New directions in earthquake seismology

Introduction to Back-projection

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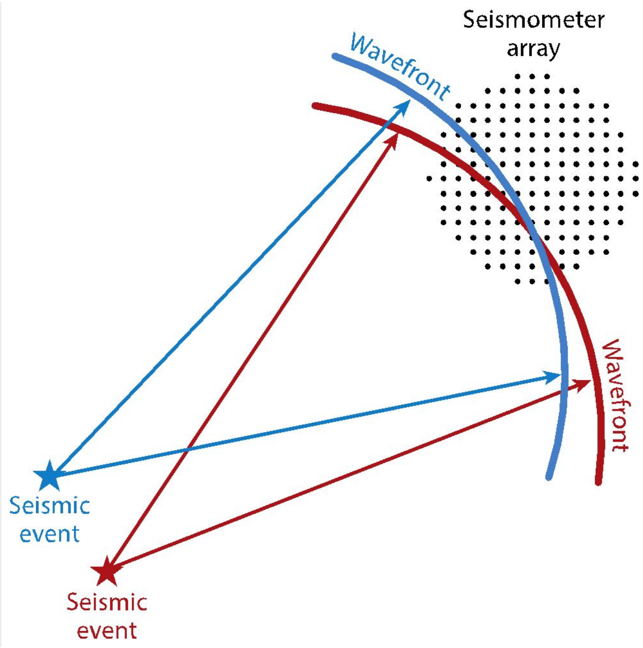
November 12, 2020

Back-projection



The dense array of seismometers that are now available around the world.

The wavefront observed by the array is collapsed back in space and time (back-projected) to the target region to determine the timing and location of the energy source that generated the seismic waves.

http://www.seismology.harvard.edu/research/backproj.html

Outline

- Array Seismology

  Introduction

  Beamforming & Beam back projection

- Back Projection

  Wave front--travel time

  Calibration

- Array Response Function (ARF)

Array definition

An array is a systematic arrangement of objects, e.g.
in computer science : a compound data type whose elements are selected by one or more indices or keys, theoretically equivalent to a vector or an n-tuple

in engineering : an antenna array (radar), a telescope array (astronomy), microphone array or directional sound speaker array (speech, acoustic)

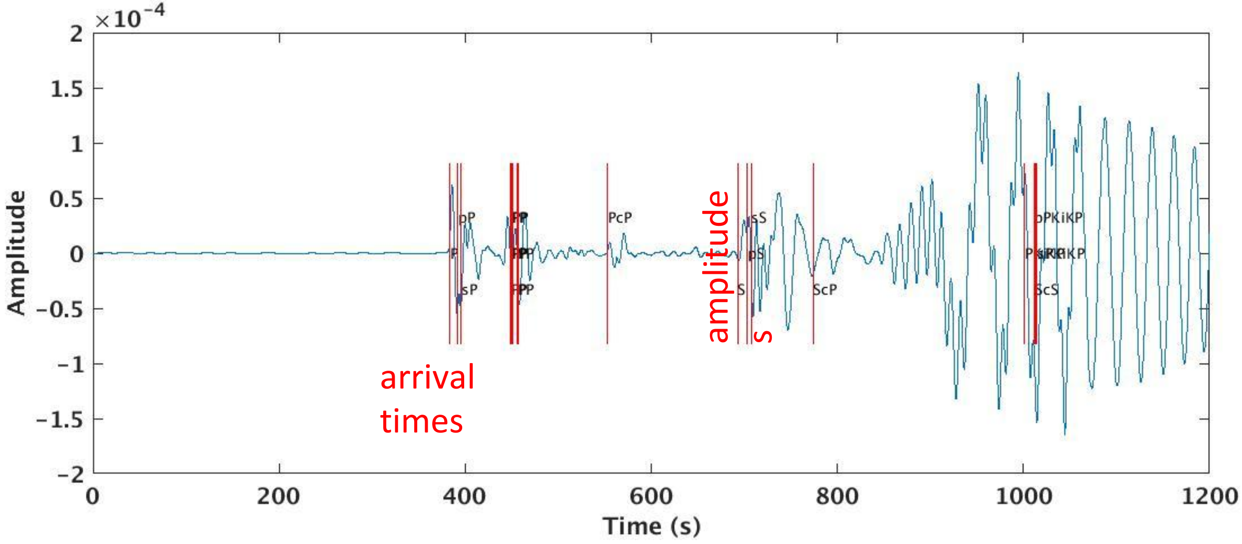
in seismology : a spatially distributed set of seismological sensors (geophones, seismometers, accelerometers usually deployed along the earth’s surface) recording with a common time base.

(Ohrnberger, 2009)

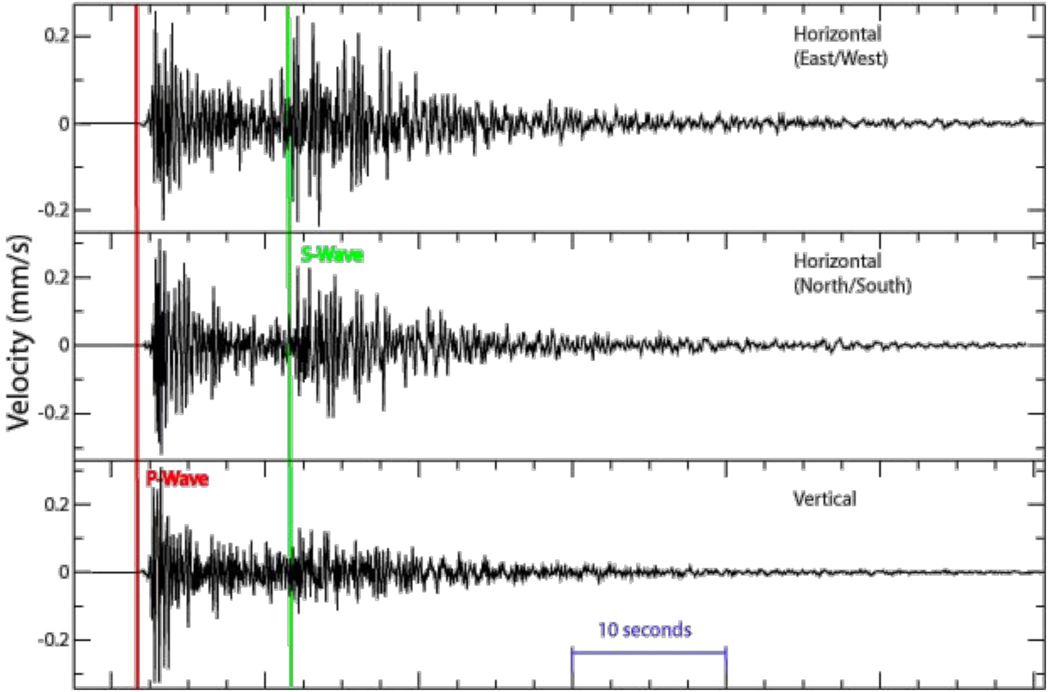
Aperture (largest horizontal distance between two sensors of the array)
Instrument Spacing
Number of array stations
Array geometry/configuration

Array Seismology

Single station & Single component :

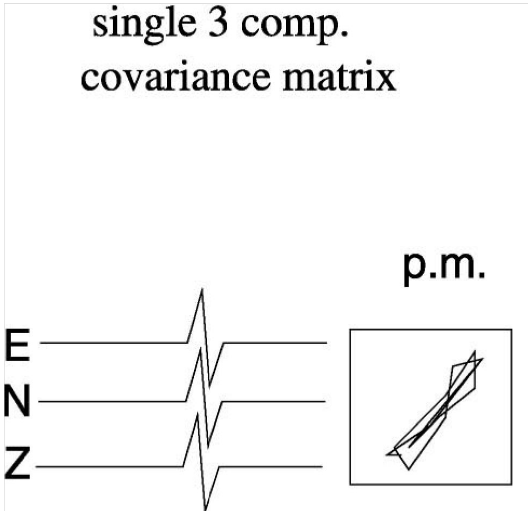


Single station & Three components



arrival times & amplitudes

https://earthscience.stackexchange.com/questions/2430/interpretation-of-a-seismogram-three-components

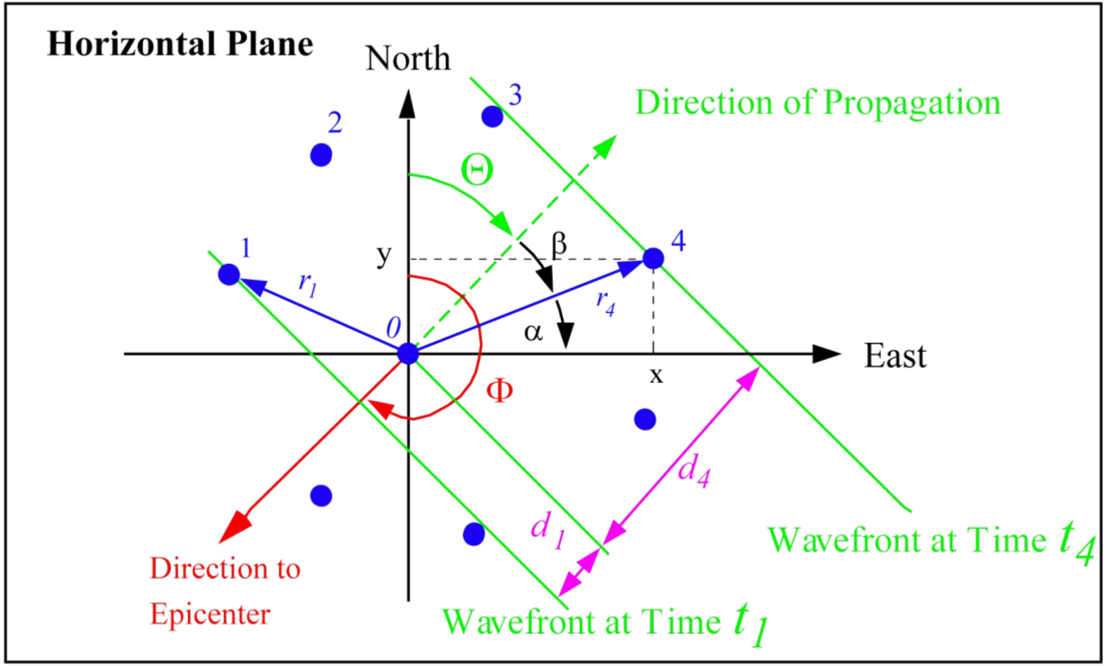


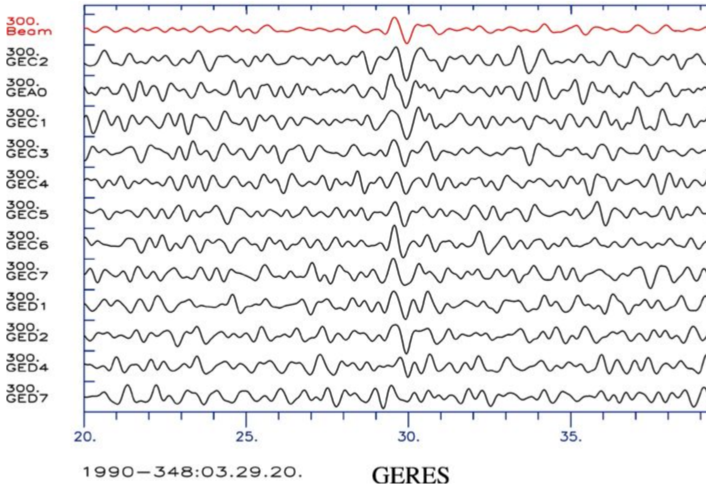
(Rost & Thomas, 2002)

polarization (local particle motion)

Array Seismology

seismic array (1 or 3 components)





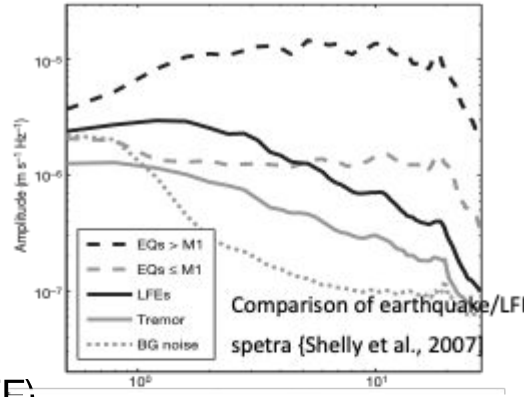
(Schweitzer et al., 2011)

arrival times, amplitudes, (polarization)

direction of wave propagation apparent velocity of wave propagation, signal to noise ratio (SNR) improvement

Array Benefits

- **Improve SNR**: $S \approx \sqrt{M}\, s$
- **Determine signal characteristics** (wave propagation direction and apparent wave speed)
- **Filter good from bad signals**



Comparison of earthquake/LFE/tremor spetra (Shelly et al., 2007)

Applications based on the benefits

- investigation of weak phase arrivals and wave propagation phenomena related to path and site
- weak signal detection (induced seismicity, tremor, LFE, VLFE)
- direct imaging of source processes by tracking spatio-temporal evolution of seismic wave radiation

Array Seismology

Array Seismology: Use multiple observations of the wavefield recorded at array stations distributed in space (usually Earth's surface) and combines those observations to an output quantity using certain predictions about the spatio-temporal characteristics of the wavefield

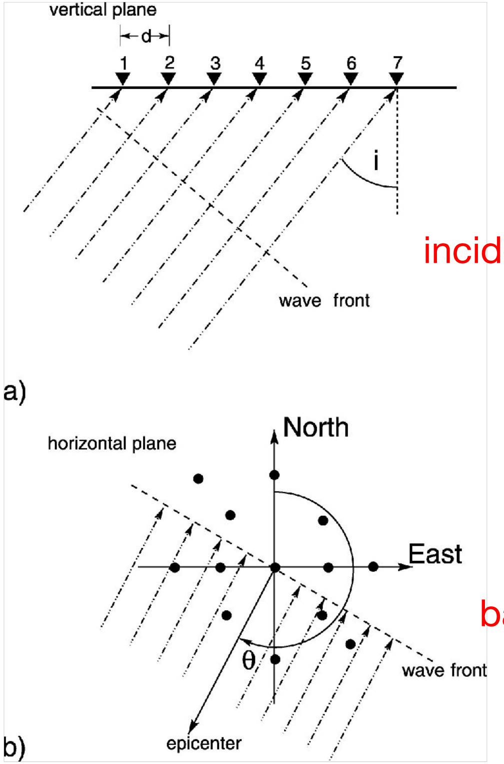
(Ohrnberger, 2009)

Wave propagation model: Plane wave propagation

>10 wavelengths

slowness *u*

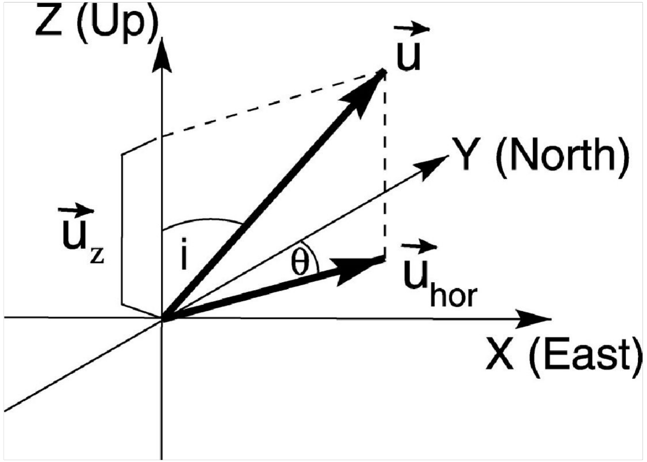
$$u = \frac{1}{v_{\mathrm{app}}} = \frac{\sin i}{v_0}$$



incident angle *i*

back azimuth θ

(a) The vertical plane of an incident wave front crossing an array at an angle of incidence *i*. (b) Sketch of the horizontal plane of an incident plane wave arriving with a back azimuth θ. (Rost & Thomas, 2002)

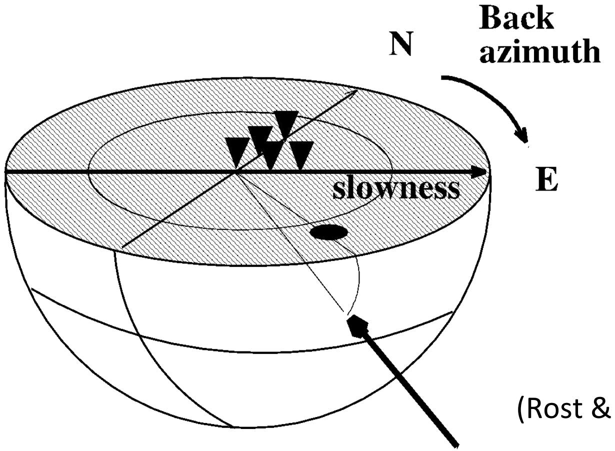


$$\mathbf{u} = (u_x, u_y, u_z)$$

$$= \left(\frac{\sin\theta}{v_{\text{app}}}, \frac{\cos\theta}{v_{\text{app}}}, \frac{1}{v_{\text{app}}\tan i}\right)$$

$$= u_{\text{hor}}\left(\sin\theta, \cos\theta, \frac{1}{\tan i}\right)$$

$$= \frac{1}{v_0}(\sin i\ \sin\theta, \sin i\ \cos\theta, \cos i)$$



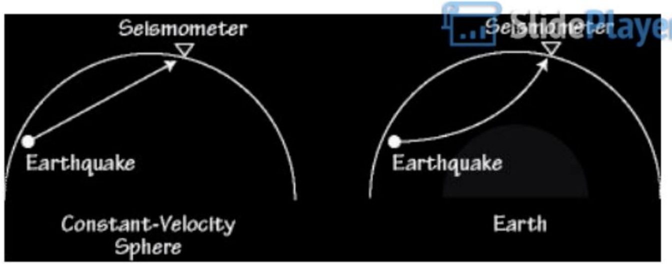
(Rost & Thomas, 2002)

direction where the wave came from

Array Seismology

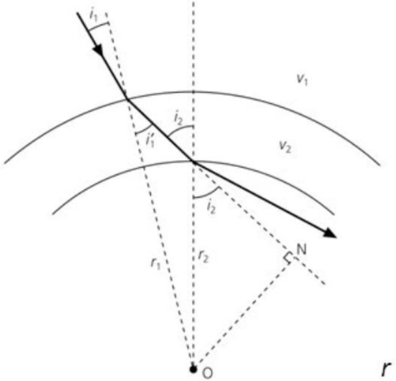
Seismic Waves in a spherical Earth

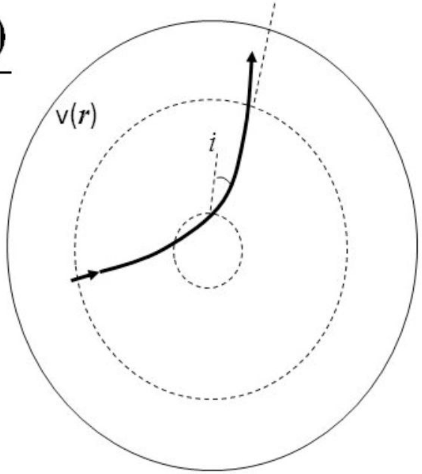
Ray parameter in a spherical coordinate system



$$p = \frac{r \sin(i)}{c(r)}$$

Figure 3.4-1: Geometry of Snell's law for a spherical earth.





