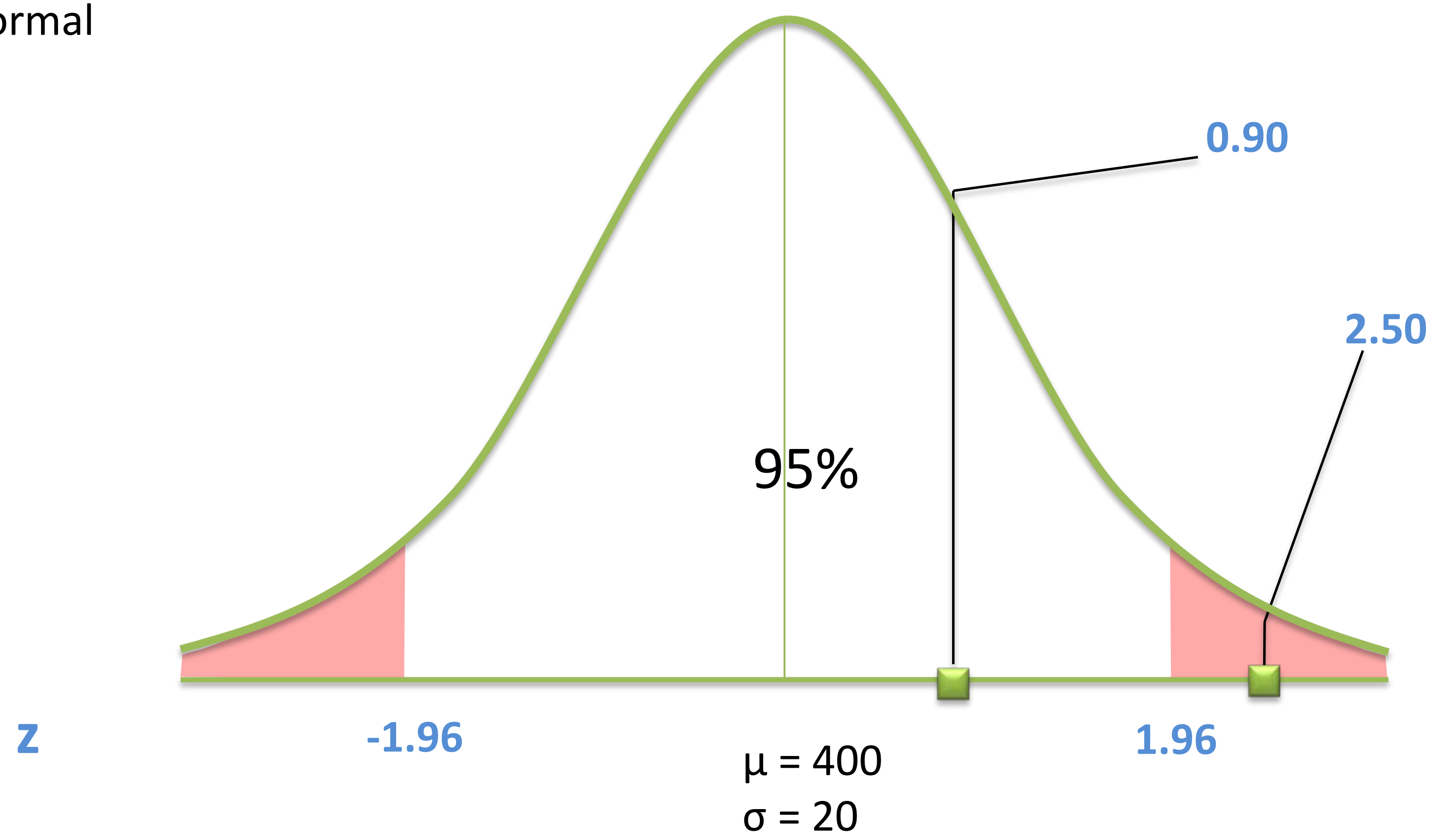
Sampling Distributions

Does treatment work?

Population of rats (weight)
Normal

Inject growth hormone



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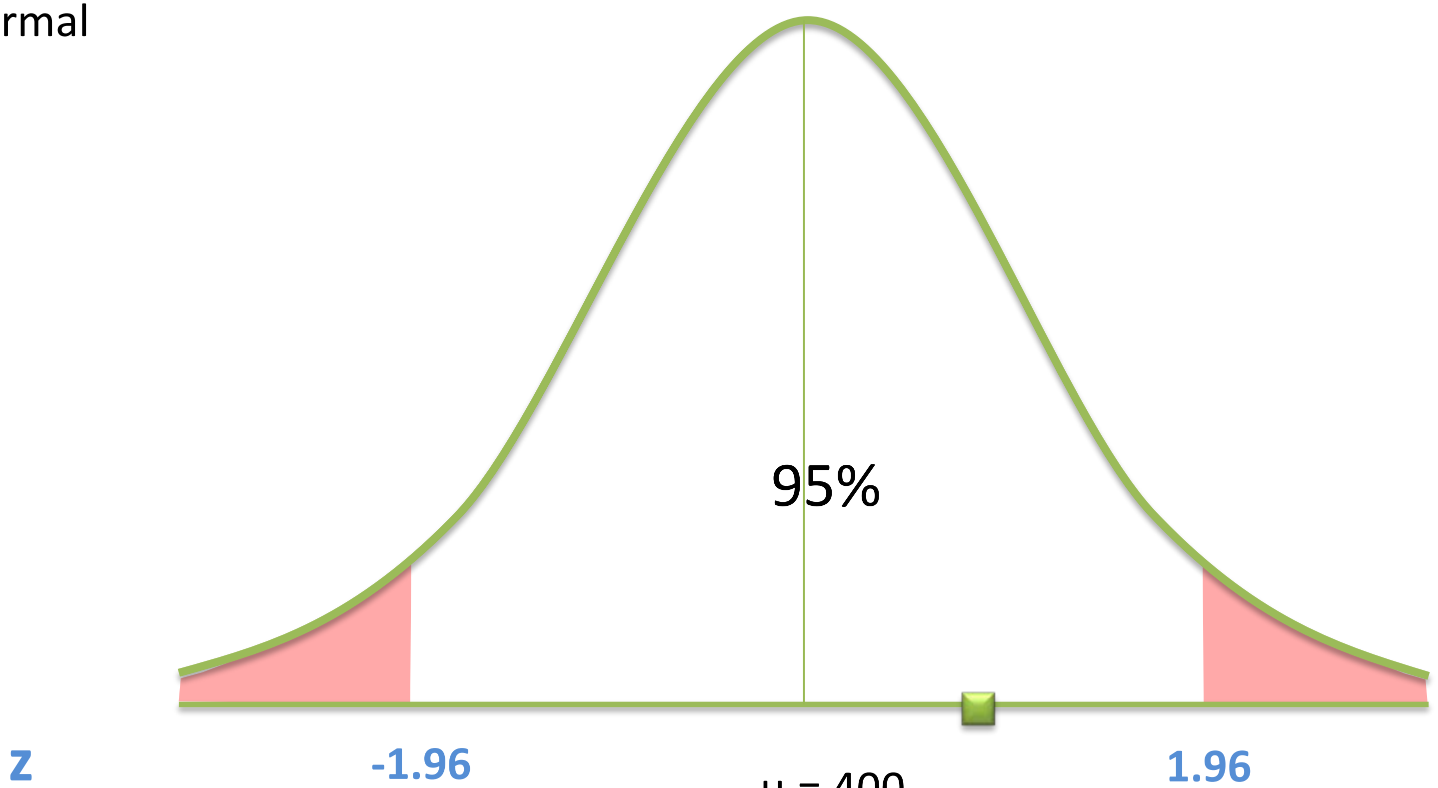

μ = 400
σ = 20

Does treatment work?

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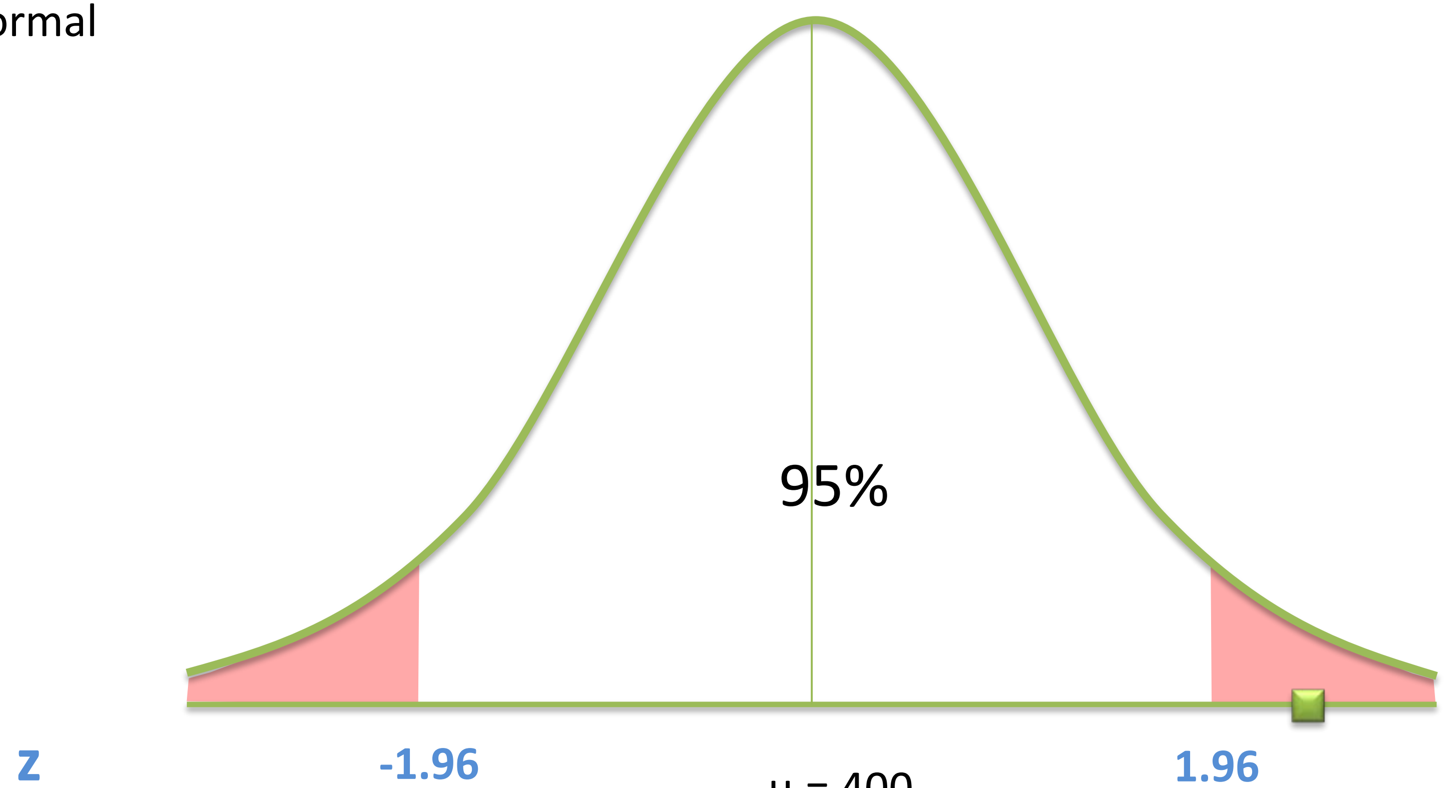

