National Research University "Higher School of Economics"

Master of Data Science Online Programme

Master's Thesis

Topic: Electricity Spot Prices Forecasting Using Stochastic Volatility Models

Computations Notebook

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Table of Contents

In [ ]:
```
import numpy as np
import pandas as pd
```

```
from sklearn.model_selection import TimeSeriesSplit, cross_validate
from sklearn.cluster import KMeans
from sklearn.metrics import root_mean_squared_error, mean_absolute_error
from statsmodels.tsa.stattools import adfuller, pacf
from scipy.stats import norm, probplot, mannwhitneyu
import matplotlib.pyplot as plt
import seaborn as sns
from tqdm import tqdm
from stan_model import StanModel
# This is needed to solve the problem with SSL certificates from www.atsenergo.r
import ssl
ssl._create_default_https_context = ssl._create_unverified_context
```

---

1. Description

There are several approaches to modeling and forecasting time series as applied to prices of services, commodities, and derivative instruments. One of the approaches is to model the price as a heteroscedastic process with changing volatility (variance of price) over time.

In our scenario we will consider a typical Stochastic Volatility model that treats the volatility as a latent stochastic process in discrete time (Kim, Shephard, and Chib 1998) Kim, Sangjoon, Neil Shephard, and Siddhartha Chib. 1998. "Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models." Review of Economic Studies 65: 361–393.

Back

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2. SV Baseline Model

First, we will examine an **SV Baseline** model. This model can be described as a set of regression-like equations, with the following 4 parameters [Kim, Stan user's guide]:

- μ, mean log volatility
- ϕ, persistence of volatility
- σ, white noise shock scale
- h_t, latent log volatility at time t

The variable ϵ_t represents the white-noise shock (i.e., multiplicative error) on the price at time t, whereas δ_t represents the shock on volatility at time t:

$\epsilon_t \sim \mathcal{N}(0,1)$; $\delta_t \sim \mathcal{N}(0,1)$.

$y_t = e^{\frac{h_t}{2}}\epsilon_t$, where

$h_t = \mu + \phi(h_{t-1} - \mu) + \delta_t\sigma$;

$h_1 \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{1-\phi^2}}\right)$.

To learn these 4 parameters from the price data, we will use the **Stan** platform for statistical modeling and high-performance statistical computation. It uses its own proprietary probabilistic language and can do Bayesian statistical inference, as well as maximum likelihood estimation (MLE) with derivative-based optimization (Newton, BFGS, etc.). https://mc-stan.org/

Other notable probabilistic programming library is PyMC https://www.pymc.io/

Rearranging the equations above yields the following equations to be used in the Stan code for the SV Baseline model:

$y_t \sim \mathcal{N}(0, e^{\frac{h_t}{2}})$, where

$$h_1 \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{1-\phi^2}}\right);$$

$$h_t \sim \mathcal{N}(\mu + \phi(h_{t-1} - \mu), \sigma),$$

2.1. Price Data

There are two price zones: 1 (European) and 2 (Siberian) in the day-ahead spot electricity markets.

1. For building and examining our models, we will load the hourly data for the price zone 1 for the period 01.05.2023 -- 30.04.2024 (one year back from now) for the peak hour.
2. For model cross-validation, we will load and examine both price zones for both peak and off-peak hours with cross-validation over a sliding window for the period 23.06.2014 -- 30.04.2024.
3. For forecasting, we will generate predictions for both price zone for both peak and off-peak hours for the period 01.05.2024 -- 07.05.2024 (one week ahead).

The data is available on the AO "ATC" (Администратор Торговой Системы) platform https://www.atsenergo.ru/results/rsv/index, starting from Aug 8th, 2013 to present. The unit of prices is RUB/MWh (руб./МВт·ч).

In [ ]:
```
# # We need to explicitly specify the column names to correctly parse the xml
# price_data = pd.read_xml(f'https://www.atsenergo.ru/market/stats.xml?period=0&
#                          names=['ROW_ID',
#                                 'DAT',
#                                 'PRICE_ZONE_CODE',
#                                 'CONSUMER_VOLUME',
#                                 'CONSUMER_PRICE',
#                                 'CONSUMER_RD_VOLUME',
#                                 'CONSUMER_SPOT_VOLUME',
#                                 'CONSUMER_PROVIDE_RD',
#                                 'CONSUMER_MAX_PRICE',
#                                 'CONSUMER_MIN_PRICE',
#                                 'SUPPLIER_VOLUME',
#                                 'SUPPLIER_PRICE',
#                                 'SUPPLIER_RD_VOLUME',
#                                 'SUPPLIER_SPOT_VOLUME',
```

```
#                               'SUPPLIER_PROVIDE_RD',
#                               'SUPPLIER_MAX_PRICE',
#                               'SUPPLIER_MIN_PRICE',
#                               'HOUR'],
#                       xpath='//row',
#                       parse_dates=['DAT'])
# # Make datetime
# price_data = price_data.set_index(pd.to_datetime(price_data['DAT'].astype(str)
# price_data.index.name ='Datetime'
# # We can now drop all unnecessary columns to reduce the dataframe
# price_data = price_data.drop(columns=['ROW_ID',
#                           'DAT',
#                           'CONSUMER_VOLUME',
#                           'CONSUMER_RD_VOLUME',
#                           'CONSUMER_SPOT_VOLUME',
#                           'CONSUMER_PROVIDE_RD',
#                           'CONSUMER_MAX_PRICE',
#                           'CONSUMER_MIN_PRICE',
#                           'SUPPLIER_VOLUME',
#                           'SUPPLIER_PRICE',
#                           'SUPPLIER_RD_VOLUME',
#                           'SUPPLIER_SPOT_VOLUME',
#                           'SUPPLIER_PROVIDE_RD',
#                           'SUPPLIER_MAX_PRICE',
#                           'SUPPLIER_MIN_PRICE']).dropna()
# price_data = price_data.loc[~price_data.index.isna()].sort_index()
# price_data['Weekday'] = price_data.index.day_of_week
# price_data.to_csv('./data/price_data_20230501_20240507.csv')
```

In [ ]:
```
price_data = pd.read_csv('./data/price_data_20230501_20240507.zip', index_col='D
price_data = price_data.loc[:'30.04.2024'] # 01.05.2024 -- 07.05.2024 is used fo
```

In [ ]:
```
price_data.shape
```

Out[ ]: (17568, 4)

In [ ]:
```
price_data.head(4)
```

Out[ ]:

Datetime	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday
2023-05-01 00:00:00	1	1517.92	0	0
2023-05-01 00:00:00	2	1517.39	0	0
2023-05-01 01:00:00	2	1512.35	1	0
2023-05-01 01:00:00	1	1413.99	1	0

In [ ]:
```
price_data.tail(4)
```

Out[ ]:

Datetime	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday
2024-04-30 22:00:00	2	1465.38	22	1
2024-04-30 22:00:00	1	1442.53	22	1
2024-04-30 23:00:00	2	1439.88	23	1
2024-04-30 23:00:00	1	1327.94	23	1

In [ ]:
```
price_data.describe()
```

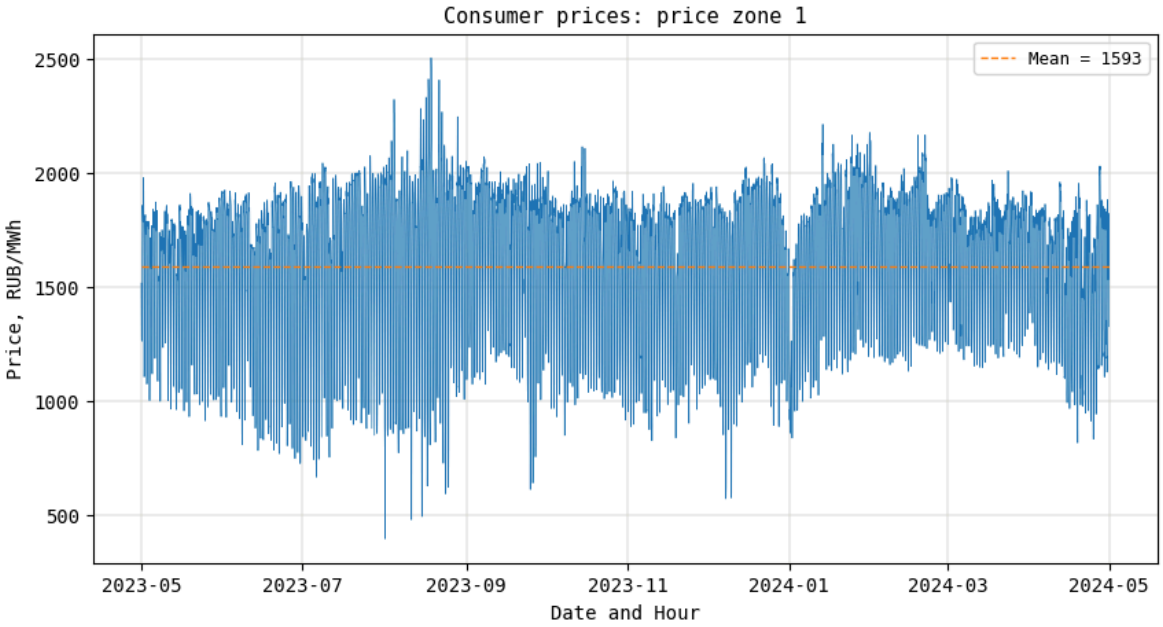
Out[ ]:

	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday
count	17568.000000	17568.000000	17568.000000	17568.000000
mean	1.500000	1424.566452	11.500000	2.986339
std	0.500014	331.979989	6.922384	2.003423
min	1.000000	398.480000	0.000000	0.000000
25%	1.000000	1173.405000	5.750000	1.000000
50%	1.500000	1394.690000	11.500000	3.000000
75%	2.000000	1726.827500	17.250000	5.000000
max	2.000000	2504.960000	23.000000	6.000000

In [ ]:
```
price_zone = 1
```

In [ ]:
```
# Price zone 1 (European)
price_data_eur = price_data[price_data['PRICE_ZONE_CODE'] == price_zone]
```

In [ ]:
```
plt.figure(figsize=(10, 5))
plt.plot(price_data_eur.index, price_data_eur['CONSUMER_PRICE'], color='C0', lw=
plt.hlines(price_data_eur['CONSUMER_PRICE'].mean(), xmin=price_data_eur.index.mi
plt.xlabel('Date and Hour', size=10, family='monospace')
plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.title(f'Consumer prices: price zone {price_zone}', size=11, family='monospac
plt.legend(prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement');
```



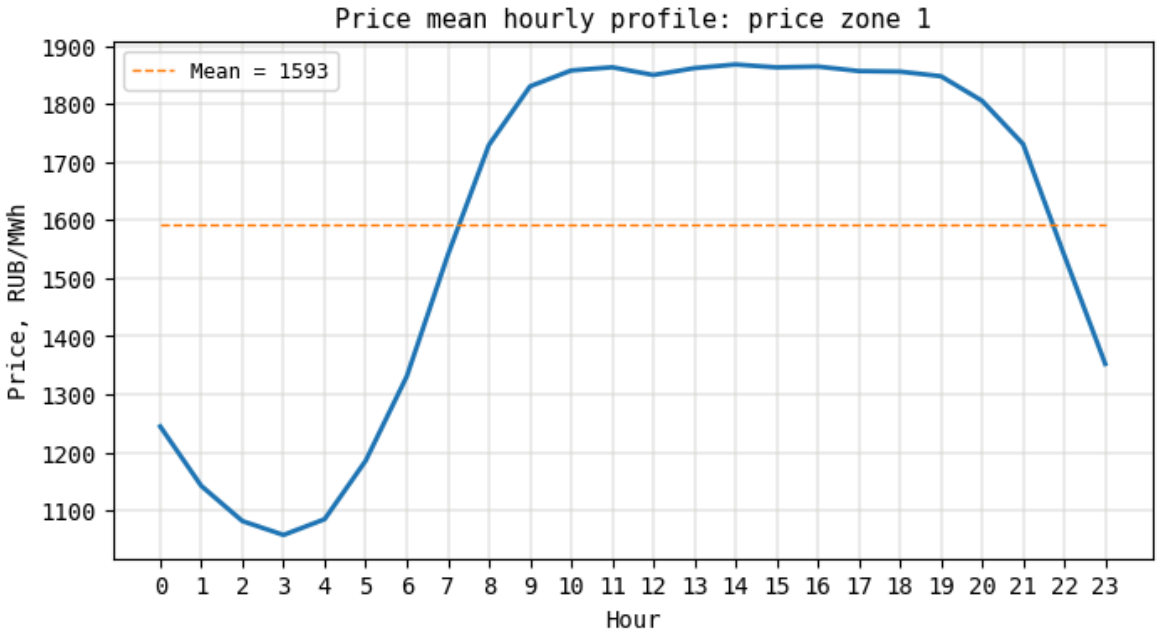
Next, we'll check the hour distribution -- consumer price mean hourly profile.

In [ ]:
```
price_data_daily_agg_eur = price_data_eur.groupby('HOUR')['CONSUMER_PRICE'].mean
price_data_daily_agg_eur
```

Out[ ]:
```
HOUR
0     1244.706885
1     1142.310683
2     1081.862240
3     1058.018962
4     1085.111284
5     1185.857131
6     1331.013224
7     1540.080082
8     1729.485519
9     1830.166311
10    1857.536230
11    1863.038005
12    1849.733060
13    1861.542568
14    1867.866858
15    1862.852432
16    1864.292022
17    1856.555902
18    1855.611311
19    1847.650683
20    1805.406421
21    1730.848115
22    1539.855820
23    1352.347623
Name: CONSUMER_PRICE, dtype: float64
```

In [ ]:
```
plt.figure(figsize=(8, 4))
plt.plot(price_data_daily_agg_eur.index, price_data_daily_agg_eur, color='C0', l
plt.hlines(price_data_daily_agg_eur.mean(), xmin=price_data_daily_agg_eur.index.
plt.xlabel('Hour', size=10, family='monospace')
plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
plt.xticks(price_data_daily_agg_eur.index, size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.title(f'Price mean hourly profile: price zone {price_zone}', size=11, family
```

```
plt.legend(prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement');
```



Let's examine the peak and off-peak hours.

