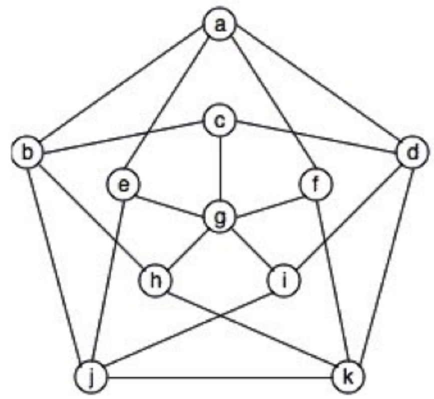
**Q5.1) Given a graph *G*, we say that *G* is *k-colorable* if every vertex of *G* can be assigned one of *k* colors so that for every pair *u*,*v* of adjacent vertices, *u* and *v* are assigned different colors.**
**The chromatic number of a graph *G*, denoted by $\chi(G)$, is the smallest integer *k* for which graph *G* is *k*-colorable. To show that $\chi(G)=k$, you must show that the graph is *k-colorable* and that the graph is not $(k-1)$-colorable.**

**Let *G* be the graph below, with 11 vertices and 20 edges. Clearly explain why $\chi(G)=4$**



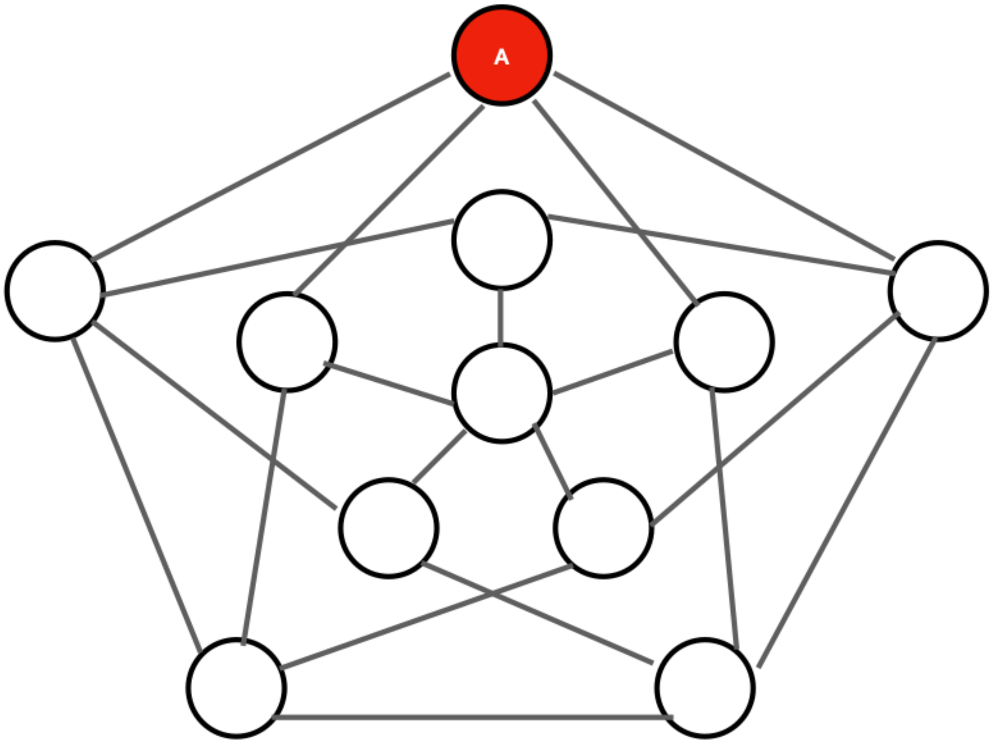

The chromatic number of the graph is the minimum number of colors needed to produce a proper coloring of the graph. A ***proper coloring*** is an assignment of colors to the vertices of a graph so that no two adjacent vertices have the same color. A ***k-coloring*** of a graph is a proper coloring involving a total of k colors. A graph that has a k-coloring is said to be ***k-colorable***.

So, what we seek is a ***k-coloring*** of our graph with k as small as possible.