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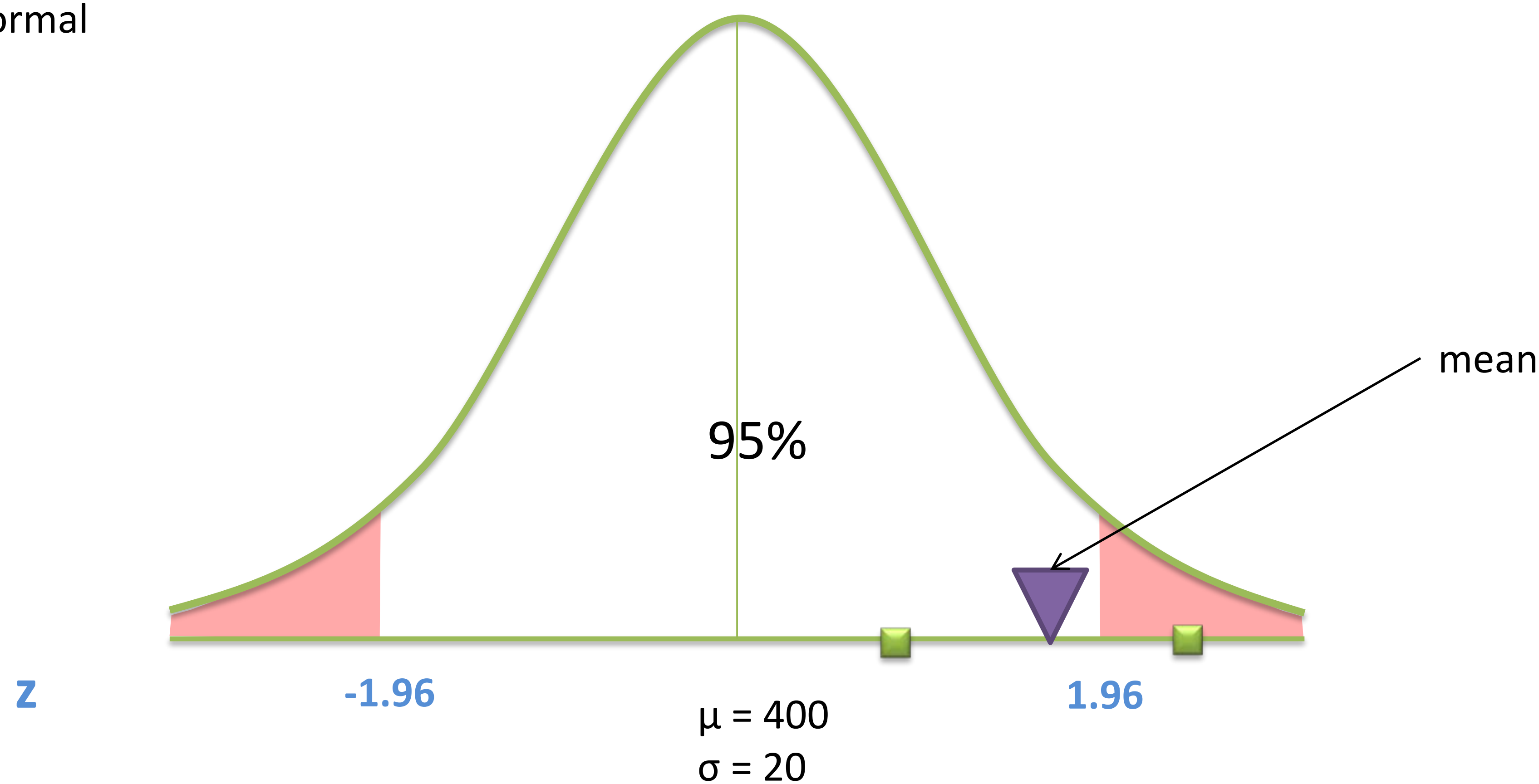


Plan for today: SHIFT

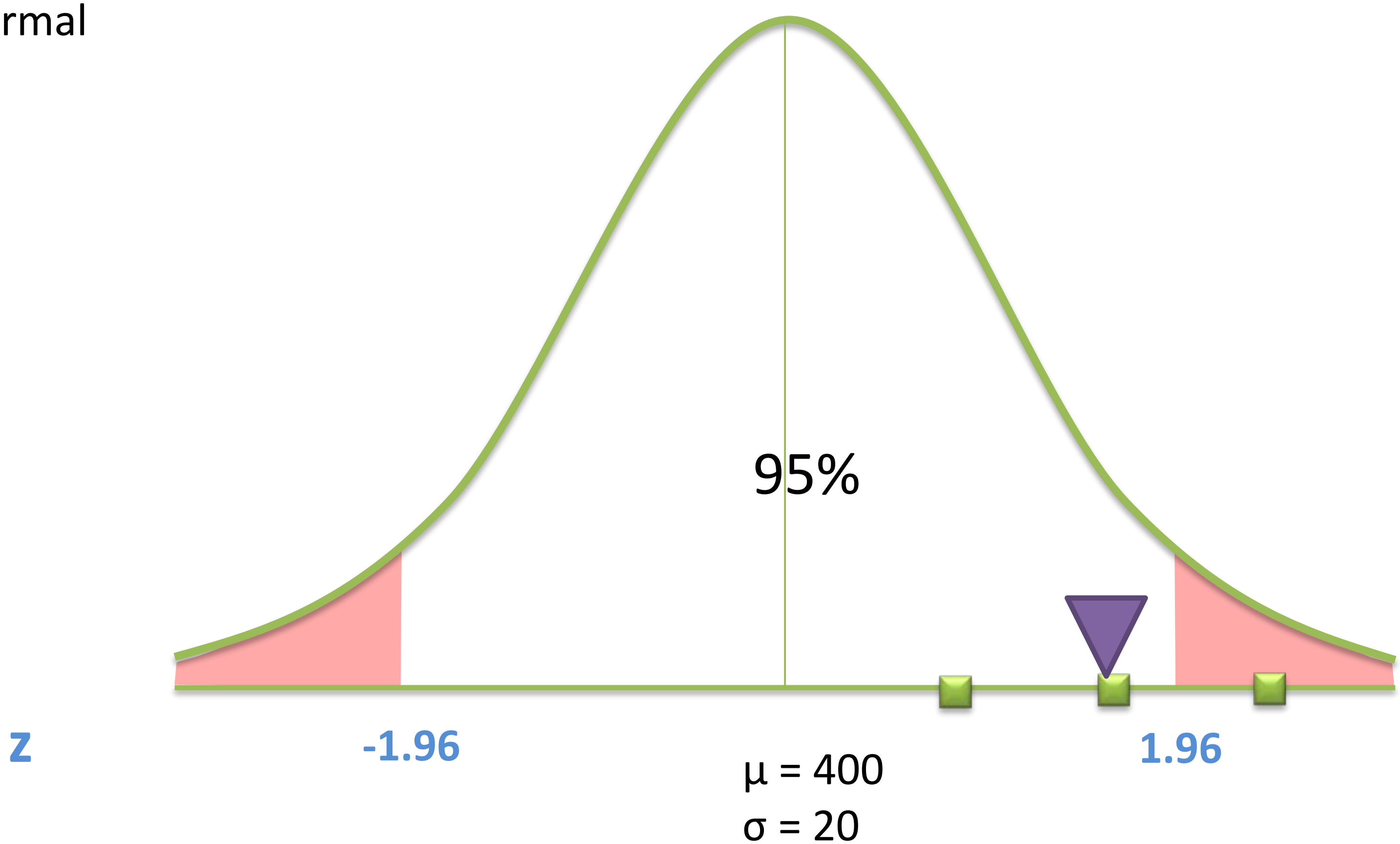
- How likely is the **score**?

- How likely is the **sample**?

Does treatment work?

