Graphics and Plotting FRE6871 & FRE7241, Spring 2023

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Plotting in R

Functions in package *graphics* for creating static plots:

plot() plots a line or scatter plot,
lines() adds lines to a plot,
curve() plots a function given by its name,
title() adds a title to a plot,
legend() adds a legend to a plot,
x11() opens a Windows graphics device,
par() sets graphical plot parameters,

Functions in package *ggplot2* for creating static plots:

ggplot() creates a new ggplot object,
aes() specifies aesthetics, i.e. mappings between
data elements (rows, columns) and plot elements
(points, lines),
geom_point() adds plot points,
geom_line() adds plot lines,
theme() customizes plot objects.

Functions in package *plotly* for creating dynamic plots:

plot_ly() creates a new plotly object,
add_trace() adds elements to a plotly plot,
layout() modifies the layout of a plotly plot,

Functions in package *dygraphs* for creating dynamic time series plots:

dygraph() creates an interactive time series plot, dyOptions() adds options (like colors, etc.) to a dygraph plot,

dyRangeSelector() adds a date range selector to the bottom of a dygraphs plot,

Package graphics Help and Documentation

General

CRAN Graphics task view: http://cran.r-project.org/web/views/Graphics.html

Tutorials on Charts and Plotting

```
http://www.statmethods.net/graphs/index.html
```

http://www.statmethods.net/advgraphs/index.html

http://en.wikibooks.org/wiki/R_Programming/Graphics#Standard_R_graphs

http://ww2.coastal.edu/kingw/statistics/R-tutorials/graphs.html

http://www.harding.edu/fmccown/r/

Graphical Parameters

```
http://www.statmethods.net/advgraphs/parameters.html
```

http://research.stowers-institute.org/efg/R/Graphics/Basics/mar-oma/index.htm

http://www.programmingr.com/content/controlling-margins-and-axes-oma-and-mgp/

Galleries of Charts

Vistat reproducible gallery of statistical graphics: http://vis.supstat.com/

Gallery of data charts: http://zoonek2.free.fr/UNIX/48_R/03.html

Gallery of data charts and R code: http://rgm3.lab.nig.ac.jp/RGM/R_image_list

Plotting Scatter Plots With Labels

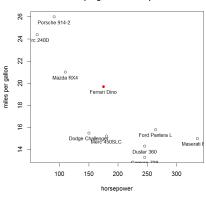
The package graphics is one of the base packages in R. and offers a number of plotting capabilities.

The function plot() by default plots a scatter plot, but can also plot lines using the argument type="1".

The function text() draws text on a plot, and can be used to draw plot labels.

```
> car_s <- mtcars[sample(NROW(mtcars), 10), ]
> # Plot scatter plot horsepower vs miles per gallon
> plot(car_s[, "hp"], car_s[, "mpg"],
      xlab="horsepower", ylab="miles per gallon",
      main="miles per gallon vs horsepower")
> # Add a solid red point (pch=16) for the last car
> points(x=car_s[NROW(car_s), "hp"],
  v=car_s[NROW(car_s), "mpg"],
  col="red", pch=16)
> # Add labels with the car names
> text(x=car_s[, "hp"], y=car_s[, "mpg"],
      labels=rownames(car s[, ]).
      pos=1, cex=0.8)
> # Labels using wordcloud, to prevent overlaps
> library(wordcloud)
> textplot(x=car_s[, "hp"], y=car_s[, "mpg"],
     words=rownames(car s))
```

miles per gallon vs horsepower



Rule of Thumb

Always plot in a large x11() window, not in the default RStudio plot panel.

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First open a large x11() window, then run your plot code.

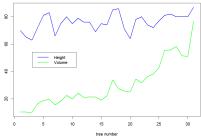
Plotting Line Plots Using the Base graphics Package

The function plot() accepts many different graphical parameters, including:

- type: type of plot,
- lwd: line width.
- col: plotting object color.
- xlab, ylab: axis titles,
- xlim, ylim: axis range,
- main: plot title.

```
> plot(trees[, "Height"],
      type="1",
      lwd=2.
      col="blue".
      main="Tree heights and volumes".
      xlab="tree number", vlab="",
      vlim=c(min(trees[, c("Height", "Volume")]),
       max(trees[, c("Height", "Volume")])))
> # Plot the tree Volume
> lines(trees[, "Volume"], lwd=2, col="green")
> # Add legend
> legend(x="left", legend=c("Height", "Volume"),
```

+ inset=0.1, cex=1.0, bg="white", bty="n", y.intersp=0.4,



Tree heights and volumes

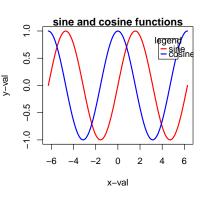
The function lines() adds lines to a plot.

The function legend() adds a legend to a plot.

+ lwd=2, lty=1, col=c("blue", "green"))

Plotting Mathematical Functions

Plotting functions in package graphics: x11() opens a Windows graphics device. plot() creates a scatter plot or line plot. lines() plots a line on an existing plot. title() adds a title to a plot.

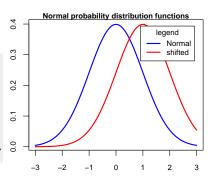


> graphics.off() # Close all graphics devices

Plotting Mathematical Functions Using curve()

 $\ensuremath{\mathtt{R}}$ has dedicated functions for plotting mathematical functions.

The function curve() plots a function defined by its name.



Plotting zoo Time Series With Custom x-axis

```
> library(zoo) # Load zoo
> load(file="/Users/jerzy/Develop/lecture_slides/data/zoo_data.RData
> zoo_series <- window(zoo_stx[, "AdjClose"],
     start=as.Date("2013-01-01"),
     end=as.Date("2013-12-31"))
> # extract time index and monthly dates
> dates <- zoo::index(zoo series)
> # Coerce index to monthly dates
> month lv <- as.vearmon(dates)
> # tick locations at beginning of month
> tick_s <- dates[match(unique(month_ly), month_ly)]
> # tick_s <- as.Date(tapply(X=dates, INDEX=month_ly, FUN=min))
> # first plot zoo without "x" axis
> plot(zoo series, xaxt="n", xlab=NA, vlab=NA, main="MSFT stock price
> # Add "x" axis with monthly ticks
> axis(side=1, at=tick s, labels=format(tick s, "%b-%v"), tcl=-0.7)
> # Add vertical lines
> abline(v=tick s, col="grev", lwd=0.5)
> # Plot zoo using base plotting functions
> plot(as.vector(zoo_series), xaxt="n",
+ xlab=NA, ylab=NA, t="1", main="MSFT stock prices")
> a t <- match(tick s. dates)
> # a t <- seg along(dates) [dates %in% tick s]
> # Add "x" axis with monthly ticks
> axis(side=1, at=a t, labels=format(tick s, "%b-%v"), tcl=-0.7)
```



> library(microbenchmark)
> summary(microbenchmark()
+ m_atch=dates[match(unique(month_ly), month_ly)],
+ t_apply=as.Date(tapply(X=dates,
+ INDEX=month_ly, FUN=min)),
+ times=10

)[, c(1, 4, 5)]

> abline(v=a t, col="grev", lwd=0.5)

Plotting Two Time Series With Two "v" Axes

The function par() sets graphical parameters used for plotting, and invisibly returns existing parameters as a named list.

