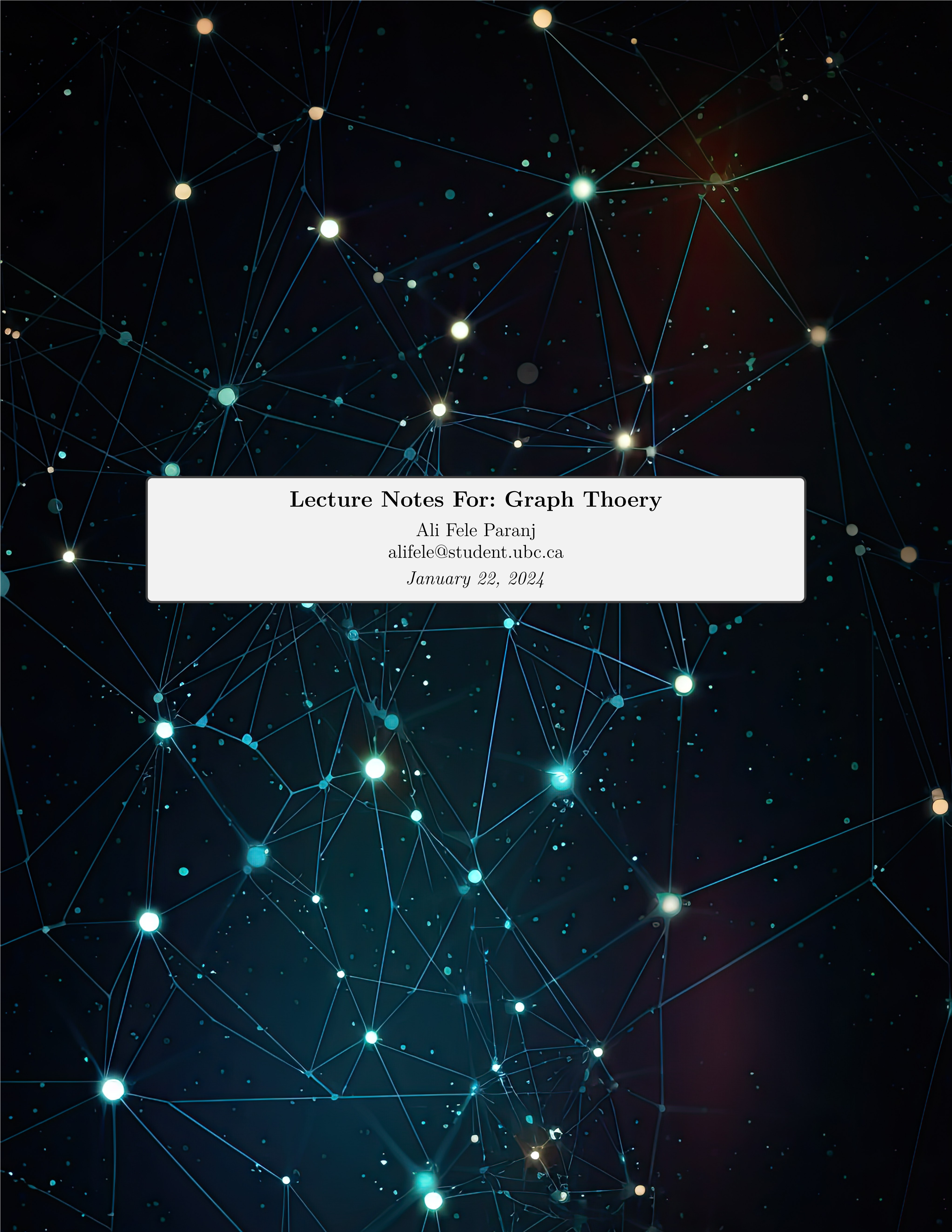

Lecture Notes For: Graph Thoery

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0. Contents

1. Basics and Definitions

Here in this chapter we will cover the sporadic definitions, theorems, and proofs for the graph theory to get ready for the upcoming chapters. So expect a high diversity and less coherency for the material covered in this chapter.

1.1 Basic Definitions and Notions of Graph Theory

Definition 1.1 The complement of graph $G = (V, E)$ is defined to be

$$\bar{G} = (V, E^c) = (V, \binom{V}{2} - E).$$

The following definition happens to be one of the central definitions in the realm of graph theory.

Definition 1.2 Two graphs G, H are isomorphic if there exists a bijection $\varphi : V(G) \to V(H)$ such that it preserves the edges, i.e.

$$xy \in E(G) \implies \varphi(x)\varphi(y) \in E(H).$$

The intuitive description of isomorphism between two graphs is that we can change one to the other one by just moving around the vertices and not connecting/disconnecting any edges. Combining the notion of the complement of a graph with the notion of isomorphic graphs we can have the following theorem.

Theorem 1.1 Two graphs are isomorphic if and only of their complement are isomorphic.

Proof. This proof will have two direction. For the forward direction, Let G and H be two isomorphic graphs (i.e. there exists a structure preserving bijection φ between the vertex set of two graphs). Let xy be a non-edge in G. I.e. $xy \in E(\bar{G})$ or equivalently $xy \notin E(G)$. Then it follows that $\varphi(x)\varphi(y) \notin E(H)$ which is equivalent to $\varphi(x)\varphi(y) \in E(\bar{H})$. Thus we can conclude that the same φ is a structure preserving map from $\bar{G}$ to $\bar{H}$ as well, thus $\bar{G}$ and $\bar{H}$ are isomorphism as well. The proof for the converse is very similar to the structure of the proof presented above. □