In [ ]:
```
hour_max_eur = price_data_daily_agg_eur.idxmax()
hour_min_eur = price_data_daily_agg_eur.idxmin()
hour_max_eur, hour_min_eur
```

Out[ ]: (14, 3)

In [ ]:
```
price_data_peak_eur = price_data_eur[price_data_eur['HOUR'] == hour_max_eur]
price_data_peak_eur
```

Out[ ]:

Datetime	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday
2023-05-01 14:00:00	1	1803.61	14	0
2023-05-02 14:00:00	1	1793.39	14	1
2023-05-03 14:00:00	1	1770.27	14	2
2023-05-04 14:00:00	1	1674.88	14	3
2023-05-05 14:00:00	1	1810.13	14	4
...	...	...	...	...
2024-04-26 14:00:00	1	1862.04	14	4
2024-04-27 14:00:00	1	2025.49	14	5
2024-04-28 14:00:00	1	1810.51	14	6
2024-04-29 14:00:00	1	1779.80	14	0
2024-04-30 14:00:00	1	1553.93	14	1

366 rows × 4 columns

In [ ]:
```
price_data_peak_eur['CONSUMER_PRICE'].describe()
```

Out[ ]:
```
count     366.000000
mean     1867.866858
std       137.479673
min      1132.110000
25%      1793.745000
50%      1869.885000
75%      1944.390000
max      2504.960000
Name: CONSUMER_PRICE, dtype: float64
```

In [ ]:
```
price_data_offpeak_eur = price_data_eur[price_data_eur['HOUR'] == hour_min_eur]
price_data_offpeak_eur
```

Out[ ]:

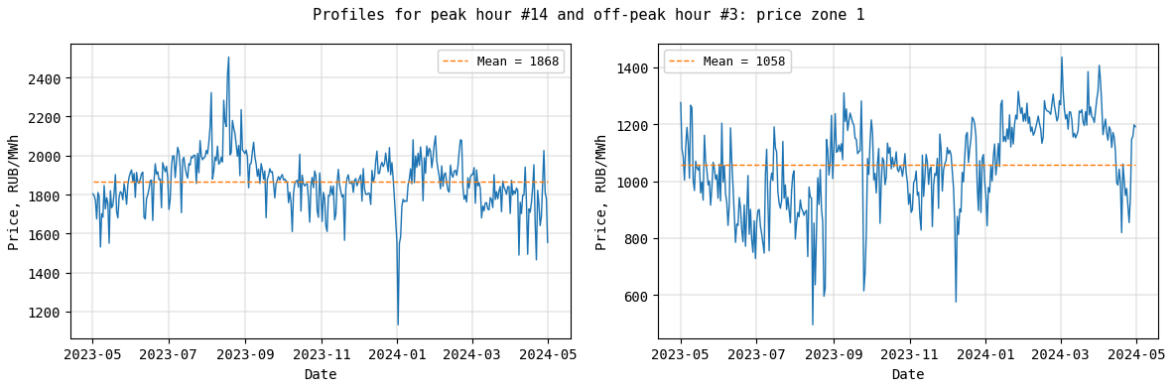
Datetime	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday
2023-05-01 03:00:00	1	1275.81	3	0
2023-05-02 03:00:00	1	1113.04	3	1
2023-05-03 03:00:00	1	1085.05	3	2
2023-05-04 03:00:00	1	1003.91	3	3
2023-05-05 03:00:00	1	1121.00	3	4
...	...	...	...	...
2024-04-26 03:00:00	1	944.52	3	4
2024-04-27 03:00:00	1	1147.59	3	5
2024-04-28 03:00:00	1	1157.11	3	6
2024-04-29 03:00:00	1	1197.78	3	0
2024-04-30 03:00:00	1	1191.04	3	1

366 rows × 4 columns

In [ ]:
```
price_data_offpeak_eur['CONSUMER_PRICE'].describe()
```

Out[ ]:
```
count     366.000000
mean     1058.018962
std       153.703503
min       495.830000
25%       956.015000
50%      1066.625000
75%      1182.727500
max      1435.760000
Name: CONSUMER_PRICE, dtype: float64
```

In [ ]:
```
plt.figure(figsize=(12, 4))
for i, hour_profile in zip(range(1, 3), [price_data_peak_eur, price_data_offpeak
    plt.subplot(1, 2, i)
    plt.plot(hour_profile.index, hour_profile['CONSUMER_PRICE'], color='C0', lw=
    plt.hlines(hour_profile['CONSUMER_PRICE'].mean(), xmin=hour_profile.index.mi
    plt.xlabel('Date', size=10, family='monospace')
    plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
    plt.xticks(size=10, family='monospace')
    plt.yticks(size=10, family='monospace')
    plt.legend(prop={'size': 9, 'family': 'monospace'})
    plt.grid(lw=0.25, color='xkcd:cement');
    plt.gca().set_axisbelow(True)
plt.suptitle(f'Profiles for peak hour #{hour_max_eur} and off-peak hour #{hour_m
plt.tight_layout();
```



For further modeling we will consider only the peak hour. During cross-validation we will consider both peak and off-peak hours.

2.2. Test For Stationarity

Apply augmented Dickey-Fuller test for stationarity of price zone 1 (European)

$\mathcal{H}_0$: non-stationary

$\mathcal{H}_1$: stationary

In [ ]:
```
alpha = 0.05
print(f'Peak hour #{hour_max_eur}:', end='    ')
adf_stats, adf_pval, _, _, _, _ = adfuller(price_data_peak_eur['CONSUMER_PRICE']
print(f'{adf_stats = :.2f}, {adf_pval = :.2f}', end=', ')
print('Stationary') if adf_pval < alpha else print('Non-stationary')
print(f'Off-peak hour #{hour_min_eur}:', end=' ')
adf_stats, adf_pval, _, _, _, _ = adfuller(price_data_offpeak_eur['CONSUMER_PRIC
print(f'{adf_stats = :.2f}, {adf_pval = :.2f}', end=', ')
print('Stationary') if adf_pval < alpha else print('Non-stationary')
```

```
Peak hour #14:    adf_stats = -2.56, adf_pval = 0.10, Non-stationary
Off-peak hour #3: adf_stats = -2.71, adf_pval = 0.07, Non-stationary
```

2.3. Modeling

The mean of y_t is modeled as 0 in the original model, which means that we have to either centralize the data to make it oscillate around 0 or change this parameter to reflect our data's real mean, which yields the following final SV Baseline model:

$$y_t \sim \mathcal{N}(\bar{y}, e^{\frac{h_t}{2}}).$$

We have created a custom wrapper class inheriting from the scikit-learn's `BaseEstimator` and `RegressorMixin` classes to work with our Stan models using the scikit-learn's interfaces: `fit()`, `predict()`, `score()`, `cross_validate()` etc.

In [ ]:
```
# SV Baseline model
with open('./models/sv_base_fit.stan', 'r') as fh:
  sv_base_code_fit = fh.read()
with open('./models/sv_base_predict.stan', 'r') as fh:
  sv_base_code_predict = fh.read()
#print(sv_base_code_fit)
#print(sv_base_code_predict)
num_samples = 1_000
```

```
model_sv_base_peak_eur = StanModel(kind='sv_base',
                                   name='SV Baseline',
                                   stan_code_fit=sv_base_code_fit,
                                   stan_code_predict=sv_base_code_predict,
                                   num_samples=num_samples)
```

In [ ]:
```
%%capture
# Learn parameters
model_sv_base_peak_eur.fit(X=price_data_peak_eur['CONSUMER_PRICE'], y=price_data
```

In [ ]:
```
model_sv_base_peak_eur
```

Out[ ]:

StanModel

```
StanModel(kind='sv_base', name='SV Baseline',
          stan_code_fit='// SV Baseline model\n'
                        '// Volatility: stochastic\n'
                        '\n'
                        'data\n'
                        '{\n'
                        '  int<lower=1> N; // Number of train time p
oints '
                        '(equally spaced)\n'
```

In [ ]:
```
fit_sv_base_peak_eur_df = model_sv_base_peak_eur.fit_result_df_
fit_sv_base_peak_eur_df
```

Out[ ]:

parameters	lp__	accept_stat__	stepsize__	treedepth__	n_leapfrog__	divergent_
draws						
0	-2048.987072	0.998808	0.162794	5.0	31.0	0
1	-2051.645520	0.785068	0.162794	5.0	31.0	0
2	-2078.617586	0.936711	0.162794	5.0	31.0	0
3	-2065.823102	0.974228	0.162794	5.0	31.0	0
4	-2078.911922	0.623916	0.162794	5.0	31.0	0
...	...	...	...	...	...	
995	-2094.658510	0.841583	0.162794	5.0	31.0	0
996	-2060.396429	0.995562	0.162794	5.0	31.0	0
997	-2056.457131	0.962217	0.162794	5.0	31.0	0
998	-2087.629346	0.661779	0.162794	5.0	31.0	0
999	-2086.687795	0.987314	0.162794	5.0	31.0	0

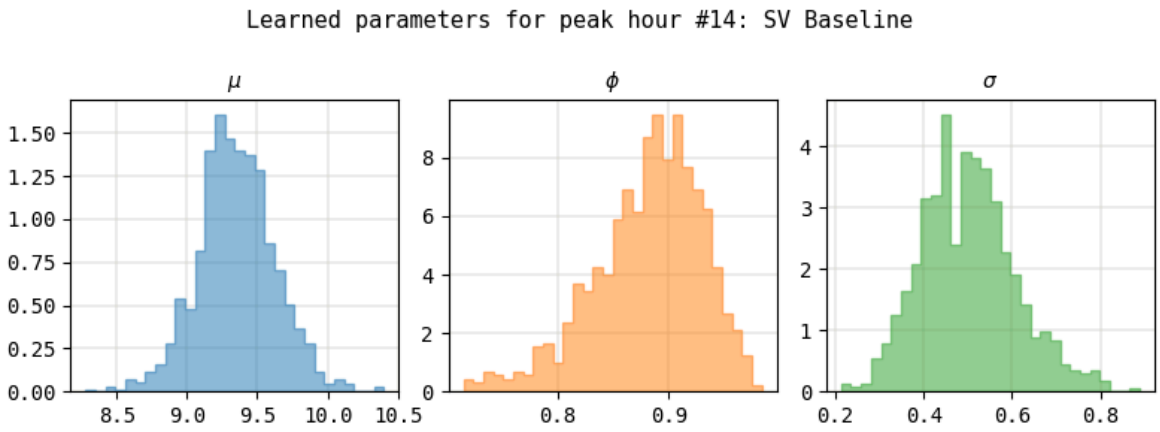
1000 rows × 742 columns

In [ ]:
```
# Learned parameters
fit_sv_base_peak_eur_df[['mu', 'phi', 'sigma']].describe()
```

Out[ ]:

parameters	mu	phi	sigma
count	1000.000000	1000.000000	1000.000000
mean	9.348617	0.881165	0.499893
std	0.275783	0.048779	0.107231
min	8.276047	0.715244	0.212606
25%	9.173458	0.853809	0.423141
50%	9.339528	0.888114	0.496753
75%	9.520876	0.916686	0.563613
max	10.401829	0.983804	0.889206

In [ ]:
```
plt.figure(figsize=(8, 3))
for i, param in zip(range(1, 4), fit_sv_base_peak_eur_df[['mu', 'phi', 'sigma']]
    plt.subplot(1, 3, i)
    plt.hist(fit_sv_base_peak_eur_df[param], bins=30, density=True, histtype='st
    plt.xticks(size=10, family='monospace')
    plt.yticks(size=10, family='monospace')
    if param == 'y_mean':
        plt.title('$\\bar{y}$', size=10, family='monospace')
    else:
        plt.title(f'$\{param}$', size=10, family='monospace')
    plt.grid(lw=0.25, color='xkcd:cement')
    plt.gca().set_axisbelow(True)
plt.suptitle(f'Learned parameters for peak hour #{hour_max_eur}: {model_sv_base_
plt.tight_layout();
```

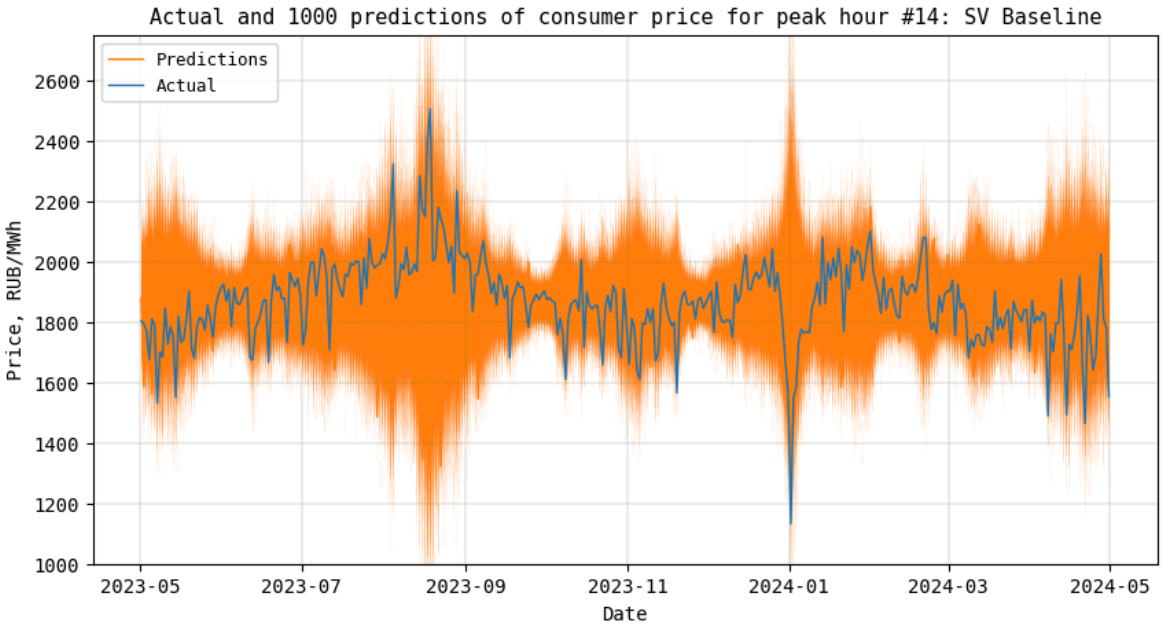


In [ ]:
```
%%capture
# Predict
predict_sv_base_peak_eur_mean = model_sv_base_peak_eur.predict(price_data_peak_e
predict_sv_base_peak_eur_mean.shape
```