Population of rats (weight)
Normal

Inject growth hormone



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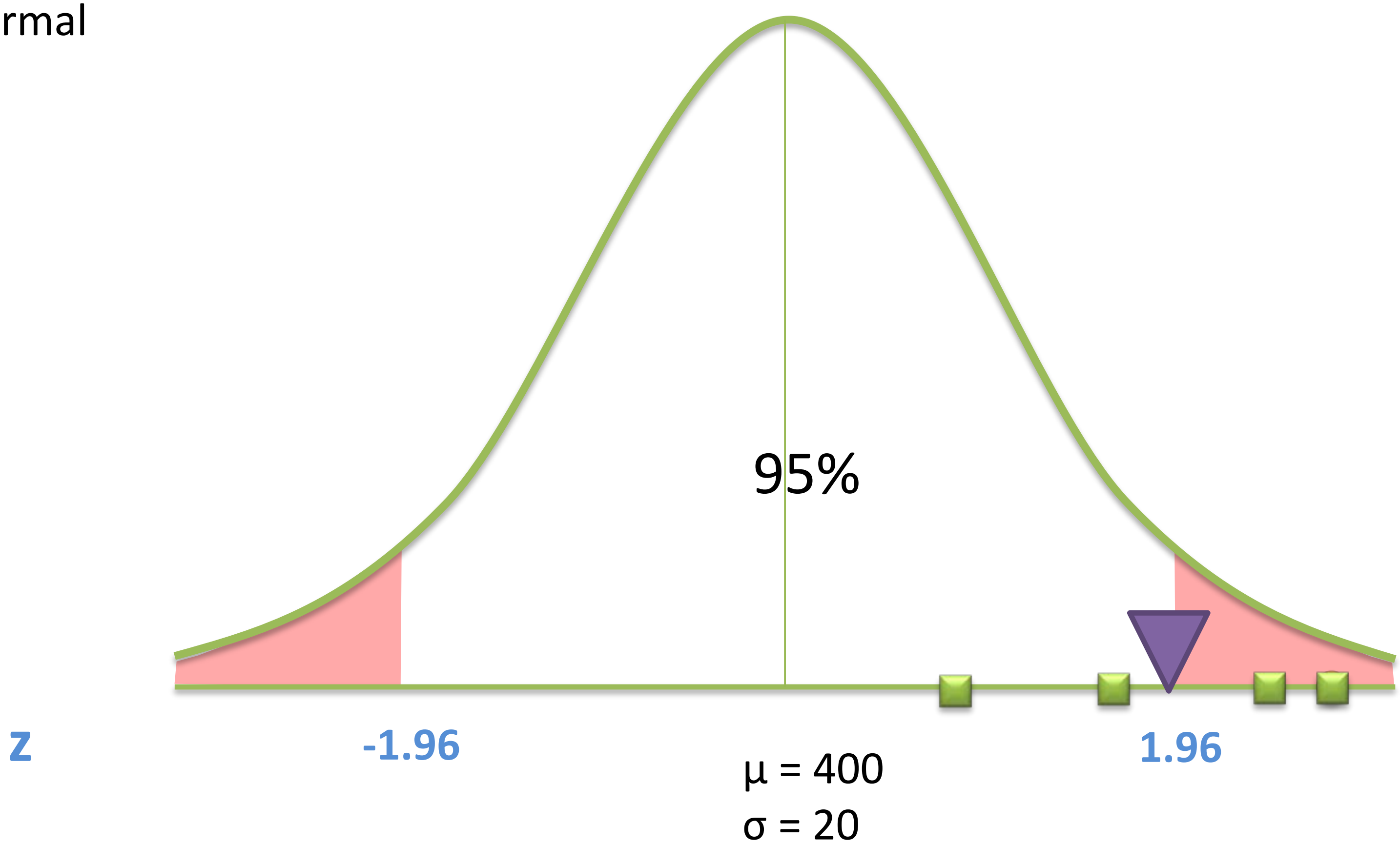