r being the distance from the centre of symmetry

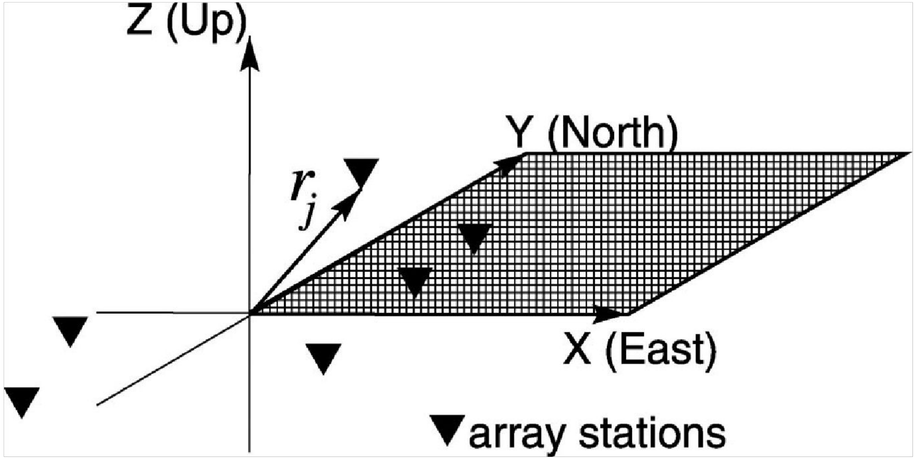
$$p = \frac{r \sin i}{v} = \frac{R_0 \sin i}{v_0} = R_0\, u_{\text{hor}}$$

R0 is the distance of the turning point of the ray from the Earth's center
The velocity at the turning point of the ray can be determined from the slowness.
The slowness is a way to identify different phases traveling through the Earth's interior

https://slideplayer.com/slide/3266366/

Beam Forming- separate coherent signals and noise

Use the differential travel times of the plane wave front due to a specific slowness and back azimuth to individual array stations. All signals with the matching back azimuth and slowness will sum constructively



(Rost & Thomas, 2002)

$$x_{\text{center}}(t) = f(t) + n_i(t)$$

Station i

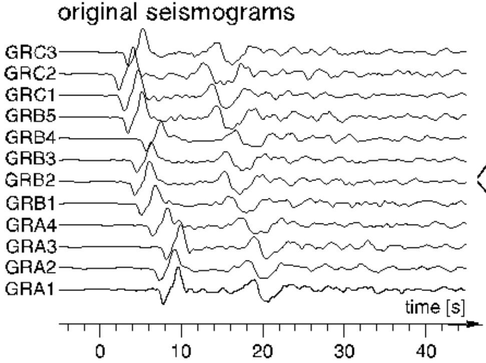
$$x_i(t) = f(t - \mathbf{r}_i \cdot \mathbf{u}_{\text{hor}}) + n_i(t)$$

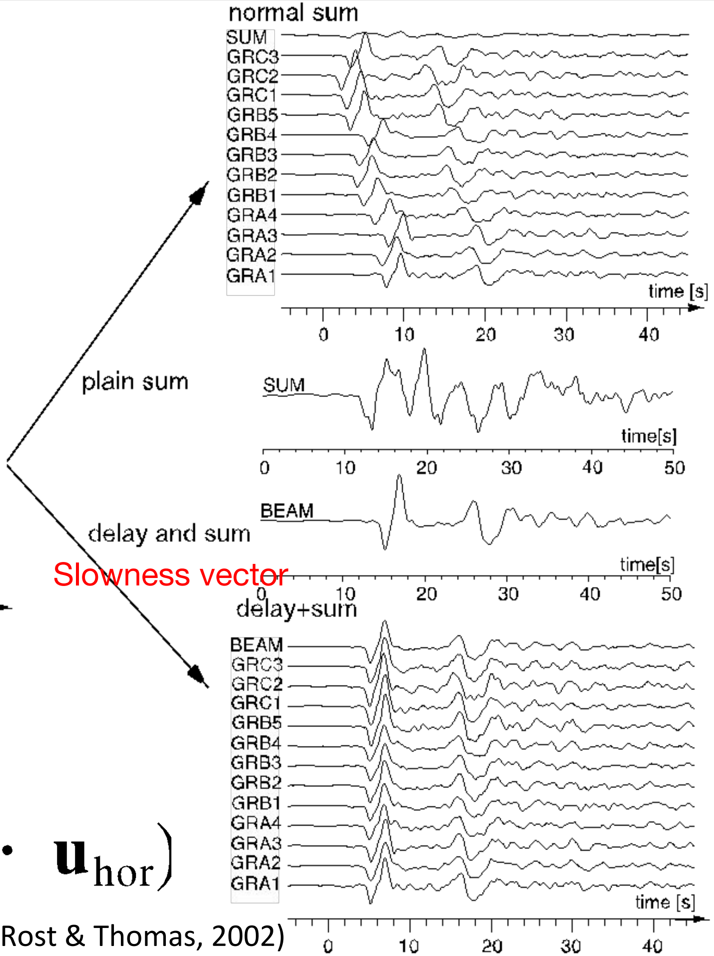
Station i remove the time shift

$$\tilde{x}_i(t) = x_i(t + \mathbf{r}_i \cdot \mathbf{u}_{\text{hor}})$$
$$= f(t) + n_i(t + \mathbf{r}_i \cdot \mathbf{u}_{\text{hor}})$$

Array Seismology

Amplify phases with the appropriate slowness, while suppressing incoherent noise and phases with different slowness





$$b(t) = \frac{1}{M}\sum_{i=1}^{M}\tilde{x}_i(t) = f(t) + \frac{1}{M}\sum_{i=1}^{M} n_i(t + \mathbf{r}_i \cdot \mathbf{u}_{\mathrm{hor}})$$

(Rost & Thomas, 2002)

Array application

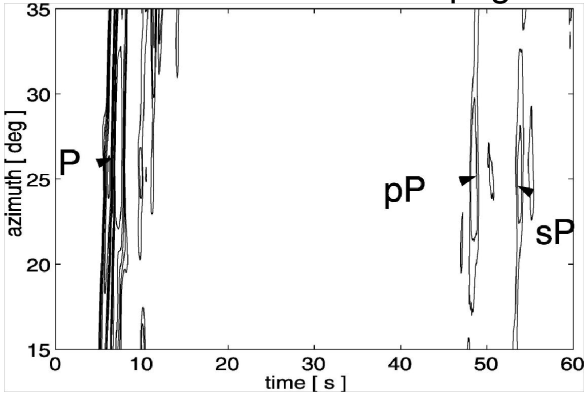
Vespa Process–Slant Stacks-determine the unknown horizontal slowness or the back azimuth

Estimate the seismic energy arriving at the array for a given back azimuth and different horizontal slownesses *u*

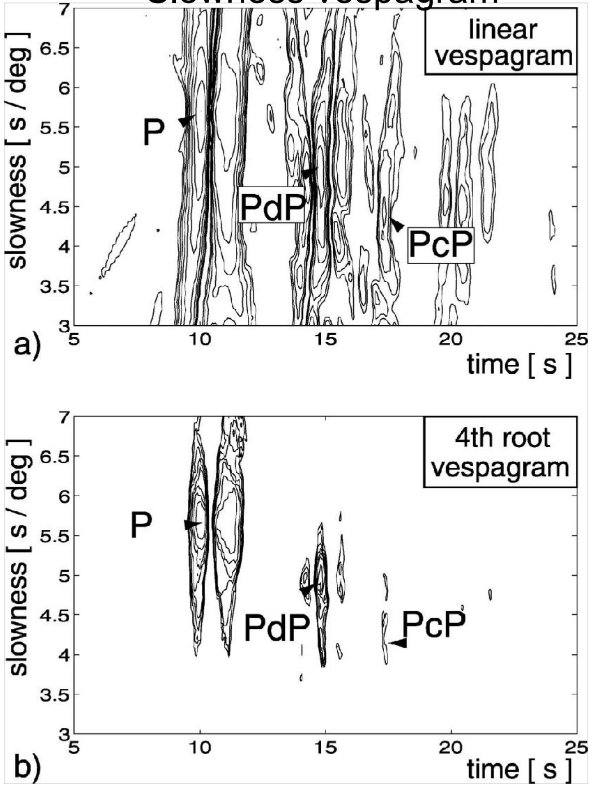
Can be used for a fixed slowness and varying back azimuths

$$v_u(t) = \frac{1}{M}\sum_{i=1}^{M} x_i(t - t_{u,i})$$

Back azimuth vespagram



Slowness vespagram



Vespagram for the event of 17 December 1991 at 0638 UT (latitude 47.27, longitude 151.64, depth h = 154.1 km, and m_b = 5.9) in the Kurile region

Theoretical back azimuth (26.6)

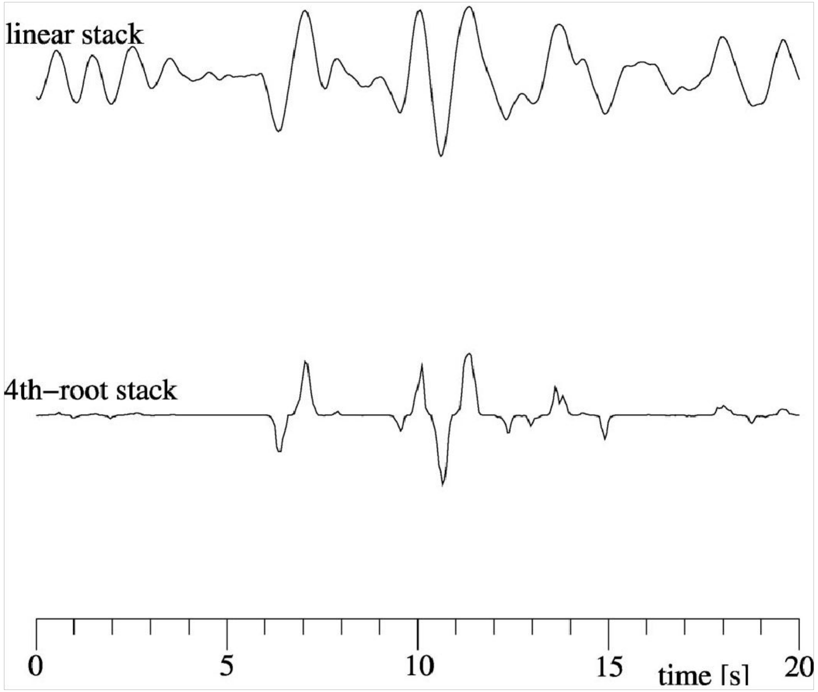
(Rost & Thomas, 2002)

Array application

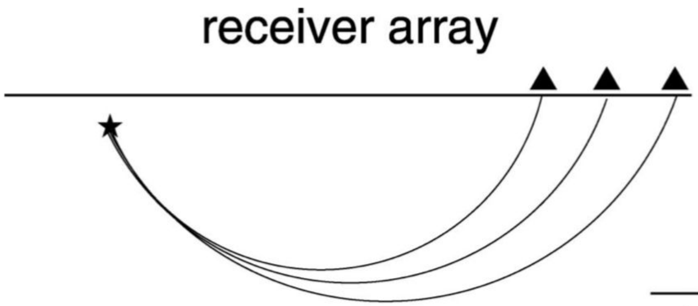
*N*th Root Process

$$v'_{u,N} = \frac{1}{M}\sum_{i=1}^{M} |x_i(t - t_{u,i})|^{1/N} \frac{x_i(t)}{|x_i(t)|}$$

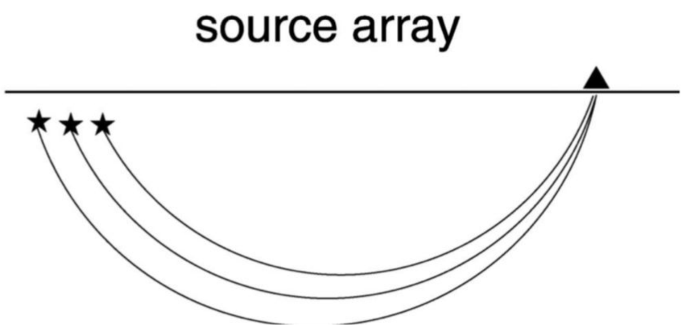
$$v_{u,N}(t) = |v'_{u,N}(t)|^N \frac{v'_{u,N}}{|v'_{u,N}|}$$



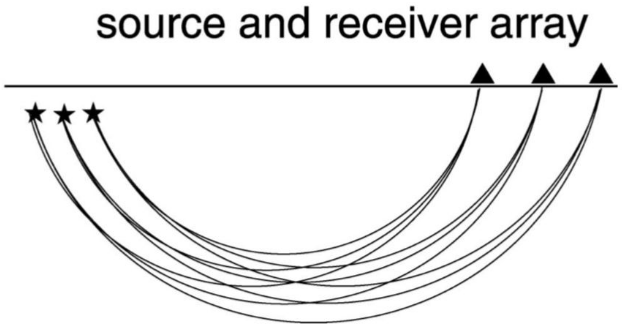
Array application



Receiver Stacks



Source Stacks



Phase-weighted Stacks

Double Beam

(Rost & Thomas, 2002)

Array application

Frequency–Wave Number Analysis (fk analysis)

Measure the complete slowness vector (i.e., back azimuth θ and horizontal slowness *u*) simultaneously
Preform grid search for all *u* and θ combinations to find the best parameter combination, producing the highest amplitudes of the summed signal

A signal arriving at a reference point within the array with a horizontal velocity v_s and a back azimuth θ is described as *s*(*t*). The *n*th seismometer with the location vector $\mathbf{r}_n$, relative to the array reference point

$$x_n(t) = s(t - \mathbf{u}_0 \cdot \mathbf{r}_n)$$

$$\mathbf{u}_0 = \frac{1}{v_0}(\cos\theta, \sin\theta)$$

v_o is the surface velocity

For the true slowness vector

$$y(t) = \frac{1}{N}\sum_{n=1}^{N} x_n(t + \mathbf{u}_0 \cdot \mathbf{r}_\mathrm{n})$$

For an different slowness vector

$$y(t) = \frac{1}{N}\sum_{n=1}^{N} s\left\{t + [(\mathbf{u}_0 - \mathbf{u}) \cdot \mathbf{r}_n]\right\}$$

Array application

Frequency–Wave Number Analysis (fk analysis)

Total enery recorded at the array:

$$E(k-k_0) = \int_{-\infty}^{\infty} y^2(t)dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} |S(\omega)|^2 \left| \frac{1}{N}\sum_{n=1}^{N} e^{2\pi i\cdot(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}_n} \right|^2 d\omega$$

S(w) is the Fourier transform of *s*(*t*)

k is the wave number vector

Name	Symbol	Relationships
Period	T	$T = 1/f = 2\pi/\omega = \Lambda/v$
Frequency	f	$f = 1/T = \omega/2\pi = v/\Lambda$
Angular frequency	ω	$\omega = 2\pi f = 2\pi/T = v\cdot k$
Velocity	v	$v = \Lambda/T = f\cdot\Lambda = \omega/k$
Wavelength	Λ	$\Lambda = v/f = v\cdot T = 2\pi/k$
Wavenumber	k	$k = \omega/v = 2\pi/\Lambda = 2\pi f/v$

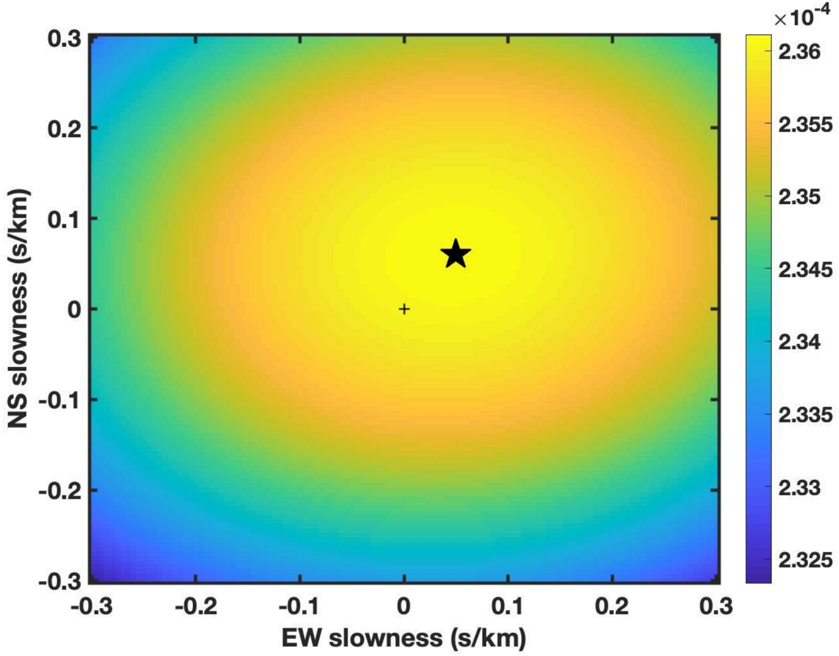
$$\mathbf{k} = (k_x, k_y) = \omega\cdot\mathbf{u} = \frac{\omega}{\nu_0}(\cos\theta, \sin\theta)$$

