2))
```

```
> # Wealth of fixed dollars equal to $0.5 each (with rebalancing)
> wealthfd <- cumsum(retp %*% weightv)
> # Calculate the Sharpe and Sortino ratios
> wealthv <- cbind(log(wealthfs), wealthfd)
> wealthv <- xts::xts(wealthv, datev)
> colnames(wealthv) <- c("Fixed shares", "Fixed dollars")
> sqrt(252)*sapply(rutils::diffit(wealthv), function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # Plot the log wealth
> colnamev <- colnames(wealthv)
> endd <- rutils::calc_endpoints(retp, interval="weeks")
> dygraphs::dygraph(wealthv[endd], main="Wealth of Weighted Portfoli
+   dySeries(name=colnamev[1], col="blue", strokeWidth=2) %>%
+   dySeries(name=colnamev[2], col="red", strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

Transaction Costs of Weighted Portfolio Rebalancing

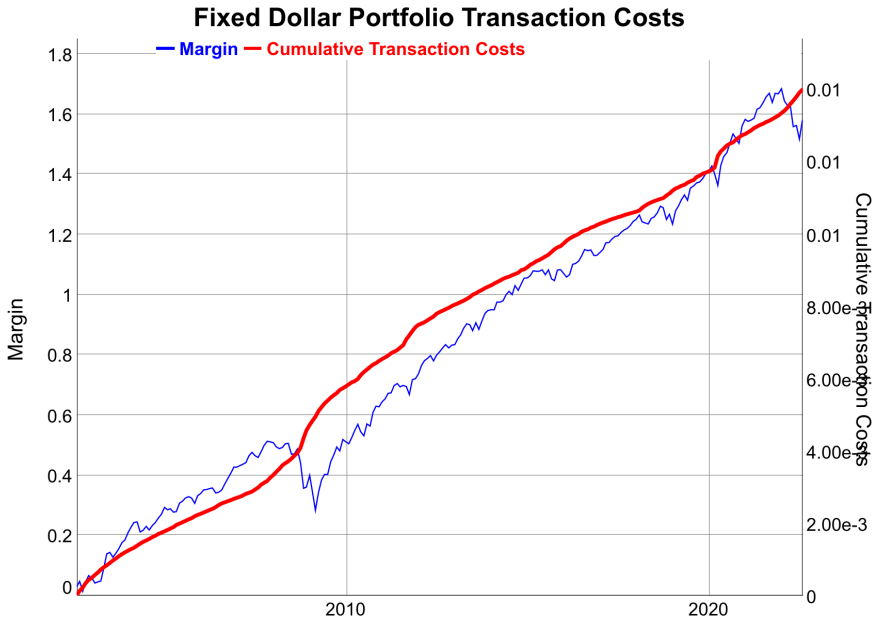
Maintaining a *fixed dollar allocation* of stock requires periodic *rebalancing*, selling shares when their price goes up, and vice versa.

Adding the weighted percentage returns is equivalent to investing in *fixed dollar amounts of stock* proportional to the weights w_1, w_2.

The dollar amount of stock that must be traded in a given period is equal to the weighted sum of the absolute percentage returns: $w_1 \left|r_t^1\right| + w_2 \left|r_t^2\right|$.

The *transaction costs* c_t^r due to rebalancing are equal to half the *bid-ask spread* δ times the dollar amount of the traded stock: $c_t^r = \frac{\delta}{2}(w_1 \left|r_t^1\right| + w_2 \left|r_t^2\right|)$.

The *cumulative transaction costs* $\sum_{i=1}^{t} c_t^r$ must be subtracted from the *margin account* m_t: $m_t - \sum_{i=1}^{t} c_t^r$.




```
> # Margin account for fixed dollars (with rebalancing)
> marginv <- cumsum(retp %*% weightv)
> # Cumulative transaction costs
> costv <- bidask*cumsum(abs(retp) %*% weightv)/2
> # Subtract transaction costs from margin account
> marginv <- (marginv - costv)
> # dygraph plot of margin and transaction costs
> datav <- cbind(marginv, costv)
> datav <- xts::xts(datav, datev)
> colnamev <- c("Margin", "Transaction Costs")
> colnames(datav) <- colnamev
> dygraphs::dygraph(datav[endd], main="Fixed Dollar Portfolio Transa
+   dyAxis("y", label=colnamev[1], independentTicks=TRUE) %>%
+   dyAxis("y2", label=colnamev[2], independentTicks=TRUE) %>%
+   dySeries(name=colnamev[1], axis="y", col="blue") %>%
+   dySeries(name=colnamev[2], axis="y2", col="red", strokeWidth=3)
+   dyLegend(show="always", width=300)
```

Proportional Wealth Allocations

In the *proportional wealth allocation* strategy (*PWA*), the total wealth Π_t is allocated to the assets Π_i proportional to the portfolio weights w_i: $\Pi_i = w_i \Pi_t$.

The total wealth Π_t is not fixed and is equal to the portfolio market value $\Pi_t = \sum \Pi_i$, so there's no margin account.

The portfolio is rebalanced daily to maintain the dollar allocations Π_i equal to the total wealth $\Pi_t = \sum \Pi_i$ times the portfolio weights: w_i: $\Pi_i = w_i \Pi_t$.

Let r_t be the percentage returns, w_i be the portfolio weights, and $\bar{r}_t = \sum_{i=1}^{n} w_i r_t$ be the weighted percentage returns at time t.

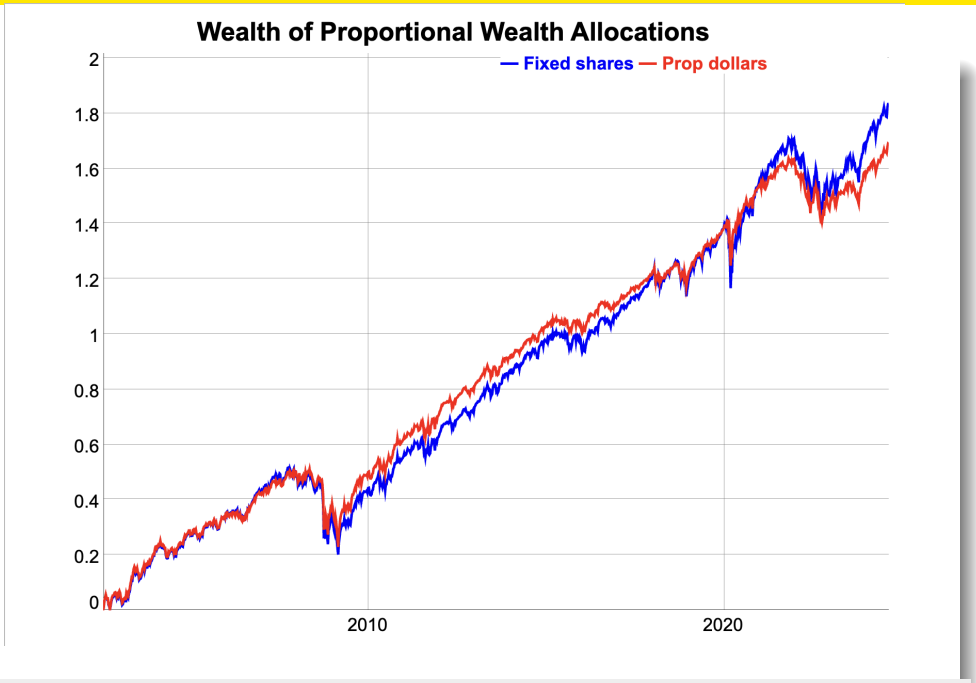
The total portfolio wealth at time t is equal to the wealth at time $t-1$ multiplied by the weighted returns: $\Pi_t = \Pi_{t-1}(1 + \bar{r}_t)$.

The dollar amount of stock i at time t increases by $w_i r_t$ so it's equal to $w_i \Pi_{t-1}(1 + r_t)$, while the target amount is $w_i \Pi_t = w_i \Pi_{t-1}(1 + \bar{r}_t)$

The dollar amount of stock i needed to trade to rebalance back to the target weight is equal to:

$$\begin{aligned} \varepsilon_i &= |w_i \Pi_{t-1}(1 + \bar{r}_t) - w_i \Pi_{t-1}(1 + r_t)| \\ &= w_i \Pi_{t-1} |\bar{r}_t - r_t| \end{aligned}$$

If $\bar{r}_t > r_t$ then an amount ε_i of the stock i needs to be bought, and if $\bar{r}_t < r_t$ then it needs to be sold.




```
> # Wealth of fixed shares (without rebalancing)
> wealthfs <- cumsum(retd %*% (weightv/prici))
> # Or compound the percentage returns
> wealthfs <- drop(cumprod(1 + retp) %*% weightv)
> # Wealth of proportional wealth strategy (with rebalancing)
> wealthpr <- cumprod(1 + retp %*% weightv)
> wealthv <- cbind(wealthfs, wealthpr)
> wealthv <- xts::xts(wealthv, datev)
> colnames(wealthv) <- c("Fixed shares", "Prop dollars")
> wealthv <- log(wealthv)
> # Calculate the Sharpe and Sortino ratios
> sqrt(252)*sapply(rutils::diffit(wealthv), function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # Plot the log wealth
> dygraphs::dygraph(wealthv[endd],
+   main="Wealth of Proportional Wealth Allocations") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

Transaction Costs With Proportional Wealth Allocations

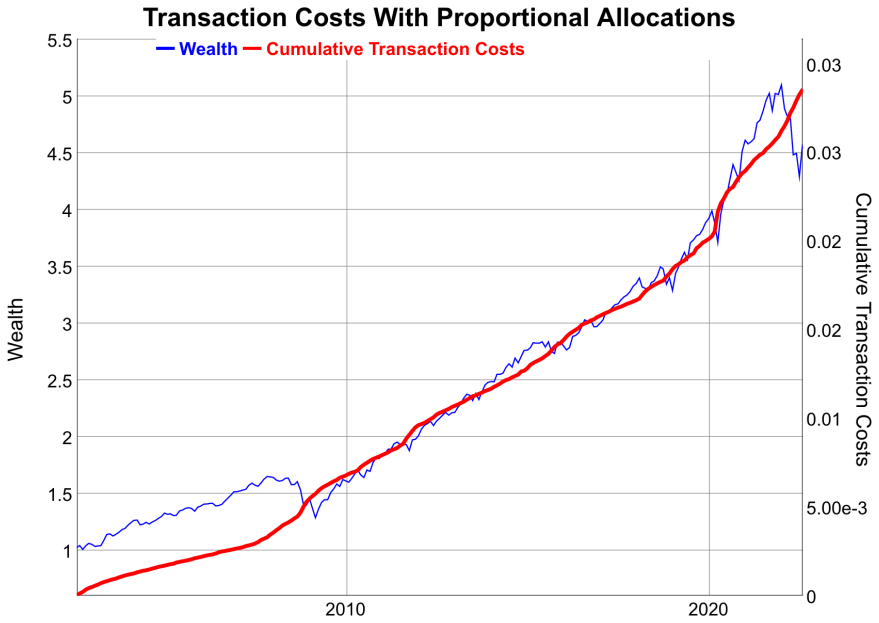
In each period the stocks must be rebalanced to maintain the proportional wealth allocations.

The total dollar amount of stocks that need to be traded to rebalance back to the target weight is equal to: $\sum_{i=1}^{n} \varepsilon_i = \Pi_{t-1} \sum_{i=1}^{n} w_i \left| \bar{r}_t - r_t \right|$

The *transaction costs* c_t^r are equal to half the *bid-ask spread* δ times the dollar amount of the traded stock: $c_t^r = \frac{\delta}{2} \sum_{i=1}^{n} \varepsilon_i$.

The *cumulative transaction costs* $\sum_{i=1}^{t} c_t^r$ must be subtracted from the *wealth* Π_t: $\Pi_t - \sum_{i=1}^{t} c_t^r$.

```
> # Returns in excess of weighted returns
> retw <- retp %*% weightv
> retx <- lapply(retp, function(x) (x - retw))
> retx <- do.call(cbind, retx)
> sum(retx %*% weightv)
> # Calculate the weighted sum of absolute excess returns
> retx <- abs(retx) %*% weightv
> # Total dollar amount of stocks that need to be traded
> retx <- retx*rutils::lagit(wealthpr)
> # Cumulative transaction costs
> costv <- bidask*cumsum(retx)/2
> # Subtract transaction costs from wealth
> wealthpr <- (wealthpr - costv)
```




```
> # dygraph plot of wealth and transaction costs
> wealthv <- cbind(wealthpr, costv)
> wealthv <- xts::xts(wealthv, datev)
> colnamev <- c("Wealth", "Transaction Costs")
> colnames(wealthv) <- colnamev
> dygraphs::dygraph(wealthv[endd],
+   main="Transaction Costs With Equal Wealths") %>%
+   dyAxis("y", label=colnamev[1], independentTicks=TRUE) %>%
+   dyAxis("y2", label=colnamev[2], independentTicks=TRUE) %>%
+   dySeries(name=colnamev[1], axis="y", col="blue") %>%
+   dySeries(name=colnamev[2], axis="y2", col="red", strokeWidth=3)
+   dyLegend(show="always", width=300)
```

Proportional Target Allocation Strategy

In the *fixed share strategy* (*FSA*), the number of shares is fixed, with their initial dollar value equal to the portfolio weights.

In the *proportional wealth allocation* strategy (*PWA*), the portfolio is rebalanced daily to maintain the dollar allocations Π_i equal to the total wealth $\Pi_t = \sum \Pi_i$ times the portfolio weights: w_i: $\Pi_i = w_i \Pi_t$.

In the *proportional target allocation* strategy (*PTA*), the portfolio is rebalanced only if the dollar allocations Π_i differ from their targets $w_i \Pi_t$ more than the threshold value τ: $\frac{\sum |\Pi_i - w_i \Pi_t|}{\Pi_t} > \tau$.

The *PTA* strategy is path-dependent so it must be simulated using an explicit loop.

The *PTA* strategy is contrarian, since it sells assets that have outperformed, and it buys assets that have underperformed.

If the threshold level is very small then the *PTA* strategy rebalances daily and it's the same as the *PWA*.

If the threshold level is very large then the *PTA* strategy does not rebalance and it's the same as the *FSA*.

```
> # Wealth of fixed shares (without rebalancing)
> wealthfs <- drop(cumprod(1 + retp) %*% weightv)-1
> # Wealth of proportional allocation (with rebalancing)
> wealthpr <- cumprod(1 + retp %*% weightv) - 1
> # Wealth of proportional target allocation (with rebalancing)
> retp <- zoo::coredata(retp)
> threshv <- 0.05
> wealthv <- matrix(nrow=NROW(retp), ncol=2)
> colnames(wealthv) <- colnames(retp)
> wealthv[1, ] <- weightv
> for (it in 2:NROW(retp)) {
+   # Accrue wealth without rebalancing
+   wealthv[it, ] <- wealthv[it-1, ]*(1 + retp[it, ])
+   # Rebalance if wealth allocations differ from weights
+   if (sum(abs(wealthv[it, ] - sum(wealthv[it, ])*weightv))/sum(wea
+     # cat("Rebalance at:", it, "\n")
+     wealthv[it, ] <- sum(wealthv[it, ])*weightv
+   } # end if
+ } # end for
> wealthv <- rowSums(wealthv) - 1
> wealthv <- cbind(wealthpr, wealthv)
> wealthv <- xts::xts(wealthv, datev)
> colnames(wealthv) <- c("Equal Wealths", "Proportional Target")
> dygraphs::dygraph(wealthv, main="Wealth of Proportional Target All
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

draft: Stock Index Weighting Methods

Split this slide to explain equal-weighted indices: https://www.investopedia.com/terms/e/equalweight.asp

Stock market indices can be capitalization-weighted (*S&P500*), price-weighted (*DJIA*), or equal-weighted.

The cap-weighted and price-weighted indices own a fixed number of shares (excluding stock splits).

Equal-weighted indices own the same dollar amount of each stock, so they must be rebalanced as market prices change.

Cap-weighted index = Sum { (Stock Price * Number of shares) / Index Divisor }

Price-weighted index = Sum {Stock Price / Index Divisor }

Equal-weighted index = Sum { (Stock Price * factor) / Index Divisor }

Cap-weighted indices are overweight large-cap stocks, while equal-weighted indices are overweight small-cap stocks.

Cap-weighted indices are *trend following*, while equal-weighted indices are *mean reverting* (contrarian).

```
> # Create name corresponding to "^GSPC" symbol
> setSymbolLookup(
+   SP500=list(name="^GSPC", src="yahoo"))
> getSymbolLookup()
> # view and clear options
> options("getSymbols.sources")
> options(getSymbols.sources=NULL)
> # Download S&P500 prices into etfenv
> quantmod::getSymbols("SP500", env=etfenv,
+     adjust=TRUE, auto.assign=TRUE, from="1990-01-01")
> quantmod::chart_Series(x=etfenv$SP500["2016/"],
+        TA="add_Vo()",
+        name="S&P500 index")
```

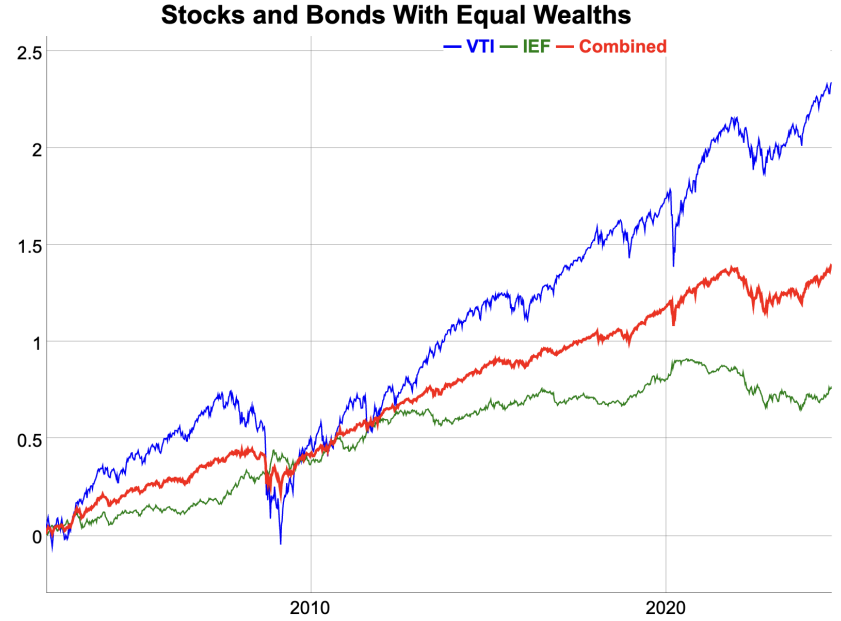
Stock and Bond Portfolio With Proportional Wealth Allocations

Portfolios combining stocks and bonds can provide a much better risk versus return tradeoff than either of the assets separately, because the returns of stocks and bonds are usually negatively correlated, so they are natural hedges of each other.

The fixed portfolio weights represent the percentage dollar allocations to stocks and bonds, while the portfolio wealth grows over time.

The weights depend on the investment horizon, with a greater allocation to bonds for a shorter investment horizon.

Active investment strategies are expected to outperform static stock and bond portfolios.




```
> # Calculate the stock and bond returns
> retp <- na.omit(rutils::etfenv$returns[, c("VTI", "IEF")])
> weightv <- c(0.4, 0.6)
> retp <- cbind(retp, retp %*% weightv)
> colnames(retp)[3] <- "Combined"
> # Calculate the correlations
> cor(retp)
> # Calculate the Sharpe ratios
> sqrt(252)*sapply(retp, function(x) mean(x)/sd(x))
> # Calculate the standard deviation, skewness, and kurtosis
> sapply(retp, function(x) {
+   # Calculate the standard deviation
+   stdev <- sd(x)
+   # Standardize the returns
+   x <- (x - mean(x))/stdev
+   c(stdev=stdev, skew=mean(x^3), kurt=mean(x^4))
+ })  # end sapply
```

```
> # Wealth of equal wealth strategy
> wealthv <- cumsum(retp)
> # Calculate the a vector of monthly end points
> endd <- rutils::calc_endpoints(retp, interval="weeks")
> # Plot cumulative log wealth
> dygraphs::dygraph(wealthv[endd],
+   main="Stocks and Bonds With Equal Wealths") %>%
+   dyOptions(colors=c("blue", "green", "blue", "red")) %>%
+   dySeries("Combined", color="red", strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

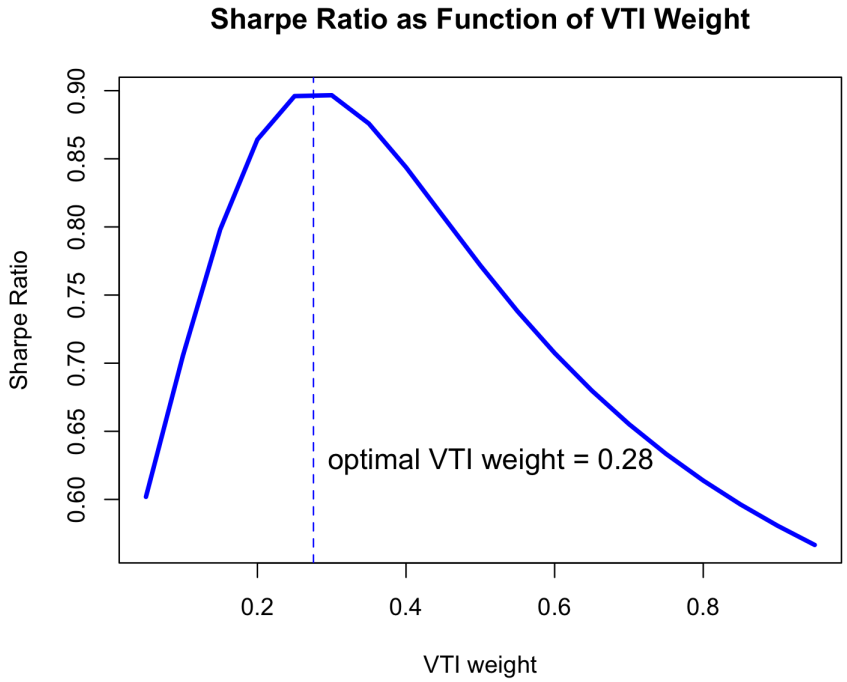
Optimal Stock and Bond Portfolio Allocations

The optimal stock and bond weights can be calculated using optimization.

Using the past 20 years of data, the optimal *VTI* weight is about 0.28.

The comments and conclusions in these slides are based on 20 years of very positive stock and bond returns, when stocks and bonds have been in a secular bull market. The conclusions would not hold if stocks and bonds had suffered from a bear market (losses) over that time.

```
> # Calculate the Sharpe ratios
> sqrt(252)*sapply(retp, function(x) mean(x)/sd(x))
> # Calculate the Sharpe ratios for vector of weights
> weightv <- seq(0.05, 0.95, 0.05)
> sharpev <- sqrt(252)*sapply(weightv, function(weight) {
+   weightv <- c(weight, 1-weight)
+   retp <- (retp[, 1:2] %*% weightv)
+   mean(retp)/sd(retp)
+ })  # end sapply
> # Calculate the optimal VTI weight
> weightm <- weightv[which.max(sharpev)]
> # Calculate the optimal weight using optimization
> calc_sharpe <- function(weight) {
+   weightv <- c(weight, 1-weight)
+   retp <- (retp[, 1:2] %*% weightv)
+   -mean(retp)/sd(retp)
+ }  # end calc_sharpe
> optv <- optimize(calc_sharpe, interval=c(0, 1))
> weightm <- optv$minimum
```




```
> # Plot Sharpe ratios
> plot(x=weightv, y=sharpev,
+      main="Sharpe Ratio as Function of VTI Weight",
+      xlab="VTI weight", ylab="Sharpe Ratio",
+      t="l", lwd=3, col="blue")
> abline(v=weightm, lty="dashed", lwd=1, col="blue")
> text(x=weightm, y=0.7*max(sharpev), pos=4, cex=1.2,
+      labels=paste("optimal VTI weight =", round(weightm, 2)))
```

Simulating Wealth Scenarios Using Bootstrap

The past data represents only one possible future scenario. We can generate more scenarios using bootstrap simulation.

The bootstrap data is a list of simulated *VTI* and *IEF* returns, which represent possible realizations of future returns, based on past history.

For sampling from rows of data, it's better to convert time series to matrices.

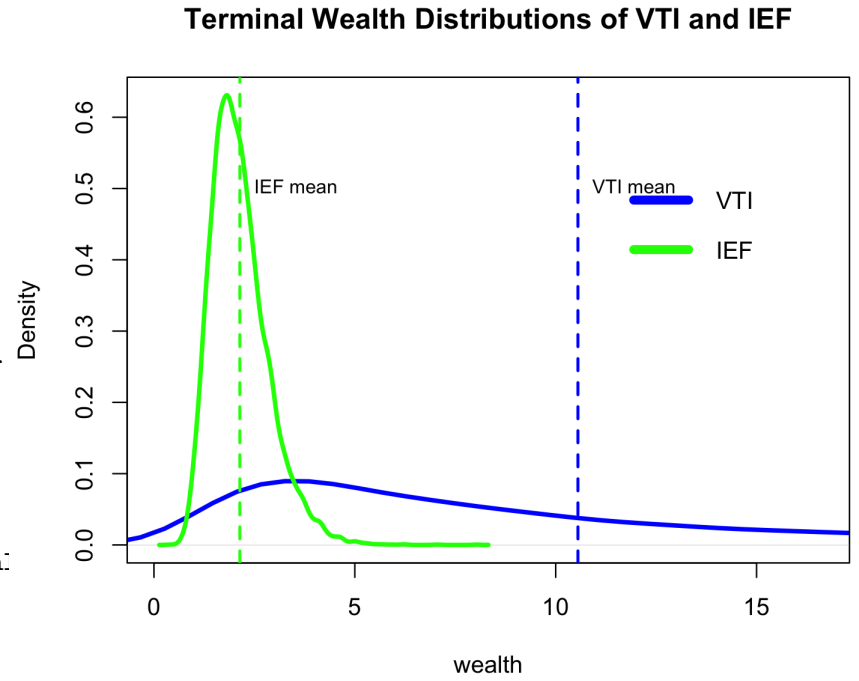
```
> # Coerce the returns from xts time series to matrix
> retp <- zoo::coredata(retp[, 1:2])
> nrows <- NROW(retp)
> # Bootstrap the returns and Calculate the a list of random returns
> nboot <- 1e4
> library(parallel)  # Load package parallel
> ncores <- detectCores() - 1  # Number of cores
> # Perform parallel bootstrap under Windows
> compclust <- makeCluster(ncores)  # Initialize compute cluster und
> clusterSetRNGStream(compclust, 1121)  # Reset random number genera
> clusterExport(compclust, c("retp", "nrows"))
> bootd <- parLapply(compclust, 1:nboot, function(x) {
+   retp[sample.int(nrows, replace=TRUE), ]
+ })  # end parLapply
> # Perform parallel bootstrap under Mac-OSX or Linux
> set.seed(1121, "Mersenne-Twister", sample.kind="Rejection")
> bootd <- mclapply(1:nboot, function(x) {
+   retp[sample.int(nrows, replace=TRUE), ]
+ }, mc.cores=ncores)  # end mclapply
> is.list(bootd); NROW(bootd); dim(bootd[[1]])
```

The Distributions of Terminal Wealth From Bootstrap

The distribution of *VTI* and *IEF* wealths can be calculated from the bootstrap data.

The distribution of *VTI* wealth is much wider than *IEF*, but it has a much greater mean value.

```
> # Calculate the distribution of terminal wealths under Windows
> wealthv <- parLapply(compclust, bootd, function(retp) {
+   apply(retp, 2, function(x) prod(1 + x))
+ })  # end parLapply
> # Calculate the distribution of terminal wealths under Mac-OSX or
> wealthv <- mclapply(bootd, function(retp) {
+   apply(retp, 2, function(x) prod(1 + x))
+ }, mc.cores=ncores)  # end mclapply
> wealthv <- do.call(rbind, wealthv)
> class(wealthv); dim(wealthv); tail(wealthv)
> # Calculate the means and standard deviations of the terminal wea
> apply(wealthv, 2, mean)
> apply(wealthv, 2, sd)
> # Extract the terminal wealths of VTI and IEF
> vtiw <- wealthv[, "VTI"]
> iefw <- wealthv[, "IEF"]
```



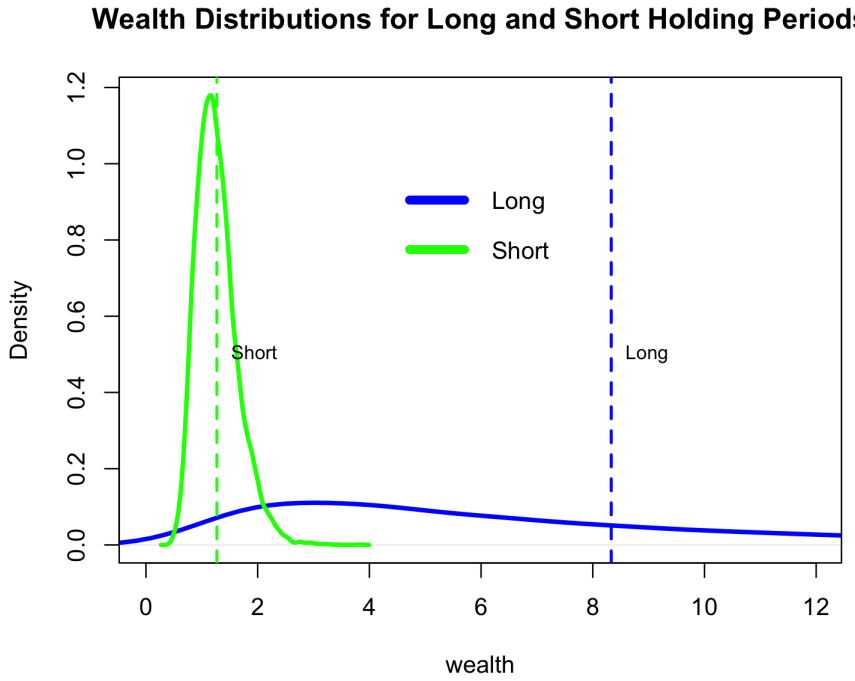

```
> # Plot the densities of the terminal wealths of VTI and IEF
> vtim <- mean(vtiw); iefm <- mean(iefw)
> vtid <- density(vtiw); iefd <- density(iefw)
> plot(vtid, col="blue", lwd=3, xlab="wealth",
+      xlim=c(0, 2*max(iefd$x)), ylim=c(0, max(iefd$y)),
+      main="Terminal Wealth Distributions of VTI and IEF")
> lines(iefd, col="green", lwd=3)
> abline(v=vtim, col="blue", lwd=2, lty="dashed")
> text(x=vtim, y=0.5, labels="VTI mean", pos=4, cex=0.8)
> abline(v=iefm, col="green", lwd=2, lty="dashed")
> text(x=iefm, y=0.5, labels="IEF mean", pos=4, cex=0.8)
> legend(x="topright", legend=c("VTI", "IEF"),
+  inset=0.1, cex=1.0, bg="white", bty="n", y.intersp=0.5,
+  lwd=6, lty=1, col=c("blue", "green"))
```

The Distribution of Stock Wealth and Holding Period

The distribution of stock wealth for short holding periods is close to symmetric around par (1).

The distribution for long holding periods is highly positively skewed with a much larger mean.

```
> # Calculate the distributions of stock wealth
> holdv <- nrows*seq(0.1, 1.0, 0.1)
> wealthm <- mclapply(bootd, function(retp) {
+   sapply(holdv, function(holdp) {
+     prod(1 + retp[1:holdp, "VTI"])
+   })  # end sapply
+ }, mc.cores=ncores)  # end mclapply
> wealthm <- do.call(rbind, wealthm)
> dim(wealthm)
```




```
> # Plot the stock wealth for long and short holding periods
> wealth1 <- wealthm[, 9]
> wealth2 <- wealthm[, 1]
> mean1 <- mean(wealth1); mean2 <- mean(wealth2)
> dens1 <- density(wealth1); dens2 <- density(wealth2)
> plot(dens1, col="blue", lwd=3, xlab="wealth",
+      xlim=c(0, 3*max(dens2$x)), ylim=c(0, max(dens2$y)),
+      main="Wealth Distributions for Long and Short Holding Periods")
> lines(dens2, col="green", lwd=3)
> abline(v=mean1, col="blue", lwd=2, lty="dashed")
> text(x=mean1, y=0.5, labels="Long", pos=4, cex=0.8)
> abline(v=mean2, col="green", lwd=2, lty="dashed")
> text(x=mean2, y=0.5, labels="Short", pos=4, cex=0.8)
> legend(x="top", legend=c("Long", "Short"),
+   inset=0.1, cex=1.0, bg="white", bty="n", y.intersp=0.5,
```

Risk-adjusted Stock Wealth and Holding Period

The downside risk is equal to the mean of the wealth below par (1).

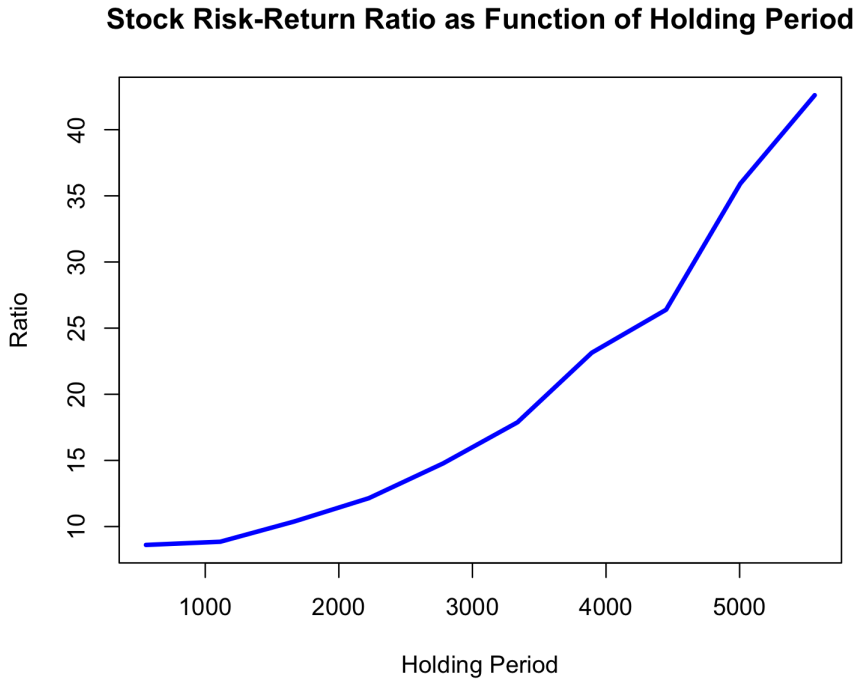
The risk-adjusted wealth measure is equal to the mean wealth divided by the downside risk.

U.S. stocks in the last 40 years have had higher risk-adjusted wealth for longer holding periods.

The risk-adjusted wealth may also be higher for very short holding periods because the risk is low - there's not enough time for the wealth to significantly drop below par (1).

The risk increases for intermediate holding periods, so the risk-adjusted wealth drops.

The mean wealth increases for longer holding periods, so the risk-adjusted wealth also increases.




