



# Manifolds

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# 1. Smooth Manifolds

## 1.1 Problems

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■ **Problem 1.1** Let  $X$  be the set of all points  $(x, y) \in \mathbb{R}^2$  such that  $y = \pm 1$ , and let  $M$  be the quotient of  $X$  by the equivalence relation generated by  $(x, -1) \sim (x, 1)$  for all  $x \neq 0$ . Show that  $M$  is locally Euclidean and second countable, but not Hausdorff. This space is called the line with two origins.

**Solution** Denote the topological space  $Y = X/\sim$ . The open sets in  $Y$  are those that their pre-image under the quotient map  $\pi : X \mapsto X/\sim$  is open. Denote the open sets around the origins as  $I, I_+$ , and  $I_-$ , where  $\pi((0, -1)) \in I_-, \pi((0, 1)) \in I_+$ , and bot holds for  $I$ .