II.1

A Brief History of Data Visualization

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It is common to think of statistical graphics and data visualization as relatively modern developments in statistics. In fact, the graphic representation of quantitative information has deep roots. These roots reach into the histories of the earliest map making and visual depiction, and later into thematic cartography, statistics and statistical graphics, medicine and other fields. Along the way, developments in technologies (printing, reproduction), mathematical theory and practice, and empirical observation and recording enabled the wider use of graphics and new advances in form and content.

This chapter provides an overview of the intellectual history of data visualization from medieval to modern times, describing and illustrating some significant advances along the way. It is based on a project, called the *Milestones Project*, to collect, catalogue and document in one place the important developments in a wide range of areas and fields that led to modern data visualization. This effort has suggested some questions concerning the use of present-day methods of analysing and understanding this history, which I discuss under the rubric of 'statistical historiography.'

1.1 Introduction

The only new thing in the world is the history you don't know. – Harry S Truman

It is common to think of statistical graphics and data visualization as relatively modern developments in statistics. In fact, the graphic portrayal of quantitative information has deep roots. These roots reach into the histories of the earliest map-making and visual depiction, and later into thematic cartography, statistics and statistical graphics, with applications and innovations in many fields of medicine and science which are often intertwined with each other. They also connect with the rise of statistical thinking and widespread data collection for planning and commerce up through the 19th century. Along the way, a variety of advancements contributed to the widespread use of data visualization today. These include technologies for drawing and reproducing images, advances in mathematics and statistics, and new developments in data collection, empirical observation and recording.

From above ground, we can see the current fruit and anticipate future growth; we must look below to understand their germination. Yet the great variety of roots and nutrients across these domains, which gave rise to the many branches we see today, are often not well known and have never been assembled in a single garden to be studied or admired.

This chapter provides an overview of the intellectual history of data visualization from medieval to modern times, describing and illustrating some significant advances along the way. It is based on what I call the Milestones Project, an attempt to provide a broadly comprehensive and representative catalogue of important developments in *all* fields related to the history of data visualization.

There are many historical accounts of developments within the fields of probability (Hald, 1990), statistics (Pearson, 1978; Porter, 1986; Stigler, 1986), astronomy (Riddell, 1980) and cartography (Wallis and Robinson, 1987), which relate to, *inter alia*, some of the important developments contributing to modern data visualization. There are other, more specialized, accounts which focus on the early history of graphic recording (Hoff and Geddes, 1959, 1962), statistical graphs (Funkhouser, 1936, 1937; Royston, 1970; Tilling, 1975), fitting equations to empirical data (Farebrother, 1999), economics and time-series graphs (Klein, 1997), cartography (Friis, 1974; Kruskal, 1977) and thematic mapping (Robinson, 1982; Palsky, 1996) and so forth; Robinson (Robinson, 1982, Chap. 2) presents an excellent overview of some of the important scientific, intellectual and technical developments of the 15th–18th centuries leading to thematic cartography and statistical thinking. Wainer and Velleman (2001) provide a recent account of some of the history of statistical graphics.

But there are no accounts which span the entire development of visual thinking and the visual representation of data and which collate the contributions of disparate disciplines. Inasmuch as their histories are intertwined, so too should be any telling of the development of data visualization. Another reason for interweaving these accounts is that practitioners in these fields today tend to be highly specialized and unaware of related developments in areas outside their domain, much less of their history.

Milestones Tour 1.2

Every picture tells a story. – Rod Stewart, 1971

In organizing this history, it proved useful to divide history into epochs, each of which turned out to be describable by coherent themes and labels. This division is, of course, somewhat artificial, but it provides the opportunity to characterize the accomplishments in each period in a general way before describing some of them in more detail. Figure 1.1, discussed in Sect. 1.3.2, provides a graphic overview of the epochs I describe in the subsections below, showing the frequency of events considered milestones in the periods of this history. For now, it suffices to note the labels attached to these epochs, a steady rise from the early 18th century to the late 19th century, with a curious wiggle thereafter.

In the larger picture – recounting the history of data visualization – it turns out that many of the milestone items have a story to be told: What motivated this development? What was the communication goal? How does it relate to other developments – What were the precursors? How has this idea been used or re-invented today? Each section below tries to illustrate the general themes with a few exemplars. In particular, this account attempts to tell a few representative stories of these periods, rather than to try to be comprehensive.

For reasons of economy, only a limited number of images could be printed here, and these only in black and white. Others are referred to by Web links, mostly from

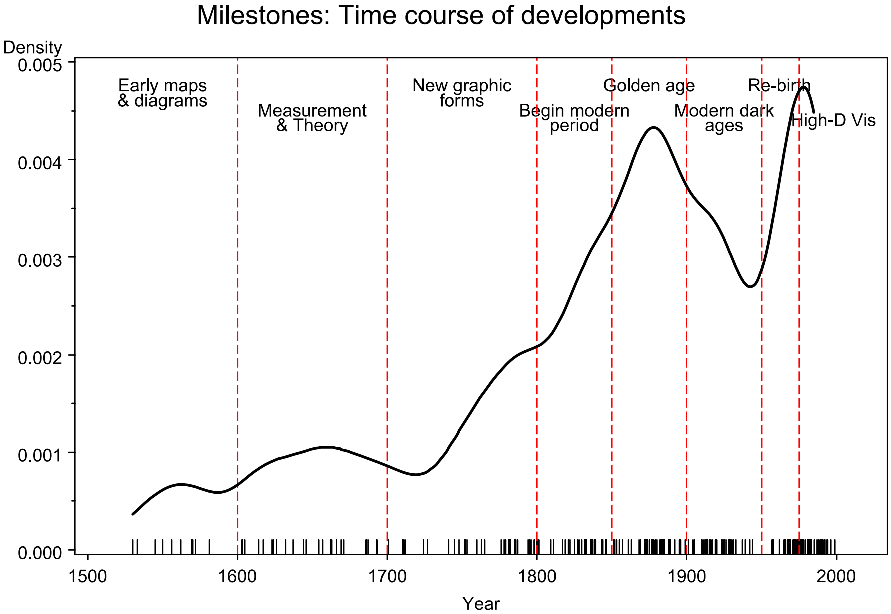

Figure 1.1. Time distribution of events considered milestones in the history of data visualization, shown by a rug plot and density estimate

the Milestones Project, http://www.math.yorku.ca/SCS/Gallery/milestone/, where a colour version of this chapter will also be found.

1.2.1 Pre-17th Century: Early Maps and Diagrams

The earliest seeds of visualization arose in geometric diagrams, in tables of the positions of stars and other celestial bodies, and in the making of maps to aid in navigation and exploration. The idea of coordinates was used by ancient Egyptian surveyors in laying out towns, earthly and heavenly positions were located by something akin to latitude and longitude by at least 200 B.C., and the map projection of a spherical earth into latitude and longitude by Claudius Ptolemy [c. 85–c. 165] in Alexandria would serve as reference standards until the 14th century.

Among the earliest graphical depictions of quantitative information is an anonymous 10th-century multiple time-series graph of the changing position of the seven most prominent heavenly bodies over space and time (Fig. 1.2), described by Funkhouser (1936) and reproduced in Tufte (1983, p. 28). The vertical axis represents the inclination of the planetary orbits; the horizontal axis shows time, divided into 30 intervals. The sinusoidal variation with different periods is notable, as is the use of a grid, suggesting both an implicit notion of a coordinate system and something akin to graph paper, ideas that would not be fully developed until the 1600–1700s.

In the 14th century, the idea of plotting a theoretical function (as a proto bar graph) and the logical relation between tabulating values and plotting them appeared in

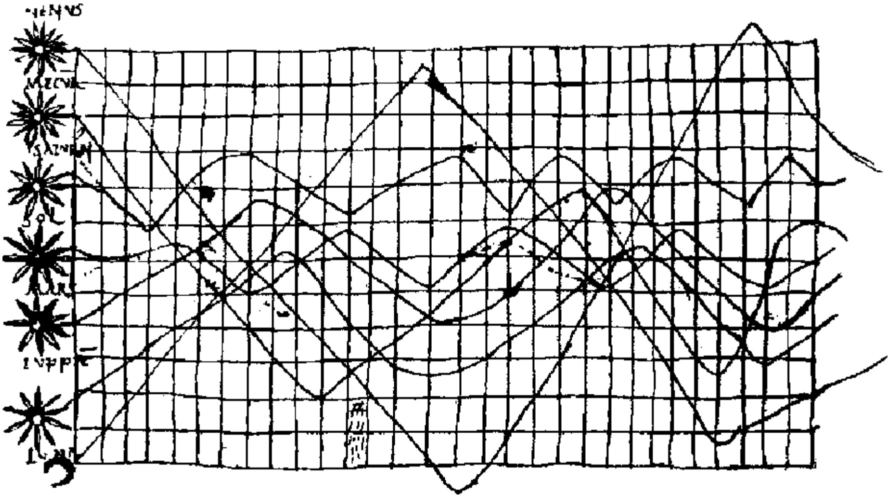

Figure 1.2. Planetary movements shown as cyclic inclinations over time, by an unknown astronomer, appearing in a 10th-century appendix to commentaries by A.T. Macrobius on Cicero's *In Somnium Sciponis*. *Source*: Funkhouser (1936, p. 261)

a work by Nicole Oresme [1323–1382] Bishop of Liseus[1] (Oresme, 1482, 1968), followed somewhat later by the idea of a theoretical graph of distance vs. speed by Nicolas of Cusa.

By the 16th century, techniques and instruments for precise observation and measurement of physical quantities and geographic and celestial position were well developed (for example, a 'wall quadrant' constructed by Tycho Brahe [1546–1601], covering an entire wall in his observatory). Particularly important were the development of triangulation and other methods to determine mapping locations accurately (Frisius, 1533; Tartaglia, 1556). As well, we see initial ideas for capturing images directly (the camera obscura, used by Reginer Gemma-Frisius in 1545 to record an eclipse of the sun), the recording of mathematical functions in tables (trigonometric tables by Georg Rheticus, 1550) and the first modern cartographic atlas (*Theatrum Orbis Terrarum* by Abraham Ortelius, 1570). These early steps comprise the beginnings of data visualization.

1600–1699: Measurement and Theory 1.2.2

Amongst the most important problems of the 17th century were those concerned with physical measurement – of time, distance and space – for astronomy, survey-

[1] Funkhouser (1936, p. 277) was sufficiently impressed with Oresme's grasp of the relation between functions and graphs that he remarked, 'If a pioneering contemporary had collected some data and presented Oresme with actual figures to work upon, we might have had statistical graphs four hundred years before Playfair.'

ing, map making, navigation and territorial expansion. This century also saw great new growth in theory and the dawn of practical application – the rise of analytic geometry and coordinate systems (Descartes and Fermat), theories of errors of measurement and estimation (initial steps by Galileo in the analysis of observations on Tycho Brahe's star of 1572 (Hald, 1990, §10.3)), the birth of probability theory (Pascal and Fermat) and the beginnings of demographic statistics (John Graunt) and 'political arithmetic' (William Petty) – the study of population, land, taxes, value of goods, etc. for the purpose of understanding the wealth of the state.

Early in this century, Christopher Scheiner (1626–1630, recordings from 1611) introduced an idea Tufte (1983) would later call the principle of 'small multiples' to show the changing configurations of sunspots over time, shown in Fig. 1.3. The multiple images depict the recordings of sunpots from 23 October 1611 until 19 December of that year. The large key in the upper left identifies seven groups of sunspots by the letters A–G. These groups are similarly identified in the 37 smaller images, arrayed left to right and top to bottom below.

