PHYS 402 (Applications of Quantum Mechanics) Notes

Ali Fele Paranj
alifele@studnet.ubc.ca

January 18, 2023

Lecture Notes For: Advanced Linear Algebra

Ali Fele Paranj
alifele@student.ubc.ca

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Definition: definition box

This is an example of a definition box

$$E = mc^2 \tag{0.1}$$

Theorem: sample theorem box

This is an example of a definition box

$$E = mc^2 \tag{0.2}$$

Lemma: sample lem box

This is an example of a definition box

$$E = mc^2 \tag{0.3}$$

Corollary: sample cor box

This is an example of a definition box

$$E = mc^2 \tag{0.4}$$

Proposition: sample prop box

This is an example of a definition box

$$E = mc^2 \tag{0.5}$$

Axiom: sample axiom box

This is an example of a definition box

$$E = mc^2 \tag{0.6}$$

Proof. Here is an example proof □

Example: sample

This is a sample problem in a box

Introduction:
The introduction about the course goes here

Contents

1 Fundamental Concepts

1.1 The Beginnings of Quantum Mechanics

Here is a sample file for a chapter tex file

Here is an example for the equation:

$$I_{\text{Wien}}(\lambda, T) \sim \frac{1}{\lambda^5} \exp(-\frac{1}{\lambda T}) \tag{1.1}$$

Looks great! In the following figure you can see a template for inserting images. Images are stored in a seperate folder in the Images.

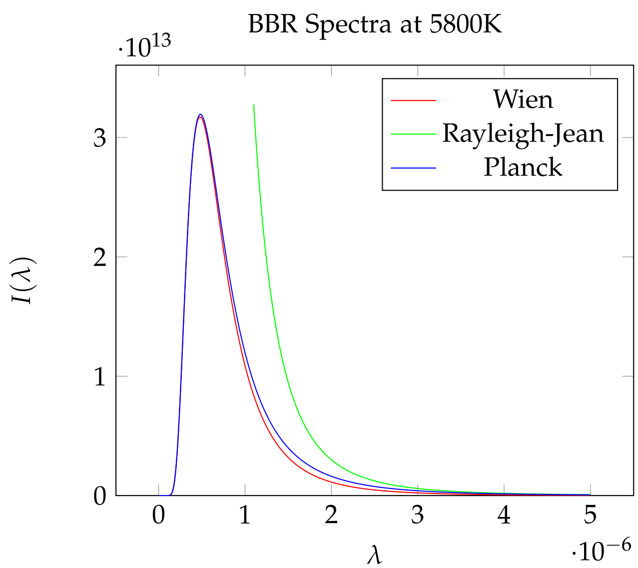

Figure 1.1: This is the caption of the sample image!

Also here is an example how to include a foot note! In the above discussion, we have introduced Planck's constant. It has numerical value[1].

1.2 Kets, Bras, and Hilbert Space

Here is an example of table!

Quantum states	$\lvert\psi\rangle \in \mathcal{H}$	
Evolution	$i\hbar\frac{\partial}{\partial t}\lvert\psi\rangle = H\lvert\psi\rangle$	
Measurement	$\lvert\psi\rangle \mapsto \frac{\Pi_j\lvert\psi\rangle}{\sqrt{\langle\psi\rvert\Pi_j\lvert\psi\rangle}}$	$p(j) = \langle\psi\rvert\Pi_j\lvert\psi\rangle$

Table 1: Axioms of quantum mechanics, concerning states, evolution, and measurement.

Here are some examples for boxes

[1] which is the set/absolute (rather than measured) value of the Planck constant as per the 2018 redefinition of SI units.

Axiom: : Quantum states

Quantum states $|\psi\rangle$ are vectors (also called "kets") in a complex Hilbert space $\mathcal{H}$.

Definition: : (Complex) Hilbert spaces

$\mathcal{H}$ is a (complex) Hilbert space if:

(i) $\mathcal{H}$ is a vector space over $\mathbb{C}$

(ii) $\mathcal{H}$ has an inner product

(iii) $\mathcal{H}$ is complete (with respect to the metric induced by the norm induced by the inner product)[2]- For the purposes of this course, this last point can be ignored.

Definition: : Dual correspondence

To each vector space $\mathcal{H}$, there exists a dual vector space $\mathcal{H}^*$. There is a one-to-one correspondence[3]between the kets $|\psi\rangle \in \mathcal{H}$ and the bras[4] $\langle\psi| \in \mathcal{H}^*$. We call this the *dual correspondence*, and write it as follows:

$$|\psi\rangle \overset{DC}{\longleftrightarrow} \langle\psi|. \tag{1.2}$$

It has the following properties:

(i) $|\psi\rangle + |\varphi\rangle \overset{DC}{\longleftrightarrow} \langle\psi| + \langle\varphi|$

(ii) $c|\psi\rangle \overset{DC}{\longleftrightarrow} c^*\langle\psi|$

where the $*$ denotes complex conjugation.

Proposition: : Resolution of the identity

For all ONBs $\left\{|b_j\rangle\right\}_j$, the following relation holds:

$$\sum_j |b_j\rangle\langle b_j| = \mathbb{I} \tag{1.3}$$

where $\mathbb{I}$ is the identity operator on the Hilbert space.

Here is an example proof

Proof. Recall that $|\psi\rangle = \sum_j \psi_j |b_j\rangle$ for any $|\psi\rangle \in \mathcal{H}$ and for any basis $\left\{|b_j\rangle\right\}_j$ of $\mathcal{H}$. Further, recall that

[2]This is a technical qualification for the mathematicians in the crowd. An intuitive explanation for the curious; the inner product on a Hilbert spaces creates a notion of distance on the space. There are sequences (of vectors) that get closer together over time; completeness tells us that any such sequences (known as Cauchy sequences) must converge to a limit.

[4]Formally, this follows from the Riesz Representation Theorem. But for the purposes of this course, we take this one-to-one correspondence as a postulate. Curious readers can find discussions/proofs of the theorem in any text on functional analysis, or mathematical quantum theory.

[4]Given such names because $\langle|\rangle$ is a bracket - bra-ket. Physicists remain unmatched in their sense of humour.

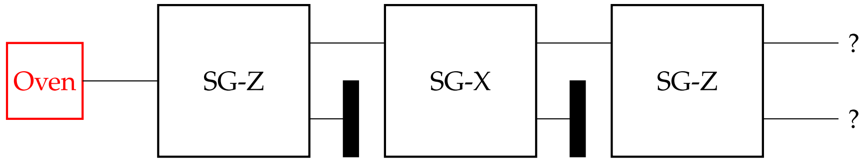

Figure 1.2: Here is an example figure using the latex utilities

$\psi_j = \langle b_j | \psi \rangle$ if the basis is orthonormal. Hence we have that:

$$|\psi\rangle = \sum_j \langle b_j|\psi\rangle |b_j\rangle = \sum_j |b_j\rangle \left(\langle b_j|\psi\rangle\right) = \sum_j \left(|b_j\rangle\langle b_j|\right)|\psi\rangle = \left(\sum_j |b_j\rangle\langle b_j|\right)|\psi\rangle. \tag{1.4}$$

Since the above relation holds for all $|\psi\rangle$, it follows then that $\sum_j |b_j\rangle\langle b_j|$ is the identity as claimed. □

At this point in the course, the reader may be wondering what happened to quantum *wavefunctions*[5]; the central objects of interest have instead been quantum states, without a wavefunction in sight. We now elucidate the connection between the two.

[5] Though this nomenclature of "wavefunction" is arguably a misnomer; the Schrödinger equation does not contain second order derivatives in time, as a wave equation would.