Lecture Notes For: Numerical Methods for Scientific Computing

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In this document, I have organized different numerical methods that are commonly used for scientific computing.

Chapter 1

System of Linear Equations

1.1 Direct Methods

1.1.1 LU Decomposition

1.1.2 RQ Decomposition

1.1.3 Guassian Elimination

1.1.4 Tridiagonal Matrix

1.1.5 Approximate Method

We want to solve the following system of equations:

$$\boldsymbol{A}x = b$$

.

We set the matrix $\boldsymbol{A}$ to be: $\boldsymbol{A} = \boldsymbol{S} - \boldsymbol{T}$, in which $\boldsymbol{S}$ and $\boldsymbol{T}$ are the some matrices which are chosed in a smart way!. Let's plug in the new value of $\boldsymbol{A}$ in the system of linear equations:

$$\begin{aligned}
(\boldsymbol{S} - \boldsymbol{T})x &= b \\
\boldsymbol{S}x &= \boldsymbol{T}x + b \\
x &= \boldsymbol{S}^{-1}(\boldsymbol{T}x + b) = \boldsymbol{S}^{-1}\boldsymbol{T}x + \boldsymbol{S}^{-1}b
\end{aligned}$$

So we will have:

$$\boxed{x = \boldsymbol{S}^{-1}\boldsymbol{T}x + \boldsymbol{S}^{-1}b} \tag{1.1.1}$$

Now let's plug in an initial guess x_0 in RHS of the the equation 1.1.1 and name it x_1:

$$
\begin{aligned}
x_1 &= \boldsymbol{S}^{-1}\boldsymbol{T}x_0 + \boldsymbol{S}^{-1}b \\
x_2 &= \boldsymbol{S}^{-1}\boldsymbol{T}x_1 + \boldsymbol{S}^{-1}b \\
&\vdots \\
x_n &= \boldsymbol{S}^{-1}\boldsymbol{T}x_{n-1} + \boldsymbol{S}^{-1}b
\end{aligned}
$$

To see if we have get closer to the actual solution of the system of equations, let's asume that the actual solution is x. So let's define the following errors:

$$
\begin{aligned}
\epsilon_0 &= x - x_0 \\
\epsilon_1 &= x - x_1 \\
\epsilon_2 &= x - x_2 \\
&\vdots \\
\epsilon_n &= x - x_n
\end{aligned}
$$

insert a guess evaluate the error through the iteration find a convergence rule

1.1.6 Jacobi Method

S is the identity matrix

1.1.7 Guass Seidel Method

S is the lower trianglar matrix

Chapter 2

Matrices

2.1 Eigenvalue and Eigenvectors

2.1.1 Power Method

This is to calculate the largest eigenvalue of a matric