Q3.4) Let *A* be a random permutation of [1,2,3,4,5,6,7,8]. Determine the probability that exactly 12 comparisons are required by Merge Sort to sort the input array *A*. Clearly and carefully justify your answer.

A random permutation of array A = [1,2,3,4,5,6,7,8] will result in 8! cases where the elements ranging from 1 to 8 will be arranged in a random manner.

Now, checking for cases where probability of Merge Sort to sort an input array with exactly 12 comparisons refers to the case of minimum number comparisons for array size 8.

As we have previously calculated the number of minimum comparisons is,
M(n) = nlogn/2

Here, M(8) = 8log(8)/2 = 24/2 =12 comparisons.

MergeSort will have **minimum number of comparisons** when the largest element of one sorted sub-list is smaller than the first element of its opposing sub-list, for every merge step that occurs.
Only one element from the opposing list is compared, which reduces the number of comparisons in each merge step from n to **n/2**.

So, we need to compute base cases for which one subarray out of the two is exhausted at every step and the opposing sorted subarray is carried over as it is, which would result in 4 comparisons each at 3 levels. So, total 12 comparisons.

A = [1,2,3,4,5,6,7,8]
Each of the four colored groups will require 2 computations each as we can swap 2,1 and 1,2.
So, cases = 2*2*2*2 = 16

Next, we check for grouping of 2 numbers within the 4 numbers keeping the order same.
A = [1,2,3,4,5,6,7,8]
So, cases will be 2*2 = 4

Next, we check for grouping of 4 numbers within the 8 numbers keeping the order same.
A = [1,2,3,4,5,6,7,8]
So, cases will be 2 which are A = [5,6,7,8,1,2,3,4] and A = [1,2,3,4,5,6,7,8].

Now, the idea here is to keep the difference between adjacent numbers 1 in the array and also to arrange the numbers 1,2,3,4 and 5,6,7,8 with each other.

So, the probability that exactly 12 comparisons are required by Merge Sort to sort the input array *A* is 16*4*2/8! = 128/40320 = 1/315.