Lecture Notes For: Mathematical Proof

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This lecture note contains the material in the course MATH 220 Mathematical Proof (UBC 2023). However have exapnded the material and examples using the following books:

- Main Teextbook: PLP (an introduction to mathematical proof). Link: PLP website
- Book of Proof (3rd Edition) By Richard Hammack.
- Mathematical Proofs: A Transition to Advanced Mathematics By Chartrand et. al.
- Math proof lectures on YouTube: YouTube Link

Also some useful information can be found here which are the course content of this course in previous years.

- https://personal.math.ubc.ca/~ilaba/teaching/math220_F2015/
- https://secure.math.ubc.ca/php/MathNet/courseinfo.php?session=2020W&t=outline&name=220:101

Some open text books also can be found here in this link: https://aimath.org/textbooks/approved-textbooks/
Also I will add some material from the book "A first course in logic" by Hedman.

1 Fundamentals

In this section we will review some basic definitions in the mathematical proof and mathematical logic.

Definition: Axiom

Axiom is a mathematical statement whose truth is accepted without proof.

For example the followings are some well-known axioms in mathematics:

- Kolmogorov axioms (axioms of probability)
- Axioms of the Euclidean geometry: For every line l and point P that is not on the line, there exists only one line l' that contains the point P and is parallel to the line l.

Definition: Theorem

A true mathematical statement whose truth can be verified using mathematical proof and following mathematical proof.

However, the mathematicians reserve the word theorem for true mathematical statements that is significant and very important. For instance the fact that $2+3=5$ is a true mathematical statement whose truth can be verified using mathematical proof. However, since it is not a significant results, it is not common to call it a theorem. Instead, alternative words are used like: proposition, results, fact, observation.