Algorithms Illuminated Part 2 - Ch. 11

Problem 11.1

a) (True) A binary search tree with h levels can have at most $\sum_{i=0}^{h-1} 2^i$ nodes (all levels fully filled). The sum of all nodes on all levels prior to the last one is no more than a constant factor < 1 of the last one, so the total size scales with $< 2^h$. Therefore, a BST with n nodes has $< 2^h$ nodes. Using 2^h as an upper bound, $n < 2^h$ we get $\log_2(n) < h$. So the height cannot be smaller than $\Theta(\log n)$.

b) (False) This is only true if the tree is relatively balanced.

c) (False) The heap property does not meet the search tree property because the heap allows both children of a node to be larger than the node.

d) (False) If the data is static, a sorted array has the same or better time complexity for all required operations.