```
> # Define the risk-adjusted wealth measure
> riskretfun <- function(wealthv) {
+   riskv <- 0.01 # Risk floor
+   if (min(wealthv) < 1) # Some wealth is below par
+     # Calculate the mean stock wealth below par
+     riskv <- mean((1-wealthv)[wealthv<1])
+   mean(wealthv)/riskv
+ }  # end riskretfun
> # Calculate the stock wealth risk-return ratios
> riskrets <- apply(wealthm, 2, riskretfun)
```

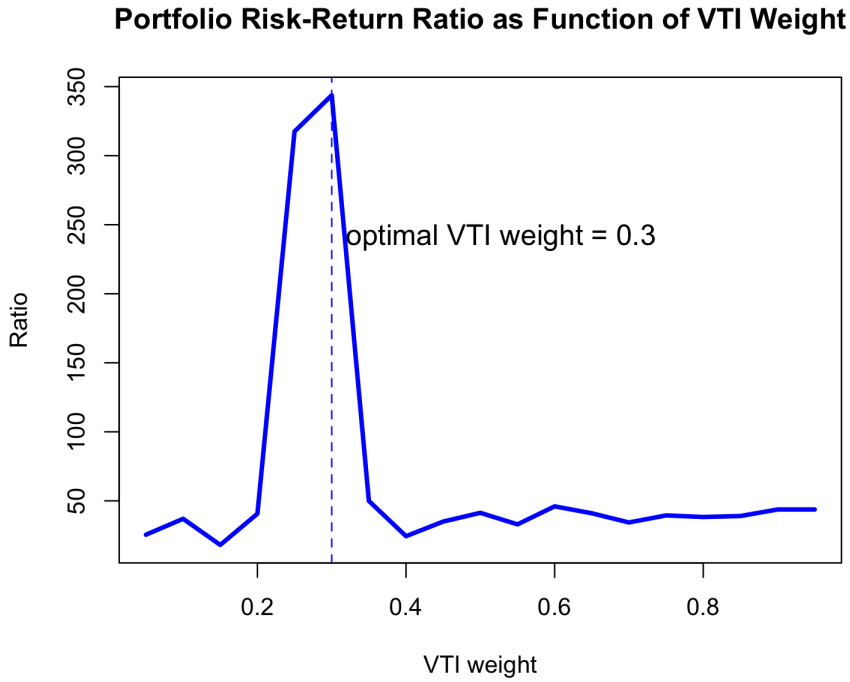
```
> # Plot the stock wealth risk-return ratios
> plot(x=holdv, y=riskrets,
+      main="Stock Risk-Return Ratio as Function of Holding Period",
+      xlab="Holding Period", ylab="Ratio",
+      t="l", lwd=3, col="blue")
```

Optimal Stock and Bond Portfolio Allocations From Bootstrap

The optimal stock and bond weights can be calculated using bootstrap simulation.

Bootstrapping the past 20 years of data, the optimal *VTI* weight is about 0.3.

```
> # Calculate the distributions of portfolio wealth
> weightv <- seq(0.05, 0.95, 0.05)
> wealthm <- mclapply(bootd, function(retp) {
+   sapply(weightv, function(weight) {
+     prod(1 + retp %*% c(weight, 1-weight))
+   })  # end sapply
+ }, mc.cores=ncores)  # end mclapply
> wealthm <- do.call(rbind, wealthm)
> dim(wealthm)
> # Calculate the portfolio risk-return ratios
> riskrets <- apply(wealthm, 2, riskretfun)
> # Calculate the optimal VTI weight
> weightm <- weightv[which.max(riskrets)]
```




```
> # Plot the portfolio risk-return ratios
> plot(x=weightv, y=riskrets,
+      main="Portfolio Risk-Return Ratio as Function of VTI Weight",
+      xlab="VTI weight", ylab="Ratio",
+      t="l", lwd=3, col="blue")
> abline(v=weightm, lty="dashed", lwd=1, col="blue")
> text(x=weightm, y=0.7*max(riskrets), pos=4, cex=1.2,
+      labels=paste("optimal VTI weight =", round(weightm, 2)))
```

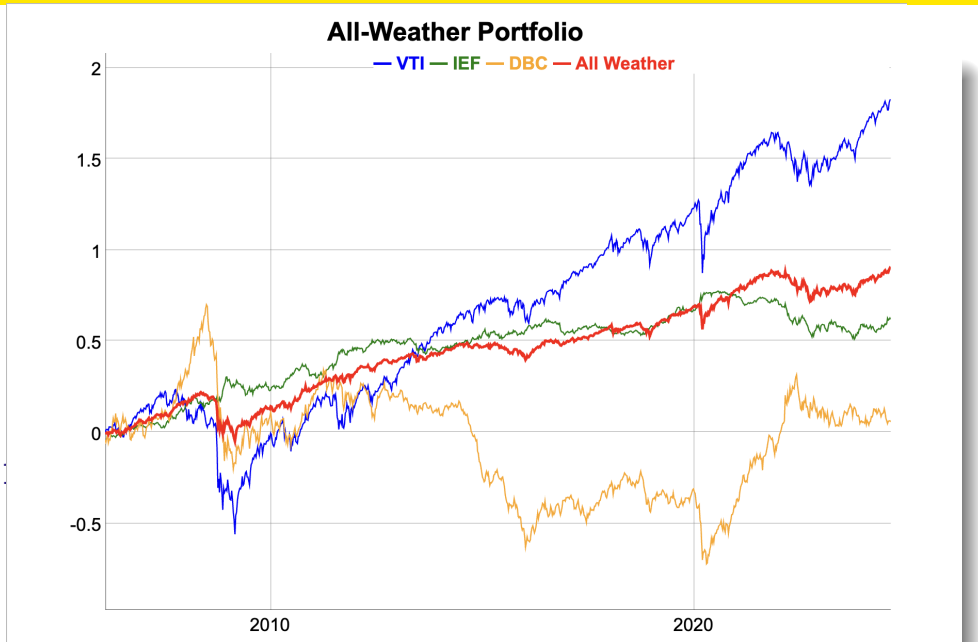
The *All-Weather* Portfolio

The *All-Weather* portfolio is a portfolio with proportional allocations of stocks (30%), bonds (55%), and commodities and precious metals (15%) (approximately).

The *All-Weather* portfolio was developed by *Bridgewater Associates*, the largest hedge fund in the world:
https://www.bridgewater.com/research-library/the-all-weather-strategy/
http://www.nasdaq.com/article/remember-the-allweather-portfolio-its-having-a-killer-year-cm6855

The three different asset classes (stocks, bonds, commodities) provide positive returns under different economic conditions (recession, expansion, inflation).

The combination of bonds, stocks, and commodities in the *All-Weather* portfolio is designed to provide positive returns under most economic conditions, without the costs of trading.




```
> # Extract the ETF returns
> symbolv <- c("VTI", "IEF", "DBC")
> retp <- na.omit(rutils::etfenv$returns[, symbolv])
> # Calculate the all-weather portfolio wealth
> weightaw <- c(0.30, 0.55, 0.15)
> retp <- cbind(retp, retp %*% weightaw)
> colnames(retp)[4] <- "All Weather"
> # Calculate the Sharpe ratios
> sqrt(252)*sapply(retp, function(x) mean(x)/sd(x))
```

```
> # Calculate the cumulative wealth from returns
> wealthv <- cumsum(retp)
> # Calculate the a vector of monthly end points
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> # dygraph all-weather wealth
> dygraphs::dygraph(wealthv[endd], main="All-Weather Portfolio") %>%
+   dyOptions(colors=c("blue", "green", "orange", "red")) %>%
+   dySeries("All Weather", color="red", strokeWidth=2) %>%
+   dyLegend(show="always", width=400)
> # Plot all-weather wealth
> plot_theme <- chart_theme()
> plot_theme$col$line.col <- c("orange", "blue", "green", "red")
> quantmod::chart_Series(wealthv, theme=plot_theme, lwd=c(2, 2, 2, 4
+        name="All-Weather Portfolio")
> legend("topleft", legend=colnames(wealthv),
+   inset=0.1, bg="white", lty=1, lwd=6, y.intersp=0.5,
+   col=plot_theme$col$line.col, bty="n")
```

Constant Proportion Portfolio Insurance Strategy

In the *Constant Proportion Portfolio Insurance* (CPPI) strategy the portfolio is rebalanced between stocks and zero-coupon bonds, to protect against the loss of principal.

A zero-coupon bond pays no coupon, but it's bought at a discount to par (100%), and pays par at maturity. The investor receives capital appreciation instead of coupons.

Let P be the investor principal amount (total initial invested dollar amount), and let F be the zero-coupon *bond floor*. The zero-coupon bond floor F is set so that its value at maturity is equal to the principal P. This guarantees that the investor is paid back at least the full principal P.

The stock investment is levered by the *CPPI multiplier* C. The initial dollar amount invested in stocks is equal to the *cushion* $(P - F)$ times the *multiplier* C: $C * (P - F)$. The remaining amount of the principal is invested in zero-coupon bonds and is equal to: $P - C * (P - F)$.

```
> # Calculate the VTI returns
> retp <- na.omit(rutils::etfenv$returns$VTI["2008/2009"])
> datev <- zoo::index(retp)
> nrows <- NROW(retp)
> retp <- drop(zoo::coredata(retp))
> # Bond floor
> bfloor <- 60
> # CPPI multiplier
> coeff <- 2
> # Portfolio market values
> portfv <- numeric(nrows)
> # Initial principal
> portfv[1] <- 100
> # Stock allocation
> stockv <- numeric(nrows)
> stockv[1] <- min(coeff*(portfv[1] - bfloor), portfv[1])
> # Bond allocation
> bondv <- numeric(nrows)
> bondv[1] <- (portfv[1] - stockv[1])
```

CPPI Strategy Dynamics

If the stock price changes and the portfolio value becomes P_t, then the dollar amount invested in stocks must be adjusted to: $C * (P_t - F)$. The amount invested in stocks changes both because the stock price changes and because of rebalancing with the zero-coupon bonds.

The amount invested in zero-coupon bonds is then equal to: $P_t - C * (P_t - F)$. If the portfolio value drops to the *bond floor* $P_t = F$, then all the stocks must be sold, with only the zero-coupon bonds remaining. But if the stock price rises, more stocks must be purchased, and vice versa.

Therefore the *CPPI* strategy is a *trend following* strategy, buying stocks when their prices are rising, and selling when their prices are dropping.

The *CPPI* strategy can be considered a dynamic replication of a portfolio with a zero-coupon bond and a stock call option.

The *CPPI* strategy is exposed to *gap risk*, if stock prices drop suddenly by a large amount. The *gap risk* is exacerbated by high leverage, when the *multiplier* C is large, say greater than 5.


```
> # Simulate CPPI strategy
> for (t in 2:nrows) {
+   portfv[t] <- portfv[t-1] + stockv[t-1]*retp[t]
+   stockv[t] <- min(coeff*(portfv[t] - bfloor), portfv[t])
+   bondv[t] <- (portfv[t] - stockv[t])
+ }  # end for
> # dygraph plot of CPPI strategy
> pricev <- 100*cumprod(1 + retp)
> datav <- xts::xts(cbind(stockv, bondv, portfv, pricev), datev)
> colnames(datav) <- c("stocks", "bonds", "CPPI", "VTI")
> endd <- rutils::calc_endpoints(datav, interval="weeks")
> dygraphs::dygraph(datav[endd], main="CPPI strategy") %>%
+   dyOptions(colors=c("red", "green", "blue", "orange"), strokeWidt
+   dyLegend(show="always", width=300)
```

Risk Parity Strategy For Stocks and Bonds

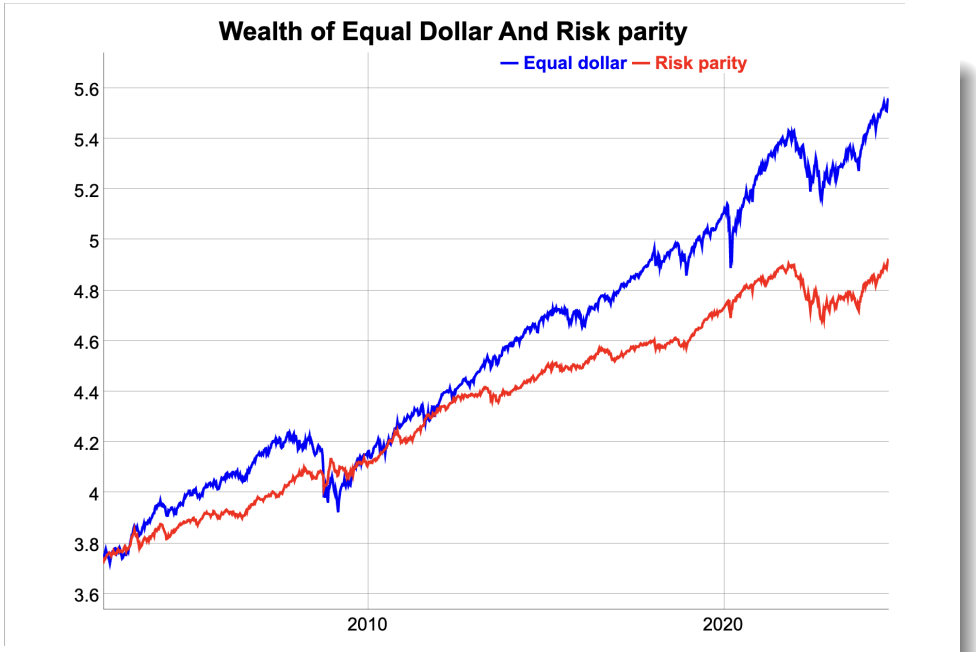
The weights of the risk parity strategy are proportional to the inverse of the stock dollar volatilities: $w_i \propto \frac{1}{\sigma_i}$.

So that the stock allocations (*dollar amounts*) have equal dollar volatilities.

The risk parity strategy has a higher Sharpe ratio than the equal dollar portfolio because it's overweight bonds. But it also has lower absolute returns.

The risk parity strategy was developed in the early 1990s by *Bridgewater Associates*.

```
> # Calculate the dollar and percentage returns of VTI and IEF
> pricev <- na.omit(rutils::etfenv$prices[, c("VTI", "IEF")])
> datev <- zoo::index(pricev)
> retd <- rutils::diffit(pricev)
> retp <- retd/rutils::lagit(pricev, lagg=1, pad_zeros=FALSE)
> # Calculate the risk parity weights
> weightv <- 1/sapply(retd, sd)
> weightv <- weightv/sum(weightv)
> # Wealth of risk parity
> wealthrp <- drop(pricev %*% weightv)
> # Wealth of equal dollar (without rebalancing)
> weightv <- 1/as.numeric(pricev[1, ])
> weightv <- weightv/sum(weightv)
> wealthed <- (pricev %*% weightv)
> wealthed <- wealthrp[1]*wealthed/wealthed[1]
```




```
> # Calculate the Sharpe and Sortino ratios
> wealthv <- xts::xts(cbind(wealthed, wealthrp), datev)
> colnames(wealthv) <- c("Equal dollar", "Risk parity")
> sqrt(252)*sapply(rutils::diffit(wealthv), function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # Plot the log wealth
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(log(wealthv[endd]),
+   main="Wealth of Equal Dollar And Risk parity") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

The Rolling Risk Parity Allocations

In practice, the dollar volatility σ_t changes over time so it must be recalculated, and the weights must be updated: $w_t \propto \frac{1}{\sigma_t}$.

The stock allocations (*dollar amounts*) increase when the volatility is low, and vice versa.

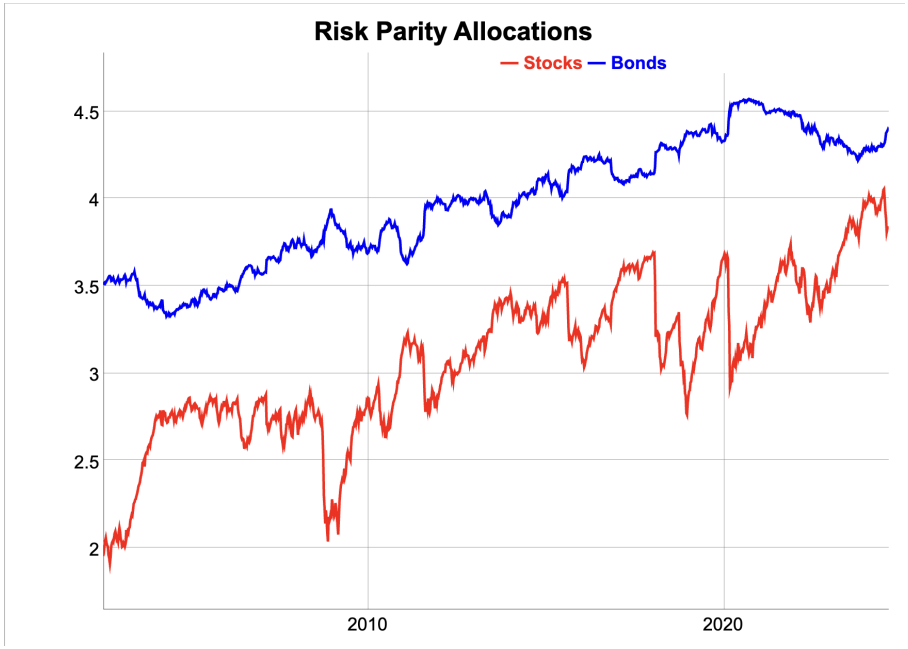
The function `HighFreq::run_var()` calculates the trailing variance of returns r_t, by recursively weighting the past variance estimates σ^2_{t-1}, with the squared differences of the returns minus the trailing means $(r_t - \bar{r}_t)^2$, using the decay factor λ:

$$\bar{r}_t = \lambda \bar{r}_{t-1} + (1 - \lambda) r_t$$

$$\sigma^2_t = \lambda \sigma^2_{t-1} + (1 - \lambda)(r_t - \bar{r}_t)^2$$

Where σ^2_t is the trailing variance at time t.

The decay factor λ determines how quickly the variance estimates are updated, with smaller values of λ producing faster updating, giving more weight to recent returns, and vice versa.




```
> # Calculate the trailing dollar volatilities
> lambdav <- 0.99
> vold <- HighFreq::run_var(retd, lambda=lambdav)
> vold <- sqrt(vold)
> # Calculate the rolling risk parity weights
> weightv <- 1/vold
> weightv <- weightv/rowSums(weightv)
> # Calculate the risk parity allocations
> pricerp <- pricev*weightv
> # Plot the risk parity allocations
> colnames(pricerp) <- c("Stocks", "Bonds")
> dygraph(log(pricerp[endd]), main="Risk Parity Allocations") %>%
+   dyOptions(colors=c("red", "blue"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

Rolling Risk Parity Strategy

In the *Risk Parity* strategy the stock allocations are rebalanced daily so that their dollar volatilities remain equal.

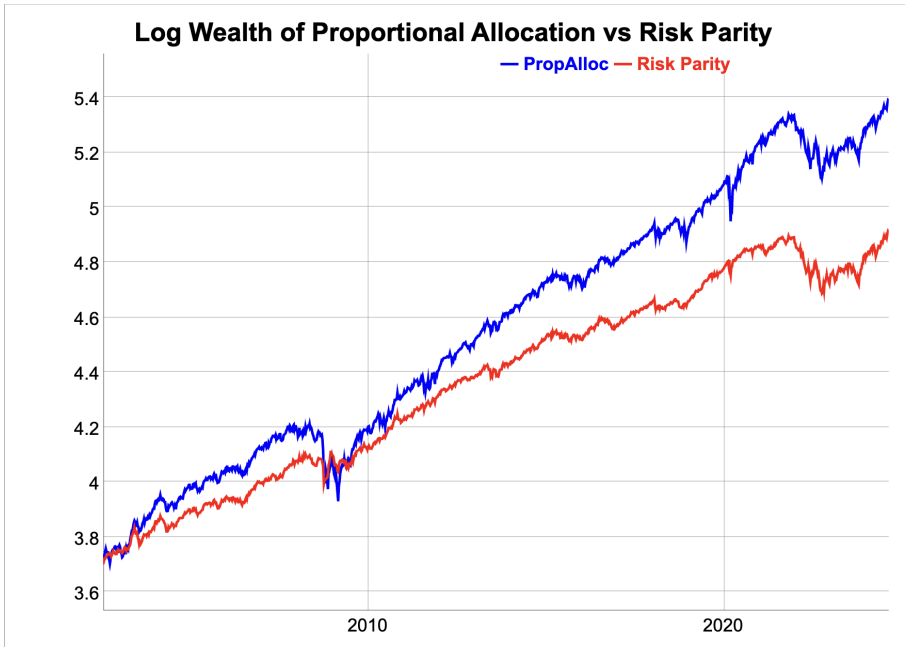
The dollar returns of risk parity r_d are equal to the percentage returns of the original prices r_t times the stock allocations p_a: $r_d = r_t p_a$.

The risk parity strategy for stocks and bonds has a higher Sharpe ratio than the proportional wealth strategy. But it also has lower absolute returns.

The absolute returns can be increased by increasing the leverage - buying more stocks and bonds with borrowed money.

Risk parity performs better for assets with negative or low correlations of returns, like stocks and bonds.

Risk parity performed poorly during the Covid crisis because of the *simultaneous losses of both stocks and bonds* (positive correlations).




```
> # Calculate the dollar returns of risk parity
> retrp <- retp*rutils::lagit(pricerp)
> retrp[1, ] <- pricerp[1, ]
> # Calculate the wealth of risk parity
> wealthrp <- cumsum(rowSums(retrp))
> # Wealth of proportional allocation (with rebalancing)
> wealthpr <- cumprod(1 + rowMeans(retp))
> wealthpr <- wealthpr*wealthrp[1]
```

```
> # Calculate the Sharpe and Sortino ratios
> wealthv <- cbind(wealthpr, wealthrp)
> wealthv <- xts::xts(wealthv, datev)
> colnames(wealthv) <- c("PropAlloc", "Risk Parity")
> sqrt(252)*sapply(rutils::diffit(wealthv), function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # Plot a dygraph of the log wealths
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(log(wealthv[endd]),
+   main="Log Wealth of Proportional Allocation vs Risk Parity") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

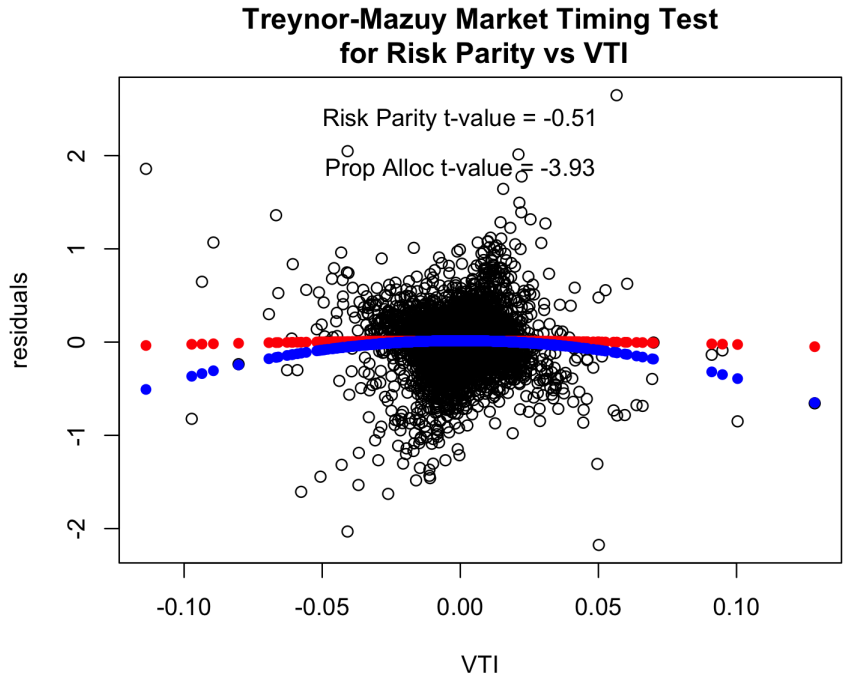
Risk Parity Strategy Market Timing Skill

The risk parity strategy reduces allocations to assets with higher volatilities, which is often accompanied by negative returns.

This allows the risk parity strategy to time the markets better - selling when prices are droppng and buying when prices are rising. But only when the changes of the volatility are significant.

But the t-value of the risk parity strategy is negative, because it's contrarian when the volatility is stable.

The t-value of the proportional wealth allocation strategy is negative, because it's contrarian - it buys stocks when their prices drop.




```
> # Test risk parity market timing of VTI using Treynor-Mazuy test
> retrp <- rutils::diffit(wealthv)
> retvti <- retp$VTI
> desm <- cbind(retrp, retvti, retvti^2)
> colnames(desm)[1:2] <- c("prop", "riskp")
> colnames(desm)[4] <- "Treynor"
> regmod <- lm(riskp ~ VTI + Treynor, data=desm)
> summary(regmod)
> # Plot residual scatterplot
> resids <- regmod$residuals
> plot.default(x=retvti, y=resids, xlab="VTI", ylab="residuals")
> title(main="Treynor-Mazuy Market Timing Test\n for Risk Parity vs
```

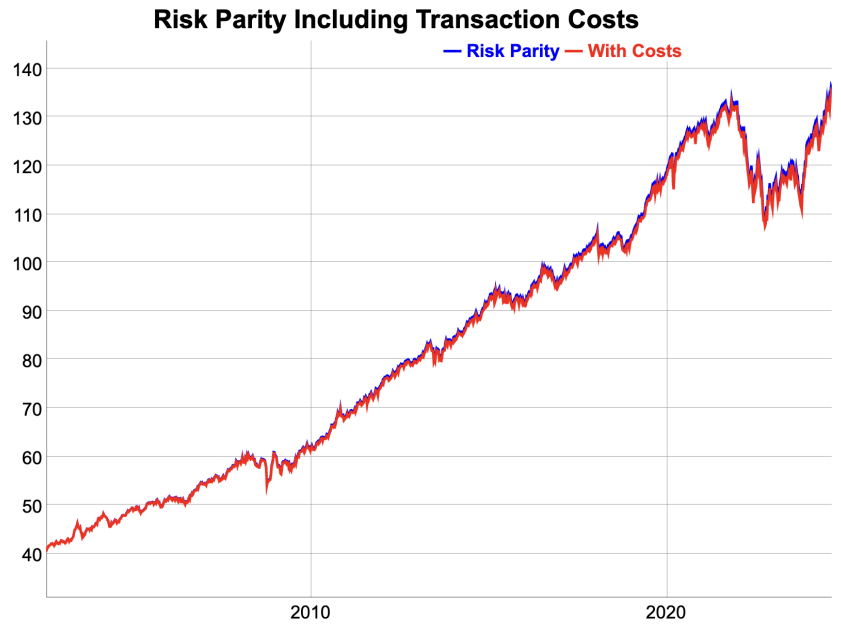
```
> # Plot fitted (predicted) response values
> coefreg <- summary(regmod)$coeff
> fitv <- regmod$fitted.values - coefreg["VTI", "Estimate"]*retvti
> tvalue <- round(coefreg["Treynor", "t value"], 2)
> points.default(x=retvti, y=fitv, pch=16, col="red")
> text(x=0.0, y=0.9*max(resids), paste("Risk Parity t-value =", tval
> # Test for equal wealth strategy market timing of VTI using Treyno
> regmod <- lm(prop ~ VTI + Treynor, data=desm)
> summary(regmod)
> # Plot fitted (predicted) response values
> coefreg <- summary(regmod)$coeff
> fitv <- regmod$fitted.values - coefreg["VTI", "Estimate"]*retvti
> points.default(x=retvti, y=fitv, pch=16, col="blue")
> text(x=0.0, y=0.7*max(resids), paste("Prop Alloc t-value =", round
```

Transaction Costs of Risk Parity Strategy

The risk parity strategy has higher transaction costs because it trades more shares than the proportional wealth strategy, and it may also use leverage to buy more shares.

But the higher transaction costs are not large enough to completely cancel the higher returns of the strategy.

```
> # Total dollar amount of stocks that need to be traded
> notx <- rutils::diffit(pricerp)
> # The bid-ask spread is equal to 1 bp for liquid ETFs
> bidask <- 0.001
> # Calculate the cumulative transaction costs
> costv <- 0.5*bidask*cumsum(rowSums(abs(notx)))
```




```
> # dygraph plot of wealth and transaction costs
> wealthv <- cbind(wealthrp, wealthrp-costv)
> wealthv <- xts::xts(wealthv, datev)
> colnamev <- c("Risk Parity", "With Costs")
> colnames(wealthv) <- colnamev
> dygraphs::dygraph(wealthv[endd], main="Risk Parity Including Trans
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

Sell in May Calendar Strategy

Sell in May is a *market timing calendar strategy*, in which stocks are sold at the beginning of May, and then bought back at the beginning of November.

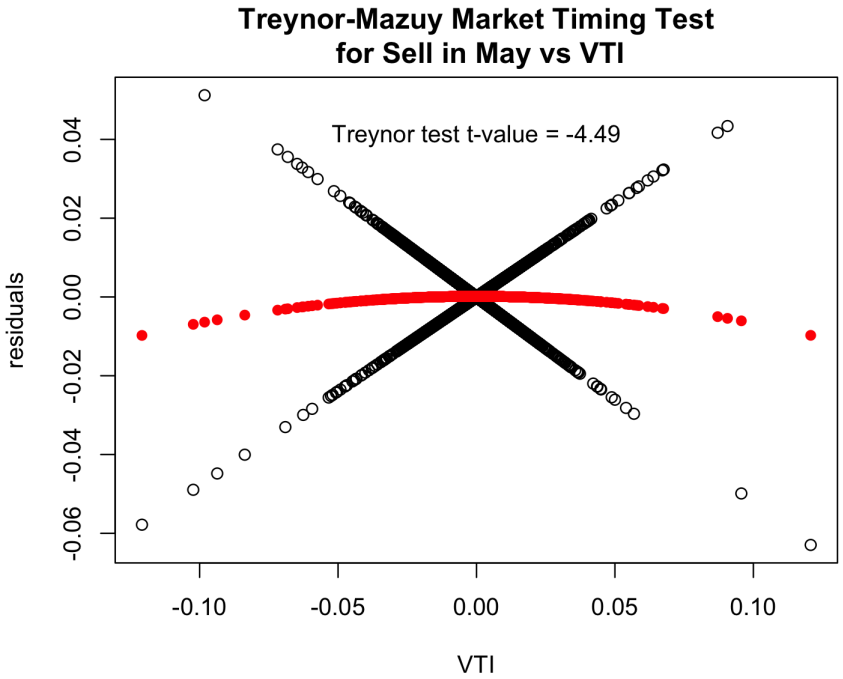
```
> # Calculate the positions
> retp <- na.omit(rutils::etfenv$returns$VTI)
> posv <- rep(NA_integer_, NROW(retp))
> datev <- zoo::index(retp)
> datev <- format(datev, "%m-%d")
> posv[datev == "05-01"] <- 0
> posv[datev == "05-03"] <- 0
> posv[datev == "11-01"] <- 1
> posv[datev == "11-03"] <- 1
> # Carry forward and backward non-NA posv
> posv <- zoo::na.locf(posv, na.rm=FALSE)
> posv <- zoo::na.locf(posv, fromLast=TRUE)
> # Calculate the strategy returns
> pnlmay <- posv*retp
> wealthv <- cbind(retp, pnlmay)
> colnames(wealthv) <- c("VTI", "sellmay")
> # Calculate the Sharpe and Sortino ratios
> sqrt(252)*sapply(wealthv, function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
```


```
> # Plot wealth of Sell in May strategy
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd], main="Sell in May Strateg
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
> # OR: Open x11 for plotting
> x11(width=6, height=5)
> par(mar=c(4, 4, 3, 1), oma=c(0, 0, 0, 0))
> plot_theme <- chart_theme()
> plot_theme$col$line.col <- c("blue", "red")
> quantmod::chart_Series(wealthv, theme=plot_theme, name="Sell in Ma
> legend("topleft", legend=colnames(wealthv),
+   inset=0.1, bg="white", lty=1, lwd=6, y.intersp=0.5,
+   col=plot_theme$col$line.col, bty="n")
```

Sell in May Strategy Market Timing

The *Sell in May* strategy doesn't demonstrate any ability of *timing* the *VTI* ETF.

```
> # Test if Sell in May strategy can time VTI
> desm <- cbind(wealthv, 0.5*(retp+abs(retp)), retp^2)
> colnames(desm) <- c(colnames(wealthv), "Merton", "Treynor")
> # Perform Merton-Henriksson test
> regmod <- lm(sellmay ~ VTI + Merton, data=desm)
> summary(regmod)
> # Perform Treynor-Mazuy test
> regmod <- lm(sellmay ~ VTI + Treynor, data=desm)
> summary(regmod)
> # Plot Treynor-Mazuy residual scatterplot
> resids <- (desm$sellmay - regmod$coeff["VTI"]*retp)
> plot.default(x=retp, y=resids, xlab="VTI", ylab="residuals")
> title(main="Treynor-Mazuy Market Timing Test\n for Sell in May vs
> # Plot fitted (predicted) response values
> coefreg <- summary(regmod)$coeff
> fitv <- regmod$fitted.values - coefreg["VTI", "Estimate"]*retp
> tvalue <- round(coefreg["Treynor", "t value"], 2)
> points.default(x=retp, y=fitv, pch=16, col="red")
> text(x=0.0, y=0.8*max(resids), paste("Treynor test t-value =", tvalue))
```



Overnight Market Anomaly

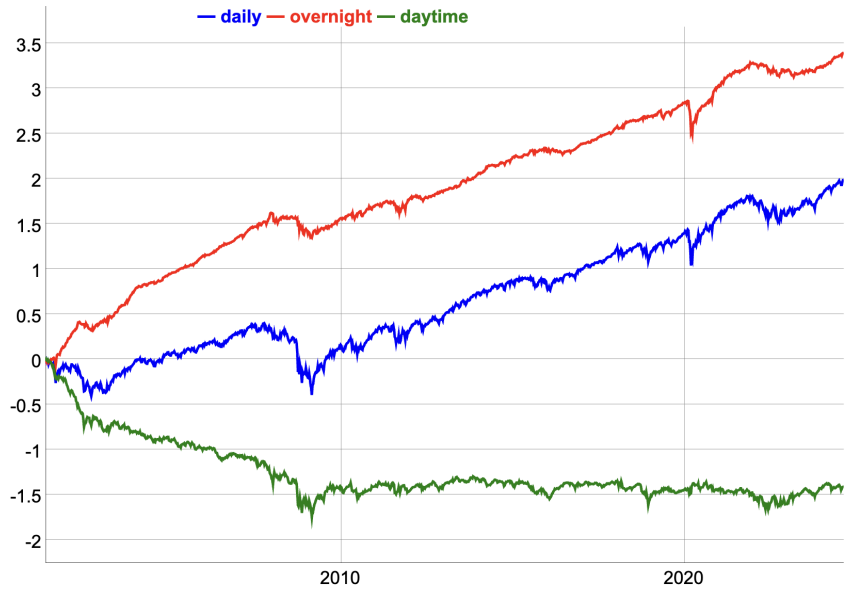
The *Overnight Market Anomaly* is the consistent outperformance of overnight returns relative to the daytime returns.

The *Overnight Market Anomaly* has been observed for many decades for most stock market indices, but not always for all stock sectors.

The Overnight Strategy consists of holding a long position only overnight (buying at the close and selling at the open the next day).

The Daytime Strategy consists of holding a long position only during the daytime (buying at the open and selling at the close the same day).

The *Overnight Market Anomaly* is not as pronounced after the 2008-2009 financial crisis.