Another noteworthy example (Fig. 1.4) shows a 1644 graphic by Michael Florent van Langren[1600–1675], a Flemish astronomer to the court of Spain, believed to be the first visual representation of statistical data (Tufte, 1997, p. 15). At that time, lack of

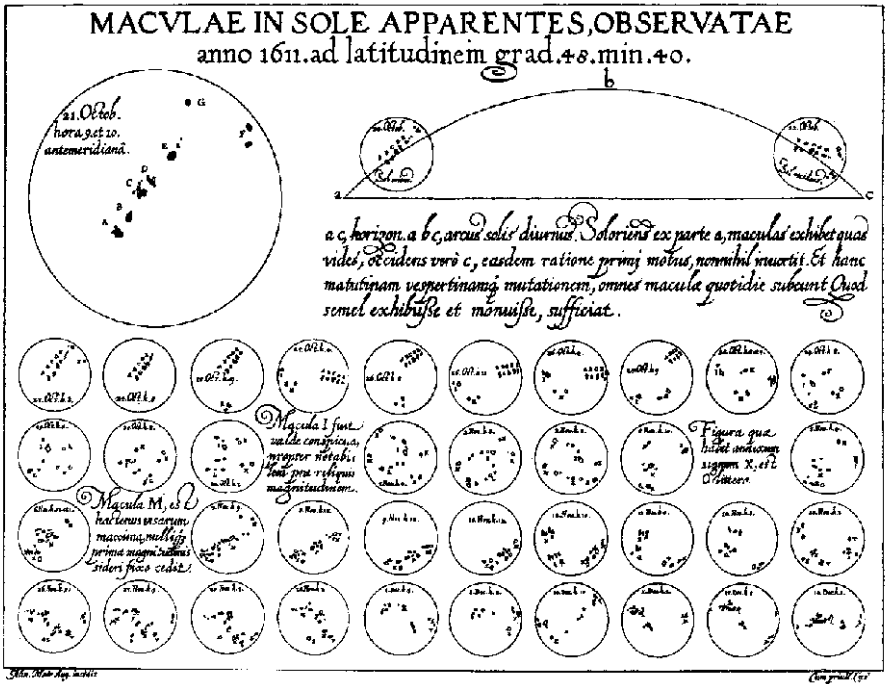

Figure 1.3. Scheiner's 1626 representation of the changes in sunspots over time. *Source*: Scheiner (1626–1630)

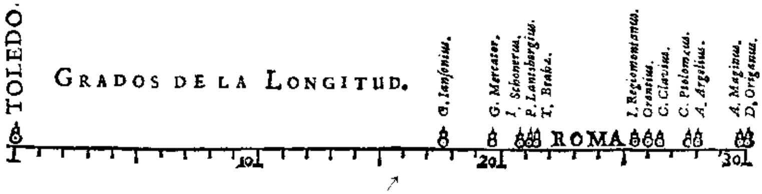

Figure 1.4. Langren's 1644 graph of determinations of the distance, in longitude, from Toledo to Rome. The correct distance is 16°30′. *Source*: Tufte (1997, p. 15)

a reliable means to determine longitude at sea hindered navigation and exploration.[2] This 1-D line graph shows all 12 known estimates of the difference in longitude between Toledo and Rome and the name of the astronomer (Mercator, Tycho Brahe, Ptolemy, etc.) who provided each observation.

What is notable is that van Langren could have presented this information in various tables – ordered by author to show provenance, by date to show priority, or by distance. However, only a graph shows the wide variation in the estimates; note that the range of values covers nearly half the length of the scale. Van Langren took as his overall summary the centre of the range, where there happened to be a large enough gap for him to inscribe 'ROMA.' Unfortunately, all of the estimates were biased upwards; the true distance (16°30′) is shown by the arrow. Van Langren's graph is also a milestone as the earliest known exemplar of the principle of 'effect ordering for data display' (Friendly and Kwan, 2003).

In the 1660s, the systematic collection and study of social data began in various European countries, under the rubric of 'political arithmetic' (John Graunt, 1662 and William Petty, 1665), with the goals of informing the state about matters related to wealth, population, agricultural land, taxes and so forth,[3] as well as for commercial purposes such as insurance and annuities based on life tables (Jan de Witt, 1671). At approximately the same time, the initial statements of probability theory around 1654 (see Ball, 1908) together with the idea of coordinate systems were applied by Christiaan Huygens in 1669 to give the first graph of a continuous distribution function[4] (from Graunt's based on the bills of mortality). The mid-1680s saw the first bivariate plot derived from empirical data, a theoretical curve relating barometric pressure to altitude, and the first known weather map,[5] showing prevailing winds on a map of the earth (Halley, 1686).

By the end of this century, the necessary elements for the development of graphical methods were at hand – some real data of significant interest, some theory to make

[2] For navigation, latitude could be fixed from star inclinations, but longitude required accurate measurement of time at sea, an unsolved problem until 1765 with the invention of a marine chronometer by John Harrison. See Sobel (1996) for a popular account.

[3] For example, Graunt (1662) used his tabulations of London births and deaths from parish records and the bills of mortality to estimate the number of men the king would find available in the event of war (Klein, 1997, pp. 43–47).

[4] Image: http://math.yorku.ca/SCS/Gallery/images/huygens-graph.gif

[5] Image: http://math.yorku.ca/SCS/Gallery/images/halleyweathermap-1686.jpg

sense of them, and a few ideas for their visual representation. Perhaps more importantly, one can see this century as giving rise to the beginnings of visual thinking, as illustrated by the examples of Scheiner and van Langren.

1.2.3 1700–1799: New Graphic Forms

With some rudiments of statistical theory, data of interest and importance, and the idea of graphic representation at least somewhat established, the 18th century witnessed the expansion of these aspects to new domains and new graphic forms. In cartography, mapmakers began to try to show more than just geographical position on a map. As a result, new data representations (isolines and contours) were invented, and thematic mapping of physical quantities took root. Towards the end of this century, we see the first attempts at the thematic mapping of geologic, economic and medical data.

Abstract graphs, and graphs of functions became more widespread, along with the early stirrings of statistical theory (measurement error) and systematic collection of empirical data. As other (economic and political) data began to be collected, some novel visual forms were invented to portray them, so the data could 'speak to the eyes.'

For example, the use of isolines to show contours of equal value on a coordinate grid (maps and charts) was developed by Edmund Halley (1701). Figure 1.5, showing isogons – lines of equal magnetic declination – is among the first examples of thematic cartography, overlaying data on a map. Contour maps and topographic maps were introduced somewhat later by Philippe Buache (1752) and Marcellin du Carla-Boniface (1782).

Timelines, or 'cartes chronologiques,' were first introduced by Jacques Barbeu-Dubourg in the form of an annotated chart of all of history (from Creation) on a 54-foot scroll (Ferguson, 1991). Joseph Priestley, presumably independently, used a more convenient form to show first a timeline chart of biography (lifespans of 2000 famous people, 1200 B.C. to A.D. 1750, Priestley, 1765), and then a detailed chart of history (Priestley, 1769).

The use of geometric figures (squares or rectangles) and cartograms to compare areas or demographic quantities by Charles de Fourcroy[6] (1782) and August F.W. Crome (1785) provided another novel visual encoding for quantitative data using superimposed squares to compare the areas of European states.

As well, several technological innovations provided necessary ingredients for the production and dissemination of graphic works. Some of these facilitated the reproduction of data images, such as three-colour printing, invented by Jacob le Blon in 1710, and lithography, invented by Aloys Senefelder in 1798. Of the latter, Robinson (1982, p. 57) says "the effect was as great as the introduction [of the Xerox machine]." Yet, likely due to expense, most of these new graphic forms appeared in publications with limited circulation, unlikely to attract wide attention.

[6] Image: http://math.yorku.ca/SCS/Gallery/images/palsky/defourcroy.jpg

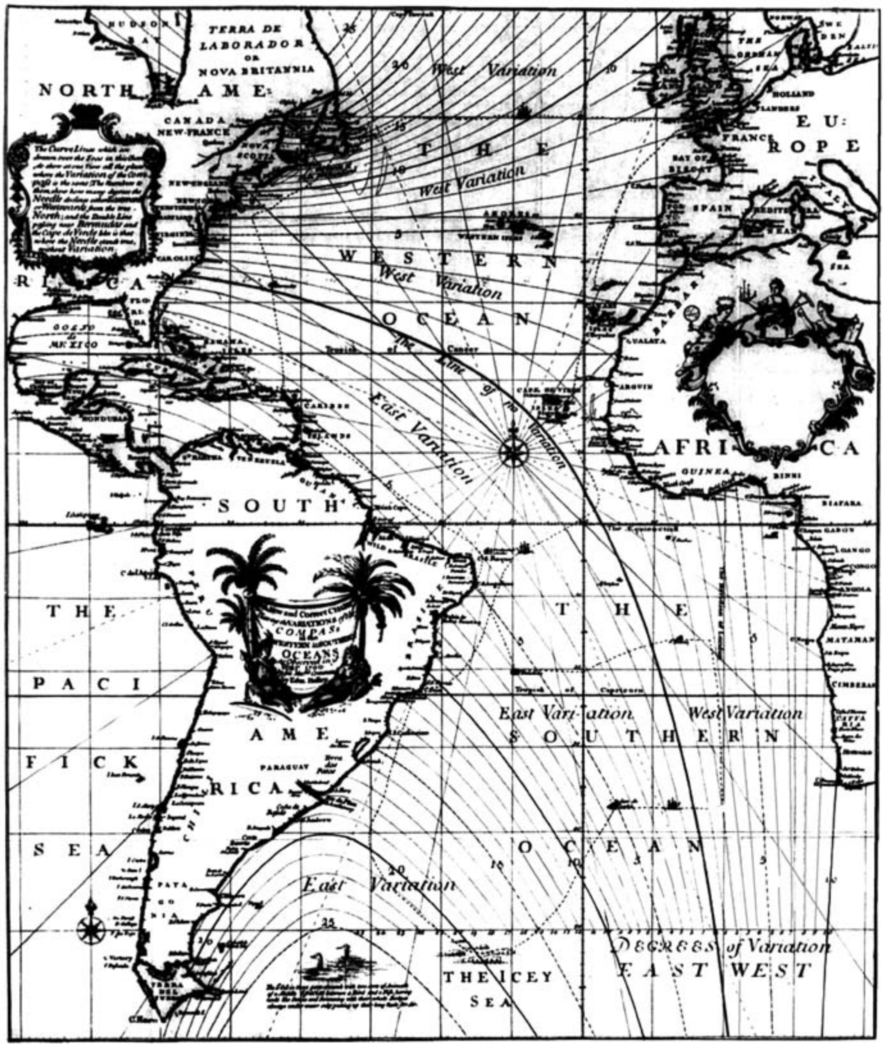

Figure 1.5. A portion of Edmund Halley's *New and Correct Sea Chart Shewing the Variations in the Compass in the Western and Southern Ocean*, 1701. *Source*: Halley (1701), image from Palsky (1996, p. 41)

A prodigious contributor to the use of the new graphical methods, Johann Lambert [1728–1777] introduced the ideas of curve fitting and interpolation from empirical data points. He used various sorts of line graphs and graphical tables to show periodic variation in, for example, air and soil temperature.[7]

William Playfair [1759–1823] is widely considered the inventor of most of the graphical forms used today – first the line graph and barchart (Playfair, 1786), later the

[7] Image: http://www.journals.uchicago.edu/Isis/journal/demo/v000n000/000000/fg7.gif

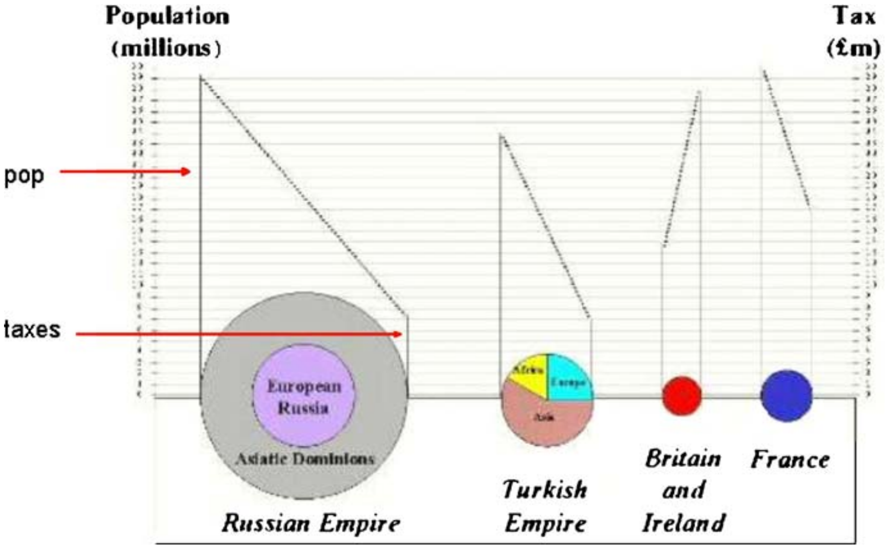

Figure 1.6. Redrawn version of a portion of Playfair's 1801 pie-circle-line chart, comparing population and taxes in several nations

piechart and circle graph (Playfair, 1801). Figure 1.6 shows a creative combination of different visual forms: circles, pies and lines, redrawn from Playfair (1801, Plate 2).

