

The background of the entire page is a fractal image of the Mandelbrot set. It features a large, bright yellow and orange central region that tapers into a long, narrow vertical strip extending downwards. This central strip is flanked by dark, intricate, and self-similar fractal patterns that resemble the edges of the Mandelbrot set. The overall effect is a symmetrical, highly detailed, and colorful mathematical visualization.

Complex Analysis

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0. Contents

1 Introduction	5
1.1 Solved Problems	5

1. Introduction

1.1 Solved Problems

■ **Problem 1.1 — Convergence of complex variables.** Let $\{z_n = a_n + ib_n\}_{n \in \mathbb{N}}$ for some $a_n \in \mathbb{R}, b_n \in \mathbb{R}$ be a sequence of complex numbers. Show that z_n converges to $w = \alpha + i\beta$ if and only if $a_n \rightarrow \alpha$ and $b_n \rightarrow \beta$ as $n \rightarrow \infty$.

Proof. The proof is as follows

\Rightarrow Let otherwise. Without loss of generality, we can assume that a_n does not converge to α . Then $\exists \epsilon > 0$ such that $\forall N > 0$ we can find $n > N$ for which $|a_n - \alpha| > \epsilon$. This implies that

$$|(a_n - \alpha) + i(b_n - \beta)|^2 = |a_n - \alpha|^2 + |b_n - \beta|^2 > \epsilon^2$$

which implies

$$|z_n - w| = |(a_n - \alpha) + i(b_n - \beta)| > \epsilon$$

for some ϵ and for some $n > N$ for any choice of N . This is a contradiction, since implies z_n is not converging to w .

\Leftarrow Assume $a_n \rightarrow a$ and $b_n \rightarrow b$ as $n \rightarrow \infty$. Fix $\epsilon > 0$. Let N be large enough such that

$$|a_n - \alpha| < \epsilon^2/2, \quad |b_n - \beta| < \epsilon^2/2.$$

Then we can write

$$|(a_n - \alpha) + i(b_n - \beta)|^2 = |a_n - \alpha|^2 + |b_n - \beta|^2 < \epsilon^2.$$

This implies that z_n converges to w .

□

■ **Problem 1.2 — Completeness of \mathbb{C} .** Prove that the set of all complex numbers \mathbb{C} is complete.

Proof. The convergence of a complex number is equivalent to the convergence of its real and imaginary parts. Since \mathbb{R} is complete, it follows that \mathbb{C} is also complete, □