```
> # Calculate the log of OHLC VTI prices
> ohlc <- log(rutils::etfenv$VTI)
> openp <- quantmod::Op(ohlc)
> highp <- quantmod::Hi(ohlc)
> lowp <- quantmod::Lo(ohlc)
> closep <- quantmod::Cl(ohlc)
> # Calculate the close-to-close log returns,
> # the daytime open-to-close returns
> # and the overnight close-to-open returns.
> retp <- rutils::diffit(closep)
> colnames(retp) <- "daily"
> retd <- (closep - openp)
> colnames(retd) <- "daytime"
> reton <- (openp - rutils::lagit(closep, lagg=1, pad_zeros=FALSE))
> colnames(reton) <- "overnight"
```

```
> # Calculate the Sharpe and Sortino ratios
> wealthv <- cbind(retp, reton, retd)
> sqrt(252)*sapply(wealthv, function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # Plot the log wealth
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd],
+   main="Wealth of Close-to-Close, Overnight, and Daytime Strategie
+   dySeries(name="daily", strokeWidth=2, col="blue") %>%
+   dySeries(name="overnight", strokeWidth=2, col="red") %>%
+   dySeries(name="daytime", strokeWidth=2, col="green") %>%
+   dyLegend(width=500)
```

Turn of the Month Effect

The *Turn of the Month* (TOM) effect is the outperformance of stocks on the last trading day of the month and on the first three days of the following month.

The *TOM* effect was observed for the period from 1928 to 1975, but it has been less pronounced since the year 2000.

The *TOM* effect has been attributed to the investment of funds deposited at the end of the month.

This would explain why the *TOM* effect has been more pronounced for less liquid small-cap stocks.


```
> # Calculate the VTI returns
> retp <- na.omit(rutils::etfenv$returns$VTI)
> datev <- zoo::index(retp)
> # Calculate the first business day of every month
> dayv <- as.numeric(format(datev, "%d"))
> indeks <- which(rutils::diffit(dayv) < 0)
> datev[head(indeks)]
> # Calculate the Turn of the Month dates
> indeks <- lapply((-1):2, function(x) indeks + x)
> indeks <- do.call(c, indeks)
> sum(indeks > NROW(datev))
> indeks <- sort(indeks)
> datev[head(indeks, 11)]
> # Calculate the Turn of the Month pnls
> pnls <- numeric(NROW(retp))
> pnls[indeks] <- retp[indeks, ]
```

```
> # Combine data
> wealthv <- cbind(retp, pnls)
> colnamev <- c("VTI", "TOM Strategy")
> colnames(wealthv) <- colnamev
> # Calculate the Sharpe and Sortino ratios
> sqrt(252)*sapply(wealthv, function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # dygraph plot VTI Turn of the Month strategy
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd],
+   main="Turn of the Month Strategy") %>%
+   dyAxis("y", label=colnamev[1], independentTicks=TRUE) %>%
+   dyAxis("y2", label=colnamev[2], independentTicks=TRUE) %>%
+   dySeries(name=colnamev[1], axis="y", strokeWidth=2, col="blue")
+   dySeries(name=colnamev[2], axis="y2", strokeWidth=2, col="red")
```

The Stop-loss Strategy

Stop-loss rules are used to reduce losses in case of a significant drawdown in prices.

For example, a simple stop-loss rule is to sell the stock if its price drops below the stop-loss level, equal to the stop-loss percentage times the previous maximum price.

The stock is bought back after the price recovers, for example after the price reaches its previous maximum price.

The stop-loss strategy trades a single $1 of stock, and is either long $1 of stock or it's flat ($0 of stock).

The stop-loss strategy is trend-following because it expects prices to continue dropping.

```
> # Calculate the VTI prices and returns
> pricev <- na.omit(rutils::etfenv$prices$VTI)
> nrows <- NROW(pricev)
> datev <- zoo::index(pricev)
> retp <- rutils::diffit(log(pricev))
> # Simulate stop-loss strategy
> stopl <- 0.05 # Stop-loss percentage
> pricem <- cummax(pricev) # Trailing maximum prices
> # Calculate the drawdown
> dd <- (pricev - pricem)
> pnls <- retp # Initialize PnLs
> for (i in 1:(nrows-1)) {
+ # Check for stop-loss
+   if (dd[i] < -stopl*pricem[i])
+     pnls[i+1] <- 0 # Set PnLs = 0 if in stop-loss
+ }  # end for
> # Same but without using loops in R
> pnls2 <- retp
> insl <- rutils::lagit(dd < -stopl*pricem)
> pnls2 <- ifelse(insl, 0, pnls2)
> all.equal(pnls, pnls2, check.attributes=FALSE)
```

Stop-loss Strategy Performance

The stop-loss strategy underperforms because it trades with a lag.

After it hits the stop-loss, it unwinds its long position with a lag. And after it recovers from the stop-loss, it re-enters the long position with a lag.

The losses are compounded when the strategy is *"whipsawed"*, when prices are range-bound without having a trend, and the strategy switches back and forth.

When prices are range-bound without a trend, the stop-loss strategy often stops because of a drawdown (goes flat risk), but if the prices soon rebound, then it's forced to buy back the stock.

The strategy enters the stop-loss just before the prices start rising, and it re-enters the long position just before the prices start dropping.


```
> # Combine the data
> wealthv <- cbind(retp, pnls)
> colnamev <- c("VTI", "Strategy")
> colnames(wealthv) <- colnamev
> # Calculate the Sharpe and Sortino ratios
> sqrt(252)*sapply(wealthv, function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # dygraph plot the stop-loss strategy
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd],
+   main="VTI Stop-loss Strategy") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

```
> # Plot dygraph with shading
> # Create colors for background shading
> indic <- (rutils::diffit(insl) != 0) # Indices of stop-loss
> crossd <- c(datev[indic], datev[nrows]) # Dates of stop-loss
> shadev <- ifelse(insl[indic] == 1, "antiquewhite", "lightgreen")
> # Create dygraph object without plotting it
> dyplot <- dygraphs::dygraph(cumsum(wealthv),
+   main="VTI Stop-loss Strategy") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
> # Add shading to dygraph object
> for (i in 1:NROW(shadev)) {
+   dyplot <- dyplot %>% dyShading(from=crossd[i], to=crossd[i+1], c
+ }  # end for
> # Plot the dygraph object
> dyplot
```

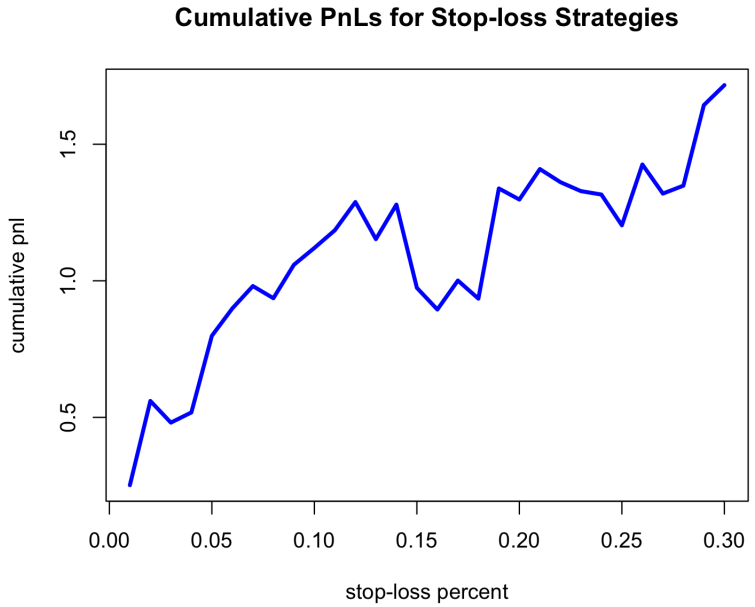
Optimal Stop-loss Strategy

Stop-loss rules can reduce the largest drawdowns but they also tend to reduce cumulative returns for stocks with good returns.

The best performing stop-loss strategy for *VTI* has the largest stop-loss percentage - i.e. it almost never enters into a stop-loss.

That's because *VTI* has had positive returns in the last 20 years, so a stop-loss rule had little benefit.

That's why many quantitative investment funds do not use stop-loss rules if they have strong convictions that the stocks will have positive returns.




```
> # Simulate multiple stop-loss strategies
> dd <- (pricev - pricem)
> stopv <- 0.01*(1:30)
> pnlc <- sapply(stopv, function(stopl) {
+   pnls <- retp
+   insl <- rutils::lagit(dd < -stopl*pricem)
+   pnls <- ifelse(insl, 0, pnls)
+   sum(pnls)
+ })  # end sapply
```

```
> # Plot cumulative pnls for stop-loss strategies
> plot(x=stopv, y=pnlc,
+    main="Cumulative PnLs for Stop-loss Strategies",
+    xlab="stop-loss percent", ylab="cumulative pnl",
+    t="l", lwd=3, col="blue")
```

Stop-Start Strategy

The stop-loss strategy can be improved by introducing a start-gain rule: buy back the stock if its price rebounds from the previous minimum and exceeds the start-gain level.

The stop-start strategy implements both stop-loss events due to price drawdowns and start-gain events due to price draw-ups.

In order to determine the stop-loss and start-gain events, the stop-start strategy follows the trailing maximum and trailing minimum prices.

A start-gain event is when the stock price rebounds from the previous minimum and exceeds the start-gain level, equal to the start-gain percentage times the previous minimum price.

After the start-gain level is crossed, the strategy buys back the stock which was sold under the stop-loss.

The stop-start strategy is trend-following because it profits if price trends are persistent. But it loses if prices are range-bound.

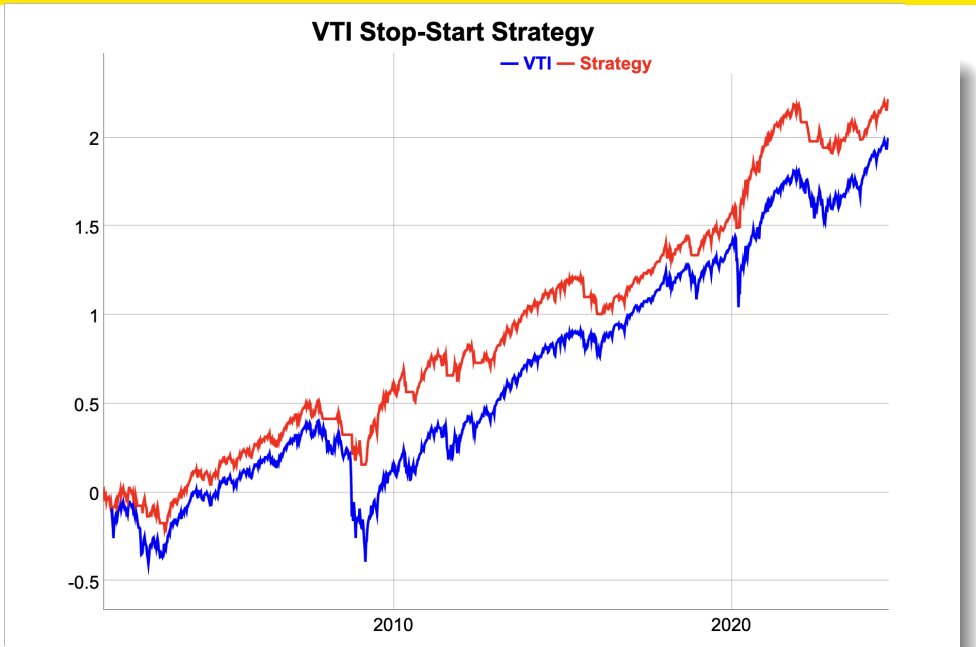
```
> # Define function for simulating a stop-start strategy
> sim_stopstart <- function(stopl) {
+   maxp <- pricev[1] # Trailing maximum price
+   minp <- pricev[1] # Trailing minimum price
+   insl <- FALSE # Is in stop-loss?
+   insg <- FALSE # Is in start-gain?
+   pnls <- retp # Initialize PnLs
+   for (i in 1:nrows) {
+     if (insl) { # In stop-loss
+ pnls[i] <- 0 # Set PnLs = 0 if in stop-loss
+ minp <- min(minp, pricev[i]) # Update minimum price to current pri
+ if (pricev[i] > ((1 + stopl)*minp)) { # Check for start-gain
+   insg <- TRUE # Is in start-gain?
+   insl <- FALSE # Is in stop-loss?
+   maxp <- pricev[i] # Reset trailing maximum price
+ }  # end if
+     } else if (insg) { # In start-gain
+ maxp <- max(maxp, pricev[i]) # Update maximum price to current pri
+ if (pricev[i] < ((1 - stopl)*maxp)) { # Check for stop-loss
+   insl <- TRUE # Is in stop-loss?
+   insg <- FALSE # Is in start-gain?
+   minp <- pricev[i] # Reset trailing minimum price
+ }  # end if
+     } else { # Warmup period
+ # Update the maximum and minimum prices
+ maxp <- max(maxp, pricev[i])
+ minp <- min(minp, pricev[i])
+ # Update the stop-loss and start-gain indicators
+ insl <- (pricev[i] < ((1 - stopl)*maxp)) # Is in stop-loss?
+ insg <- (pricev[i] > ((1 + stopl)*minp)) # Is in start-gain?
+     }  # end if
+   }  # end for
+   return(pnls)
+ } # end sim_stopstart
```

Stop-Start Strategy Performance

The stop-start strategy spends less time in a stop-loss because prices tend to rebound sharply after a steep loss.

The start-gain rule tends to quickly override the stop-loss rule, so that the strategy is long the stock after it recovers after a stop-loss.

```
> # Simulate the stop-start strategy
> pnls <- sim_stopstart(0.1)
> # Combine the data
> wealthv <- cbind(retp, pnls)
> colnamev <- c("VTI", "Strategy")
> colnames(wealthv) <- colnamev
> # Calculate the Sharpe and Sortino ratios
> sqrt(252)*sapply(wealthv, function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # dygraph plot the stop-loss strategy
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd],
+   main="VTI Stop-Start Strategy") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

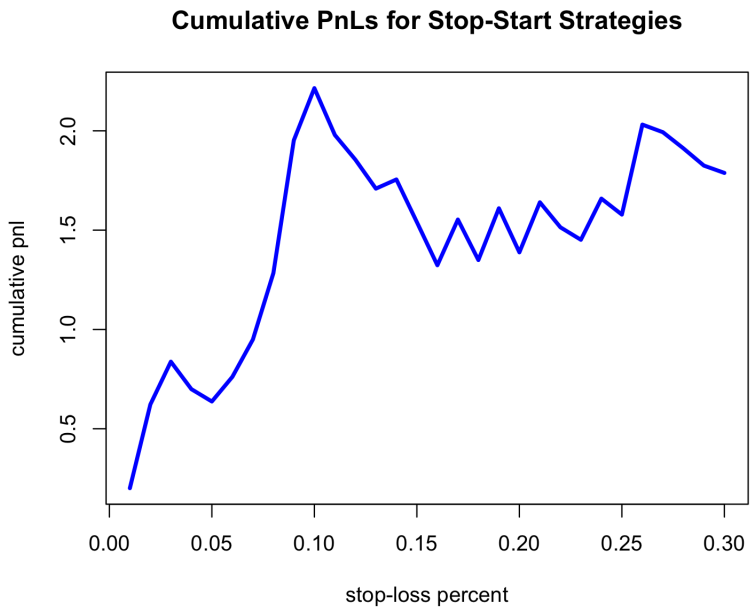



```
> # Plot dygraph with shading
> # Create colors for background shading
> insl <- (pnls == 0) # Is in stop-loss?
> indic <- (rutils::diffit(insl) != 0) # Indices of crosses
> crossd <- c(datev[indic], datev[nrows]) # Dates of crosses
> shadev <- ifelse(insl[indic] == 1, "antiquewhite", "lightgreen")
> # Create dygraph object without plotting it
> dyplot <- dygraphs::dygraph(cumsum(wealthv),
+   main="VTI Stop-Start Strategy") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
> # Add shading to dygraph object
> for (i in 1:NROW(shadev)) {
+   dyplot <- dyplot %>% dyShading(from=crossd[i], to=crossd[i+1], c
+ }  # end for
> # Plot the dygraph object
> dyplot
```

Optimal Stop-Start Strategy

The stop-start strategy performs much better than the stop-loss strategy because it captures stock gains.

```
> # Simulate multiple stop-loss strategies
> stopv <- 0.01*(1:30)
> pnlc <- sapply(stopv, function(stopl) {
+   sum(sim_stopstart(stopl))
+ })  # end sapply
> stopl <- stopv[which.max(pnlc)]
```

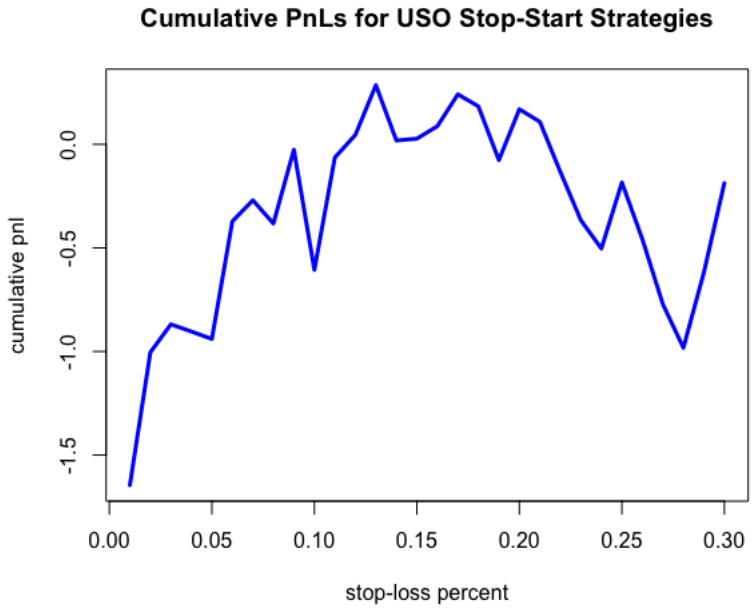



```
> # Plot cumulative pnls for stop-loss strategies
> plot(x=stopv, y=pnlc,
+    main="Cumulative PnLs for Stop-Start Strategies",
+    xlab="stop-loss percent", ylab="cumulative pnl",
+    t="l", lwd=3, col="blue")
```

Stop-Start Strategy for Other ETFs

The stop-start strategy can prevent losses for stocks with significant negative returns, like the *USO* ETF (oil fund).

```
> # Calculate the USO prices and returns
> pricev <- na.omit(rutils::etfenv$prices$USO)
> nrows <- NROW(pricev)
> datev <- zoo::index(pricev)
> retp <- rutils::diffit(log(pricev))
> # Simulate multiple stop-start strategies
> stopv <- 0.01*(1:30)
> pnlc <- sapply(stopv, function(stopl) {
+   sum(sim_stopstart(stopl))
+ })  # end sapply
```




```
> # Plot cumulative pnls for stop-start strategies
> plot(x=stopv, y=pnlc,
+    main="Cumulative PnLs for USO Stop-Start Strategies",
+    xlab="stop-loss percent", ylab="cumulative pnl",
+    t="l", lwd=3, col="blue")
```

Optimal Stop-Start Strategy For *USO*

The stop-start strategy for the *USO* ETF has performed well because *USO* has had very negative returns in the last 20 years.

Stop-start strategies are able to preserve profits for the best performing stocks, and avoid losses for the worst performing stocks.

```
> # Simulate optimal stop-start strategy for USO
> stopl <- stopv[which.max(pnlc)]
> pnls <- sim_stopstart(stopl)
> # Combine the data
> wealthv <- cbind(retp, pnls)
> colnamev <- c("USO", "Strategy")
> colnames(wealthv) <- colnamev
> # Calculate the Sharpe and Sortino ratios
> sqrt(252)*sapply(wealthv, function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # dygraph plot the stop-start strategy
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd],
+   main="USO Stop-Start Strategy") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```


```
> # Plot dygraph with shading
> # Create colors for background shading
> insl <- (pnls == 0) # Is in stop-loss?
> indic <- (rutils::diffit(insl) != 0) # Indices of crosses
> crossd <- c(datev[indic], datev[nrows]) # Dates of crosses
> shadev <- ifelse(insl[indic] == 1, "antiquewhite", "lightgreen")
> # Create dygraph object without plotting it
> dyplot <- dygraphs::dygraph(cumsum(wealthv),
+   main="USO Stop-Start Strategy") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
> # Add shading to dygraph object
> for (i in 1:NROW(shadev)) {
+   dyplot <- dyplot %>% dyShading(from=crossd[i], to=crossd[i+1], c
+ }  # end for
> # Plot the dygraph object
> dyplot
```

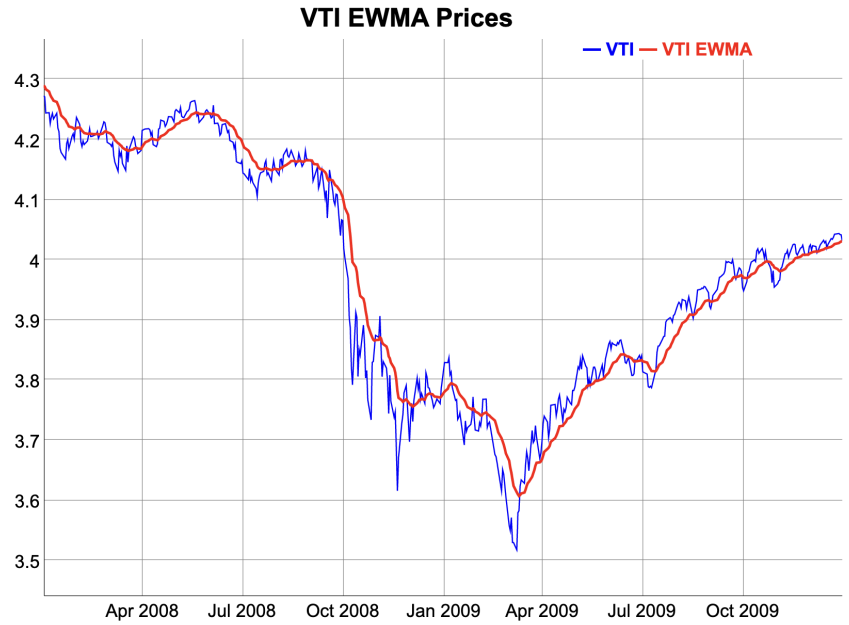
EMA Price Technical Indicator

The *Exponentially Weighted Moving Average Price* (*EMA*) is defined as the weighted average of prices over a rolling interval:

$$p_i^{EMA} = (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j p_{i-j}$$

Where the decay factor λ determines the rate of decay of the *EMA* weights, with smaller values of λ producing faster decay, giving more weight to recent prices, and vice versa.

The function `HighFreq::roll_wsum()` calculates the convolution of a time series with a vector of weights.




```
> # Extract the log VTI prices
> ohlc <- log(rutils::etfenv$VTI)
> closep <- quantmod::Cl(ohlc)
> colnames(closep) <- "VTI"
> nrows <- NROW(closep)
> # Calculate the EMA weights
> lookb <- 111
> lambdaf <- 0.9
> weightv <- lambdaf^(0:lookb)
> weightv <- weightv/sum(weightv)
> # Calculate the EMA prices as a convolution
> pricema <- HighFreq::roll_sumw(closep, weightv=weightv)
> pricev <- cbind(closep, pricema)
> colnames(pricev) <- c("VTI", "VTI EMA")
```

```
> # Dygraphs plot with custom line colors
> colnamev <- colnames(pricev)
> dygraphs::dygraph(pricev["2008/2009"], main="VTI EMA Prices") %>%
+   dySeries(name=colnamev[1], strokeWidth=1, col="blue") %>%
+   dySeries(name=colnamev[2], strokeWidth=2, col="red") %>%
+   dyLegend(show="always", width=300)
> # Standard plot of  EMA prices with custom line colors
> x11(width=6, height=5)
> plot_theme <- chart_theme()
> colorv <- c("blue", "red")
> plot_theme$col$line.col <- colorv
> quantmod::chart_Series(pricev["2009"], theme=plot_theme,
+        lwd=2, name="VTI EMA Prices")
> legend("topleft", legend=colnames(pricev), y.intersp=0.5,
+  inset=0.1, bg="white", lty=1, lwd=6, cex=0.8,
+  col=plot_theme$col$line.col, bty="n")
```

Recursive EMA Price Indicator

The *EMA* prices can be calculated recursively as follows:

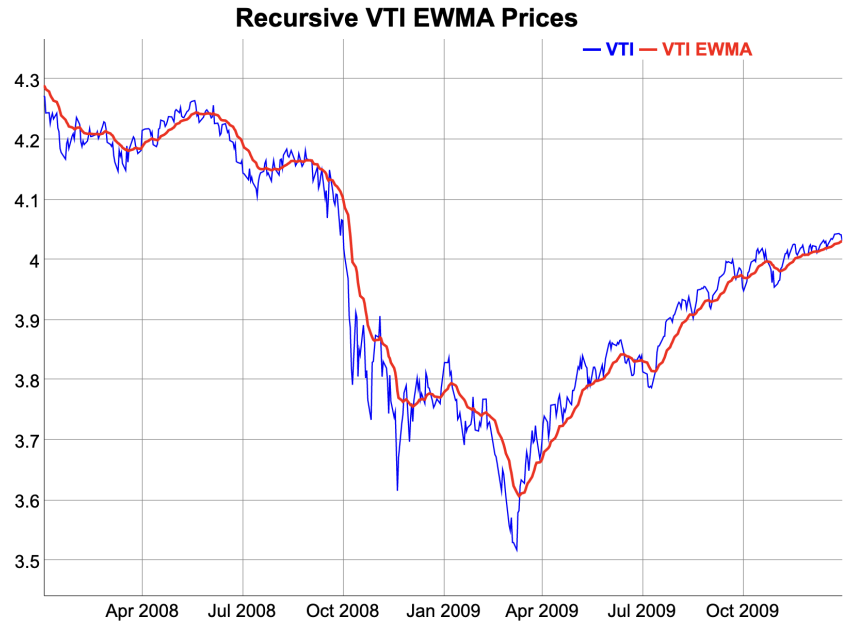
$$p_i^{EMA} = (1 - \lambda)p_i + \lambda p_{i-1}^{EMA}$$

Where the decay factor λ determines the rate of decay of the *EMA* weights, with smaller values of λ producing faster decay, giving more weight to recent prices, and vice versa.

The recursive *EMA* prices are slightly different from those calculated as a convolution, because the convolution uses a fixed look-back interval.

The compiled `C++` function `stats:::C_rfilter()` calculates the exponentially weighted moving average prices recursively.

The function `HighFreq::run_mean()` calculates the exponentially weighted moving average prices recursively.




```
> # Calculate the EMA prices recursively using C++ code
> emar <- .Call(stats:::C_rfilter, closep, lambdaf, c(as.numeric(cl
> # Or R code
> # emar <- filter(closep, filter=lambdaf, init=as.numeric(closep[1
> emar <- (1-lambdaf)*emar
> # Calculate the EMA prices recursively using RcppArmadillo
> pricema <- HighFreq::run_mean(closep, lambda=lambdaf)
> all.equal(drop(pricema), emar)
> # Compare the speed of C++ code with RcppArmadillo
> library(microbenchmark)
> summary(microbenchmark(
+   Rcpp=HighFreq::run_mean(closep, lambda=lambdaf),
+   rfilter=.Call(stats:::C_rfilter, closep, lambdaf, c(as.numeric
+   times=10))[, c(1, 4, 5)]
```

```
> # Dygraphs plot with custom line colors
> pricev <- cbind(closep, pricema)
> colnames(pricev) <- c("VTI", "VTI EMA")
> colnamev <- colnames(pricev)
> dygraphs::dygraph(pricev["2008/2009"], main="Recursive VTI EMA Pri
+   dySeries(name=colnamev[1], strokeWidth=1, col="blue") %>%
+   dySeries(name=colnamev[2], strokeWidth=2, col="red") %>%
+   dyLegend(show="always", width=300)
> # Standard plot of  EMA prices with custom line colors
> plot_theme <- chart_theme()
> colorv <- c("blue", "red")
> plot_theme$col$line.col <- colorv
> quantmod::chart_Series(pricev["2009"], theme=plot_theme,
+        lwd=2, name="VTI EMA Prices")
> legend("topleft", legend=colnames(pricev), y.intersp=0.5,
+  inset=0.1, bg="white", lty=1, lwd=6, cex=0.8,
+  col=plot_theme$col$line.col, bty="n")
```

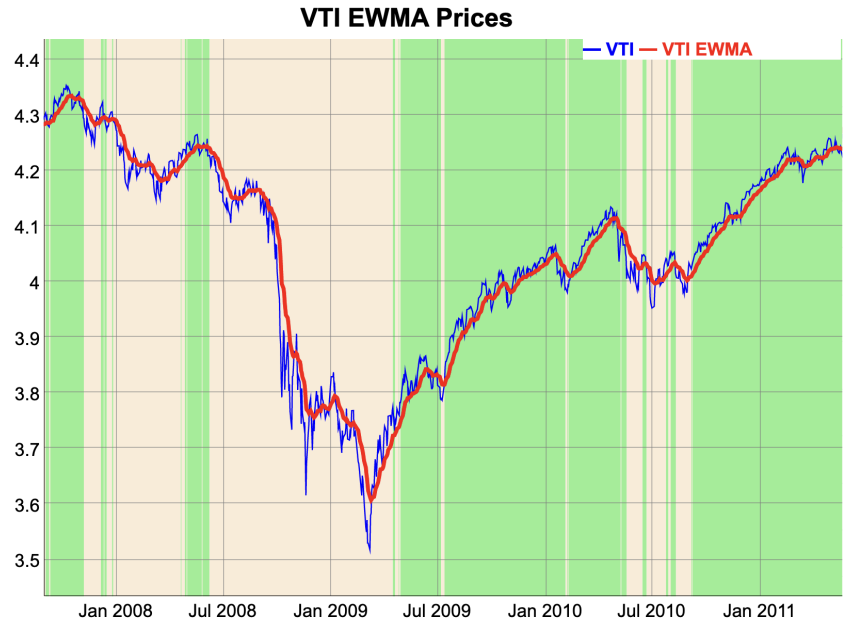
The EMA Crossover Strategy

The trend following *EMA Crossover* strategy switches its stock position depending if the current price is above or below the *EMA*.

If the stock price is above the *EMA* price, then the strategy switches to long $1 dollar of stock, and if it is below, to short $1 dollar of stock.

The strategy holds the same position until the *EMA* crosses over the current price (either from above or below), and then it switches its position.

The strategy is therefore always either long $1 dollar of stock or short $1 dollar of stock.




```
> # Calculate the EMA prices recursively using C++ code
> lambdaf <- 0.984
> pricema <- HighFreq::run_mean(closep, lambda=lambdaf)
> pricev <- cbind(closep, pricema)
> colnames(pricev) <- c("VTI", "VTI EMA")
> colnamev <- colnames(pricev)
> # Calculate the positions, either: -1, 0, or 1
> indic <- sign(closep - pricema)
> posv <- rutils::lagit(indic, lagg=1)
> # Create colors for background shading
> crossd <- (rutils::diffit(posv) != 0)
> shadev <- posv[crossd]
> crossd <- c(zoo::index(shadev), end(posv))
> shadev <- ifelse(drop(zoo::coredata(shadev)) == 1, "lightgreen",
> # Create dygraph object without plotting it
> dyplot <- dygraphs::dygraph(pricev, main="VTI EMA Prices") %>%
+   dySeries(name=colnamev[1], strokeWidth=1, col="blue") %>%
+   dySeries(name=colnamev[2], strokeWidth=3, col="red") %>%
+   dyLegend(show="always", width=300)
```

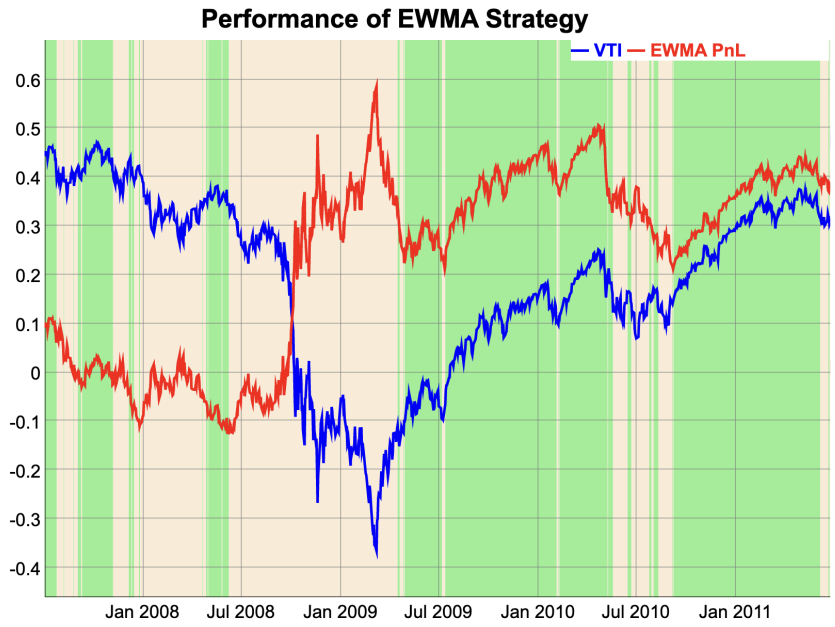
```
> # Add shading to dygraph object
> for (i in 1:NROW(shadev)) {
+   dyplot <- dyplot %>% dyShading(from=crossd[i], to=crossd[i+1], c
+ }  # end for
> # Plot the dygraph object
> dyplot
> # Standard plot of EMA prices with position shading
> quantmod::chart_Series(pricev, theme=plot_theme,
+        lwd=2, name="VTI EMA Prices")
> add_TA(posv > 0, on=-1, col="lightgreen", border="lightgreen")
> add_TA(posv < 0, on=-1, col="lightgrey", border="lightgrey")
> legend("topleft", legend=colnames(pricev),
+  inset=0.1, bg="white", lty=1, lwd=6, y.intersp=0.5,
+  col=plot_theme$col$line.col, bty="n")
```

EMA Crossover Strategy Performance

The crossover strategy trades at the *Close* price on the same day that prices cross the *EMA*, which may be difficult in practice.