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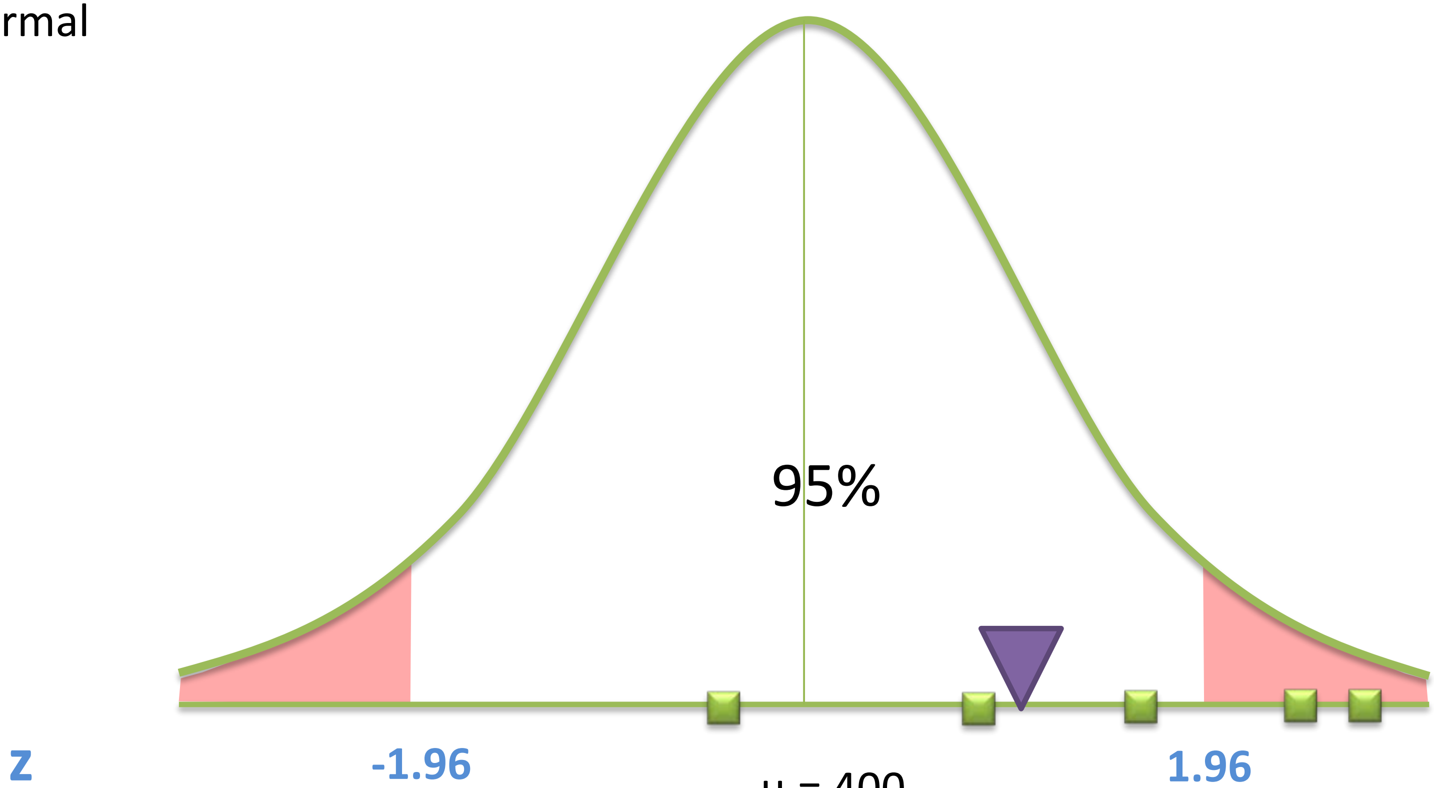
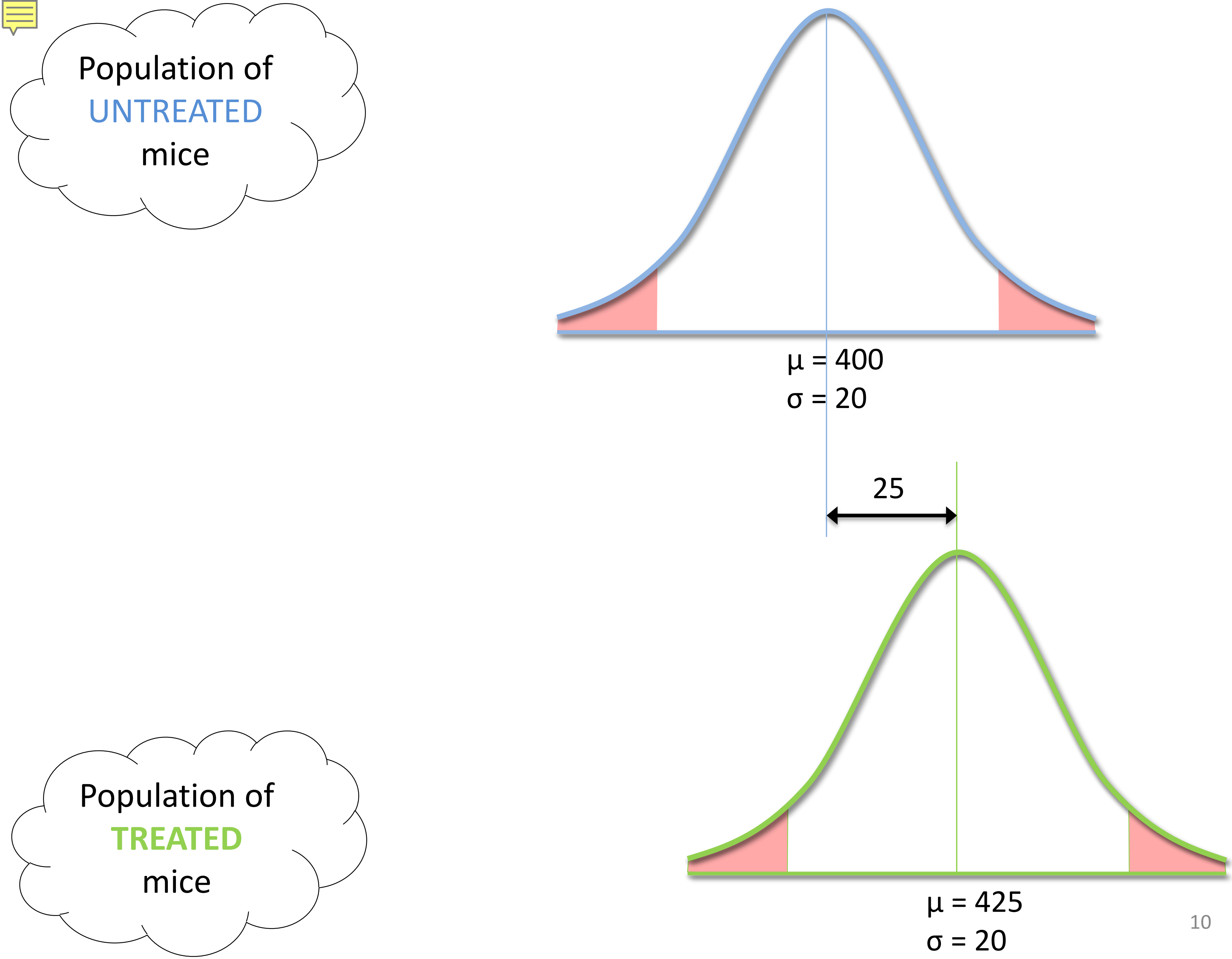
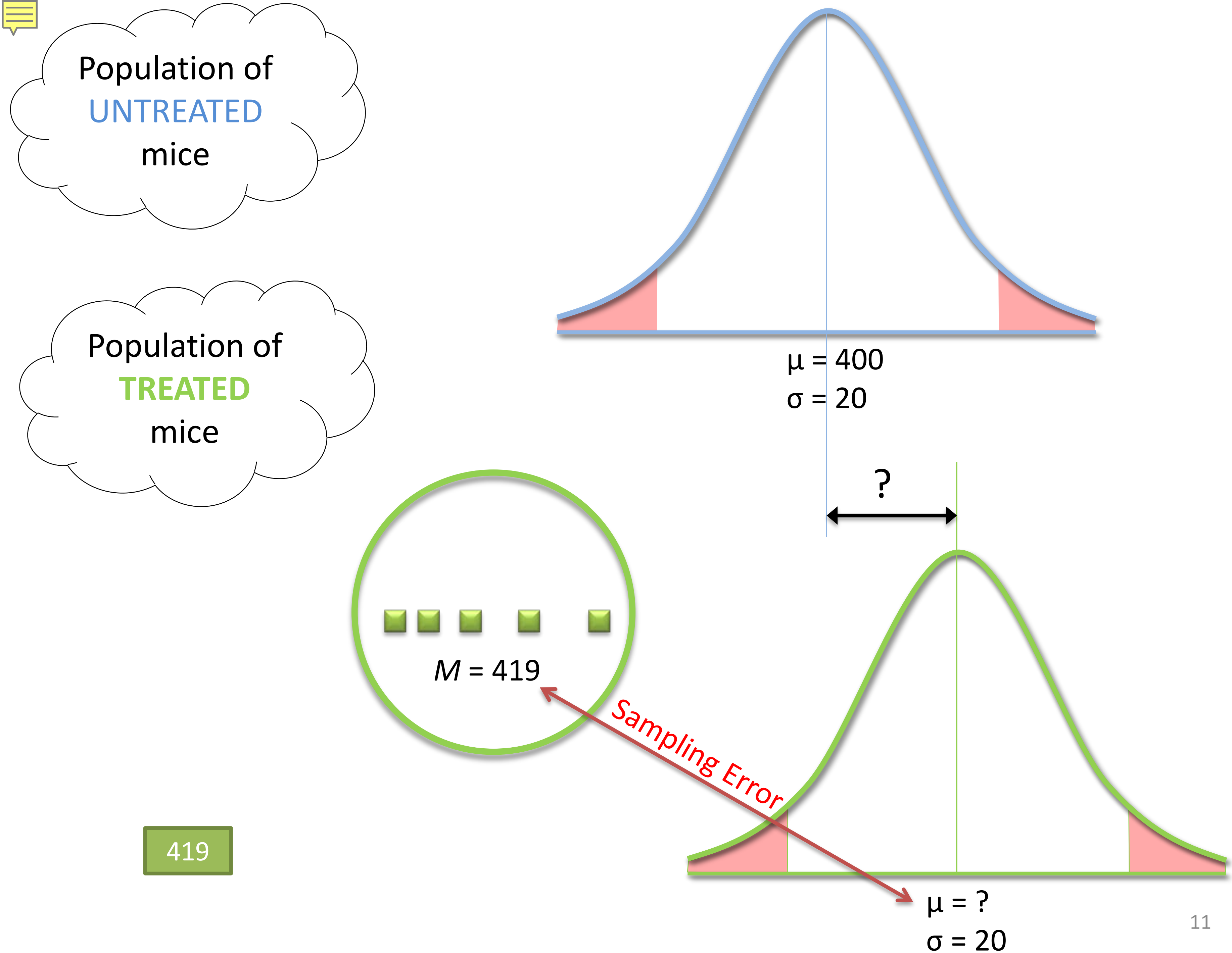
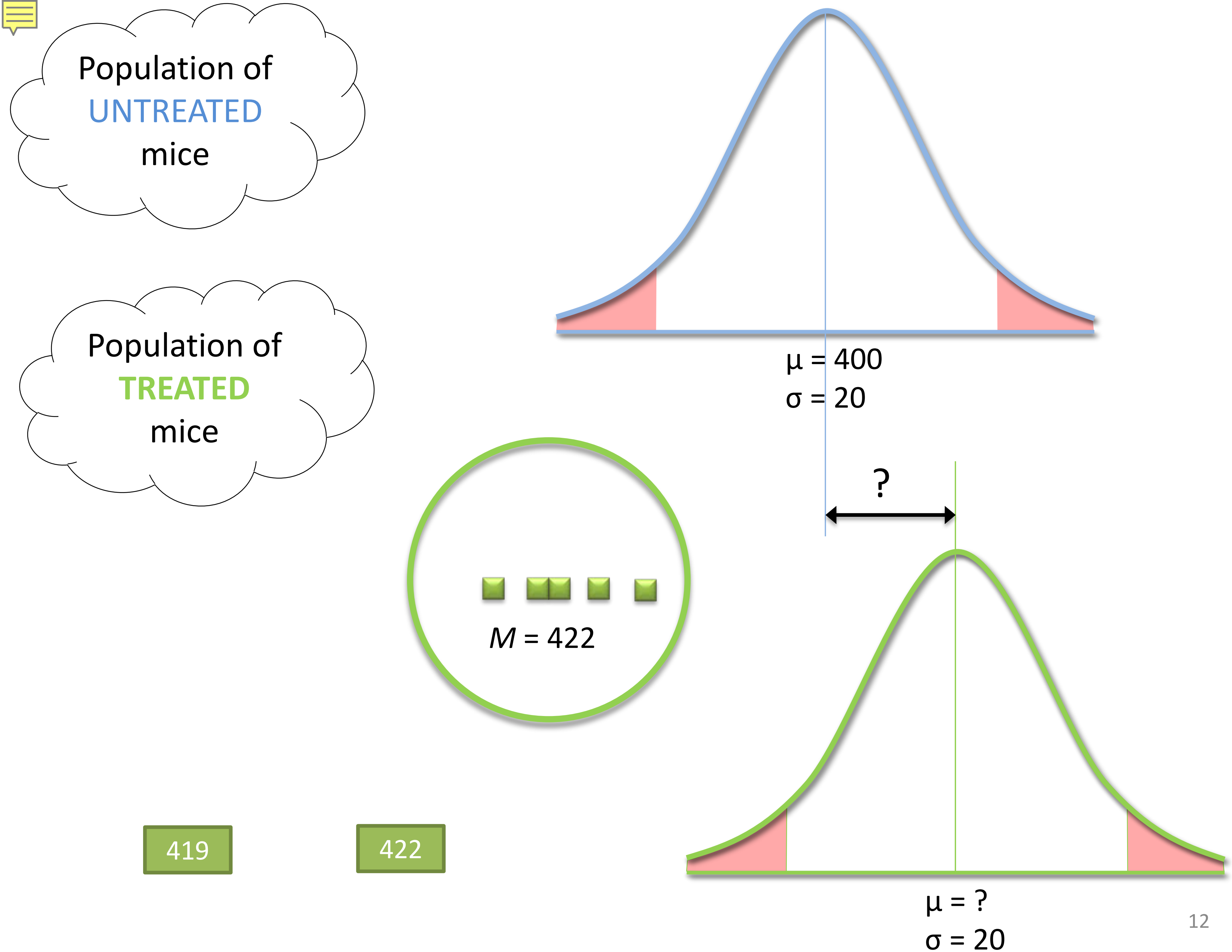


