Tensor Decomposition For Logic Circuits

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February 6, 2025

Let A represent a boolean variable. Then observe that

$$A + \overline{A} = 1.$$

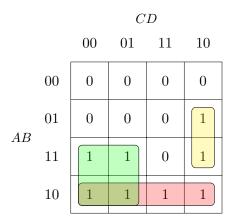
We can use this fact to simplify the logical circuits. For instance to implement $f(A, B, C, D) = \overline{ABCD} + A\overline{B}CD$ as it is we need more logic gates to implement its equivalent f(A, B, C, D) = CD. Karnaugh map makes such simplification possible for any logic function. For instance, consider the following truth table:

A	В	С	D	F(A,B,C,D)
0	0	0	0	0
0	0	0	1	0
0	0	1 1	0	0
0	0	1	1	0
0 0 0 0 0 0 0 0 1 1	1	0	0	0
0	1 1 1	0	1	0
0		1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1 1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

We can write this function as

$$F(A,B,C,D) = \overline{A}BC\overline{D} + A\overline{B}C\overline{D} + A\overline{B}C\overline{D} + A\overline{B}C\overline{D} + A\overline{B}C\overline{D} + AB\overline{C}D + AB\overline{C}$$

To decompose 4-linear form and then use the identity above simplify the function we can use the Karnaugh map.



The grouped boxes above shows which terms can be grouped to make a simpler term. For instance, the green box above show the following terms that can be grouped easily:

$$\begin{split} AB\overline{CD} + AB\overline{C}D + A\overline{B}\overline{C}D + A\overline{B}\overline{C}D \\ &= AB\overline{C}(D + \overline{D}) + A\overline{B}\overline{C}(D + \overline{D}) \\ &= A\overline{C}(B + \overline{B}) \\ &= A\overline{C}. \end{split}$$

With a similar reasoning, the red box results in the term $A\overline{B}$ and the yellow box results in the term $BC\overline{D}$. So the simplified function will be

$$F(A, B, C, D) = A\overline{C} + A\overline{B} + BC\overline{D}.$$