The crossover strategy performance is worse than the underlying asset (*VTI*), but it has a negative correlation to it, which is very valuable when building a portfolio.

```
> # Calculate the daily profits and losses of crossover strategy
> retp <- rutils::diffit(closep)  # VTI returns
> pnls <- retp*posv
> colnames(pnls) <- "EMA"
> wealthv <- cbind(retp, pnls)
> colnames(wealthv) <- c("VTI", "EMA PnL")
> # Annualized Sharpe ratio of crossover strategy
> sqrt(252)*sapply(wealthv, function (x) mean(x)/sd(x))
> # The crossover strategy has a negative correlation to VTI
> cor(wealthv)[1, 2]
> # Plot dygraph of crossover strategy wealth
> # Create dygraph object without plotting it
> colorv <- c("blue", "red")
> dyplot <- dygraphs::dygraph(cumsum(wealthv), main="Performance o
+   dyOptions(colors=colorv, strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
> # Add shading to dygraph object
> for (i in 1:NROW(shadev)) {
+   dyplot <- dyplot %>%
+     dyShading(from=crossd[i], to=crossd[i+1], color=shadev[i])
+ }  # end for
> # Plot the dygraph object
> dyplot
```



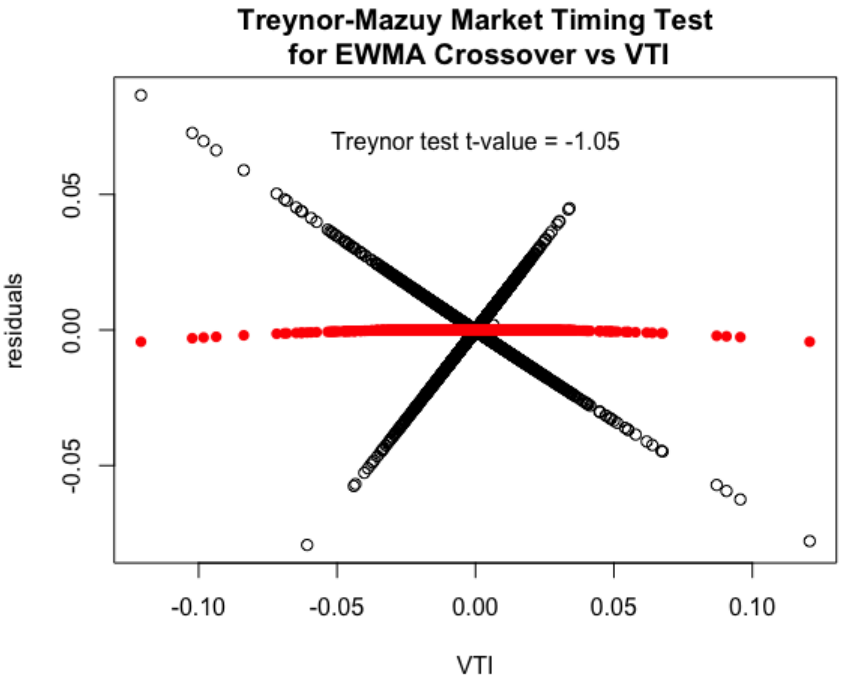

```
> # Standard plot of crossover strategy wealth
> x11(width=6, height=5)
> plot_theme <- chart_theme()
> plot_theme$col$line.col <- colorv
> quantmod::chart_Series(cumsum(wealthv), theme=plot_theme,
+        name="Performance of Crossover Strategy")
> add_TA(posv > 0, on=-1, col="lightgreen", border="lightgreen")
> add_TA(posv < 0, on=-1, col="lightgrey", border="lightgrey")
> legend("top", legend=colnames(wealthv), y.intersp=0.5,
+  inset=0.05, bg="white", lty=1, lwd=6,
+  col=plot_theme$col$line.col, bty="n")
```

EMA Crossover Strategy Market Timing Skill

The EMA crossover strategy shorts the market during significant selloffs, but otherwise doesn't display market timing skill.

The t-value of the *Treynor-Mazuy* test is negative, but not statistically significant.

```
> # Test EMA crossover market timing of VTI using Treynor-Mazuy test
> desm <- cbind(pnls, retp, retp^2)
> desm <- na.omit(desm)
> colnames(desm) <- c("EMA", "VTI", "Treynor")
> regmod <- lm(EMA ~ VTI + Treynor, data=desm)
> summary(regmod)
> # Plot residual scatterplot
> resids <- (desm$EMA - regmod$coeff["VTI"]*retp)
> resids <- regmod$residuals
> plot.default(x=retp, y=resids, xlab="VTI", ylab="residuals")
> title(main="Treynor-Mazuy Market Timing Test\n for EMA Crossover
> # Plot fitted (predicted) response values
> coefreg <- summary(regmod)$coeff
> fitv <- regmod$fitted.values - coefreg["VTI", "Estimate"]*retp
> tvalue <- round(coefreg["Treynor", "t value"], 2)
> points.default(x=retp, y=fitv, pch=16, col="red")
> text(x=0.0, y=0.8*max(resids), paste("Treynor test t-value =", tvalue))
```

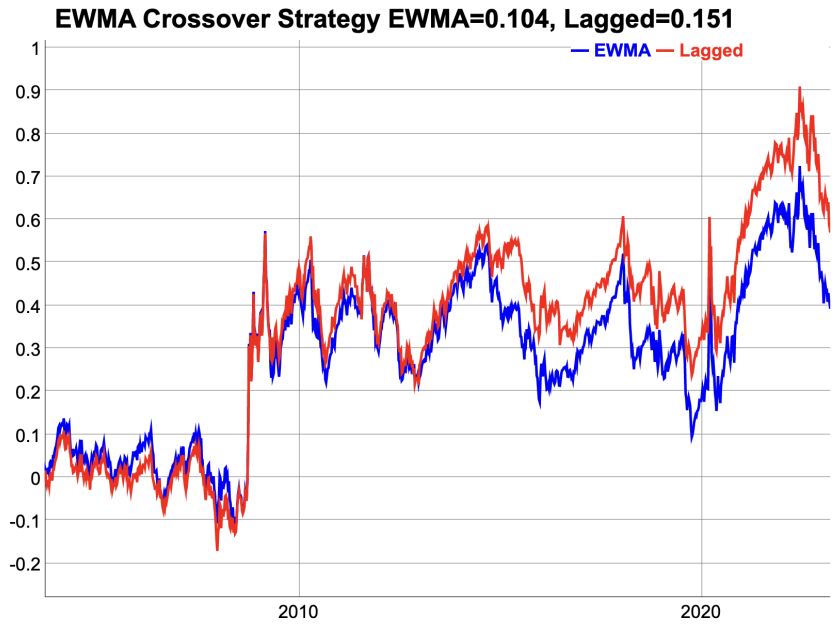


EMA Crossover Strategy With Lag

The crossover strategy suffers losses when prices are range-bound without a trend, because whenever it switches position the prices soon change direction. (This is called a *"whipsaw"*.)

To prevent whipsaws and over-trading, the crossover strategy may choose to delay switching positions until the indicator repeats the same value for several periods.

There's a tradeoff between switching positions too early and risking a whipsaw, and waiting too long and missing an emerging trend.




```
> # Determine the trade dates right after EMA has crossed prices
> indic <- sign(closep - pricema)
> # Calculate the positions from lagged indicator
> lagg <- 2
> indic <- HighFreq::roll_sum(indic, lagg)
> # Calculate the positions, either: -1, 0, or 1
> posv <- rep(NA_integer_, nrows)
> posv[1] <- 0
> posv <- ifelse(indic == lagg, 1, posv)
> posv <- ifelse(indic == (-lagg), -1, posv)
> posv <- zoo::na.locf(posv, na.rm=FALSE)
> posv <- xts::xts(posv, order.by=zoo::index(closep))
> # Lag the positions to trade in next period
> posv <- rutils::lagit(posv, lagg=1)
> # Calculate the PnLs of lagged strategy
> pnlslag <- retp*posv
> colnames(pnlslag) <- "Lagged Strategy"
```

```
> wealthv <- cbind(pnls, pnlslag)
> colnames(wealthv) <- c("EMA", "Lagged")
> # Annualized Sharpe ratios of crossover strategies
> sharper <- sqrt(252)*sapply(wealthv, function (x) mean(x)/sd(x))
> # Plot both strategies
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd], main=paste("EMA Crossover
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

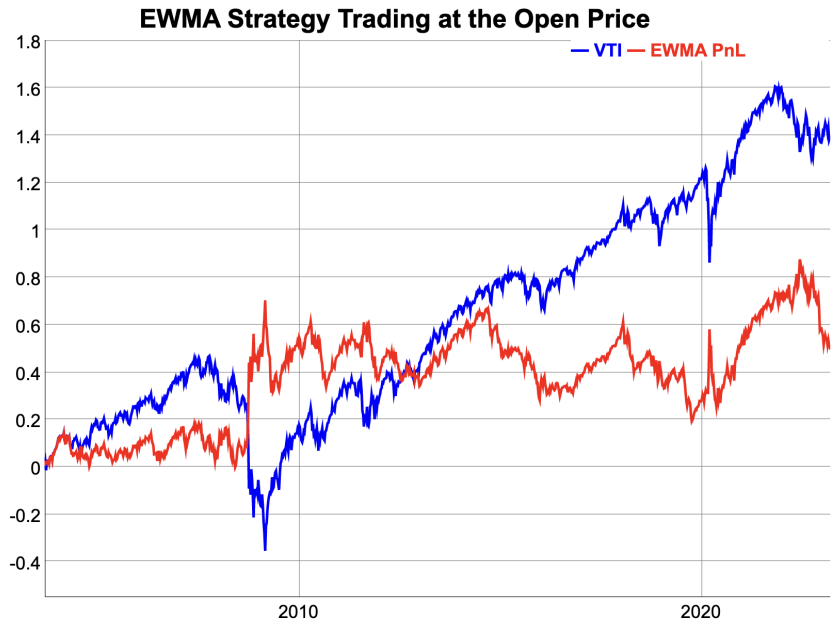
Crossover Strategy Trading at the Open Price

In practice it may not be possible to trade immediately at the *Close* price on the same day that prices cross the *EMA*.

Then the strategy may trade at the *Open* price on the next day.

The Profit and Loss (*PnL*) on a trade date is the sum of the realized *PnL* from closing the old position, plus the unrealized *PnL* after opening the new position.

```
> # Calculate the positions, either: -1, 0, or 1
> indic <- sign(closep - pricema)
> posv <- rutils::lagit(indic, lagg=1)
> # Calculate the daily pnl for days without trades
> pnls <- retp*posv
> # Determine the trade dates right after EMA has crossed prices
> crossd <- which(rutils::diffit(posv) != 0)
> # Calculate the realized pnl for days with trades
> openp <- quantmod::Op(ohlc)
> closelag <- rutils::lagit(closep)
> poslag <- rutils::lagit(posv)
> pnls[crossd] <- poslag[crossd]*(openp[crossd] - closelag[crossd])
> # Calculate the unrealized pnl for days with trades
> pnls[crossd] <- pnls[crossd] +
+   posv[crossd]*(closep[crossd] - openp[crossd])
> # Calculate the wealth
> wealthv <- cbind(retp, pnls)
> colnames(wealthv) <- c("VTI", "EMA PnL")
> # Annualized Sharpe ratio of crossover strategy
> sqrt(252)*sapply(wealthv, function (x) mean(x)/sd(x))
> # The crossover strategy has a negative correlation to VTI
> cor(wealthv)[1, 2]
```




```
> # Plot dygraph of crossover strategy wealth
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd], main="Crossover Strategy
+   dyOptions(colors=colorv, strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
> # Standard plot of crossover strategy wealth
> quantmod::chart_Series(cumsum(wealthv)[endd], theme=plot_theme,
+        name="Crossover Strategy Trading at the Open Price")
> legend("top", legend=colnames(wealthv),
+  inset=0.05, bg="white", lty=1, lwd=6,
+  col=plot_theme$col$line.col, bty="n")
```

EMA Crossover Strategy With Transaction Costs

The *bid-ask spread* is the percentage difference between the *ask* (offer) minus the *bid* prices, divided by the *mid* price.

The bid-ask spread for many liquid ETFs is about 1 basis point. For example the *XLK ETF*

Let n_t be the number of shares of the stock owned at time t, and let p_t be their price.

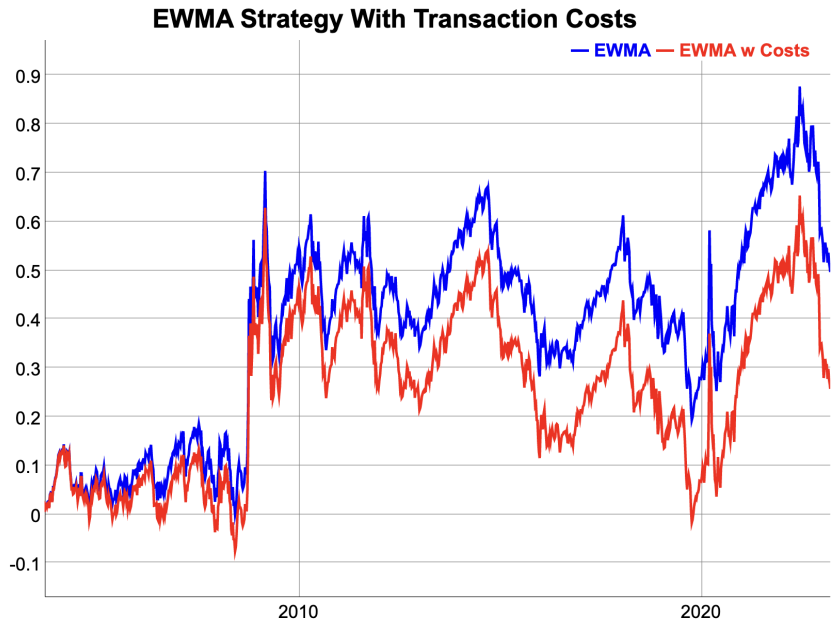
Then the traded dollar amount of the stock is equal to the change in the number of shares times the stock price: $\Delta n_t p_t$.

The the *transaction costs* c^r due to the *bid-ask spread* are equal to half the *bid-ask spread* δ times the absolute value of the traded dollar amount of the stock:

$$c^r = \frac{\delta}{2} |\Delta n_t| \, p_t$$

If d_t is the dollar amount of the stock owned at time t then the *transaction costs* c^r are equal to:

$$c^r = \frac{\delta}{2} |\Delta d_t|$$




```
> # The bid-ask spread is equal to 1 bp for liquid ETFs
> bidask <- 0.001
> # Calculate the transaction costs
> costv <- 0.5*bidask*abs(poslag - posv)
> # Plot strategy with transaction costs
> wealthv <- cbind(pnls, pnls - costv)
> colnames(wealthv) <- c("EMA", "EMA w Costs")
> colorv <- c("blue", "red")
> dygraphs::dygraph(cumsum(wealthv)[endd], main="Crossover Strategy
+   dyOptions(colors=colorv, strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

Simulation Function for EMA Crossover Strategy

The *EMA* strategy can be simulated by a single function, which allows the analysis of its performance depending on its parameters.

The function `sim_ema()` performs a simulation of the *EMA* strategy, given an *OHLC* time series of prices, and a decay factor λ.

The function `sim_ema()` returns the *EMA* strategy positions and returns, in a two-column *xts* time series.

```
> sim_ema <- function(closep, lambdaf=0.9, bidask=0.001, trend=1, la
+   retp <- rutils::diffit(closep)
+   nrows <- NROW(closep)
+   # Calculate the EMA prices
+   pricema <- HighFreq::run_mean(closep, lambda=lambdaf)
+   # Calculate the indicator
+   indic <- trend*sign(closep - pricema)
+   if (lagg > 1) {
+     indic <- HighFreq::roll_sum(indic, lagg)
+     indic[1:lagg] <- 0
+   }  # end if
+   # Calculate the positions, either: -1, 0, or 1
+   posv <- rep(NA_integer_, nrows)
+   posv[1] <- 0
+   posv <- ifelse(indic == lagg, 1, posv)
+   posv <- ifelse(indic == (-lagg), -1, posv)
+   posv <- zoo::na.locf(posv, na.rm=FALSE)
+   posv <- xts::xts(posv, order.by=zoo::index(closep))
+   # Lag the positions to trade on next day
+   posv <- rutils::lagit(posv, lagg=1)
+   # Calculate the PnLs of strategy
+   pnls <- retp*posv
+   costv <- 0.5*bidask*abs(rutils::diffit(posv))
+   pnls <- (pnls - costv)
+   # Calculate the strategy returns
+   pnls <- cbind(posv, pnls)
+   colnames(pnls) <- c("positions", "pnls")
+   pnls
+ }  # end sim_ema
```

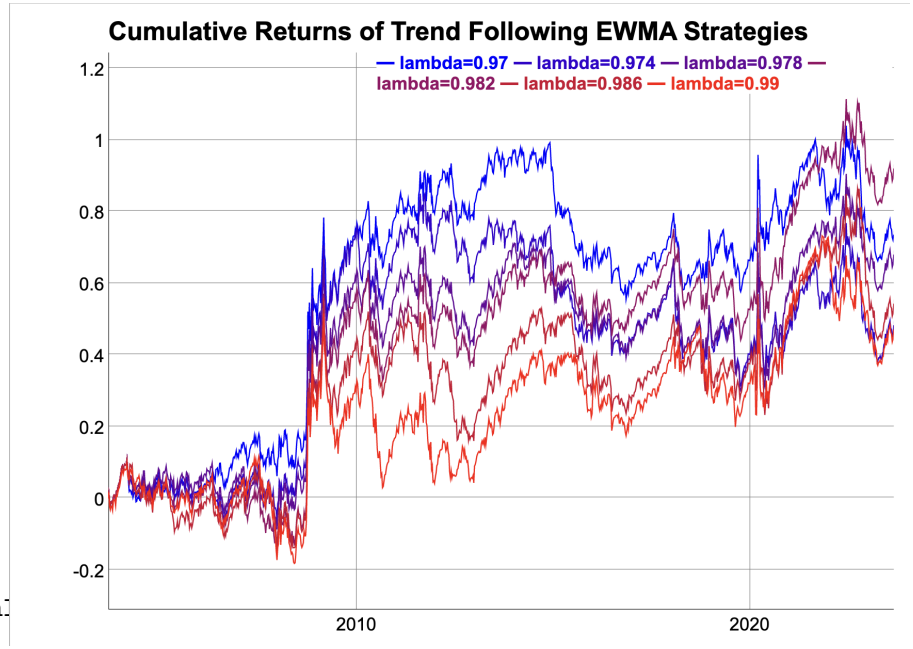
Simulating Multiple Trend Following Crossover Strategies

Multiple *EMA* strategies can be simulated by calling the function `sim_ema()` in a loop over a vector of λ parameters.

But `sim_ema()` returns an *xts* time series, and `sapply()` cannot merge *xts* time series together.

So instead the loop is performed using `lapply()` which returns a list of *xts*, and the list is merged into a single *xts* using the functions `do.call()` and `cbind()`.

```
> lambdav <- seq(from=0.97, to=0.99, by=0.004)
> # Perform lapply() loop over lambdav
> pnltrend <- lapply(lambdav, function(lambdaf) {
+   # Simulate crossover strategy and calculate the returns
+   sim_ema(closep=closep, lambdaf=lambdaf, bidask=0, lagg=2)[, "pnl
+ })  # end lapply
> pnltrend <- do.call(cbind, pnltrend)
> colnames(pnltrend) <- paste0("lambda=", lambdav)
```




```
> # Plot dygraph of multiple crossover strategies
> colorv <- colorRampPalette(c("blue", "red"))(NCOL(pnltrend))
> endd <- rutils::calc_endpoints(pnltrend, interval="weeks")
> dygraphs::dygraph(cumsum(pnltrend)[endd], main="Cumulative Returns
+   dyOptions(colors=colorv, strokeWidth=1) %>%
+   dyLegend(show="always", width=400)
> # Plot crossover strategies with custom line colors
> plot_theme <- chart_theme()
> plot_theme$col$line.col <- colorv
> quantmod::chart_Series(cumsum(pnltrend), theme=plot_theme,
+   name="Cumulative Returns of Crossover Strategies")
> legend("topleft", legend=colnames(pnltrend), inset=0.1,
+   bg="white", cex=0.8, lwd=rep(6, NCOL(pnltrend)),
+   col=plot_theme$col$line.col, bty="n")
```

Simulating Crossover Strategies Using Parallel Computing

Simulating *EMA* strategies naturally lends itself to parallel computing, since the simulations are independent from each other.

The function `parLapply()` is similar to `lapply()`, and performs loops under *Windows* using parallel computing on several CPU cores.

The resulting list of time series can then be collapsed into a single *xts* series using the functions `rutils::do_call()` and `cbind()`.

```
> # Initialize compute cluster under Windows
> library(parallel)
> ncores <- detectCores() - 1  # Number of cores
> compclust <- makeCluster(detectCores()-1)
> clusterExport(compclust,
+   varlist=c("ohlc", "lookb", "sim_ema"))
> # Perform parallel loop over lambdav under Windows
> pnltrend <- parLapply(compclust, lambdav, function(lambdaf) {
+   library(quantmod)
+   # Simulate crossover strategy and calculate the returns
+   sim_ema(closep=closep, lambdaf=lambdaf, bidask=0, lagg=2)[, "pnl
+ })  # end parLapply
> stopCluster(compclust)  # Stop R processes over cluster under Wind
> # Perform parallel loop over lambdav under Mac-OSX or Linux
> pnltrend <- mclapply(lambdav, function(lambdaf) {
+   library(quantmod)
+   # Simulate crossover strategy and calculate the returns
+   sim_ema(closep=closep, lambdaf=lambdaf, bidask=0, lagg=2)[, "pnl
+ }, mc.cores=ncores)  # end mclapply
> pnltrend <- do.call(cbind, pnltrend)
> colnames(pnltrend) <- paste0("lambda=", lambdav)
```

Optimal Decay Factor of Trend Following Crossover Strategies

The performance of trend following *EMA* strategies depends on the λ decay factor, with smaller λ parameters performing better than larger ones.

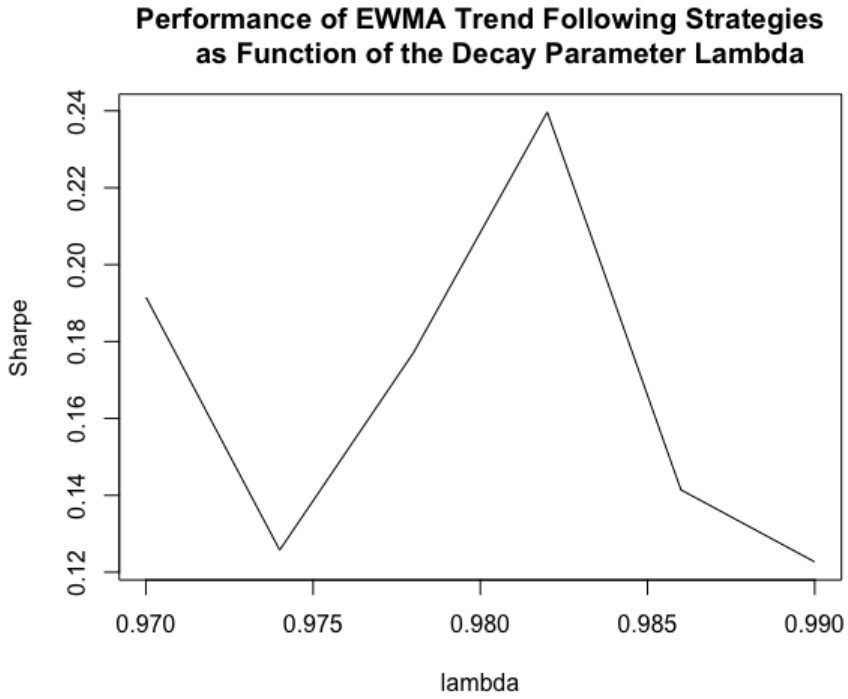
The optimal λ parameter applies significant weight to returns $8 - 12$ months in the past, which is consistent with research on trend following strategies.

The *Sharpe ratios* of *EMA* strategies with different λ parameters can be calculated by performing an `sapply()` loop over the *columns* of returns.

`sapply()` treats the columns of *xts* time series as list elements, and loops over the columns.

Performing loops in `R` over the *columns* of returns is acceptable, but `R` loops over the *rows* of returns should be avoided.

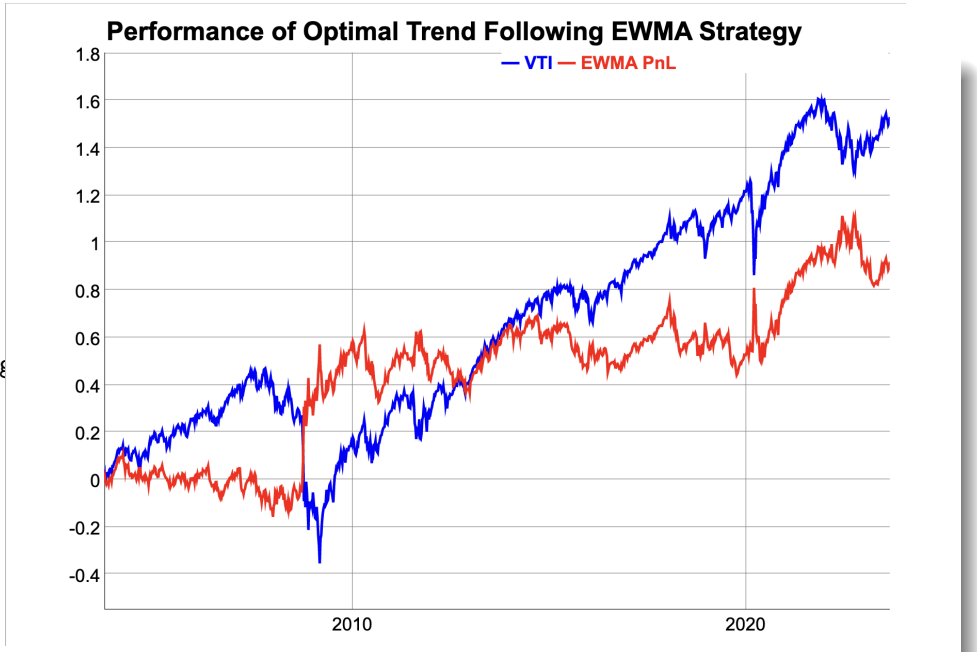
```
> # Calculate the annualized Sharpe ratios of strategy returns
> sharpetrend <- sqrt(252)*sapply(pnltrend, function(xtsv) {
+   mean(xtsv)/sd(xtsv)
+ })  # end sapply
> # Plot Sharpe ratios
> dev.new(width=6, height=5, noRStudioGD=TRUE)
> plot(x=lambdav, y=sharpetrend, t="l",
+      xlab="lambda", ylab="Sharpe",
+      main="Performance of EMA Trend Following Strategies
+      as Function of the Decay Factor Lambda")
```



Optimal Trend Following Crossover Strategy

The best performing trend following *EMA* strategy has a relatively small λ parameter, corresponding to slower weight decay (giving more weight to past pricev), and producing less frequent trading.

```
> # Calculate the optimal lambda
> lambdaf <- lambdav[which.max(sharpetrend)]
> # Simulate best performing strategy
> ematrend <- sim_ema(closep=closep, lambdaf=lambdaf, bidask=0, lagg
> posv <- ematrend[, "positions"]
> trendopt <- ematrend[, "pnls"]
> wealthv <- cbind(retp, trendopt)
> colnames(wealthv) <- c("VTI", "EMA PnL")
> # Calculate the Sharpe and Sortino ratios
> sqrt(252)*sapply(wealthv, function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> cor(wealthv)[1, 2]
> # Plot dygraph of crossover strategy wealth
> dygraphs::dygraph(cumsum(wealthv)[endd], main="Performance of Opt
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```



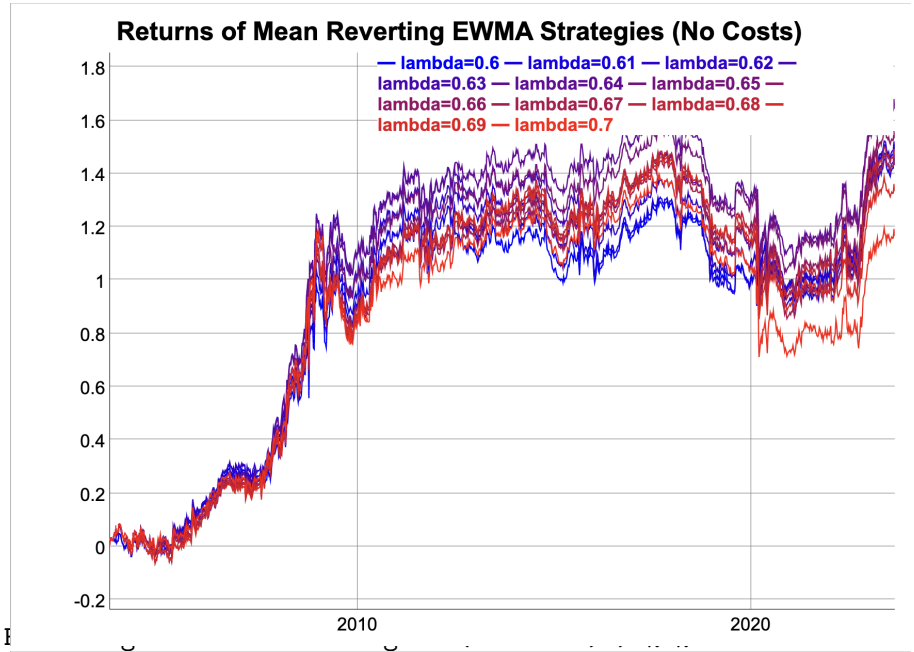

```
> # Plot EMA PnL with position shading
> # Standard plot of crossover strategy wealth
> x11(width=6, height=5)
> plot_theme <- chart_theme()
> plot_theme$col$line.col <- colorv
> quantmod::chart_Series(cumsum(wealthv), theme=plot_theme,
+        name="Performance of Crossover Strategy")
> add_TA(posv > 0, on=-1, col="lightgreen", border="lightgreen")
> add_TA(posv < 0, on=-1, col="lightgrey", border="lightgrey")
> legend("top", legend=colnames(wealthv),
+  inset=0.05, bg="white", lty=1, lwd=6,
+  col=plot_theme$col$line.col, bty="n")
```

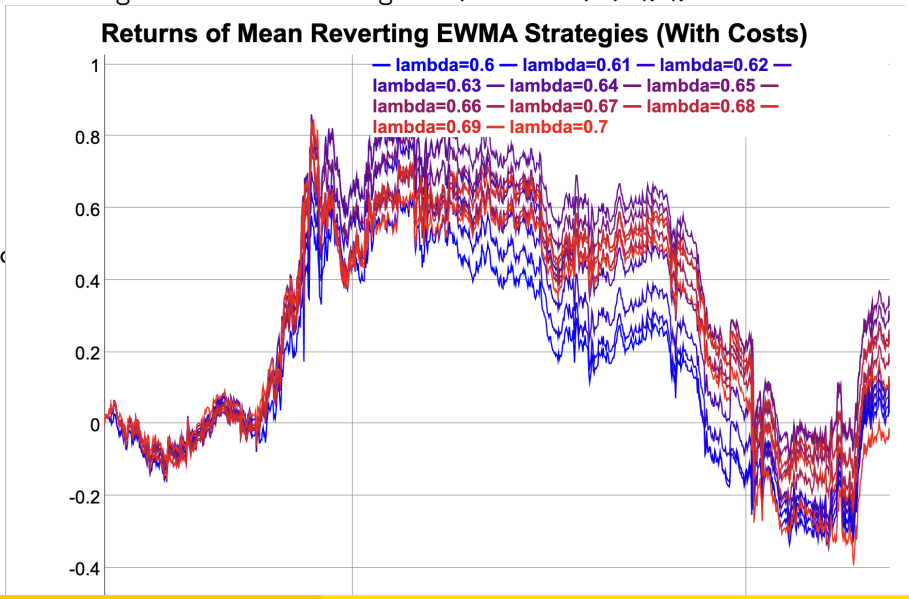
Mean Reverting EMA Crossover Strategies

Mean reverting EMA crossover strategies can be simulated using function `sim_ema()` with argument `trend=(-1)`.

The profitability of mean reverting strategies can be significantly improved by using limit orders, to reduce transaction costs.