The use of two separate vertical scales for different quantities (population and taxes) is today considered a sin in statistical graphics (you can easily jiggle either scale to show different things). But Playfair used this device to good effect here to try to show taxes per capita in various nations and argue that the British were over-taxed, compared with others. But, alas, showing simple numbers by a graph was hard enough for Playfair – he devoted several pages of text in Playfair (1786) describing how to read and understand a line graph. The idea of calculating and graphing rates and other indirect measurements was still to come.

In this figure, the left axis and line on each circle/pie graph shows population, while the right axis and line shows taxes. Playfair intended that the *slope* of the line connecting the two would depict the rate of taxation directly to the eye; but, of course, the slope also depends on the diameters of the circles. Playfair's graphic sins can perhaps be forgiven here, because the graph clearly shows the slope of the line for Britain to be in the opposite direction of those for the other nations.

A somewhat later graph (Playfair, 1821), shown in Fig. 1.7, exemplifies the best that Playfair had to offer with these graphic forms. Playfair used three parallel time series to show the price of wheat, weekly wages and reigning ruler over a 250-year span from 1565 to 1820 and used this graph to argue that workers had become better off in the most recent years.

By the end of this century (1794), the utility of graphing in scientific applications prompted a Dr Buxton in London to patent and market printed coordinate paper; curiously, a patent for lined notepaper was not issued until 1815. The first known

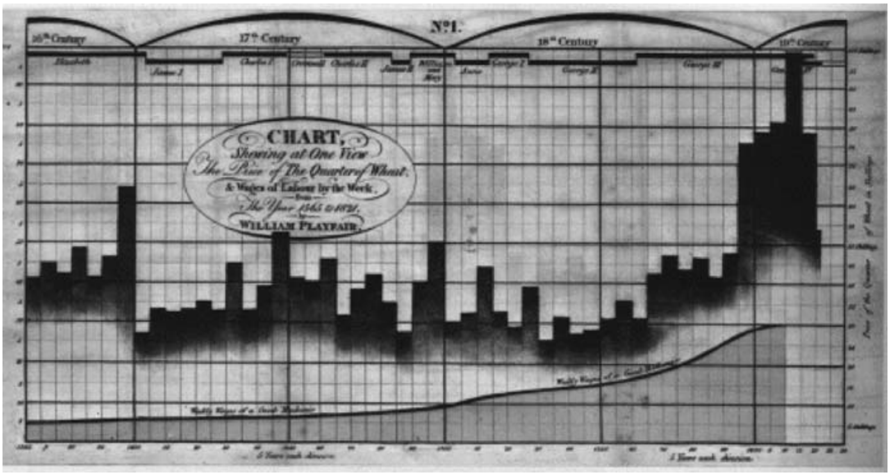

Figure 1.7. William Playfair's 1821 time-series graph of prices, wages and reigning ruler over a 250-year period. *Source*: Playfair (1821), image from Tufte (1983, p. 34)

published graph using coordinate paper is one of periodic variation in barometric pressure (Howard, 1800). Nevertheless, graphing of data would remain rare for another 30 or so years,[8] perhaps largely because there wasn't much quantitative information (apart from widespread astronomical, geodetic and physical measurement) of sufficient complexity to require new methods and applications. Official statistics, regarding population and mortality, and economic data were generally fragmentary and often not publicly available. This would soon change.

1800–1850: Beginnings of Modern Graphics 1.2.4

With the fertilization provided by the previous innovations of design and technique, the first half of the 19th century witnessed explosive growth in statistical graphics and thematic mapping, at a rate which would not be equalled until modern times.

In statistical graphics, all of the modern forms of data display were invented: bar- and piecharts, histograms, line graphs and time-series plots, contour plots, scatterplots and so forth. In thematic cartography, mapping progressed from single maps to comprehensive atlases, depicting data on a wide variety of topics (economic, social, moral, medical, physical, etc.), and introduced a wide range of novel forms of symbolism. During this period graphical analysis of natural and physical phenomena (lines of magnetism, weather, tides, etc.) began to appear regularly in scientific publications as well.

In 1801, the first geological maps were introduced in England by William Smith [1769–1839], setting the pattern for geological cartography or 'stratigraphic geology'

[8] William Herschel (1833), in a paper that describes the first instance of a modern scatterplot, devoted three pages to a description of plotting points on a grid.

(Smith, 1815). These and other thematic maps soon led to new ways of showing quantitative information on maps and, equally importantly, to new domains for graphically based inquiry.

In the 1820s, Baron Charles Dupin [1784–1873] invented the use of continuous shadings (from white to black) to show the distribution and degree of illiteracy in France (Dupin, 1826) – the first unclassed choropleth map,[9] and perhaps the first modern-style thematic statistical map (Palsky, 1996, p. 59). Later given the lovely title 'Carte de la France obscure et de la France éclairée,' it attracted wide attention, and was also perhaps the first application of graphics in the social realm.

More significantly, in 1825, the ministry of justice in France instituted the first centralized national system of crime reporting, collected quarterly from all departments and recording the details of every charge laid before the French courts. In 1833, André-Michel Guerry, a lawyer with a penchant for numbers, used these data (along with other data on literacy, suicides, donations to the poor and other 'moral' variables) to produce a seminal work on the moral statistics of France (Guerry, 1833) – a work that (along with Quételet, 1831, 1835) can be regarded as the foundation of modern social science.[10]

Guerry used maps in a style similar to Dupin to compare the ranking of departments on pairs of variables, notably crime vs. literacy, but other pairwise variable comparisons were made.[11] He used these to argue that the lack of an apparent (negative) relation between crime and literacy contradicted the armchair theories of some social reformers who had argued that the way to reduce crime was to increase education.[12] Guerry's maps and charts made somewhat of an academic sensation both in France and the rest of Europe; he later exhibited several of these at the 1851 London Exhibition and carried out a comparative study of crime in England and France (Guerry, 1864) for which he was awarded the Moynton Prize in statistics by the French Academy of Sciences.[13] But Guerry's systematic and careful work was unable

[9] Image: http://math.yorku.ca/SCS/Gallery/images/dupin1826-map_200.jpg

[10] Guerry showed that rates of crime, when broken down by department, type of crime, age and gender of the accused and other variables, remained remarkably consistent from year to year, yet varied widely across departments. He used this to argue that such regularity implied the possibility of establishing social laws, much as the regularity of natural phenomena implied physical ones. Guerry also pioneered the study of suicide, with tabulations of suicides in Paris, 1827–1830, by sex, age, education, profession, etc., and a content analysis of suicide notes as to presumed motives.

[11] Today, one would use a scatterplot, but that graphic form had only just been invented (Herschel, 1833) and would not enter common usage for another 50 years; see Friendly and Denis (2005).

[12] Guerry seemed reluctant to take sides. He also contradicted the social conservatives who argued for the need to build more prisons or impose more severe criminal sentences. See Whitt (2002).

[13] Among the 17 plates in this last work, seven pairs of maps for England and France each included sets of small line graphs to show trends over time, decompositions by subtype of crime and sex, distributions over months of the year, and so forth. The final plate, on general causes of crime, is an incredibly detailed and complex multivariate semi-graphic display attempting to relate various types of crimes to each other, to various social and moral aspects (instruction, religion, population) as well as to their geographic distribution.

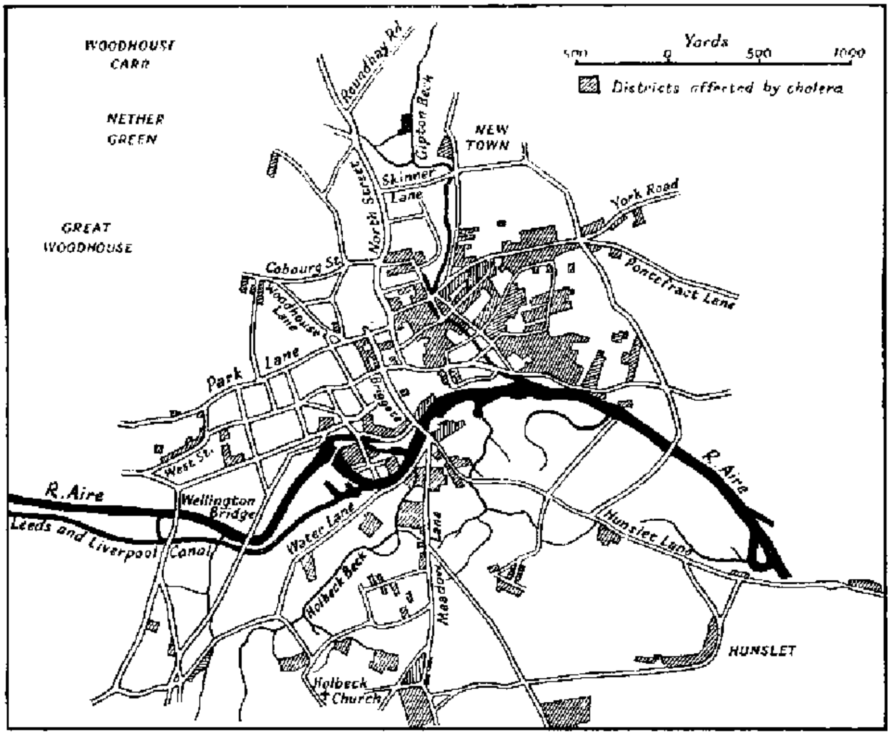

Figure 1.8. A portion of Dr Robert Baker's cholera map of Leeds, 1833, showing the districts affected by cholera. *Source*: Gilbert (1958, Fig. 2)

to shine in the shadows cast by Adolphe Quételet, who regarded moral and social statistics as his own domain.

In October 1831, the first case of asiatic cholera occurred in Great Britain, and over 52 000 people died in the epidemic that ensued over the next 18 months or so (Gilbert, 1958). Subsequent cholera epidemics in 1848–1849 and 1853–1854 produced similarly large death tolls, but the water-borne cause of the disease was unknown until 1855 when Dr John Snow produced his famous dot map[14] (Snow, 1855) showing deaths due to cholera clustered around the Broad Street pump in London. This was indeed a landmark graphic discovery, but it occurred at the end of the period, roughly 1835–1855, which marks a high point in the application of thematic cartography to human (social, medical, ethnic) topics. The first known disease map of cholera (Fig. 1.8), due to Dr Robert Baker (1833), shows the districts of Leeds 'affected by cholera' in the particularly severe 1832 outbreak.

I show this figure to make another point – why Baker's map did not lead to a 'eureka' experience, while John Snow's did. Baker used a town plan of Leeds that had been divided into districts. Of a population of 76 000 in all of Leeds, Baker mapped

[14] Image: http://www.math.yorku.ca/SCS/Gallery/images/snow4.jpg

the 1800 cholera cases by hatching in red 'the districts in which the cholera had prevailed.' In his report, he noted an association between the disease and living conditions: 'how exceedingly the disease has prevailed in those parts of the town where there is a deficiency, often an entire want of sewage, drainage and paving' (Baker, 1833, p. 10). Baker did not indicate the incidence of disease on his map, nor was he equipped to display *rates* of disease (in relation to population density),[15] and his knowledge of possible causes, while definitely on the right track, was both weak and implicit (not analysed graphically or by other means). It is likely that some, perhaps tenuous, causal indicants or evidence were available to Baker, but he was unable to connect the dots or see a geographically distributed outcome in relation to geographic factors in even the simple ways that Guerry had tried.

At about the same time, 1830–1850, the use of graphs began to become recognized in some official circles for economic and state planning – where to build railroads and canals? What is the distribution of imports and exports? This use of graphical methods is no better illustrated than in the works of Charles Joseph Minard [1781–1870], whose prodigious graphical inventions led Funkhouser (1937) to call him the Playfair of France. To illustrate, we choose (with some difficulty) an 1844 'tableau-graphique' (Fig. 1.9) by Minard, an early progenitor of the modern mosaicplot (Friendly, 1994). On the surface, mosaicplots descend from bar charts, but Minard introduced two simultaneous innovations: the use of divided and proportional-width bars so that area had a concrete visual interpretation. The graph shows the transportation of commercial goods along one canal route in France by variable-width, divided bars (Minard, 1844). In this display the width of each vertical bar shows distance along this route; the divided-bar segments have height proportional to amount of goods of various types (shown by shading), so the area of each rectangular segment is proportional to the cost of transport. Minard, a true visual engineer (Friendly, 2000), developed such diagrams to argue visually for setting differential price rates for partial vs. complete runs. Playfair had tried to make data 'speak to the eyes,' but Minard wished to make them 'calculer par l'œil' as well.