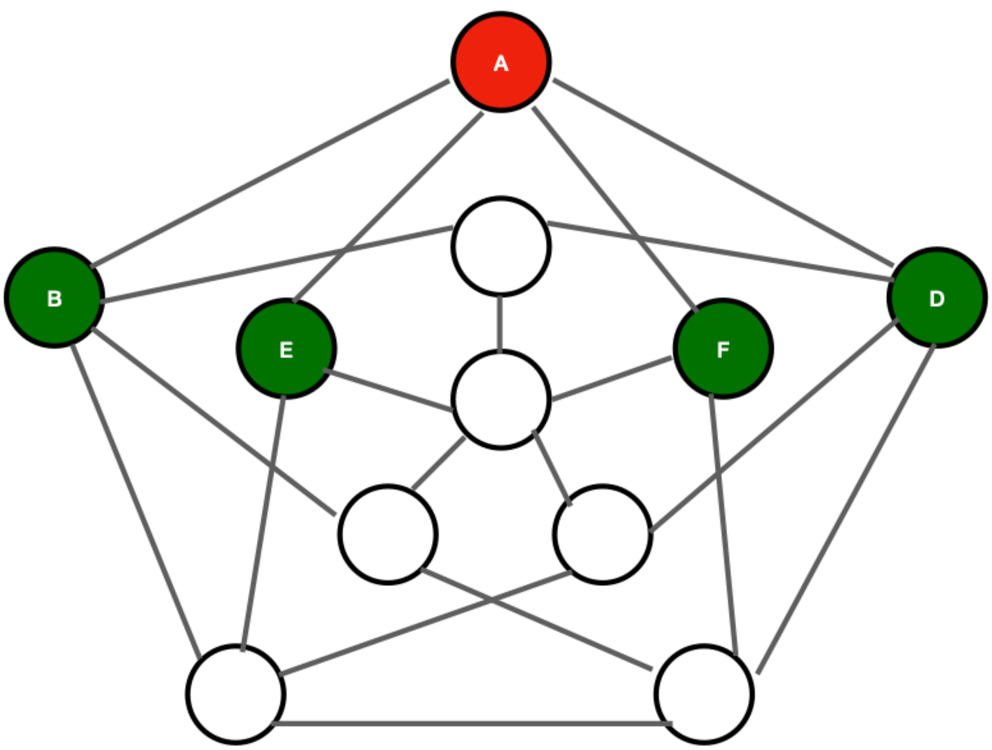
Now, we know that a vertex is assigned one of the k colors and every pair u,v of adjacent vertices, u and v are assigned different colors.

We define the k-color list as follows,
k-colors = [red, green, yellow, blue, pink, purple, gray, orange, brown, black]

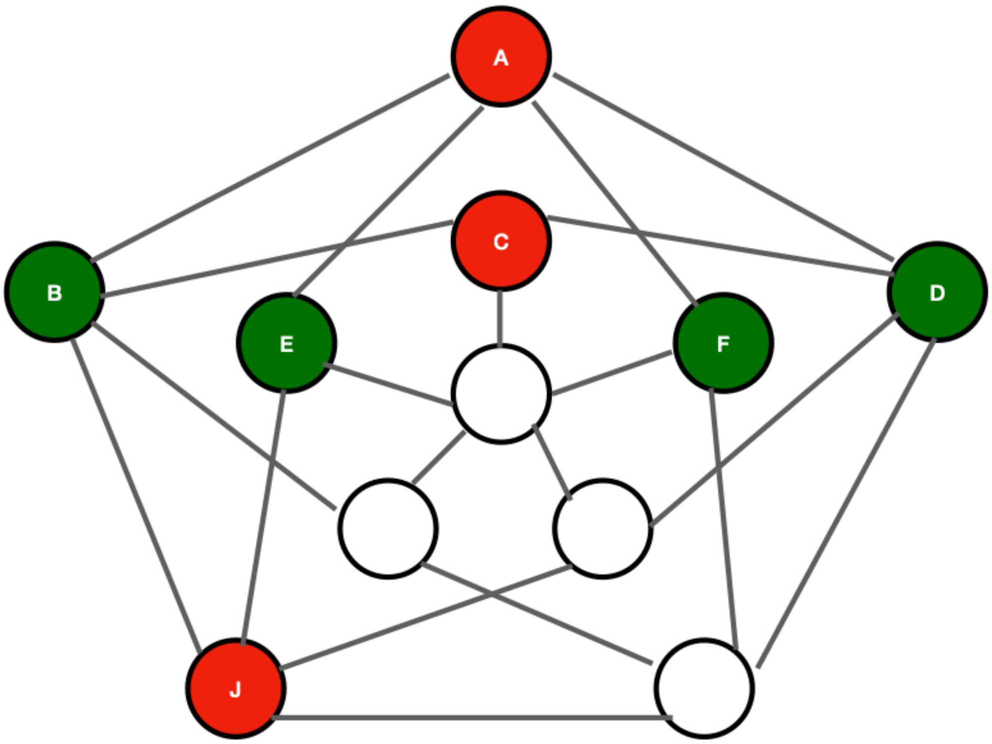
So, starting with vertex A, let us assign the vertex one of the k colors., which is red. So, the graph G would look like,



