

$n = 1 :$

we only have one answer  $sum = |s[1] - h[1]|$

$n > 1 :$

first sort array  $s$  and  $h$  in ascending order

$$s = [s_1, s_2, \dots, s_n]$$

$$h = [h_1, h_2, \dots, h_n]$$

after sorting we have  $s_a < s_b, h_a < h_b$  ( $a < b$ )

Upon applying the algorithm, we have  $sum = \sum_{i=1}^n |s[i] - h[i]|$

### Proving by contradiction

Assume we can find a pair  $|s_a - h_b| + |s_b - h_a|$  ( $a < b$ ) to lower the sum

Which means  $|s_a - h_a| + |s_b - h_b| \geq |s_a - h_b| + |s_b - h_a|$

$$s_b - s_a = X, X \geq 0$$

$$h_b - h_a = Y, Y \geq 0$$

$$|s_a - h_a| + |s_b - h_b| = |s_a - h_a| + |s_a - h_a + X - Y|$$

$$|s_a - h_b| + |s_b - h_a| = |s_a - h_a - Y| + |s_a + X - h_a|$$

if  $s_a \geq h_a$

$$|s_a - h_a| + |s_b - h_b| = s_a - h_a + |s_a - h_a + X - Y|$$

$$|s_a - h_b| + |s_b - h_a| = s_a - h_a + X + |s_a - h_a - Y|$$

$$\text{since } X + |s_a - h_a - Y| \geq |s_a - h_a + X - Y|$$

$$\text{Therefore } |s_a - h_a| + |s_b - h_b| \leq |s_a - h_b| + |s_b - h_a|$$

which contradict with our assumption  $|s_a - h_a| + |s_b - h_b| \geq |s_a - h_b| + |s_b - h_a|$

if  $s_a < h_a$

$$|s_a - h_a| + |s_b - h_b| = h_a - s_a + |s_a - h_a + X - Y|$$

$$|s_a - h_b| + |s_b - h_a| = h_a - s_a + X + |s_a - h_a - Y|$$

$$\text{since } X + |s_a - h_a - Y| \geq |s_a - h_a + X - Y|$$

$$\text{Therefore } |s_a - h_a| + |s_b - h_b| \leq |s_a - h_b| + |s_b - h_a|$$

which contradict with our assumption  $|s_a - h_a| + |s_b - h_b| \geq |s_a - h_b| + |s_b - h_a|$

Thus, changing any pair  $s[i]$  and  $h[i]$  from sorted arrays will make the sum same or bigger