It is no accident that, in England, outside the numerous applications of graphical methods in the sciences, there was little interest in or use of graphs amongst statisticians (or 'statists' as they called themselves). If there is a continuum ranging from 'graph people' to 'table people,' British statisticians and economists were philosophically more table-inclined and looked upon graphs with suspicion up to the time of William Stanley Jevons around 1870 (Maas and Morgan, 2005). Statistics should be concerned with the recording of 'facts relating to communities of men which are capable of being expressed by numbers' (Mouat, 1885, p. 15), leaving the generalization to laws and theories to others. Indeed, this view was made abundantly clear in the logo of the Statistical Society of London (now the Royal Statistical Society): a banded

[15] The German geographer Augustus Petermann produced a 'Cholera map of the British Isles' in 1852 using national data from the 1831–1832 epidemic (image: http://images.rgs.org/webimages/0/0/10000/1000/800/S0011888.jpg) shaded in proportion to the relative rate of mortality using class intervals ($< 1/35, 1/35 : 1/100, 1/100 : 1/200, \ldots$). No previous disease map had allowed determination of the range of mortality in any given area.

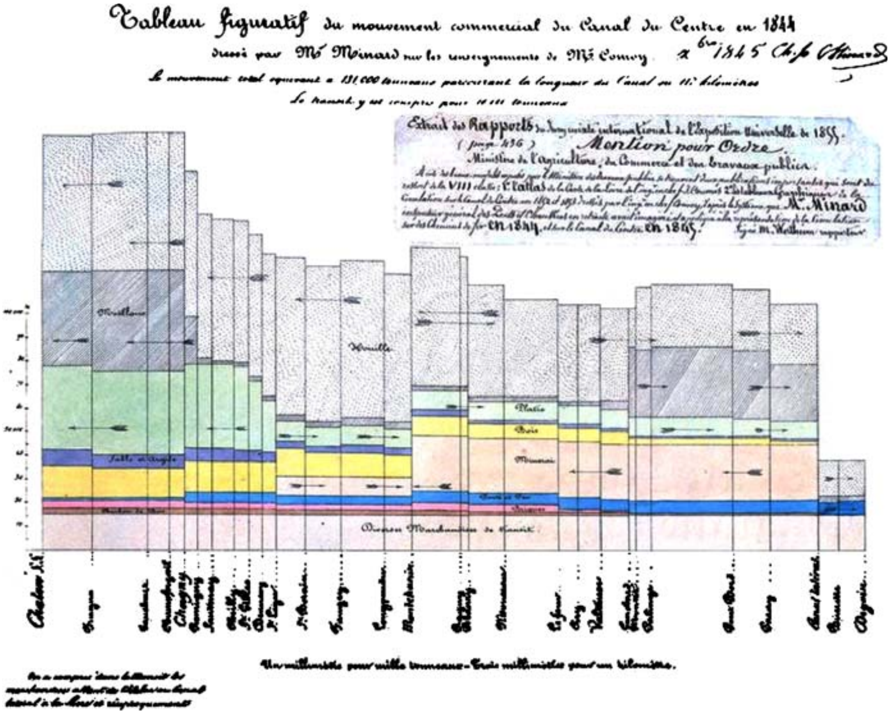

Figure 1.9. Minard's Tableau Graphique, showing the transportation of commercial goods along the Canal du Centre (Chalon–Dijon). Intermediate stops are spaced by distance, and each *bar* is divided by type of goods, so the area of each tile represents the cost of transport. *Arrows* show the direction of transport. *Source*: ENPC:5860/C351 (Col. et cliché ENPC; used by permission)

sheaf of wheat, with the motto *Aliis Exterendum* – to others to flail the wheat. Making graphs, it seemed, was too much like breadmaking.

1850–1900: The Golden Age of Statistical Graphics 1.2.5

By the mid-1800s, all the conditions for the rapid growth of visualization had been established – a 'perfect storm' for data graphics. Official state statistical offices were established throughout Europe, in recognition of the growing importance of numerical information for social planning, industrialization, commerce and transportation. Statistical theory, initiated by Gauss and Laplace and extended to the social realm by Guerry and Quételet, provided the means to make sense of large bodies of data.

What started as the *Age of Enthusiasm* (Funkhouser, 1937; Palsky, 1996) for graphics ended with what can be called the *Golden Age*, with unparalleled beauty and many innovations in graphics and thematic cartography. So varied were these developments that it is difficult to be comprehensive, but a few themes stand out.

Escaping Flatland

Although some attempts to display more than two variables simultaneously had occurred earlier in multiple time series (Playfair, 1801; Minard, 1826), contour graphs (Vauthier, 1874) and a variety of thematic maps, (e.g. Berghaus (1838)) a number of significant developments extended graphics beyond the confines of a flat piece of paper. Gustav Zeuner [1828–1907] in Germany (Zeuner, 1869), and later Luigi Perozzo [1750–1875] in Italy (Perozzo, 1880) constructed 3-D surface plots of population data.[16] The former was an axonometric projection showing various slices, while the latter (a 3-D graph of population in Sweden from 1750–1875 by year and age group) was printed in red and black and designed as a stereogram.[17]

Contour diagrams, showing isolevel curves of 3-D surfaces, had also been used earlier in mapping contexts (Nautonier, 1602–1604; Halley, 1701; von Humboldt, 1817), but the range of problems and data to which they were applied expanded considerably over this time in attempts to understand relations among more than two data-based variables, or where the relationships are statistical, rather than functional or measured with little error. It is more convenient to describe these under Galton, below. By 1884, the idea of visual and imaginary worlds of varying numbers of dimensions found popular expression in Edwin Abbott's (1884) *Flatland*, implicitly suggesting possible views in four and more dimensions.

Graphical Innovations

With the usefulness of graphical displays for understanding complex data and phenomena established, many new graphical forms were invented and extended to new areas of inquiry, particularly in the social realm.

Minard (1861) developed the use of divided circle diagrams on maps (showing both a total, by area, and subtotals, by sectors, with circles for each geographic region on the map). Later he developed to an art form the use of flow lines on maps of width proportional to quantities (people, goods, imports, exports) to show movement and transport geographically. Near the end of his life, the flow map would be taken to its highest level in his famous depiction of the fate of the armies of Napoleon and Hannibal, in what Tufte (1983) would call the 'best graphic ever produced.' See Friendly (2002) for a wider appreciation of Minard's work.

The social and political uses of graphics is also evidenced in the polar area charts (called 'rose diagrams' or 'coxcombs') invented by Florence Nightingale [1820–1910] to wage a campaign for improved sanitary conditions in battlefield treatment of soldiers (Nightingale, 1857). They left no doubt that many more soldiers died from disease and the consequences of wounds than at the hands of the enemy. From around the same time, Dr John Snow [1813–1858] is remembered for his use of a dot map of deaths from cholera in an 1854 outbreak in London. Plotting the residence of each

[16] Image: http://math.yorku.ca/SCS/Gallery/images/stereo2.jpg

[17] Zeuner used one axis to show year of birth and another to show present age, with number of surviving persons on the third, vertical, axis giving a 3-D surface. One set of curves thus showed the distribution of population for a given generation; the orthogonal set of curves showed the distributions across generations at a given point in time, e.g. at a census.

deceased provided the insight for his conclusion that the source of the outbreak could be localized to contaminated water from a pump on Broad Street, the founding innovation for modern epidemiological mapping.

Scales and shapes for graphs and maps were also transformed for a variety of purposes, leading to semi-logarithmic graphs (Jevons, 1863, 1879) to show percentage change in commodities over time, log-log plots to show multiplicative relations, anamorphic maps by Émile Cheysson (Palsky, 1996, Figs. 63–64) using deformations of spatial size to show a quantitative variable (e.g. the decrease in time to travel from Paris to various places in France over 200 years) and alignment diagrams or nomograms using sets of parallel axes. We illustrate this slice of the Golden Age with Fig. 1.10, a tour-de-force graphic for determination of magnetic deviation at sea in relation to latitude and longitude without calculation ('L' Abaque Triomphe') by Charles Lallemand (1885), director general of the geodetic measurement of altitudes throughout France, which combines many variables into a multifunction nomogram, using 3-D, juxtaposition of anamorphic maps, parallel coordinates and hexagonal grids.

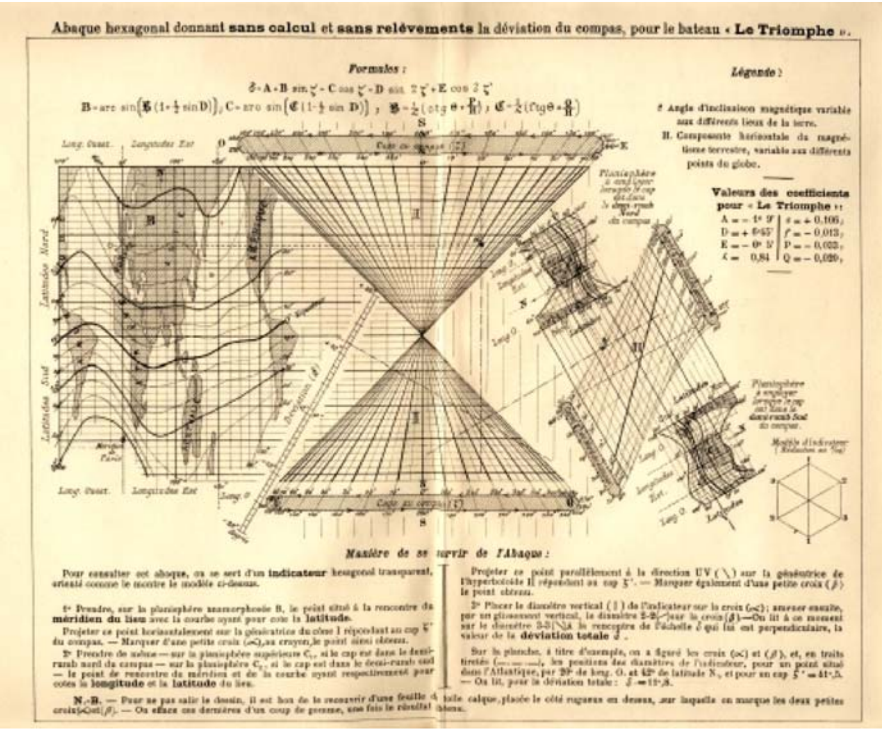

Figure 1.10. Lallemand's *L' abaque du bateau "Le Triomphe"*, allowing determination of magnetic deviation at sea without calculation. *Source*: courtesy Mme Marie-Noëlle Maisonneuve, Les fonds anciens de la bibliothèque de l'École des Mines de Paris

Galton's Contributions

Special note should be made of the varied contributions of Francis Galton [1822–1911] to data visualization and statistical graphics. Galton's role in the development of the ideas of correlation and regression are well known. Less well known is the role that visualization and graphing played in his contributions and discoveries.

Galton's statistical insight (Galton, 1886) – that, in a bivariate (normal) distribution, (say, height of a child against height of parents), (a) The isolines of equal frequency would appear as concentric ellipses and (b) The locus of the (regression) lines of means of $y|x$ and of $x|y$ were the conjugate diameters of these ellipses – was based largely on visual analysis from the application of smoothing to his data. Karl Pearson would later say 'that Galton should have evolved all this from his observations is to my mind one of the most noteworthy scientific discoveries arising from pure analysis of observations.' (Pearson, 1920, p. 37). This was only one of Galton's discoveries based on graphical methods.

In earlier work, Galton had made wide use of isolines, contour diagrams and smoothing in a variety of areas. An 1872 paper showed the use of 'isodic curves' to portray the joint effects of wind and current on the distance ships at sea could travel in any direction. An 1881 'isochronic chart' (Galton, 1881) showed the time it took to reach any destination in the world from London by means of coloured regions on a world map. Still later, he analysed rates of fertility in marriages in relation to the ages of father and mother using 'isogens,' curves of equal percentage of families having a child (Galton, 1894).

But perhaps the most notable non-statistical graphical discovery was that of the "anti-cyclonic" (anticlockwise) pattern of winds around low-pressure regions, combined with clockwise rotations around high-pressure zones. Galton's work on weather patterns began in 1861 and was summarized in *Meteorographica* (1863). It contained a variety of ingenious graphs and maps (over 600 illustrations in total), one of which is shown in Fig. 1.11. This remarkable chart, one of a two-page Trellis-style display, shows observations on barometric pressure, wind direction, rain and temperature from 15 days in December 1861.[18] For each day, the 3×3 grid shows schematic maps of Europe, mapping pressure (row 1), wind and rain (row 2) and temperature (row 3), in the morning, afternoon and evening (columns). One can clearly see the series of black areas (low pressure) on the barometric charts for about the first half of the month, corresponding to the anticlockwise arrows in the wind charts, followed by a shift to red areas (high pressure) and more clockwise arrows. Wainer (2005, p. 56) remarks, 'Galton did for the collectors of weather data what Kepler did for Tycho Brahe. This is no small accomplishment.'