The function axis() plots an axis on the current plot.

The function lines() plots a line on the current plot.

```
> par(las=1) # Set text printing to horizontal
> ## Plot with two y-axes - plot first time series
> zoo::plot.zoo(zoo_stxeur[, 1], lwd=2, xlab=NA, ylab=NA)
> par(new=TRUE) # Allow new plot on same chart
```

- > # Plot second time series without y-axis
- > zoo::plot.zoo(zoo_stxeur[, 2], xlab=NA, ylab=NA, lwd=2, yaxt="n", col="red")
- > # Plot second y-axis on right
- > axis(side=4, col="red")
- > # Add axis labels
- > colnamev <- colnames(zoo_stxeur)
- > mtext(colnamev[1], side=2, adj=-0.5)
- > mtext(colnamev[2], side=4, adj=1.5, col="red")
- > # Add title and legend
- > title(main=paste0(colnamev, collapse=" and "),
- + line=0.5
- > legend("top", legend=colnamev,
- bg="white", ltv=1, lwd=6, v.intersp=0.4,
- col=c("black", "red"), btv="n")

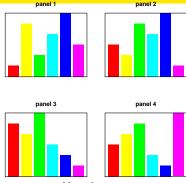


- > par(las=1) # Set text printing to horizontal
- > zoo::plot.zoo(zoo_stxeur[, 1], xlab=NA, ylab=NA, lwd=2)
- > # Set range of "y" coordinates for second axis
- > par(usr=c(par("usr")[1:2], range(zoo stxeur[,2]))) > lines(zoo stxeur[, 2], col="red", lwd=2) # Second plot > axis(side=4, col="red") # Second y-axis on right
- > # Add axis labels
- > mtext(colnamev[1], side=2, adj=-0.5) > mtext(colnamev[2], side=4, adj=1.5, col="red")
- > # Add title and legend
- > title(main=pasteO(colnamev, collapse=" and "),
- + line=0.5)
- > legend("top", legend=colnamev,
- bg="white", lty=1, lwd=6, y.intersp=0.4, col=c("black", "red"), bty="n")

Graphical Parameters

The function par() sets graphical parameters used for plotting, and invisibly returns existing parameters as a named list.

```
> graph params <- par() # get existing parameters
> par("mar") # get plot margins
> par(mar=c(2, 1, 2, 1)) # Set plot margins
> par(oma=c(1, 1, 1, 1)) # Set outer margins
> par(mgp=c(2, 1, 0)) # Set title and label margins
> par(cex.lab=0.8, # Set font scales
     cex.axis=0.8, cex.main=0.8, cex.sub=0.5)
> par(las=1) # Set axis labels to horizontal
> par(ask=TRUE) # Pause, ask before plotting
> par(mfrow=c(2, 2)) # Plot on 2x2 grid by rows
> for (i in 1:4) { # Plot 4 panels
   barplot(sample(1:6), main=paste("panel", i),
     col=rainbow(6), border=NA, axes=FALSE)
   box()
+ } # end for
> par(ask=FALSE) # Restore automatic plotting
> par(new=TRUE) # Allow new plot on same chart
> par(graph_params) # Restore original parameters
```



- cex set graphic scales,
- mar & oma set plot margins,
- mgp set title and label margins,
- las set orientation of axis labels,
- mfcol & mfrow set number of plot panels,

Normal (Gaussian) Probability Distribution

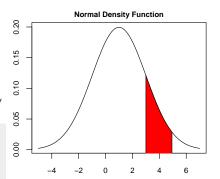
The Normal (Gaussian) probability density function is given by:

$$\phi(x,\mu,\sigma) = \frac{e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$$

The Standard Normal distribution $\phi(0,1)$ is a special case of the *Normal* $\phi(\mu, \sigma)$ with $\mu = 0$ and $\sigma = 1$.

The function dnorm() calculates the Normal probability density.

```
> xvar <- seq(-5, 7, length=100)
> yvar <- dnorm(xvar, mean=1.0, sd=2.0)
> plot(xvar, yvar, type="l", lty="solid", xlab="", ylab="")
> title(main="Normal Density Function", line=0.5)
> startp <- 3; endd <- 5 # Set lower and upper bounds
> # Set polygon base
> subv <- ((xvar >= startp) & (xvar <= endd))
> polygon(c(startp, xvar[subv], endd), # Draw polygon
 c(-1, yvar[subv], -1), col="red")
```

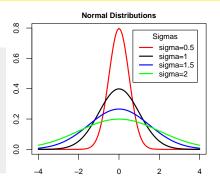


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Normal (Gaussian) Probability Distributions

Plots of several *Normal* distributions with different values of σ , using the function curve() for plotting functions given by their name.

```
> sigmavs <- (c0.5, 1, 1.5, 2) # Sigma values
> # Create plot colors
> colorv <- c("red", "black", "blue", "green")
> # Create legend labels
> labelv <- paste("sigma", sigmavs, sep="=")
> for (indeks in 1:4) { # Plot four curves
+ curve(exprednorm(x, sd=sigmavs[indeks]),
+ xlime(-4, 4), xlab="", ylab="", lwd=2,
+ col=colorv[indeks], add=as.logical(indeks-1))
+ } # end for
> # Add title
> title(main="Normal Distributions", line=0.5)
> # Add legend
> legend("topright", inset=0.05, title="Sigmas", y.intersp=0.4,
+ labelv, cex=0.8, lwd=2, lty=1, bty="n", col=colory)
```

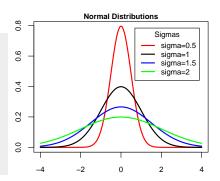


Normal Probability Distributions Plotted as Lines

Plots of several Normal distributions with different values of $\sigma.$

```
> xvar <- seq(-4, 4, length=100)
> sigmavs <- c(0.5, 1, 1.5, 2) # Sigma values
> # Create plot colors
> colorv <- c("red", "black", "blue", "green")
> # Create legend labels
> labelv <- paste("sigma", sigmavs, sep="=")
> # Plot the first chart
> plot(xvar, dnorm(xvar, sd=sigmavs[1]),
      type="n", xlab="", ylab="", main="Normal Distributions")
> # Add lines to plot
> for (indeks in 1:4) {
 lines(xvar, dnorm(xvar, sd=sigmavs[indeks]),
 lwd=2, col=colorv[indeks])
+ } # end for
> # Add legend
> legend("topright", inset=0.05, title="Sigmas", y.intersp=0.4,
```

