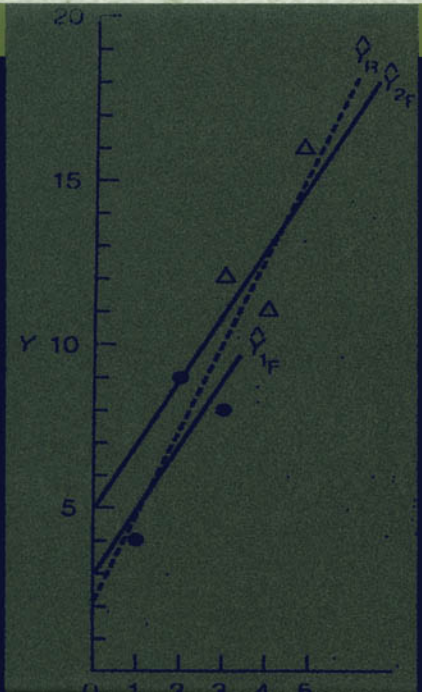
CD Enclosed

Designing Experiments and Analyzing Data

Second Edition

A Model Comparison Perspective



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