Chapter 7 Andrew Doss

7.1) a) At least one vertex of G has degree at most 10

For a large graph ($\geq 10{,}000$ nodes), 10 edges is very small (as each node could have up to $\geq 10{,}000-1$ edges). Therefore, the existence of one node with < 10 edges is possible in a sparse graph. In addition, this condition still permits all other nodes to have up to $\geq 10{,}000-1$ edges, which is consistent with a dense graph. Therefore, both sparse and dense graphs can meet this condition.

b) Every vertex of G has degree at most 10.

For a graph with $n \geq 10{,}000$ nodes, this condition results in $|E|$ being $\leq 10\frac{n}{2}$ which is $O(n)$. Sparse graphs have edges that scale as $O(n)$ while dense graphs have edges that scale as $O(n^2)$. $O(n^2)$ is not $O(n)$, so this condition only permits sparse graphs.

c) At least one vertex of G has degree $n-1$.

Consider a graph where one node has an edge to every other node, and no other nodes have edges between them. This graph has $n-1$ edges, making the graph a sparse graph. In contrast, a graph meeting the above condition could have $n-1$ edges incident to every node, resulting in $\binom{n}{2} = \frac{n(n-1)}{2}$ edges which is $O(n^2)$. Therefore, this condition can be met by both sparse and dense graphs.

d) Every vertex of G has degree $n-1$.

This condition is described in the second example in the answer to part c. This condition requires that the graph be dense.

7.2) Assuming that the nodes are indexed such that I can get either the row or column of the vertex v in constant time, a single linear scan over all nodes (or rather $n-1$) is required to identify the edges incident to v, so $n-1$ operations are required. c

7.3) To find all incoming edges, a linear search must be run over every array of every node to check for references to v. Therefore, m operations are required to search all edges. d