+ labely, cex=0.8, lwd=2, ltv=1, btv="n", col=colory)



The Log-normal Probability Distribution

If x follows the *Normal* distribution $\phi(x, \mu, \sigma)$, then the exponential of x: $y = e^x$ follows the Log-normal distribution $\log \phi()$:

$$\log \phi(y, \mu, \sigma) = \frac{\exp(-(\log y - \mu)^2/2\sigma^2)}{y\sigma\sqrt{2\pi}}$$

With mean equal to: $\bar{y} = \mathbb{E}[y] = \mathrm{e}^{(\mu + \sigma^2/2)}$, and median equal to: $\tilde{v} = e^{\mu}$

With variance equal to: $\sigma_v^2 = (e^{\sigma^2} - 1)e^{(2\mu + \sigma^2)}$, and skewness (third moment) equal to:

$$\varsigma = \mathbb{E}[(y - \mathbb{E}[y])^3] = (e^{\sigma^2} + 2)\sqrt{e^{\sigma^2} - 1}$$

```
> # Standard deviations of log-normal distribution
> sigmays <- c(0.5, 1, 1.5)
> # Create plot colors
```

> colory <- c("black", "red", "blue")

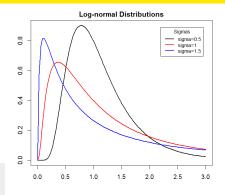
Plot all curves

> for (indeks in 1:NROW(sigmays)) {

curve(expr=dlnorm(x, sdlog=sigmavs[indeks]), type="1", xlim=c(0, 3), lwd=2,

xlab="", vlab="", col=colorv[indeks], add=as.logical(indeks-1))

end for



- > # Add title and legend
- > title(main="Log-normal Distributions", line=0.5)
- > legend("topright", inset=0.05, title="Sigmas",
- + paste("sigma", sigmavs, sep="="), v.intersp=0.4.
- + cex=0.8, lwd=2, ltv=rep(1, NROW(sigmays)), col=colory)

Chi-squared Distribution

Let z_1, \ldots, z_k be independent standard *Normal* random variables.

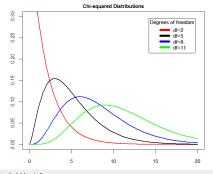
Then the random variable $X = \sum_{i=1}^k z_i^2$ is distributed according to the *Chi-squared* distribution with k degrees of freedom: $X \sim \chi_k^2$, and its probability density function is given by:

$$f(x) = \frac{x^{k/2-1} e^{-x/2}}{2^{k/2} \Gamma(k/2)}$$

The *Chi-squared* distribution with k degrees of freedom has mean equal to k and variance equal to 2k.

```
> # Degrees of freedom
> degf < c(2, 5, 8, 11)
> # Plot four curves in loop
> colorv <- c("red", "black", "blue", "green")
> for (indeks in 1:4) {
+ curve(expr=dchisq(x, df=degf[indeks]),
+ xlim=c(0, 20), ylim=c(0, 0.3),
+ xlab="", ylab="", col=colorv[indeks],
+ lyd=2, add=ss.lorical(indeks-1))
```

+ } # end for



```
> # Add title
> title(main="Chi-squared Distributions", line=0.5)
> # Add legend
> labelv <- paste("df", degf, sep="=")</pre>
```

> legend("topright", inset=0.05, bty="n", y.intersp=0.4,
+ title="Degrees of freedom", labely,

+ cex=0.8, lwd=6, lty=1, col=colorv)

4□ > 4□ > 4□ > 4□ > 4□ > 4□ > 9

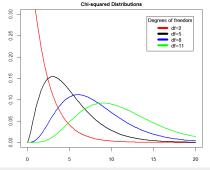
Chi-squared Distribution Plotted as Line

Let z_1, \ldots, z_k be independent standard *Normal* random variables.

Then the random variable $X = \sum_{i=1}^k z_i^2$ is distributed according to the *Chi-squared* distribution with k degrees of freedom: $X \sim \chi_k^2$, and its probability density function is given by:

$$f(x) = \frac{x^{k/2-1} e^{-x/2}}{2^{k/2} \Gamma(k/2)}$$

```
> degf <- c(2, 5, 8, 11) # df values
> # Create plot colors
> colorv <- c("red", "black", "blue", "green")
> # Create legend labels
> labelv <- paste("df", degf, sep="=")
> # Plot an empty chart
> xwar <- seq(0, 20, length=100)
> plot(xvar, dchisq(xvar, df=degf[1]),
+ type="n", xlab="", ylab="", ylim=c(0, 0.3))
> # Add lines to plot
> for (indeks in 1:4) {
+ lines(xvar, dchisq(xvar, df=degf[indeks]),
+ lwd=2, col=colorv[indeks])
+ } # end for
```



> # Add title
> title(main="Chi-squared Distributions", line=0.5)
> # Add legend
> legend("topright", inset=0.05, y.intersp=0.4,
+ title="Degrees of freedom", labely,
+ cs=0.8. lyd=6. lty=1, btv="m". col=colory)

Fisher's F-distribution

Let χ_n^2 and χ_n^2 be independent random variables following chi-squared distributions with m and n degrees of freedom.

Then the random variable:

$$F = \frac{\chi_m^2/m}{\chi_n^2/n}$$

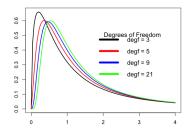
Follows the F-distribution with m and n degrees of freedom, with the probability density function:

$$f(F) = \frac{\Gamma((m+n)/2)m^{m/2}n^{n/2}}{\Gamma(m/2)\Gamma(n/2)} \frac{F^{m/2-1}}{(n+mF)^{(m+n)/2}}$$

The F-distribution depends on the ratio F and also on the degrees of freedom, m and n.

