ECON-7022MK PhD ECONOMETRICS I (Fall 2017)

Quiz 2 (September 8, 2017)
Due: September 10, 5.00pm.
Late Due: September 12, 7.00pm.

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I. Theory-based problems: .

For these excercises, also revise and draw from Appendices A, B1-B3 and B5 in Hansen, B. (2015). "Econometrics", Lecture notes University of Wisconsin. [HaL]

Question 1: Revise the Slides

Using only the properties of the expected value (E.1 – E3; But make sure to highlight/note which property you are using for each step!),

1. Show that
$$Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X,Y)$$
2. Verify the claims about independence on p. 31, SLIDESET 002A:
   - Claim:
     (a) If $P(A) = 0$, then A and B are independent.
     (b) If $P(B) > 0$, then independence of A and B implies that
     $$P(A|B) = \frac{P(A \bigcap B)}{P(B)} = P(A)$$
     (c) If A and B are independent, then so are A^C and B, A^C and B^C and A and B^C.
3. Prove the first 2 properties of the CDF (p. 15, Slidesest 002B) – Try by yourself first and get inspiration from the course materials/readings, only after you get stuck.
4. Suppose X is distributed uniformly on [0,1], i.e. (FX (x) = x for x in [0;1]). Calculate E[X], E[X^2] and Var(X).
5. Now consider 4 equal random variables X_1, X_2, X_3, X_4 distributed uniformly on [0,1] (same as in previous excercise). consider:
$$Y = \frac{(X_1 + X_2 + X_3 + X_4)}{4}$$
   - Assume X_1, X_2, X_3, X_4 are independent: Calculate E[Y] and Var[Y]
   - Now assume Cov$(X_i, X_j) = 0.2$, for any two X-Variables. Compute E[Y] and Var[Y]

Which of the random variables has the smaller Variance?

For all these excercises make sure to highlight/note which property you are using for each step where you use them!

Question 2: Matrix Algebra

Take the annihilator matrix $\mathbf{M} = \mathbf{I}_n - \mathbf{P}$ where $\mathbf{P} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$

a.) Show that $\mathbf{M}$ is symmetric and idempotent.

b.) Show that $tr\mathbf{M} = n - k$.

c.) Prove that $\mathbf{X}'\hat{\mathbf{u}} = \mathbf{0}$ where $\hat{\mathbf{u}}$ is the least square residual from the linear model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$.

More Theory: (Bonus)

1. Practice Question on p22, slideset 002A ("Review of Probability Theory"). Show that the conditional probability P' is a probability measure
2. On slides 36 and 37, slideset 002A: Verify the illustrative examples that show that conditional independence need not imply independence and vice-verca.
3. Show that, for $x_2 > x_1$:
$$F_x(x_2) - F_x(x_1) = P(x_1 < X < x_2):$$
4. Marginal Distribution: Attempt the proof on p. 27 (SLIDESET 002B) for the discrete case, Try by yourself first and get inspiration from the course materials/readings, only after you get stuck.

Empirical Problem .

Grades [8pts]

For this excercise, refer to the R-primer slides that I uploaded with this problem set. To open a dataset use the command "load", or the mouse-click menu of R-studio.

a.) Write a single source-files that performs all the commands shown for analyzing the course grades, including the scatter plot with the linear fit and the conditional means for rounded values of the x-variable. As some of you noted already, some of the commands are outdated and you have to find the updated commands (online, etc...)

b.) Carefully comment the steps you take in the file (i.e. write 1-3 short lines of explanation what each codeblock is doing)