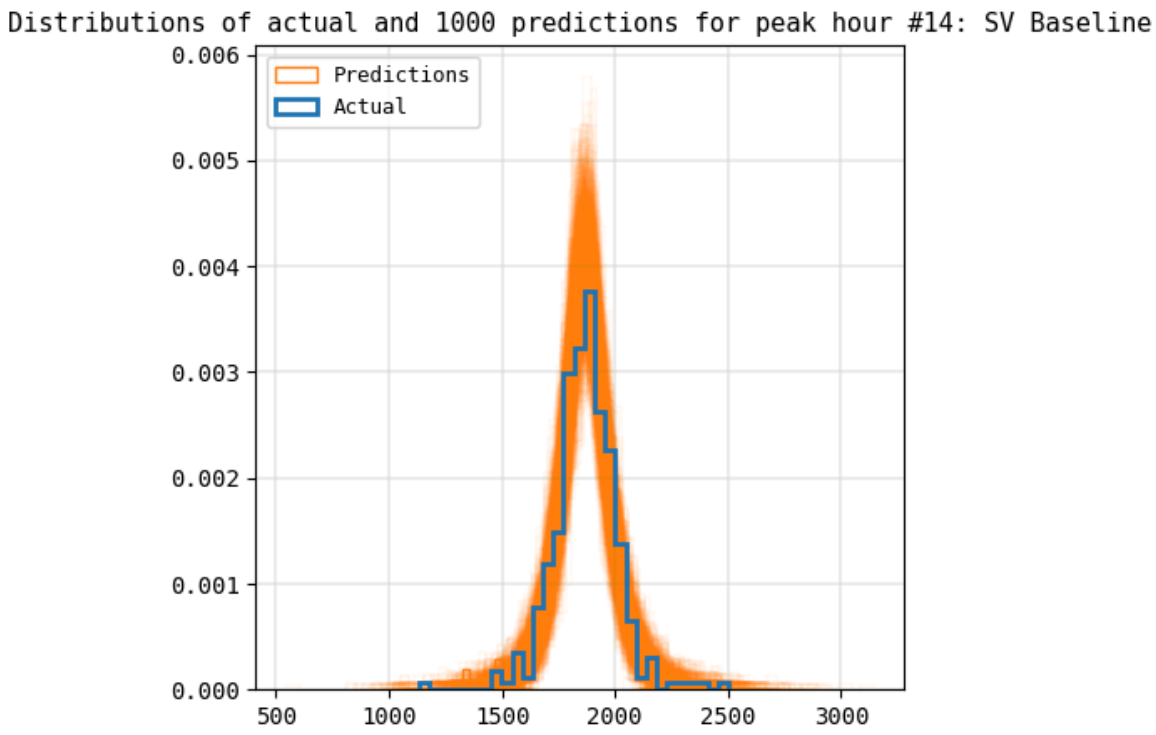
In [ ]:
```
%%capture
predict_sv_base_peak_eur_many = model_sv_base_peak_eur.predict_many(price_data_p
predict_sv_base_peak_eur_many.shape
```

In [ ]:
```
%%capture
predict_sv_base_peak_eur_ci = model_sv_base_peak_eur.predict_ci(price_data_peak_
len(predict_sv_base_peak_eur_ci)
```

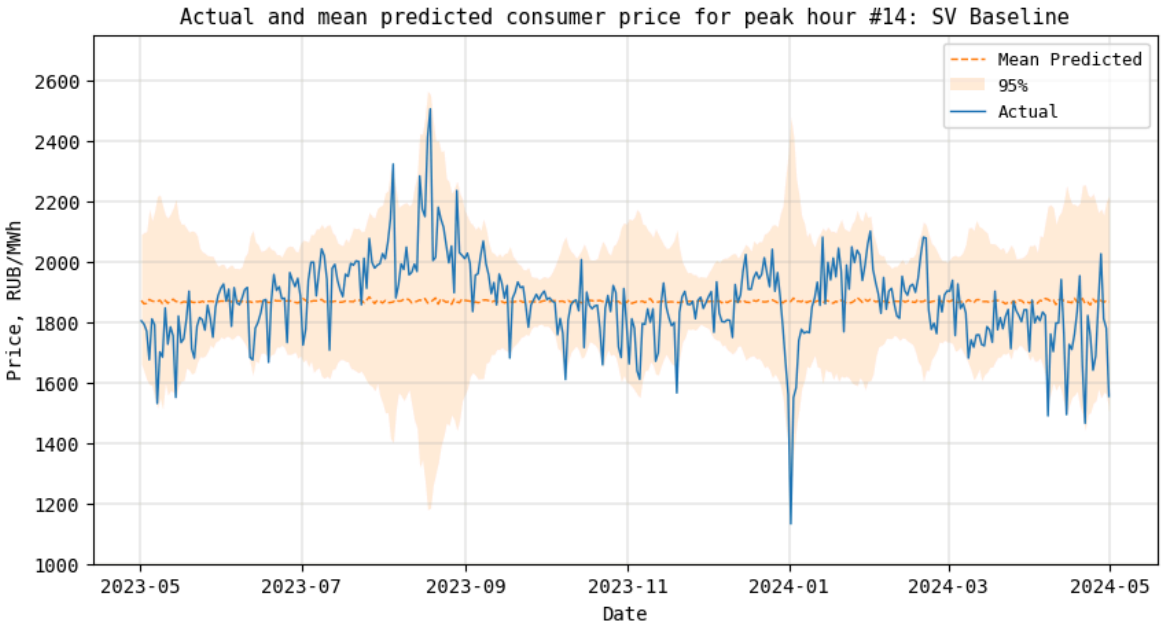
In [ ]:
```
plt.figure(figsize=(10, 5))
plt.plot(price_data_peak_eur.index, predict_sv_base_peak_eur_many.iloc[0], color
for i in range(1, predict_sv_base_peak_eur_many.shape[0]):
    plt.plot(price_data_peak_eur.index, predict_sv_base_peak_eur_many.iloc[i], c
plt.plot(price_data_peak_eur.index, price_data_peak_eur['CONSUMER_PRICE'], color
plt.xlabel('Date', size=10, family='monospace')
plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.ylim([1000, 2750])
plt.title(f'Actual and {num_samples} predictions of consumer price for peak hour
plt.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement');
```



In [ ]:
```
plt.figure(figsize=(5, 5))
plt.hist(predict_sv_base_peak_eur_many.iloc[0], bins=30, density=True, histtype=
for i in range(1, predict_sv_base_peak_eur_many.shape[0]):
    plt.hist(predict_sv_base_peak_eur_many.iloc[i], bins=30, density=True, histt
plt.hist(price_data_peak_eur['CONSUMER_PRICE'], bins=30, density=True, histtype=
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.grid(lw=0.25, color='xkcd:cement')
plt.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'})
plt.gca().set_axisbelow(True)
plt.title(f'Distributions of actual and {num_samples} predictions for peak hour
```



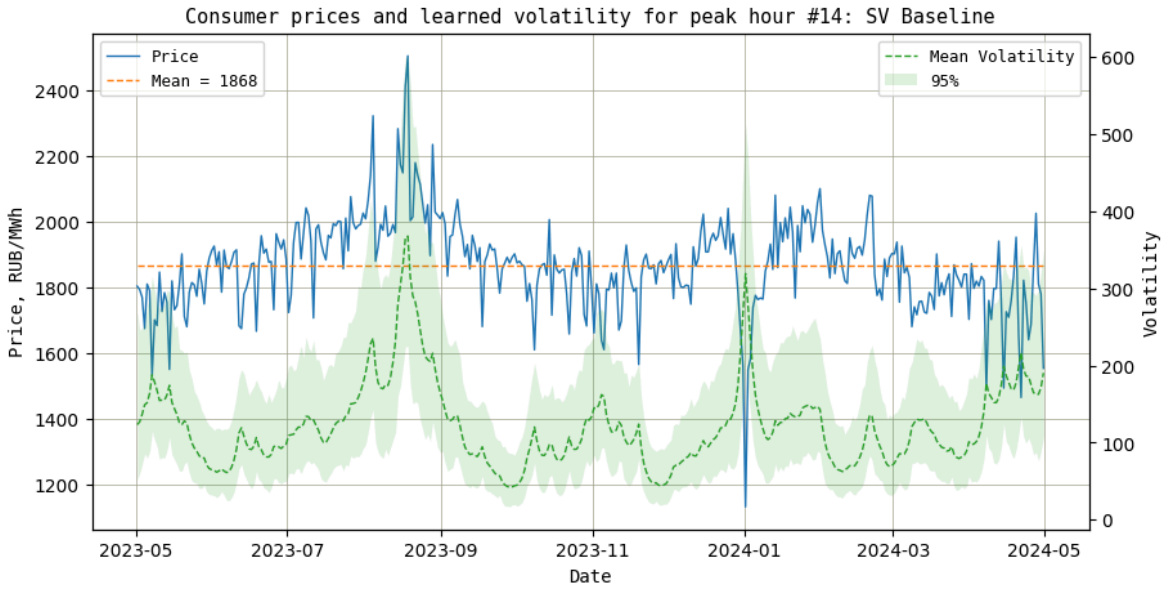
In [ ]:
```
plt.figure(figsize=(10, 5))
plt.plot(price_data_peak_eur.index, predict_sv_base_peak_eur_mean, color='C1', l
plt.fill_between(price_data_peak_eur.index, predict_sv_base_peak_eur_ci[0], pred
plt.plot(price_data_peak_eur.index, price_data_peak_eur['CONSUMER_PRICE'], color
plt.xlabel('Date', size=10, family='monospace')
plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.ylim([1000, 2750])
plt.title(f'Actual and mean predicted consumer price for peak hour #{hour_max_eu
plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'});
plt.grid(lw=0.25, color='xkcd:cement');
```



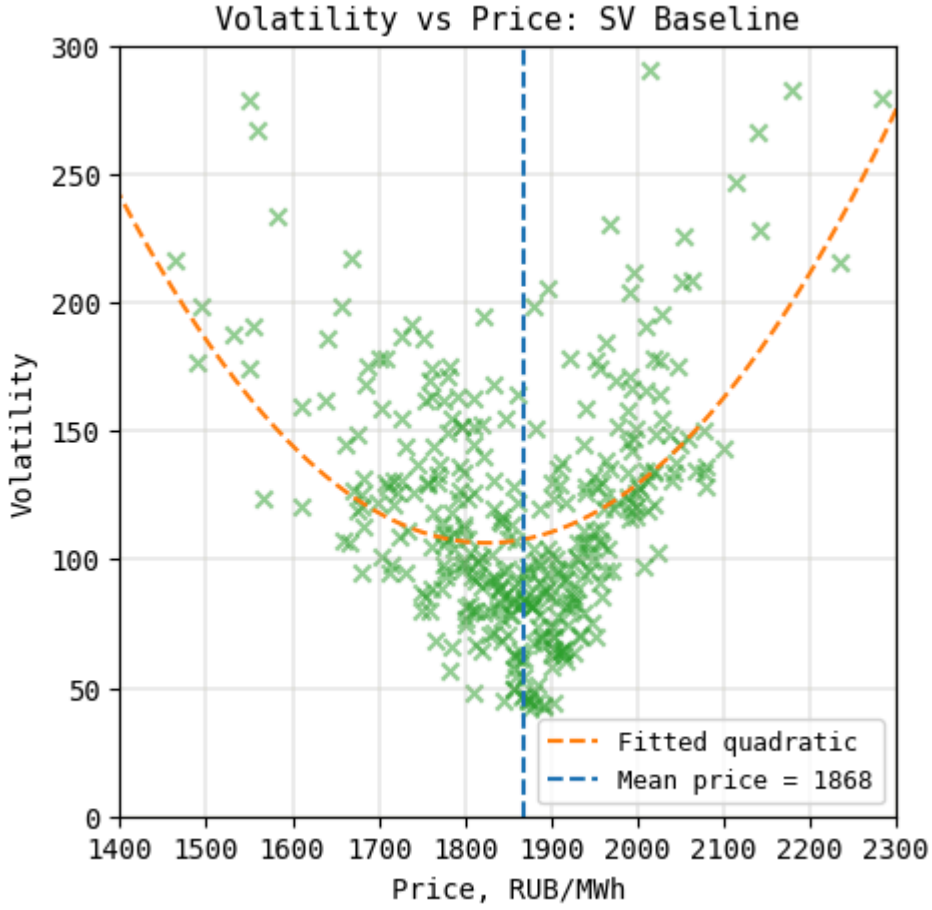
In [ ]:
```
# Volatility at time t over all predictions
vol_sv_base_peak_eur = model_sv_base_peak_eur.get_volatility()
vol_sv_base_peak_eur.shape
```

Out[ ]: (1000, 366)

