Manifolds

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1. Smooth Manifolds

1.1 Problems

■ **Problem 1.1** Let X ge the set of all points $(x, y) \in \mathbb{R}^2$ such that $y = \pm 1$, and let M be the quotient of X be the equivalence relation generated by $(x, -1) \sim (x, 1)$ for all $x \neq 0$. Show that M is locally Euclidean and second countable, but not Hausdorff. This space is called the line with two origins.

Solution Denote the topological space $Y = X/\sim$. The open sets in Y are those that their pre-image under the quotient map $\pi : X \mapsto X/\sim$ is open. Denote the open sets around the origins as I, I_+, and I_-, where $\pi((0, -1)) \in I_-$, $\pi((0, 1)) \in I_+$, and bot holds for I.