Statistical Atlases

The collection, organization and dissemination of official government statistics on population, trade and commerce, social, moral and political issues became wide-

[18] In July 1861, Galton distributed a circular to meterologists throughout Europe, asking them to record these data synchonously, three times a day for the entire month of December 1861. About 50 weather stations supplied the data; see Pearson (1914–1930, pp. 37–39).

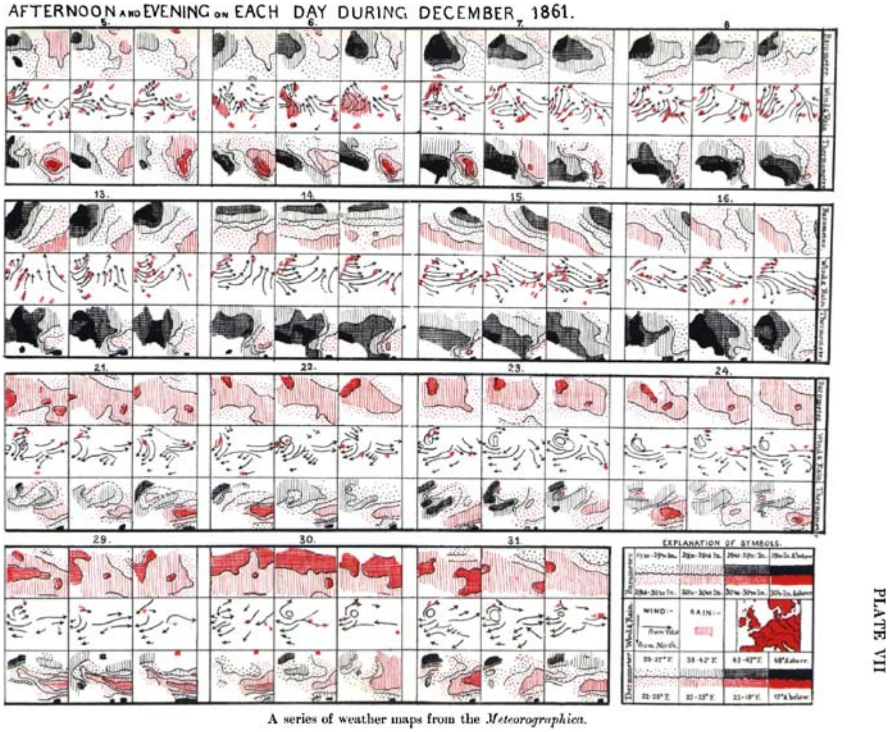

Figure 1.11. One page of Galton's 1863 multivariate weather chart of Europe showing barometric pressure, wind direction, rain and temperature for the month of December 1861. *Source*: Pearson (1914–1930, pl. 7)

spread in most of the countries of Europe from about 1825 to 1870 (Westergaard, 1932). Reports containing data graphics were published with some regularity in France, Germany, Hungary and Finland, and with tabular displays in Sweden, Holland, Italy and elsewhere. At the same time, there was an impetus to develop standards for graphical presentation at the International Statistical Congresses which had begun in 1853 in Belgium (organized by Quételet), and these congresses were closely linked with state statistical bureaus. The main participants in the graphics section included Georg von Mayr, Hermann Schwabe, Pierre Émile Levasseur and Émile Cheysson. Among other recommendations was one from the 7th Statistical Congress in 1869 that official publications be accompanied by maps and diagrams. The state-sponsored statistical atlases that ensued provide additional justification to call this period the golden age of graphics, and some of its most impressive exemplars.

The pinnacle of this period of state-sponsored statistical albums is undoubtedly the *Albums de statistique graphique* published annually by the French ministry of public works from 1879 to 1897 under the direction of Émile Cheysson.[19] They were

[19] Cheysson had been one of the major participants in committees on the standardization of graphical methods at the International Statistical Congresses from 1872 on. He was trained

published as large-format books (about 11 × 17 in.), and many of the plates folded out to four or six times that size, all printed in colour and with great attention to layout and composition. We concur with Funkhouser (1937, p. 336) that "the *Albums* present the finest specimens of French graphic work in the century and considerable pride was taken in them by the French people, statisticians and laymen alike."

The subject matter of the albums largely concerned economic and financial data related to the planning, development and administration of public works – transport of passengers and freight, by rail, on inland waterways and through seaports, but also included such topics as revenues in the major theaters of Paris, attendance at the universal expositions of 1867, 1878 and 1889, changes in populations of French departments over time and so forth.

More significantly for this account the *Albums* can also be viewed as an exquisite sampler of all the graphical methods known at the time, with significant adaptations to the problem at hand. The majority of these graphs used and extended the flow map pioneered by Minard. Others used polar forms – variants of pie and circle diagrams, star plots and rose diagrams, often overlaid on a map and extended to show additional variables of interest. Still others used subdivided squares in the manner of modern mosaic displays (Friendly, 1994) to show the breakdown of a total (passengers, freight) by several variables. It should be noted that in almost all cases the graphical representation of the data was accompanied by numerical annotations or tables, providing precise numerical values.

The *Albums* are discussed extensively by Palsky (1996), who includes seven representative illustrations. It is hard to choose a single image here, but my favourites are surely the recursive, multimosaic of rail transportation for the 1884–1886 volumes, the first of which is shown in Fig. 1.12. This cartogram uses one large mosaic (in the lower left) to show the numbers of passengers and tons of freight shipped from Paris from the four principal train stations. Of the total leaving Paris, the amounts going to each main city are shown by smaller mosaics, coloured according to railway lines; of those amounts, the distribution to smaller cities is similarly shown, connected by lines along the rail routes.

Among the many other national statistical albums and atlases, those from the US Census bureau also deserve special mention. The *Statistical Atlas of the Ninth Census*, produced in 1872–1874 under the direction of Francis A. Walker [1840–1897], contained 60 plates, including several novel graphic forms. The ambitious goal was to present a graphic portrait of the nation, and it covered a wide range of physical and human topics: geology, minerals and weather; population by ethnic origin, wealth, illiteracy, school attendance and religious affiliation; death rates by age, sex, race and cause; prevalence of blindness, deaf mutism and insanity; and so forth. 'Age pyramids' (back-to-back, bilateral frequency histograms and polygons) were used effectively to compare age distributions of the population for two classes (gender, married/single, etc.). Subdivided squares and area-proportional pies of various forms were also used to provide comparisons among the states on multiple dimensions simultaneously

as an engineer at the ENPC and later became a professor of political economy at the École des Mines.

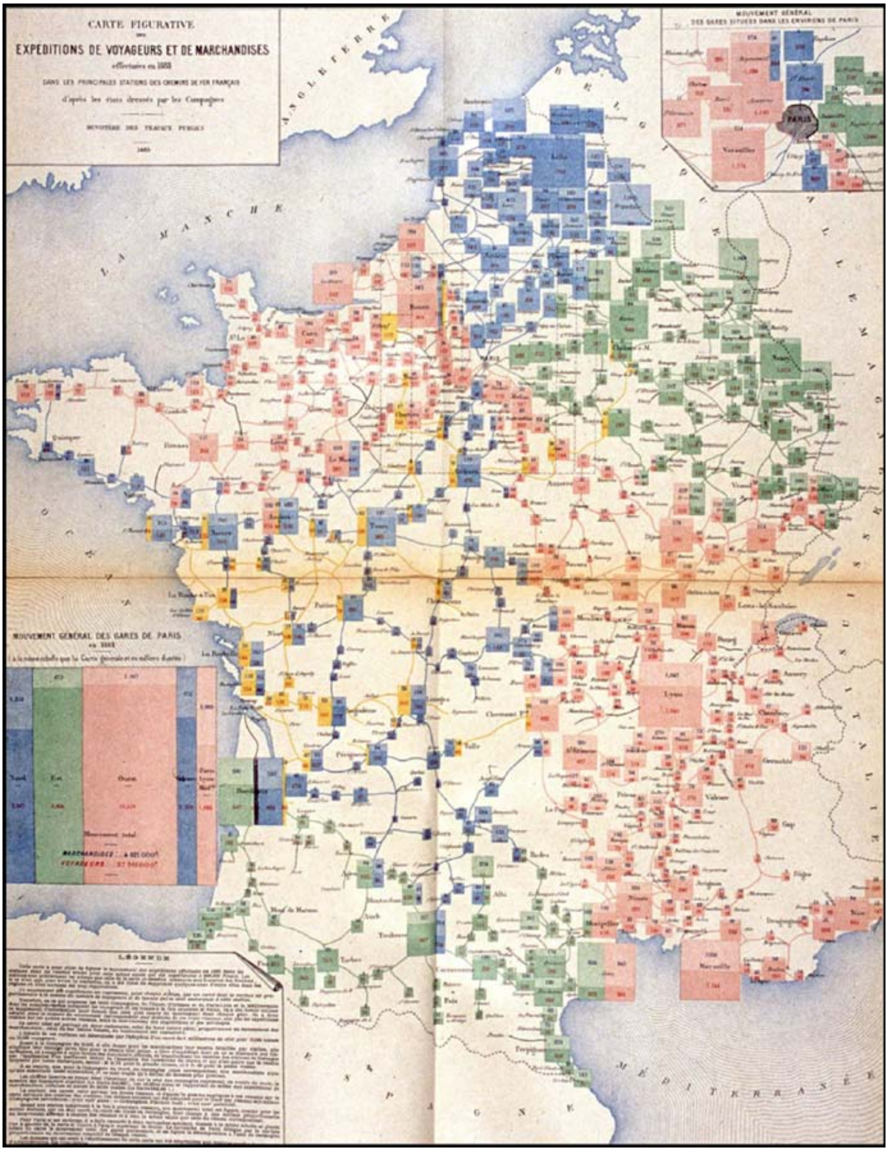

Figure 1.12. [This figure also appears in the color insert.] *Mouvement des voyageurs et des marchandises dans les principales stations de chemins de fer en 1882*. Scale: 2 mm^2 = 10 000 passengers or tons of freight. *Source*: Album, 1884, Plate 11 (author's collection)

(employed/unemployed, sex, schooling, occupational categories). The desire to provide for easy comparisons among states and other categorizations was expressed by arranging multiple subfigures as 'small multiples' in many plates.

Following each subsequent decennial census for 1880 to 1900, reports and statistical atlases were produced with more numerous and varied graphic illustrations. The 1898 volume from the Eleventh Census (1890), under the direction of Henry Gannett [1846–1914], contained over 400 graphs, cartograms and statistical diagrams. There were several ranked parallel coordinate plots comparing states and cities over all censuses from 1790–1890. Trellis-like collections of shaded maps showed interstate migration, distributions of religious membership, deaths by known causes and so forth.

The 1880 and 1890 volumes produced under Gannett's direction are also notable for (a) the multimodal combination of different graphic forms (maps, tables, barcharts, bilateral polygons) in numerous plates and (b) the consistent use of effect-order sorting (Friendly and Kwan, 2003) to arrange states or other categories in relation to what was to be shown, rather than for lookup (e.g. Alabama–Wyoming).

For example, Fig. 1.13 shows interstate immigration in relation to emigration for the 49 states and territories in 1890. The right side shows population loss sorted by emigration, ranging from New York, Ohio, Pennsylvania and Illinois at the top to Idaho, Wyoming and Arizona at the bottom. The left side shows where the emigrants went: Illinois, Missouri, Kansas and Texas had the biggest gains, Virginia the biggest net loss. It is clear that people were leaving the eastern states and were attracted to those of the Midwest Mississippi valley. Other plates showed this data in map-based formats.

However, the Age of Enthusiasm and the Golden Age were drawing to a close. The French *Albums de statistique graphique* were discontinued in 1897 due to the high cost of production; statistical atlases appeared in Switzerland in 1897 and 1914, but never again. The final two US Census atlases, issued after the 1910 and 1920 censuses, 'were both routinized productions, largely devoid of colour and graphic imagination' (Dahmann, 2001).