In [ ]:
```
_, ax = plt.subplots(figsize=(10, 5))
ax.plot(price_data_peak_eur.index, price_data_peak_eur['CONSUMER_PRICE'], color=
ax.hlines(price_data_peak_eur['CONSUMER_PRICE'].mean(), xmin=price_data_peak_eur
ax.set_xlabel('Date', size=10, family='monospace')
ax.set_ylabel('Price, RUB/MWh', size=10, family='monospace')
ax.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'})
ax.grid(lw=0.5, color='xkcd:cement')
ax2 = ax.twinx()
ax2.plot(price_data_peak_eur.index, vol_sv_base_peak_eur.mean(axis=0), color='C2
ax2.fill_between(price_data_peak_eur.index, np.percentile(vol_sv_base_peak_eur,
ax2.set_ylabel('Volatility', size=10, family='monospace')
ax2.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
ax.set_title(f'Consumer prices and learned volatility for peak hour #{hour_max_e
```



In [ ]:
```
plt.figure(figsize=(5, 5))
fitted_quadratic = np.poly1d(np.polyfit(price_data_peak_eur['CONSUMER_PRICE'], v
x = np.linspace(price_data_peak_eur['CONSUMER_PRICE'].min(), price_data_peak_eur
plt.scatter(price_data_peak_eur['CONSUMER_PRICE'], vol_sv_base_peak_eur.mean(axi
plt.plot(x, fitted_quadratic(x), color='C1', ls='--', label='Fitted quadratic')
plt.vlines(price_data_peak_eur['CONSUMER_PRICE'].mean(), ymin=0, ymax=350, color
plt.xlabel('Price, RUB/MWh', size=10, family='monospace')
plt.ylabel('Volatility', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.xlim([1400, 2300])
plt.ylim([0, 300])
plt.title(f'Volatility vs Price: {model_sv_base_peak_eur.name}', size=11, family
plt.legend(loc='lower right', prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement');
```



In [ ]:
```
alpha = 0.05
adf_stats, adf_pval, _, _, _, _ = adfuller(vol_sv_base_peak_eur.mean(axis=0))
print(f'{adf_stats = :.2f}, {adf_pval = :.2f}', end=', ')
print('Stationary') if adf_pval < alpha else print('Non-stationary')
```

```
adf_stats = -2.83, adf_pval = 0.05, Non-stationary
```

We can clearly see that volatility *does* depend on consumer price, and our **SV Baseline** model discovered this relation.

We can see a rather V-shaped dependency: the volatility tends to increase for the prices higher and lower than the mean price, while it's minimal around the mean price.

2.4. Goodness-of-fit

Consider the following metrics for evaluating the quality of the model.

1. Mean Absolute Error: $MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$, where

   n is the number of target points, y_i are the true target values, $\hat{y}_i$ are the model's predictions.

2. Rooted Mean Squared Error: $RMSE = \sqrt{MSE}$, where

   Mean Squared Error: $MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$.

In [ ]:
```
# MAE for mean predictions over all predictions
mae_sv_base = mean_absolute_error(price_data_peak_eur['CONSUMER_PRICE'], predict
mae_sv_base
```

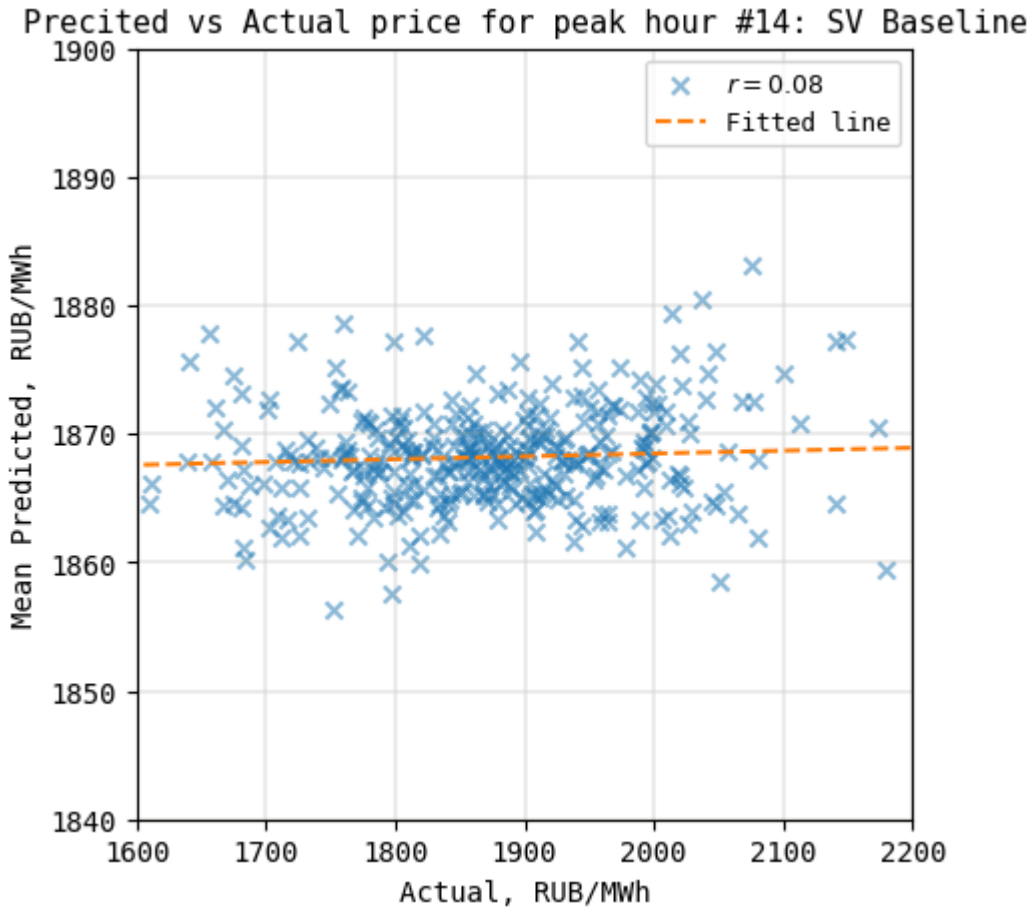
Out[ ]:
```
99.18284666027084
```

In [ ]:
```
# RMSE for mean predictions over all predictions
rmse_sv_base = root_mean_squared_error(price_data_peak_eur['CONSUMER_PRICE'], pr
rmse_sv_base
```

Out[ ]:
```
137.04160239198836
```

Let's check the correlation between mean predicted and actual prices.

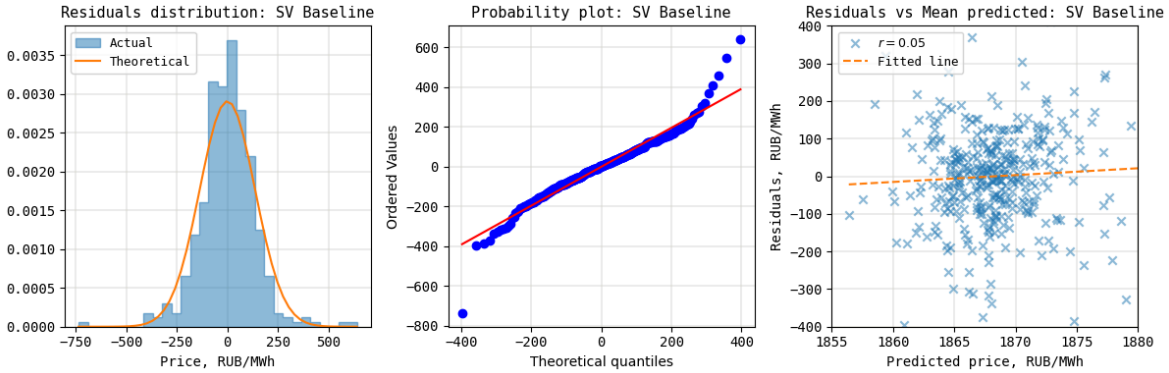
In [ ]:
```
r = np.corrcoef(price_data_peak_eur['CONSUMER_PRICE'], predict_sv_base_peak_eur_
fitted_line = np.poly1d(np.polyfit(price_data_peak_eur['CONSUMER_PRICE'], predic
x = np.linspace(price_data_peak_eur['CONSUMER_PRICE'].min(), price_data_peak_eur
plt.figure(figsize=(5, 5))
plt.scatter(price_data_peak_eur['CONSUMER_PRICE'], predict_sv_base_peak_eur_mean
plt.plot(x, fitted_line(x), color='C1', ls='--', label='Fitted line')
plt.xlabel('Actual, RUB/MWh', size=10, family='monospace')
plt.ylabel('Mean Predicted, RUB/MWh', size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.xlim([1600, 2200])
plt.ylim([1840, 1900])
plt.title(f'Precited vs Actual price for peak hour #{hour_max_eur}: {model_sv_ba
plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement');
```



Though the model was able to learn heteroscedasity of the volatility, the correlation between the mean predictions and actual prices is rather weak.

We should also check the distribution of residuals, probability plot (similar to Q-Q plot), and residuals vs mean predicted prices.

In [ ]:
```
residuals_sv_base = price_data_peak_eur['CONSUMER_PRICE'] - predict_sv_base_peak
x = np.linspace(residuals_sv_base.min(), residuals_sv_base.max())
residuals_sv_base_theor = norm(residuals_sv_base.mean(), residuals_sv_base.std(d
plt.figure(figsize=(12, 4))
plt.subplot(1, 3, 1)
plt.hist(residuals_sv_base, bins=30, density=True, histtype='stepfilled', color=
plt.plot(x, residuals_sv_base_theor.pdf(x), color='C1', label='Theoretical')
plt.xlabel('Price, RUB/MWh', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.title(f'Residuals distribution: {model_sv_base_peak_eur.name}', size=11, fam
plt.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement')
plt.gca().set_axisbelow(True)
plt.subplot(1, 3, 2)
probplot(residuals_sv_base, dist=residuals_sv_base_theor, plot=plt)
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.title(f'Probability plot: {model_sv_base_peak_eur.name}', size=11, family='m
plt.grid(lw=0.25, color='xkcd:cement')
plt.gca().set_axisbelow(True)
plt.subplot(1, 3, 3)
r = np.corrcoef(residuals_sv_base, predict_sv_base_peak_eur_mean)[0, 1]
fitted_line = np.poly1d(np.polyfit(predict_sv_base_peak_eur_mean, residuals_sv_b
x = np.linspace(predict_sv_base_peak_eur_mean.min(), predict_sv_base_peak_eur_me
plt.scatter(predict_sv_base_peak_eur_mean, residuals_sv_base, color='C0', marker
plt.plot(x, fitted_line(x), color='C1', ls='--', label='Fitted line')
plt.xlabel('Predicted price, RUB/MWh', size=10, family='monospace')
plt.ylabel('Residuals, RUB/MWh', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.xlim([1855, 1880])
plt.ylim([-400, 400])
plt.title(f'Residuals vs Mean predicted: {model_sv_base_peak_eur.name}', size=11
plt.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement')
plt.gca().set_axisbelow(True)
plt.tight_layout();
```



Residuals look to be distributed more or less normally. Also they look rather homoscedastic w.r.t predictions, which means that the model is rather acceptable.

Back

---

3. SV Exogenous Model

Next, we will consider an extension of the SV Baseline model. The idea of adding exogenous regressor(s) is inspired by the paper "Probabilistic electricity price forecasting with Bayesian stochastic volatility models" by Maciej Kostrzewski, Jadwiga Kostrzewska, 2019.

The authors propose a stochastic volatility model with a double exponential distribution of jumps, a leverage effect and exogenous variables (in short, the SVDEJX model). This model introduces one exogenous factor -- the logarithm of the hourly air temperature at time t. The model also introduces indicator variables for 3 days of the week: Sat, Sun, Mon.

First, we'll examine the first possible exogenous regressor -- air temperature.

3.1. Temperature Data

In [ ]:
```
# Price zone 1: Moscow
temp_data_eur = pd.read_csv(f'./data/UUEE.01.05.2023.07.05.2024.1.0.0.ru.utf8.00
                            sep=';',
                            encoding='utf-8',
                            skiprows=7,
                            index_col=0,
                            dayfirst=True,
                            usecols=[0, 1],
                            names=['Datetime', 'Temperature'],
                            parse_dates=True).dropna().sort_index()
```

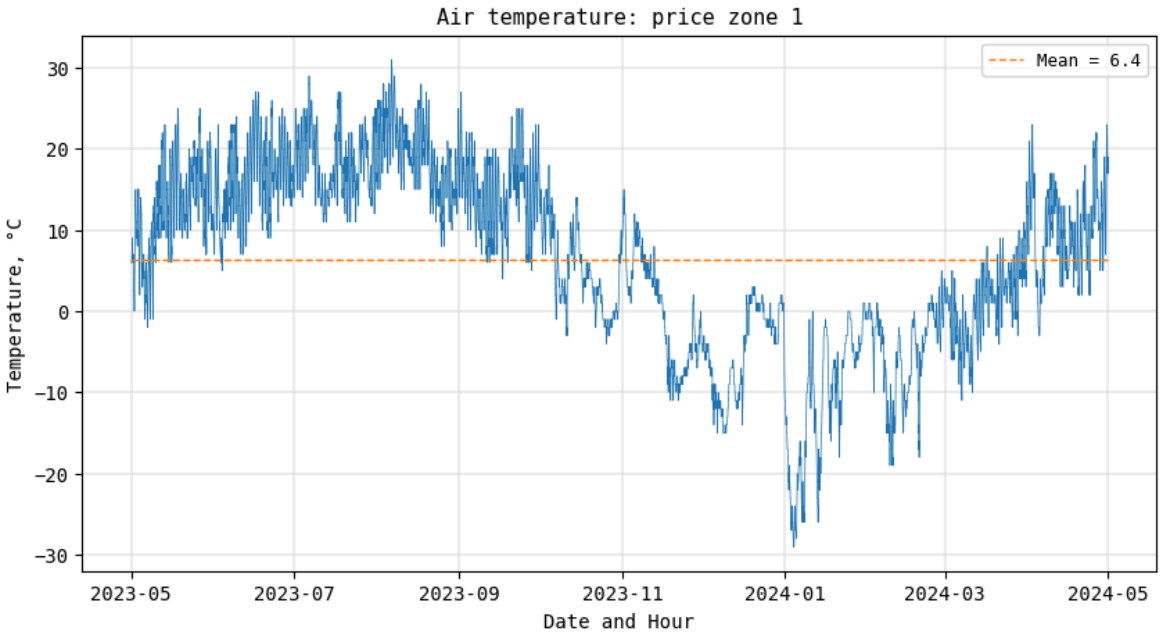
In [ ]:
```
temp_data_eur = temp_data_eur.loc[:'30.04.2024'] # 01.05.2024 -- 07.05.2024 is u
temp_data_eur
```

