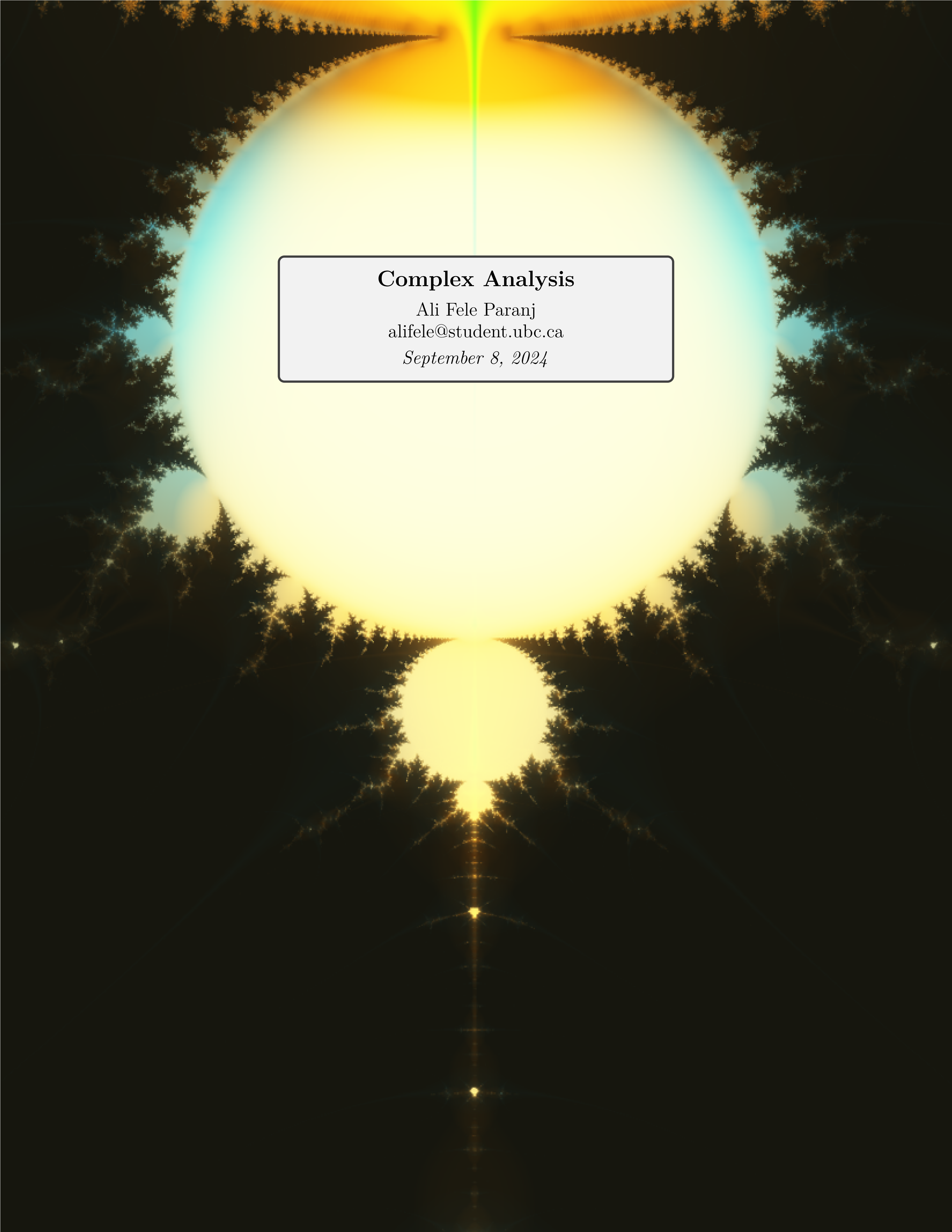

Complex Analysis

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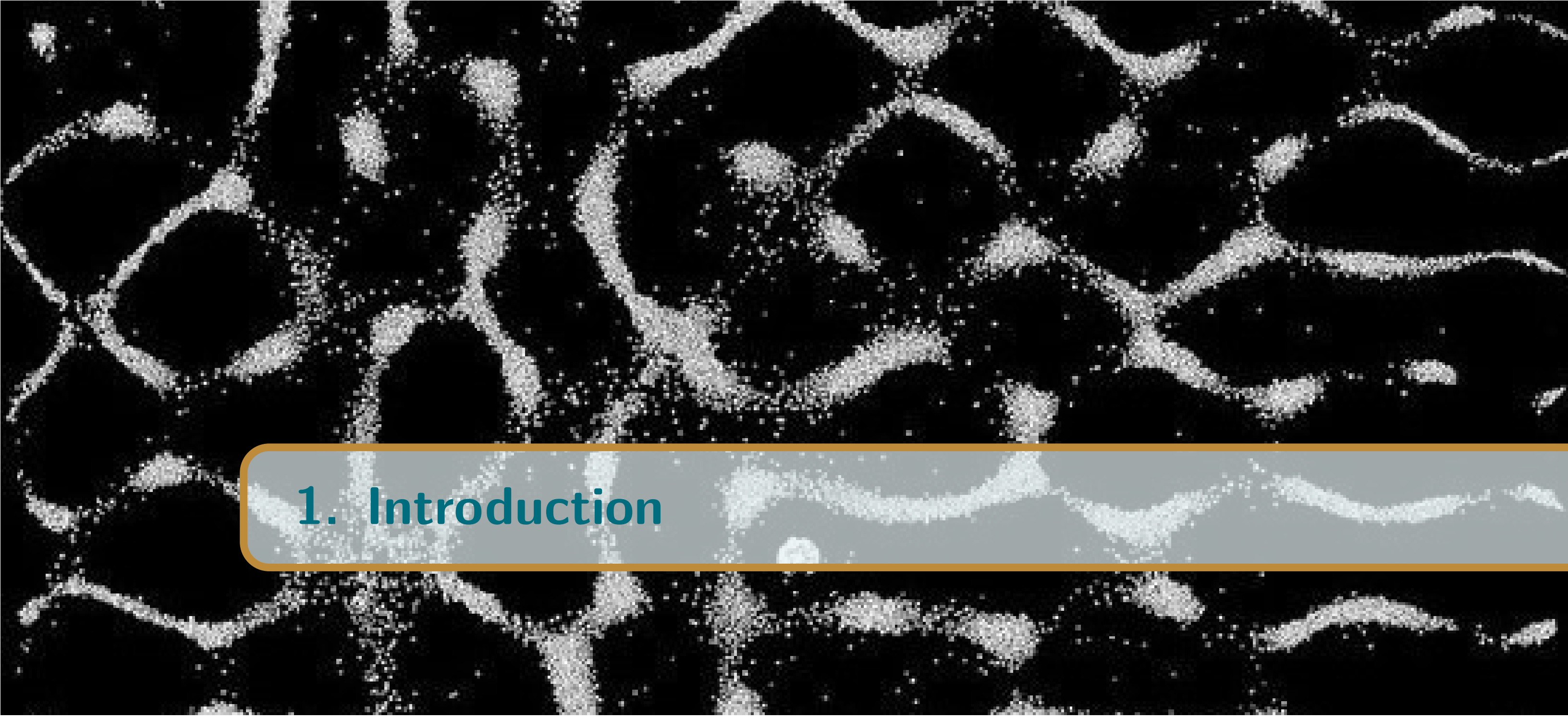
0. Contents



1. Introduction

1.1 Solved Problems

■ **Problem 1.1 — Convergence of complex variables.** Let $\{z_n = a_n + ib_n\}_{n\in\mathbb{N}}$ for some $a_n \in \mathbb{R}, b_n \in \mathbb{R}$ be a sequence of complex numbers. Show that z_n converges to $w = \alpha + i\beta$ if and only if $a_n \to \alpha$ and $b_n \to \beta$ as $n \to \infty$.

Proof. The proof is as follows

$\boxed{\Longrightarrow}$ Let otherwise. Without loss of generality, we can assume that a_n does not converge to α. Then $\exists\epsilon > 0$ such that $\forall N > 0$ we can find $n > N$ for which $|a_n - \alpha| > \epsilon$. This implies that

$$|(a_n - \alpha) + i(b_n - \beta)|^2 = |a_n - \alpha|^2 + |b_n - \beta|^2 > \epsilon^2$$

which implies

$$|z_n - \omega| = |(a_n - \alpha) + i(b_n - \beta)| > \epsilon$$

for some ϵ and for some $n > N$ for any choice of N. This is a contradiction, since implies z_n is not converging to w.

$\boxed{\Longleftarrow}$ Assume $\alpha_n \to a$ and $\beta_n \to b$ as $n \to \infty$. Fix $\epsilon > 0$. Let N be large enough such that

$$|a_n - \alpha| < \epsilon^2/2, \qquad |b_n - \beta| < \epsilon^2/2.$$

Then we can write

$$|(a_n - \alpha) + i(b_n - \beta)|^2 = |a_n - \alpha|^2 + |b_n - \beta|^2 < \epsilon^2.$$

This implies that z_n converges to w.

□

■ **Problem 1.2 — Completeness of $\mathbb{C}$.** Prove that the set of all complex numbers $\mathbb{C}$ is complete.

Proof. The convergence of a complex number is equivalent to the convergence of its real and imaginary parts. Since $\mathbb{R}$ is complete, it follows that $\mathbb{C}$ is also complete, □