```
> lambdav <- seq(0.6, 0.7, 0.01)
> # Perform lapply() loop over lambdav
> pnlrevert <- lapply(lambdav, function(lambdaf) {
+   # Simulate crossover strategy and calculate the returns
+   sim_ema(closep=closep, lambdaf=lambdaf, bidask=0, trend=(-1))[,
+ })  # end lapply
> pnlrevert <- do.call(cbind, pnlrevert)
> colnames(pnlrevert) <- paste0("lambda=", lambdav)
> # Plot dygraph of mean reverting crossover strategies
> colorv <- colorRampPalette(c("blue", "red"))(NROW(lambdav))
> dygraphs::dygraph(cumsum(pnlrevert)[endd], main="Returns of Mean R
+   dyOptions(colors=colorv, strokeWidth=1) %>%
+   dyLegend(show="always", width=400)
> # Plot crossover strategies with custom line colors
> x11(width=6, height=5)
> plot_theme <- chart_theme()
> plot_theme$col$line.col <- colorv
> quantmod::chart_Series(pnlrevert,
+   theme=plot_theme, name="Cumulative Returns of Mean Reverting Cro
> legend("topleft", legend=colnames(pnlrevert),
+   inset=0.1, bg="white", cex=0.8, lwd=6,
+   col=plot_theme$col$line.col, bty="n")
```





Performance of Mean Reverting Crossover Strategies

The *Sharpe ratios* of *EMA* strategies with different λ parameters can be calculated by performing an `sapply()` loop over the *columns* of returns.

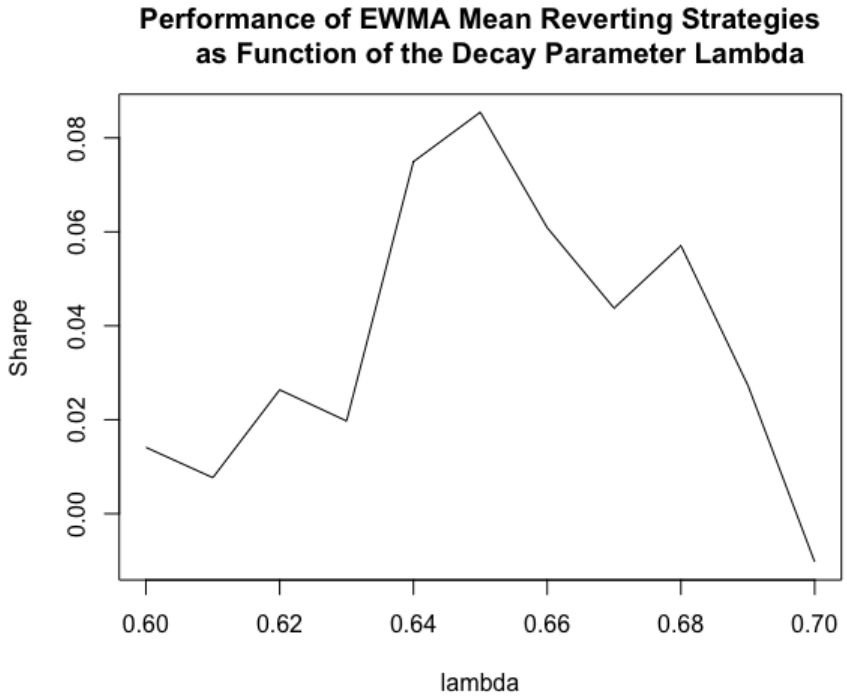
`sapply()` treats the columns of *xts* time series as list elements, and loops over the columns.

Performing loops in `R` over the *columns* of returns is acceptable, but `R` loops over the *rows* of returns should be avoided.

The performance of mean reverting *EMA* strategies depends on the λ parameter, with performance decreasing for very small or very large λ parameters.

For too large λ parameters, the trading frequency is too high, causing high transaction costs.

For too small λ parameters, the trading frequency is too low, causing the strategy to miss profitable trades.



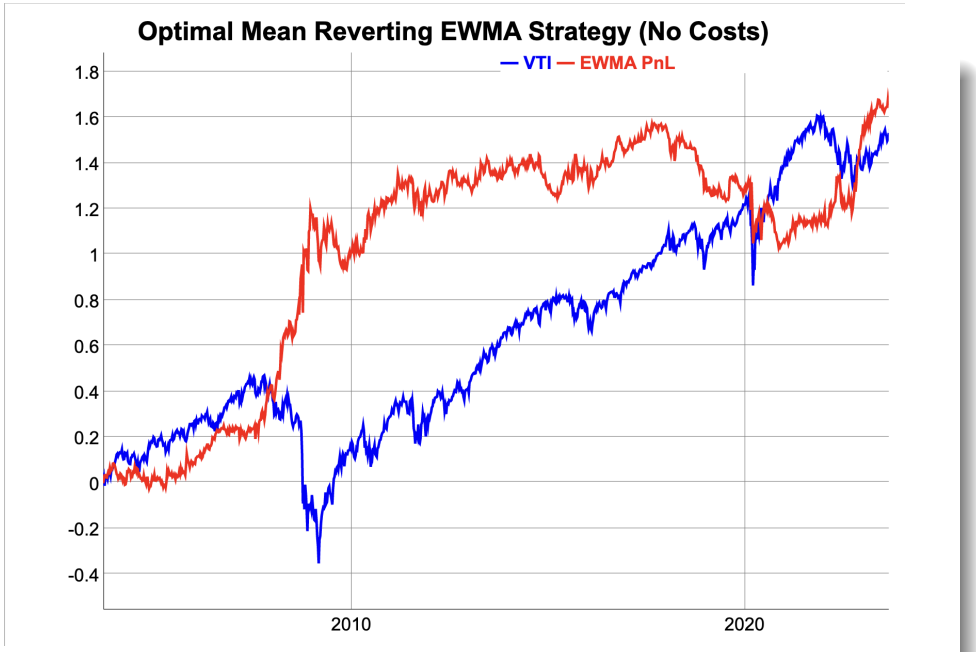

```
> # Calculate the Sharpe ratios of strategy returns
> sharperevert <- sqrt(252)*sapply(pnlrevert, function(xtsv) {
+   mean(xtsv)/sd(xtsv)
+ })  # end sapply
> plot(x=lambdav, y=sharperevert, t="l",
+      xlab="lambda", ylab="Sharpe",
+      main="Performance of EMA Mean Reverting Strategies
+      as Function of the Decay Factor Lambda")
```

Optimal Mean Reverting Crossover Strategy

Reverting the direction of the trend following *EMA* strategy creates a mean reverting strategy.

The best performing mean reverting *EMA* strategy has a relatively large λ parameter, corresponding to faster weight decay (giving more weight to recent prices), and producing more frequent trading.

But a too large λ parameter also causes very high trading frequency, and high transaction costs.




```
> # Calculate the optimal lambda
> lambdaf <- lambdav[which.max(sharperevert)]
> # Simulate best performing strategy
> emarevert <- sim_ema(closep=closep, bidask=0.0,
+   lambdaf=lambdaf, trend=(-1))
> posv <- emarevert[, "positions"]
> revertopt <- emarevert[, "pnls"]
> wealthv <- cbind(retp, revertopt)
> colnames(wealthv) <- c("VTI", "EMA PnL")
> # Plot dygraph of crossover strategy wealth
> colorv <- c("blue", "red")
> dygraphs::dygraph(cumsum(wealthv)[endd], main="Optimal Mean Reve
+   dyOptions(colors=colorv, strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

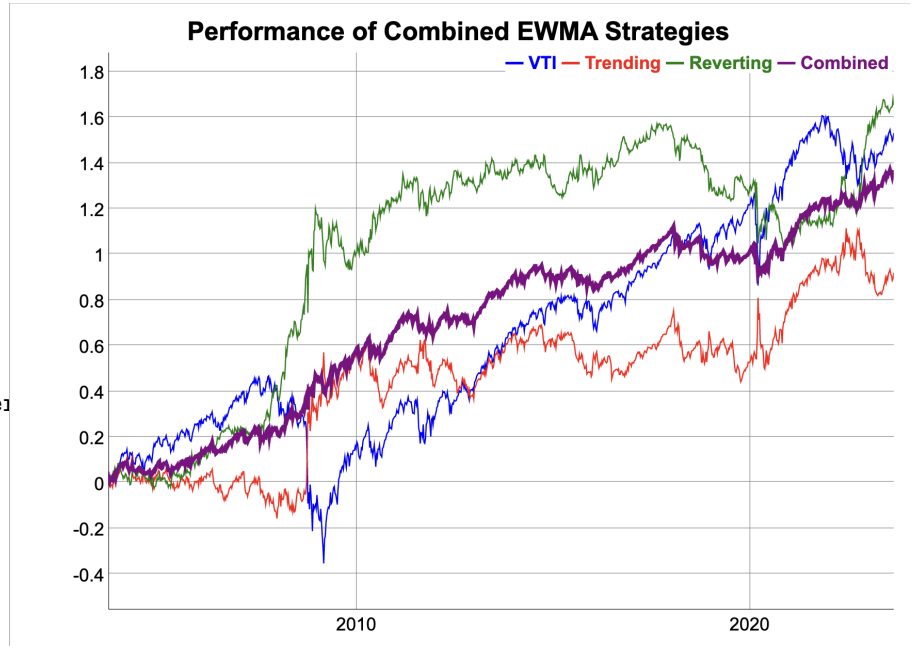
```
> # Standard plot of crossover strategy wealth
> x11(width=6, height=5)
> plot_theme <- chart_theme()
> plot_theme$col$line.col <- colorv
> quantmod::chart_Series(cumsum(wealthv), theme=plot_theme,
+        name="Optimal Mean Reverting Crossover Strategy")
> add_TA(posv > 0, on=-1, col="lightgreen", border="lightgreen")
> add_TA(posv < 0, on=-1, col="lightgrey", border="lightgrey")
> legend("top", legend=colnames(wealthv),
+  inset=0.05, bg="white", lty=1, lwd=6,
+  col=plot_theme$col$line.col, bty="n")
```

Combining Trend Following and Mean Reverting Strategies

The returns of trend following and mean reverting strategies are usually negatively correlated to each other, so combining them can achieve significant diversification of risk.

The main advantage of EMA crossover strategies is that they provide positive returns and a diversification of risk with respect to static stock portfolios.

```
> # Calculate the correlation between trend following and mean rever
> trendopt <- ematrend[, "pnls"]
> colnames(trendopt) <- "trend"
> revertopt <- emarevert[, "pnls"]
> colnames(revertopt) <- "revert"
> cor(cbind(retp, trendopt, revertopt))
> # Calculate the combined strategy
> combstrat <- (retp + trendopt + revertopt)/3
> colnames(combstrat) <- "combined"
> # Calculate the annualized Sharpe ratio of strategy returns
> retc <- cbind(retp, trendopt, revertopt, combstrat)
> colnames(retc) <- c("VTI", "Trending", "Reverting", "Combined")
> sqrt(252)*sapply(retc, function(xtsv) mean(xtsv)/sd(xtsv))
```

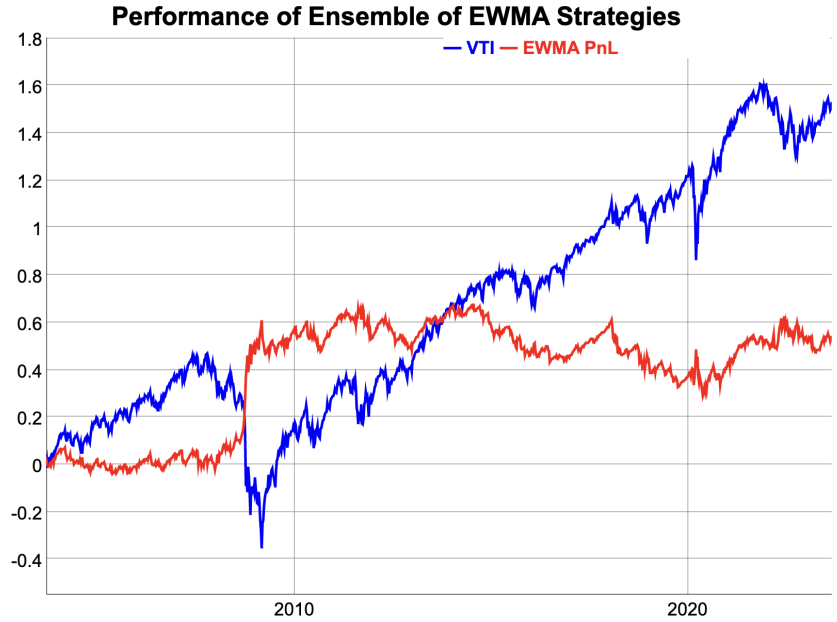



```
> # Plot dygraph of crossover strategy wealth
> colorv <- c("blue", "red", "green", "purple")
> dygraphs::dygraph(cumsum(retc)[endd], main="Performance of Combine
+   dyOptions(colors=colorv, strokeWidth=1) %>%
+   dySeries(name="Combined", strokeWidth=3) %>%
+   dyLegend(show="always", width=300)
> # Standard plot of crossover strategy wealth
> plot_theme <- chart_theme()
> plot_theme$col$line.col <- colorv
> quantmod::chart_Series(pnls, theme=plot_theme,
+   name="Performance of Combined Crossover Strategies")
> legend("topleft", legend=colnames(pnls),
+   inset=0.05, bg="white", lty=1, lwd=6,
+   col=plot_theme$col$line.col, bty="n")
```

Ensemble of Crossover Strategies

Instead of selecting the best performing *EMA* strategy, one can choose a weighted average of strategies (ensemble), which corresponds to allocating positions according to the weights.

The weights can be chosen to be proportional to the Sharpe ratios of the *EMA* strategies.



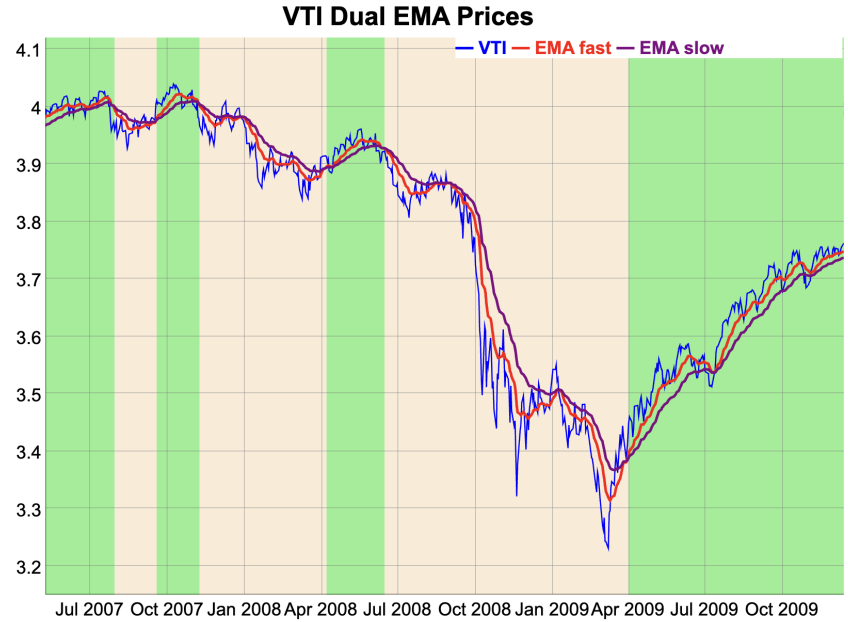

```
> # Calculate the weights proportional to Sharpe ratios
> weightv <- c(sharpetrend, sharperevert)
> weightv[weightv < 0] <- 0
> weightv <- weightv/sum(weightv)
> retc <- cbind(pnltrend, pnlrevert)
> retc <- retc %*% weightv
> retc <- cbind(retp, retc)
> colnames(retc) <- c("VTI", "EMA PnL")
> # Plot dygraph of crossover strategy wealth
> colorv <- c("blue", "red")
> dygraphs::dygraph(cumsum(retc)[endd], main="Performance of Ensemble of Crossover Strategies") %>%
+   dyOptions(colors=colorv, strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
> # Standard plot of crossover strategy wealth
> plot_theme <- chart_theme()
> plot_theme$col$line.col <- colorv
> quantmod::chart_Series(cumsum(retc), theme=plot_theme,
+        name="Performance of Ensemble of Crossover Strategies")
> legend("topleft", legend=colnames(pnls),
+  inset=0.05, bg="white", lty=1, lwd=6,
+  col=plot_theme$col$line.col, bty="n")
```

Dual EMA Crossover Strategy

In the *Dual EMA Crossover* strategy, the stock position depends on the difference between two moving averages.

The stock position flips when the fast moving *EMA* crosses the slow moving *EMA*.

```
> # Extract the log VTI prices
> ohlc <- log(rutils::etfenv$VTI)
> datev <- zoo::index(ohlc)
> nrows <- NROW(ohlc)
> closep <- quantmod::Cl(ohlc)
> colnames(closep) <- "VTI"
> retp <- rutils::diffit(closep)  # VTI returns
> # Calculate the fast and slow EMAs
> lambdafa <- 0.89
> lambdasl <- 0.95
> # Calculate the EMA prices
> emaf <- HighFreq::run_mean(closep, lambda=lambdafa)
> emas <- HighFreq::run_mean(closep, lambda=lambdasl)
> pricev <- cbind(closep, emaf, emas)
> colnames(pricev) <- c("VTI", "EMA fast", "EMA slow")
> # Determine the positions and the trade dates right after the EMA
> indic <- sign(emaf - emas)
> posv <- rutils::lagit(indic)
> crossd <- (rutils::diffit(posv) != 0)
> # Create colors for background shading
> shadev <- posv[crossd]
> shadev <- ifelse(shadev == 1, "lightgreen", "antiquewhite")
> crossd <- c(datev[crossd], end(closep))
```



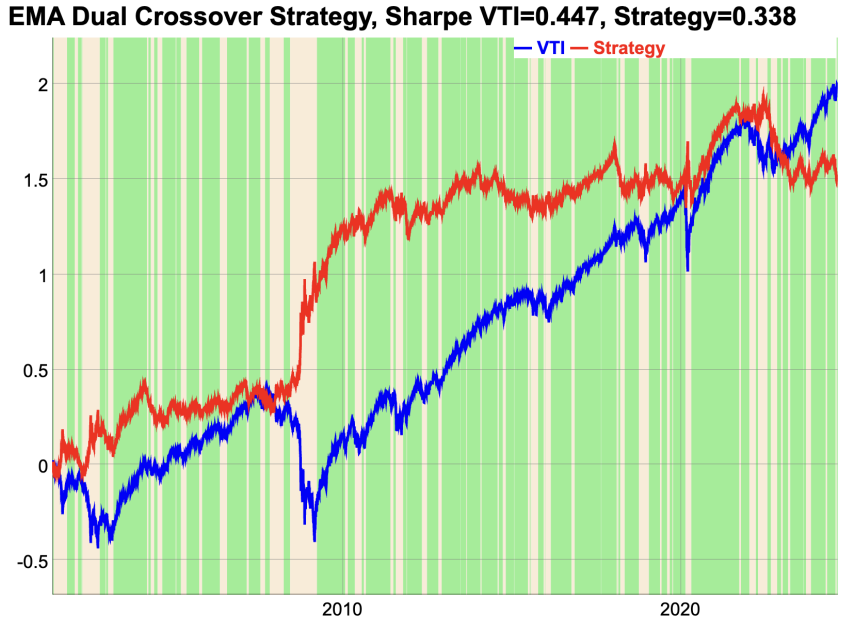

```
> # Plot dygraph
> colnamev <- colnames(pricev)
> dyplot <- dygraphs::dygraph(pricev, main="VTI Dual EMA Prices") %>
+   dySeries(name=colnamev[1], strokeWidth=1, col="blue") %>%
+   dySeries(name=colnamev[2], strokeWidth=2, col="red") %>%
+   dySeries(name=colnamev[3], strokeWidth=2, col="purple") %>%
+   dyLegend(show="always", width=300)
> for (i in 1:NROW(shadev)) {
+   dyplot <- dyplot %>% dyShading(from=crossd[i], to=crossd[i+1], c
+ }  # end for
> dyplot
```

Dual EMA Crossover Strategy Performance

The crossover strategy suffers losses when prices are range-bound without a trend, because whenever it switches position the prices soon change direction. (This is called a *"whipsaw"*.)

The crossover strategy performance is worse than the underlying asset (*VTI*), but it has a negative correlation to it, which is very valuable when building a portfolio.

```
> # Calculate the daily profits and losses of strategy
> pnls <- retp*posv
> colnames(pnls) <- "Strategy"
> wealthv <- cbind(retp, pnls)
> # Annualized Sharpe ratio of dual crossover strategy
> sharper <- sqrt(252)*sapply(wealthv, function (x) mean(x)/sd(x))
> # The crossover strategy has a negative correlation to VTI
> cor(wealthv)[1, 2]
```




```
> # Plot Dual crossover strategy
> dyplot <- dygraphs::dygraph(cumsum(wealthv),
+   main=paste("EMA Dual Crossover Strategy, Sharpe", paste(paste(na
+   dyOptions(colors=c("blue", "red"), strokeWidth=2)
> # Add shading to dygraph object
> for (i in 1:NROW(shadev)) {
+   dyplot <- dyplot %>% dyShading(from=crossd[i], to=crossd[i+1], c
+ }  # end for
> # Plot the dygraph object
> dyplot
```

Simulation Function for the Dual EMA Crossover Strategy

The *Dual EMA* strategy can be simulated by a single function, which allows the analysis of its performance depending on its parameters.

The function `sim_ema2()` performs a simulation of the *Dual EMA* strategy, given an *OHLC* time series of prices, and two decay factors λ_1 and λ_2.

The function `sim_ema2()` returns the *EMA* strategy positions and returns, in a two-column *xts* time series.

```
> sim_ema2 <- function(closep, lambdafa=0.1, lambdasl=0.01,
+                bidask=0.001, trend=1, lagg=1) {
+   if (lambdafa >= lambdasl) return(NA)
+   retp <- rutils::diffit(closep)
+   nrows <- NROW(closep)
+   # Calculate the EMA prices
+   emaf <- HighFreq::run_mean(closep, lambda=lambdafa)
+   emas <- HighFreq::run_mean(closep, lambda=lambdasl)
+   # Calculate the positions, either: -1, 0, or 1
+   indic <- sign(emaf - emas)
+   if (lagg > 1) {
+     indic <- HighFreq::roll_sum(indic, lagg)
+     indic[1:lagg] <- 0
+   }  # end if
+   posv <- rep(NA_integer_, nrows)
+   posv[1] <- 0
+   posv <- ifelse(indic == lagg, 1, posv)
+   posv <- ifelse(indic == (-lagg), -1, posv)
+   posv <- zoo::na.locf(posv, na.rm=FALSE)
+   posv <- xts::xts(posv, order.by=zoo::index(closep))
+   # Lag the positions to trade on next day
+   posv <- rutils::lagit(posv, lagg=1)
+   # Calculate the PnLs of strategy
+   pnls <- retp*posv
+   costv <- 0.5*bidask*abs(rutils::diffit(posv))
+   pnls <- (pnls - costv)
+   # Calculate the strategy returns
+   pnls <- cbind(posv, pnls)
+   colnames(pnls) <- c("positions", "pnls")
+   pnls
+ }  # end sim_ema2
```

Dual Crossover Strategy Performance Matrix

Multiple *Dual EMA* strategies can be simulated by calling the function `sim_ema2()` in two loops over the vectors of λ parameters.

The function `outer()` calculates the values of a function over a grid spanned by two variables, and returns a matrix of function values.

The function `Vectorize()` performs an `apply()` loop over the arguments of a function, and returns a vectorized version of the function.

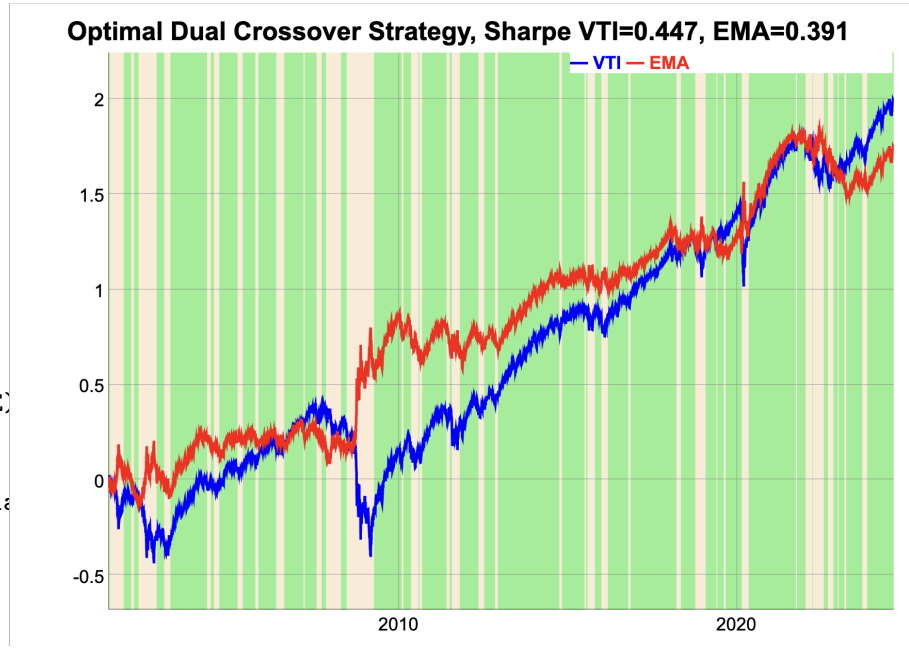
```
> lambdafv <- seq(from=0.85, to=0.99, by=0.01)
> lambdasv <- seq(from=0.85, to=0.99, by=0.01)
> # Calculate the Sharpe ratio of dual crossover strategy
> calc_sharpe <- function(closep, lambdafa, lambdasl, bidask, trend,
+   if (lambdafa >= lambdasl) return(NA)
+   pnls <- sim_ema2(closep=closep, lambdafa=lambdafa, lambdasl=lamb
+     bidask=bidask, trend=trend, lagg=lagg)[, "pnls"]
+   sqrt(252)*mean(pnls)/sd(pnls)
+ }  # end calc_sharpe
> # Vectorize calc_sharpe with respect to lambdafa and lambdasl
> calc_sharpe <- Vectorize(FUN=calc_sharpe,
+   vectorize.args=c("lambdafa", "lambdasl"))
> # Calculate the matrix of PnLs
> sharpem <- outer(lambdafv, lambdasv, FUN=calc_sharpe,
+            closep=closep, bidask=0.0, trend=1, lagg=1)
> # Or perform two sapply() loops over lambda vectors
> sharpem <- sapply(lambdasv, function(lambdasl) {
+   sapply(lambdafv, function(lambdafa) {
+     if (lambdafa >= lambdasl) return(NA)
+     calc_sharpe(closep=closep, lambdafa=lambdafa,
+       lambdasl=lambdasl, bidask=0.0, trend=1, lagg=1)
+   })  # end sapply
+ })  # end sapply
> colnames(sharpem) <- lambdasv
> rownames(sharpem) <- lambdafv
```

Optimal Dual Crossover Strategy

The best *Dual EMA* strategy performs better than the best *single EMA* strategy, because it has an extra parameter that can be adjusted to improve in-sample performance.

But this doesn't guarantee better out-of-sample performance.

```
> # Calculate the PnLs for the optimal crossover strategy
> whichv <- which(sharpem == max(sharpem, na.rm=TRUE), arr.ind=TRUE)
> lambdafa <- lambdafv[whichv[1]]
> lambdasl <- lambdasv[whichv[2]]
> crossopt <- sim_ema2(closep=closep, lambdafa=lambdafa, lambdasl=la
+   bidask=0.0, trend=1, lagg=1)
> pnls <- crossopt[, "pnls"]
> wealthv <- cbind(retp, pnls)
> colnames(wealthv)[2] <- "EMA"
> # Calculate the Sharpe and Sortino ratios
> sqrt(252)*sapply(wealthv, function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # Annualized Sharpe ratio of dual crossover strategy
> sharper <- sqrt(252)*sapply(wealthv, function (x) mean(x)/sd(x))
> # The crossover strategy has a negative correlation to VTI
> cor(wealthv)[1, 2]
```

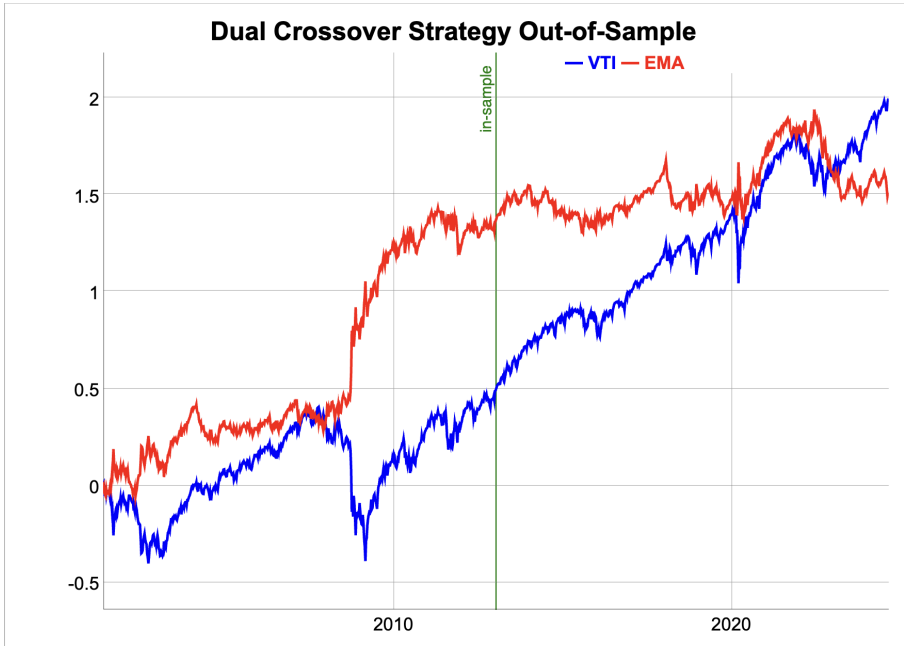



```
> # Create colors for background shading
> posv <- crossopt[, "positions"]
> crossd <- (rutils::diffit(posv) != 0)
> shadev <- posv[crossd]
> crossd <- c(zoo::index(shadev), end(posv))
> shadev <- ifelse(drop(zoo::coredata(shadev)) == 1, "lightgreen",
> # Plot Optimal Dual crossover strategy
> dyplot <- dygraphs::dygraph(cumsum(wealthv), main=paste("Optimal
+   dyOptions(colors=c("blue", "red"), strokeWidth=2)
> # Add shading to dygraph object
> for (i in 1:NROW(shadev)) {
+   dyplot <- dyplot %>% dyShading(from=crossd[i], to=crossd[i+1],
+ }  # end for
> # Plot the dygraph object
> dyplot
```

Performance of Dual Crossover Strategy *Out-of-Sample*

In-sample, the best *Dual EMA* strategy performs better than *VTI*, because it has two parameters that can be adjusted to improve performance.

But out-of-sample, the best *Dual EMA* strategy performs worse than *VTI*, because it's been *overfitted* in-sample.




```
> # Define in-sample and out-of-sample intervals
> insample <- 1:(nrows %/% 2)
> outsample <- (nrows %/% 2 + 1):nrows
> # Calculate the matrix of PnLs
> sharpem <- outer(lambdafv, lambdasv,
+            FUN=calc_sharpe, closep=closep[insample, ],
+            bidask=0.0, trend=1, lagg=1)
> colnames(sharpem) <- lambdasv
> rownames(sharpem) <- lambdafv
> # Calculate the PnLs for the optimal strategy
> whichv <- which(sharpem == max(sharpem, na.rm=TRUE), arr.ind=TRU
> lambdafa <- lambdafv[whichv[1]]
> lambdasl <- lambdasv[whichv[2]]
> pnls <- sim_ema2(closep=closep, lambdafa=lambdafa, lambdasl=lamb
+            bidask=0.0, trend=1, lagg=1)[, "pnls"]
> wealthv <- cbind(retp, pnls)
> colnames(wealthv)[2] <- "EMA"
> # Calculate the Sharpe and Sortino ratios in-sample and out-of-sample
> sqrt(252)*sapply(wealthv[insample, ], function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> sqrt(252)*sapply(wealthv[outsample, ], function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
```

```
> # Dygraphs plot with custom line colors
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd], main="Dual Crossover Stra
+   dyEvent(zoo::index(wealthv[last(insample)]), label="in-sample",
+   dyOptions(colors=c("blue", "red"), strokeWidth=2)
```

Volume-Weighted Average Price Indicator

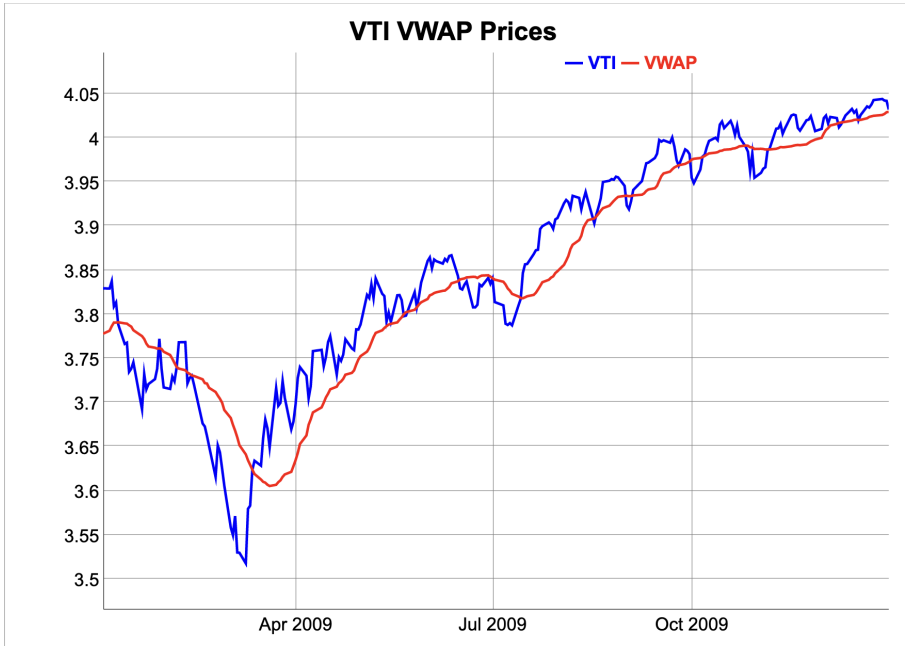
The Volume-Weighted Average Price (*VWAP*) is defined as the sum of prices multiplied by trading volumes, divided by the sum of volumes:

$$p_t^{VWAP} = \frac{\sum_{j=0}^{n} v_{t-j} p_{t-j}}{\sum_{j=0}^{n} v_{t-j}}$$

The *VWAP* applies more weight to prices with higher trading volumes, which allows it to react more quickly to recent market volatility.

The drawback of the *VWAP* indicator is that it applies large weights to prices far in the past.