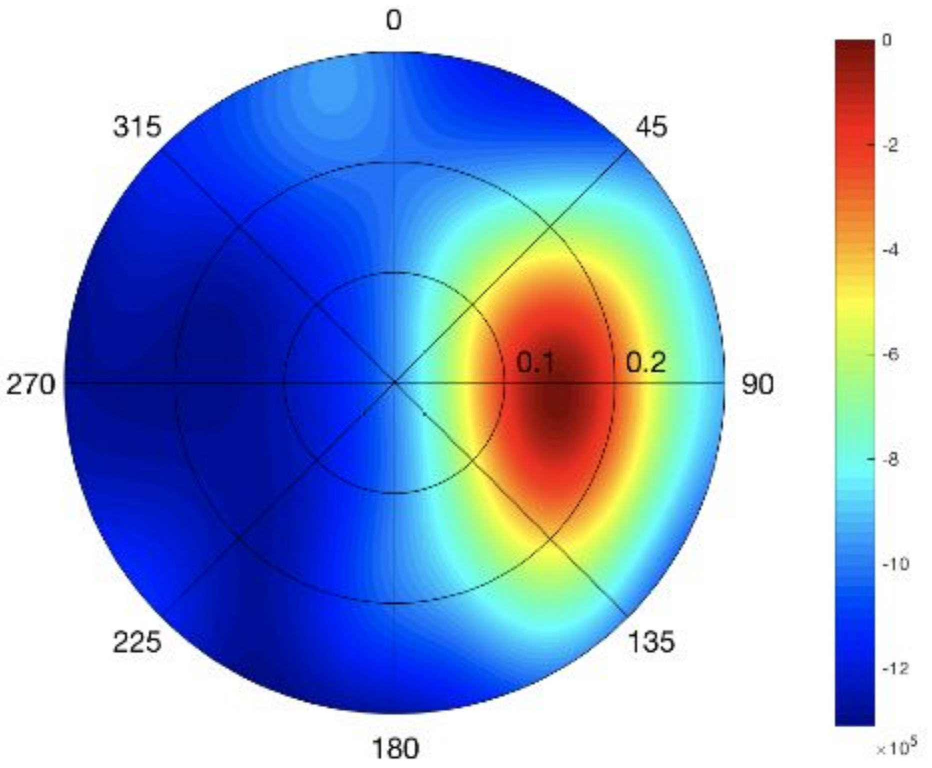
Grid search for slowness vector

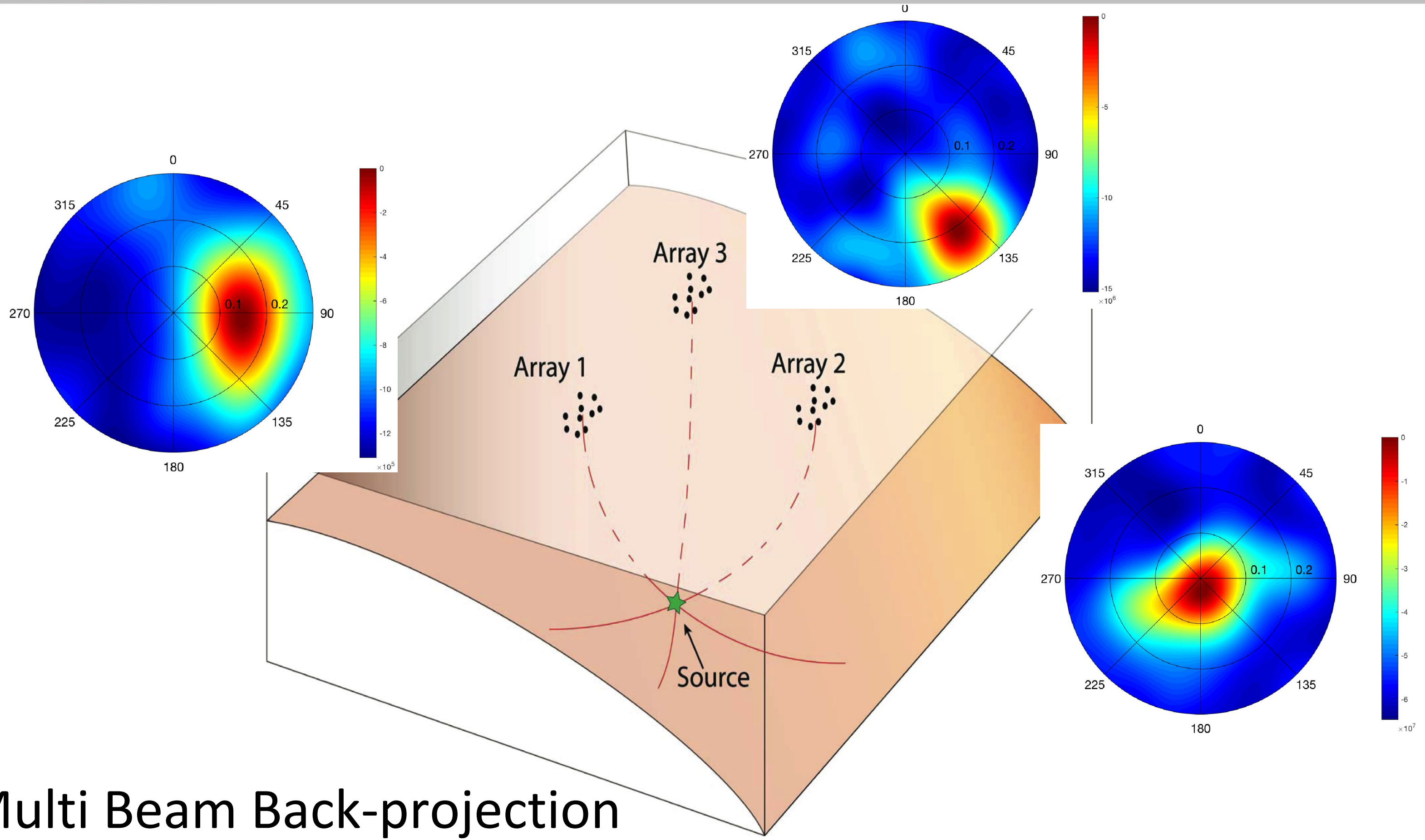
Acoor: array coordinates in longitude & latitude
Acent: array center

```
distN = (acoor(:,2) - acent(2))*111.0; % NS offset in km
distE = (acoor(:,1) - acent(1))*111.0 * cos(acent(2)*pi/180); % EW offset in km

% Slowness
NsloV = [-0.3:0.005:0.3];
EsloV = NsloV;

beam = NaN*ones(length(NsloV));

for ind1 = 1: length(NsloV)
    for ind2 = 1: length(EsloV)

        stdel = datacs(1).data*0;
        for ind3 = 1: size(acoor1,1)

            delay = distN(ind3)*NsloV(ind1) + distE(ind3)*EsloV(ind2);  % compute the time delay from slownesses

            rep = exp(1i*2*pi*frq*-delay);
            vhelp = fft(datacs(ind3).data);
            dshft = rep.*shiftdim(vhelp(1:length(frq)),1); % phase-shift(shifting in frequency domain)
            s_sh = double(2*real(ifft(dshft, datacs(1).recNumData))); % taking it back to the time domain with correct amplitude

            stdel = stdel + s_sh';
            clear delay rep dshft s_sh

        end
        stpower = sum((stdel/ind3).^2);
        beam(ind1,ind2) = stpower;
        clear stpower stdel vhelp
    end
```

Array Seismology

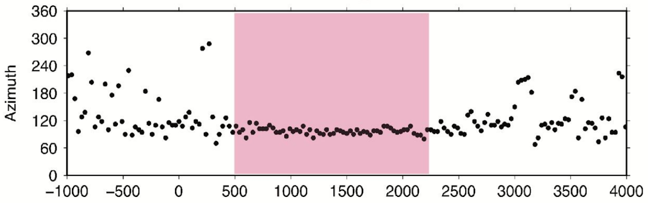


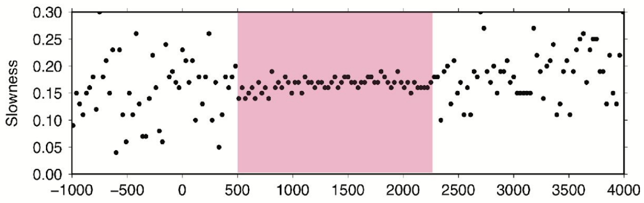


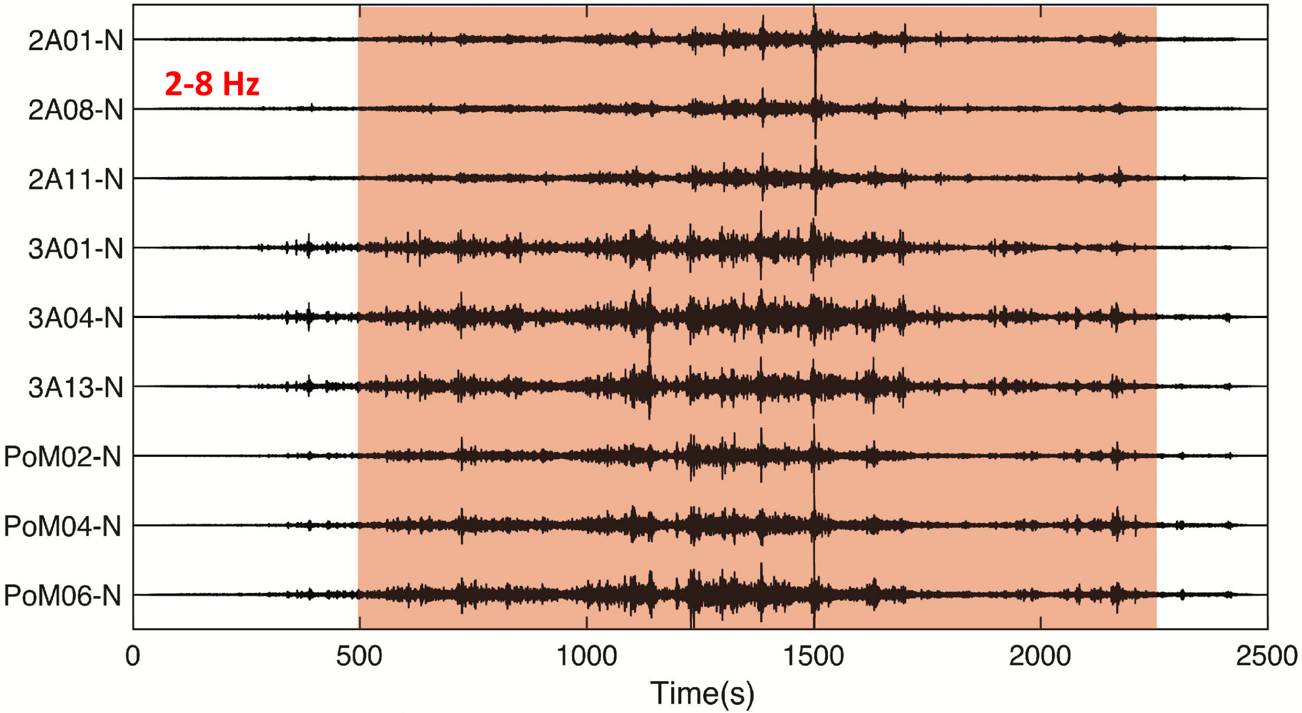


Multi Beam Back-projection

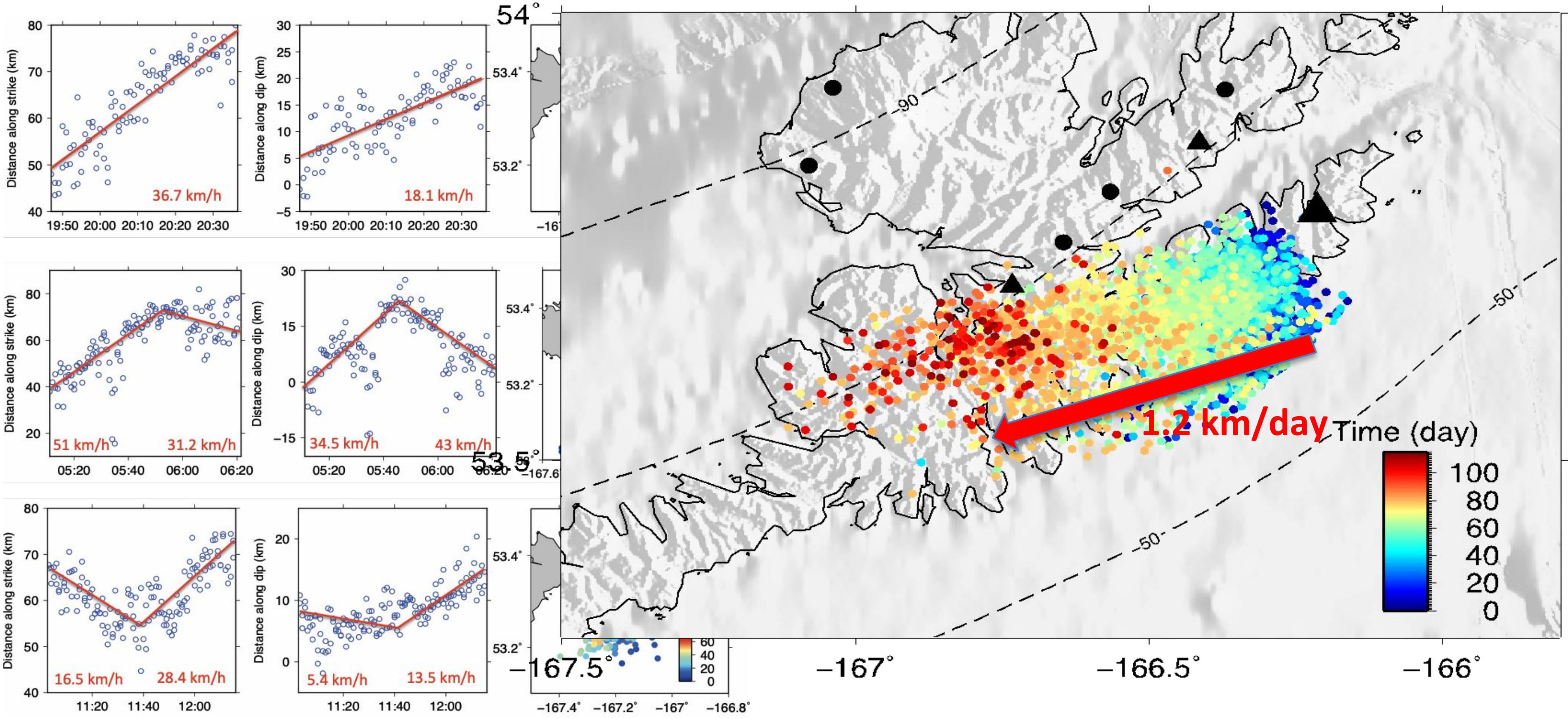
Tremor Detection Example



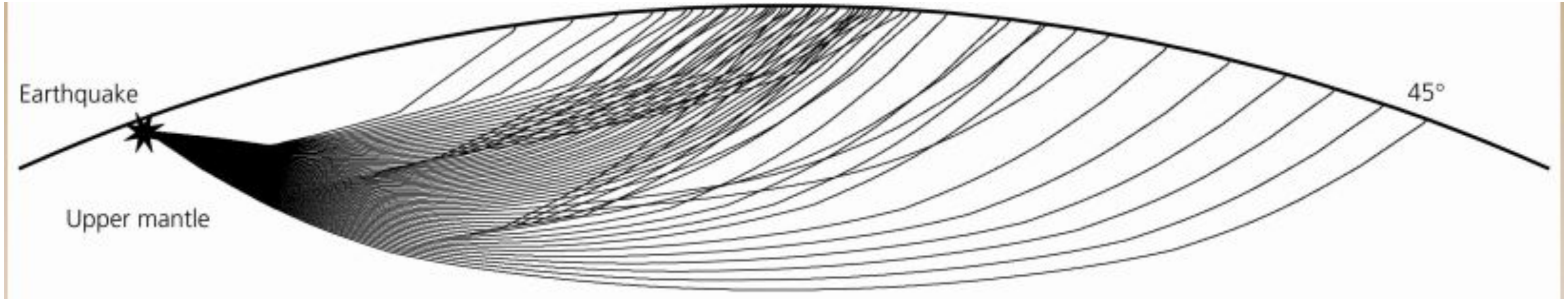




Tremor migrations



Beamforming Limitations



http://rses.anu.edu.au/~nick/teachdoc/lecture7.pdf

When the source & array are far away:
Large uncertainties in locations with small slowness uncertainties
Small array ≈ single station
Large array: can not use plane wave front
Unable to track ruptures of large EQs

Back-projection

Rupture propagation animation of the 2004 Sumatra-Andaman earthquake as imaged by the Hi-net array

http://www.seismology.harvard.edu/research/backproj.html

Advantages

Simplicity of the calculations

It doesn't require extensive computation of the wave field and not require a priori knowledge of the duration and geometry of the event

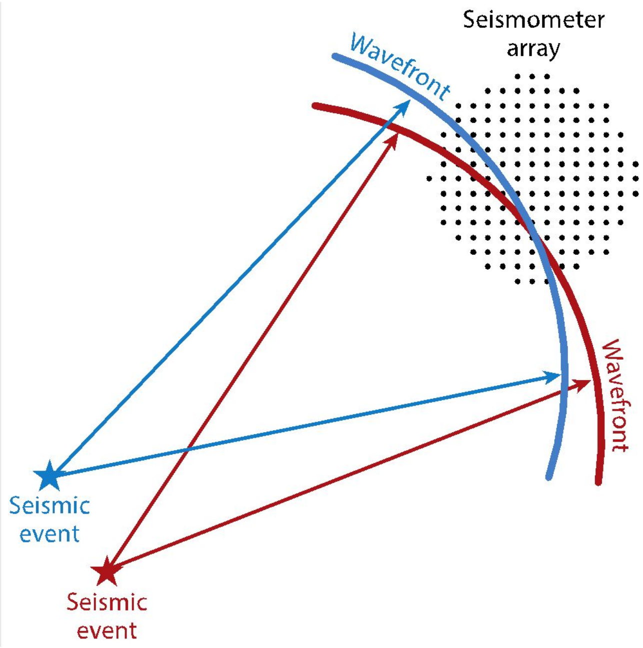
Be applied to data at higher frequency

These high frequencies are sensitive to dynamics of earthquake, such as acceleration of rupture and rapid changes in slip amplitude

Seismic phase filter

Because the collapsing of the wavefront is performed with travel times of a target seismic phase (e.g., the first-arriving P waves), there is an implicit slowness filter.

Back-projection



The dense array of seismometers that are now available around the world.

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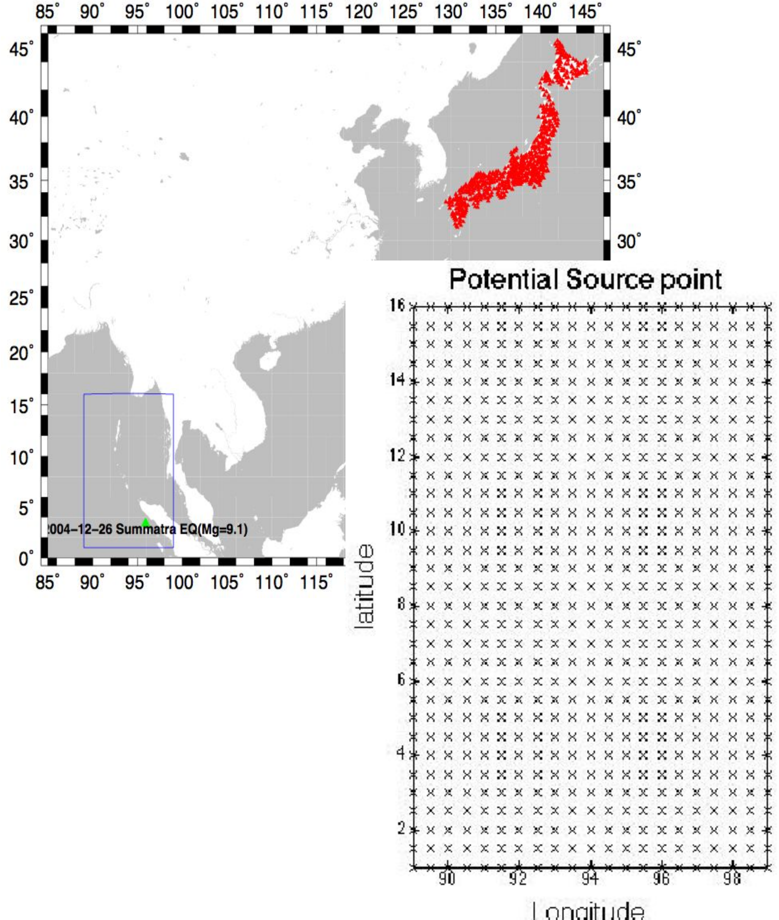
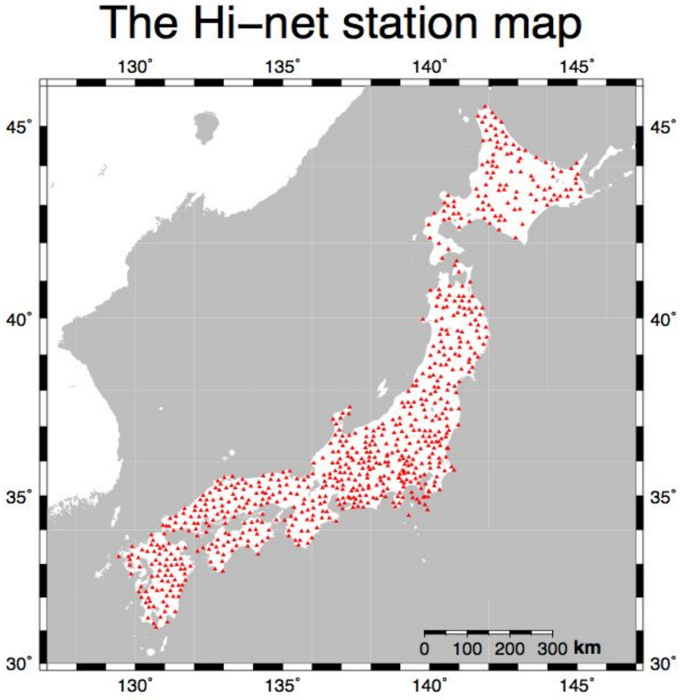
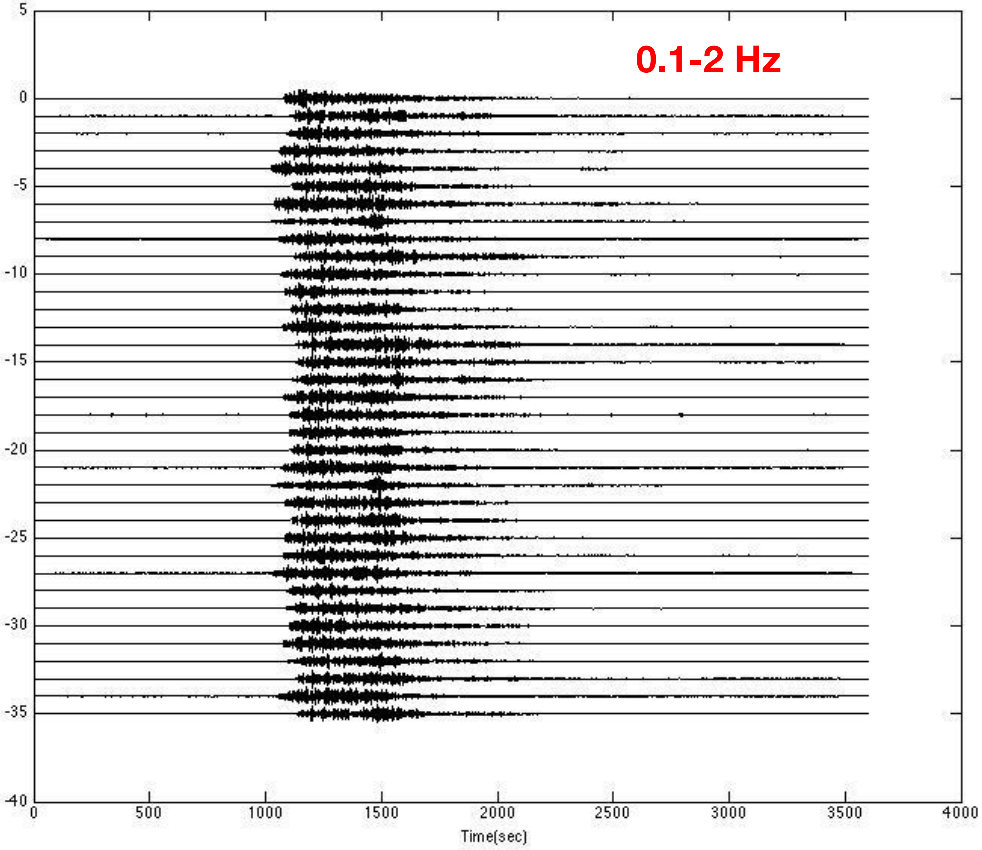
Back-projection