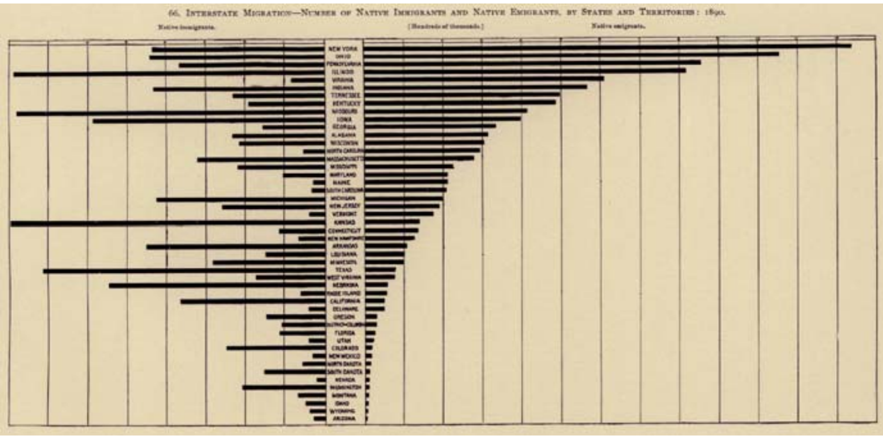

Figure 1.13. Interstate migration shown by back-to-back barcharts, sorted by emigration. *Source*: Statistical Atlas of the Eleventh Census, 1890, diagram 66, p. 23 (author's collection)

1900–1950: The Modern Dark Ages 1.2.6

If the late 1800s were the 'golden age' of statistical graphics and thematic cartography, the early 1900s can be called the 'modern dark ages' of visualization (Friendly and Denis, 2000).

There were few graphical innovations, and by the mid-1930s the enthusiasm for visualization which characterized the late 1800s had been supplanted by the rise of quantification and formal, often statistical, models in the social sciences. Numbers, parameter estimates and, especially, those with standard errors were precise. Pictures were – well, just pictures: pretty or evocative, perhaps, but incapable of stating a 'fact' to three or more decimals. Or so it seemed to many statisticians.

But it is equally fair to view this as a time of necessary dormancy, application and popularization rather than one of innovation. In this period statistical graphics became mainstream. Graphical methods entered English[20] textbooks (Bowley, 1901; Peddle, 1910; Haskell, 1919; Karsten, 1925), the curriculum (Costelloe, 1915; Warne, 1916) and standard use in government (Ayres, 1919), commerce (Gantt charts and Shewart's control charts) and science.

These textbooks contained rather detailed descriptions of the graphic method, with an appreciative and often modern flavour. For example, Sir Arthur Bowley's (1901) *Elements of Statistics* devoted two chapters to graphs and diagrams and discussed frequency and cumulative frequency curves (with graphical methods for finding the median and quartiles), effects of choice of scales and baselines on visual estimation of differences and ratios, smoothing of time-series graphs, rectangle diagrams in which three variables could be shown by height, width and area of bars, and 'historical diagrams' in which two or more time series could be shown on a single chart for comparative views of their histories.

Bowley's (1901, pp. 151–154) example of smoothing (Fig. 1.14) illustrates the character of his approach. Here he plotted the total value of exports from Britain and Ireland over the period 1855–1899. At issue was whether exports had become stationary in the most recent years, and the conclusion by Sir Robert Giffen (1899), based solely on tables of averages for successive 5-year periods,[21] that 'the only sign of stationariness is an increase at a less rate in the last periods than in the earlier periods' (p. 152). To answer this, he graphed the raw data, together with curves of the moving average over 3-, 5- and 10-year periods. The 3- and 5-year moving averages show strong evidence of an approximately 10-year cycle, and he noted, 'no argument can stand which does not take account of the cycle of trade, which is not eliminated until we take decennial averages' (p. 153). To this end, he took averages of successive 10-year periods starting 1859 and drew a freehand curve 'keeping as close [to the points] as possible,

[20] The first systematic attempt to survey, describe and illustrate available graphic methods for experimental data was that of Étienne Jules Marey's (1878) *La Méthode Graphique*. Marey [1830–1904] also invented several devices for visual recording, including the sphymograph and chronophotography to record the motion of birds in flight, people running and so forth.

[21] Giffen, an early editor of *The Statist*, also wrote a statistical text published posthumously in 1913; it contained an entire chapter on constructing tables, but not a single graph (Klein, 1997, p. 17).

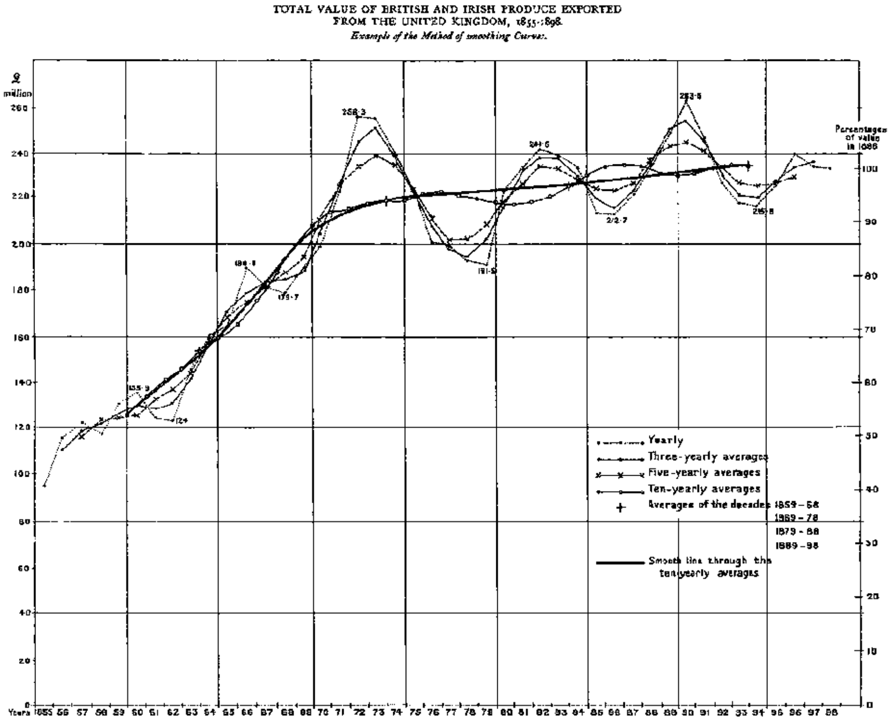

Figure 1.14. Arthur Bowley's demonstration of methods of smoothing a time-series graph. Moving averages of 3, 5 and 10 years are compared with a freehand curve drawn through four *points* representing the averages of successive 10-year periods. *Source*: Bowley (1901, opposite p. 151)

without making sudden changes in curvature,' giving the thick curve in Fig. 1.14.[22] Support for Sir Robert's conclusion and the evidence for a 10-year cycle owe much to this graphical treatment.

Moreover, perhaps for the first time, graphical methods proved crucial in a number of new insights, discoveries and theories in astronomy, physics, biology and other sciences. Among these, one may refer to (a) E.W. Maunder's (1904) 'butterfly diagram' to study the variation of sunspots over time, leading to the discovery that they were markedly reduced in frequency from 1645–1715; (b) the Hertzsprung–Russell diagram (Hertzsprung, 1911; Spence and Garrison, 1993), a log-log plot of luminosity as a function of temperature for stars, used to explain the changes as a star evolves and laying the groundwork for modern stellar physics; (c) the discovery of the concept of atomic number by Henry Moseley (1913) based largely on graphical analysis. See Friendly and Denis (2005) for more detailed discussion of these uses.

[22] A reanalysis of the data using a loess smoother shows that this is in fact oversmoothed and corresponds closely to a loess window width of $f = 0.50$. The optimal smoothing parameter, minimizing AIC_C is $f = 0.16$, giving a smooth more like Bowley's 3- and 5-year moving averages.

As well, experimental comparisons of the efficacy of various graphics forms were begun (Eells, 1926; von Huhn, 1927; Washburne, 1927), a set of standards and rules for graphic presentation was finally adopted by a joint committee (Joint Committee on Standards for Graphic Presentation, 1914) and a number of practical aids to graphing were developed. In the latter part of this period, new ideas and methods for multidimensional data in statistics and psychology would provide the impetus to look beyond the 2-D plane.

Graphic innovation was also awaiting new ideas and technology: the development of the machinery of modern statistical methodology, and the advent of the computational power and display devices which would support the next wave of developments in data visualization.

1950–1975: Rebirth of Data Visualization 1.2.7

Still under the influence of the formal and numerical zeitgeist from the mid-1930s on, data visualization began to rise from dormancy in the mid-1960s. This was spurred largely by three significant developments:

- In the USA, John W. Tukey [1915–2000], in a landmark paper, *The Future of Data Analysis* (Tukey, 1962), issued a call for the recognition of data analysis as a legitimate branch of statistics distinct from mathematical statistics; shortly later, he began the invention of a wide variety of new, simple and effective graphic displays, under the rubric of 'exploratory data analysis' (EDA) – stem-leaf plots, boxplots, hanging rootograms, two-way table displays and so forth, many of which entered the statistical vocabulary and software implementation. Tukey's stature as a statistician and the scope of his informal, robust and graphical approach to data analysis were as influential as his graphical innovations. Although not published until 1977, chapters from Tukey's EDA book (Tukey, 1977) were widely circulated as they began to appear in 1970–1972 and began to make graphical data analysis both interesting and respectable again.
- In France, Jacques Bertin [1918–] published the monumental *Sémiologie graphique* (Bertin, 1967). To some, this appeared to do for graphics what Mendeleev had done for the organization of the chemical elements, that is, to organize the visual and perceptual elements of graphics according to the features and relations in data. In a parallel but separate stream, an exploratory and graphical approach to multidimensional data ('L'analyse des données') begun by Jean-Paul Benzécri [1932–] provided French and other European statisticians with an alternative, visually based view of what statistics was about. Other graphically minded schools of data-thought would later arise in the Netherlands (Gifi), Germany and elsewhere in Europe.
- But the skills of hand-drawn maps and graphics had withered during the dormant 'modern dark ages' of graphics (though nearly every figure in Tukey's *EDA* (Tukey, 1977) was, by intention, hand-drawn). Computer processing of statistical data began in 1957 with the creation of FORTRAN, the first high-level language for computing. By the late 1960s, widespread mainframe university computers offered the possibility to construct old and new graphic forms by computer

programs. Interactive statistical applications, e.g. Fowlkes (1969); Fishkeller et al. (1974), and true high-resolution graphics were developed but would take a while to enter common use.

By the end of this period significant intersections and collaborations would begin: (a) Computer science research (software tools, C language, UNIX, etc.) at Bell Laboratories (Becker, 1994) and elsewhere would combine forces with (b) Developments in data analysis (EDA, psychometrics, etc.) and (c) Display and input technology (pen plotters, graphic terminals, digitizer tablets, the mouse, etc.). These developments would provide new paradigms, languages and software packages for expressing statistical ideas and implementing data graphics. In turn, they would lead to an explosive growth in new visualization methods and techniques.

Other themes began to emerge, mostly as initial suggestions: (a) Various novel visual representations of multivariate data (Andrews' (1972) Fourier function plots, Chernoff (1973) faces, star plots, clustering and tree representations); (b) The development of various dimension-reduction techniques (biplot (Gabriel, 1971), multidimensional scaling, correspondence analysis), providing visualization of multidimensional data in a 2-D approximation; (c) Animations of a statistical process; and (d) Perceptually based theory and experiments related to how graphic attributes and relations might be rendered to better convey data visually.

By the close of this period, the first exemplars of modern GIS and interactive systems for 2-D and 3-D statistical graphics would appear. These would set goals for future development and extension.

1.2.8 1975–present: High-D, Interactive and Dynamic Data Visualization

During the last quarter of the 20th century data visualization blossomed into a mature, vibrant and multidisciplinary research area, as may be seen in this Handbook, and software tools for a wide range of visualization methods and data types are available for every desktop computer. Yet it is hard to provide a succinct overview of the most recent developments in data visualization because they are so varied and have occurred at an accelerated pace and across a wider range of disciplines. It is also more difficult to highlight the most significant developments which may be seen as such in a subsequent history focusing on this recent period.

With this disclaimer, a few major themes stand out.

