

Q2.4) Suppose I give you \$15 to play the online Guess My Word game. Every time you make a guess, you give me \$1.

If you agree to play this game with me, do you expect to win money or lose money? Clearly justify your answer.

(Assume that each of the 267,751 words is equally likely to be chosen.)

Here, the algorithm is recursively calling the function by dividing the array in to two halves at each iteration in $n/2$ and checking the middle word. If a match occurs, then the word is guessed. If the middle word comes after the word that needs to be guessed, then the word is searched in the sub-array to the left of the middle word. Otherwise, the word is searched for in the sub-array to the right of the middle word. This process continues on the sub-array as well until the size of the subarray reduces to 1 and the word is found.

So, it is given in the question that each of the 267,751 words is equally likely to be chosen.

According to the recurrence relation, $T(n) = \Theta(\log n)$ where n is the number of words in the dictionary, the running time of the game would be $\log(267,751) = 18.0305$ which means that the word can be guessed in **at most 19 guesses** in the worst case.

I have \$15 to play the online Guess My Word game. Every time I make a guess, I give \$1 back. This means that the average guesses I will have to make to guess the word correctly will be 19 guesses. Therefore, I will have to give \$19, $\$15 - \$19 = \$-4$, which means I will **lose \$4**.

Considering all the possible cases to play and win this game. If the word is a middle word then the word can be guessed in 1 guess. So, if the guesses taken is anything from the range 1 to 14, then I will win the money which be $(\$15 - \text{guessesTaken})$. If I guess the word in 16 guesses, then I will lose \$1. Similarly, if I guess the word in 17 guesses, then I will lose \$2. And, if I guess the word in 18 guesses, then I will lose \$3. And, finally, if I guess the word in 19 guesses, then I will lose \$4.