Back-projection

Seismograms recordings in Hi-net array for the Sumatra EQs



Back-Projection

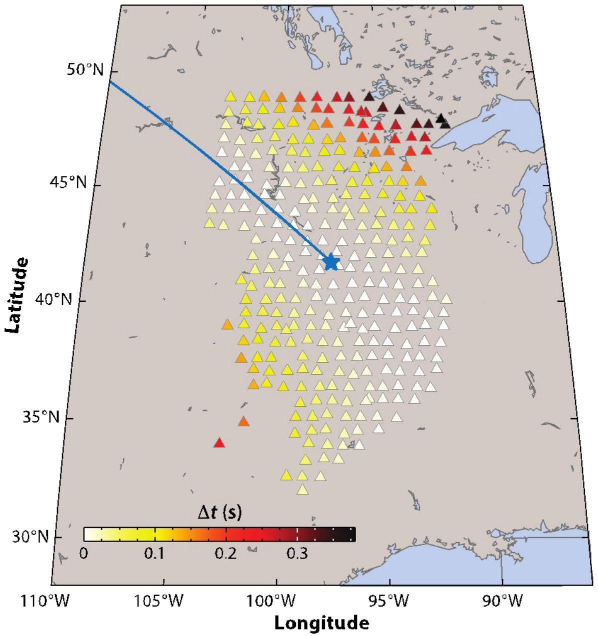
Back-projection is similar to the beamforming approach, but it does not assume a planar wavefront

$$s_i(t) = \sum_{k=1}^{n} \alpha_k \, u_k\left(t - t_{ik}^{p}\right)$$

α_k weighting factor $\qquad \alpha_k = \dfrac{p_k}{A_k}$

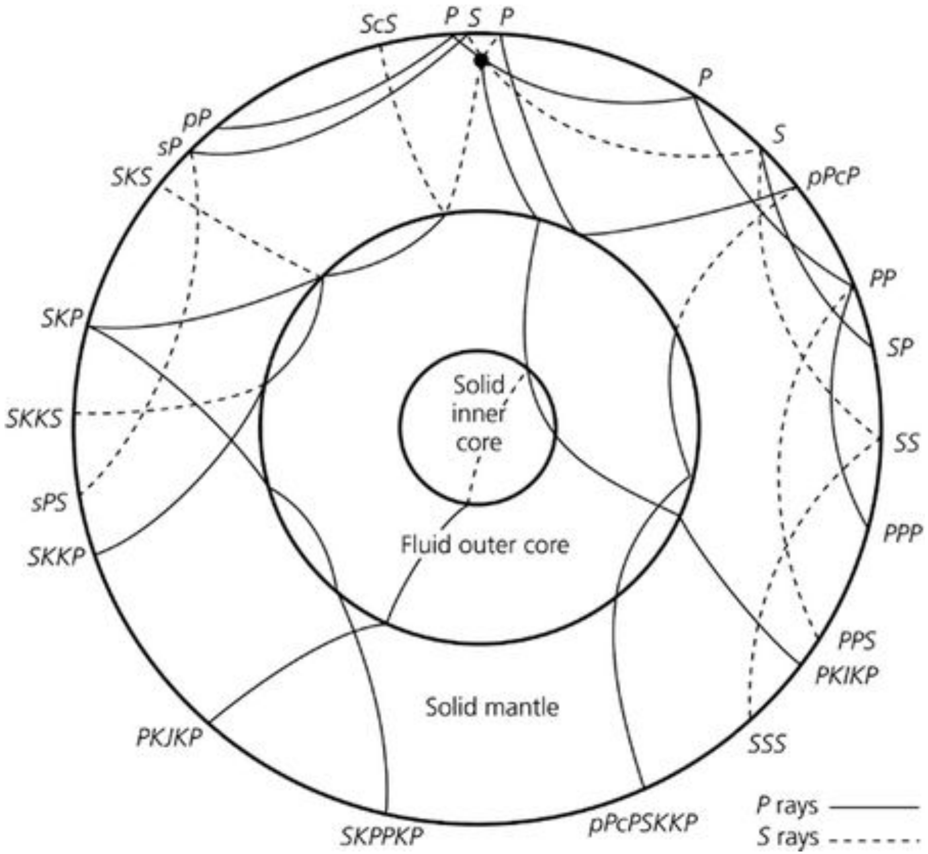
$u_k(t)$ the seismogram at the kth station

t_{ik}^{p} predicted travel time between the grid point i and station k



Difference between the travel times calculated for back-projection (*curved wavefront*) and the plane wave approximation for Transportable Array stations (*triangles*) with respect to the hypocenter of the earthquake on March 11, 2011, Tohoku-oki, Japan. (Kiser & Ishii, 2017)

Back-projection



(Havskov & Ottemöller, 2010)

In Back-projection:

Direct P wave

30-95 degrees distance range

- TauP toolkit developed by:
  H. Philip Crotwell, Thomas J. Owens, Jeroen Ritsema
  Department of Geological Sciences
  University of South Carolina
  http://www.seis.sc.edu

Back-projection

The TauP Toolkit

Time | Pierce | Path

Dist	Dist (km)	Depth	Name	Time	Ray Param	Purist Dist	
50.00	5559.75	20.0	P	532.72	7.597	50.00	=
50.00	5559.75	20.0	pP	539.05	7.609	50.00	=
50.00	5559.75	20.0	PcP	612.36	3.669	50.00	=
50.00	5559.75	20.0	S	963.13	13.956	50.00	=
50.00	5559.75	20.0	sS	973.92	13.974	50.00	=
50.00	5559.75	20.0	PKiKP	1018.31	1.057	50.00	=
50.00	5559.75	20.0	ScS	1122.90	6.805	50.00	=
50.00	5559.75	20.0	SKiKS	1443.07	1.162	50.00	=

TauP-travel time calculation

Model iasp91 | Distance (deg) 50.0 | Calculate

Phases | Depth 20.0 | Quit

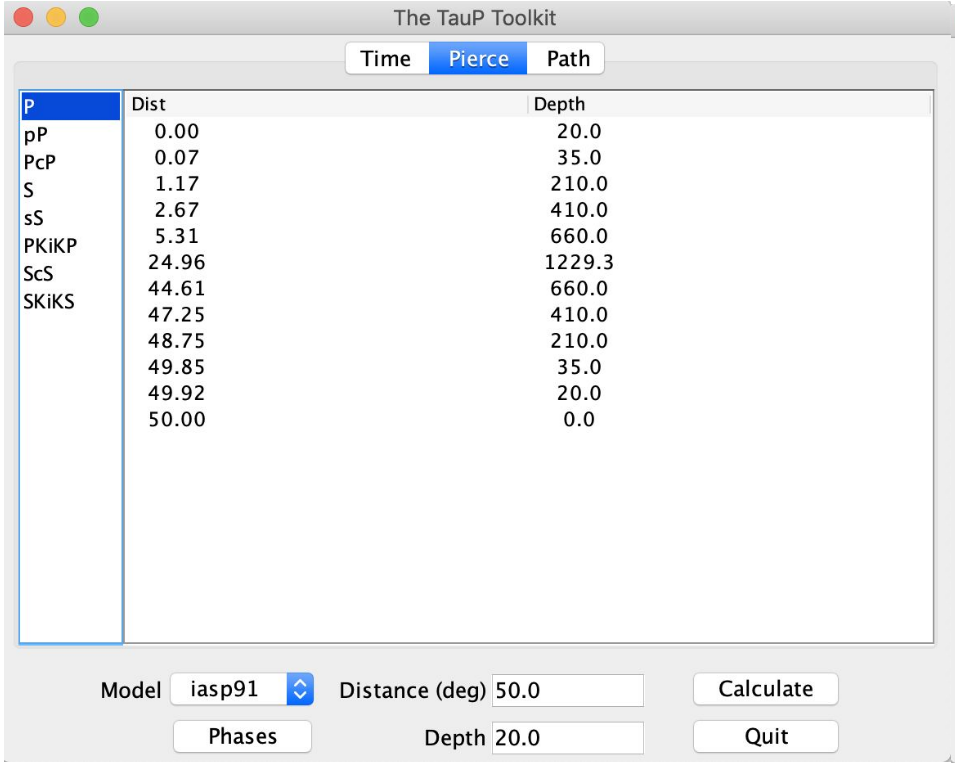
Velocity Model

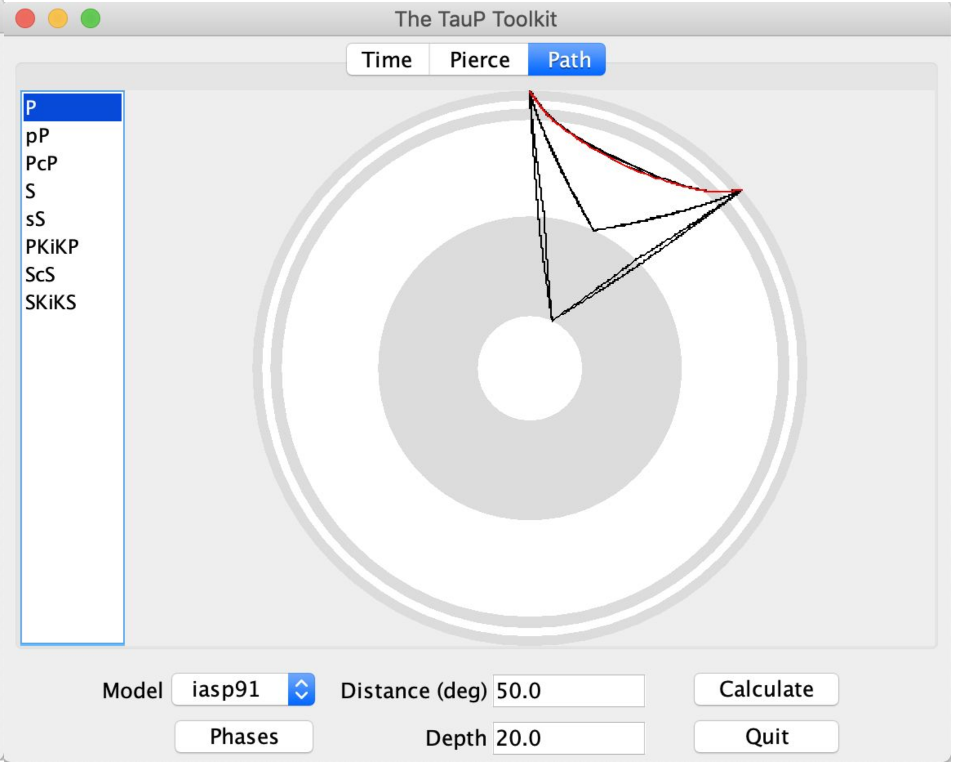
Isap91

Prem

ak135

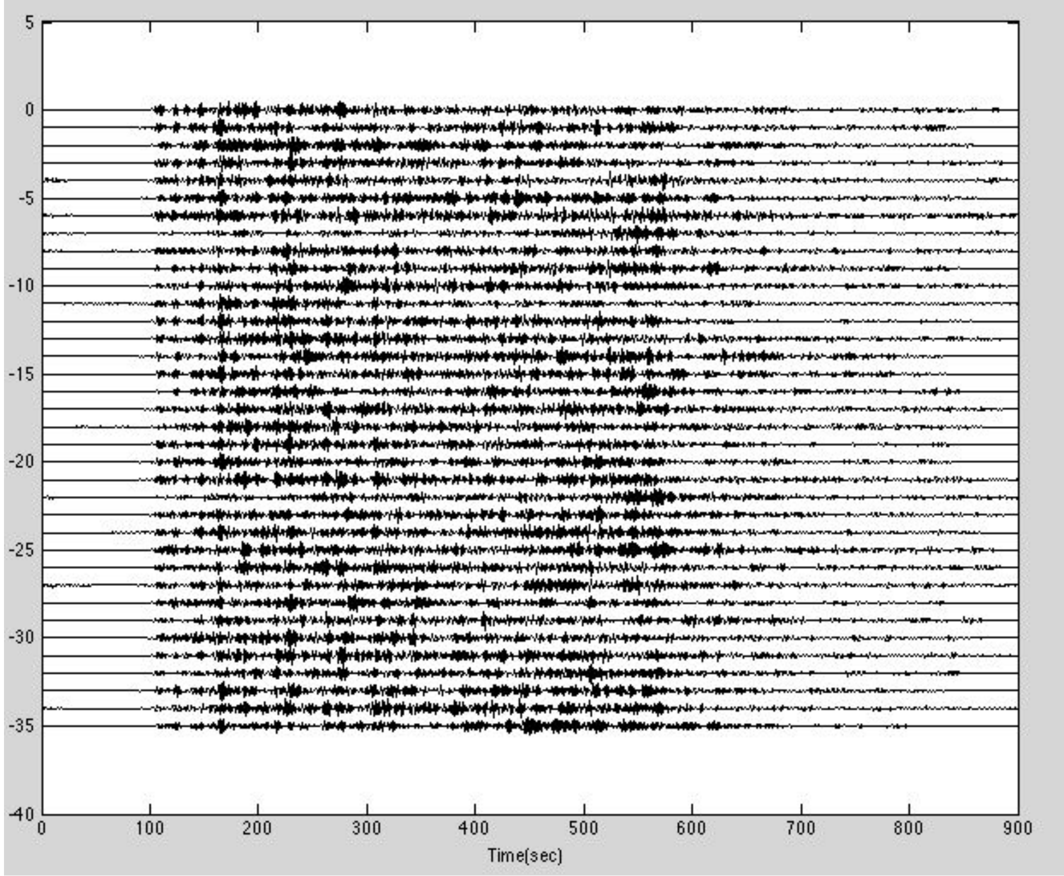
TauP-travel time calculation





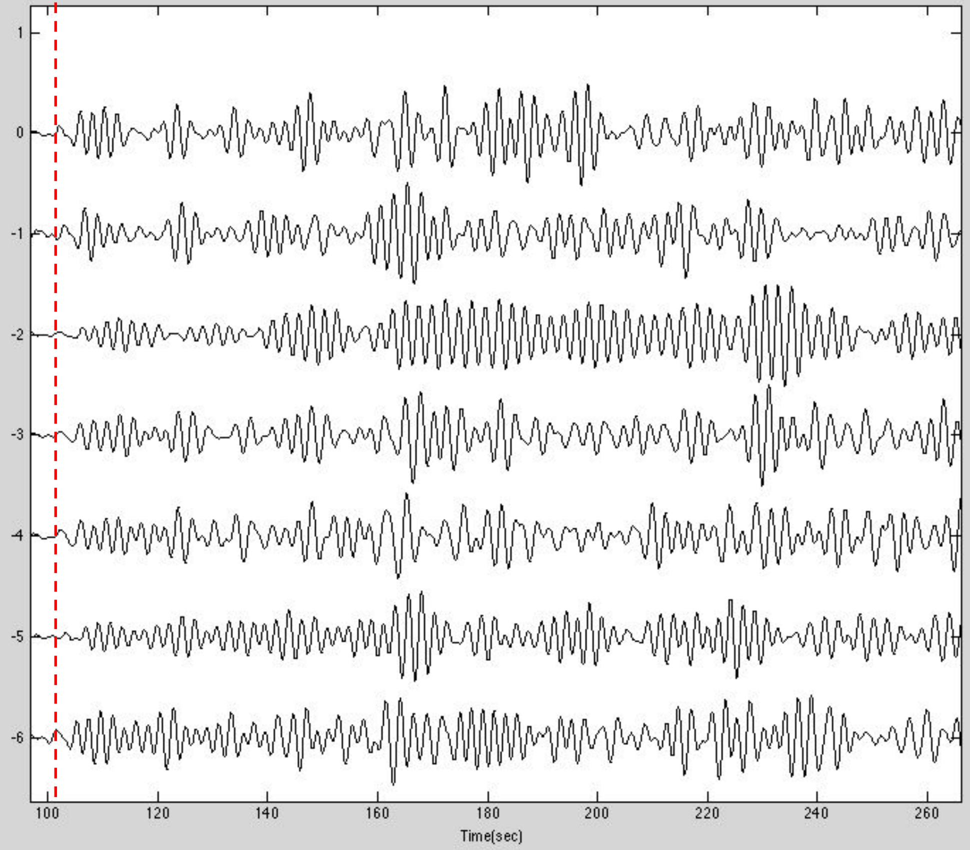
Back-projection

Seismograms after theoretical time shift



Back-Projection

For real data, the wavefront is distorted by lateral variations in wave speed within the Earth.