μ = 400
σ = 20



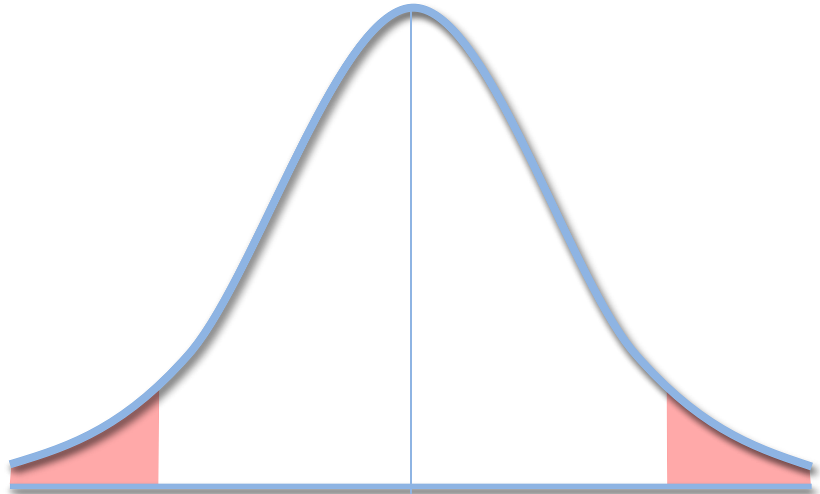
Population of
UNTREATED
mice
μ = 400
σ = 20
25
Population of
TREATED
mice
μ = 425
σ = 20


Population of UNTREATED mice
Population of TREATED mice
μ = 400
σ = 20
?
M = 419
Sampling Error
μ = ?
σ = 20
419


Population of
UNTREATED
mice
Population of
TREATED
mice
μ = 400
σ = 20
?
M = 422
419
422
μ = ?
σ = 20

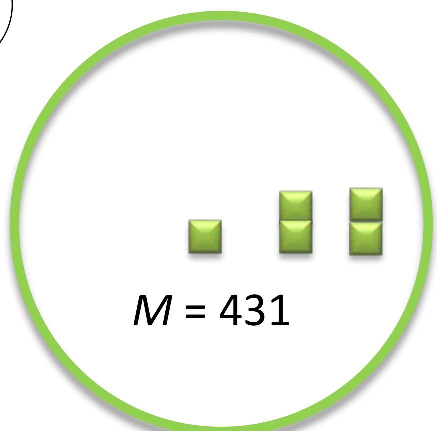
Population of
UNTREATED
mice

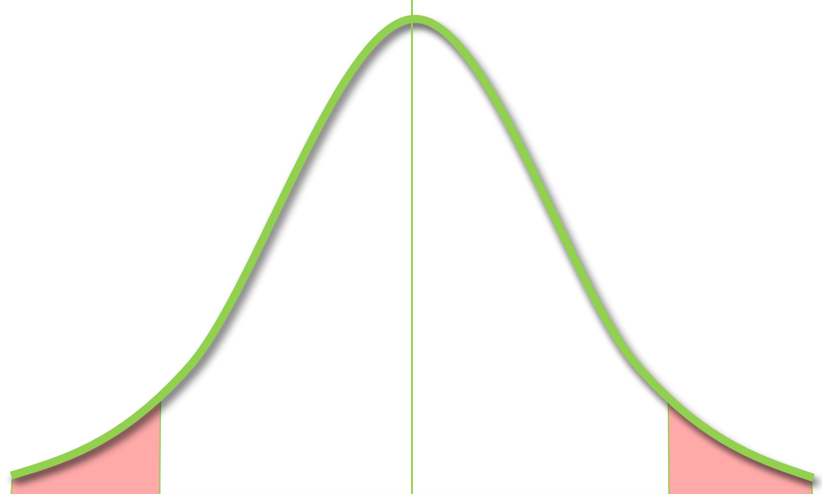
Population of
TREATED
mice



$\mu = 400$
$\sigma = 20$

?



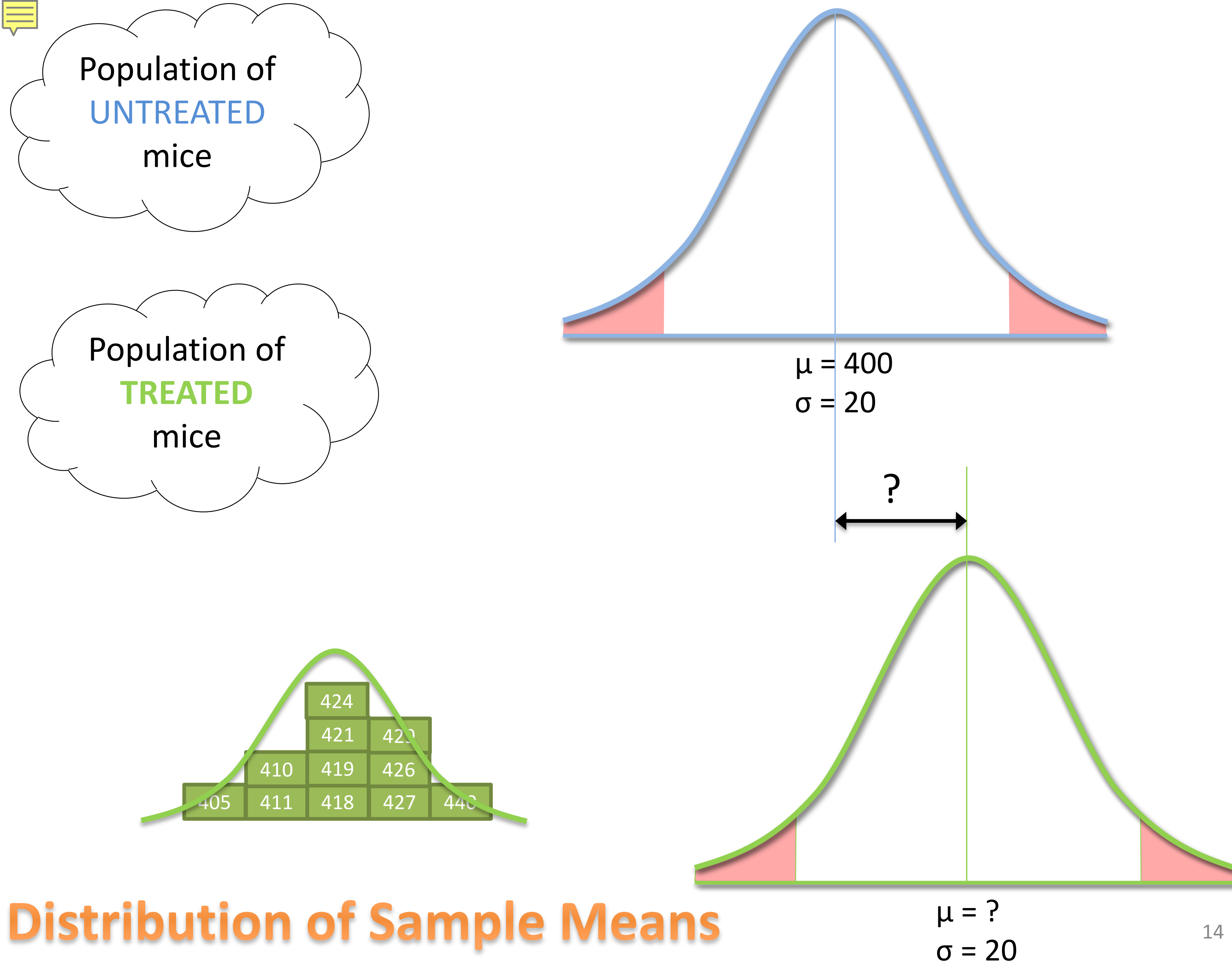


$\mu = ?$
$\sigma = 20$

419 422 431

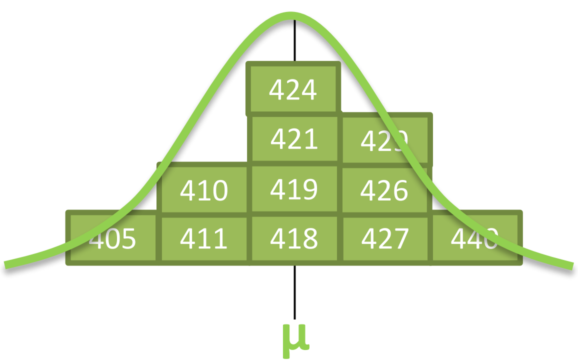
Sampling Variability

Distribution of Sample Means



Distribution of Samples Means

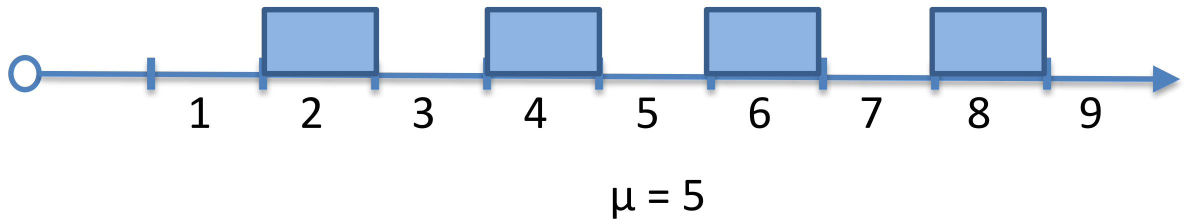
1. Piles up around μ
2. Appears normal in shape