Out[ ]:

Datetime	Temperature
2023-05-01 00:00:00	6.0
2023-05-01 00:30:00	6.0
2023-05-01 01:00:00	6.0
2023-05-01 01:30:00	6.0
2023-05-01 02:00:00	6.0
...	...
2024-04-30 21:30:00	18.0
2024-04-30 22:00:00	19.0
2024-04-30 22:30:00	19.0
2024-04-30 23:00:00	18.0
2024-04-30 23:30:00	17.0

17519 rows × 1 columns

In [ ]:
```
plt.figure(figsize=(10, 5))
plt.plot(temp_data_eur.index, temp_data_eur['Temperature'], color='C0', lw=0.5)
plt.hlines(temp_data_eur['Temperature'].mean(), xmin=temp_data_eur.index.min(),
plt.xlabel('Date and Hour', size=10, family='monospace')
plt.ylabel('Temperature, °C', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.title(f'Air temperature: price zone {price_zone}', size=11, family='monospac
plt.legend(prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement');
```



In [ ]:
```
# Join consumer price and air temperature data
price_temp_data_eur = pd.merge(price_data_eur, temp_data_eur, on='Datetime')
```

```
price_temp_data_eur.shape
```

Out[ ]: (8760, 5)

In [ ]:
```
price_temp_data_eur.head(4)
```

Out[ ]:

Datetime	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday	Temperature
2023-05-01 00:00:00	1	1517.92	0	0	6.0
2023-05-01 01:00:00	1	1413.99	1	0	6.0
2023-05-01 02:00:00	1	1345.22	2	0	6.0
2023-05-01 03:00:00	1	1275.81	3	0	6.0

In [ ]:
```
price_temp_data_eur.tail(4)
```

Out[ ]:

Datetime	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday	Temperature
2024-04-30 20:00:00	1	1822.91	20	1	18.0
2024-04-30 21:00:00	1	1742.75	21	1	17.0
2024-04-30 22:00:00	1	1442.53	22	1	19.0
2024-04-30 23:00:00	1	1327.94	23	1	18.0

In [ ]:
```
price_temp_data_peak_eur = price_temp_data_eur[price_temp_data_eur['HOUR'] == ho
price_temp_data_peak_eur
```

Out[ ]:

Datetime	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday	Temperature
2023-05-01 14:00:00	1	1803.61	14	0	7.0
2023-05-02 14:00:00	1	1793.39	14	1	14.0
2023-05-03 14:00:00	1	1770.27	14	2	12.0
2023-05-04 14:00:00	1	1674.88	14	3	13.0
2023-05-05 14:00:00	1	1810.13	14	4	4.0
...	...	...	...	...	...
2024-04-26 14:00:00	1	1862.04	14	4	22.0
2024-04-27 14:00:00	1	2025.49	14	5	10.0
2024-04-28 14:00:00	1	1810.51	14	6	15.0
2024-04-29 14:00:00	1	1779.80	14	0	18.0
2024-04-30 14:00:00	1	1553.93	14	1	22.0

366 rows × 5 columns

In [ ]:
```
price_temp_data_offpeak_eur = price_temp_data_eur[price_temp_data_eur['HOUR'] ==
price_temp_data_offpeak_eur
```

Out[ ]:

Datetime	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday	Temperature
2023-05-01 03:00:00	1	1275.81	3	0	6.0
2023-05-02 03:00:00	1	1113.04	3	1	2.0
2023-05-03 03:00:00	1	1085.05	3	2	9.0
2023-05-04 03:00:00	1	1003.91	3	3	4.0
2023-05-05 03:00:00	1	1121.00	3	4	5.0
...	...	...	...	...	...
2024-04-26 03:00:00	1	944.52	3	4	11.0
2024-04-27 03:00:00	1	1147.59	3	5	15.0
2024-04-28 03:00:00	1	1157.11	3	6	6.0
2024-04-29 03:00:00	1	1197.78	3	0	7.0
2024-04-30 03:00:00	1	1191.04	3	1	8.0

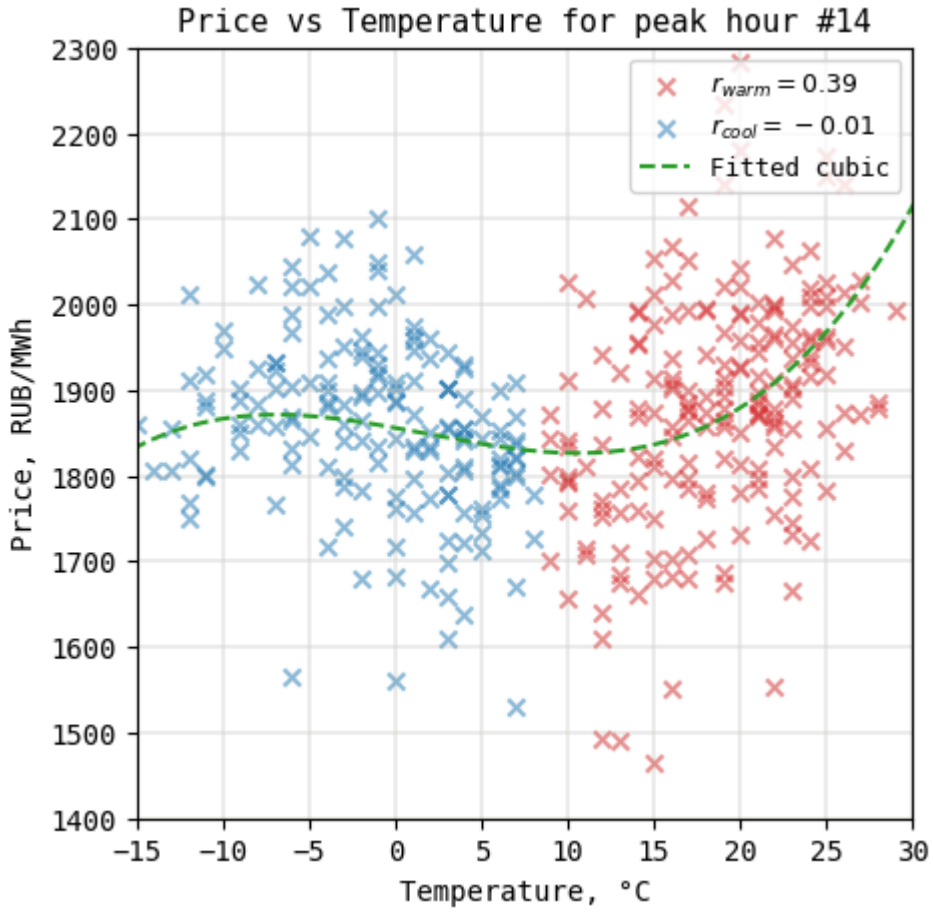
366 rows × 5 columns

Let's check the correlation between the Price and Temperature. We are expecting a rather V-shaped dependency, since both high and low air temperatures should increase electricity demand for cooling and heating and thus the price.

In [ ]:
```
price_temp_data_peak_eur.loc[:, 'Cluster'] = KMeans(n_clusters=2, random_state=0
r_warm = price_temp_data_peak_eur[price_temp_data_peak_eur['Cluster'] == price_t
r_cool = price_temp_data_peak_eur[price_temp_data_peak_eur['Cluster'] == price_t
fitted_cubic = np.poly1d(np.polyfit(price_temp_data_peak_eur['Temperature'], pri
x = np.linspace(price_temp_data_peak_eur['Temperature'].min(), price_temp_data_p
plt.figure(figsize=(5, 5))
plt.scatter(price_temp_data_peak_eur[price_temp_data_peak_eur['Cluster'] == pric
plt.scatter(price_temp_data_peak_eur[price_temp_data_peak_eur['Cluster'] == pric
plt.plot(x, fitted_cubic(x), color='C2', ls='--', label='Fitted cubic')
plt.xlabel('Temperature, °C', size=10, family='monospace')
plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.xlim([-15, 30])
plt.ylim([1400, 2300])
plt.title(f'Price vs Temperature for peak hour #{hour_max_eur}', size=11, family
plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement');
```

```
/tmp/ipykernel_10106/4233981735.py:1: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  price_temp_data_peak_eur.loc[:, 'Cluster'] = KMeans(n_clusters=2, random_state=0).fit_predict(price_temp_data_peak_eur[['Temperature']])
```



We can confirm that the price *does* depend on the air temperature:

- coefficient of correlation (Pearson) for high air temperatures is 0.39,
- coefficient of correlation (Pearson) for low air temperatures is -0.01.

The price tends to be higher for higher and lower air temperatures, which looks reasonable: electricity demand is higher during the hot and cold season of the year (both cooling and heating are required), while the demand is minimal during semi-seasons when minimal heating and cooling are required. At the same time, the price responds more to the higher air temperatures. This might be due to the fact that in the studied scenario electricity is used more for cooling than for heating.

Next, let's examine the second possible exogenous regressor -- the day of week.

3.2. Weekday

In [ ]:
```
price_temp_data_peak_eur['day_name'] = price_temp_data_peak_eur.index.day_name()
price_temp_data_peak_eur
```

```
/tmp/ipykernel_10106/884510011.py:1: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  price_temp_data_peak_eur['day_name'] = price_temp_data_peak_eur.index.day_name()
```

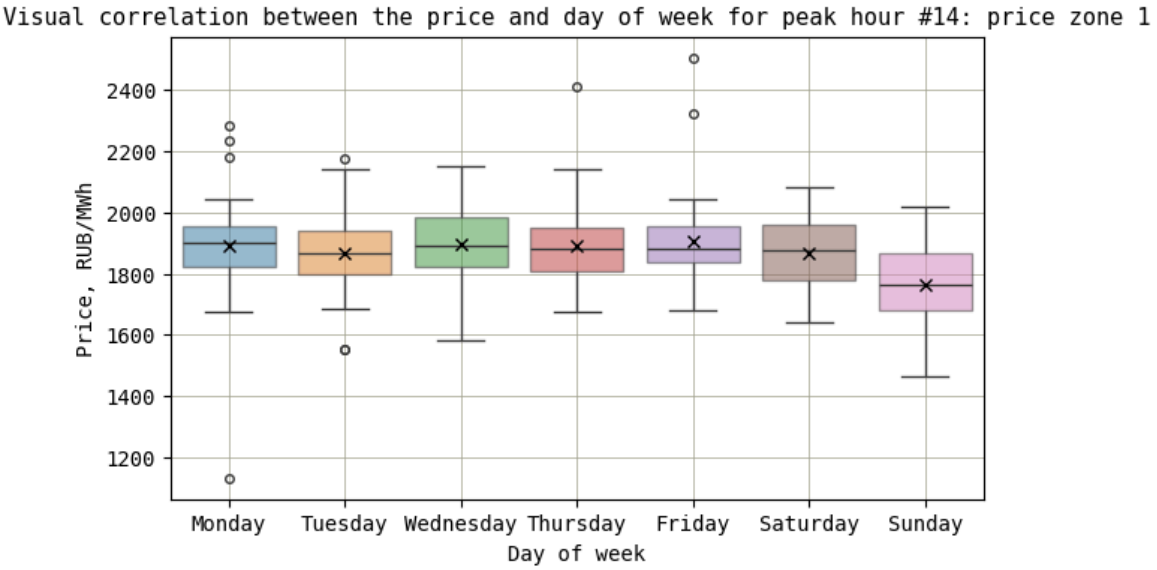
Out[ ]:

Datetime	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday	Temperature	Cluste
2023-05-01 14:00:00	1	1803.61	14	0	7.0	
2023-05-02 14:00:00	1	1793.39	14	1	14.0	
2023-05-03 14:00:00	1	1770.27	14	2	12.0	
2023-05-04 14:00:00	1	1674.88	14	3	13.0	
2023-05-05 14:00:00	1	1810.13	14	4	4.0	
...	...	...	...	...	...	
2024-04-26 14:00:00	1	1862.04	14	4	22.0	
2024-04-27 14:00:00	1	2025.49	14	5	10.0	
2024-04-28 14:00:00	1	1810.51	14	6	15.0	
2024-04-29 14:00:00	1	1779.80	14	0	18.0	
2024-04-30 14:00:00	1	1553.93	14	1	22.0	

366 rows × 7 columns

In [ ]:
```
plt.figure(figsize=(7, 4))
sns.boxplot(data=price_temp_data_peak_eur[['CONSUMER_PRICE', 'Weekday', 'day_nam
plt.xlabel('Day of week', size=10, family='monospace')
plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
```

```
plt.yticks(size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.grid(lw=0.5, color='xkcd:cement')
plt.title(f'Visual correlation between the price and day of week for peak hour #
```



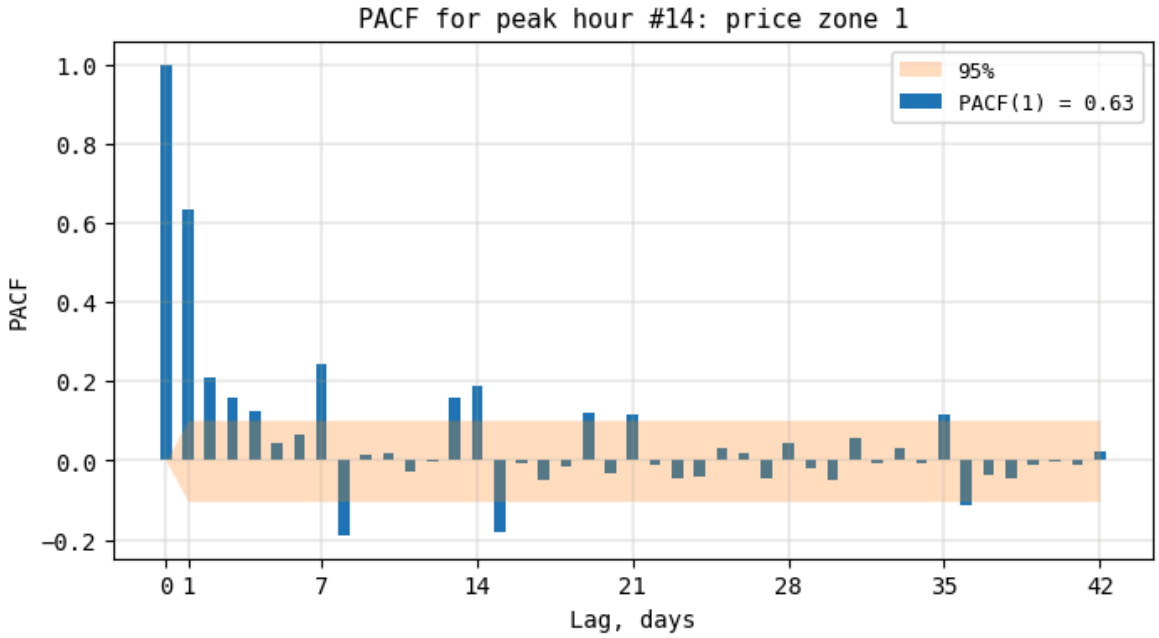
Though the mean values look equal, at least on Sundays the price tends to be lower. Let's introduce the second new regressor -- day of week.