The function df() calculates the probability density of the F-distribution





- > # Add title
- > title(main="F-Distributions", line=0.5)
- > # Add legend > labely <- paste("degf", degf, sep=" = ")
- > legend("topright", title="Degrees of Freedom", inset=0.0, btv="n"
- v.intersp=0.4, labely, cex=1.2, lwd=6, ltv=1, col=colory)

Student's t-distribution

Let z_1, \ldots, z_{ν} be independent standard normal random variables, with sample mean: $\bar{z} = \frac{1}{i!} \sum_{i=1}^{\nu} z_i$ $(\mathbb{E}[\bar{z}] = \mu)$ and sample variance:

$$\hat{\sigma}^2 = \frac{1}{\nu - 1} \sum_{i=1}^{\nu} (z_i - \bar{z})^2$$

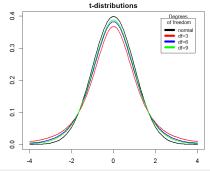
Then the random variable (t-ratio):

$$t = \frac{\bar{z} - \mu}{\hat{\sigma} / \sqrt{\nu}}$$

Follows the *t-distribution* with ν degrees of freedom. with the probability density function:

$$f(t) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu} \, \Gamma(\nu/2)} \, (1 + t^2/\nu)^{-(\nu+1)/2}$$

- > degf <- c(3, 6, 9) # df values
- > colorv <- c("black", "red", "blue", "green")
- > labelv <- c("normal", paste("df", degf, sep="="))
- > # Plot a Normal probability distribution
- > curve(expr=dnorm, xlim=c(-4, 4), xlab="", ylab="", lwd=2)
- > for (indeks in 1:3) { # Plot three t-distributions
- + curve(expr=dt(x, df=degf[indeks]),
- + lwd=2, col=colorv[indeks+1], add=TRUE)
- + } # end for



- > # Add title
- > title(main="t-distributions", line=0.5)
- > # Add legend
- > legend("topright", inset=0.05, bty="n",
- title="Degrees\n of freedom", labely,
 - v.intersp=0.4, cex=0.8, lwd=6, ltv=1, col=colorv)

Student's t-distribution Plotted as Line

Let z_1, \ldots, z_{ν} be independent standard normal random variables, with sample mean: $\bar{z} = \frac{1}{i!} \sum_{i=1}^{\nu} z_i$ $(\mathbb{E}[\bar{z}] = \mu)$ and sample variance:

$$\hat{\sigma}^2 = \frac{1}{\nu - 1} \sum_{i=1}^{\nu} (z_i - \bar{z})^2$$

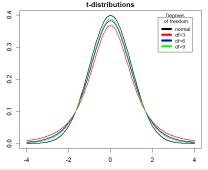
Then the random variable (t-ratio):

$$t = \frac{\bar{z} - \mu}{\hat{\sigma} / \sqrt{\nu}}$$

Follows the *t-distribution* with ν degrees of freedom. with the probability density function:

$$f(t) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu} \, \Gamma(\nu/2)} \, (1 + t^2/\nu)^{-(\nu+1)/2}$$

- > xvar <- seq(-4, 4, length=100)
- > degf <- c(3, 6, 9) # df values
- > colorv <- c("black", "red", "blue", "green") > labelv <- c("normal", paste("df", degf, sep="="))
- > # Plot chart of normal distribution
- > plot(xvar, dnorm(xvar), type="1", lwd=2, xlab="", ylab="") > for (indeks in 1:3) { # Add lines for t-distributions
- + lines(xvar, dt(xvar, df=degf[indeks]),
- + lwd=2, col=colorv[indeks+1])
- + } # end for



- > # Add title
- > title(main="t-distributions", line=0.5)
- > # Add legend
- > legend("topright", inset=0.05, bty="n",
 - title="Degrees\n of freedom", labely,
- v.intersp=0.4, cex=0.8, lwd=6, ltv=1, col=colorv)

Non-standard Student's t-distribution

The non-standard Student's t-distribution has the probability density function:

$$f(t) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu} \sigma \Gamma(\nu/2)} \left(1 + \left(\frac{t-\mu}{\sigma}\right)^2/\nu\right)^{-(\nu+1)/2}$$

It has non-zero mean equal to the location parameter μ , and a standard deviation proportional to the scale parameter σ .

```
> dev.new(width=6, height=5, noRStudioGD=TRUE)
> # x11(width=6, height=5)
> # Define density of non-standard t-distribution
> tdistr <- function(x, dfree, loc=0, scalev=1) {
   dt((x-loc)/scalev, df=dfree)/scalev
+ } # end tdistr
> # Or
> tdistr <- function(x, dfree, loc=0, scalev=1) {
   gamma((dfree+1)/2)/(sqrt(pi*dfree)*gamma(dfree/2)*scalev)*
      (1+((x-loc)/scaley)^2/dfree)^(-(dfree+1)/2)
+ } # end tdistr
> # Calculate vector of scale values
> scalev <- c(0.5, 1.0, 2.0)
> colorv <- c("blue", "black", "red")
> labelv <- paste("scale", format(scalev, digits=2), sep="=")
> # Plot three t-distributions
> for (indeks in 1:3) {
   curve(expr=tdistr(x, dfree=3, scalev=scalev[indeks]), xlim=c(-3, 3),
+ xlab="", vlab="", lwd=2, col=colorv[indeks], add=(indeks>1))
```

t-distributions with Different Scale Parameters Scale Parameters scale=0.5 0.4 0.2

- > # Add title
- > title(main="t-distributions with Different Scale Parameters", lin-
- > # Add legend
- > legend("topright", inset=0.05, bty="n", title="Scale Parameters"
 - y.intersp=0.4, cex=0.8, lwd=6, lty=1, col=colorv)

+ } # end for

Cauchy Distribution

The Cauchy distribution is Student's t-distribution with one degree of freedom $\nu = 1$, with the probability density function:

$$f(x) = \frac{1}{\pi\sigma} \frac{1}{((x-\mu)/\sigma)^2 + 1}$$

Where μ is the location parameter (equal to the mean) and σ is the scale parameter.

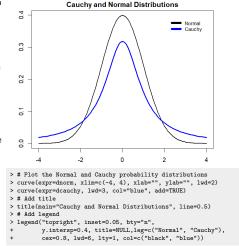
Since the Cauchy distribution has an infinite standard deviation, its measure of dispersion is the interquartile range (IQR), which is equal to σ .

The interquartile range is a robust measure of dispersion (scale), defined as the difference between the 75th minus the 25th percentiles.

The function dcauchy() calculates the Cauchy probability density.

The probability density of the Cauchy distribution decreases as the second power for large values of x:

$$f(x) \propto 1/x^2$$



Pareto Distribution and Zipf's Law

The probability density of Student's t-distribution decreases as a power for large values of x:

$$f(x) \propto |x|^{-(\nu+1)}$$

The probability density of the *Pareto* distribution decreases as a power of the random variable *x*:

$$f(x) = \alpha x^{-(\alpha+1)}$$

For x > 1 and decay parameter $\alpha > 1$.

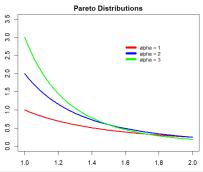
The mean μ and variance σ^2 of the *Pareto* distribution are equal to:

$$\mu = \frac{\alpha}{\alpha - 1}$$
 $\sigma^2 = \frac{\alpha}{(\alpha - 1)^2(\alpha - 2)}$

Zipf's law is analogous to the Pareto distribution, and applies to discrete variables.

Zipf's law states that the frequency f of a given value is inversely proportional to its rank n in the frequency table: $f(n) \propto n^{-s}$.