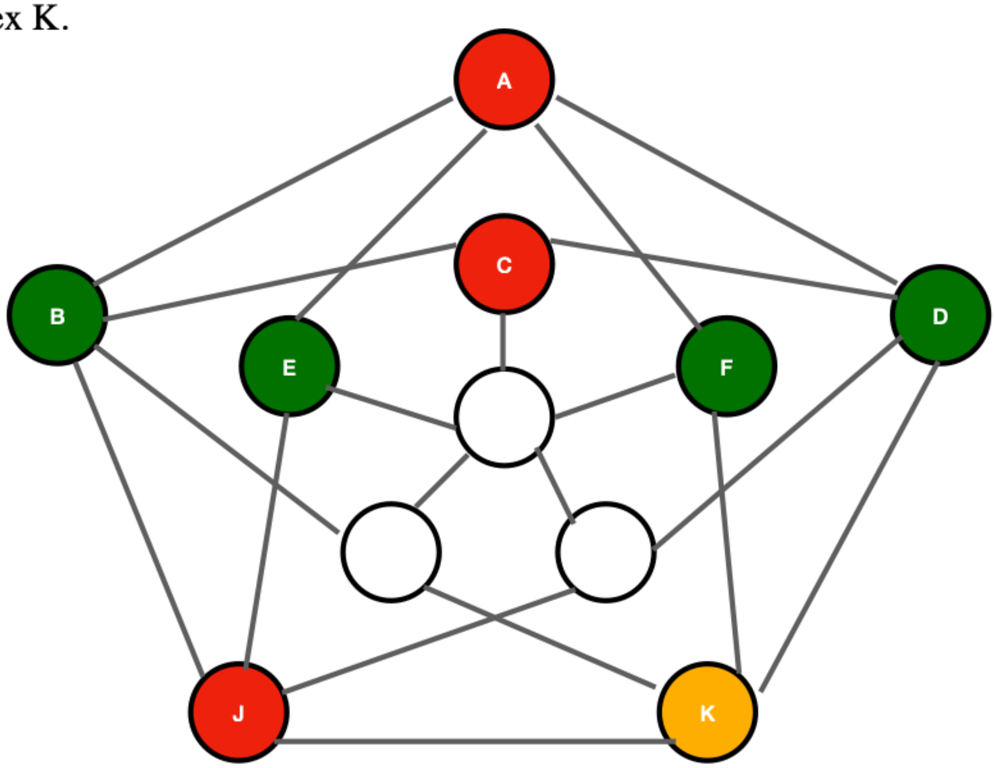

So, now adjacent vertex A are vertices {B, E, F, D}. So, we cannot assign the same color. Let us assign the next green k-color from the list to the vertices B, E, F and D as they do not share edges with each other.



