Tensor Decomposition For Logic Circuits

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February 6, 2025

Let A represent a boolean variable. Then observe that

$$A + \overline{A} = 1.$$

We can use this fact to simplify the logical circuits. For instance to implement $f(A, B, C, D) = \overline{A}BCD + A\overline{B}CD$ as it is we need more logic gates to implement its equivalent $f(A, B, C, D) = CD$. Karnaugh map makes such simplification possible for any logic function. For instance, consider the following truth table:

A	B	C	D	F(A,B,C,D)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

We can write this function as

$$F(A, B, C, D) = \overline{A}BC\overline{D} + A\overline{B}\overline{C}\overline{D} + A\overline{B}\overline{C}D + A\overline{B}C\overline{D} + A\overline{B}CD + AB\overline{C}\overline{D} + AB\overline{C}D + ABC\overline{D}.$$

To decompose 4-linear form and then use the identity above simplify the function we can use the Karnaugh map.

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	1
11	1	1	0	1
10	1	1	1	1

The grouped boxes above shows which terms can be grouped to make a simpler term. For instance, the green box above show the following terms that can be grouped easily:

$$
\begin{aligned}
&AB\overline{C}\overline{D} + AB\overline{C}D + A\overline{B}\overline{C}\overline{D} + A\overline{B}\overline{C}D \\
&= AB\overline{C}(D + \overline{D}) + A\overline{B}\overline{C}(D + \overline{D}) \\
&= A\overline{C}(B + \overline{B}) \\
&= A\overline{C}.
\end{aligned}
$$

With a similar reasoning, the red box results in the term $A\overline{B}$ and the yellow box results in the term $BC\overline{D}$. So the simplified function will be

$$F(A, B, C, D) = A\overline{C} + A\overline{B} + BC\overline{D}.$$