For example, Zipf's law applies to the frequency of words in a natural language.



```
> # Define Pareto function
```

Poisson Probability Distribution

The *Poisson* distribution gives the probability of the number of events observed in an interval of space or time.

The Poisson probability function is given by:

$$f(n;\lambda) = \frac{\lambda^n \cdot e^{-\lambda}}{n!}$$

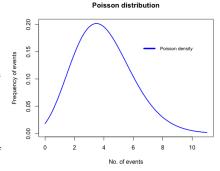
The Poisson random variable ${\tt n}$ is the number of events observed in the interval.

The parameter λ is the average number of events that are observed in the interval.

An example of a *Poisson* distribution is the number of mail items received each day.

The function dpois() returns the probability density of the *Poisson* distribution.

The function rpois() returns random numbers following the *Poisson* distribution.



- > # Poisson frequency
- > eventv <- 0:11 # Poisson events
- > poissonf <- dpois(eventy, lambda=4)
- > names(poissonf) <- as.character(eventv)
- > # Poisson function
- > poissonfun <- function(x, lambda) {exp(-lambda)*lambda^x/factorial(x)}
- > curve(expr=poissonfun(x, lambda=4), xlim=c(0, 11), main="Poisson distribution",
- + xlab="No. of events", ylab="Frequency of events", lwd=2, col="blue")
- > legend(x="topright", legend="Poisson density", title="", bty="n",
- + inset=0.05, cex=0.8, bg="white", lwd=6, ltv=1, col="blue")
- ----- ---- ---- --- --- ---- , --- --, --, --, ---

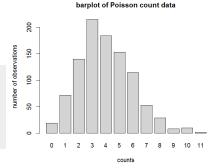
Plotting Bar Charts of Table Data

The function barplot() plots a bar chart for a table of data.

The function rpois() produces random numbers from the *Poisson* distribution.

The function table() calculates the frequency distribution of categorical data.

```
> # Simulate Poisson variables
> poissonv <- rpois(1000, lambda=4)
> head(poissonv)
[1] 5 2 8 9 3 4
> # Calculate contingency table
> poissonf <- table(poissonv)
> poissonf
poissonv
0 1 2 3 4 5 6 7 8 9 10 11 13
19 73 148 197 194 150 93 67 33 14 8 2 2
```



- > # Create barplot of table data
- > barplot(poissonf, col="lightgrey",
- + xlab="counts", ylab="number of observations",
- + main="Barplot of Poisson Count Data")

Plotting Histograms of Frequency Data

The function hist() calculates and plots a histogram, and returns its data *invisibly*.

If the argument freq is TRUE then the frequencies (counts) are plotted, and if it's FALSE then the probability density is plotted (with total area equal to 1).

The function density() calculates a kernel estimate of the probability density for a sample of data.

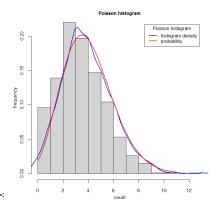
The function lines() draws a line through specified points.

```
> # Create histogram of Poisson variables
> histy <= hist(poissony, col="lightgrey", xlab="count",
+ ylab="frequency", freq=FALSE, main="Poisson histogram")
> # Poisson probability distribution function
> poissonfun <= function(x, lambda)
+ {exp(-lambda)*lambda*x/factorial(x)}
> curve(expr=poissonfun(x, lambda=4), xlim=c(0, 11), add=TRUE, lvd=faraderend
```

> legend("topright", inset=0.01, title="Poisson histogram",
+ c("histogram density", "probability"), cex=1.1, lwd=6,
+ y.intersp=0.4, lty=1, bty="n", col=c("blue", "red"))

> # total area under histogram

> diff(histp\$breaks) %*% histp\$density



Plotting Boxplots of Distributions of Values

Box-and-whisker plots (boxplots) are graphical representations of a distribution of values.

The bottom and top box edges (hinges) are equal to the first and third quartiles, and the box width is equal to the interquartile range (IQR).

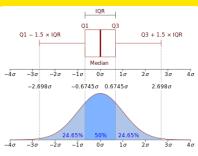
The nominal range is equal to 1.5 times the IQR above and below the box hinges.

The whiskers are dashed vertical lines representing values beyond the first and third quartiles, but within the nominal range.

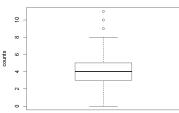
The *whiskers* end at the last values within the nominal range, while the open circles represent outlier values beyond the nominal range.

The function boxplot() has two methods: one for vectors and data frames, and another for formula objects (for categorical variables).

- > # boxplot of Poisson count data
 > boxplot(x=poissony, vlab="counts",
- main="Poisson box plot")
- > # boxplot method for formula
- > boxplot(formula=mpg ~ cyl, data=mtcars,
- + main="Mileage by number of cylinders",
- + xlab="Cylinders", ylab="Miles per gallon")







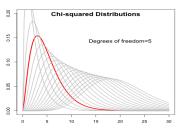
Plotting Using Expression Objects

It's sometimes convenient to create an *expression* object containing plotting commands, to be able to later create plots using it.

The function quote() produces an *expression* object without evaluating it.

The function eval() evaluates an expression in a specified environment.

```
> # Create a plotting expression
> expv <- quote(f
   degf <- 2:20
   rangev <- (1:NROW(degf))
  indeks <- 4
   # Plot a curve
   curve(expr=dchisg(x, df=degf[indeks]).
+ xlim=c(0, 30), ylim=c(0, 0.2),
+ xlab="", vlab="", lwd=3, col="red")
   # Add grey lines to plot
   for (it in rangev[-indeks]) {
      curve(expr=dchisq(x, df=degf[it]),
   xlim=c(0, 30), vlim=c(0, 0.2),
   xlab="", ylab="", lwd=2, col="grey80", add=TRUE)
      # end for
    # Add title
   title(main="Chi-squared Distributions", line=-1.5, cex.main=1.5)
   # Add legend
   text(x=20, y=0.15, labels=paste0("Degrees of freedom=",
       degf[indeks]), pos=1, cex=1.3)
+ }) # end quote
```



```
> # View the plotting expression
> expv
> # Create plot by evaluating the plotting expression
> x11(width=6, height=4)
> eval(exp)
```

Animated Plots Using Package animation

The package *animation* allows creating animated plots in the form of *gif* and *html* documents.

The function saveGIF() produces a $\it gif$ image with an animated plot.