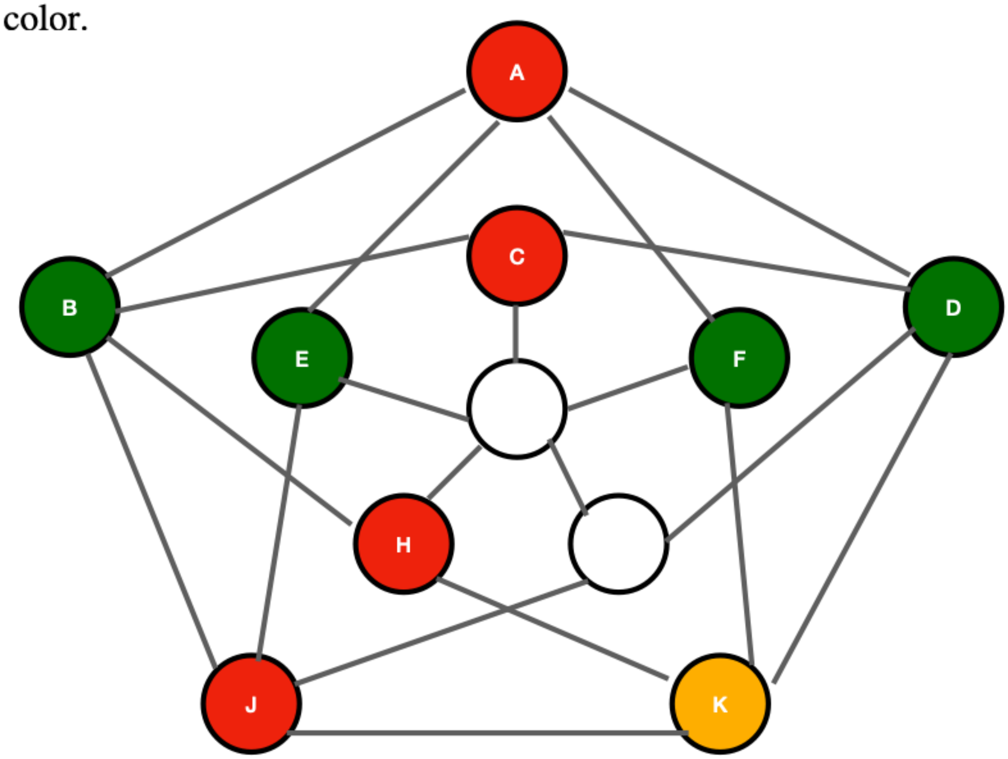
Now, vertex B shares an edge with vertex J and C and both them do not share an edge with vertex A. So, we can assign the red color to vertex J and C.



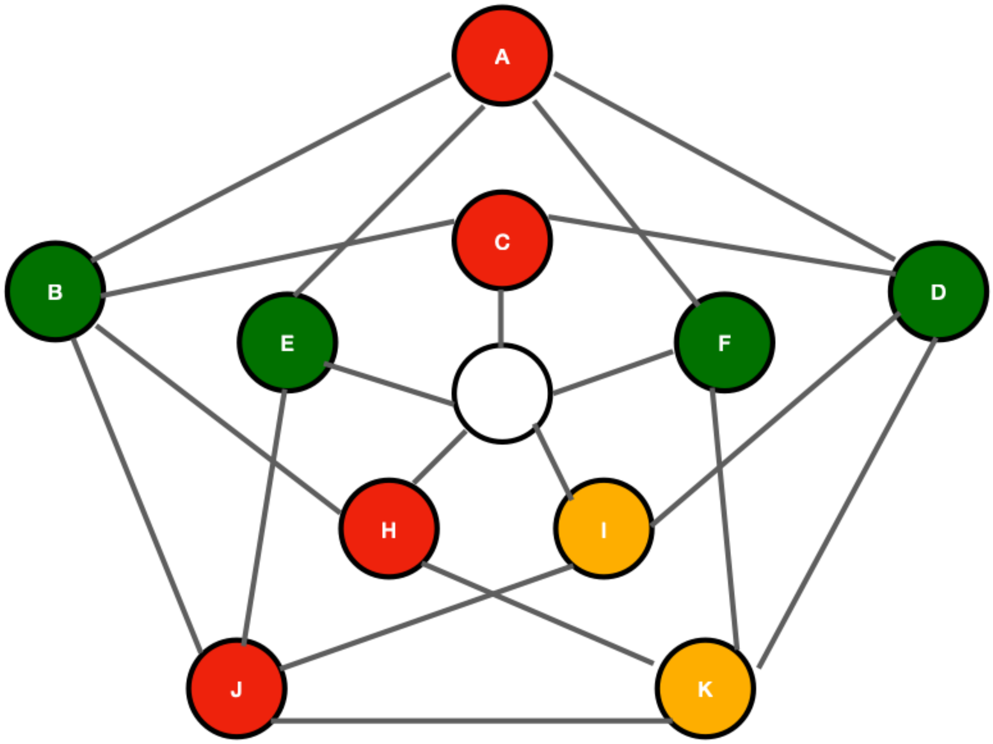

Now, vertex J and D in outer ring shares an edge with vertex K, so we cannot assign red and green color to K. Lets assign a new yellow k-color from the list to vertex K.