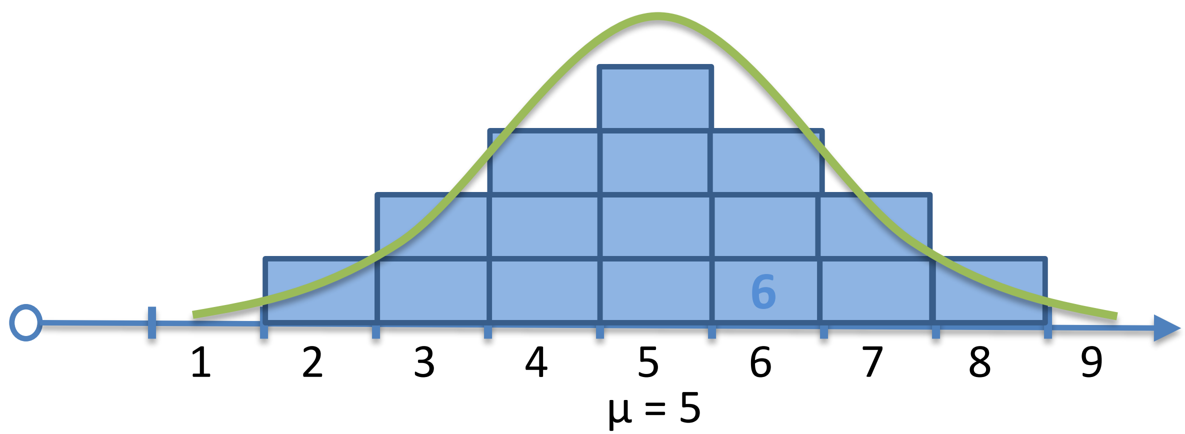
Distribution of Sample Means

Empirical Sampling Distribution

Sample # (n =2)	X1	X2	Mean
1	2	2	**2**
2	2	4	**3**
3	2	6	**4**
4	2	8	**5**
5	4	2	**3**
6	4	4	**4**
7	4	6	**5**
8	4	8	**6**
9	6	2	**4**
10	6	4	**5**
11	6	6	**6**
12	6	8	**7**
13	8	2	**5**
14	8	4	**6**
15	8	6	**7**
16	8	8	**8**