The *VWAP* is often used as a technical indicator in trend following strategies.




```
> # Calculate the log OHLC prices and volumes
> ohlc <- rutils::etfenv$VTI
> closep <- log(quantmod::Cl(ohlc))
> colnames(closep) <- "VTI"
> volumv <- quantmod::Vo(ohlc)
> colnames(volumv) <- "Volume"
> nrows <- NROW(closep)
> # Calculate the VWAP prices
> lookb <- 21
> vwap <- HighFreq::roll_sum(closep, lookb=lookb, weightv=volumv)
> colnames(vwap) <- "VWAP"
> pricev <- cbind(closep, vwap)
```

```
> # Dygraphs plot with custom line colors
> colorv <- c("blue", "red")
> dygraphs::dygraph(pricev["2009"], main="VTI VWAP Prices") %>%
+   dyOptions(colors=colorv, strokeWidth=2)
> # Plot VWAP prices with custom line colors
> x11(width=6, height=5)
> plot_theme <- chart_theme()
> plot_theme$col$line.col <- colors
> quantmod::chart_Series(pricev["2009"], theme=plot_theme,
+        lwd=2, name="VTI VWAP Prices")
> legend("bottomright", legend=colnames(pricev),
+  inset=0.1, bg="white", lty=1, lwd=6, cex=0.8,
+  col=plot_theme$col$line.col, bty="n")
```

Recursive VWAP Price Indicator

The *VWAP* prices p^{VWAP} can also be calculated as the ratio of the volume weighted prices μ^{pv} divided by the mean trading volumes μ^{v}:

$$p^{VWAP} = \frac{\mu^{pv}}{\mu^{v}}$$

The volume weighted prices μ^{pv} and the mean trading volumes μ^{v} are both calculated recursively:

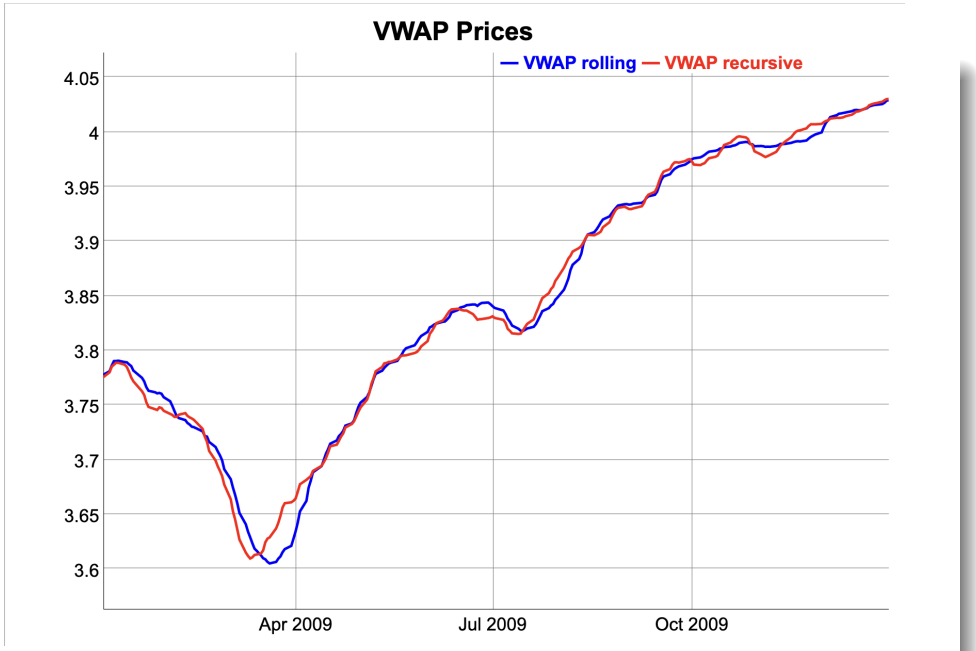
$$\mu_t^{v} = \lambda \mu_{t-1}^{v} + (1-\lambda) v_t$$
$$\mu_t^{pv} = \lambda \mu_{t-1}^{pv} + (1-\lambda) v_t p_t$$

The recursive *VWAP* prices are slightly different from those calculated as a convolution, because the convolution uses a fixed look-back interval.

The advantage of the recursive *VWAP* indicator is that it gradually "forgets" about large trading volumes far in the past.

The compiled C++ function `stats:::C_rfilter()` calculates the trailing weighted values recursively.

The function `HighFreq::run_mean()` also calculates the trailing weighted values recursively.




```
> # Calculate the VWAP prices recursively using C++ code
> lambdaf <- 0.9
> volumer <- .Call(stats:::C_rfilter, volumv, lambdaf, c(as.numeric(
> pricer <- .Call(stats:::C_rfilter, volumv*closep, lambdaf, c(as.nu
> vwapr <- pricer/volumer
> # Calculate the VWAP prices recursively using RcppArmadillo
> vwapc <- HighFreq::run_mean(closep, lambda=lambdaf, weightv=volumv
> all.equal(vwapr, drop(vwapc))
> # Dygraphs plot the VWAP prices
> pricev <- xts(cbind(vwap, vwapr), zoo::index(ohlc))
> colnames(pricev) <- c("VWAP rolling", "VWAP recursive")
> dygraphs::dygraph(pricev["2009"], main="VWAP Prices") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

Simulating the VWAP Crossover Strategy

In the trend following *VWAP Crossover* strategy, the stock position switches depending if the current price is above or below the *VWAP*.

If the current price crosses above the *VWAP*, then the strategy switches its stock position to a fixed unit of long risk, and if it crosses below, to a fixed unit of short risk.

To prevent whipsaws and over-trading, the crossover strategy delays switching positions until the indicator repeats the same value for several periods.


```
> # Calculate the VWAP prices recursively using RcppArmadillo
> lambdaf <- 0.99
> vwapc <- HighFreq::run_mean(closep, lambda=lambdaf, weightv=volumv
> # Calculate the positions from lagged indicator
> indic <- sign(closep - vwapc)
> lagg <- 2
> indic <- HighFreq::roll_sum(indic, lagg)
> # Calculate the positions, either: -1, 0, or 1
> posv <- rep(NA_integer_, nrows)
> posv[1] <- 0
> posv <- ifelse(indic == lagg, 1, posv)
> posv <- ifelse(indic == (-lagg), -1, posv)
> posv <- zoo::na.locf(posv, na.rm=FALSE)
> posv <- xts::xts(posv, order.by=zoo::index(closep))
> # Lag the positions to trade in next period
> posv <- rutils::lagit(posv, lagg=1)
> # Calculate the PnLs of VWAP strategy
> retp <- rutils::diffit(closep)  # VTI returns
> pnls <- retp*posv
> colnames(pnls) <- "VWAP"
> wealthv <- cbind(retp, pnls)
> colnamev <- colnames(wealthv)
```

```
> # Annualized Sharpe ratios of VTI and VWAP strategy
> sharper <- sqrt(252)*sapply(wealthv, function (x) mean(x)/sd(x))
> # Create colors for background shading
> crossd <- (rutils::diffit(posv) != 0)
> shadev <- posv[crossd]
> crossd <- c(zoo::index(shadev), end(posv))
> shadev <- ifelse(drop(zoo::coredata(shadev)) == 1, "lightgreen", "
> # Plot dygraph of VWAP strategy
> # Create dygraph object without plotting it
> dyplot <- dygraphs::dygraph(cumsum(wealthv),
+   main=paste("VWAP Crossover Strategy, Sharpe", paste(paste(names(
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
> # Add shading to dygraph object
> for (i in 1:NROW(shadev)) {
+   dyplot <- dyplot %>% dyShading(from=crossd[i], to=crossd[i+1], c
+ }  # end for
> # Plot the dygraph object
```

Combining VWAP Crossover Strategy with Stocks

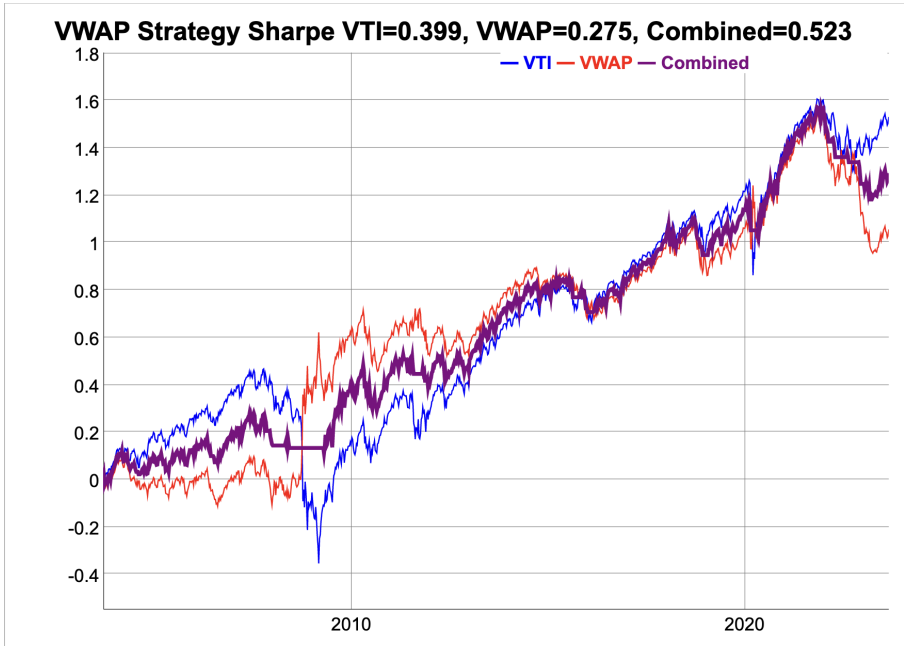
Even though the *VWAP* strategy doesn't perform as well as a static buy-and-hold strategy, it can provide risk reduction when combined with it.

This is because the *VWAP* strategy has a negative correlation with respect to the underlying asset.

In addition, the *VWAP* strategy performs well in periods of extreme market selloffs, so it can provide a hedge for a static buy-and-hold strategy.

The *VWAP* strategy serves as a dynamic put option in periods of extreme market selloffs.

```
> # Calculate the correlation of VWAP strategy with VTI
> cor(retp, pnls)
> # Combine VWAP strategy with VTI
> wealthv <- cbind(retp, pnls, 0.5*(retp+pnls))
> colnames(wealthv) <- c("VTI", "VWAP", "Combined")
> sharper <- sqrt(252)*sapply(wealthv, function (x) mean(x)/sd(x))
```



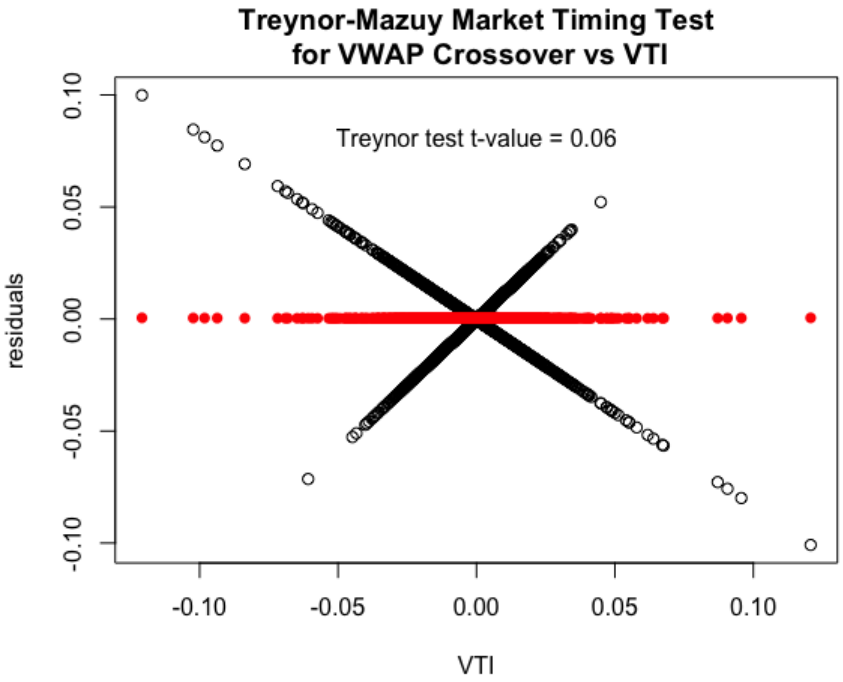

```
> # Plot dygraph of VWAP strategy combined with VTI
> colorv <- c("blue", "red", "purple")
> dygraphs::dygraph(cumsum(wealthv)[endd],
+   paste("VWAP Strategy Sharpe", paste(paste(names(sharper), round(
+   dyOptions(colors=colorv, strokeWidth=1) %>%
+   dySeries(name="Combined", strokeWidth=3) %>%
+   dyLegend(show="always", width=300)
```

VWAP Crossover Strategy Market Timing Skill

The VWAP crossover strategy shorts the market during significant selloffs, but otherwise doesn't display market timing skill.

The t-value of the *Treynor-Mazuy* test is negative, but not statistically significant.

```
> # Test VWAP crossover market timing of VTI using Treynor-Mazuy tes
> desm <- cbind(pnls, retp, retp^2)
> desm <- na.omit(desm)
> colnames(desm) <- c("VWAP", "VTI", "Treynor")
> regmod <- lm(VWAP ~ VTI + Treynor, data=desm)
> summary(regmod)
> # Plot residual scatterplot
> resids <- (desm$VWAP - regmod$coeff["VTI"]*retp)
> resids <- regmod$residuals
> # x11(width=6, height=6)
> plot.default(x=retp, y=resids, xlab="VTI", ylab="residuals")
> title(main="Treynor-Mazuy Market Timing Test\n for VWAP Crossover
> # Plot fitted (predicted) response values
> coefreg <- summary(regmod)$coeff
> fitv <- regmod$fitted.values - coefreg["VTI", "Estimate"]*retp
> tvalue <- round(coefreg["Treynor", "t value"], 2)
> points.default(x=retp, y=fitv, pch=16, col="red")
> text(x=0.0, y=0.8*max(resids), paste("Treynor test t-value =", tvalue))
```



Simulation Function for VWAP Crossover Strategy

The *VWAP* strategy can be simulated by a single function, which allows the analysis of its performance depending on its parameters.

The function `sim_vwap()` performs a simulation of the *VWAP* strategy, given an *OHLC* time series of prices, and the length of the look-back interval (`lookb`).

The function `sim_vwap()` returns the *VWAP* strategy positions and returns, in a two-column *xts* time series.

```
> sim_vwap <- function(ohlc, lambdaf=0.9, bidask=0.001, trend=1, lag
+   closep <- log(quantmod::Cl(ohlc))
+   volumv <- quantmod::Vo(ohlc)
+   retp <- rutils::diffit(closep)
+   nrows <- NROW(ohlc)
+   # Calculate the VWAP prices
+   vwap <- HighFreq::run_mean(closep, lambda=lambdaf, weightv=volum
+   # Calculate the indicator
+   indic <- trend*sign(closep - vwap)
+   if (lagg > 1) {
+     indic <- HighFreq::roll_sum(indic, lagg)
+     indic[1:lagg] <- 0
+   }  # end if
+   # Calculate the positions, either: -1, 0, or 1
+   posv <- rep(NA_integer_, nrows)
+   posv[1] <- 0
+   posv <- ifelse(indic == lagg, 1, posv)
+   posv <- ifelse(indic == (-lagg), -1, posv)
+   posv <- zoo::na.locf(posv, na.rm=FALSE)
+   posv <- xts::xts(posv, order.by=zoo::index(closep))
+   # Lag the positions to trade on next day
+   posv <- rutils::lagit(posv, lagg=1)
+   # Calculate the PnLs of strategy
+   pnls <- retp*posv
+   costv <- 0.5*bidask*abs(rutils::diffit(posv))
+   pnls <- (pnls - costv)
+   # Calculate the strategy returns
+   pnls <- cbind(posv, pnls)
+   colnames(pnls) <- c("positions", "pnls")
+   pnls
+ }  # end sim_vwap
```

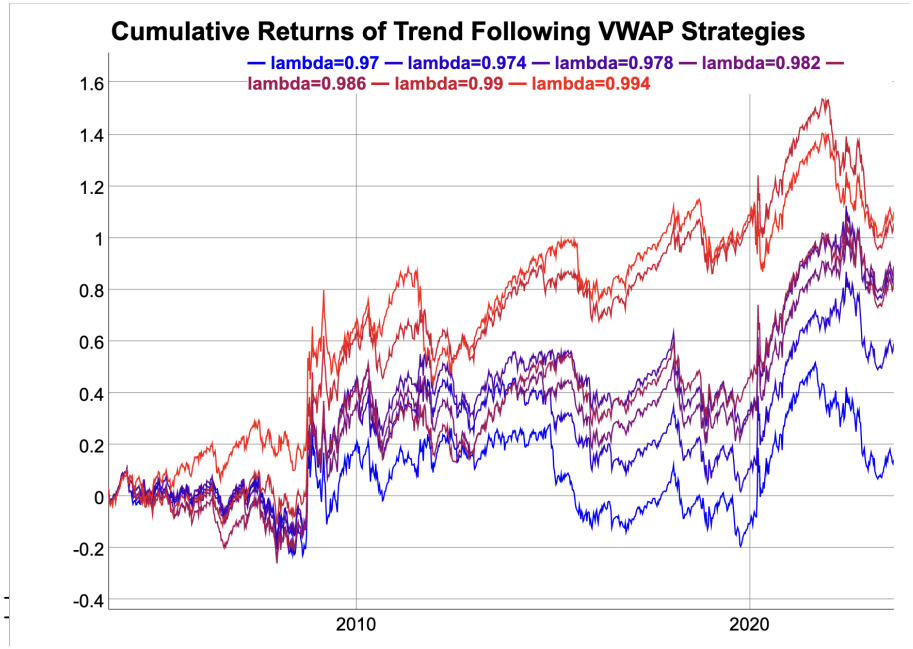
Simulating Multiple Trend Following VWAP Strategies

Multiple *VWAP* strategies can be simulated by calling the function `sim_vwap()` in a loop over a vector of λ parameters.

But `sim_vwap()` returns an *xts* time series, and `sapply()` cannot merge *xts* time series together.

So instead the loop is performed using `lapply()` which returns a list of *xts*, and the list is merged into a single *xts* using the functions `do.call()` and `cbind()`.

```
> lambdav <- seq(from=0.97, to=0.995, by=0.004)
> # Perform lapply() loop over lambdav
> pnls <- lapply(lambdav, function(lambdaf) {
+   # Simulate VWAP strategy and Calculate the returns
+   sim_vwap(ohlc=ohlc, lambdaf=lambdaf, bidask=0, lagg=2)[, "pnls"]
+ })  # end lapply
> pnls <- do.call(cbind, pnls)
> colnames(pnls) <- paste0("lambda=", lambdav)
```




```
> # Plot dygraph of multiple VWAP strategies
> colorv <- colorRampPalette(c("blue", "red"))(NCOL(pnls))
> dygraphs::dygraph(cumsum(pnls)[endd], main="Cumulative Returns of
+   dyOptions(colors=colorv, strokeWidth=1) %>%
+   dyLegend(show="always", width=500)
> # Plot VWAP strategies with custom line colors
> x11(width=6, height=5)
> plot_theme <- chart_theme()
> plot_theme$col$line.col <- colorv
> quantmod::chart_Series(cumsum(pnls), theme=plot_theme,
+   name="Cumulative Returns of VWAP Strategies")
> legend("topleft", legend=colnames(pnls), inset=0.1,
+   bg="white", cex=0.8, lwd=rep(6, NCOL(pnls)),
+   col=plot_theme$col$line.col, bty="n")
```

Backtesting of Rolling Strategies

Backtesting is the simulation of a trading strategy on historical data.

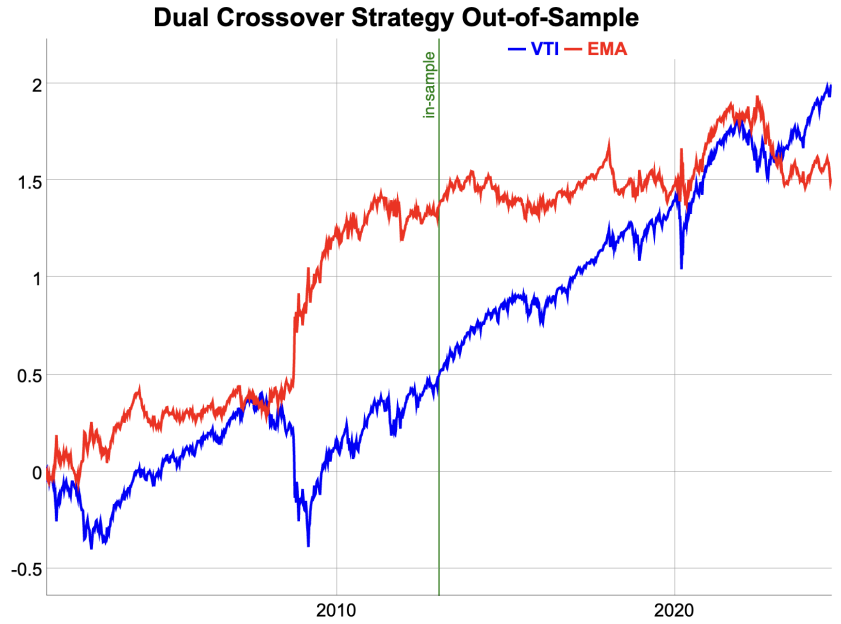
A *rolling strategy* can be *backtested* by specifying the parameter updating frequency, the formation interval, and the holding period:

- Calculate the *end points* for parameter updating,
- Define an objective function for parameter optimization,
- Calculate the optimal parameters in the in-sample formation interval,
- Calculate the out-of-sample strategy returns,
- Calculate the transaction costs and subtract them from the strategy returns.

The *backtesting* redefines the problem of finding (tuning) the optimal trading strategy parameters, into the problem of finding the optimal *backtest* (meta-model) parameters: the updating frequency, the formation interval, and the holding period.

The advantage of using the *backtest* meta-model is that it can reduce the number of parameters that need to be optimized.

Using a different updating frequency in the *backtest* can produce different values for the optimal trading strategy parameters.




```
> # Dygraphs plot with custom line colors
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd], main="Dual Crossover Stra
+   dyEvent(zoo::index(wealthv[last(insample)]), label="in-sample",
+   dyOptions(colors=c("blue", "red"), strokeWidth=2)
```

Bollinger Bands

The Bollinger Bands are three time series, with the middle band equal to the trailing mean prices, the upper band equal to the mean prices plus the trailing standard deviation, and the lower band equal to the mean prices minus the standard deviation.

The Bollinger Bands are often used to indicate that prices are cheap if they are below the lower band, and rich (expensive) if they are above the upper band.

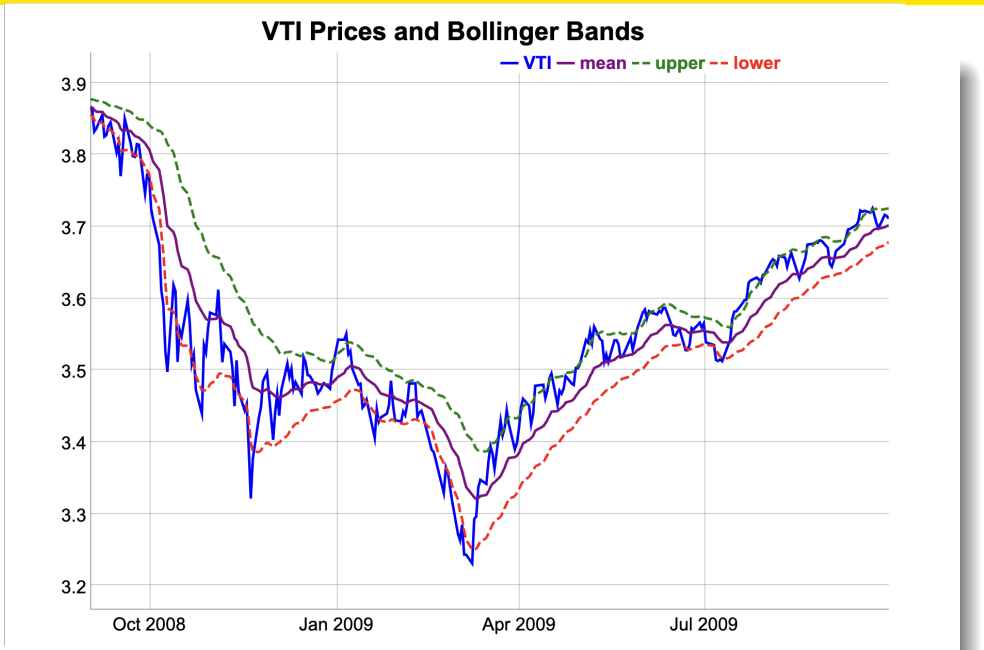
The function `HighFreq::run_var()` calculates the trailing mean and variance of the prices p_t, by recursively weighting the past variance estimates σ^2_{t-1}, with the squared differences of the prices minus the trailing means $(p_t - \bar{p}_t)^2$, using the decay factor λ:

$$\bar{p}_t = \lambda \bar{p}_{t-1} + (1 - \lambda) p_t$$

$$\sigma^2_t = \lambda \sigma^2_{t-1} + (1 - \lambda)(p_t - \bar{p}_t)^2$$

Where $\bar{p}_t$ and σ^2_t are the trailing mean and variance at time t.

The decay factor λ determines how quickly the mean and variance estimates are updated, with smaller values of λ producing faster updating, giving more weight to recent prices, and vice versa.




```
> # Calculate the trailing mean prices and volatilities
> pricev <- log(na.omit(rutils::etfenv$prices$VTI))
> lambdaf <- 0.9
> pricem <- HighFreq::run_mean(pricev, lambda=lambdaf)
> volp <- HighFreq::run_var(pricev, lambda=lambdaf)
> volp <- sqrt(volp)
> # Dygraphs plot of Bollinger bands
> priceb <- cbind(pricev, pricem, pricem+volp, pricem-volp)
> colnames(priceb)[2:4] <- c("mean", "upper", "lower")
> colnamev <- colnames(priceb)
> dygraphs::dygraph(priceb["2008-09/2009-09"], main="VTI Prices and
+   dySeries(name=colnamev[1], strokeWidth=2, col="blue") %>%
+   dySeries(name=colnamev[2], strokeWidth=2, col="purple") %>%
+   dySeries(name=colnamev[3], strokeWidth=2, strokePattern="dashed"
+   dySeries(name=colnamev[4], strokeWidth=2, strokePattern="dashed"
+   dyLegend(show="always", width=300)
```

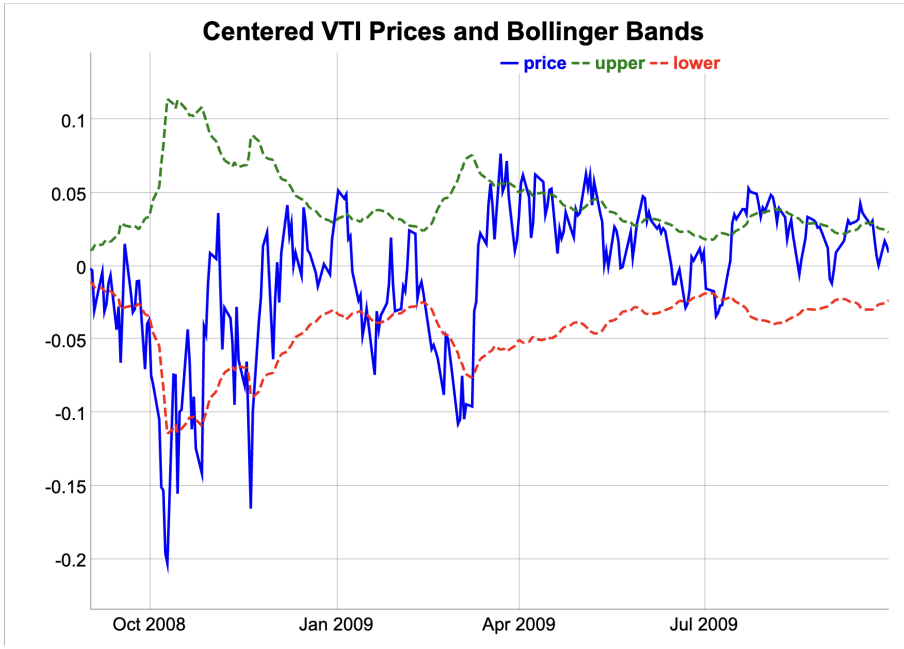
Bollinger Bands With Centered Prices

Centering (de-meaning) the prices provides a better illustration of the *Bollinger Bands*.

The centered prices tend to be range-bound between the *Bollinger Bands*.

When the centered price is below the lower band it's considered cheap, and if it's above the upper band it's considered rich (expensive).

When the centered price is close to zero it's considered fair (neutral).




```
> # Center the prices
> pricec <- pricev - pricem
> # Dygraphs plot of Bollinger bands
> priceb <- cbind(pricec, volp, -volp)
> colnames(priceb) <- c("price", "upper", "lower")
> colnamev <- colnames(priceb)
> dygraphs::dygraph(priceb["2008-09/2009-09"],
+   main="Centered VTI Prices and Bollinger Bands") %>%
+   dySeries(name=colnamev[1], strokeWidth=2, col="blue") %>%
+   dySeries(name=colnamev[2], strokeWidth=2, strokePattern="dashed"
+   dySeries(name=colnamev[3], strokeWidth=2, strokePattern="dashed"
+   dyLegend(show="always", width=300)
```

The Bollinger Band Strategy

The *Bollinger Strategy* switches to long \$1 dollar of the stock when prices are cheap (below the lower band), and sells short -\$1 dollar when prices are rich (expensive - above the upper band). It goes flat (unwinds) if the stock reaches a fair (mean) price.

The strategy is therefore always either long \$1 dollar of stock, or short -\$1 dollar, or flat the stock (\$0 dollars).

The upper and lower Bollinger bands can be chosen to be a multiple of the standard deviations above and below the mean prices.

The *Bollinger Strategy* is a *mean reverting* (contrarian) strategy because it bets on prices reverting to their mean value.

The *Bollinger Strategy* can be considered an extension of the crossover strategy, which utilizes information about the volatility of the returns.

The *Bollinger Strategy* switches its position only after the stock price crosses the Bollinger band, instead of the moving average price, which delays trades and reduces *"whipsaws"*.

The *Bollinger Strategy* is a type of *statistical arbitrage* strategy.

Statistical arbitrage strategies try to exploit short-term anomalies in prices, when prices diverge from their equilibrium values and then revert back to them.

```
> # Calculate the trailing mean prices and volatilities
> lambdaf <- 0.1
> pricem <- HighFreq::run_mean(pricev, lambda=lambdaf)
> volp <- HighFreq::run_var(pricev, lambda=lambdaf)
> volp <- sqrt(volp)
> # Prepare the simulation parameters
> pricen <- as.numeric(pricev) # Numeric price
> pricec <- pricen - pricem # Centered price
> threshv <- volp
> nrows <- NROW(pricev)
> posv <- integer(nrows) # Stock positions
> posv[1] <- 0 # Initial position
> # Calculate the positions from Bollinger bands
> for (it in 2:nrows) {
+   if (pricec[it-1] > threshv[it-1]) {
+     # Enter short
+     posv[it] <- (-1)
+   } else if (pricec[it-1] < (-threshv[it-1])) {
+     # Enter long
+     posv[it] <- 1
+   } else if ((posv[it-1] < 0) && (pricec[it-1] < 0)) {
+     # Unwind short
+     posv[it] <- 0
+   } else if ((posv[it-1] > 0) && (pricec[it-1] > 0)) {
+     # Unwind long
+     posv[it] <- 0
+   } else {
+     # Do nothing
+     posv[it] <- posv[it-1]
+   }  # end if
+ }  # end for
> # Calculate the number of trades
> ntrades <- sum(abs(rutils::diffit(posv)) > 0)
> # Calculate the pnls
> retp <- rutils::diffit(pricev)
> pnls <- retp*posv
```

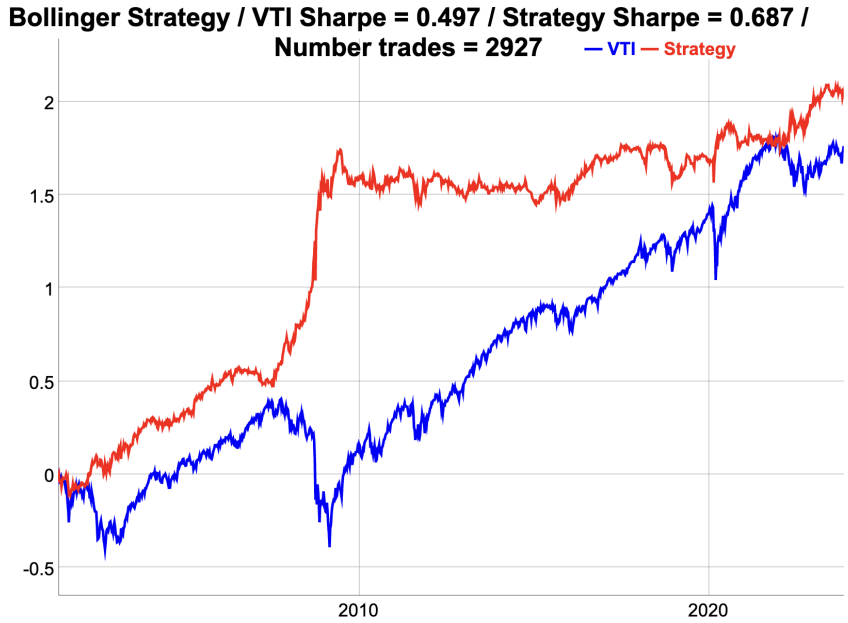
Bollinger Strategy Performance

The *Bollinger Band* strategy has two parameters: the decay factor λ and the standard deviation multiple n.

The best strategy parameters can be found using backtest simulation, but it risks overfitting the parameters to the in-sample data, and poor performance out-of-sample.

The *Bollinger Band* strategy has performed well for *VTI* with $\lambda = 0.1$, but it hasn't performed well for most other stocks.

The *Bollinger Band* strategy had its best performance for *VTI* prior to the financial crisis of 2008-2009.



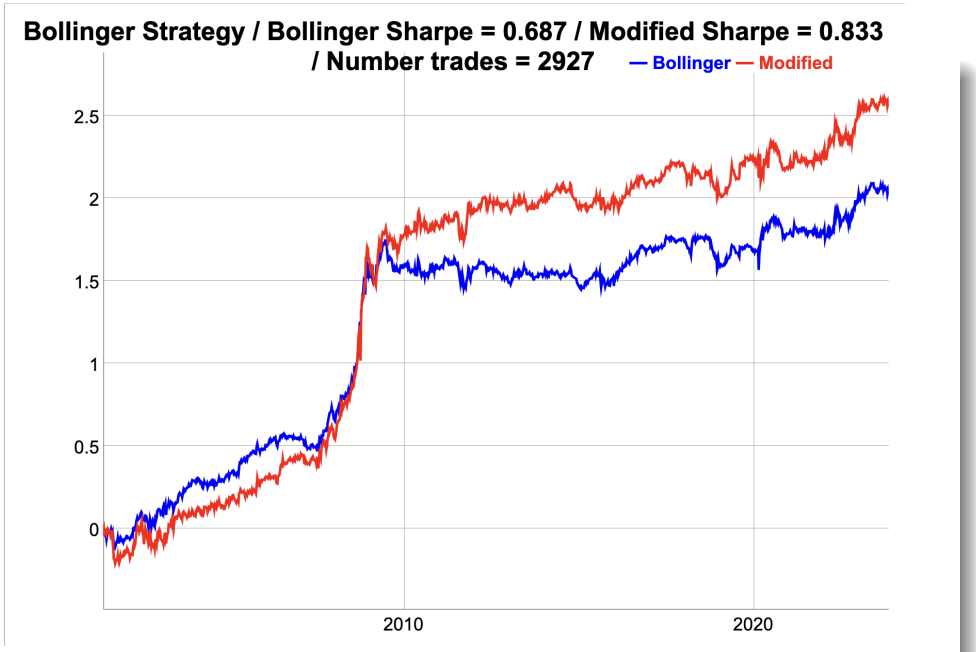

```
> # Calculate the Sharpe ratios
> wealthv <- cbind(retp, pnls)
> colnames(wealthv) <- c("VTI", "Strategy")
> sharper <- sqrt(252)*sapply(wealthv, function(x) mean(x)/sd(x[x<0]
> sharper <- round(sharper, 3)
> # Dygraphs plot of Bollinger strategy
> colnamev <- colnames(wealthv)
> captiont <- paste("Bollinger Strategy", "/ \n",
+   paste0(paste(colnamev[1:2], "Sharpe =", sharper), collapse=" / "
+   "Number trades =", ntrades)
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd], main=captiont) %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

The Modified Bollinger Strategy

Unwinding the position when the stock reaches a fair (mean) price doesn't necessarily produce better performance.