- The development of highly interactive statistical computing systems. Initially, this meant largely command-driven, directly programmable systems (APL, S), as opposed to compiled, batch processing;
- New paradigms of direct manipulation for visual data analysis (linking, brushing (Becker and Cleveland, 1987), selection, focusing, etc.);
- New methods for visualizing high-dimensional data (the grand tour (Asimov, 1985), scatterplot matrix (Tukey and Tukey, 1981), parallel coordinates plot (Inselberg, 1985; Wegman, 1990), spreadplots (Young, 1994a), etc.);

- The invention (or re-invention) of graphical techniques for discrete and categorical data;
- The application of visualization methods to an ever-expanding array of substantive problems and data structures; and
- Substantially increased attention to the cognitive and perceptual aspects of data display.

These developments in visualization methods and techniques arguably depended on advances in theoretical and technological infrastructure, perhaps more so than in previous periods. Some of these are:

- Large-scale statistical and graphics software engineering, both commercial (e.g. SAS) and non-commercial (e.g. Lisp-Stat, the R project). These have often been significantly leveraged by open-source standards for information presentation and interaction (e.g. Java, Tcl/Tk);
- Extensions of classical linear statistical modelling to ever-wider domains (generalized linear models, mixed models, models for spatial/geographical data and so forth);
- Vastly increased computer processing speed and capacity, allowing computationally intensive methods (bootstrap methods, Bayesian MCMC analysis, etc.), access to massive data problems (measured in terabytes) and real-time streaming data. Advances in this area continue to press for new visualization methods.

From the early 1970s to mid-1980s, many of the advances in statistical graphics concerned static graphs for multidimensional quantitative data, designed to allow the analyst to see relations in progressively higher dimensions. Older ideas of dimension-reduction techniques (principal component analysis, multidimensional scaling, discriminant analysis, etc.) led to generalizations of projecting a high-dimensional dataset to 'interesting' low-dimensional views, as expressed by various numerical indices that could be optimized (projection pursuit) or explored interactively (grand tour).

The development of general methods for multidimensional contingency tables began in the early 1970s, with Leo Goodman (1970), Shelly Haberman (1973) and others (Bishop et al., 1975) laying out the fundamentals of log-linear models. By the mid-1980s, some initial, specialized techniques for visualizing such data were developed (four-fold display (Fienberg, 1975), association plot (Cohen, 1980), mosaicplot (Hartigan and Kleiner, 1981) and sieve diagram (Riedwyl and Schüpbach, 1983)), based on the idea of displaying frequencies by area (Friendly, 1995). Of these, extensions of the mosaicplot (Friendly, 1994, 1999) have proved most generally useful and are now widely implemented in a variety of statistical software, most completely in the `vcd` package (Meyer et al., 2005) in R and interactive software from the Augsburg group (MANET, Mondrian).

It may be argued that the greatest potential for recent growth in data visualization came from the development of interactive and dynamic graphic methods, allowing instantaneous and direct manipulation of graphical objects and related statistical properties. One early instance was a system for interacting with probability plots (Fowlkes, 1969) in real time, choosing a shape parameter of a reference distribution and power transformations by adjusting a control. The first general system for manipulating high-dimensional data was PRIM-9, developed by Fishkeller, Friedman and Tukey (1974), and providing dynamic tools for projecting, rotating (in 3-D), isolating (identifying subsets) and masking data in up to 9 dimensions. These were quite influential, but remained one-of-a-kind, 'proof-of-concept' systems. By the mid-1980s, as workstations and display technology became cheaper and more powerful, desktop software for interactive graphics became more widely available (e.g. MacSpin, Xgobi). Many of these developments to that point are detailed in the chapters of *Dynamic Graphics for Statistics* (Cleveland and McGill, 1988).

In the 1990s, a number of these ideas were brought together to provide more general systems for dynamic, interactive graphics, combined with data manipulation and analysis in coherent and extensible computing environments. The combination of all these factors was more powerful and influential than the sum of their parts. Lisp-Stat (Tierney, 1990) and its progeny (Arc, Cook and Weisberg, 1999; ViSta, Young, 1994b), for example, provided an easily extensible object-oriented environment for statistical computing. In these systems, widgets (sliders, selection boxes, pick lists, etc.), graphs, tables, statistical models and the user all communicated through messages, acted upon by whoever was a designated 'listener,' and had a method to respond. Most of the ideas and methods behind present-day interactive graphics are described and illustrated in Young et al. (2006). Other chapters in this Handbook provide current perspectives on other aspects of interactive graphics.

1.3 Statistical Historiography

As mentioned at the outset, this review is based on the information collected for the Milestones Project, which I regard (subject to some caveats) as a relatively comprehensive corpus of the significant developments in the history of data visualization. As such, it is of interest to consider what light modern methods of statistics and graphics can shed on this history, a self-referential question we call 'statistical historiography' (Friendly, 2005). In return, this offers other ways to view this history.

1.3.1 History as 'Data'

Historical events, by their nature, are typically discrete, but marked with dates or ranges of dates, and some description – numeric, textual, or classified by descriptors (who, what, where, how much and so forth). Amongst the first to recognize that history could be treated as data and portrayed visually, Joseph Priestley (1765; 1769) developed the idea of depicting the lifespans of famous people by horizontal lines along a time scale. His enormous (2 × 3 ft., or .75 × 1 m) and detailed *Chart of Biography* showed two thousand names from 1200 B.C. to A.D. 1750 by horizontal lines from birth to death, using dots at either end to indicate ranges of uncertainty. Along the vertical dimension, Priestly classified these individuals, e.g., as statesmen or men of learning. A small fragment of this chart is shown in Fig. 1.15.

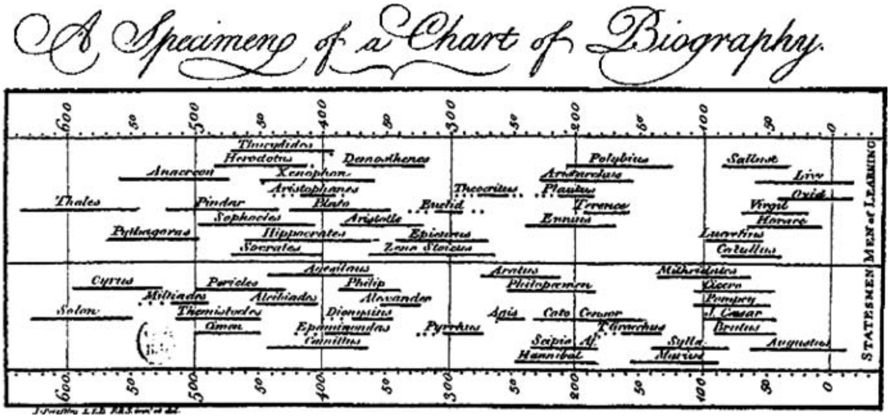

Figure 1.15. A specimen version of Priestley's *Chart of Biography*. *Source*: Priestley (1765)

Priestley's graphical representations of time and duration apparently influenced Playfair's introduction of time-series charts and barcharts (Funkhouser, 1937, p. 280). But these inventions did not inspire the British statisticians of his day, as noted earlier; historical events and statistical facts were seen as separate, rather than as data arrayed along a time dimension. In 1885, at the Jubilee meeting of the Royal Statistical Society, Alfred Marshall (1885) argued that the causes of historical events could be understood by the use of statistics displayed by 'historical curves' (time-series graphs): 'I wish to argue that the graphic method may be applied as to enable history to do this work better than it has hitherto' (p. 252). Maas and Morgan (2005) discuss these issues in more detail.

Analysing Milestones Data 1.3.2

The information collected in the Milestones Project is rendered in print and Web forms as a chronological list but is maintained as a relational database (historical items, references, images) in order to be able to work with it as 'data.' The simplest analyses examine trends over time. Figure 1.1 shows a density estimate for the distribution of 248 milestone items from 1500 to the present, keyed to the labels for the periods in history. The bumps, peaks and troughs all seem interpretable: note particularly the steady rise up to about 1880, followed by a decline through the 'modern dark ages' to 1945, then the steep rise up to the present. In fact, it is slightly surprising to see that the peak in the Golden Age is nearly as high as that at present, but this probably just reflects underrepresentation of the most recent events.[23]

[23] Technical note: In this figure an optimal bandwidth for the kernel density estimate was selected (using the Sheather–Jones plug-in estimate) for each series separately. The smaller range and sample size of the entries for Europe vs. North America gives a smaller bandwidth for the former, by a factor of about 3. Using a common bandwidth fixed to that determined for the whole series (Fig. 1.1) undersmoothes the more extensive data on European develop-

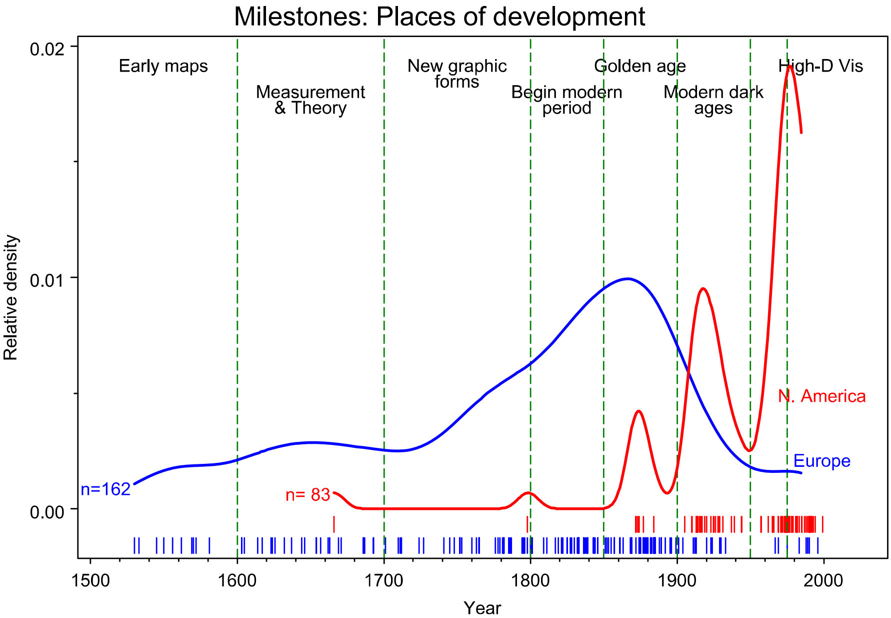

Figure 1.16. The distribution of milestone items over time, comparing trends in Europe and North America

Other historical patterns can be examined by classifying the items along various dimensions (place, form, content and so forth). If we classify the items by place of development (Europe vs. North America, ignoring Other), interesting trends appear (Fig. 1.16). The greatest peak in Europe around 1875–1880 coincided with a smaller peak in North America. The decline in Europe following the Golden Age was accompanied by an initial rise in North America, largely due to popularization (e.g. textbooks) and significant applications of graphical methods, then a steep decline as mathematical statistics held sway.

Finally, Fig. 1.17 shows two mosaicplots for the milestone items classified by Epoch, Subject matter and Aspect. Subject was classed as having to do with human (e.g. mortality, disease), physical or mathematical characteristics of what was represented in the innovation. Aspect classed each item according to whether it was primarily map-based, a diagram or statistical innovation or a technological one. The left mosaic shows the shifts in Subject over time: most of the early innovations concerned physical subjects, while the later periods shift heavily to mathematical ones. Human topics are not prevalent overall but were dominant in the 19th century. The right mosaic, for Subject × Aspect, indicates that, unsurprisingly, map-based innovations were mainly about physical and human subjects, while diagrams and statistical ones were largely about mathematical subjects. Historical classifications clearly rely on more

ments and oversmoothes the North American ones. The details differ, but most of the points made in the discussion about what was happening when and where hold.

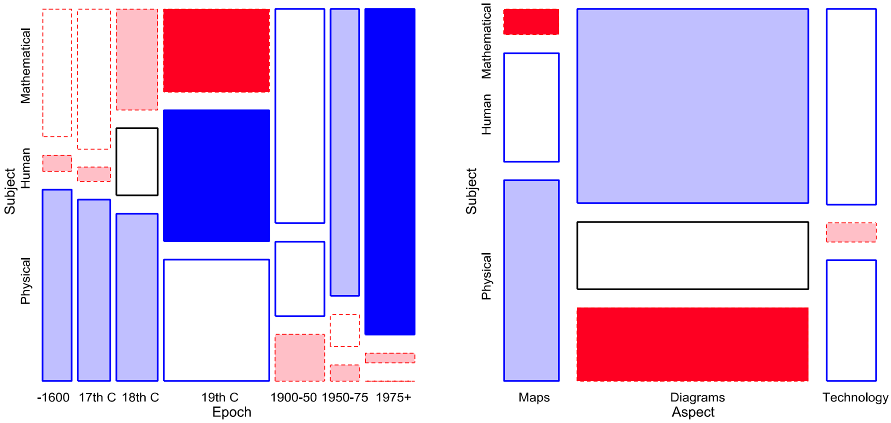

Figure 1.17. [This figure also appears in the color insert.] Mosaic plots for milestones items, classified by Subject, Aspect and Epoch. *Cells* with greater (less) frequency than expected under independence are coloured *blue* (*red*), with intensity proportional to the deviation from independence

detailed definitions than described here; however, it seems reasonable to suggest that such analyses of history as 'data' are a promising direction for future work.