Population

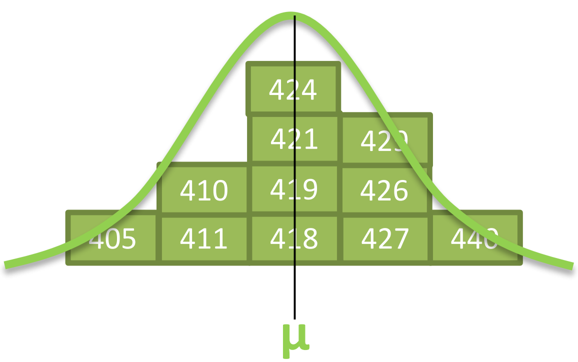


Sampling Distribution

Distribution of Samples Means

1. Piles up around μ
2. Normal in shape

The larger the sample size, the closer to μ

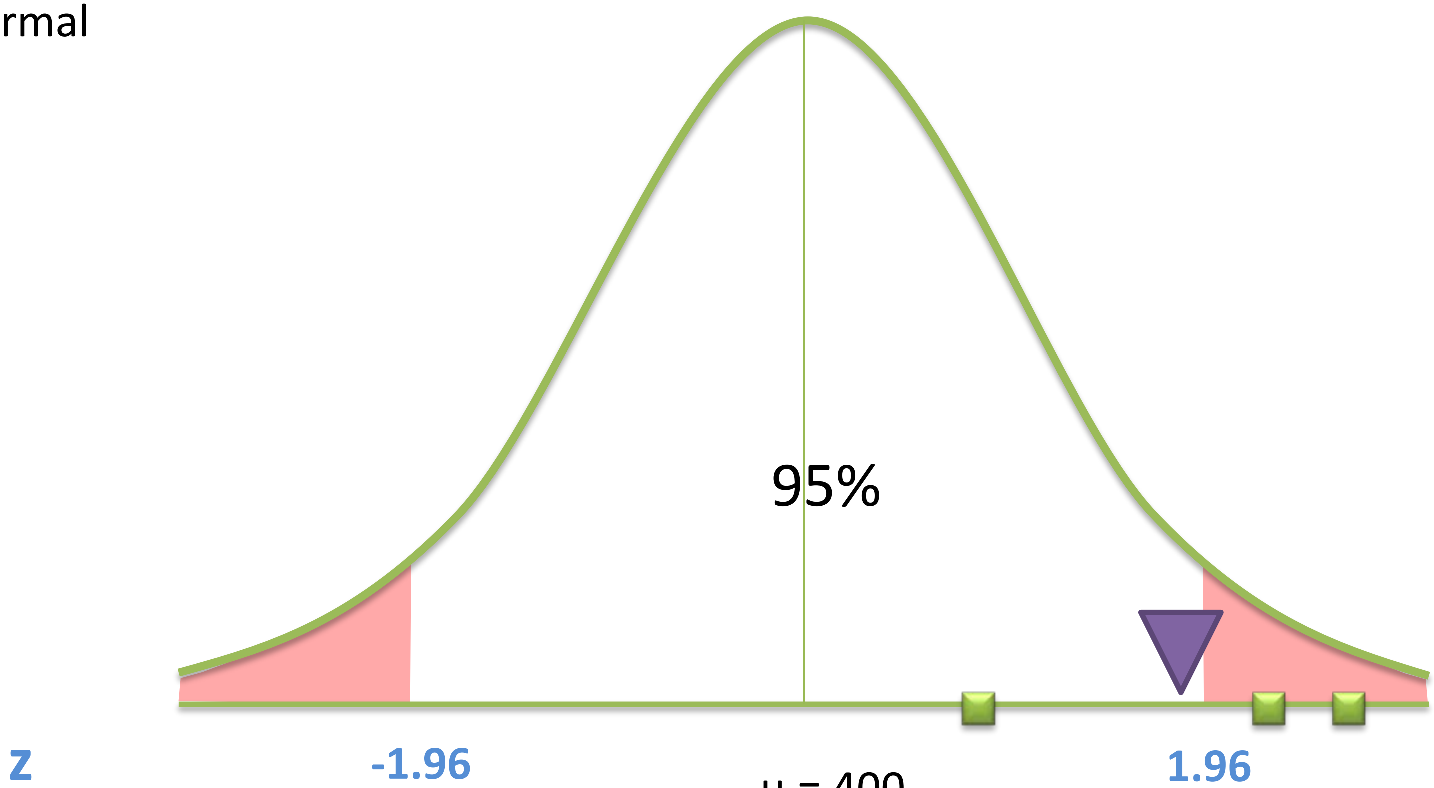


Distribution of Sample Means

Does treatment work? Sample 1

Population of rats (weight)
Normal

Inject growth hormone

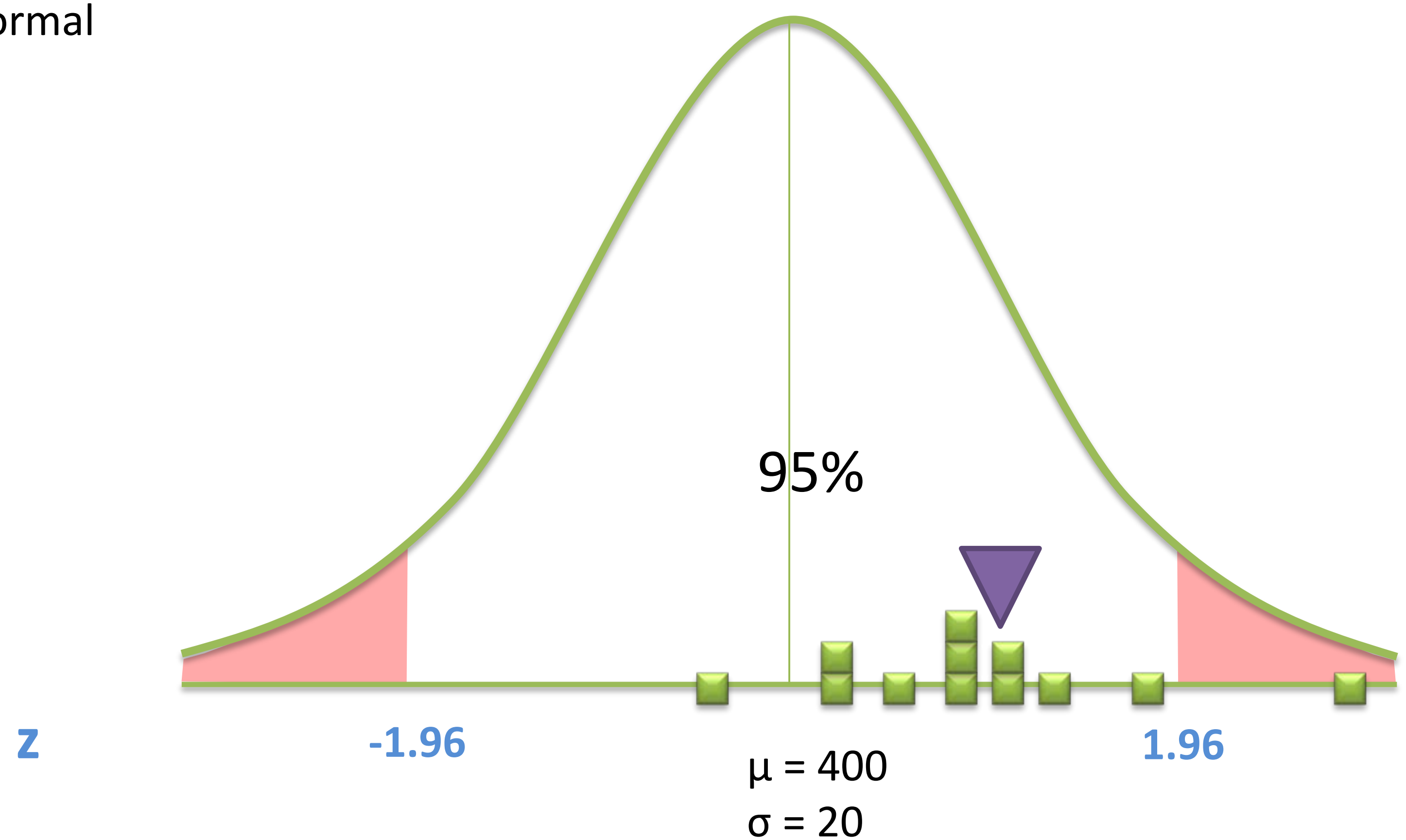


μ = 400
σ = 20

Does treatment work? Sample 2

Population of rats (weight)
Normal

Inject growth hormone



The law of large numbers

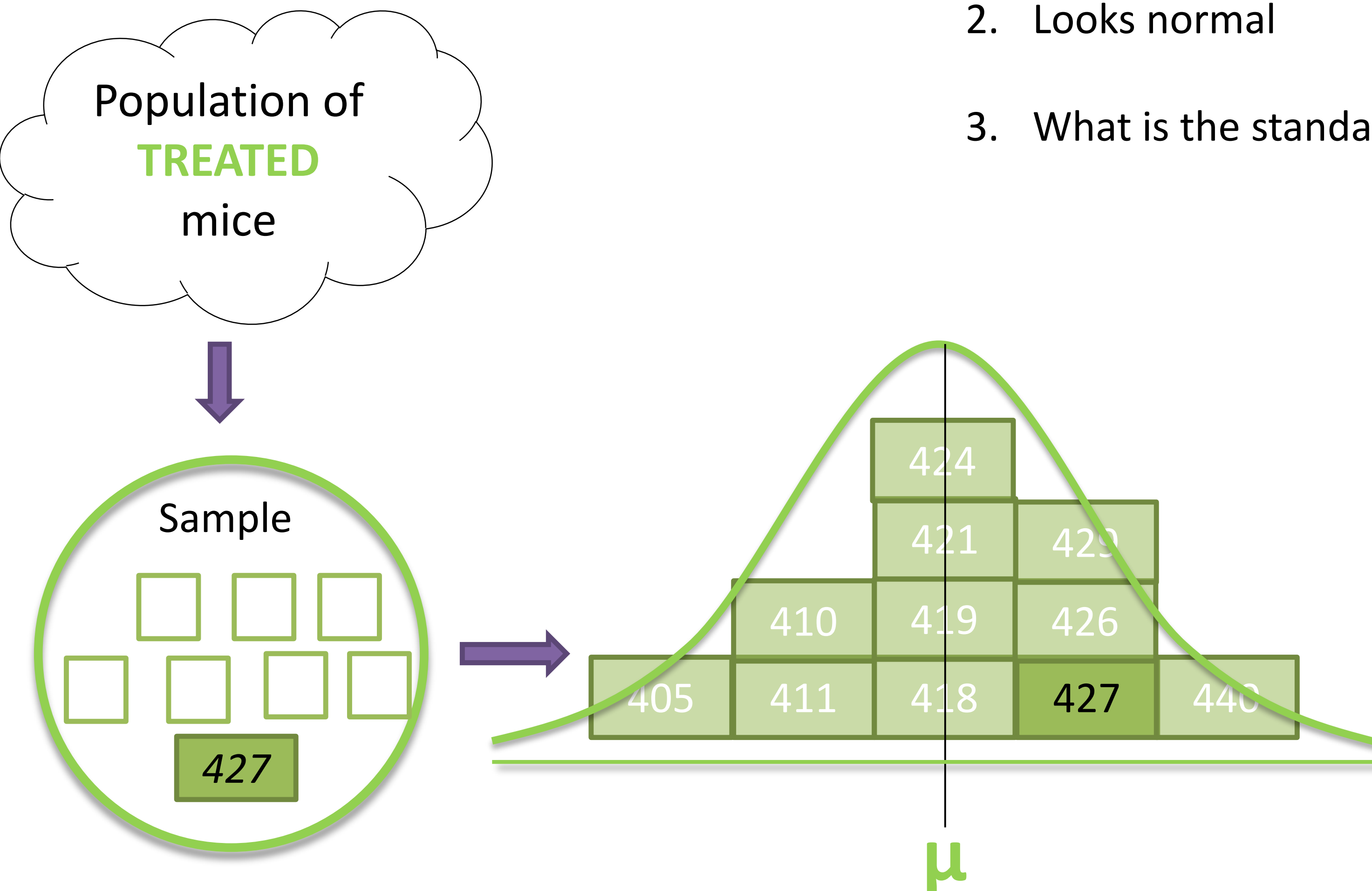
- The larger the sample size (*n*) the more probable that the sample mean (*M*) would be similar to the population mean (μ)

Each sample mean is an element in hypothetical Sampling Distribution

1. Mean of Sampling distribution piles around the mean of the population

2. Looks normal

3. What is the standard deviation?



Central Limit Theory

- For any population with mean μ and standard deviation σ, the distribution of sample means for sample size n will have a mean of μ and standard deviation of $\frac{\sigma}{\sqrt{n}}$ and will approach normality as n approaches infinity

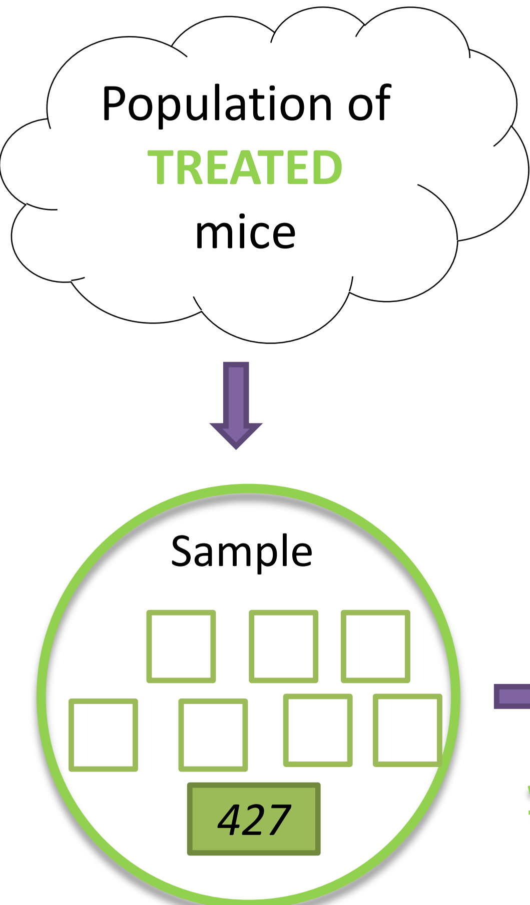
Central Limit Theory

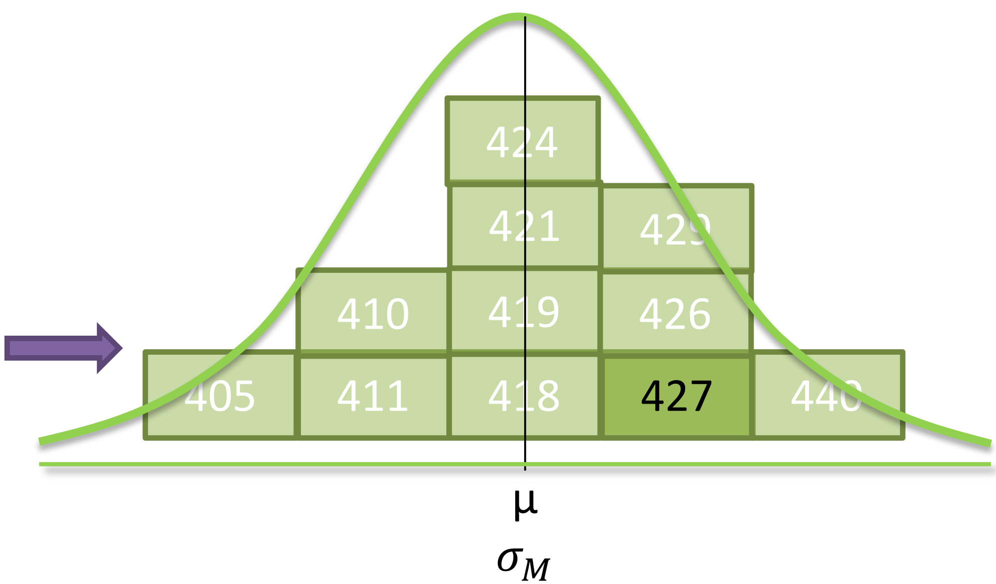
Each sample mean is an element in hypothetical Sampling Distribution

1. Mean of Sampling distribution is the mean of the population

2. Normal in shape

3. $\sigma_M = \dfrac{\boldsymbol{\sigma}}{\sqrt{\boldsymbol{n}}}$ - Standard Error