3.3. Autoregressive Component

Kostrzewski et al. extend their model with the minimum of the previous day's 24 hourly log prices [p. 615]. Based om this idea, we will check the correlation between the prices at time t (today) and $t-1$ (yesterday). For that, we'll compute the partial autocorrelation function which is the correlation between two observations that the shorter lags between those observations do not explain, i.e. the partial correlation for each lag is the unique correlation between those two observations after removing out the intervening correlations.

In [ ]:
```
# PACF for Peak hour + Price zone 1 with CI at 5% significance level for 42 day
pacf_corr_price_temp_data_peak_eur, pacf_ci_price_temp_data_peak_eur = pacf(pric
```

In [ ]:
```
plt.figure(figsize=(8, 4))
plt.bar(x=range(0, 43), height=pacf_corr_price_temp_data_peak_eur, width=0.5, al
plt.fill_between(range(0, 43), pacf_corr_price_temp_data_peak_eur - pacf_ci_pric
plt.xlabel('Lag, days', size=10, family='monospace')
plt.ylabel('PACF', size=10, family='monospace')
plt.xticks([0, 1, 7, 14, 21, 28, 35, 42], size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.title(f'PACF for peak hour #{hour_max_eur}: price zone 1', size=11, family='
plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement');
```



Since our price is not stationary [2.2], the PACF decays rather slowly during the period of 42 days (6 weeks). We have shown above [3.2] that the price *is* correlated with the day of week. Observe that the PACF has spikes exactly every 7 days (1 week) which further supports the correctness of inclusion of the day of week as an exogenous regressor.

PACF suggests that using at least 7 or more lags (new features) might be possible. We already decided to include the day of week to explain the seasonality of the price. The strongest correlation is seen for the lag of one (0.63), while other lags are much weaker correlated. Taking into account computational compexity of inference with Stan, using only the lag of one day looks to be a reasonable compromise. In other words, we will consider an autoregressive model with lag of one AR(1) as the autoregressive component.

A first-order autoregressive model AR(1) is the following:

$y_t = \mathcal{N}(\alpha + \beta y_{t-1}, \sigma)$, where

α and β are the intercept and slope of autoregression and $\sigma \sim \mathcal{N}(0, 1)$ (constant normal volatility). Since we model volatility as a stochastic process (not simply constant), we will use only the expected value of this model.

We have shown that the consumer price is correlated with both the air temperature [3.1], day of week [3.2], and with itself with at least lag of one day [3.3]. Let's propose a new model **SV X**, which extends the ideas of our regression-like SV Baseline model and [Kostrzewski] with three exogenous regressors -- air temperature, day of week, and autoregressive component:

$y_t \sim \mathcal{N}(\bar{y} + \alpha y_{t-1} + \beta_3 X_{t-1}^3 + \beta_2 X_{t-1}^2 + \beta_1 X_{t-1} + \gamma D_t + \xi, e^{\frac{h_t}{2}})$, where

X_{t-1} is the hourly air temperature at time $t-1$. To prevent target leakage, the temperature readings are lagged one day behind $(t-1)$;

D_t is the day of week at time t;

$$h_t \sim \mathcal{N}(\mu + \phi(h_{t-1} - \mu), \sigma);$$

$$h_1 \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{1-\phi^2}}\right).$$

Thus, we are introducing:

- 1 new parameter: α for the autoregressive component;
- 3 new parameters: $\beta_{i=1...3}$ of the air temperature regressor. The unit of air temperature is °C;
- 1 new parameter: γ for the day of week regressor. The weekdays are numbered from 0 (Monday) to 6 (Sunday);
- 1 new parameter: ξ for the constant term (intercept) for all exogenous regressors.

3.4. Modeling

In [ ]:
```
# SV Exogenous model
with open('./models/sv_x_fit.stan', 'r') as fh:
  sv_x_code_fit = fh.read()
with open('./models/sv_x_predict.stan', 'r') as fh:
  sv_x_code_predict = fh.read()
#print(sv_x_code_fit)
#print(sv_x_code_predict)
num_samples = 1_000
model_sv_x_peak_eur = StanModel(kind='sv_x',
                                name='SV X',
                                stan_code_fit=sv_x_code_fit,
                                stan_code_predict=sv_x_code_predict,
                                num_samples=num_samples)
```

In [ ]:
```
%%capture
# Learn parameters
model_sv_x_peak_eur.fit(X=price_temp_data_peak_eur[['Temperature', 'Weekday']],
```

In [ ]:
```
model_sv_x_peak_eur
```

Out[ ]:
```
StanModel
StanModel(kind='sv_x', name='SV X',
          stan_code_fit='// SV Exogenous model\n'
                        '// Volatility: stochastic\n'
                        '// Exogenous regressors\n'
                        '\n'
                        'data\n'
                        '{\n'
                        '  int<lower=1> N;        // Number of train
time '
```

In [ ]:
```
fit_sv_x_peak_eur_df = model_sv_x_peak_eur.fit_result_df_
fit_sv_x_peak_eur_df
```

Out[ ]:

parameters	lp__	accept_stat__	stepsize__	treedepth__	n_leapfrog__	divergent
draws						
0	-1958.858614	0.968849	0.026799	7.0	127.0	0
1	-1981.437741	0.962916	0.026799	7.0	127.0	0
2	-1991.197662	0.997666	0.026799	7.0	127.0	0
3	-1948.205885	0.928831	0.026799	7.0	127.0	0
4	-1963.159089	0.986114	0.026799	7.0	127.0	0
...	...	...	...	...	...	
995	-1941.548370	1.000000	0.026799	7.0	127.0	0
996	-1920.339048	0.993652	0.026799	7.0	127.0	0
997	-1945.402821	0.991550	0.026799	7.0	127.0	0
998	-1954.025128	0.803253	0.026799	7.0	127.0	0
999	-1944.601411	0.936501	0.026799	7.0	127.0	0

1000 rows × 748 columns

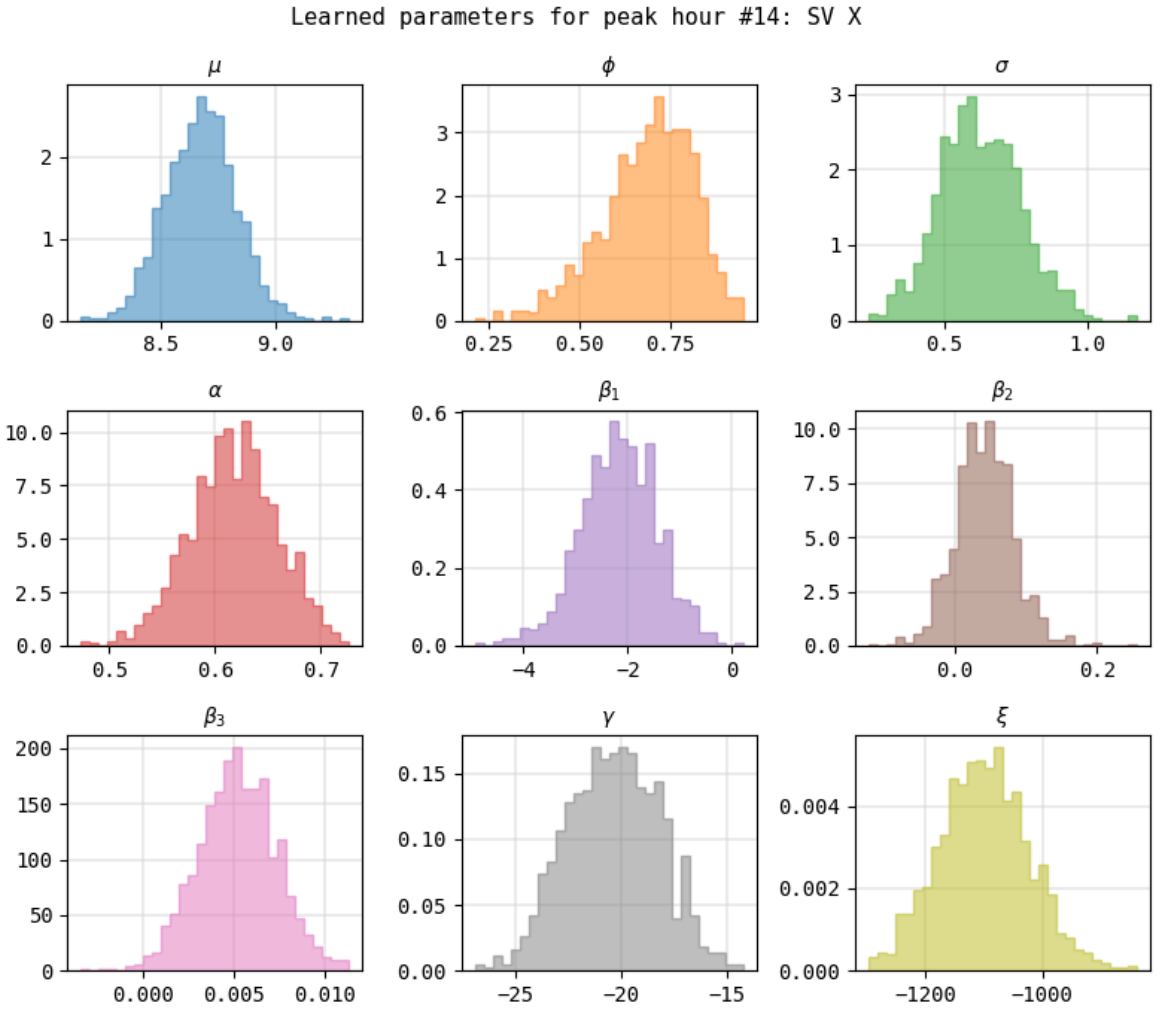
In [ ]:
```
# Learned parameters
fit_sv_x_peak_eur_df[['mu', 'phi', 'sigma', 'alpha', 'beta_1', 'beta_2', 'beta_3
```

Out[ ]:

parameters	mu	phi	sigma	alpha	beta_1	beta
count	1000.000000	1000.000000	1000.000000	1000.000000	1000.000000	1000.0000
mean	8.674955	0.685212	0.625996	0.617247	-2.181777	0.0413
std	0.154522	0.124779	0.142119	0.041495	0.737174	0.0412
min	8.151622	0.212766	0.237326	0.472789	-4.939281	-0.1208
25%	8.573767	0.608953	0.524341	0.589491	-2.652985	0.0151
50%	8.678518	0.700238	0.618743	0.617323	-2.171834	0.0410
75%	8.774036	0.778162	0.721254	0.644932	-1.665593	0.0678
max	9.319936	0.949155	1.174375	0.726671	0.212885	0.2557

In [ ]:
```
plt.figure(figsize=(8, 7))
ncols = 3
nrows = int(np.ceil(9 / ncols))
for i, param in zip(range(1, 10), fit_sv_x_peak_eur_df[['mu', 'phi', 'sigma', 'a
    plt.subplot(nrows, ncols, i)
    plt.hist(fit_sv_x_peak_eur_df[param], bins=30, density=True, histtype='stepf
    plt.xticks(size=10, family='monospace')
    plt.yticks(size=10, family='monospace')
    if param == 'y_mean':
        plt.title('$\\bar{y}$', size=10, family='monospace')
```

```
    else:
        plt.title(f'$\{param}$', size=10, family='monospace')
    plt.grid(lw=0.25, color='xkcd:cement')
    plt.gca().set_axisbelow(True)
plt.suptitle(f'Learned parameters for peak hour #{hour_max_eur}: {model_sv_x_pea
plt.tight_layout();
```



