Math Journey

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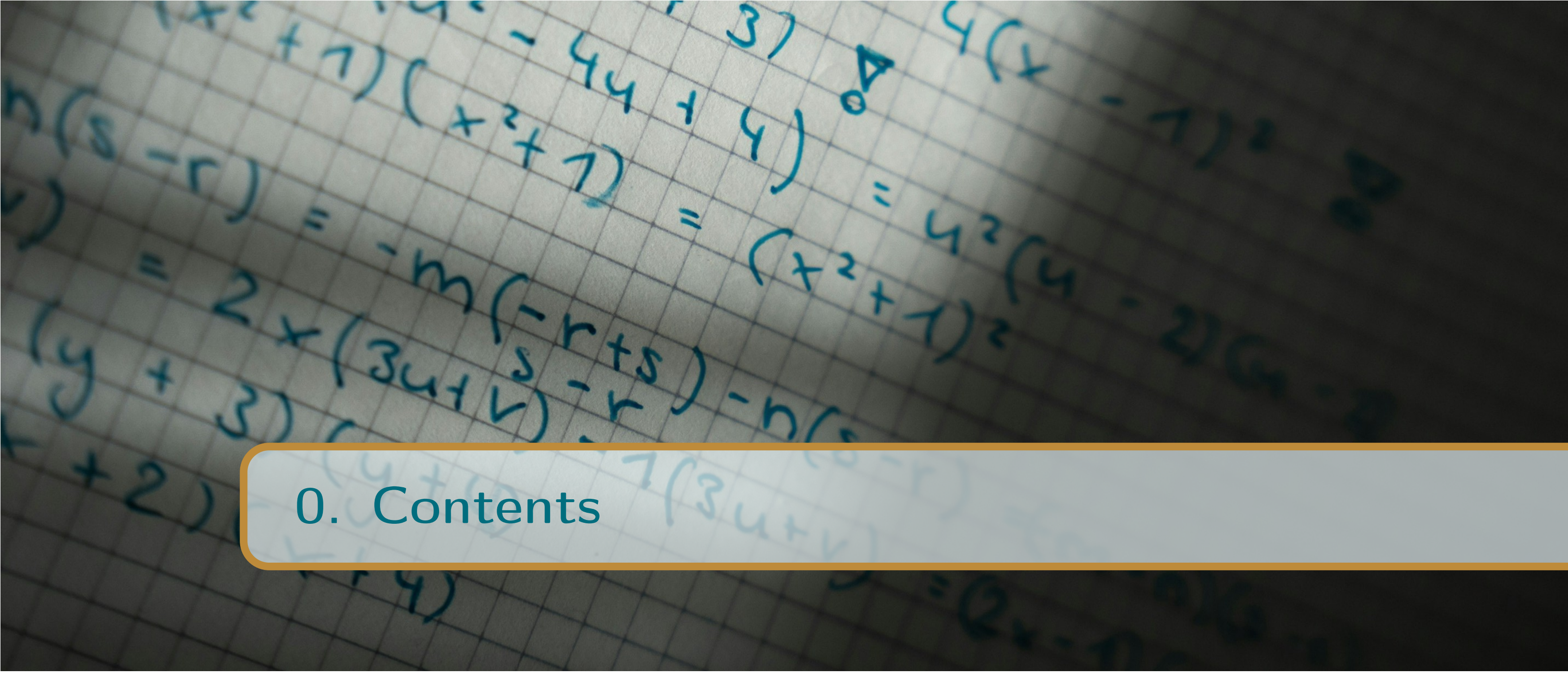
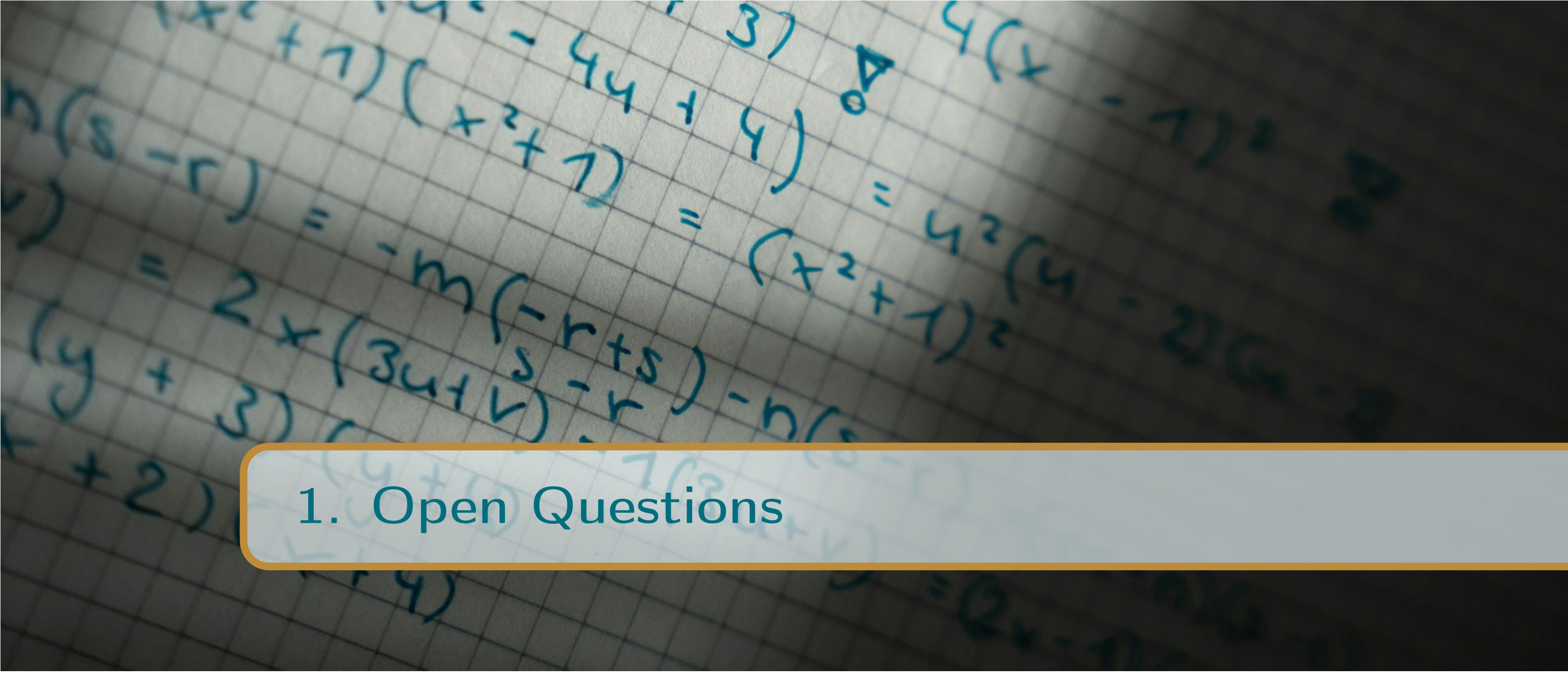


0. Contents



1. Open Questions

In this chapter I am collecting my open questions and problems.

Quesiton 1.1 — Finite elements, weak derivartives. Let $L_2(\Omega)$ be the space of square integrable functions defined on $\Omega \subset \mathbb{R}^n$ open, where we identify $u, v \in L_2(\Omega)$ if they are different on a set of measure zero. Then we define the weak derivative as the following.
Let $u \in L_2(\Omega)$. According to Brass, We say that u posses the weak derivative in $L_2(\Omega)$ if there exists $v \in L_2(\Omega)$ such that

$$\int_\Omega \varphi v \; dx = \int_\Omega \varphi' u \; dx, \qquad \forall \varphi \in C_0^\infty(\Omega)$$

where $C_0^\infty(\Omega)$ is the set of all smooth functions defined on Ω that has compact support.

Now, my question is that why φ needs to have compact support? Why $\varphi(\partial\Omega) = 0$ is not enough? I think the requirement $\varphi(\partial\Omega) = 0$ is enough to remove the boundary term in the integration by parts and arrive at the definition above.