$\sigma_M = \frac{\boldsymbol{\sigma}}{\sqrt{\boldsymbol{n}}}$ - Standard Error

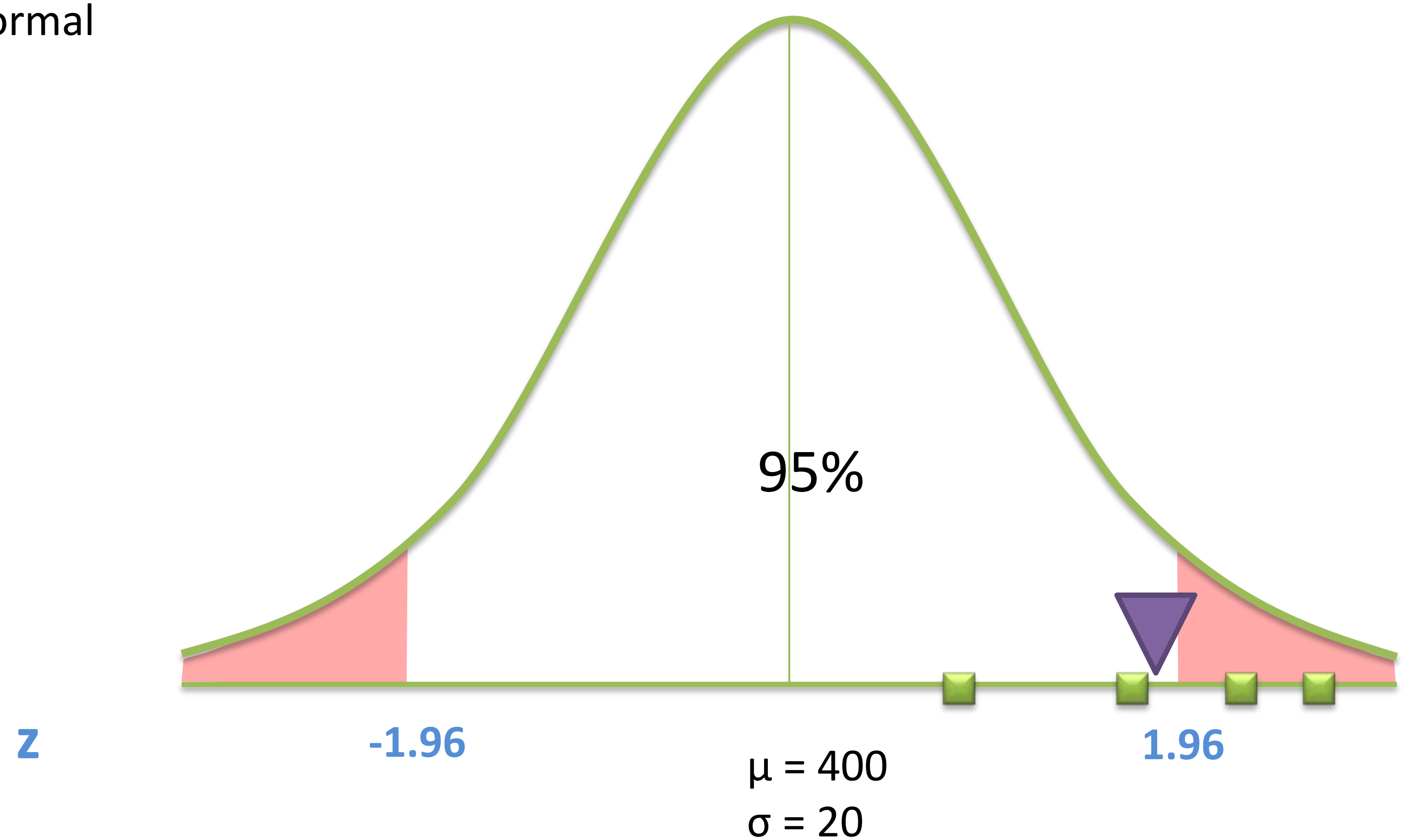
1. SD in (hypothetical) distribution of sample means
2. Average sampling error
3. Expected difference between *M* and μ

Tells us how well the sample mean estimates the population mean.

Sample 1: What is standard error?

Population of rats (weight)
Normal

Inject growth hormone

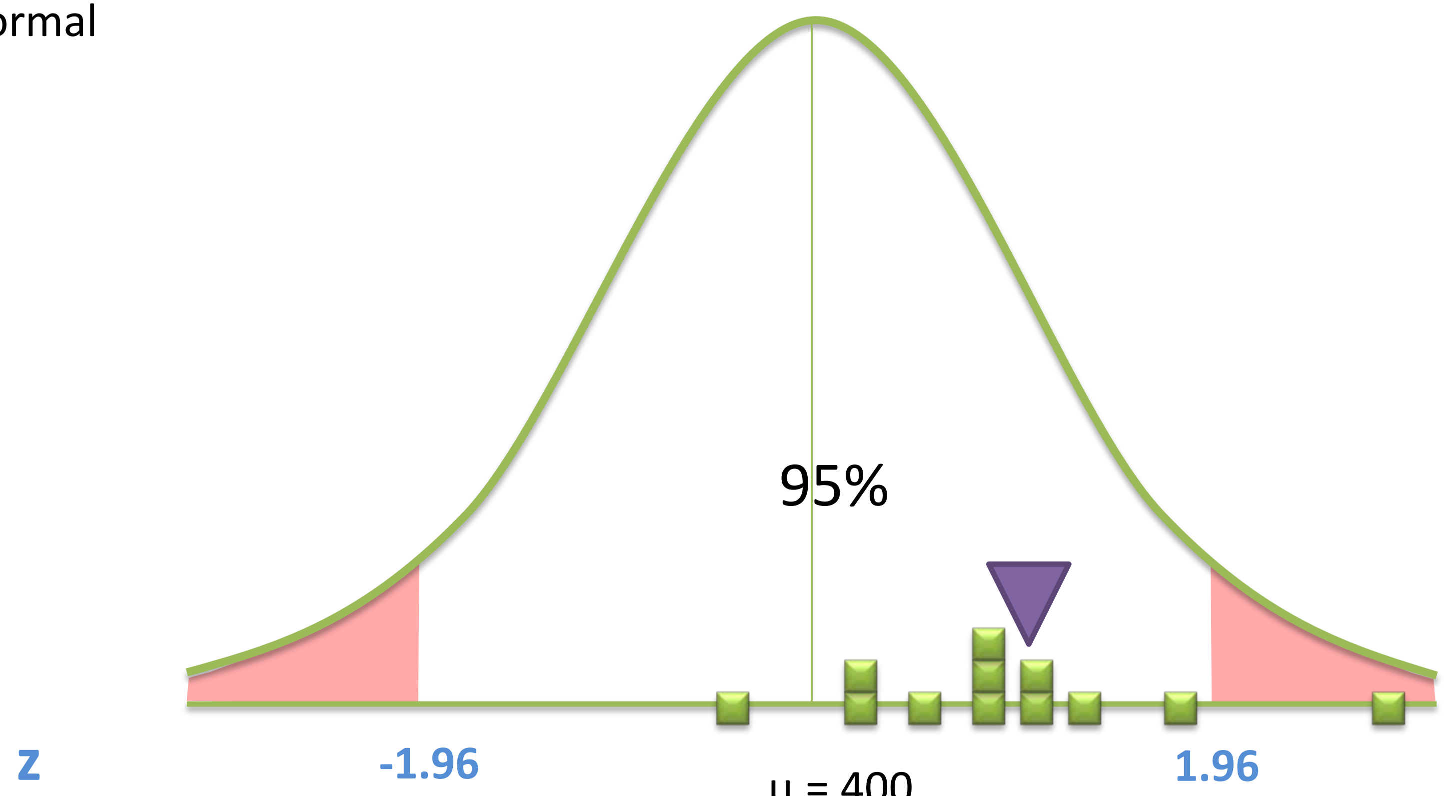


$$\sigma_M = \frac{\boldsymbol{\sigma}}{\sqrt{\boldsymbol{n}}} = \frac{\mathbf{20}}{\sqrt{\mathbf{4}}} = 10$$

Sample 1: What is standard error?

Population of rats (weight)
Normal

Inject growth hormone



μ = 400
σ = 20

$$\sigma_M = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{12}} = 5.77$$

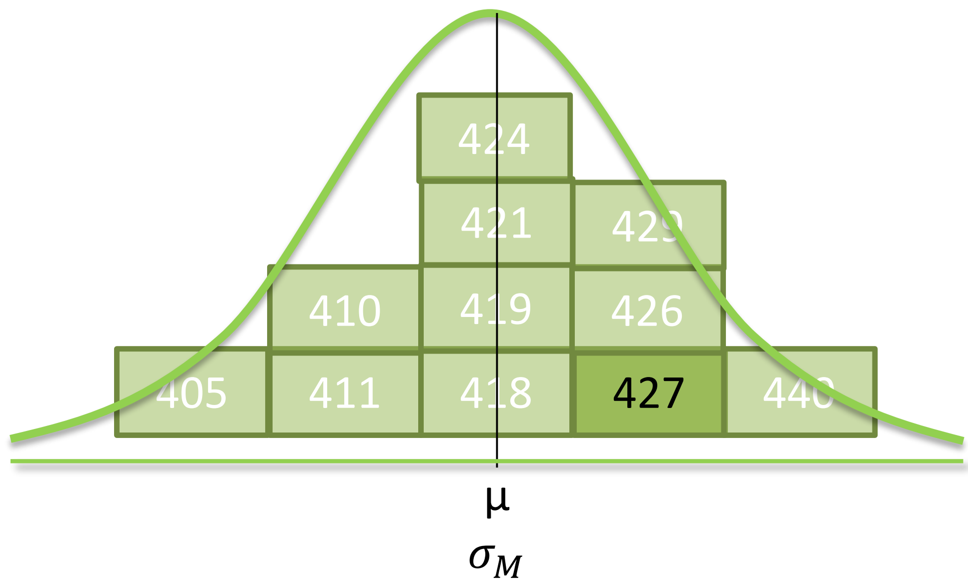
Consequences of CLT

Check

☐ Population from which samples are taken is normal

☐ Sample size is greater than 30

NORMAL



Standard Error and Sample size

How much every element in population is expected to differ from μ

$$\sigma_M = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sigma^2}{n}}$$

Divide the responsibility for error among many observations

What can we do with all of this?

- Law of large numbers
- Central Limit Theorem
- Consequences of CLM
- Properties of Normal Distribution

How "normal" the sample mean is

Is it within the middle 95%?