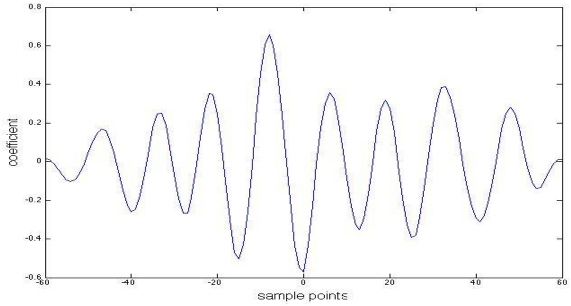


$$s_i(t) = \sum_{k=1}^{n} \alpha_k \, u_k\left(t - t_{ik}^{p} + \Delta t_k\right)$$

theoretical

empirical

The empirical time corrections are often obtained using the initial few seconds of the first-arriving P waveforms



The aligned seismograms are associated with the hypocentral location

Back-Projection

theoretical

$$s_i(t) = \sum_{k=1}^{n} \alpha_k \, u_k\left(t - t_{ik}^{p} + \Delta t_k\right)$$

empirica

One could attempt to achieve spatial coverage by combining empi
by analyzing seismograms from other earthquakes in the source re

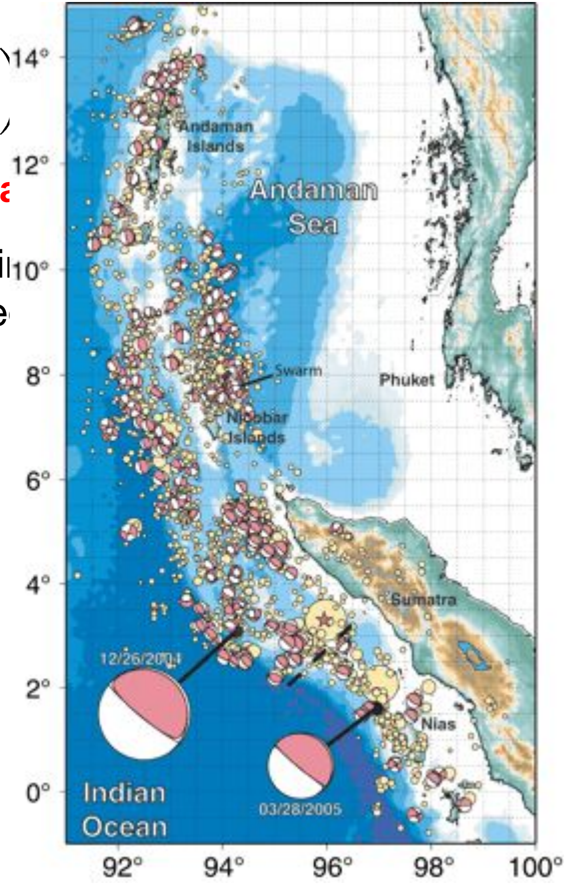
$$\Delta t_{ik} = \left(\sum_{j=1}^{m} \frac{\omega_j \Delta t_{jk}}{\Delta_{ij}} \right) \Bigg/ \left(\sum_{j=1}^{m} \frac{\omega_j}{\Delta_{ij}} \right)$$

Δt_{ik} Time shift from grid point i and station k

ω_j Measure of data quality for the aftershock

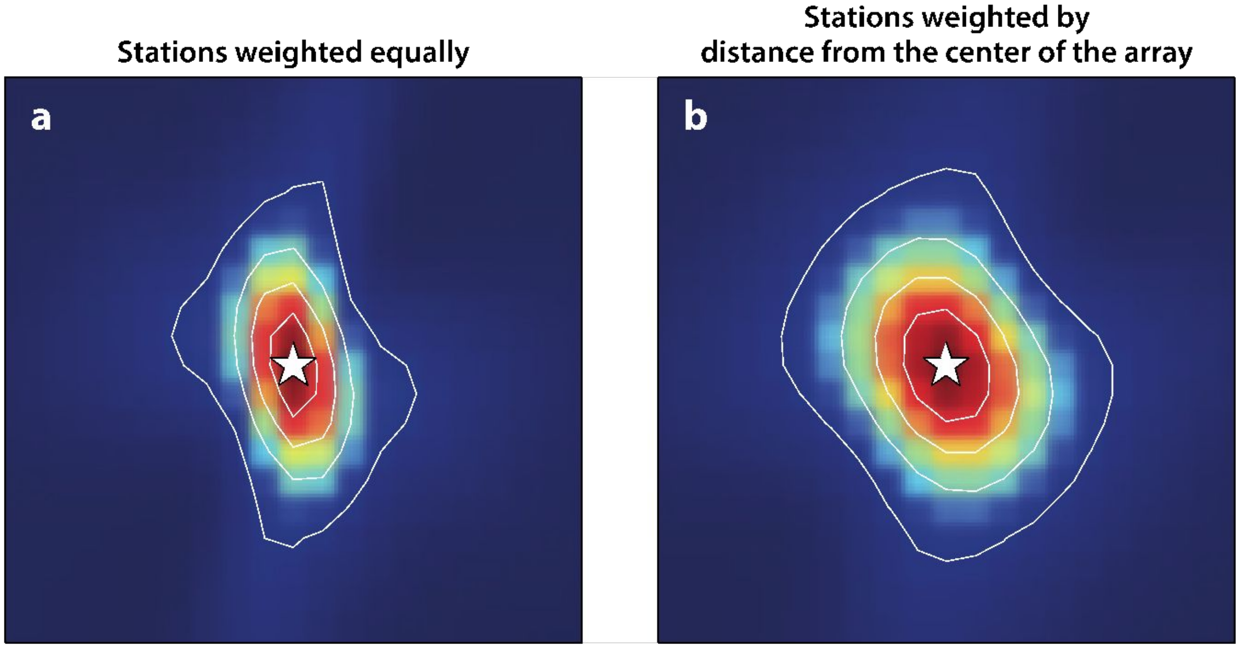
Δ_{ij} Distance between each aftershock j and grid point i

Δt_{jk} Time shift from the j'th aftershock



(Lay et al., 2005)

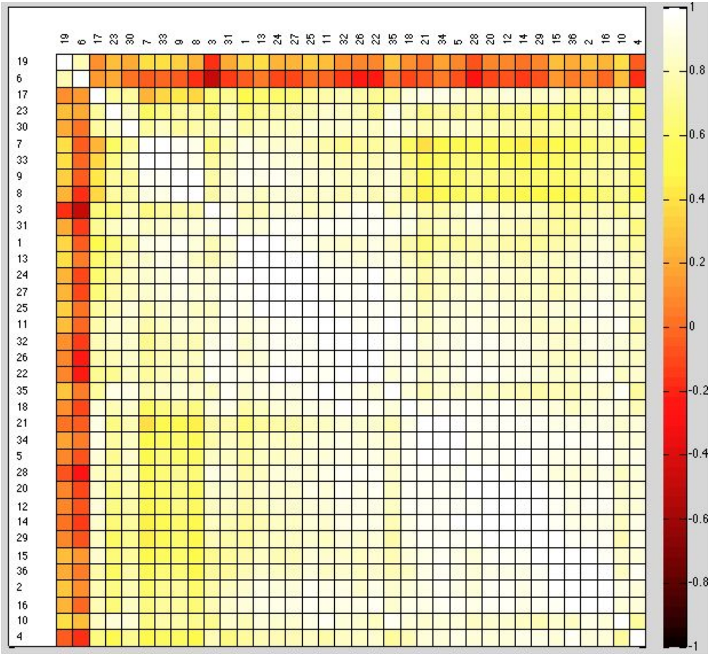
Back-Projection

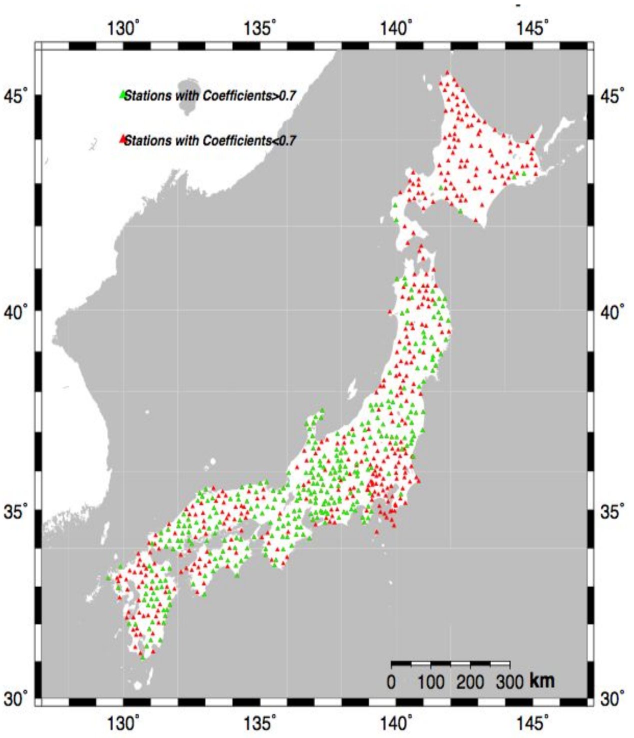


(*a*) Back-projected image obtained using the Hi-net array geometry with stations weighted equally. (*b*) Same as in panel *a* except a filter that mimics a circular array geometry is applied, i.e., each station is weighted differently on the basis of their distance from the center of the array. The resolution becomes more symmetrical. (Kiser & Ishii, 2017)

Back-Projection

Cross-correlation between station pairs



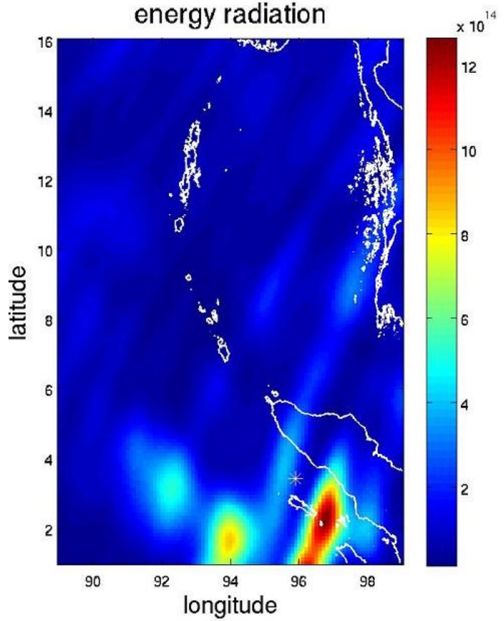


321/702

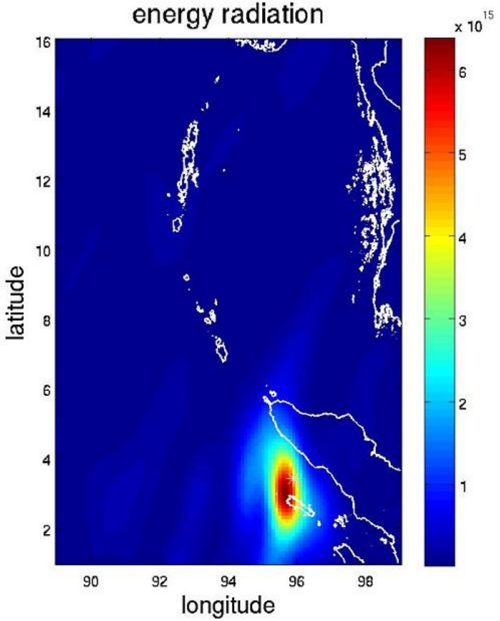
Co>=0.7

Back-Projection

60s after the initiation

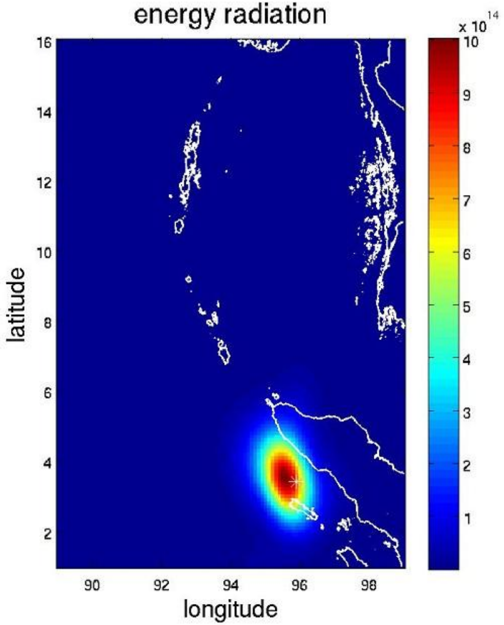


TauP shift data



TauP shift data

cross-correlation

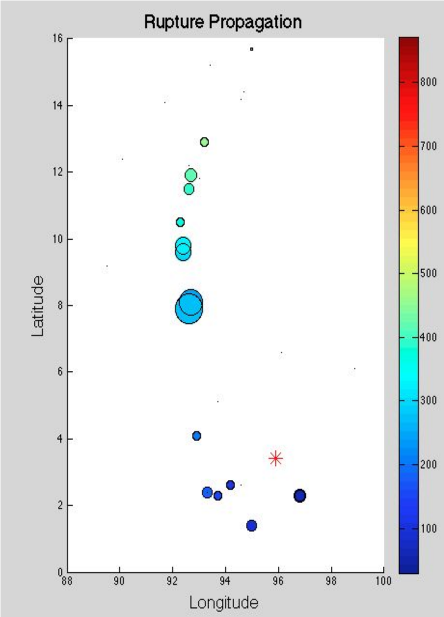


TauP shift data

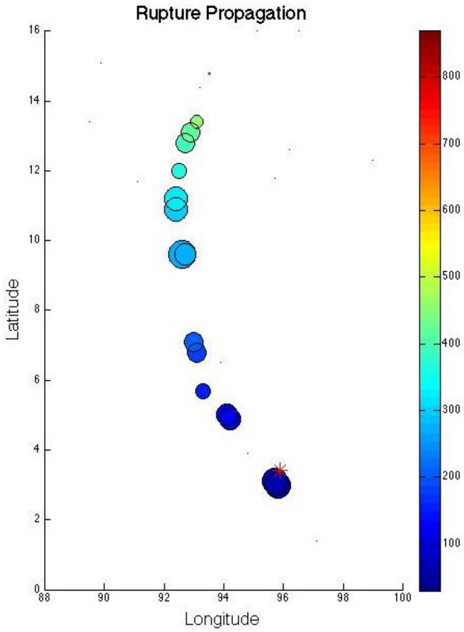
cross-correlation

Co>=0.7

Back-Projection

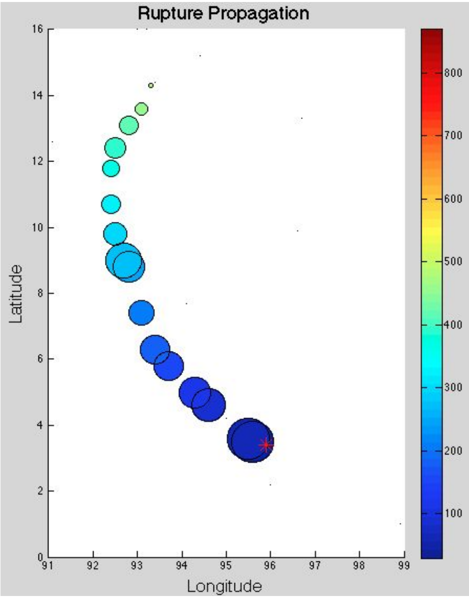


TauP shift data



TauP shift data

cross-correlation

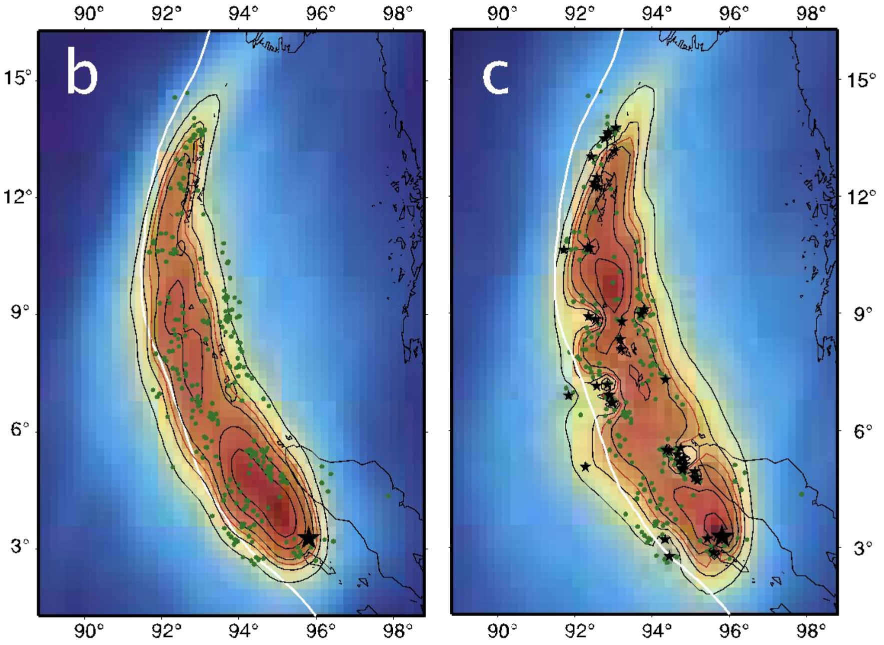


TauP shift data

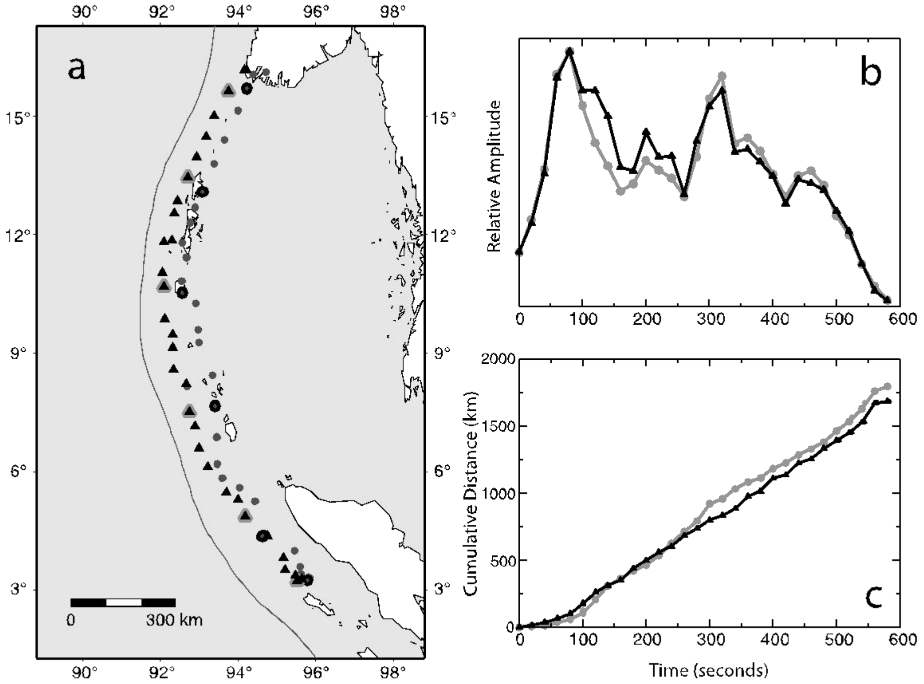
cross-correlation

Co>=0.7

Back-Projection



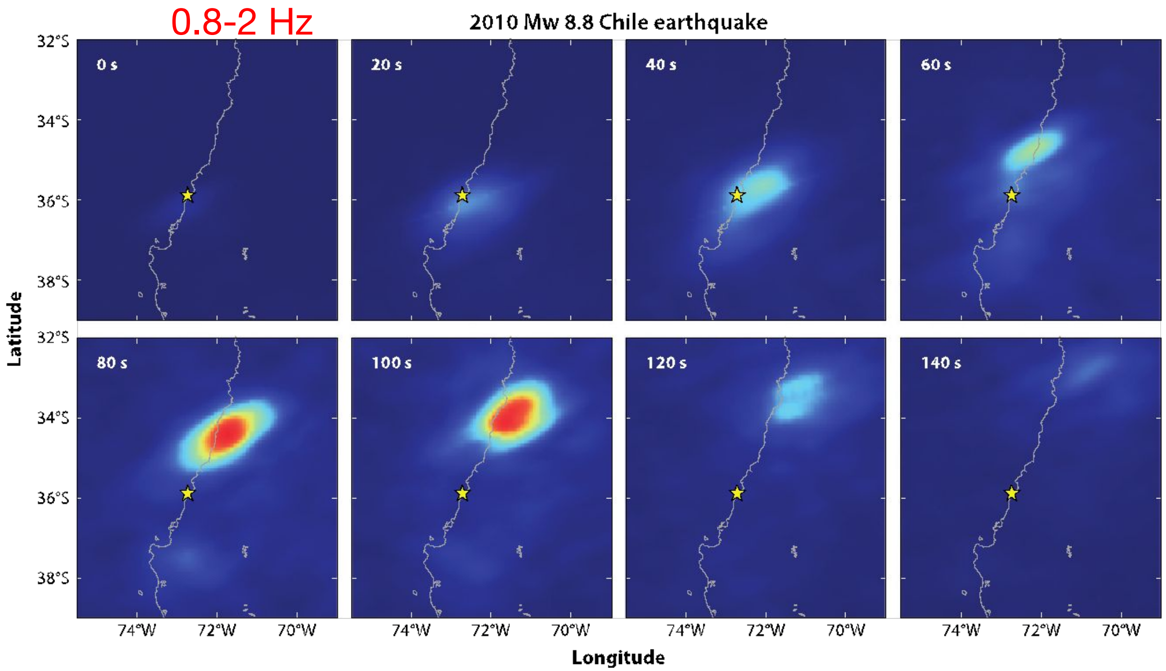
Distribution of relative energy radiation (approximated by the squared amplitude of stacked seismograms) obtained by integrating the first 600 s of the stacked time series. (b) with a hypocentral time correction. (c) with 46 aftershocks calibration. (Ishii et al., 2007)



Back-projection results of the 2004 Sumatra-Andaman Earthquake (Ishii et al., 2007)

Applications

Image detailed rupture propagation/properties of large earthquakes



(Kiser & Ishii, 2017)