The function saveHTML() produces an html document with an animated plot.

```
> library(animation)
> # Create an expression for creating multiple plots
> expv <- quote({
    degf <- 2:20
   rangev <- (1:NROW(degf))
   # Set image refesh interval
    animation::ani.options(interval=0.5)
   # Create multiple plots with curves
   for (indeks in rangev) {
      curve(expr=dchisq(x, df=degf[indeks]),
   xlim=c(0, 30), ylim=c(0, 0.2),
   xlab="", ylab="", lwd=3, col="red")
      # Add grey lines to plot
      for (it in rangev[-indeks]) {
        curve(expr=dchisq(x, df=degf[it]),
      xlim=c(0, 30), vlim=c(0, 0.2),
      xlab="", ylab="", lwd=2, col="grey80", add=TRUE)
        # end for
      # Add title
      title(main="Chi-squared Distributions", line=-1.5, cex.main=: +
      # Add legend
      text(x=20, y=0.15, labels=paste0("Degrees of freedom=",
       degf[indeks]), pos=1, cex=1.3)
```

```
Chi-squared Distributions

Degrees of freedom=5

0 5 10 15 20 25 30
```

```
> # Create plot by evaluating the plotting expression
> x11(width=6, height=4)
> eval(expv)
> # Create gif with animated plot
> animation::saveGIF(expr=eval(expv),
+ movie.name="chi.squared.gif",
+ img.name="chi.squared")
> # Create html with animated plot
> animation::saveGITM(expr=eval(expv),
+ img.name="chi.squared",
+ htmlfile="chi.squared.html",
+ description="Chi-squared html",
+ description="Chi-squared html",
```

end for # end quote

Dynamic Documents Using R markdown

markdown is a simple markup language designed for creating documents in different formats, including pdf and html.

R Markdown is a modified version of markdown, which allows creating documents containing math formulas and R code embedded in them.

An *R Markdown* document (with extension .Rmd) contains:

- A YAML header.
- Text in R Markdown code format.
- Math formulas (equations), delimited using either single "\$" symbols (for inline formulas), or double "\$\$" symbols (for display formulas),
- R code chunks, delimited using either single "."
 backtick symbols (for inline code), or triple ".""
 backtick symbols (for display code).

The packages *rmarkdown* and *knitr* compile R documents into either *pdf*, *html*, or *MS Word* documents

```
title: "My First R Markdown Document"
author: Jerzy Pawlowski
date: 'f format(Sys.time(), "%m/%d/%Y")''
output: html_document
---
''''{r setup, include=FALSE}
knitr::opts_chunk$set(echo = TRUE)

# install package quantmod if it can't be loaded success
if (!require("quantmod"))
   install.packages("quantmod")
'''
```

R Markdown
This is an *R Markdown* document. Markdown is a simple i

One of the advantages of writing documents *R Markdown*

You can read more about publishing documents using *R* h
https://algoquant.github.io/r,/markdown/2016/07/02/Publi
You can read more about using *R* to create *HTML* docum

https://algoquant.github.io/2016/07/05/Interactive-Plots

Clicking the **Knit** button in *RStudio*, compiles the

```
Example of an *R* code chunk:
'''{r cars}
summary(cars)
'''
```

Plots in *R Markdown* documents

Plots can also be embedded, for example:
'''{r pressure, echo=FALSE}

plot(pressure)

draft: Online markdown Tutorials

Datacamp Interactive Courses

Datacamp introduction to R: https://www.datacamp.com/courses/introduction-to-r/

Datacamp list of free courses: https://www.datacamp.com/community/open-courses

Datacamp basic statistics in R: https://www.datacamp.com/community/open-courses/basic-statistics

Datacamp computational finance in R:

https://www.datacamp.com/community/open-courses/computational-finance- and-financial-econometrics- with-respect to the computational of the computation of the computational of

Datacamp machine learning in ${\tt R}$:

https://www.datacamp.com/community/open-courses/kaggle-r-tutorial-on-machine-learning and the state of the

Try R

Interactive R tutorial, but rather basic: http://tryr.codeschool.com/

Package shiny for Creating Interactive Applications

The package *shiny* creates interactive applications running in R, with their outputs presented as live visualizations.

Shiny allows changing the model parameters, recalculating the model, and displaying the resulting outputs as plots and charts.

A shiny app is a file with shiny commands and R code.

The shiny code consists of a shiny interface and a shiny server.

The *shiny interface* contains widgets for data input and an area for plotting.

The *shiny server* contains the R model code and the plotting code.

The function shiny::fluidPage() creates a GUI layout for the user inputs of model parameters and an area for plots and charts.

The function shiny::renderPlot() renders a plot from the outputs of a live model.

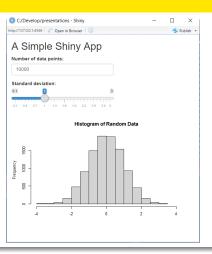
The function shiny::shinyApp() creates a shiny app from a shiny interface and a shiny server.

```
> ## App setup code that runs only once at startup.
> ndata <- 1e4
> stdev <- 1.0
> ## Define the user interface
> uiface <- shiny::fluidPage(
    # Create numeric input for the number of data points.
    numericInput("ndata", "Number of data points:", value=ndata),
    # Create slider input for the standard deviation parameter.
    sliderInput("stdev", label="Standard deviation:",
          min=0.1, max=3.0, value=stdev, step=0.1),
    # Render plot in a panel.
    plotOutput("plotobj", height=300, width=500)
+ ) # end user interface
> ## Define the server function
> servfun <- function(input, output) {
    output$plotobj <- shiny::renderPlot({
      # Simulate the data
      datav <- rnorm(input$ndata, sd=input$stdev)
      # Plot the data
      par(mar=c(2, 4, 4, 0), oma=c(0, 0, 0, 0))
      hist(datay, xlim=c(-4, 4), main="Histogram of Random Data")
    }) # end renderPlot
     # end servfun
> # Return a Shiny app object
> shinv::shinvApp(ui=uiface, server=servfun)
```

Running Shiny Apps in RStudio

A *shiny app* can be run by pressing the "Run App" button in *RStudio*.

When the *shiny app* is run, the *shiny* commands are translated into *JavaScript* code, which creates a graphical user interface (GUI) with buttons, sliders, and boxes for data input, and also with the output plots and charts.



Positioning and Sizing Widgets Within the Shiny GUI

The functions shiny::fluidRow() and shiny::column() allow positioning and sizing widgets within the *shiny* GUI.

mainPanel(dygraphs::dygraphOutput("dyplot"), width=12)



Create output plot panel

end fluidPage interface

Shiny Apps With Reactive Expressions

The package *shiny* allows specifying reactive expressions which are evaluated only when their input data is updated.

Reactive expressions avoid performing unnecessary calculations.

If the reactive expression is invalidated (recalculated), then other expressions that depend on its output are also recalculated.

This way calculations cascade through the expressions that depend on each other.