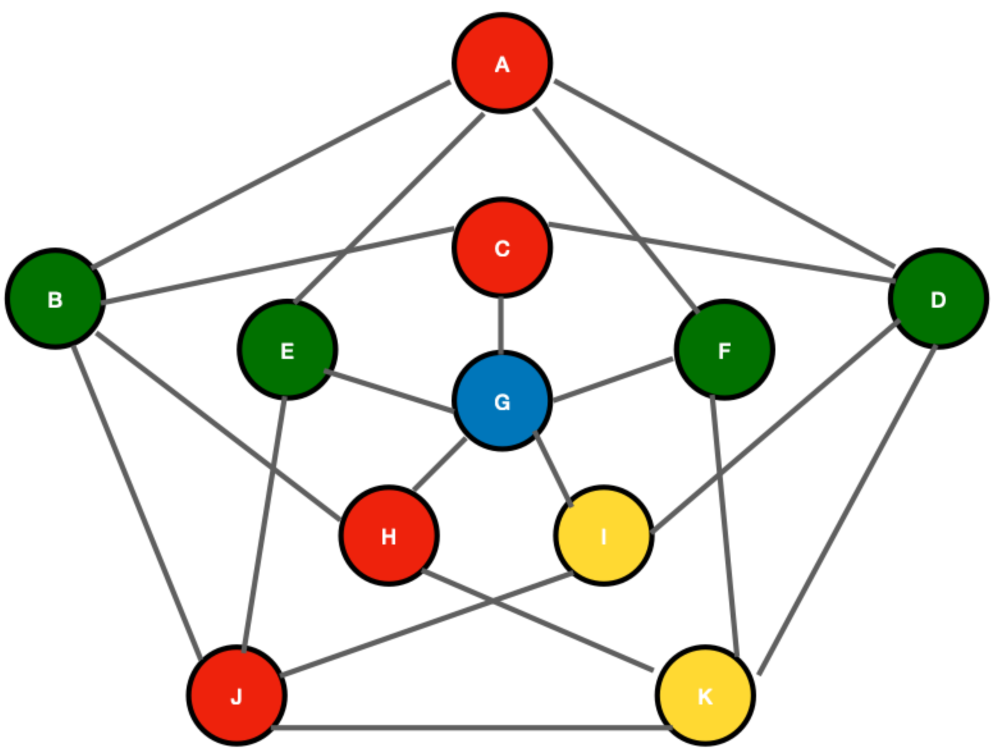
Now, we can see that the outer ring is closed with 3-colors. Moving to the inner ring, we can see that vertex B and K share an edge with vertex H in the inner ring, so we cannot assign green and yellow color to the vertex H. But we can assign red color from the visited k-colors as it does not share an edge with the vertex which has red color.




Now, we can see that vertex J and D in the outer ring share an edge with vertex I in the inner ring, so we cannot assign green and red color to the vertex H. But we can assign yellow color from the visited k-colors as it does not share an edge with the vertex which has yellow color.




Now, we can outer and inner ring is closed with 3-colors, and the vertex left is G which has to be assigned a color. But we cannot assign the 3 already visited colors from the list, as vertex G shares an edge with vertices which has red, green and yellow colors which are {C, E, F, H, I}. So, we have no other choice but to assign a new blue k-color from the list to vertex G.



**Finally, we can see the graph is k-colored as every vertex of G is assigned one of the k-colors and number of colors needed for the proper coloring of the graph is 4.**

We know that the chromatic number of a graph $G$, denoted by $\chi(G)$, is the smallest integer $k$ for which graph $G$ is $k$-colorable.
**In this graph, we used 4 different colors to assign colors to all the vertices for which the graph G is proper colored. Hence, $\chi(G) = 4$, for graph G with 11 vertices and 20 edges.**