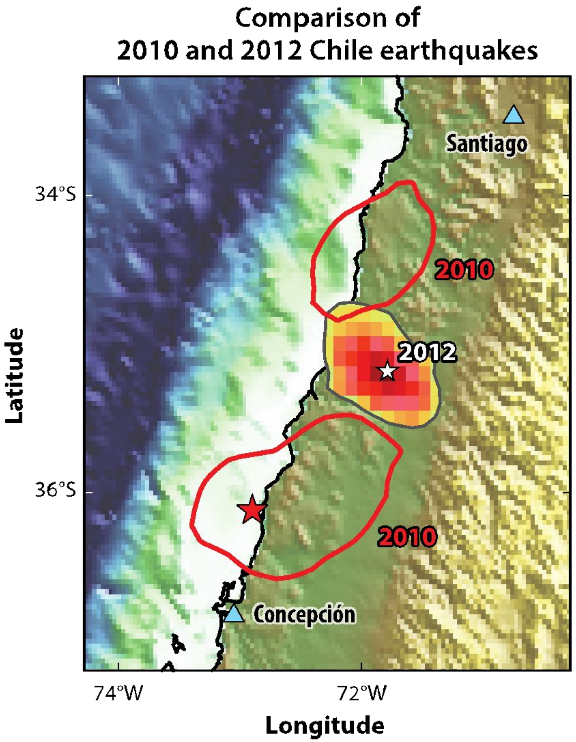
The rupture initiates at the epicenter and propagates both north and south (i.e., bilateral rupture) for about 150 s

Energy release are mostly downdip of the epicenter

The northern component of the rupture is divided into two parts (downdip and updip)

the southern component of rupture shows only weak energy release, peaking at about 80 s from event initiation

Back-Projection



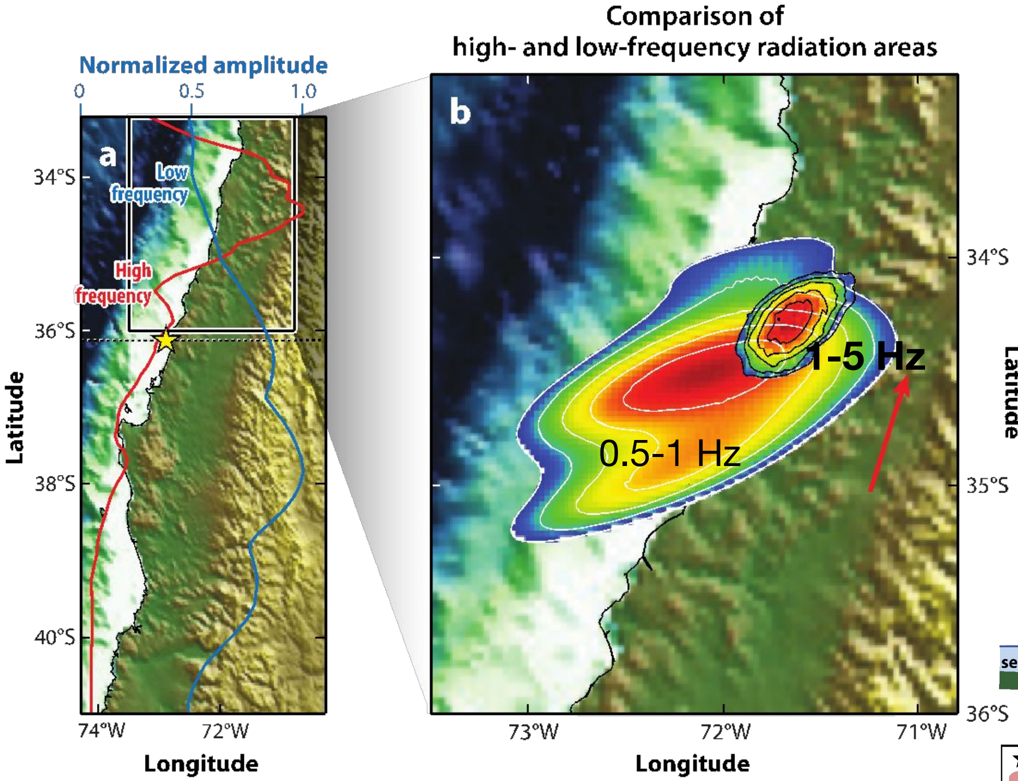
(Kiser & Ishii, 2017)

This gap in high-frequency energy radiation also corresponds to an area of reduced aftershock occurrences compared with the rest of the source area.

The 2012 Mw 7.1 earthquake ruptured an area that covers the gap observed during the 2010 mainshock

These observations indicate that the plate interface is segmented

Back-Projection

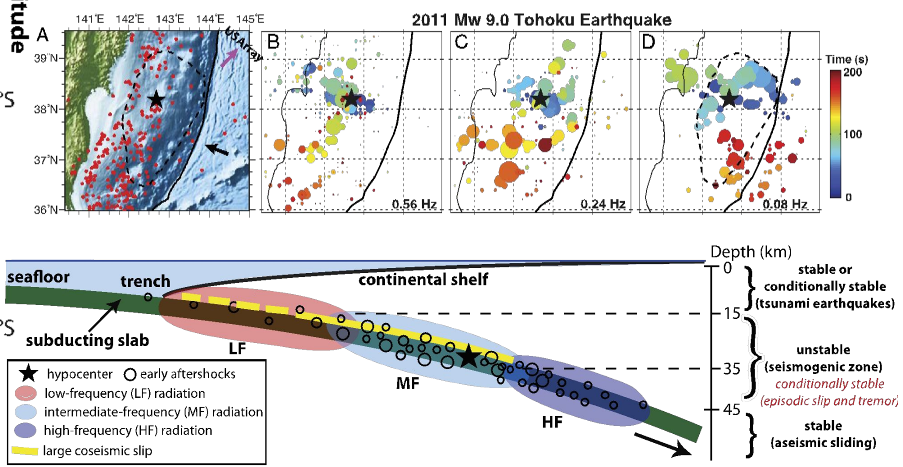


(Kiser & Ishii, 2017)

Frequency dependent rupture

Lower frequency results show the overall energy release is larger to the south

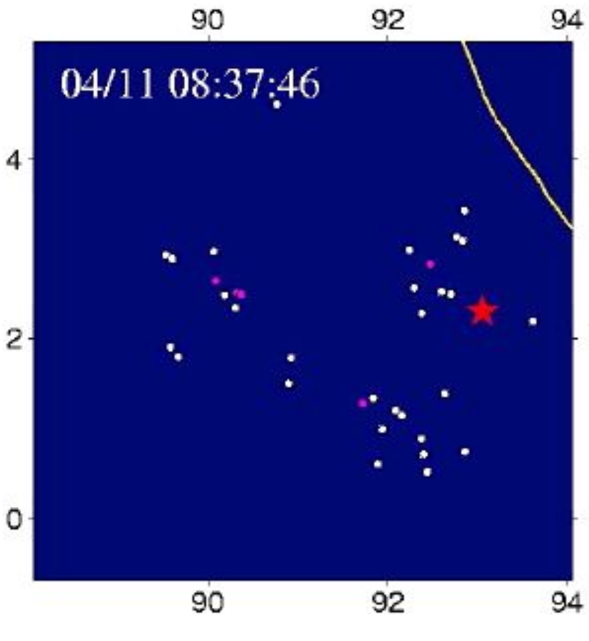
Lower-frequency rupture occurred updip



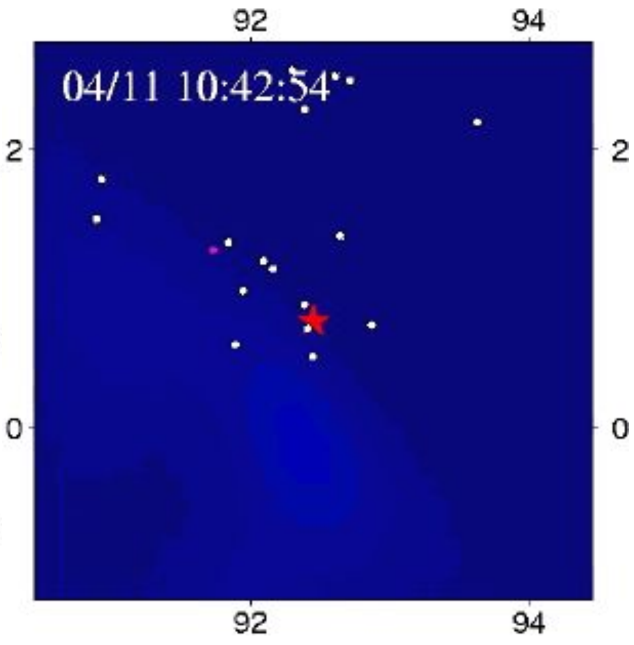
(Yao et al., 2013)

Back-Projection

Complex rupture pattern

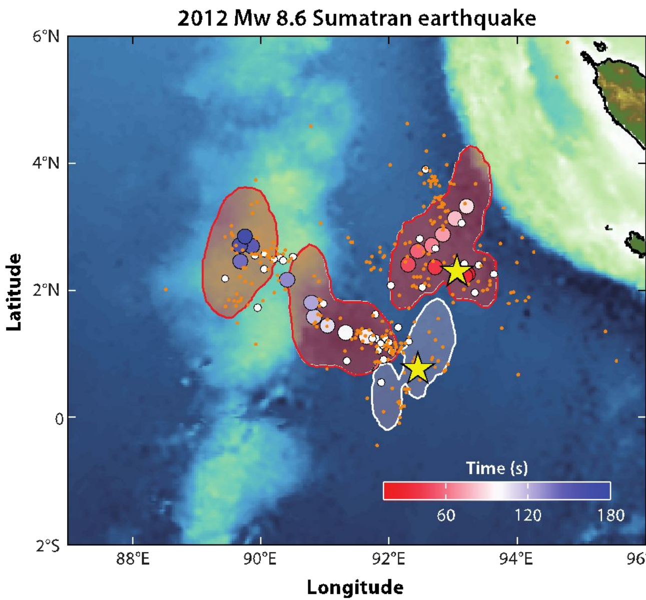


Mw 8.6



Mw 8.2

http://www.seismology.harvard.edu/research_sumatra2012.html



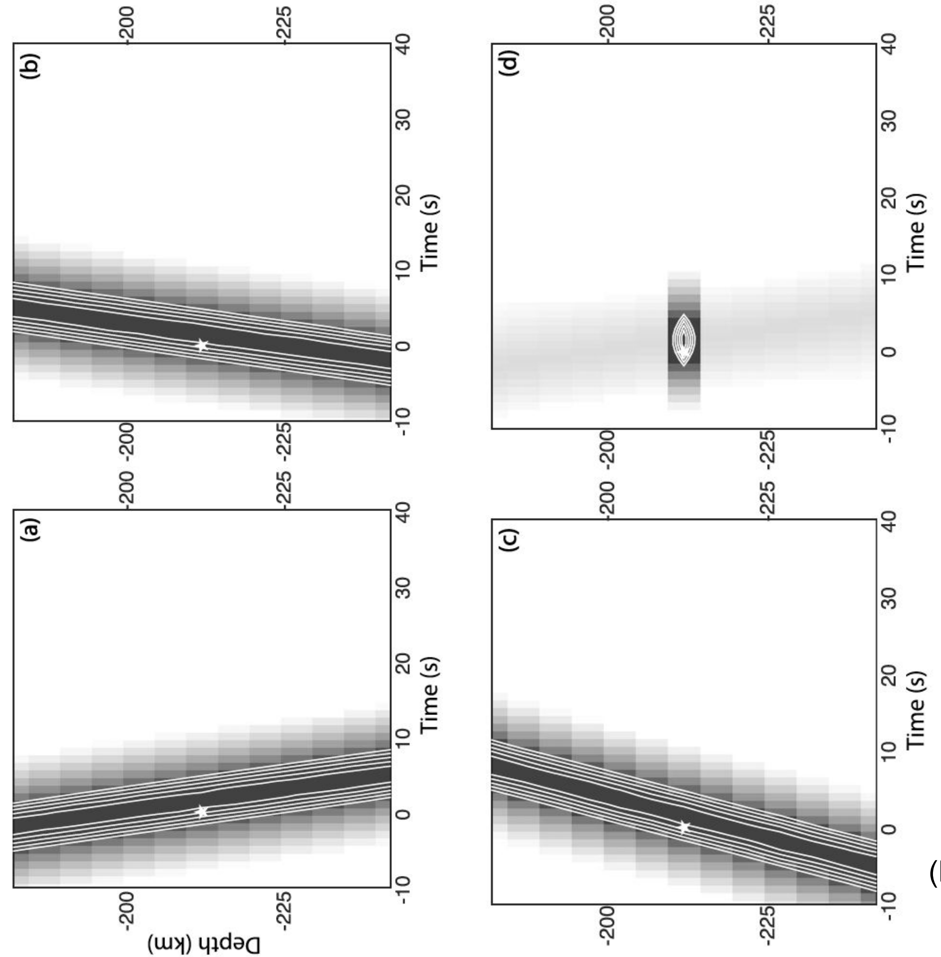
The regions colored and outlined by red show different parts of the rupture of Mw 8.6 event

The white area indicates the rupture area that was inferred from a back-projection analysis of the Mw 8.2 (Kiser & Ishii, 2017)

Back-Projection

Low Depth Resolution

Limited frequency bands



Depth-time resolution. The white stars are the depths and times of the point sources, and the white lines are 5% contours between 75% and 100% of the maximum value of the squared stack. The background colors indicate high (black) and low (white) stack amplitudes.

(a) Synthetic back projection result of a point source located at 21.08°S, 176.59°W, and 212 km depth using only the P phase.

(b) use only the pP phase.

(c) use only the sP phase.

(d) Synthetic back projection result using all three seismic phases.

(Kiser et al., 2011)

Back-Projection

Advanced BP

Yao et al., 2011:
Compressive Sensing BP

Meng et al., 2012:
MUltiple SIgnal Classification BP

Yagi et al., 2012:
Hybrid BP

Fan & Shear, 2017:
Phase-weighted stacking BP

Wang et al., 2019:
Imaging Deconvolution BP
...

Array Response Function (ARF)

Target applications

Appropriate equipment (geophones, seismometers, accelerometers…)

Array size and configuration

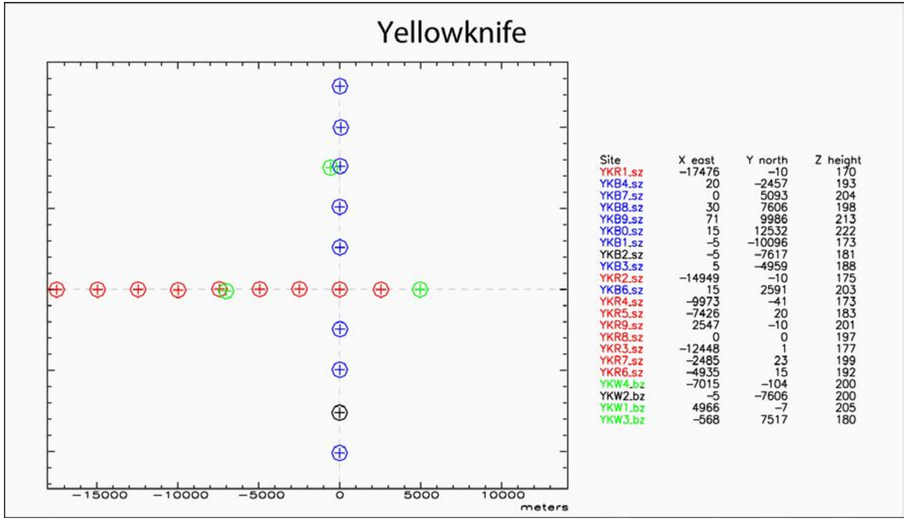
Principal prerequisites

The ARF should have a sharp main lobe, ideally a delta pulse with a strong suppression of the energy next to the main lobe.

The sidelobes due to spatial aliasing should not be within the wave number window of interest.

Array Response Function (ARF)

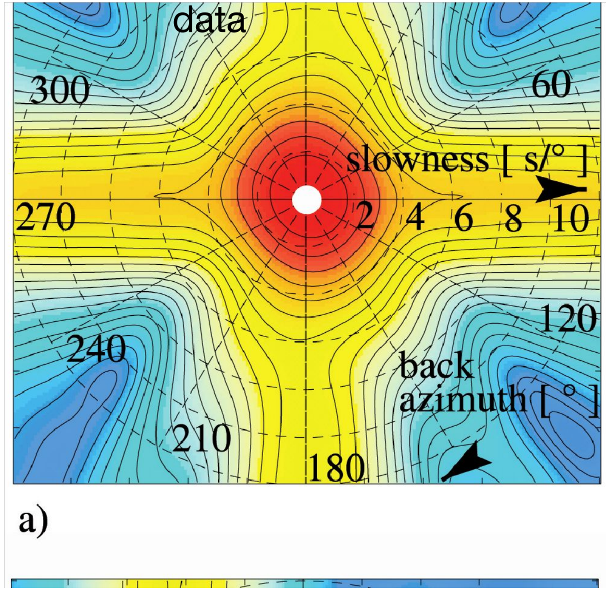
The ratio of the amplitude of the output of the array to that of the same number of elements concentrated at one location (Sherrif & Geldart, 1995)



Site	X east	Y north	Z height
YKR1_sz	−17476	−10	170
YKB4_sz	20	−2457	193
YKB7_sz	0	5093	204
YKB8_sz	30	7606	198
YKB9_sz	71	9986	213
YKB0_sz	15	12532	222
YKB1_sz	−5	−10096	173
YKB2_sz	−5	−7617	181
YKB3_sz	5	−4959	188
YKR2_sz	−14949	−10	175
YKB6_sz	15	2591	203
YKR4_sz	−9973	−41	173
YKR5_sz	−7426	20	183
YKR9_sz	2547	−10	201
YKR8_sz	0	0	197
YKR3_sz	−12448	1	177
YKR7_sz	−2485	23	199
YKR6_sz	−4935	15	192
YKW4_bz	−7015	−104	200
YKW2_bz	−5	−7606	200
YKW1_bz	4966	−7	205
YKW3_bz	−568	7517	180

(Schweitzer et al., 2011)

ARF-1 Hz



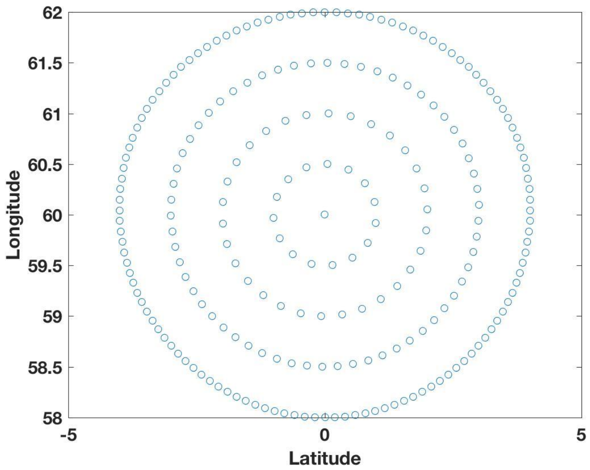
(Rost & Thomas, 2002)

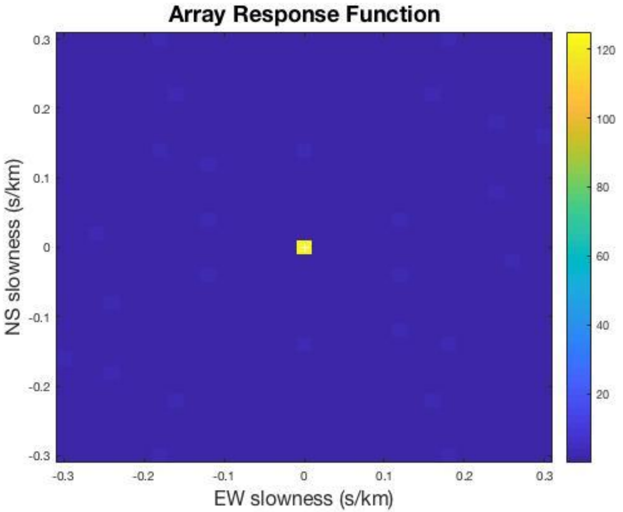
ARF

- The aperture of an array controls the sharpness of the main lobe and restricts the resolution of the wavenumber
- The inter-station spacing defines the location of side lobes of the ARF and the largest resolvable wavenumber
- The number of elements controls the suppression of energy crossing the array at the same time with a different slowness. It defines the resolution of velocity and act as a wavenumber filter
- The geometry of the array controls the azimuthal dependency of the resolution and the quality and the position of the sidelobes