In the modified Bollinger Strategy the positions are held until the price reaches the opposite extreme, so that the strategy is always either long \$1 dollar of stock or short \$1 dollar of stock.

```
> # Simulate the modified Bollinger strategy
> posv <- integer(nrows) # Stock positions
> posv[1] <- 0 # Initial position
> for (it in 2:nrows) {
+   if (pricec[it-1] > threshv[it-1]) {
+     # Enter short
+     posv[it] <- (-1)
+   } else if (pricec[it-1] < (-threshv[it-1])) {
+     # Enter long
+     posv[it] <- 1
+   } else {
+     # Do nothing
+     posv[it] <- posv[it-1]
+   }  # end if
+ }  # end for
> # Calculate the PnLs
> pnls2 <- retp*posv
```




```
> # Calculate the Sharpe ratios
> wealthv <- cbind(pnls, pnls2)
> colnames(wealthv) <- c("Bollinger", "Modified")
> sharper <- sqrt(252)*sapply(wealthv, function(x) mean(x)/sd(x[x<0]
> sharper <- round(sharper, 3)
> # Dygraphs plot of Bollinger strategy
> colnamev <- colnames(wealthv)
> captiont <- paste("Bollinger Strategy", "/ \n",
+   paste0(paste(colnamev[1:2], "Sharpe =", sharper), collapse=" / "
+   "Number trades =", ntrades)
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd], main=captiont) %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

Fast Bollinger Strategy Simulation

The Bollinger Strategy is path-dependent so simulating it requires performing a loop, which can be slow in `R`.

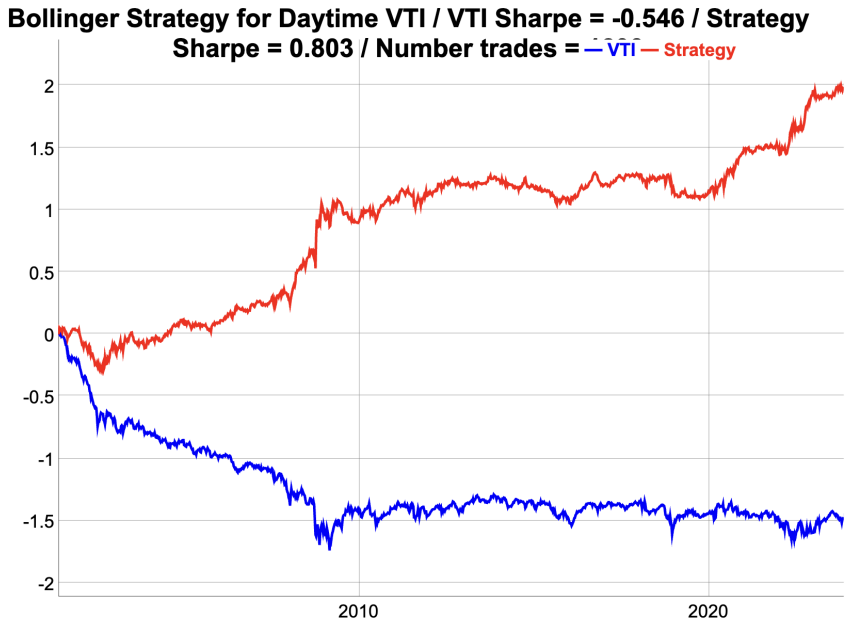
The modified Bollinger Strategy can be simulated quickly using the compiled `C++` functions `ifelse()` and `zoo::na.locf()`.

```
> # Simulate the modified Bollinger strategy quickly
> posf <- rep(NA_integer_, nrows)
> posf[1] <- 0
> posf <- ifelse(pricec > threshv, -1, posf)
> posf <- ifelse(pricec < -threshv, 1, posf)
> posf <- zoo::na.locf(posf)
> # Lag the positions to trade in the next period
> posf <- rutils::lagit(posf, lagg=1)
> # Compare the positions
> all.equal(posv, posf)
```

Bollinger Strategy For Daytime VTI Returns

The Bollinger Strategy has performed well for daytime returns of the *VTI* ETF, because daytime returns exhibit significant mean-reversion.

This simulation doesn't account for transaction costs, which would likely erase all profits if market orders were used for trade executions. But the strategy could be profitable if limit orders were used for trade executions.



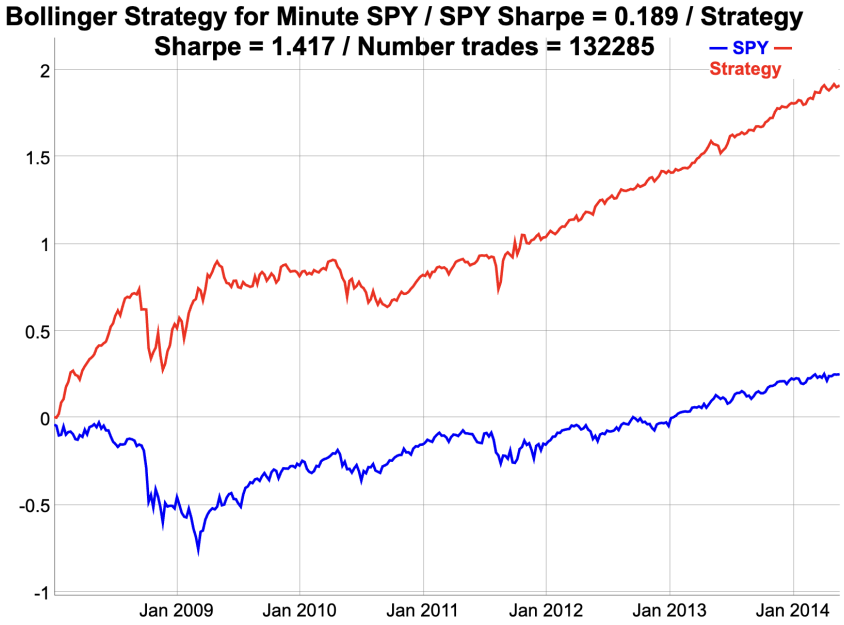

```
> # Calculate the daytime open-to-close VTI returns
> ohlc <- log(rutils::etfenv$VTI)
> nrows <- NROW(ohlc)
> openp <- quantmod::Op(ohlc)
> highp <- quantmod::Hi(ohlc)
> lowp <- quantmod::Lo(ohlc)
> closep <- quantmod::Cl(ohlc)
> retd <- (closep - openp)
> # Calculate the cumulative daytime VTI returns
> priced <- cumsum(retd)
> lambdaf <- 0.1
> pricem <- HighFreq::run_mean(priced, lambda=lambdaf)
> volp <- HighFreq::run_var(priced, lambda=lambdaf)
> volp <- sqrt(volp)
> # Calculate the positions from Bollinger bands
> threshv <- volp
> pricec <- zoo::coredata(priced - pricem)
> posv <- rep(NA_integer_, nrows)
> posv[1] <- 0
> posv <- ifelse(pricec > threshv, -1, posv)
> posv <- ifelse(pricec < -threshv, 1, posv)
> posv <- zoo::na.locf(posv)
> # Lag the positions to trade in the next period
> posv <- rutils::lagit(posv, lagg=1)
> # Calculate the number of trades and the PnLs
> ntrades <- sum(abs(rutils::diffit(posv)) > 0)
> pnls <- retd*posv
```

```
> # Calculate the Sharpe ratios
> wealthv <- cbind(retd, pnls)
> colnames(wealthv) <- c("VTI", "Strategy")
> nyears <- as.numeric(end(priced)-start(priced))/365
> sharper <- sqrt(nrows/nyears)*sapply(wealthv, function(x) mean(x)/
> sharper <- round(sharper, 3)
> # Dygraphs plot of Bollinger strategy
> colnamev <- colnames(wealthv)
> captiont <- paste("Bollinger Strategy for Daytime VTI", "/ \n",
+   paste0(paste(colnamev[1:2], "Sharpe =", sharper), collapse=" / "
+   "Number trades =", ntrades)
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd], main=captiont) %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

Bollinger Strategy For Intraday SPY Returns

The Bollinger Strategy has performed well for 1-minute prices of the *SPY* ETF, because intraday returns exhibit significant mean-reversion.

This simulation doesn't account for transaction costs, which would likely erase all profits if market orders were used for trade executions. But the strategy could be profitable if limit orders were used for trade executions.




```
> # Calculate the trailing mean prices and volatilities of SPY
> pricev <- log(quantmod::Cl(HighFreq::SPY))
> nrows <- NROW(pricev)
> lambdaf <- 0.1
> pricem <- HighFreq::run_mean(pricev, lambda=lambdaf)
> volp <- HighFreq::run_var(pricev, lambda=lambdaf)
> volp <- sqrt(volp)
> # Calculate the positions from Bollinger bands
> threshv <- volp
> pricec <- zoo::coredata(pricev - pricem)
> posv <- rep(NA_integer_, nrows)
> posv[1] <- 0
> posv <- ifelse(pricec > threshv, -1, posv)
> posv <- ifelse(pricec < -threshv, 1, posv)
> posv <- zoo::na.locf(posv)
> # Lag the positions to trade in the next period
> posv <- rutils::lagit(posv, lagg=1)
> # Calculate the number of trades and the PnLs
> ntrades <- sum(abs(rutils::diffit(posv)) > 0)
> retp <- rutils::diffit(pricev)
> pnls <- retp*posv
> # Subtract transaction costs from the pnls
> bidask <- 0.0001 # Bid-ask spread equal to 1 basis point
> costv <- 0.5*bidask*abs(rutils::diffit(posv))
> pnls <- (pnls - costv)
```

```
> # Calculate the Sharpe ratios
> wealthv <- cbind(retp, pnls)
> colnames(wealthv) <- c("SPY", "Strategy")
> nyears <- as.numeric(end(pricev)-start(pricev))/365
> sharper <- sqrt(nrows/nyears)*sapply(wealthv, function(x) mean(x)/
> sharper <- round(sharper, 3)
> # Dygraphs plot of Bollinger strategy
> colnamev <- colnames(wealthv)
> captiont <- paste("Bollinger Strategy for Minute SPY", "/ \n",
+   paste0(paste(colnamev[1:2], "Sharpe =", sharper), collapse=" / "
+   "Number trades =", ntrades)
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd], main=captiont) %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=100)
```

The Hampel Filter Bands

The Bollinger bands can be improved by using nonparametric measures of location (*median*) and dispersion (*MAD*).

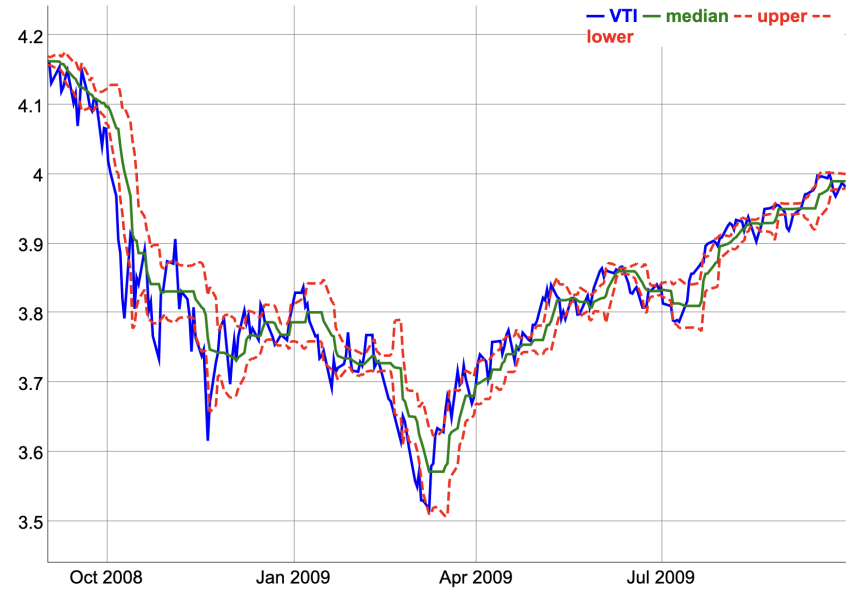
The *Median Absolute Deviation* (*MAD*) is a nonparametric measure of dispersion (variability):

$$\text{MAD} = \text{median}(\text{abs}(p_t - \text{median}(\mathbf{p})))$$

The *Hampel z-score* is equal to the deviation from the median divided by the *MAD*:

$$z_i = \frac{p_t - \text{median}(\mathbf{p})}{\text{MAD}}$$

A time series of *z-scores* over past data can be calculated using a trailing look-back window.




```
> # Extract time series of VTI log prices
> pricev <- log(na.omit(rutils::etfenv$prices$VTI))
> nrows <- NROW(pricev)
> # Define look-back window
> lookb <- 11
> # Calculate time series of trailing medians
> medianv <- HighFreq::roll_mean(pricev, lookb, method="nonparametr
> # medianv <- TTR::runMedian(pricev, n=lookb)
> # Calculate time series of MAD
> madv <- HighFreq::roll_var(pricev, lookb=lookb, method="nonparam
> # madv <- TTR::runMAD(pricev, n=lookb)
> # Calculate time series of z-scores
> zscores <- ifelse(madv > 0, (pricev - medianv)/madv, 0)
> zscores[1:lookb, ] <- 0
> tail(zscores, lookb)
> range(zscores)
```

```
> # Plot histogram of z-scores
> histp <- hist(zscores, col="lightgrey",
+   xlab="z-scores", breaks=50, xlim=c(-4, 4),
+   ylab="frequency", freq=FALSE, main="Hampel Z-Scores histogram")
> lines(density(zscores, adjust=1.5), lwd=3, col="blue")
> # Dygraphs plot of Hampel bands
> priceb <- cbind(pricev, medianv, medianv+madv, medianv-madv)
> colnames(priceb)[2:4] <- c("median", "upper", "lower")
> colnamev <- colnames(priceb)
> dygraphs::dygraph(priceb["2008-09/2009-09"], main="VTI Prices and
+   dySeries(name=colnamev[1], strokeWidth=2, col="blue") %>%
+   dySeries(name=colnamev[2], strokeWidth=2, col="green") %>%
+   dySeries(name=colnamev[3], strokeWidth=2, strokePattern="dashed"
+   dySeries(name=colnamev[4], strokeWidth=2, strokePattern="dashed"
+   dyLegend(show="always", width=300)
```

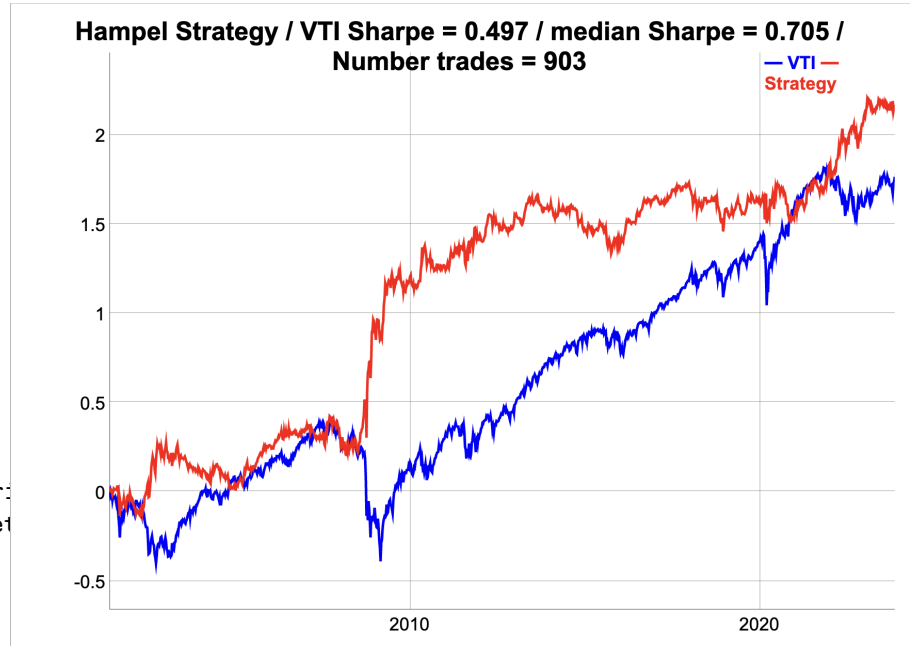
Hampel Filter Strategy

The Hampel filter strategy is a contrarian strategy that uses Hampel z-scores to establish long and short positions.

The Hampel strategy has two meta-parameters: the look-back interval and the threshold level.

The best choice of the meta-parameters can be determined through simulation.

```
> # Calculate the time series of trailing medians and MAD
> lookb <- 3
> medianv <- HighFreq::roll_mean(pricev, lookb, method="nonparametri
> madv <- HighFreq::roll_var(pricev, lookb=lookb, method="nonparamet
> # Calculate the time series of z-scores
> zscores <- ifelse(madv > 0, (pricev - medianv)/madv, 0)
> zscores[1:lookb, ] <- 0
> range(zscores)
> # Calculate the positions
> threshv <- 1
> posv <- rep(NA_integer_, nrows)
> posv[1] <- 0
> posv[zscores > threshv] <- (-1)
> posv[zscores < -threshv] <- 1
> posv <- zoo::na.locf(posv)
> posv <- rutils::lagit(posv)
> # Calculate the number of trades and the PnLs
> ntrades <- sum(abs(rutils::diffit(posv)) > 0)
> retp <- rutils::diffit(pricev)
> pnls <- retp*posv
```



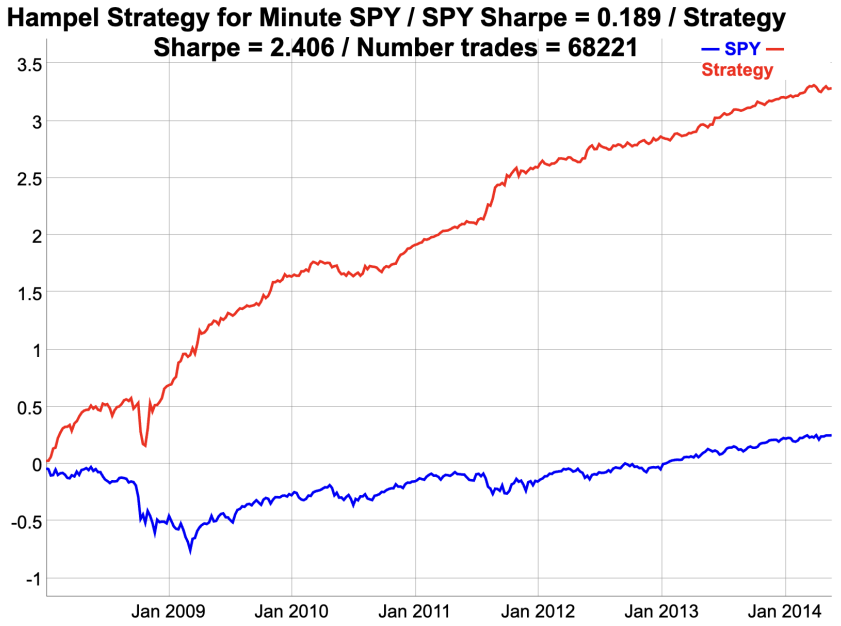

```
> # Calculate the Sharpe ratios
> wealthv <- cbind(retp, pnls)
> colnames(wealthv) <- c("VTI", "Strategy")
> sharper <- sqrt(252)*sapply(wealthv, function(x) mean(x)/sd(x[x<0]
> sharper <- round(sharper, 3)
> # Dygraphs plot of Hampel strategy
> captiont <- paste("Hampel Strategy", "/ \n",
+   paste0(paste(colnamev[1:2], "Sharpe =", sharper), collapse=" / "
+   "Number trades =", ntrades)
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> colnamev <- colnames(wealthv)
> dygraphs::dygraph(cumsum(wealthv)[endd], main=captiont) %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=100)
```

Hampel Filter Strategy For Intraday SPY Returns

The Hampel Filter strategy has performed well for 1-minute prices of the *SPY* ETF, because intraday returns exhibit significant mean-reversion.

This simulation doesn't account for transaction costs, which would likely erase all profits if market orders were used for trade executions. But the strategy could be profitable if limit orders were used for trade executions.

```
> # Calculate the trailing mean prices and volatilities of SPY
> pricev <- log(quantmod::Cl(HighFreq::SPY))
> nrows <- NROW(pricev)
> # Calculate the price medians and MAD
> lookb <- 3
> medianv <- HighFreq::roll_mean(pricev, lookb, method="nonparametri
> madv <- HighFreq::roll_var(pricev, lookb=lookb, method="nonparamet
> # Calculate the time series of z-scores
> zscores <- ifelse(madv > 0, (pricev - medianv)/madv, 0)
> zscores[1:lookb, ] <- 0
> # Calculate the positions
> threshv <- 1
> posv <- rep(NA_integer_, nrows)
> posv[1] <- 0
> posv[zscores < -threshv] <- 1
> posv[zscores > threshv] <- (-1)
> posv <- zoo::na.locf(posv)
> posv <- rutils::lagit(posv)
> # Calculate the number of trades and the PnLs
> ntrades <- sum(abs(rutils::diffit(posv)) > 0)
> retp <- rutils::diffit(pricev)
> pnls <- retp*posv
> # Subtract transaction costs from the pnls
> costv <- 0.5*bidask*abs(rutils::diffit(posv))
> pnls <- (pnls - costv)
```




```
> # Calculate the Sharpe ratios
> wealthv <- cbind(retp, pnls)
> colnames(wealthv) <- c("SPY", "Strategy")
> nyears <- as.numeric(end(pricev)-start(pricev))/365
> sharper <- sqrt(nrows/nyears)*sapply(wealthv, function(x) mean(x)/
> sharper <- round(sharper, 3)
> # Dygraphs plot of Hampel strategy
> colnamev <- colnames(wealthv)
> captiont <- paste("Hampel Strategy for Minute SPY", "/ \n",
+   paste0(paste(colnamev[1:2], "Sharpe =", sharper), collapse=" / "
+   "Number trades =", ntrades)
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd], main=captiont) %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=100)
```

Centered Price Z-scores

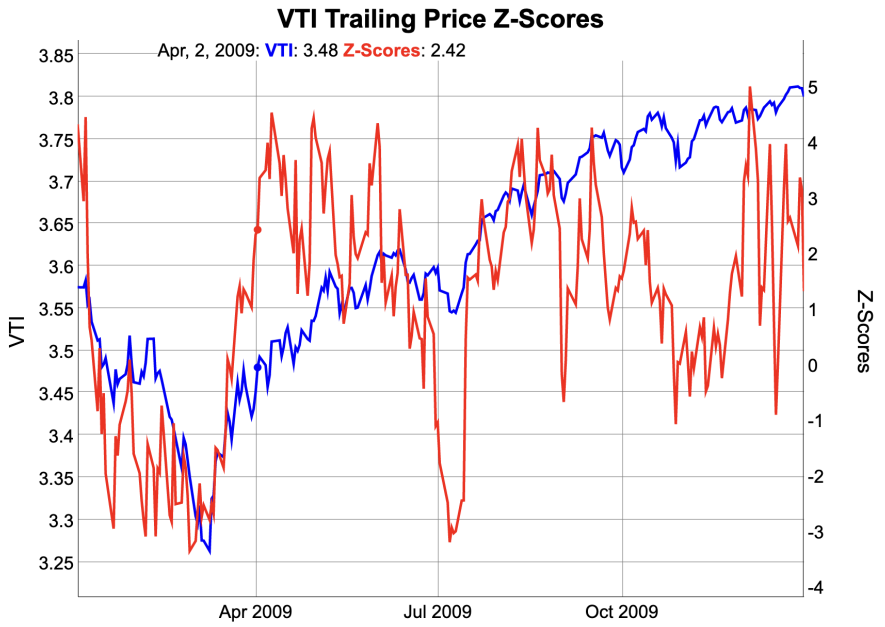
An extreme local price is a price which differs significantly from neighboring prices.

Extreme prices can be identified in-sample using the centered *price z-score* equal to the price difference with neighboring prices divided by the volatility of returns σ_i:

$$z_i = \frac{2p_i - p_{i-k} - p_{i+k}}{\sigma_i}$$

Where p_{i-k} and p_{i+k} are the lagged and advanced prices.

The lag parameter k determines the scale of the extreme local prices, with smaller k producing larger z-scores for more local price extremes.




```
> # Extract the VTI log OHLC prices
> ohlc <- log(rutils::etfenv$VTI)
> nrows <- NROW(ohlc)
> closep <- quantmod::Cl(ohlc)
> retp <- rutils::diffit(closep)
> # Calculate the centered volatility
> lookb <- 7
> halfb <- lookb %/% 2
> stdev <- sqrt(HighFreq::roll_var(retp, lookb))
> stdev <- rutils::lagit(stdev, lagg=(-halfb))
> # Calculate the z-scores of prices
> pricez <- (2*closep -
+   rutils::lagit(closep, halfb, pad_zeros=FALSE) -
+   rutils::lagit(closep, -halfb, pad_zeros=FALSE))
> pricez <- ifelse(stdev > 0, pricez/stdev, 0)
```

```
> # Plot dygraph of z-scores of VTI prices
> pricev <- cbind(closep, pricez)
> colnames(pricev) <- c("VTI", "Z-scores")
> colnamev <- colnames(pricev)
> dygraphs::dygraph(pricev["2009"], main="VTI Price Z-Scores") %>%
+   dyAxis("y", label=colnamev[1], independentTicks=TRUE) %>%
+   dyAxis("y2", label=colnamev[2], independentTicks=TRUE) %>%
+   dySeries(name=colnamev[1], axis="y", strokeWidth=2, col="blue")
+   dySeries(name=colnamev[2], axis="y2", strokeWidth=2, col="red")
```

Labeling the Tops and Bottoms of Prices

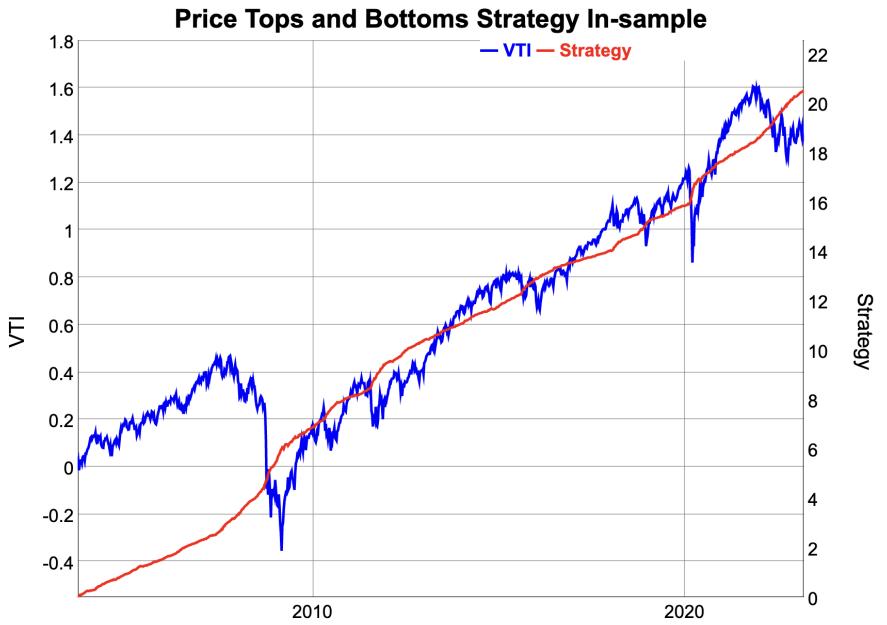
The local tops and bottoms of prices can be labeled approximately in-sample using the z-scores of prices and threshold values.

The local tops of prices represent *overbought* conditions, while the bottoms represent *oversold* conditions.

The labeled data can be used as a response or target variable in machine learning classifier models.

But it's not feasible to classify the prices out-of-sample exactly according to their in-sample labels.

```
> # Calculate the thresholds for labeling tops and bottoms
> confl <- c(0.2, 0.8)
> threshv <- quantile(pricez, confl)
> # Calculate the vectors of tops and bottoms
> topl <- zoo::coredata(pricez > threshv[2])
> bottoml <- zoo::coredata(pricez < threshv[1])
> # Simulate in-sample VTI strategy
> posv <- rep(NA_integer_, nrows)
> posv[1] <- 0
> posv[topl] <- (-1)
> posv[bottoml] <- 1
> posv <- zoo::na.locf(posv)
> posv <- rutils::lagit(posv)
> pnls <- retp*posv
```




```
> # Plot dygraph of in-sample VTI strategy
> wealthv <- cbind(retp, pnls)
> colnames(wealthv) <- c("VTI", "Strategy")
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd],
+   main="Price Tops and Bottoms Strategy In-Sample") %>%
+   dyAxis("y", label="VTI", independentTicks=TRUE) %>%
+   dyAxis("y2", label="Strategy", independentTicks=TRUE) %>%
+   dySeries(name="VTI", axis="y", strokeWidth=2, col="blue") %>%
+   dySeries(name="Strategy", axis="y2", strokeWidth=2, col="red")
```

Predictors of Price Extremes

The return volatility and trading volumes may be used as predictors in a classification model, in order to identify *overbought* and *oversold* conditions.

The trailing *volume z-score* is equal to the volume v_i minus the trailing average volumes $\bar{v}_i$ divided by the volatility of the volumes σ_i:

$$z_i = \frac{v_i - \bar{v}_i}{\sigma_i}$$

Trading volumes are typically higher when prices drop and they are also positively correlated with the return volatility.

The *volatility z-score* is equal to the spot volatility v_i minus the trailing average volatility $\bar{v}_i$ divided by the standard deviation of the volatility σ_i:

$$z_i = \frac{v_i - \bar{v}_i}{\sigma_i}$$

Volatility is typically higher when prices drop and it's also positively correlated with the trading volumes.

```
> # Calculate the volatility z-scores
> volp <- HighFreq::roll_var_ohlc(ohlc=ohlc, lookb=lookb, scale=FALS
> volatm <- HighFreq::roll_mean(volp, lookb)
> volatsd <- sqrt(HighFreq::roll_var(rutils::diffit(volp), lookb))
> volatsd[1] <- 0
> volatz <- ifelse(volatsd > 0, (volp - volatm)/volatsd, 0)
> colnames(volatz) <- "volp"
> # Calculate the volume z-scores
> volumv <- quantmod::Vo(ohlc)
> volumean <- HighFreq::roll_mean(volumv, lookb)
> volumsd <- sqrt(HighFreq::roll_var(rutils::diffit(volumv), lookb))
> volumsd[1] <- 0
> volumz <- ifelse(volumsd > 0, (volumv - volumean)/volumsd, 0)
> colnames(volumz) <- "volume"
```

Regression Z-Scores

The trailing *z-score* z_i of a price p_i can be defined as the *standardized residual* of the linear regression with respect to time t_i or some other variable:

$$z_i = \frac{p_i - (\alpha + \beta t_i)}{\sigma_i}$$

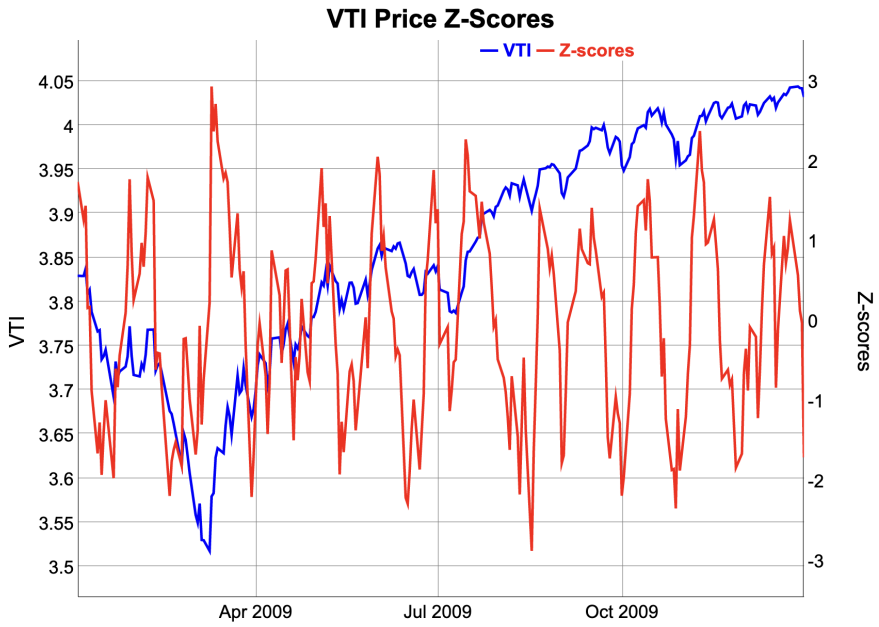
Where α and β are the *regression coefficients*, and σ_i is the standard deviation of the residuals.

The regression *z-scores* can be used as rich or cheap indicators, either relative to past prices, or relative to prices in a stock pair.

The regression residuals must be calculated in a loop, so it's much faster to Calculate the them using functions written in C++ code.