What Was He Thinking? – Understanding Through Reproduction 1.3.3

Historical graphs were created using available data, methods, technology and understanding current at the time. We can often come to a better understanding of intellectual, scientific and graphical questions by attempting a re-analysis from a modern perspective.

Earlier, we showed Playfair's time-series graph (Fig. 1.7) of wages and prices and noted that Playfair wished to show that workers were better off at the end of the period shown than at any earlier time. Presumably he wished to draw the reader's eye to the narrowing of the gap between the bars for prices and the line graph for wages. Is this what you see?

What this graph shows directly is quite different from Playfair's intention. It appears that wages remained relatively stable while the price of wheat varied greatly. The inference that wages increased relative to prices is indirect and not visually compelling.

We cannot resist the temptation to give Playfair a helping hand here – by graphing the ratio of wages to prices (labour cost of wheat), as shown in Fig. 1.18. But this would not have occurred to Playfair because the idea of relating one time series to another by ratios (index numbers) would not occur for another half-century (due to Jevons). See Friendly and Denis (2005) for further discussion of Playfair's thinking.

As another example, we give a brief account of an attempt to explore Galton's discovery of regression and the elliptical contours of the bivariate normal surface,

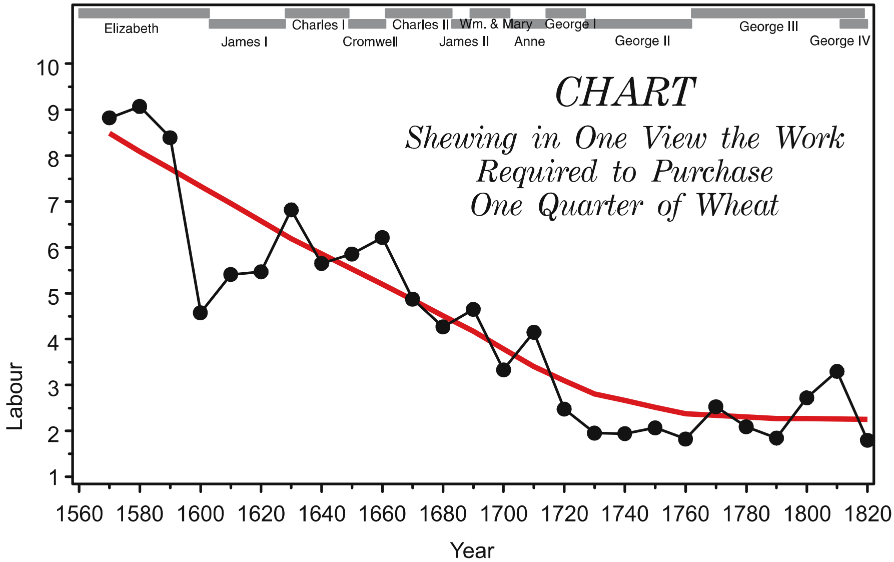

Figure 1.18. Redrawn version of Playfair's time-series graph showing the ratio of price of wheat to wages, together with a loess smoothed curve

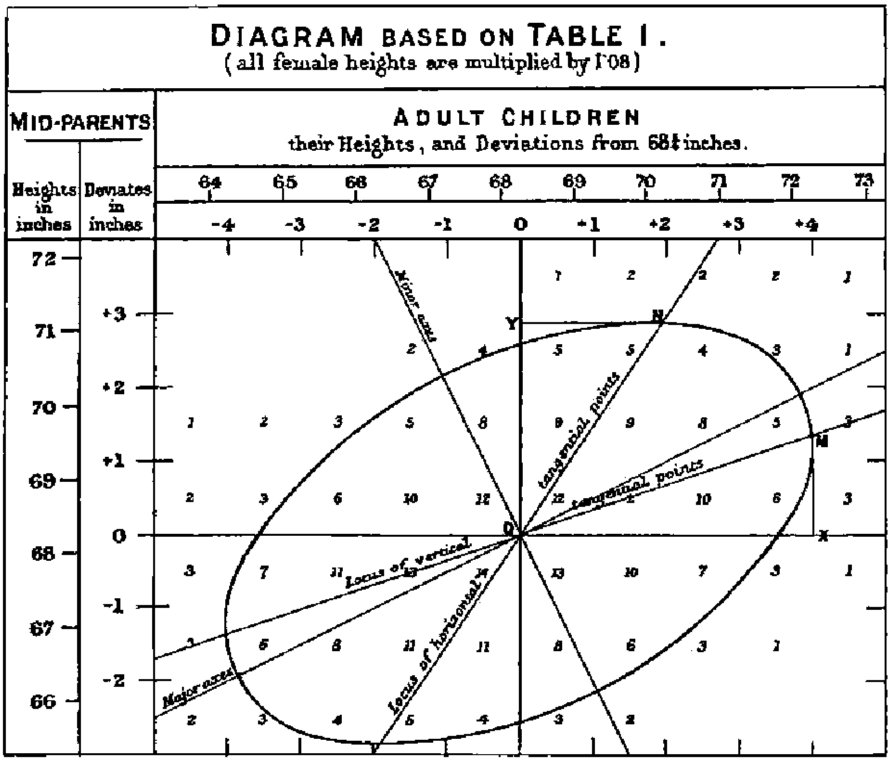

Figure 1.19. Galton's smoothed correlation diagram for the data on heights of parents and children, showing one ellipse of equal frequency. *Source*: (Galton, 1886, Plate X)

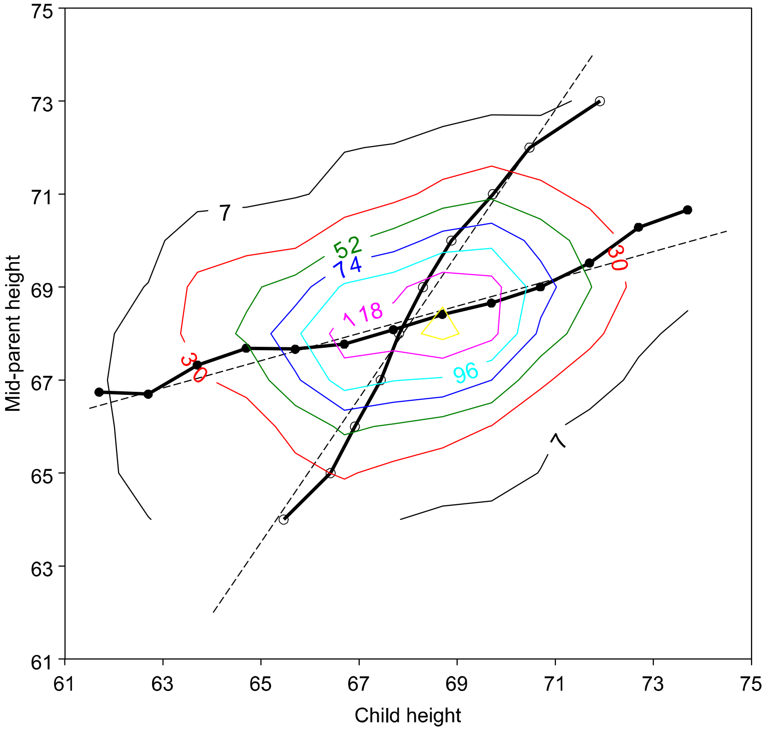

Figure 1.20. Contour plot of Galton's smoothed data, showing the curves of $\bar{y}|x$ (*filled circles, solid line*), $\bar{x}|y$ (*open circles, solid line*) and the corresponding regression lines (*dashed*)

treated in more detail in Friendly and Denis (2005). Galton's famous graph showing these relations (Fig. 1.19) portrays the joint frequency distribution of the height of children and the average height of their parents. It was produced from a 'semi-graphic table' in which Galton averaged the frequencies in each set of four adjacent cells, drew isocurves of equal smoothed value and noted that these formed 'concentric and similar ellipses.'

A literal transcription of Galton's method, using contour curves of constant average frequency and showing the curves of the means of $y|x$ and $x|y$, is shown in Fig. 1.20. It is not immediately clear that the contours are concentric ellipses, nor that the curves of means are essentially linear and have horizontal and vertical tangents to the contours.

A modern data analyst following the spirit of Galton's method might substitute a smoothed bivariate kernel density estimate for Galton's simple average of adjacent cells. The result, using jittered points to depict the cell frequencies, and a smoothed loess curve to show $\mathcal{E}(y|x)$ is shown in Fig. 1.21. The contours now *do* emphatically suggest concentric similar ellipses, and the regression line is near the points of vertical tangency. A reasonable conclusion from these figures is that Galton did not slavishly interpolate isofrequency values as is done in the contour plot shown in

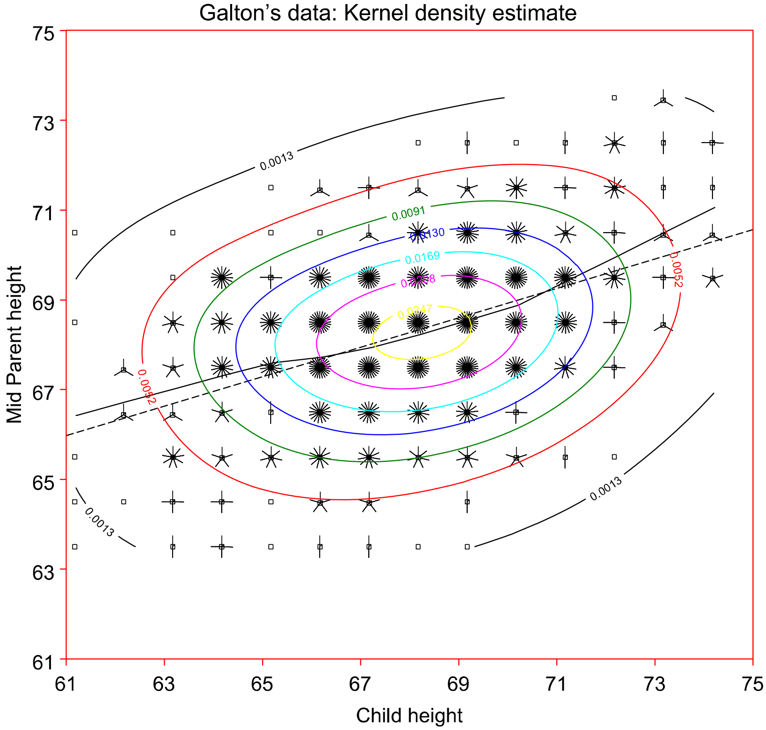

Figure 1.21. Bivariate kernel density estimate of Galton's data, using jittered points for the data, and a smoothed loess curve for $\mathcal{E}(y|x)$ (*solid*) and regression line (*dashed*)

Fig. 1.20. Rather, he drew his contours to the smoothed data by eye and brain (as he had done earlier with maps of weather patterns), with knowledge that he could, as one might say today, trade some increase in bias for a possible decrease in variance, and so achieve a greater smoothing.

1.4 Final Thoughts

This chapter is titled 'A brief history…' out of recognition that it it impossible to do full justice to the history of data visualization in such a short account. This is doubly so because I have attempted to present a broad view spanning the many areas of application in which data visualization took root and developed. That being said, it is hoped that this overview will lead modern readers and developers of graphical methods to appreciate the rich history behind the latest hot new methods. As we have seen, almost all current methods have a much longer history than is commonly thought. Moreover, as I have surveyed this work and travelled to many libraries to view original works and read historical sources, I have been struck by the exquisite beauty and attention to graphic detail seen in many of these images, particularly those from the 19th century. We would be hard-pressed to recreate many of these today.

From this history one may also see that most of the innovations in data visualization arose from concrete, often practical, goals: the need or desire to see phenomena and relationships in new or different ways. It is also clear that the development of graphic methods depended fundamentally on parallel advances in technology, data collection and statistical theory. Finally, I believe that the application of modern methods of data visualization to its own history, in this self-referential way I call 'statistical historiography,' offers some interesting views of the past and challenges for the future.

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