The function shiny::reactive() transforms an expression into a reactive expression.

```
> ## Define the server function
> servfun <- shiny::shinyServer(function(input, output) {
    # Get the close and volume data in a reactive environment
    closep <- shiny::reactive({
      # Get the data
     ohlc <- get(input$symbol, data_env)
     closep <- log(quantmod::Cl(ohlc))
     volum <- quantmod::Vo(ohlc)
      # Return the data
      cbind(closep, volum)
    }) # end reactive code
    # Calculate the VWAP indicator in a reactive environment
    vwapv <- shinv::reactive({
     # Get model parameters from input argument
     look back <- input$look back
      # Calculate the VWAP indicator
     closep <- closep()[, 1]
     volum <- closep()[, 2]
      vwapv <- HighFreq::roll_sum(tseries=closep*volum, look_back=1
      volumroll <- HighFreq::roll_sum(tseries=volum, look_back=look
      vwapv <- vwapv/volumroll
     vwapv[is.na(vwapv)] <- 0
      # Return the plot data
     datav <- cbind(closep, vwapv)
     colnames(datav) <- c(input$symbol, "VWAP")
      datav
    # end reactive code
   # Return the dygraph plot to output argument
   output$dyplot <- dygraphs::renderDygraph({
     colnamev <- colnames(vwapv())
     dygraphs::dygraph(vwapv(), main=paste(colnamev[1], "VWAP")) %
+ dyAxis("y", label=colnamev[1], independentTicks=TRUE) %>%
+ dyAxis("y2", label=colnamev[2], independentTicks=TRUE) %>%
+ dySeries(name=colnamev[1], axis="y", label=colnamev[1], strokeWid
+ dySeries(name=colnamev[2], axis="y2", label=colnamev[2], strokeWi
+ }) # end output plot
+ }) # end server code
```

Reactive Event Handlers

Event handlers are functions which evaluate expressions when an event occurs (like a button press).

The functions shiny::observeEvent() and shiny::eventReactive() are event handlers.

The function shiny::eventReactive() returns a value, while shiny::observeEvent() produces a side-effect, without returning a value.

The function shiny::reactiveValues() creates a list for storing reactive values, which can be updated by event handlers.

```
> ## Define the server function
> servfun <- shiny::shinyServer(function(input, output) {
   # Create an empty list of reactive values.
    value s <- reactiveValues()
   # Get input parameters from the user interface.
    nrows <- reactive({
      # Add nrows to list of reactive values.
      value s*nrows <- input$nrows
      input$nrows
    }) # end reactive code
   # Broadcast a message to the console when the button is pressed
   observeEvent(eventExpr=input$button, handlerExpr={
      cat("Input button pressed\n")
    }) # end observeEvent
    # Send the data when the button is pressed.
   datav <- eventReactive(eventExpr=input$button, valueExpr={
     # eventReactive() executes on input$button, but not on nrows(
     cat("Sending", nrows(), "rows of data\n")
     datav <- head(mtcars, input$nrows)
     value_s$mpg <- mean(datav$mpg)
      datav
    }) # end eventReactive
       datav
   # Draw table of the data when the button is pressed.
   observeEvent(eventExpr=input$button, handlerExpr={
     datay <- datay()
      cat("Received", value_s*nrows, "rows of data\n")
     cat("Average mpg = ", value_s$mpg, "\n")
     cat("Drawing table\n")
     output$tablev <- renderTable(datav)
    # end observeEvent.
+ }) # end server code
```

draft: Interactive Plots Using Markdown and shiny

The package shiny creates interactive plots that display the outputs of live models running in R.

The function inputPanel() creates a panel for user input of model parameters.

The function renderPlot() renders a plot from the outputs of a live model running in R.

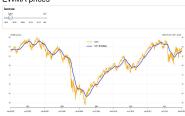
To create a shiny chart, you can first create an .Rmd file, embed the shiny code in an R chunk, and then compile the .Rmd file into an html document, using the knitr package.

```
> # R startup chunk
> # '''{r setup, include=FALSE}
> library(shiny)
> library(quantmod)
> interval <- 31
> closep <- quantmod::Cl(rutils::etfenv$VTI)
> plot_theme <- chart_theme()
> plot theme$col$line.col <- c("orange", "blue")
> ### end R startup chunk
> inputPanel(
   sliderInput("lambda", label="lambda;",
     min=0.01, max=0.2, value=0.1, step=0.01)
+ ) # end inputPanel
```

EWMA prices

+ }) # end renderPlot

Graphics and Plotting



```
> renderPlot({
    # Calculate EWMA prices
   lambda <- input$lambda
   weightv <- exp(-lambda*1:interval)
   weightv <- weights/sum(weights)
   ewmacpp <- .Call(stats:::C_cfilter, closep, filter=weightv, sid
   ewmacpp[1:(interval-1)] <- ewmacpp[interval]
   ewmacpp <- xts(cbind(closep , ewmacpp), order.by=zoo::index(closep
   colnames(ewmacpp) <- c("VTI", "VTI EWMA")
    # Plot EWMA prices
   chobj <- chart_Series(ewmacpp, theme=plot_theme, name="EWMA pri
   plot(chobj)
   legend("top", legend=colnames(ewmacpp),
    y.intersp=0.4, inset=0.1, bg="white", lty=1, lwd=2,
    col=plot_theme$col$line.col, bty="n")
```

ggplot2 Package for Plotting

The package *ggplot2* allows plotting using the *grammar* of *graphics* framework, where the plot is specified by combining different geometric objects (lines, rectangles, etc.).

In the *ggplot2* framework a plot object is created by combining the outputs of several plotting functions using the "+" operator.

The function ggplot() defines the plot data and creates a plot template.

The function aes() specifies the plot "aesthetics", which are mappings between data elements (rows, columns) and plot elements (points, lines).

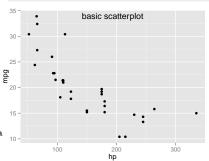
 ${\tt geom_point()} \ \ {\tt adds} \ \ {\tt plot} \ \ {\tt points}.$

geom_line() adds plot lines.

The function theme() customizes plot objects.

Simply calling the plot object renders the plot.

ggplot2 is designed for plotting data formatted as data frames.

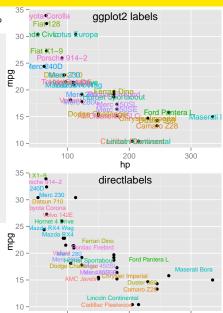


ggplot2 Scatter Plots with Labels

The package *directlabels* allows adding flexible lables to *ggplot2* scatter plots.
> library(directlabels) # Load directlabels

```
data=mtcars, mapping=aes(x=hp, y=mpg)) +
    geom point() + # Plot points
   theme( # Customize plot object
   legend.position="none",
   plot.title=element_text(vjust=-2.0),
   plot.margin=unit(c(-0.0,0.0,-0.5,0.0), "cm"),
   plot.background=element_blank()
    scale_colour_discrete(guide="none") # no label guide
> car names <- rownames(mtcars)</p>
> gg_labels <- geom_text(aes( # ggplot2 labels
   label=car_names, color=car_names, size=5))
> d_labels <- geom_dl(mapping=aes( # Directlabels
   label=car_names, color=car_names),
   method=list("last.bumpup", cex=0.7,
         hjust=1))
> # Render plots in single column
> grid.arrange(my_ggplot +
    ggtitle("ggplot2 labels") + gg_labels,
   my_ggplot + ggtitle("directlabels") +
      d_labels, ncol=1) # end grid.arrange
```

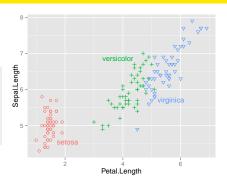
> mv ggplot <- ggplot(# Data and aesthetics



ggplot2 Scatter Plots with Group Labels

The *Poisson* distribution gives the probability of the number of events in intervals of space or time.