ARF

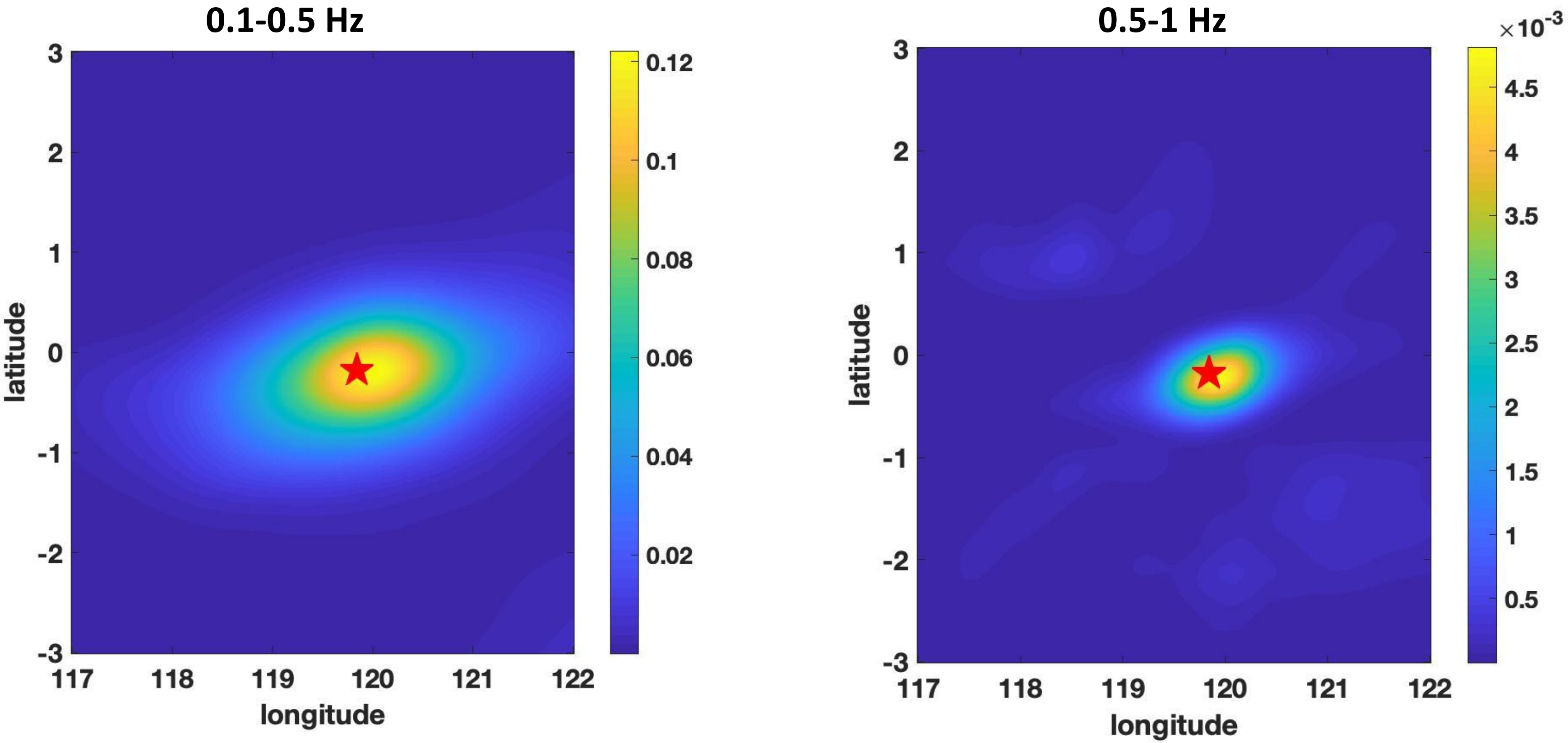
How about the source is not beneath the array?





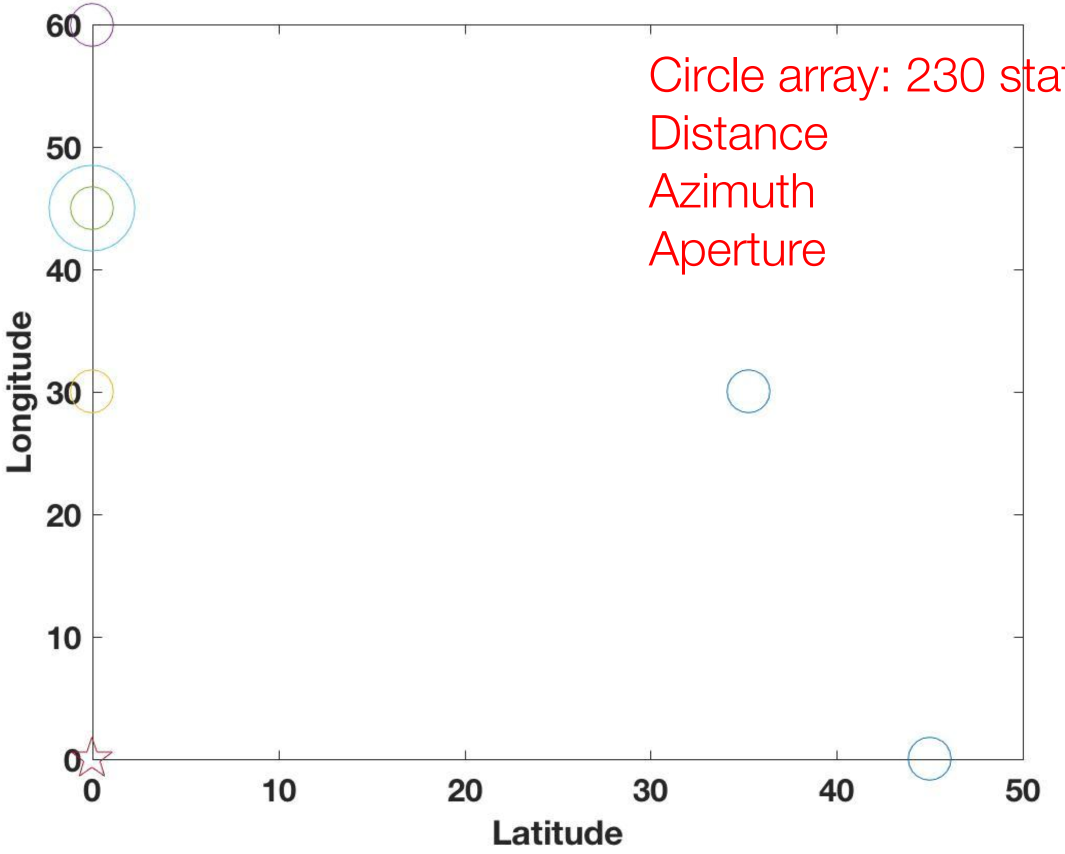
ARF

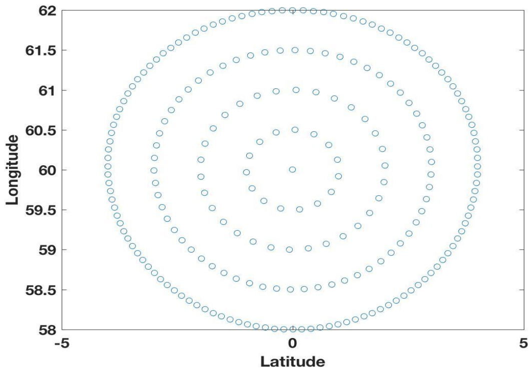
Palu BP example

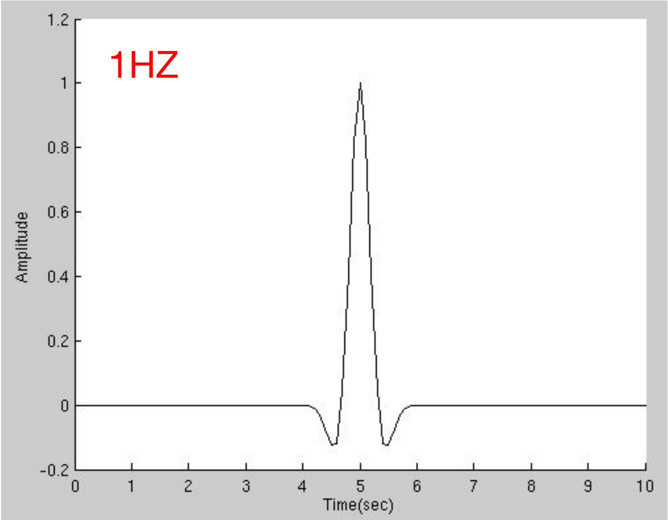


ARF

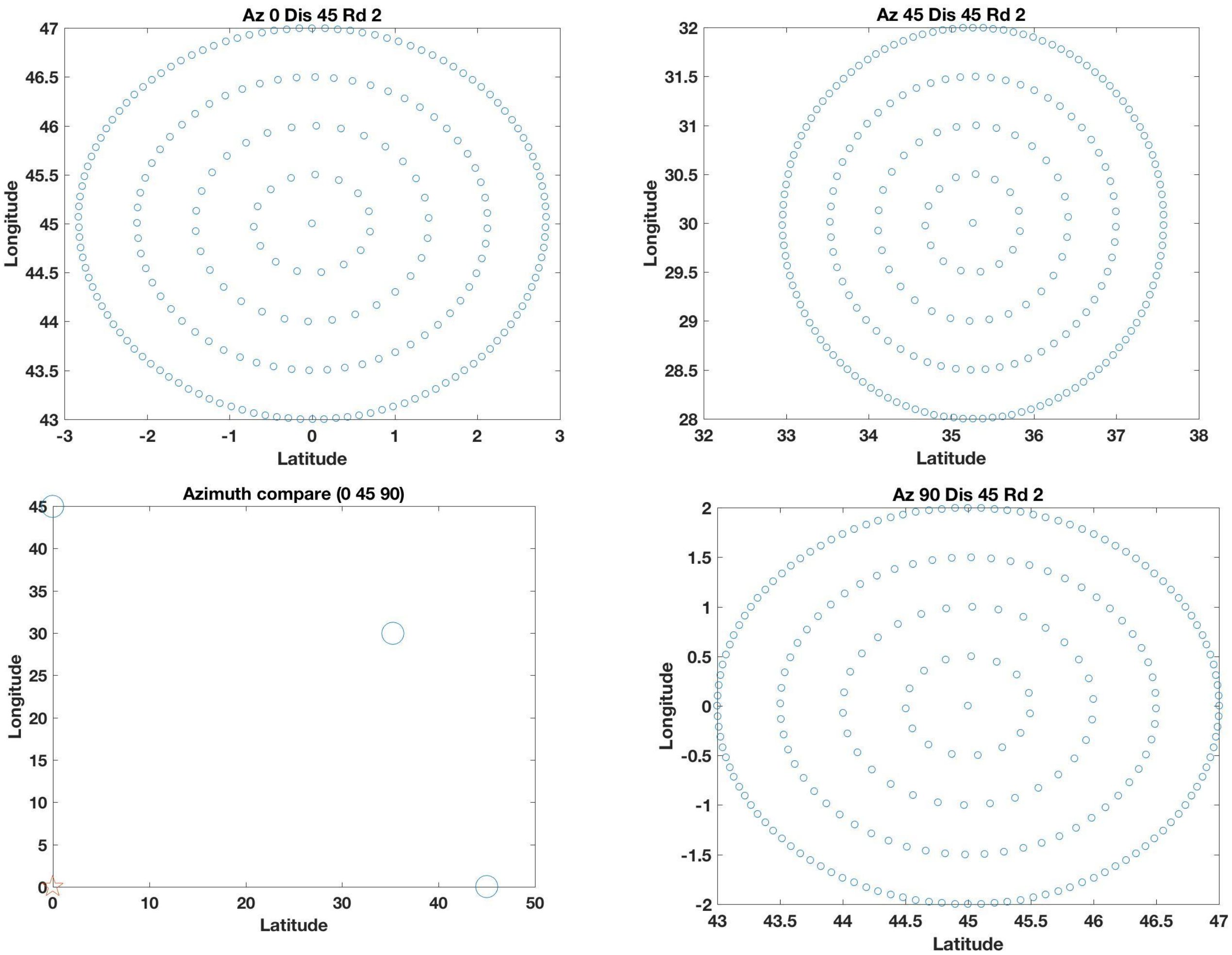
Circle array: 230 stations
Distance
Azimuth
Aperture



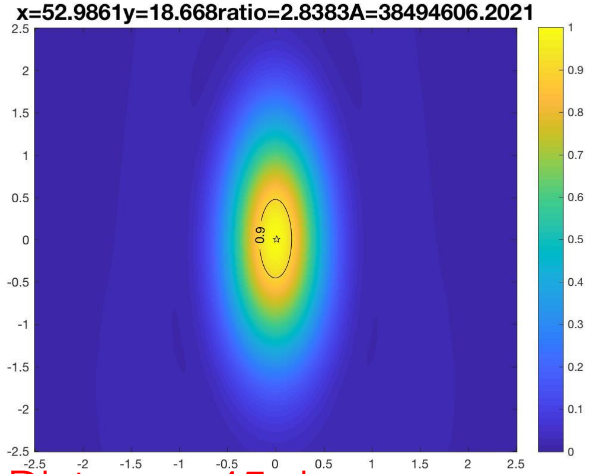




ARF-azimuth

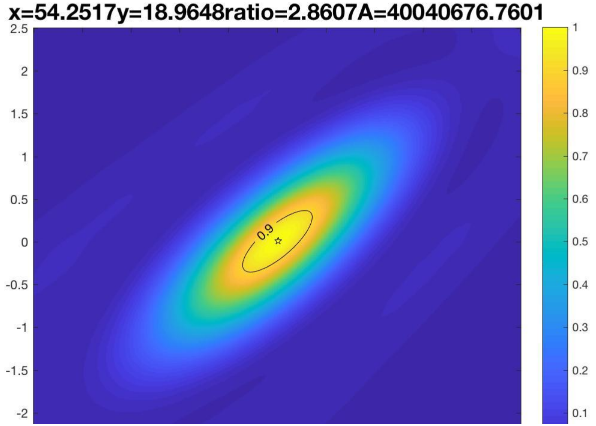

Az 0 Dis 45 Rd 2
Longitude
Latitude
Az 45 Dis 45 Rd 2
Longitude
Latitude
Azimuth compare (0 45 90)
Longitude
Latitude
Az 90 Dis 45 Rd 2
Longitude
Latitude

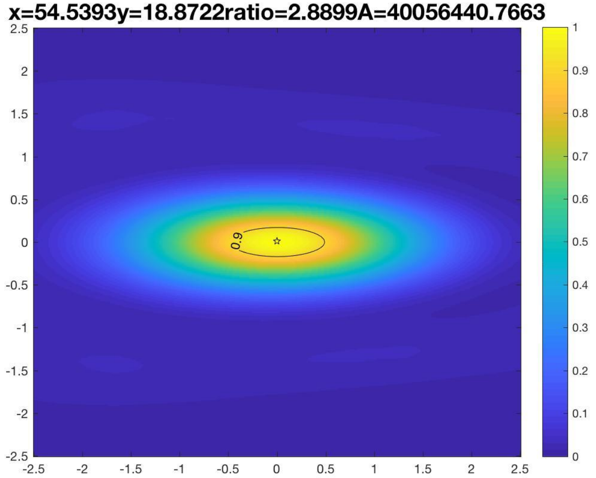
ARF-azimuth



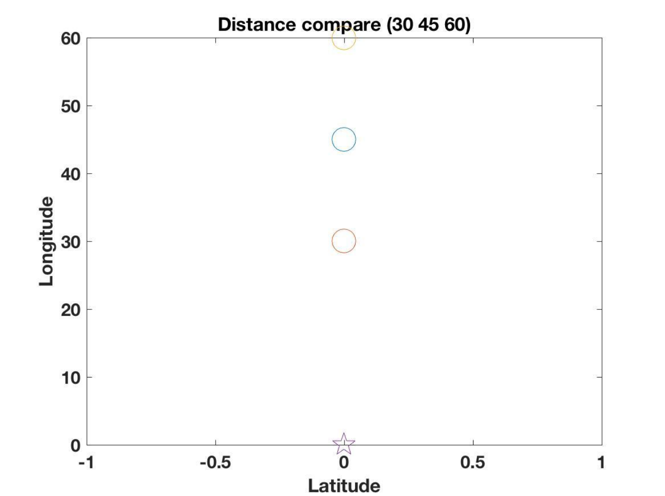
Distance: 45 degrees
Azimuth 0 45 90 degrees
Radius of the array: 2 degrees