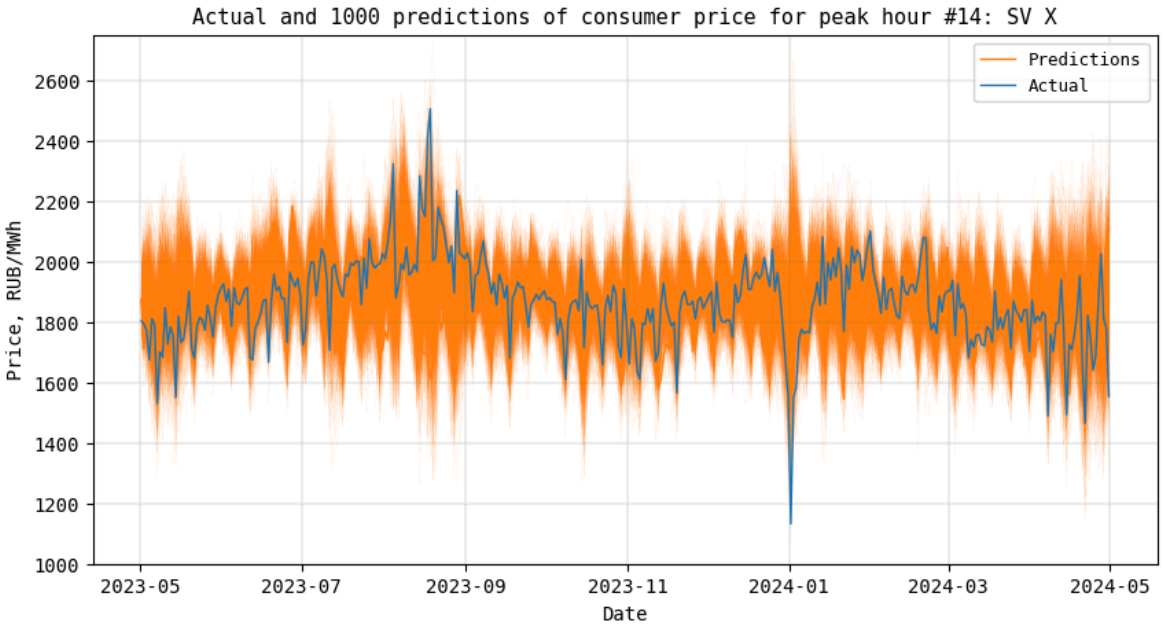
In [ ]:
```
%%capture
# Predict
predict_sv_x_peak_eur_mean = model_sv_x_peak_eur.predict(price_temp_data_peak_eu
predict_sv_x_peak_eur_mean.shape
```

In [ ]:
```
%%capture
predict_sv_x_peak_eur_many = model_sv_x_peak_eur.predict_many(price_temp_data_pe
predict_sv_x_peak_eur_many.shape
```

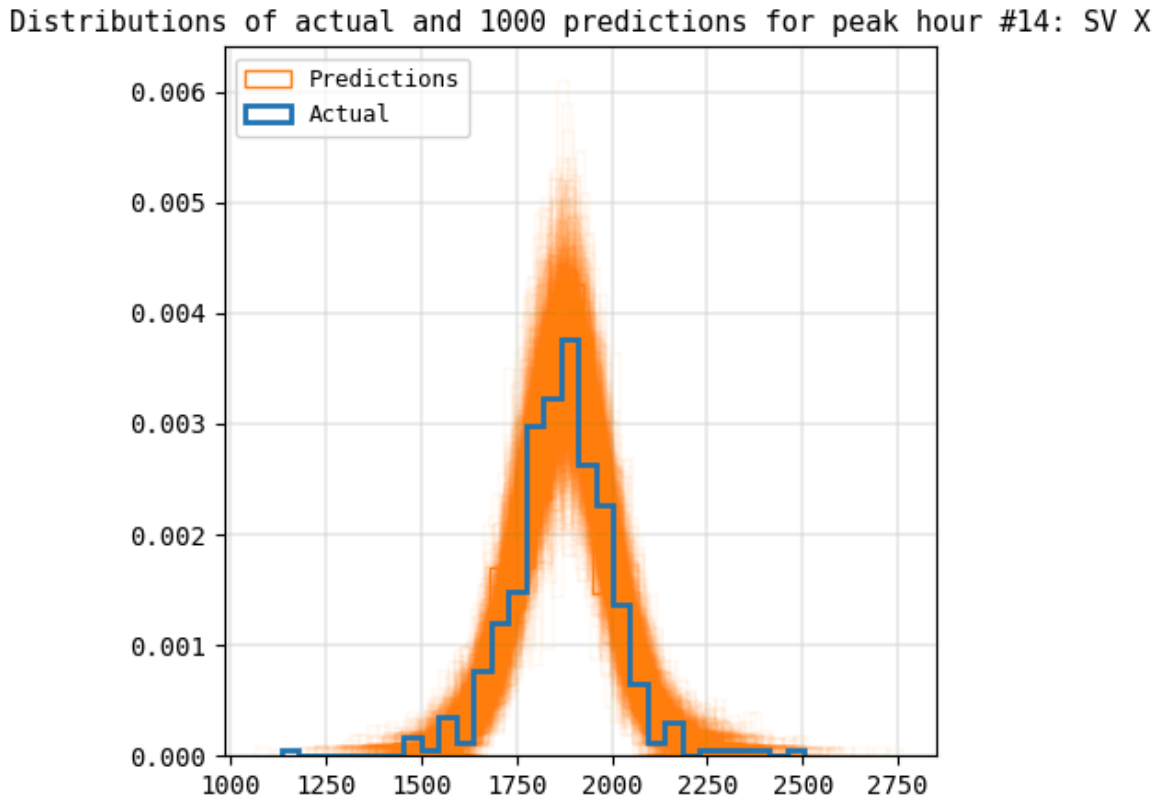
In [ ]:
```
%%capture
predict_sv_x_peak_eur_ci = model_sv_x_peak_eur.predict_ci(price_temp_data_peak_e
len(predict_sv_x_peak_eur_ci)
```

In [ ]:
```
plt.figure(figsize=(10, 5))
plt.plot(price_temp_data_peak_eur.index, predict_sv_x_peak_eur_many.iloc[0], col
for i in range(1, predict_sv_x_peak_eur_many.shape[0]):
    plt.plot(price_temp_data_peak_eur.index, predict_sv_x_peak_eur_many.iloc[i],
plt.plot(price_temp_data_peak_eur.index, price_temp_data_peak_eur['CONSUMER_PRIC
plt.xlabel('Date', size=10, family='monospace')
plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
```

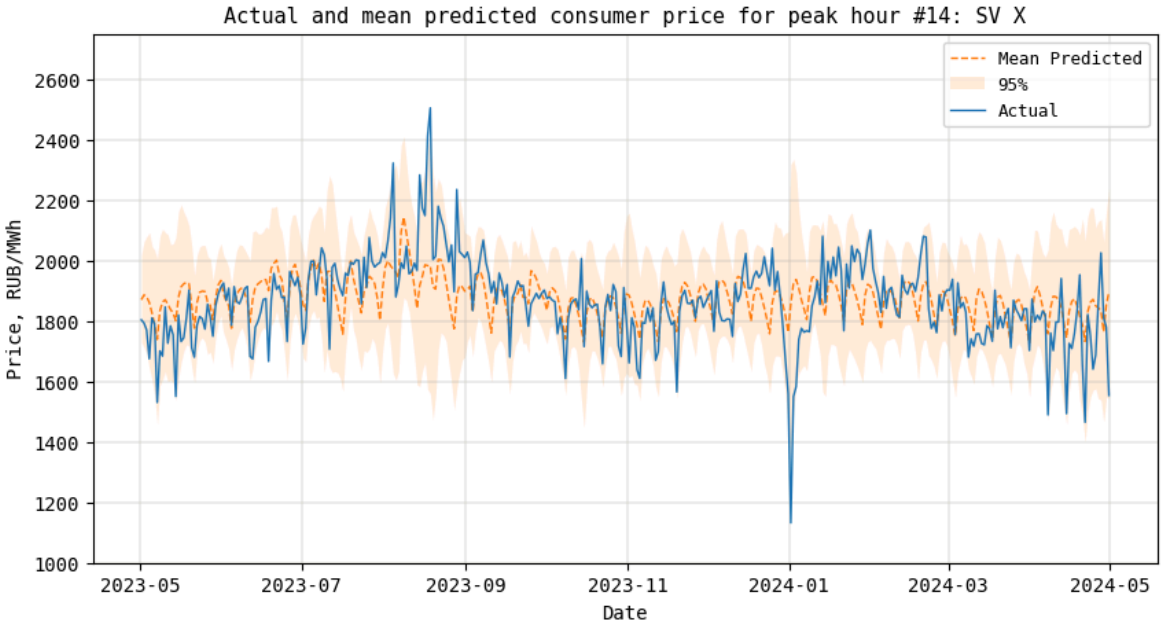
```
plt.yticks(size=10, family='monospace')
plt.ylim([1000, 2750])
plt.title(f'Actual and {num_samples} predictions of consumer price for peak hour
plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'});
plt.grid(lw=0.25, color='xkcd:cement');
```



In [ ]:
```
plt.figure(figsize=(5, 5))
plt.hist(predict_sv_x_peak_eur_many.iloc[0], bins=30, density=True, histtype='st
for i in range(1, predict_sv_x_peak_eur_many.shape[0]):
    plt.hist(predict_sv_x_peak_eur_many.iloc[i], bins=30, density=True, histtype
plt.hist(price_temp_data_peak_eur['CONSUMER_PRICE'], bins=30, density=True, hist
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.grid(lw=0.25, color='xkcd:cement')
plt.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'})
plt.gca().set_axisbelow(True)
plt.title(f'Distributions of actual and {num_samples} predictions for peak hour
```



In [ ]:
```
plt.figure(figsize=(10, 5))
plt.plot(price_temp_data_peak_eur.index, predict_sv_x_peak_eur_mean, color='C1',
plt.fill_between(price_temp_data_peak_eur.index, predict_sv_x_peak_eur_ci[0], pr
plt.plot(price_temp_data_peak_eur.index, price_temp_data_peak_eur['CONSUMER_PRIC
plt.xlabel('Date', size=10, family='monospace')
plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.ylim([1000, 2750])
plt.title(f'Actual and mean predicted consumer price for peak hour #{hour_max_eu
plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'});
plt.grid(lw=0.25, color='xkcd:cement');
```

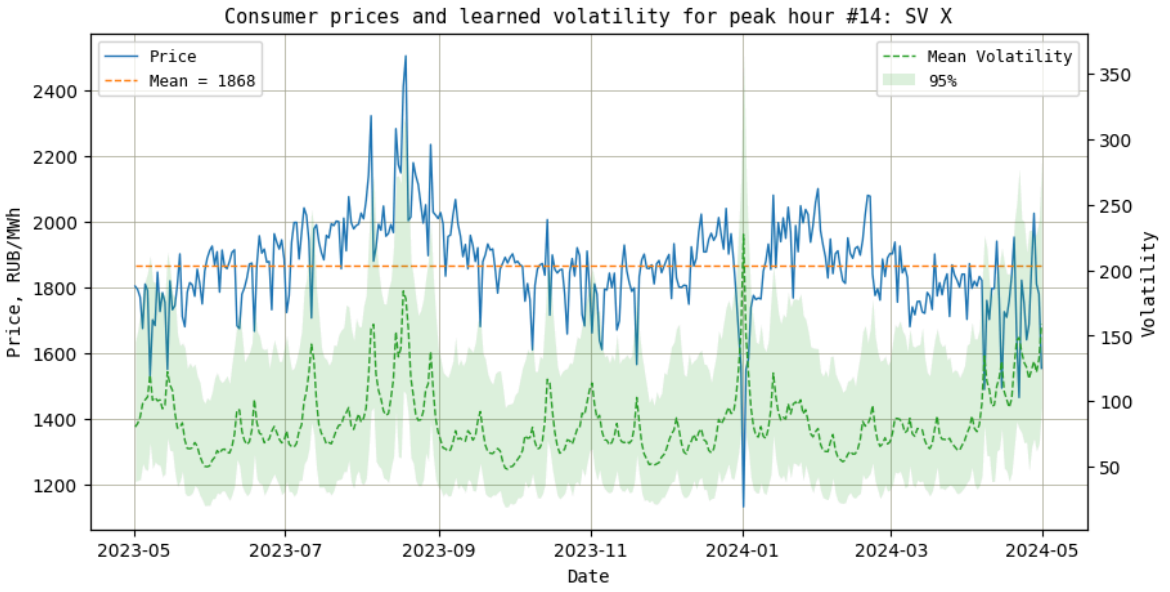


In [ ]:
```
# Volatility at time t over all predictions
vol_sv_x_peak_eur = model_sv_x_peak_eur.get_volatility()
```

