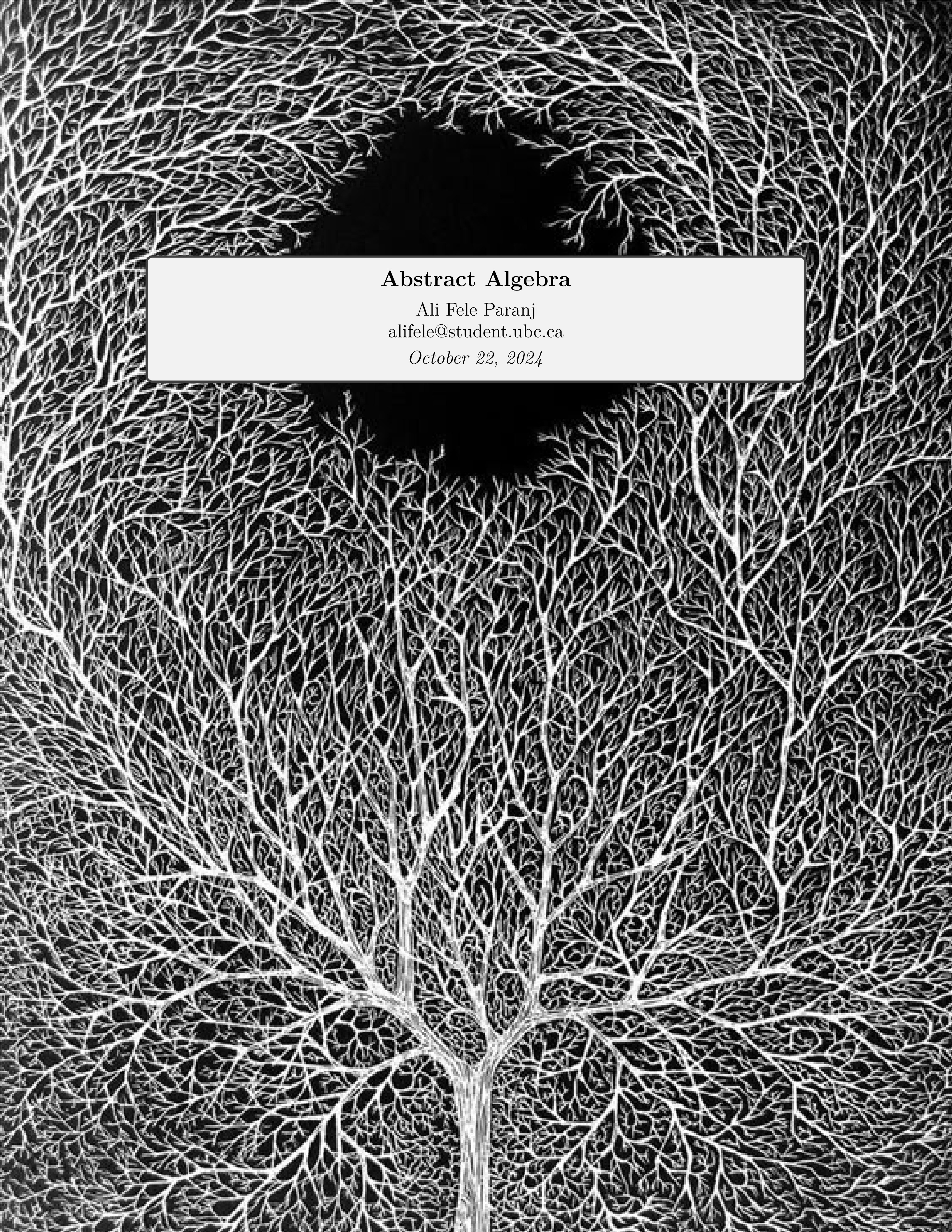

Abstract Algebra

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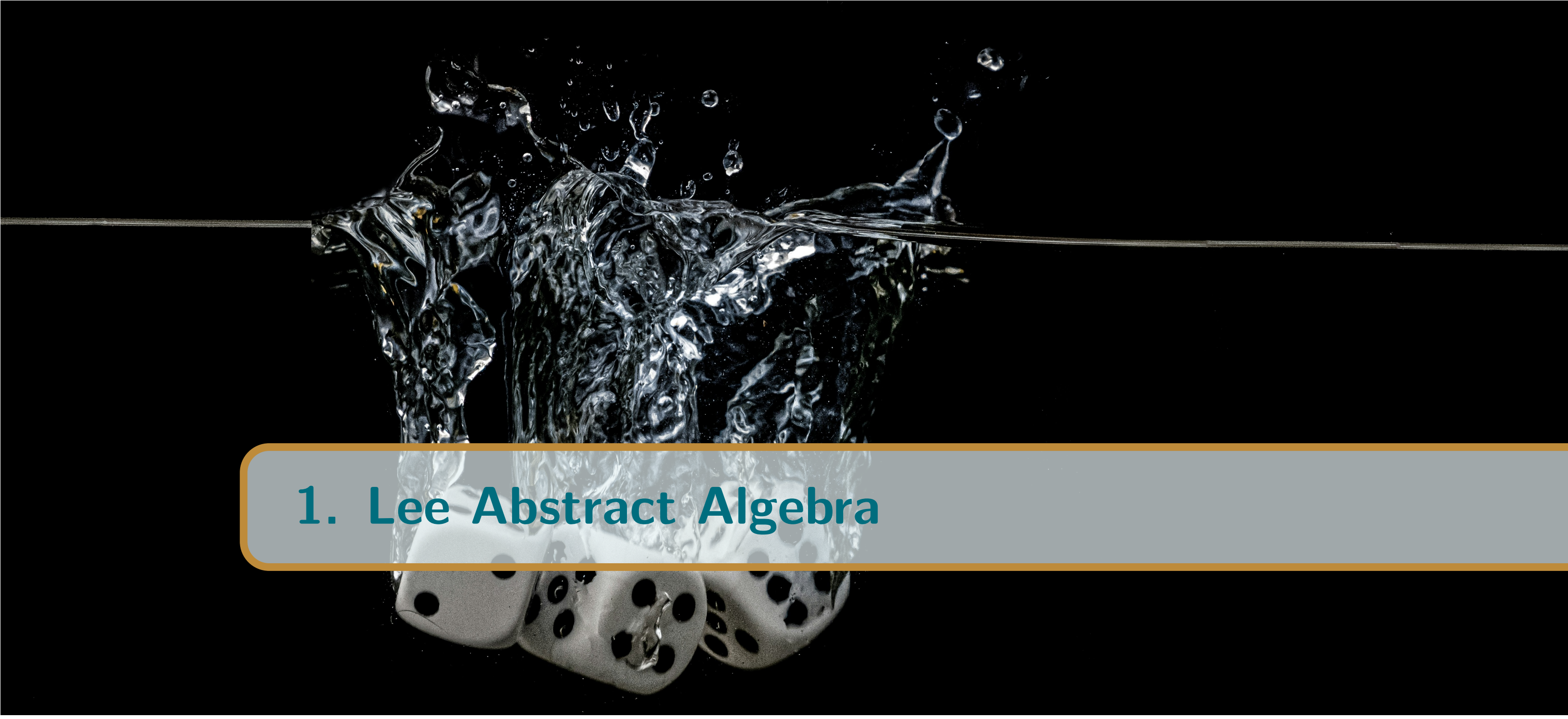
0. Contents



1. Lee Abstract Algebra

1.1 A Simple Example

1.1.1 Solved Problems

■ **Problem 1.1** In S_4 let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$. Calculate the followings.

(a) $\sigma\tau$

(b) $\tau\sigma$

(c) the inverse of σ

Solution (a)

$$\sigma\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 3 & 1 \end{pmatrix}.$$

(b)

$$\tau\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}.$$

(c)

$$\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}.$$

■ **Problem 1.2** In S_5, let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 2 & 1 & 4 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 3 & 5 \end{pmatrix}$. Calculate the following.

(a) $\sigma\tau\sigma$

(b) $\sigma\sigma\tau$

(c) the inverse of σ

Solution (a)

$$\sigma\tau\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 1 & 3 & 2 \end{pmatrix}.$$

(b)

$$\sigma\sigma\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 4 & 3 & 1 \end{pmatrix}.$$

(c)

$$\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 2 & 5 & 1 \end{pmatrix}.$$

■ **Problem 1.3** How many permutations are there in S_n? How many of those permutation satisfy $\alpha(2) = 2$?

Solution There are n choices for $\alpha(1)$, $n-1$ choices for $\alpha(2)$, and so on. So there are in total $n!$ elements in S_n. Fixing the value of $\alpha(2) = 2$ will leave 4 possible values for $\alpha(1)$, 3 possible values for $\alpha(3)$, and so on. Thus there will be $4! = 24$ permutations satisfying $\alpha(2) = 2$.

■ **Problem 1.4** Let H be the set of all permutations $\alpha \in S_5$ satisfying $\alpha(2) = 2$. Which of the properties, closure, associativity, identity, and inverse does H enjoy under composition of functions?

Solution Closure is satisfied: Let $\alpha, \beta \in H$. Then $\alpha(\beta(2)) = \alpha(2) = 2$ and also $\beta(\alpha(2)) = \beta(2) = 2$. Associativity is satisfied which follows from the axioms of the group. The identity of the group is in H, which is given by

$$e = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}.$$

Every element in H also has an inverse. Let $\alpha \in H$. Let $\tau \in S_5$ be its inverse. We have

$$\tau(2) = \tau(\alpha(2)) = e(2) = 2.$$

Thus $\tau \in H$.