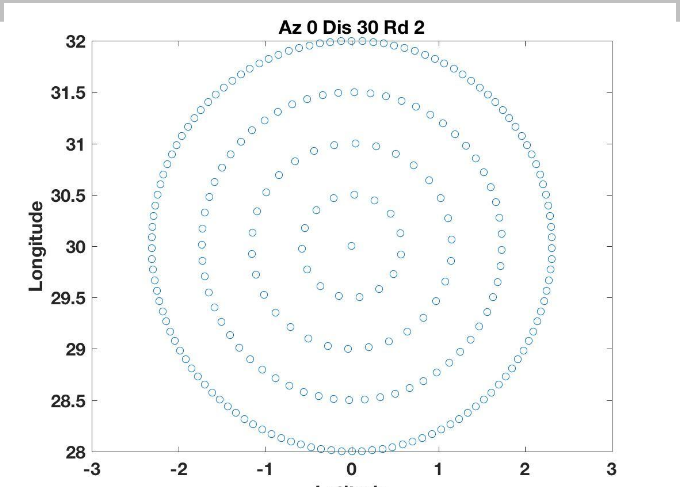
Azimuth doesn’t affect the resolution

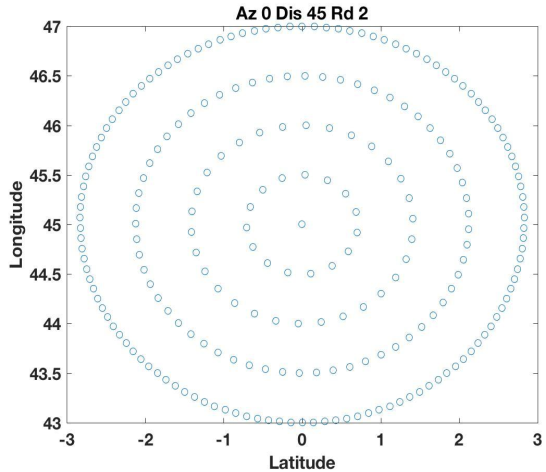


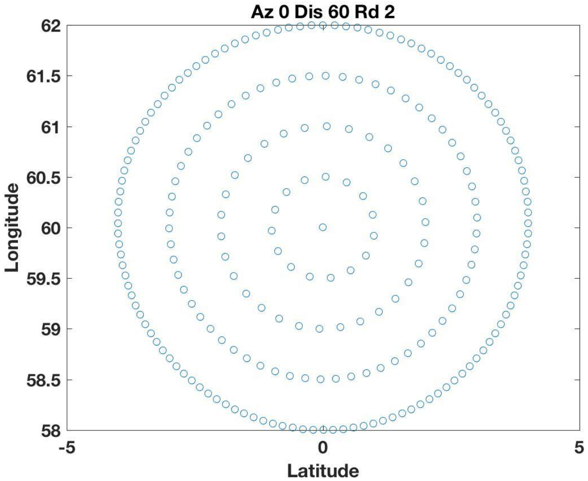


ARF-distance

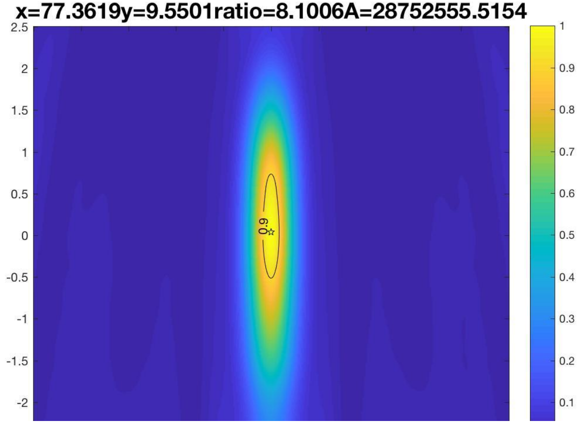


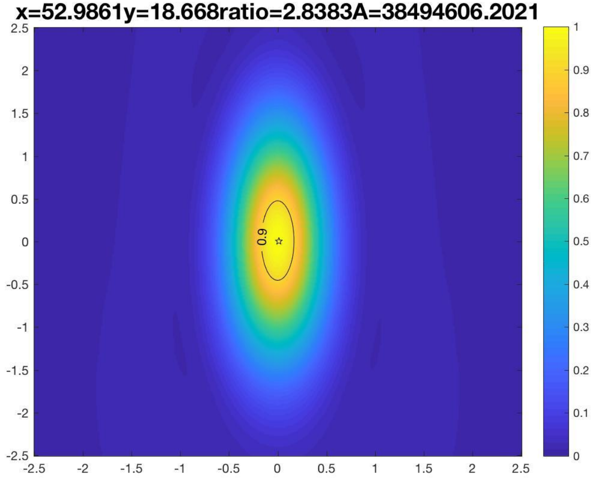


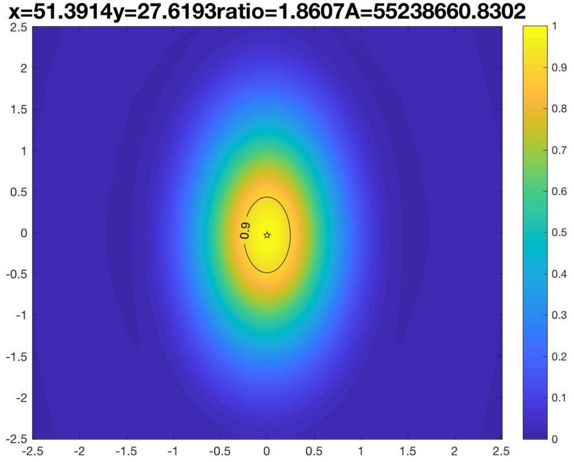




ARF-distance



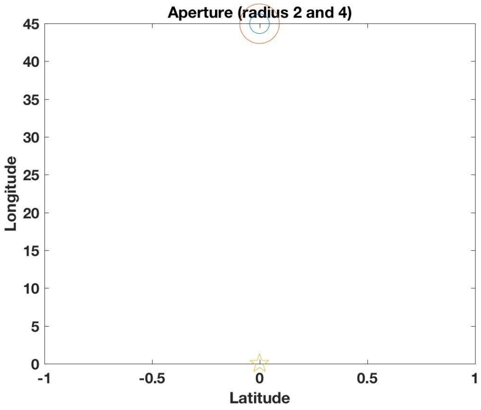




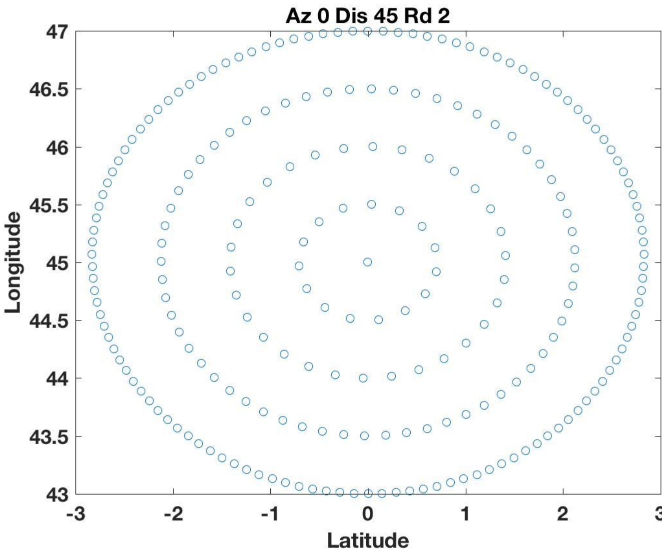
Distance: 30 45 60 degrees
Azimuth 0 degrees
Radius of the array: 2 degrees

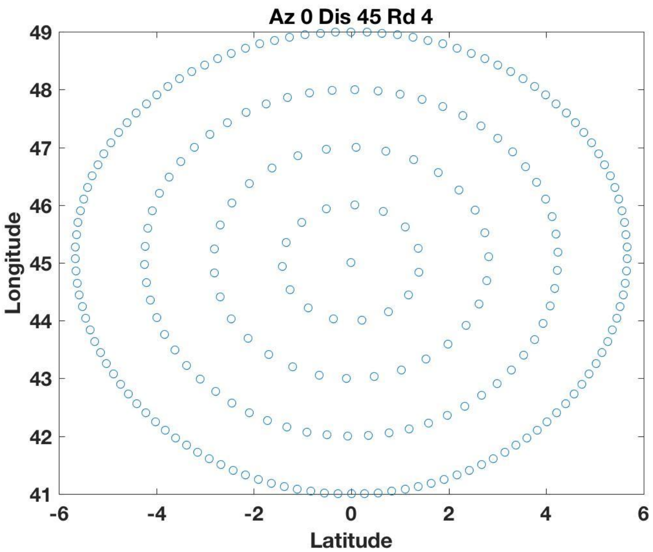
The resolution of the direction that perpendicular to the azimuth decreases proportionally to the D/R

ARF-aperture



Distance: 45 degrees
Azimuth 0
Radius of the array: 2 and 4 degrees

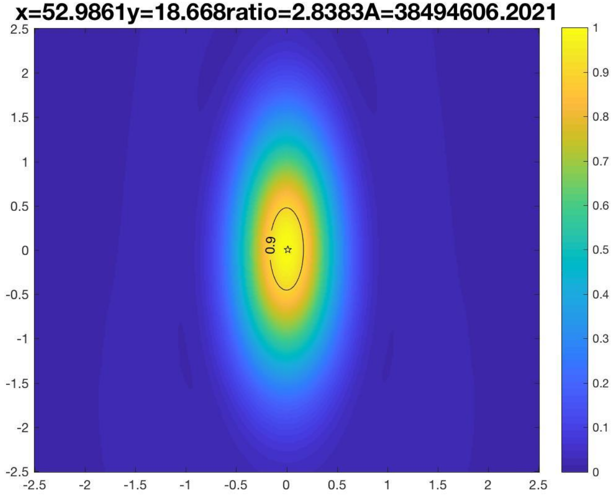


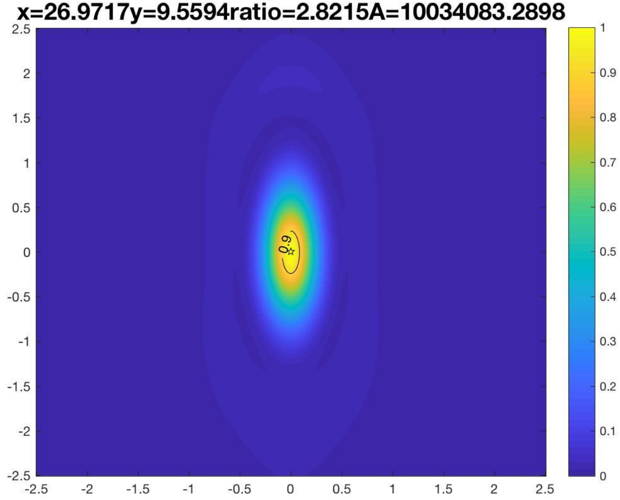


ARF-aperture

Distance: 45 degrees
Azimuth 0

Aperture improve the resolution proportionally

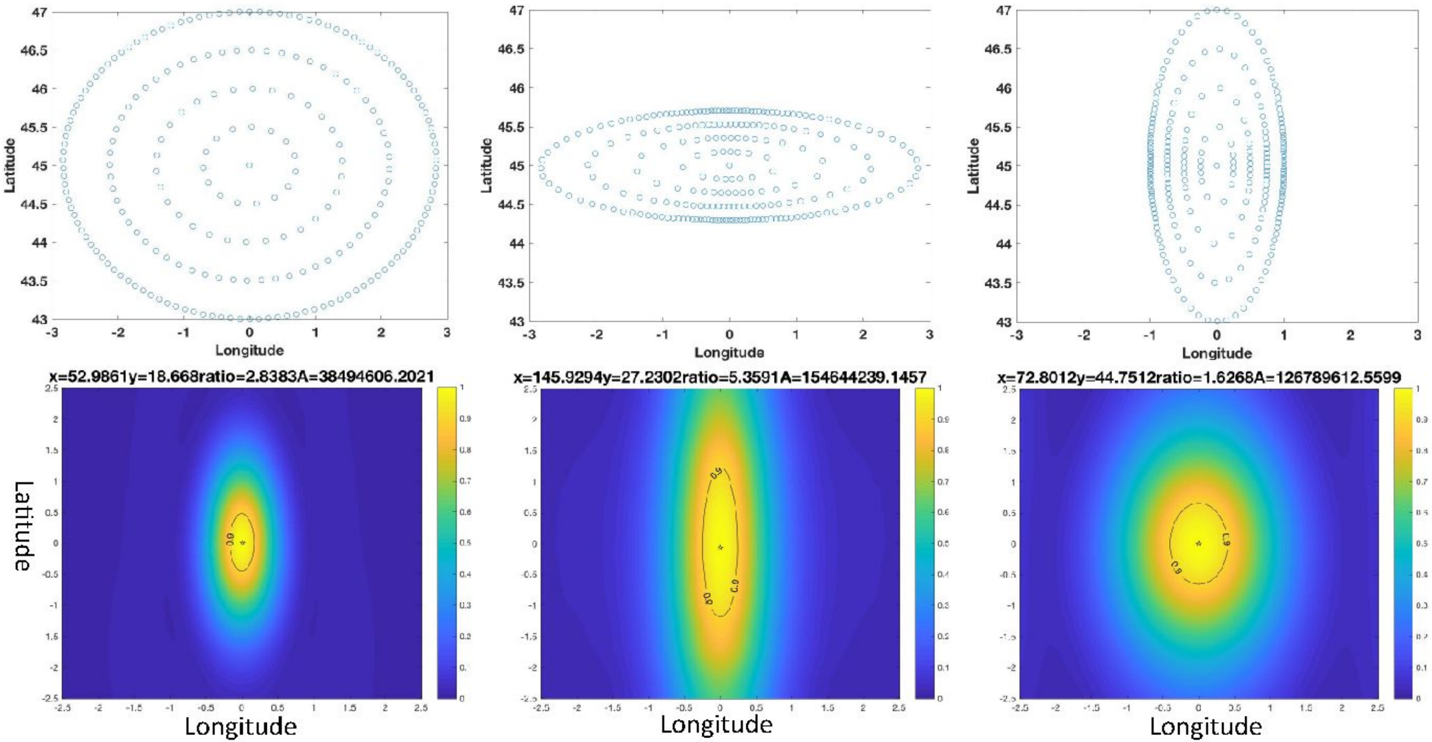




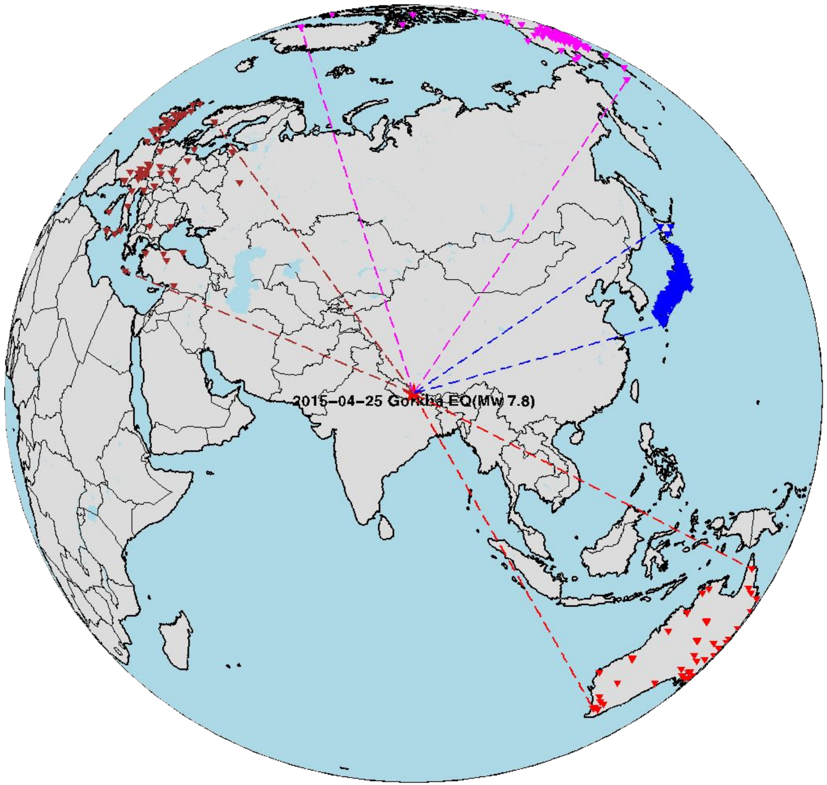
Array radius: 2 degrees

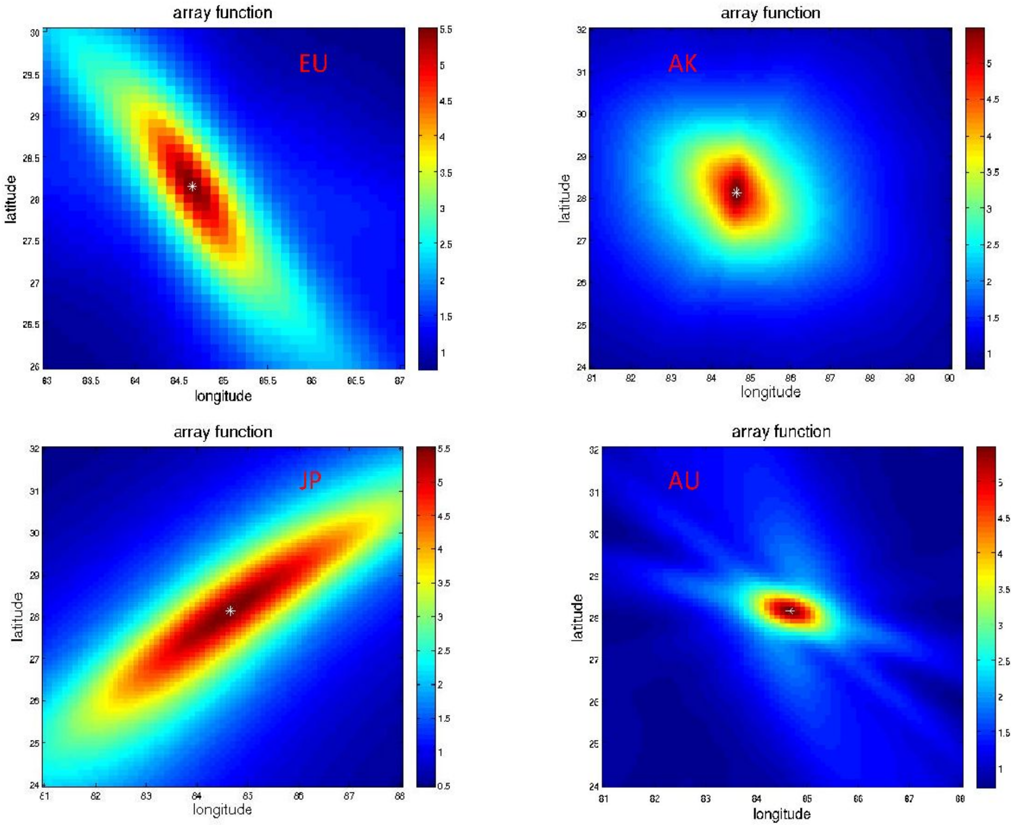
Array radius: 4 degrees

ARF-array geometry

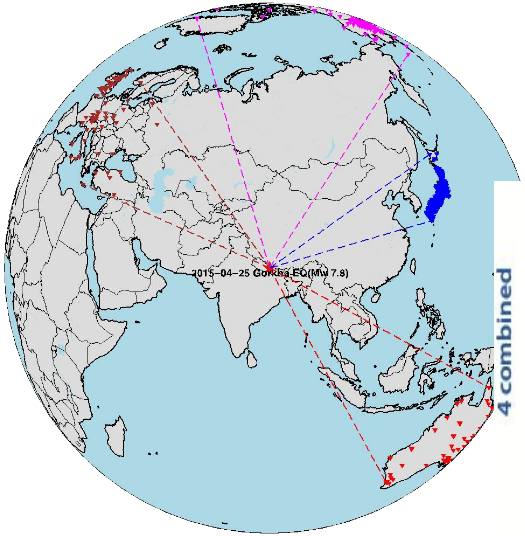


ARF

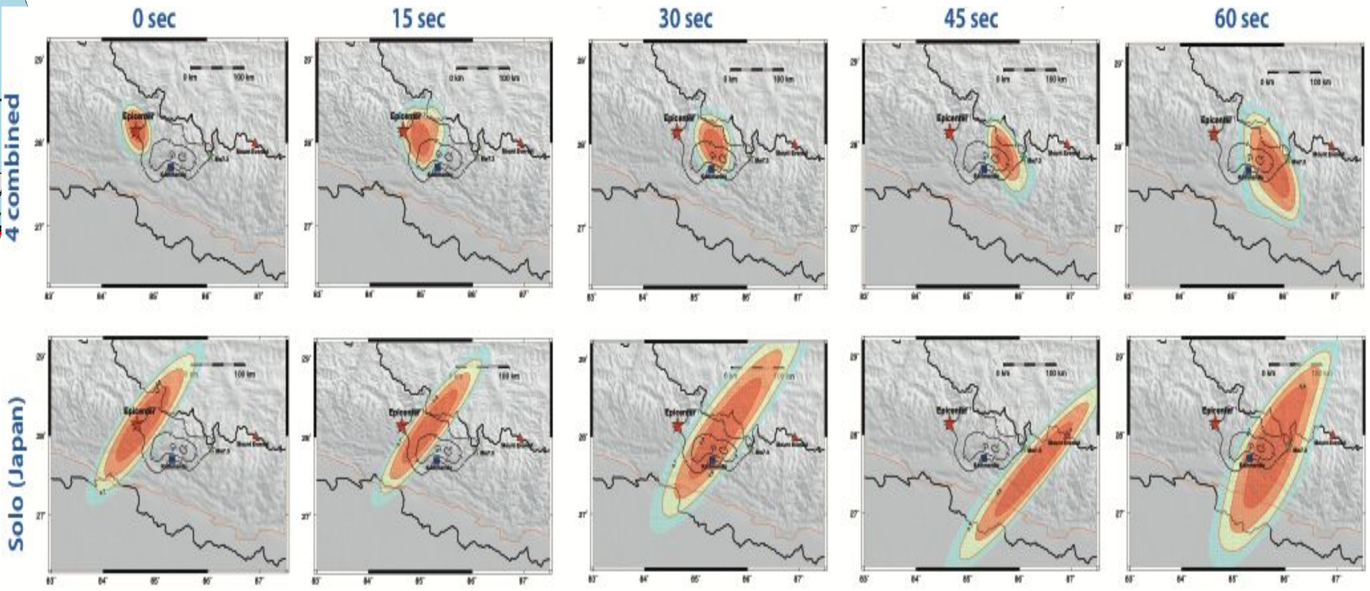

2015-04-25 Gorkha EQ(Mw 7.8)


array function
EU
array function
AK
array function
JP
array function
AU
latitude
longitude

Back-Projection & ARF



Combining arrays to improve the resolution



Exercise 1:
- Data request and processing
- ARF test
- Locate a point source using array data (beamforming)

Exercise 2:
- Pick up one earthquake and do BP