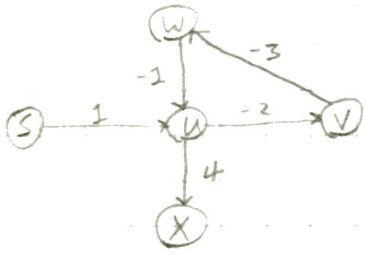
# Algorithms Illuminated Ch. 18

## Problem 18.1




Manually execute Bellman-Ford.

| i | s (1) | u (2) | v (3) | w (4) | x (5) |
|---|---|---|---|---|---|
| 0 | 0 | ∞ | ∞ | ∞ | ∞ |
| 1 | 0 | 1 | ∞ | ∞ | ∞ |
| 2 | 0 | 1 | -1 | ∞ | 5 |
| 3 | 0 | 1 | -1 | -4 | 5 |
| 4 | 0 | -5 | -1 | -4 | 5 |
| 5 | 0 | -5 | -7 | -4 | -1 |

recurrence

$$A[i][v] = \min\left(A[i-1][v],\ \min_{(w,v)\in E}\{A[i-1][w] + l_{wv}\}\right)$$

negative cycle, not stable

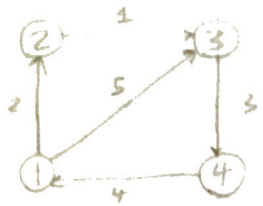
## Problem 18.2

This is not true. Problem 18.1, vertex u, provides a counter example. The shortest path stays at 1 for $1 \le i \le 3$, but then decreases to -5.

## Problem 18.3

In this case, Bellman-Ford only requires K outer iterations. Each iteration must check every incoming edge for every node, which sums to m. $\Rightarrow O(Km)$

## Problem 18.4




Manually execute Floyd-Warshall

(base case)

Step 0: K=0

| source \ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 2 | 5 | ∞ |
| 2 | ∞ | 0 | 1 | ∞ |
| 3 | ∞ | ∞ | 0 | 3 |
| 4 | 4 | ∞ | ∞ | 0 |

5 ∧ K

Step 1: K=1

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 2 | 5 | ∞ |
| 2 | ∞ | 0 | 1 | ∞ |
| 3 | ∞ | ∞ | 0 | 3 |
| 4 | 4 | 6 | 9 | 0 |

Step 2: K=2

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 2 | 3 | ∞ |
| 2 | ∞ | 0 | 1 | ∞ |
| 3 | ∞ | ∞ | 0 | 3 |
| 4 | 4 | 6 | 7 | 0 |

Step 3: K=3

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 2 | 3 | 6 |
| 2 | ∞ | 0 | 1 | 4 |
| 3 | ∞ | ∞ | 0 | 3 |
| 4 | 4 | 6 | 7 | 0 |

Step 4: K=4

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 2 | 3 | 6 |
| 2 | 8 | 0 | 1 | 4 |
| 3 | 7 | 9 | 0 | 3 |
| 4 | 4 | 6 | 7 | 0 |

## Problem 18.5

(base case)

Step 0: K=0

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 2 | 5 | ∞ |
| 2 | ∞ | 0 | 1 | ∞ |
| 3 | ∞ | ∞ | 0 | -3 |
| 4 | -4 | ∞ | ∞ | 0 |

Step 1: K=1

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 2 | 5 | ∞ |
| 2 | ∞ | 0 | 1 | ∞ |
| 3 | ∞ | ∞ | 0 | -3 |
| 4 | -4 | -2 | 1 | 0 |

Step 2: K=2

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 2 | 3 | ∞ |
| 2 | ∞ | 0 | 1 | ∞ |
| 3 | ∞ | ∞ | 0 | -3 |
| 4 | -4 | -2 | -1 | 0 |

Step 3: K=3

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0 | 2 | 3 | 0 |
| 2 | ∞ | 0 | 1 | -2 |
| 3 | ∞ | ∞ | 0 | -3 |
| 4 | 4 | 2 | -1 | -4 |

Step 4: K=4 etc., I get the idea.

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |