

Hausdorff Spaces and Categorical Products

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Abstract

Do not think too hard. This is a short note about some intuitive remarks on the Hausdorff spaces and the Categorical products, in a disjoint way!

1 Categorical Product

We start with a definition.

Definition 1. Let C be a category. “The” categorical product of $X, Y \in \text{Obj}(C)$, if it exists, is $X \times Y \in \text{Obj}(C)$ along with two morphisms $\pi_1 : X \times Y \rightarrow X$ and $\pi_2 : X \times Y \rightarrow Y$ with the following universal property: For any other $Z \in \text{Obj}(C)$, along with two maps $f_1 : Z \rightarrow X$ and $f_2 : Z \rightarrow Y$, there exists a unique morphism $f : Z \rightarrow X \times Y$ such that the following diagram commutes.

$$\begin{array}{ccccc} & & & & X \\ & & & \nearrow \pi_1 & \\ & f_1 \nearrow & & & \\ Z & \text{---} f \text{---} & X \times Y & \searrow \pi_2 & \\ & f_2 \searrow & & & Y \end{array}$$

The categorical product of X and Y is unique up to isomorphism. In fact, more is true. They are unique up to unique isomorphism.