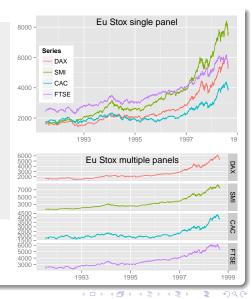
An example of a *Poisson* distribution is the number of mail items received each day.



Plotting zoo Time Series Using autoplot()

autoplot() is a convenience wrapper for ggplot2 functions, designed to plot zoo time series.

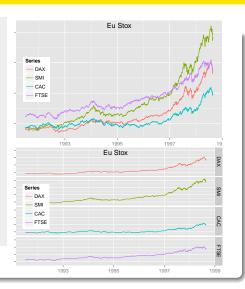
```
> zoo series <- as.zoo(EuStockMarkets)
> # Create ggplot2 theme object
> auto_theme <- theme(
   legend.position="none",
   plot.title=element_text(vjust=-2.0),
   plot.margin=unit(c(-0.0,0.0,-0.5,0.0), "cm"),
+ # axis.text.y=element_blank(),
    plot.background=element_blank()
     # end theme
   ggplot2 object for plotting in single panel
 ggp_zoo_single <- autoplot(zoo_series,
              main="Eu Stox single panel",
              facets=NULL) + xlab("") +
              auto theme
   ggplot2 object for plotting in multiple panels
 ggp_zoo_multiple <- autoplot(zoo_series,
             main="Eu Stox multiple panels",
              facets="Series ~ .") + xlab("") +
              facet_grid("Series ~ .",
              scales="free_v") + auto_theme
  # Render plots in single column
 grid.arrange(ggp_zoo_single +
          theme(legend.position=c(0.1, 0.5)).
```



ggp_zoo_multiple, ncol=1)

legacy - Plotting zoo Time Series Using ggplot2

```
> #
> auto_theme <- theme(legend.position=c(0.1, 0.5),
    plot.title=element text(viust=-2.0).
   plot.margin=unit(c(-0.5, 0.0, -0.5, 0.0), "cm"),
   plot.background=element_blank(),
   axis.text.v=element blank()
   Plot ggplot2 in single pane
 ggp.zoo1 <- autoplot(zoo series, main="Eu Stox",
     facets=NULL) + xlab("") +
   theme(legend.position=c(0.1, 0.5).
   plot.title=element text(viust=-2.0).
   plot.margin=unit(c(-0.5, 0.0, -0.5, 0.0), "cm"),
   plot.background=element blank().
    axis.text.y=element_blank()
   Plot ggplot2 in multiple panes
> ggp.zoo2 <- autoplot(zoo_series, main="Eu Stox",
     facets=Series ~ .) + xlab("") +
   theme(legend.position=c(0.1, 0.5),
   plot.title=element_text(vjust=-2.0),
   plot.margin=unit(c(-0.5, 0.0, -0.5, 0.0), "cm"),
   plot.background=element_blank(),
    axis.text.y=element_blank()
> # Create plot ggplot2 in multiple panes
> grid.arrange(ggp.zoo1, ggp.zoo2, ncol=1)
```



Plotting 3d Perspective Surface Plots

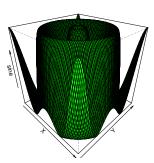
The function persp() plots a 3d perspective surface plot of a function specified over a grid of argument values.

The argument z accepts a matrix containing the function values.

persp() belongs to the base *graphics* package, and doesn't create interactive plots.

```
> # Define function of two variables
> fun2d <- function(x, y) sin(sqrt(x^2+y^2))
> # Calculate function over matrix grid
> xlim <- seq(from=-10, to=10, by=0.2)
> ylim <- seq(from=-10, to=10, by=0.2)
> # Draw 3d surface plot of function
> persp(z=outer(xlim, ylim, FUN=fun2d),
+ theta=45, phi=30, zlab="sine",
+ shade=0.1, col="green",
+ main="radial sine function")
```

radial sine function



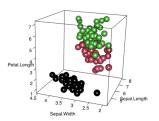
Interactive 3d Scatter Plots Using Package rgl

The package *rgl* creates *interactive* 3d scatter plots and surface plots by calling the *WebGL JavaScript* library.

WebGL is a JavaScript library which creates plots by calling graphics code written in the OpenGL language.

The function rgl::plot3d() plots an *interactive* 3d scatter plot from three vectors of data.

```
> # Set rgl options
> options(rgl.useNULL=TRUE)
> # Load package rgl
> library(rgl)
> # Create 3d scatter plot of function
> with(rirs, rgl::plot3d(Sepal.Length, Sepal.Width, Petal.Length,
+ type="s", col=as.numeric(Species)))
> # Render the 3d scatter plot of function
> rgl::rglvidget(elementId="plot3drgl", width=1000, height=1000)
```



Interactive 3d Surface Plots Using Package rgl

The function rgl::persp3d() plots an *interactive* 3d surface plot of a *vectorized* function or a matrix.

 rgI is an R package for 3d and perspective plotting, based on the OpenGL framework.

```
> library(rgl) # Load rgl
> # Define function of two variables
> fun2d <- function(x, y) y*sin(x)
> # Create 3d surface plot of function
> rgl::persp3d(x=fun2d, xlim=c(-5, 5), ylim=c(-5, 5),
+ col="green", axes=FALSE)
> # Render the 3d surface plot of function
> rgl::rglwidget(elementId="surfacergl", width=500, height=500)
> # Draw 3d surface plot of matrix
> xlim <- seq(from=-5, to=5, by=0.1)
> ylim <- seq(from=-5, to=5, by=0.1)
> rgl::persp3d(z=outer(xlim, ylim, FUN=fun2d),
+ xlab="x", ylab="y", zlab="fun2d", col="green")
> # Save current view to png file
> rgl::rgl.snapshot("surface_plot.png")
> # Define function of two variables and two parameters
> fun2d <- function(x, y, lambda1=1, lambda2=1)
   sin(lambda1*x)*sin(lambda2*y)
> # Draw 3d surface plot of function
> rgl::persp3d(x=fun2d, xlim=c(-5, 5), ylim=c(-5, 5),
```

col="green", axes=FALSE, lambda1=1, lambda2=2)

