Portfolio Analysis Report

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Introduction

Portfolio theory, introduced by Harry Markowitz in 1952, transformed investment strategies by emphasizing the importance of diversification. It highlights that asset returns should be considered within the context of the entire portfolio, not in isolation. Combining assets with different risk and return profiles can minimize overall portfolio risk and achieve a better risk-return balance. The theory proposes the "efficient frontier," representing optimal portfolios that deliver the highest expected return for a given level of risk. This systematic approach empowers investors to make informed, data-driven decisions to manage uncertainty and align investments with their risk tolerance and financial goals.

Historical Annual Returns

- X values: [6.6, 5.6, -9.0, 12.6, 14.0]
- Y values: [24.5, -5.9, 19.9, -7.8, 14.8]
- Weights: $w_X = 0.6$, $w_Y = 0.4$

Calculated Statistics

- Mean of X: 5.96
- Mean of Y: 9.1
- Std Dev of X: 8.16
- Std Dev of Y: 13.39
- Pearson Correlation: -0.39

Pearson Correlation Coefficient

The Pearson correlation coefficient (r_{XY}) measures the linear relationship between two variables, X and Y. It is calculated as:

$$r_{XY} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{(n-1)\sigma_X \sigma_Y}$$

where:

- X_i and Y_i are individual data points of X and Y,
- $\bar{X}$ and $\bar{Y}$ are the sample means,
- σ_X and σ_Y are the sample standard deviations,
- n is the number of observations.

Portfolio Statistics

$$\bar{P} = w_X \bar{X} + w_Y \bar{Y} = 7.22$$

$$\sigma_P = \sqrt{(w_X \sigma_X)^2 + (w_Y \sigma_Y)^2 + 2 w_X w_Y \sigma_X \sigma_Y r_{XY}} = 5.68$$

Where:

- w_X and w_Y are the weights of assets X and Y in the portfolio.
- σ_X and σ_Y are the standard deviations (risks) of assets X and Y.
- r_{XY} is the Pearson correlation coefficient between X and Y.

Efficient Frontier Data Table

	All X	**A**	**B**	**C**	**D**	**All Y**
X Weight	1.0	0.8	0.6	0.4	0.2	0.0
Y Weight	0.0	0.2	0.4	0.6	0.8	1.0
Return	5.96	6.59	7.22	7.84	8.47	9.1
Risk	8.16	6.02	5.68	7.4	10.19	13.39

Table 1: Efficient Frontier Data

Efficient Frontier Graph

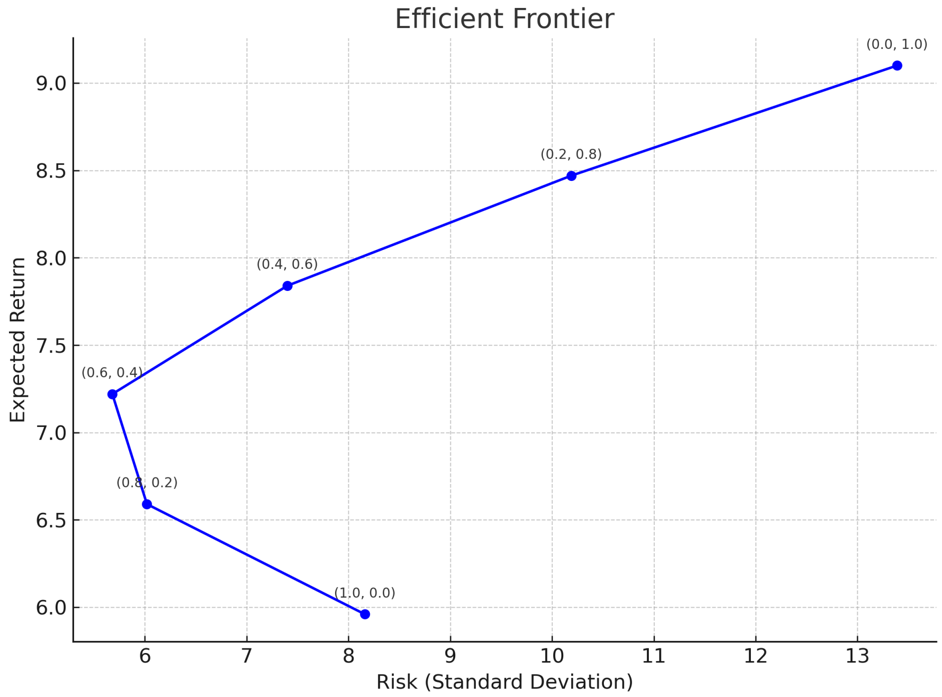

Figure 1: Efficient Frontier

Conclusion

This analysis demonstrates how combining two assets, X and Y, with different risk and return profiles can significantly reduce overall portfolio risk. Although asset X has a standard deviation of 8.16 and asset Y has a standard deviation of 13.39, their negative correlation ($r_{XY} = -0.39$) allows for risk reduction when combined in a portfolio. Specifically, by allocating 60% of the portfolio to asset X and 40% to asset Y, the portfolio's overall risk is reduced to 5.68.

This reduction is a result of diversification: the assets' returns do not move perfectly together, so their combined fluctuations offset each other to some extent. The efficient frontier data illustrates the trade-off between expected return and risk for different asset weightings, with the portfolio lying on the frontier where return is maximized for a given level of risk. Investors can use this trade-off to choose a portfolio that aligns with their risk tolerance and desired return, striking a balance between higher expected returns and the comfort of reduced risk.