## Hausdorff Spaces and Categorical Products

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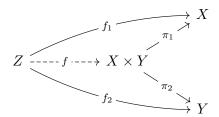
## Abstract

Do not think too hard. This is a short note about some intuitive remarks on the Hausdorff spaces and the Categorical products, in a disjoint way!

## 1 Categorical Product

We start with a definition.

**Definition 1.** Let C be a category. "The" categorical product of  $X, Y \in \text{Obj}(C)$ , if it exists, is  $X \times Y \in \text{Obj}(C)$  along with two morphisms  $\pi_1 : X \times Y \to X$  and  $p_2 : X \times Y \to Y$  with the following universal property: For any other  $Z \in \text{Obj}(C)$ , along with two maps  $f_1 : Z \to X$  and  $f_2 : Z \to Y$ , there exists a unique morphism  $f : Z \to X \times Y$  such that the following diagram commutes.



The categorical product of X and Y is unique up to isomorphism. In fact, more is true. They are unique up to unique isomorphism.