The function `HighFreq::roll_reg()` calculates the residuals of a rolling regression.

```
> # Calculate the trailing price regression z-scores
> datev <- matrix(zoo::index(closep))
> lookb <- 21
> controlv <- HighFreq::param_reg()
> regs <- HighFreq::roll_reg(respv=closep, predm=datev,
+    lookb=lookb, controlv=controlv)
> regs <- drop(regs[, NCOL(regs)])
> regs[1:lookb] <- 0
```




```
> # Plot dygraph of z-scores of VTI prices
> pricev <- cbind(closep, regs)
> colnames(pricev) <- c("VTI", "Z-scores")
> colnamev <- colnames(pricev)
> dygraphs::dygraph(pricev["2009"], main="VTI Price Z-Scores") %>%
+   dyAxis("y", label=colnamev[1], independentTicks=TRUE) %>%
+   dyAxis("y2", label=colnamev[2], independentTicks=TRUE) %>%
+   dySeries(name=colnamev[1], axis="y", strokeWidth=2, col="blue")
+   dySeries(name=colnamev[2], axis="y2", strokeWidth=2, col="red")
```

The *Logistic* Function

The *logistic* function expresses the probability of a numerical variable ranging over the whole interval of real numbers:

$$p(x) = \frac{1}{1 + \exp(-\lambda x)}$$

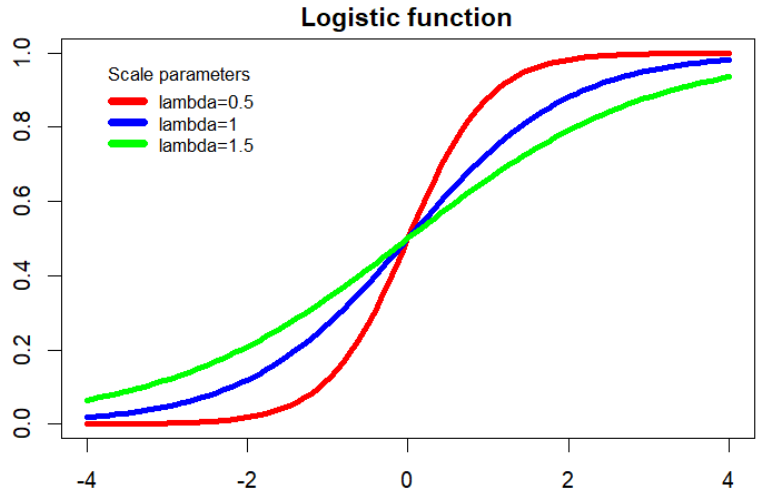
Where λ is the scale (dispersion) parameter.

The *logistic* function is often used as an activation function in neural networks, and logistic regression can be viewed as a perceptron (single neuron network).

The *logistic* function can be inverted to obtain the *Odds Ratio* (the ratio of probabilities for favorable to unfavorable outcomes):

$$\frac{p(x)}{1 - p(x)} = \exp(\lambda x)$$

The function `plogis()` gives the cumulative probability of the *Logistic* distribution,




```
> lambdav <- c(0.5, 1, 1.5)
> colorv <- c("red", "blue", "green")
> # Plot three curves in loop
> for (it in 1:3) {
+   curve(expr=plogis(x, scale=lambdav[it]),
+ xlim=c(-4, 4), type="l", xlab="", ylab="", lwd=4,
+ col=colorv[it], add=(it>1))
+ }  # end for
> # Add title
> title(main="Logistic function", line=0.5)
> # Add legend
> legend("topleft", title="Scale parameters",
+        paste("lambda", lambdav, sep="="), y.intersp=0.4,
+        inset=0.05, cex=0.8, lwd=6, bty="n", lty=1, col=colorv)
```

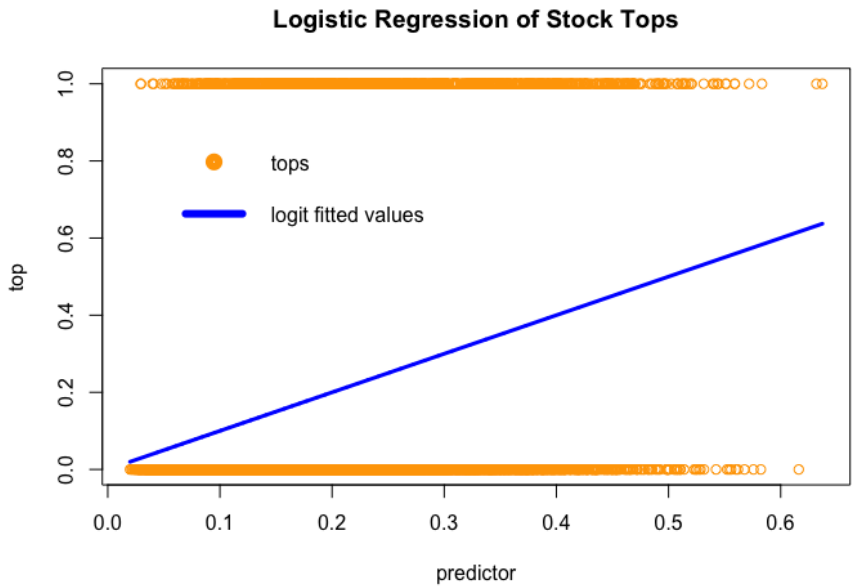
Forecasting Stock Price Tops and Bottoms Using Logistic Regression

Consider a model which uses the weighted average of the volatility, trading volume, and regression z-scores, to forecast a stock top (overbought condition) or a bottom (oversold condition).

The residuals are the differences between the actual response values (0 and 1), and the calculated probabilities of default.

The residuals are not normally distributed, so the data is fitted using the *maximum likelihood* method, instead of least squares.

```
> # Define predictor for tops including intercept column
> predm <- cbind(volatz, volumz, regs)
> predm[1, ] <- 0
> predm <- rutils::lagit(predm)
> # Fit in-sample logistic regression for tops
> logmod <- glm(topl ~ predm, family=binomial(logit))
> summary(logmod)
> coeff <- logmod$coefficients
> fcasts <- drop(cbind(rep(1, nrows), predm) %*% coeff)
> ordern <- order(fcasts)
> # Calculate the in-sample forecasts from logistic regression mode
> fcasts <- 1/(1 + exp(-fcasts))
> all.equal(logmod$fitted.values, fcasts, check.attributes=FALSE)
> hist(fcasts)
```




```
> plot(x=fcasts[ordern], y=topl[ordern],
+      main="Logistic Regression of Stock Tops",
+      col="orange", xlab="predictor", ylab="top")
> lines(x=fcasts[ordern], y=logmod$fitted.values[ordern], col="blue"
> legend(x=0.1, y=1.2, inset=0.0, bty="n", lwd=6,
+  legend=c("tops", "logit fitted values"), y.intersp=0.3,
+  col=c("orange", "blue"), lty=c(NA, 1), pch=c(1, NA))
```

Forecasting Errors of Stock Tops and Bottoms

A *binary classification model* categorizes cases based on its forecasts whether the *null hypothesis* is `TRUE` or `FALSE`.

Let the *null hypothesis* be that the data point is not a top: `tops = FALSE`.

A *positive* result corresponds to rejecting the null hypothesis (`tops = TRUE`), while a *negative* result corresponds to accepting the null hypothesis (`tops = FALSE`).

The forecasts are subject to two different types of errors: *type I* and *type II* errors.

A *type I* error is the incorrect rejection of a `TRUE` *null hypothesis* (i.e. a "false positive"), when `tops = FALSE` but it's classified as `tops = TRUE`.

A *type II* error is the incorrect acceptance of a `FALSE` *null hypothesis* (i.e. a "false negative"), when `tops = TRUE` but it's classified as `tops = FALSE`.

```
> # Define discrimination threshold value
> threshv <- quantile(fcasts, confl[2])
> # Calculate the confusion matrix in-sample
> confmat <- table(actual=!topl, forecast=(fcasts < threshv))
> confmat
> # Calculate the FALSE positive (type I error)
> sum(topl & (fcasts < threshv))
> # Calculate the FALSE negative (type II error)
> sum(!topl & (fcasts > threshv))
```

The Confusion Matrix of a Binary Classification Model

The confusion matrix summarizes the performance of a classification model on a set of test data for which the actual values of the *null hypothesis* are known.

		Forecast: Null is FALSE	Forecast: Null is TRUE
Actual	Null is FALSE	True Positive (sensitivity)	False Negative (type II error)
	Null is TRUE	False Positive (type I error)	True Negative (specificity)

```
> # Calculate the FALSE positive and FALSE negative rates
> confmat <- confmat / rowSums(confmat)
> c(typeI=confmat[2, 1], typeII=confmat[1, 2])
```

Let the *null hypothesis* be that the data point is not a top: `top1 = FALSE`.

The *true positive* rate (known as the *sensitivity*) is the fraction of `FALSE` *null hypothesis* cases that are correctly classified as `FALSE`.

The *false negative* rate is the fraction of `FALSE` *null hypothesis* cases that are incorrectly classified as `TRUE` (*type II* error).

The sum of the *true positive* plus the *false negative* rate is equal to 1.

The *true negative* rate (known as the *specificity*) is the fraction of `TRUE` *null hypothesis* cases that are correctly classified as `TRUE`.

The *false positive* rate is the fraction of `TRUE` *null hypothesis* cases that are incorrectly classified as `FALSE` (*type I* error).

The sum of the *true negative* plus the *false positive* rate is equal to 1.

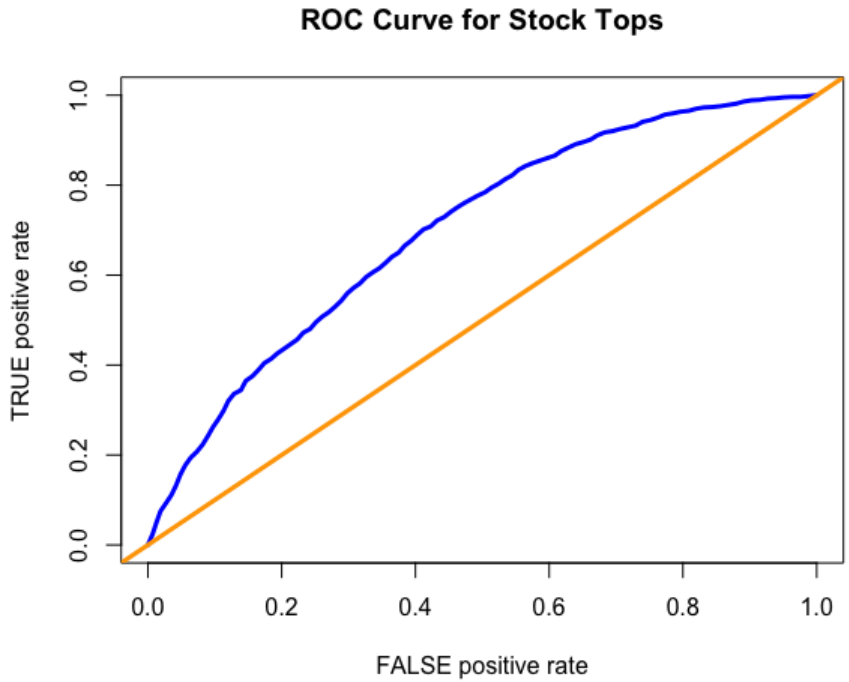
Receiver Operating Characteristic (ROC) Curve for Stock Tops

The *ROC curve* is the plot of the *true positive* rate, as a function of the *false positive* rate, and illustrates the performance of a binary classifier.

The area under the *ROC curve* (AUC) measures the classification ability of a binary classifier.

The *informedness* is equal to the sum of the sensitivity plus the specificity, and measures the performance of a binary classification model.

```
> # Confusion matrix as function of threshold
> confun <- function(actual, fcasts, threshv) {
+   forb <- (fcasts < threshv)
+   conf <- matrix(c(sum(!actual & !forb), sum(actual & !forb),
+              sum(!actual & forb), sum(actual & forb)), ncol=2)
+   conf <- conf / rowSums(conf)
+   c(typeI=conf[2, 1], typeII=conf[1, 2])
+ }  # end confun
> confun(!topl, fcasts, threshv=threshv)
> # Define vector of discrimination thresholds
> threshv <- quantile(fcasts, seq(0.01, 0.99, by=0.01))
> # Calculate the error rates
> errorr <- sapply(threshv, confun,
+   actual=!topl, fcasts=fcasts)  # end sapply
> errorr <- t(errorr)
> rownames(errorr) <- threshv
> # Calculate the informedness
> informv <- 2 - rowSums(errorr)
> plot(threshv, informv, t="l", main="Informedness")
> # Find the threshold corresponding to highest informedness
> threshm <- threshv[which.max(informv)]
> topf <- (fcasts > threshm)
```




```
> # Calculate the area under ROC curve (AUC)
> errorr <- rbind(c(1, 0), errorr)
> errorr <- rbind(errorr, c(0, 1))
> truepos <- (1 - errorr[, "typeII"])
> truepos <- (truepos + rutils::lagit(truepos))/2
> falsepos <- rutils::diffit(errorr[, "typeI"])
> abs(sum(truepos*falsepos))
> # Plot ROC Curve for stock tops
> plot(x=errorr[, "typeI"], y=1-errorr[, "typeII"],
+      xlab="FALSE positive rate", ylab="TRUE positive rate",
+      main="ROC Curve for Stock Tops", type="l", lwd=3, col="blue")
> abline(a=0.0, b=1.0, lwd=3, col="orange")
```

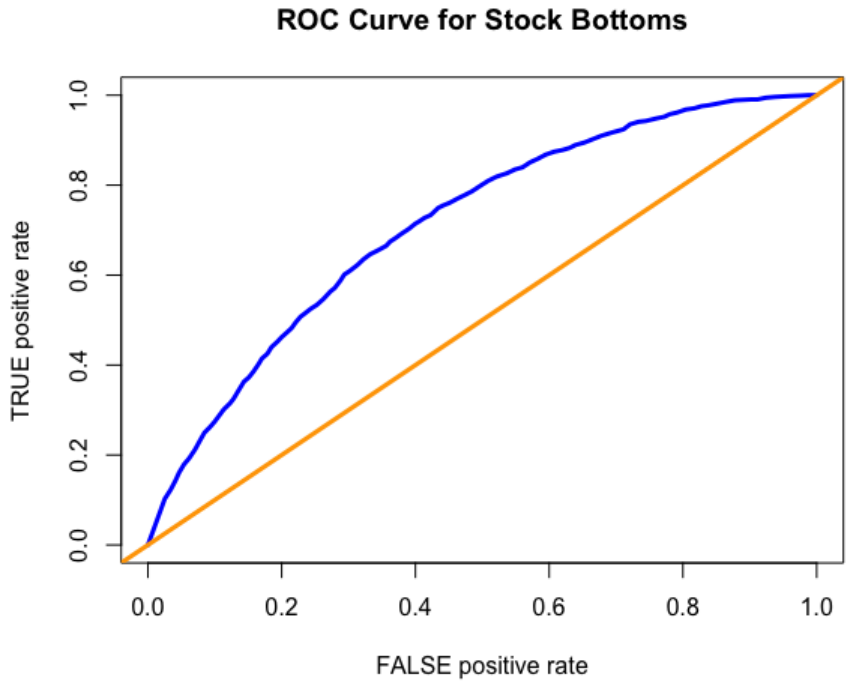
Receiver Operating Characteristic (ROC) Curve for Stock Bottoms

The *ROC curve* is the plot of the *true positive* rate, as a function of the *false positive* rate, and illustrates the performance of a binary classifier.

The area under the *ROC curve* (AUC) measures the classification ability of a binary classifier.

The *informedness* is equal to the sum of the sensitivity plus the specificity, and measures the performance of a binary classification model.

```
> # Fit in-sample logistic regression for bottoms
> logmod <- glm(bottoml ~ predm, family=binomial(logit))
> summary(logmod)
> # Calculate the in-sample forecast from logistic regression model
> coeff <- logmod$coefficients
> fcasts <- drop(cbind(rep(1, nrows), predm) %*% coeff)
> fcasts <- 1/(1 + exp(-fcasts))
> # Calculate the error rates
> errorr <- sapply(threshv, confun,
+   actual=!bottoml, fcasts=fcasts)  # end sapply
> errorr <- t(errorr)
> rownames(errorr) <- threshv
> # Calculate the informedness
> informv <- 2 - rowSums(errorr)
> plot(threshv, informv, t="l", main="Informedness")
> # Find the threshold corresponding to highest informedness
> threshm <- threshv[which.max(informv)]
> botf <- (fcasts > threshm)
```




```
> # Calculate the area under ROC curve (AUC)
> errorr <- rbind(c(1, 0), errorr)
> errorr <- rbind(errorr, c(0, 1))
> truepos <- (1 - errorr[, "typeII"])
> truepos <- (truepos + rutils::lagit(truepos))/2
> falsepos <- rutils::diffit(errorr[, "typeI"])
> abs(sum(truepos*falsepos))
> # Plot ROC Curve for stock tops
> plot(x=errorr[, "typeI"], y=1-errorr[, "typeII"],
+      xlab="FALSE positive rate", ylab="TRUE positive rate",
+      main="ROC Curve for Stock Bottoms", type="l", lwd=3, col="blu
> abline(a=0.0, b=1.0, lwd=3, col="orange")
```

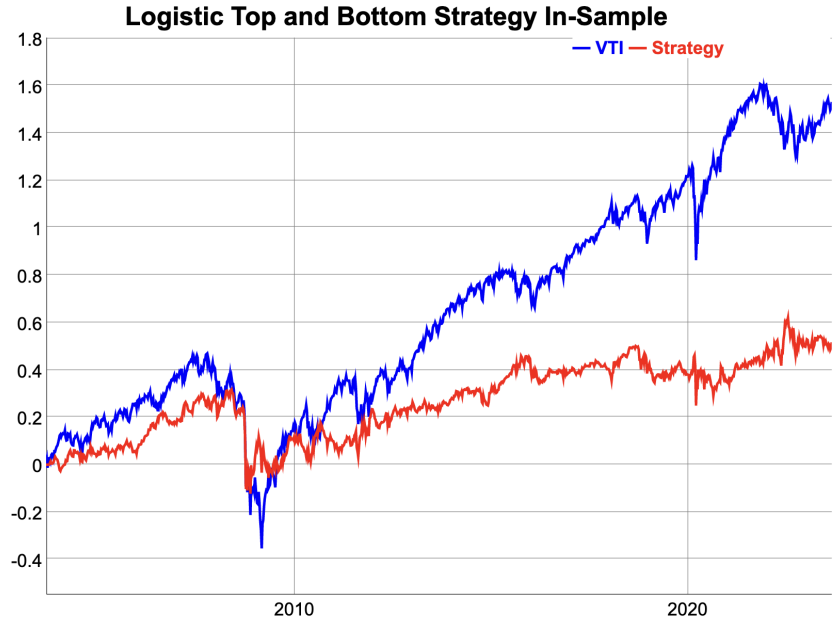
Logistic Tops and Bottoms Strategy In-sample

The logistic strategy forecasts the tops and bottoms of prices, using a logistic regression model with the volatility and trading volumes as predictors.

Averaging the forecasts over time improves strategy performance because of the bias-variance tradeoff.

It makes sense to average the forecasts over time because they are forecasts for future time intervals, not just a single point in time.

```
> # Average the signals over time
> topsav <- HighFreq::roll_sum(matrix(topf), 5)/5
> botsav <- HighFreq::roll_sum(matrix(botf), 5)/5
> # Simulate in-sample VTI strategy
> posv <- (botsav - topsav)
> # Standard strategy
> # posv <- rep(NA_integer_, NROW(retp))
> # posv[1] <- 0
> # posv[topf] <- (-1)
> # posv[botf] <- 1
> # posv <- zoo::na.locf(posv)
> posv <- rutils::lagit(posv)
> pnls <- retp*posv
```

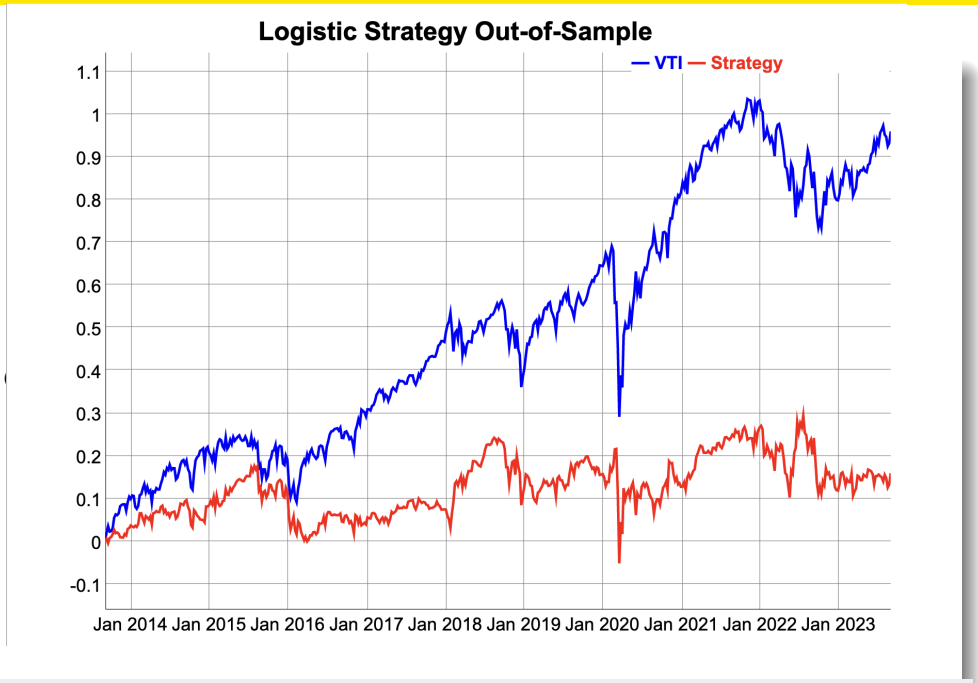



```
> # Calculate the Sharpe and Sortino ratios
> wealthv <- cbind(retp, pnls)
> colnames(wealthv) <- c("VTI", "Strategy")
> sqrt(252)*sapply(wealthv, function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # Plot dygraph of in-sample VTI strategy
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd],
+   main="Logistic Top and Bottom Strategy In-Sample") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

Logistic Tops and Bottoms Strategy Out-of-Sample

The logistic strategy forecasts the tops and bottoms of prices, using a logistic regression model with the volatility and trading volumes as predictors.

```
> # Define in-sample and out-of-sample intervals
> insample <- 1:(nrows %/% 2)
> outsample <- (nrows %/% 2 + 1):nrows
> # Fit in-sample logistic regression for tops
> logmod <- glm(topl[insample] ~ predm[insample, ], family=binomial(
> fitv <- logmod$fitted.values
> coefftop <- logmod$coefficients
> # Calculate the error rates and best threshold value
> errorr <- sapply(threshv, confun,
+   actual=!topl[insample], fcasts=fitv)  # end sapply
> errorr <- t(errorr)
> informv <- 2 - rowSums(errorr)
> threshtop <- threshv[which.max(informv)]
> # Fit in-sample logistic regression for bottoms
> logmod <- glm(bottoml[insample] ~ predm[insample, ], family=binor
> fitv <- logmod$fitted.values
> coeffbot <- logmod$coefficients
> # Calculate the error rates and best threshold value
> errorr <- sapply(threshv, confun,
+   actual=!bottoml[insample], fcasts=fitv)  # end sapply
> errorr <- t(errorr)
> informv <- 2 - rowSums(errorr)
> threshbot <- threshv[which.max(informv)]
> # Calculate the out-of-sample forecasts from logistic regression
> predictos <- cbind(rep(1, NROW(outsample)), predm[outsample, ])
> fcasts <- drop(predictos %*% coefftop)
> fcasts <- 1/(1 + exp(-fcasts))
> topf <- (fcasts > threshtop)
> fcasts <- drop(predictos %*% coeffbot)
> fcasts <- 1/(1 + exp(-fcasts))
> botf <- (fcasts > threshbot)
```




```
> # Simulate in-sample VTI strategy
> topsav <- HighFreq::roll_sum(matrix(topf), 5)/5
> botsav <- HighFreq::roll_sum(matrix(botf), 5)/5
> posv <- (botsav - topsav)
> posv <- rutils::lagit(posv)
> pnls <- retp[outsample, ]*posv
> # Calculate the Sharpe and Sortino ratios
> wealthv <- cbind(retp[outsample, ], pnls)
> colnames(wealthv) <- c("VTI", "Strategy")
> sqrt(252)*sapply(wealthv, function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # Plot dygraph of in-sample VTI strategy
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd],
+   main="Logistic Strategy Out-of-Sample") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

draft: Forecasting Stock Tops and Bottoms Out-of-Sample

The function `predict()` is a *generic function* for forecasting based on a given model.

The method `predict.glm()` produces forecasts for a generalized linear (*glm*) model, in the form of `numeric` probabilities, not the `Boolean` response variable.

The `Boolean` forecasts are obtained by comparing the *forecast probabilities* with a *discrimination threshold*.

Let the *null hypothesis* be that the data point is not a top: `tops = FALSE`.

If the *forecast probability* is greater than the *discrimination threshold*, then the forecast is that the data point is not a top and that the *null hypothesis* is `TRUE`.

The *in-sample forecasts* are just the *fitted values* of the *glm* model.

```
> # Fit logistic regression over training data
> # Initialize the random number generator
> set.seed(1121, "Mersenne-Twister", sample.kind="Rejection")
> nrows <- NROW(Default)
> samplev <- sample.int(n=nrows, size=nrows/2)
> trainset <- Default[samplev, ]
> logmod <- glm(formulav, data=trainset, family=binomial(logit))
> # Forecast over test data out-of-sample
> testset <- Default[-samplev, ]
> fcasts <- predict(logmod, newdata=testset, type="response")
> # Calculate the confusion matrix out-of-sample
> table(actual=!testset$default,
+ forecast=(fcasts < threshv))
```

draft: Forecasting Returns Using Logistic Regression

The weighted average of the volatility, trading volume, and regression z-scores can be used to forecast the sign of future returns.

The residuals are the differences between the actual response values (0 and 1), and the calculated probabilities of default.

The residuals are not normally distributed, so the data is fitted using the *maximum likelihood* method, instead of least squares.

```
> # Define response as the multi-day returns
> lagg <- 5
> retsf <- rutils::diffit(closep, lagg=5)
> retsf <- drop(coredata(retsf))
> # Fit in-sample logistic regression for positive returns
> retos <- (retsf > 0)
> logmod <- glm(retspos ~ predm - 1, family=binomial(logit))
> summary(logmod)
> coeff <- logmod$coefficients
> fcasts <- predm %*% coeff
> fcasts <- 1/(1 + exp(-fcasts))
> # Calculate the error rates
> threshv <- quantile(fcasts, seq(0.01, 0.99, by=0.01))
> errorr <- sapply(threshv, confun,
+   actual=!retspos, fcasts=fcasts)  # end sapply
> errorr <- t(errorr)
> # Calculate the threshold corresponding to highest informedness
> informv <- 2 - rowSums(errorr)
> plot(threshv, informv, t="l", main="Informedness")
> threshm <- threshv[which.max(informv)]
> forecastpos <- (fcasts > threshm)
> # Fit in-sample logistic regression for negative returns
> retsneg <- (retsf < 0)
> logmod <- glm(retsneg ~ predm - 1, family=binomial(logit))
> summary(logmod)
> coeff <- logmod$coefficients
> fcasts <- predm %*% coeff
> fcasts <- 1/(1 + exp(-fcasts))
> # Calculate the error rates
> errorr <- sapply(threshv, confun,
+   actual=!retsneg, fcasts=fcasts)  # end sapply
> errorr <- t(errorr)
> # Calculate the threshold corresponding to highest informedness
> informv <- 2 - rowSums(errorr)
> plot(threshv, informv, t="l", main="Informedness")
> threshm <- threshv[which.max(informv)]
> forecastneg <- (fcasts > threshm)
```

draft: Logistic Forecasting Returns Strategy In-sample

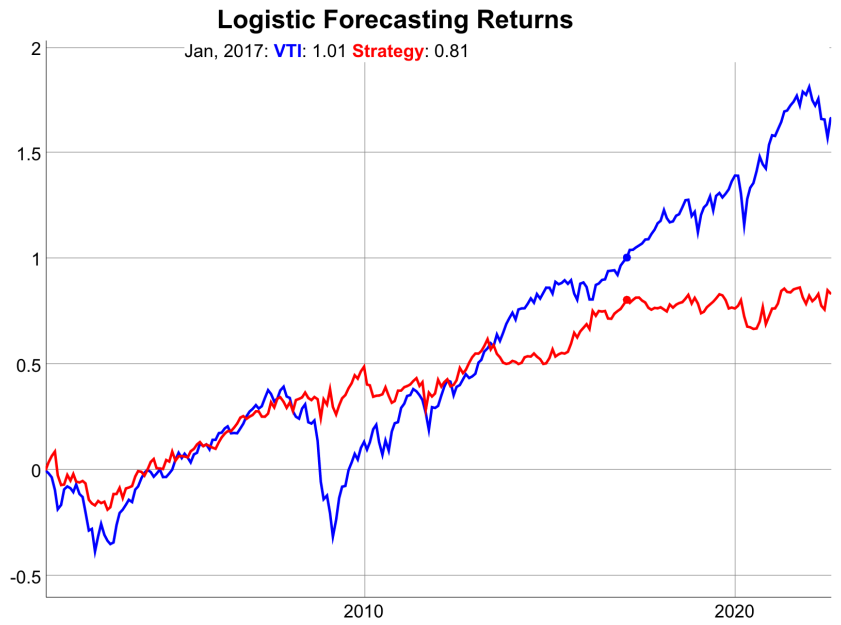
Explain why the strategy uses the minus of the signals.

The logistic strategy forecasts the sign of returns, using a logistic regression model with the volatility and trading volumes as predictors.

Averaging the forecasts over time improves strategy performance because of the bias-variance tradeoff.

It makes sense to average the forecasts over time because they are forecasts for future time intervals, not just a single point in time.

```
> # Simulate in-sample VTI strategy
> negav <- HighFreq::roll_sum(matrix(forecastneg), lagg)/lagg
> posav <- HighFreq::roll_sum(matrix(forecastpos), lagg)/lagg
> posv <- (negav - posav)
> # posv <- ifelse(forecastpos, 1, 0)
> # posv <- ifelse(forecastneg, -1, posv)
> posv <- rutils::lagit(posv)
> pnls <- retp*posv
> # Calculate the Sharpe and Sortino ratios
> wealthv <- cbind(retp, pnls)
> colnames(wealthv) <- c("VTI", "Strategy")
> sqrt(252)*sapply(wealthv, function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
```

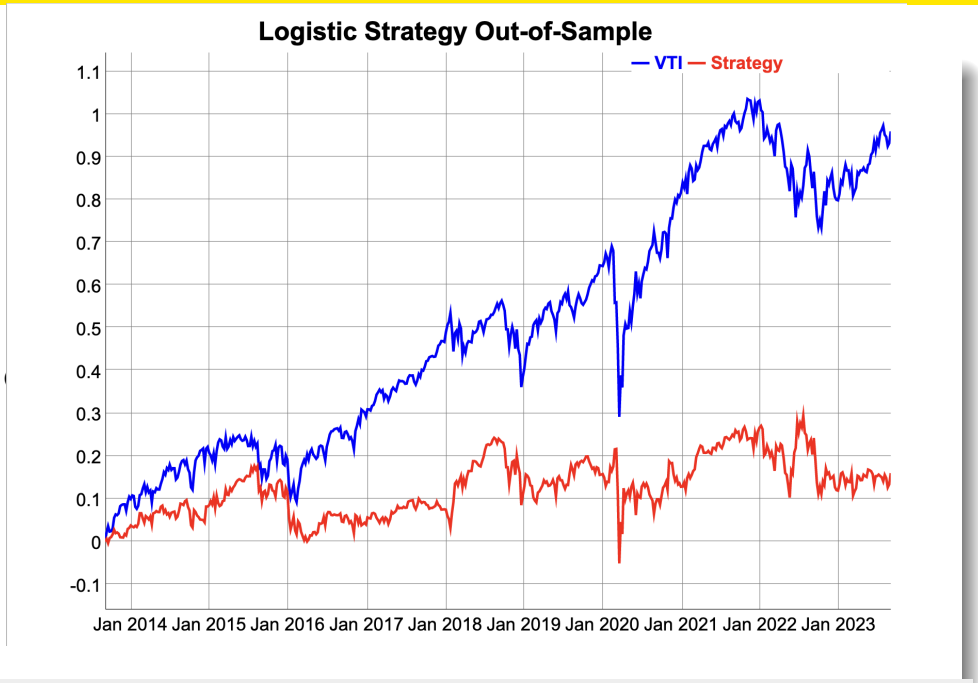



```
> # Plot dygraph of in-sample VTI strategy
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd],
+   main="Logistic Forecasting Returns") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
+   dyLegend(show="always", width=300)
```

draft: Logistic Strategy Out-of-Sample

The logistic strategy forecasts the tops and bottoms of prices, using a logistic regression model with the volatility and trading volumes as predictors.

```
> # Define in-sample and out-of-sample intervals
> insample <- 1:(nrows %/% 2)
> outsample <- (nrows %/% 2 + 1):nrows
> # Fit in-sample logistic regression for tops
> logmod <- glm(topl[insample] ~ predm[insample, ], family=binomial(
> fitv <- logmod$fitted.values
> coefftop <- logmod$coefficients
> # Calculate the error rates and best threshold value
> errorr <- sapply(threshv, confun,
+   actual=!topl[insample], fcasts=fitv)  # end sapply
> errorr <- t(errorr)
> informv <- 2 - rowSums(errorr)
> threshtop <- threshv[which.max(informv)]
> # Fit in-sample logistic regression for bottoms
> logmod <- glm(bottoml[insample] ~ predm[insample, ], family=binor
> fitv <- logmod$fitted.values
> coeffbot <- logmod$coefficients
> # Calculate the error rates and best threshold value
> errorr <- sapply(threshv, confun,
+   actual=!bottoml[insample], fcasts=fitv)  # end sapply
> errorr <- t(errorr)
> informv <- 2 - rowSums(errorr)
> threshbot <- threshv[which.max(informv)]
> # Calculate the out-of-sample forecasts from logistic regression
> predictos <- cbind(rep(1, NROW(outsample)), predm[outsample, ])
> fcasts <- drop(predictos %*% coefftop)
> fcasts <- 1/(1 + exp(-fcasts))
> topf <- (fcasts > threshtop)
> fcasts <- drop(predictos %*% coeffbot)
> fcasts <- 1/(1 + exp(-fcasts))
> botf <- (fcasts > threshbot)
```




```
> # Simulate out-of-sample VTI strategy
> posv <- rep(NA_integer_, NROW(outsample))
> posv[1] <- 0
> posv[topf] <- (-1)
> posv[botf] <- 1
> posv <- zoo::na.locf(posv)
> posv <- rutils::lagit(posv)
> pnls <- retp[outsample, ]*posv
> # Calculate the Sharpe and Sortino ratios
> wealthv <- cbind(retp[outsample, ], pnls)
> colnames(wealthv) <- c("VTI", "Strategy")
> sqrt(252)*sapply(wealthv, function(x)
+   c(Sharpe=mean(x)/sd(x), Sortino=mean(x)/sd(x[x<0])))
> # Plot dygraph of in-sample VTI strategy
> endd <- rutils::calc_endpoints(wealthv, interval="weeks")
> dygraphs::dygraph(cumsum(wealthv)[endd],
+   main="Logistic Strategy Out-of-Sample") %>%
+   dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
```