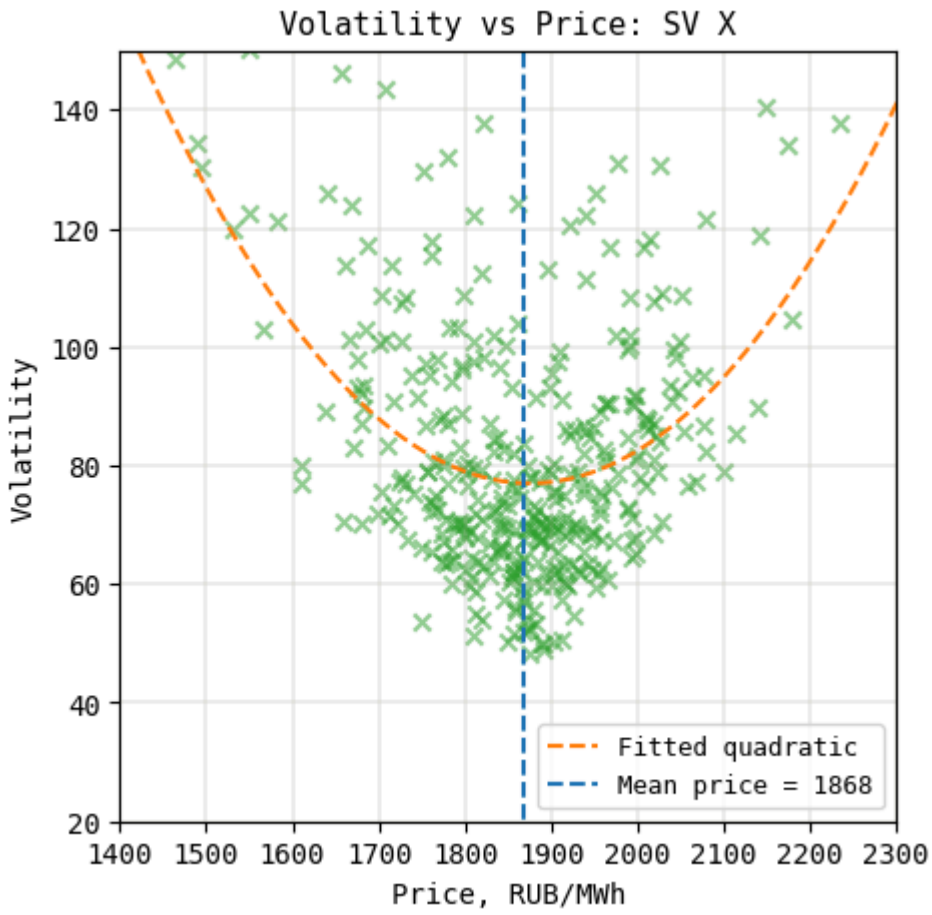
```
vol_sv_x_peak_eur.shape
```

Out[ ]: (1000, 366)

In [ ]:
```
_, ax = plt.subplots(figsize=(10, 5))
ax.plot(price_temp_data_peak_eur.index, price_temp_data_peak_eur['CONSUMER_PRICE
ax.hlines(price_temp_data_peak_eur['CONSUMER_PRICE'].mean(), xmin=price_temp_dat
ax.set_xlabel('Date', size=10, family='monospace')
ax.set_ylabel('Price, RUB/MWh', size=10, family='monospace')
ax.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'})
ax.grid(lw=0.5, color='xkcd:cement')
ax2 = ax.twinx()
ax2.plot(price_temp_data_peak_eur.index, vol_sv_x_peak_eur.mean(axis=0), color='
ax2.fill_between(price_temp_data_peak_eur.index, np.percentile(vol_sv_x_peak_eur
ax2.set_ylabel('Volatility', size=10, family='monospace')
ax2.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
ax.set_title(f'Consumer prices and learned volatility for peak hour #{hour_max_e
```



In [ ]:
```
plt.figure(figsize=(5, 5))
fitted_quadratic = np.poly1d(np.polyfit(price_temp_data_peak_eur['CONSUMER_PRICE
x = np.linspace(price_temp_data_peak_eur['CONSUMER_PRICE'].min(), price_temp_dat
plt.scatter(price_temp_data_peak_eur['CONSUMER_PRICE'], vol_sv_x_peak_eur.mean(a
plt.plot(x, fitted_quadratic(x), color='C1', ls='--', label='Fitted quadratic')
plt.vlines(price_temp_data_peak_eur['CONSUMER_PRICE'].mean(), ymin=0, ymax=350,
plt.xlabel('Price, RUB/MWh', size=10, family='monospace')
plt.ylabel('Volatility', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.xlim([1400, 2300])
plt.ylim([20, 150])
plt.title(f'Volatility vs Price: {model_sv_x_peak_eur.name}', size=11, family='m
plt.legend(loc='lower right', prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement');
```



In [ ]:
```
alpha = 0.05
adf_stats, adf_pval, _, _, _, _ = adfuller(vol_sv_x_peak_eur.mean(axis=0))
print(f'{adf_stats = :.2f}, {adf_pval = :.2f}', end=', ')
print('Stationary') if adf_pval < alpha else print('Non-stationary')
```

```
adf_stats = -5.66, adf_pval = 0.00, Stationary
```

As with SV Baseline, we can clearly see that volatility *does* depend on consumer price, and our **SV X** model discovered this relation.

Again, as with SV Baseline, we can see a rather V-shaped dependency: the volatility tends to increase for the prices higher and lower than the mean price, while it's minimal around the mean price.

3.5. Goodness-of-fit

In [ ]:
```
# MAE for mean predictions over all draws
mae_sv_x = mean_absolute_error(price_temp_data_peak_eur['CONSUMER_PRICE'], predi
mae_sv_x
```

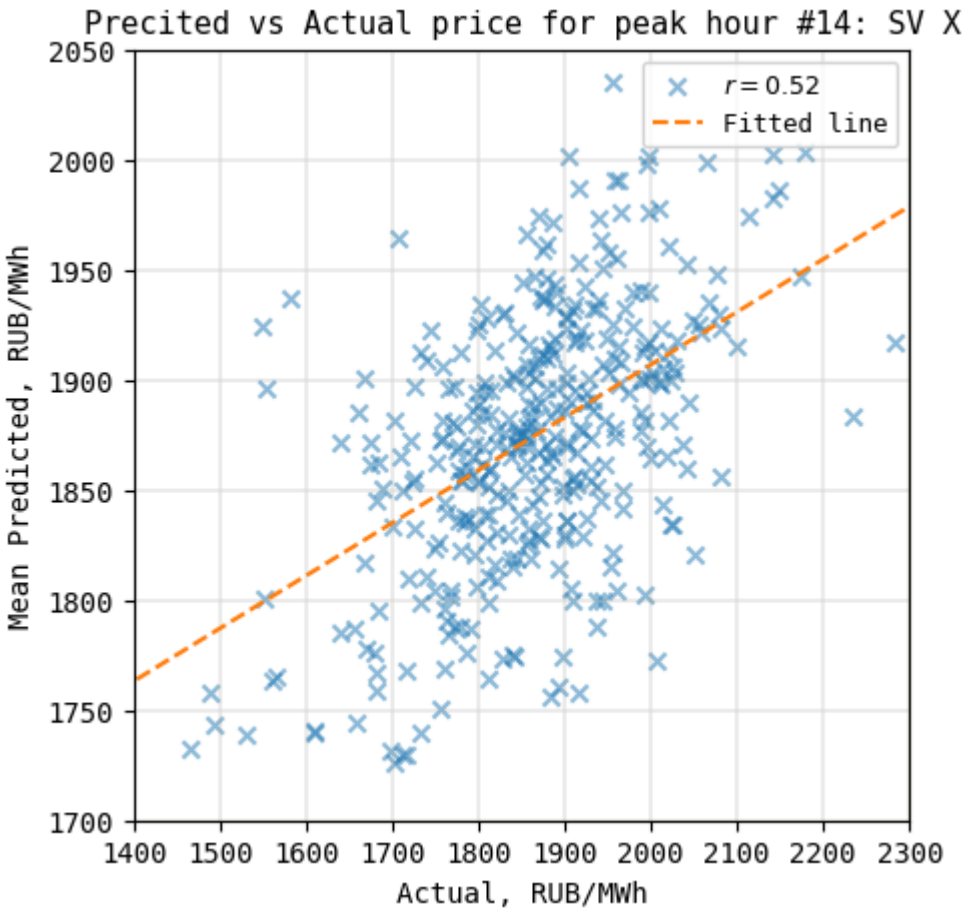
Out[ ]:
```
85.23342134206784
```

In [ ]:
```
# RMSE for mean predictions over all draws
rmse_sv_x = root_mean_squared_error(price_temp_data_peak_eur['CONSUMER_PRICE'],
rmse_sv_x
```

Out[ ]:
```
117.91113672531587
```

Let's check the correlation between mean predicted and actual prices.

In [ ]:
```
r = np.corrcoef(price_temp_data_peak_eur['CONSUMER_PRICE'], predict_sv_x_peak_eu
fitted_line = np.poly1d(np.polyfit(price_temp_data_peak_eur['CONSUMER_PRICE'], p
x = np.linspace(price_temp_data_peak_eur['CONSUMER_PRICE'].min(), price_temp_dat
plt.figure(figsize=(5, 5))
plt.scatter(price_temp_data_peak_eur['CONSUMER_PRICE'], predict_sv_x_peak_eur_me
plt.plot(x, fitted_line(x), color='C1', ls='--', label='Fitted line')
plt.xlabel('Actual, RUB/MWh', size=10, family='monospace')
plt.ylabel('Mean Predicted, RUB/MWh', size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.xlim([1400, 2300])
plt.ylim([1700, 2050])
plt.title(f'Precited vs Actual price for peak hour #{hour_max_eur}: {model_sv_x_
plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement');
```

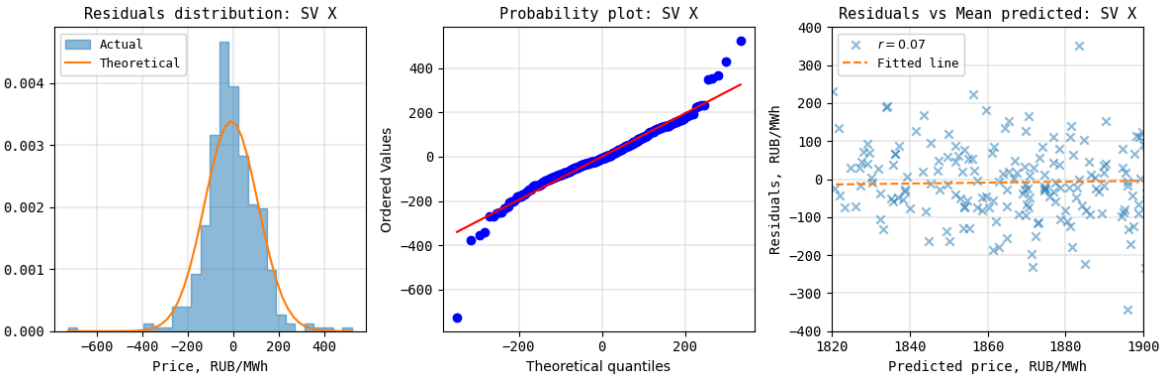


This time the correlation is stronger, as compared to the SV Baseline model.

We should also check the distribution of residuals, probability plot (similar to Q-Q plot), and residuals vs mean predicted prices.

In [ ]:
```
residuals_sv_x = price_data_peak_eur['CONSUMER_PRICE'] - predict_sv_x_peak_eur_m
x = np.linspace(residuals_sv_x.min(), residuals_sv_x.max())
residuals_sv_x_theor = norm(residuals_sv_x.mean(), residuals_sv_x.std(ddof=1))
plt.figure(figsize=(12, 4))
plt.subplot(1, 3, 1)
plt.hist(residuals_sv_x, bins=30, density=True, histtype='stepfilled', color='C0
plt.plot(x, residuals_sv_x_theor.pdf(x), color='C1', label='Theoretical')
plt.xlabel('Price, RUB/MWh', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
```

```
plt.yticks(size=10, family='monospace')
plt.title(f'Residuals distribution: {model_sv_x_peak_eur.name}', size=11, family
plt.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement')
plt.gca().set_axisbelow(True)
plt.subplot(1, 3, 2)
probplot(residuals_sv_x, dist=residuals_sv_x_theor, plot=plt)
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.title(f'Probability plot: {model_sv_x_peak_eur.name}', size=11, family='mono
plt.grid(lw=0.25, color='xkcd:cement')
plt.gca().set_axisbelow(True)
plt.subplot(1, 3, 3)
r = np.corrcoef(residuals_sv_x, predict_sv_x_peak_eur_mean)[0, 1]
fitted_line = np.poly1d(np.polyfit(predict_sv_x_peak_eur_mean, residuals_sv_x, d
x = np.linspace(predict_sv_x_peak_eur_mean.min(), predict_sv_x_peak_eur_mean.max
plt.scatter(predict_sv_x_peak_eur_mean, residuals_sv_x, color='C0', marker='x',
plt.plot(x, fitted_line(x), color='C1', ls='--', label='Fitted line')
plt.xlabel('Predicted price, RUB/MWh', size=10, family='monospace')
plt.ylabel('Residuals, RUB/MWh', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(size=10, family='monospace')
plt.xlim([1820, 1900])
plt.ylim([-400, 400])
plt.title(f'Residuals vs Mean predicted: {model_sv_x_peak_eur.name}', size=11, f
plt.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'})
plt.grid(lw=0.25, color='xkcd:cement')
plt.gca().set_axisbelow(True)
plt.tight_layout();
```



Residuals look to be distributed more or less normally. Also they look rather homoscedastic w.r.t. predictions, which means that the model is rather acceptable.

Overall, the SV X model looks to be a better fit for our price data with the air temperature and day of week as exogenous regressors, as compared to the SV Baseline model.

Back

---

4. Cross-validation

We have trained and tested our models on the same fixed year-long time frame. It is a standard practice to train and test models on different portions of the whole data to generate the distribution of metric(s) and ensure model robustness. This process is

known as cross-validation: all data is divided into a number of non-intersecting subsets of samples (folds) some of which are used for training and the rest for testing. For data with i.i.d. samples, i.e. when the samples are independent and thus the order of samples is not important, the order of folds is also not important. This is not the case for time series data, where training folds must not be precede the testing folds, otherwise the target leakage will occur and the model cannot be trusted.

Cross-validation setup.

Models:

1. SV Baseline + Peak hour + Price zone 1 (European)
2. SV Baseline + Peak hour + Price zone 2 (Siberian)
3. SV Baseline + Off-peak hour + Price zone 1
4. SV Baseline + Off-peak hour + Price zone 2
5. SV X + Peak hour + Price zone 1
6. SV X + Peak hour + Price zone 2
7. SV X + Off-peak hour + Price zone 1
8. SV X + Off-peak hour + Price zone 2

Time frame: 23.06.2014 -- 30.04.2024 (3600 days), roughly 10 years.

4.1. SV Baseline

Load price data for the whole time frame and both price zones.

In [ ]:
```
# # We need to explicitly specify the column names to correctly parse the xml
# price_data = pd.read_xml(f'https://www.atsenergo.ru/market/stats.xml?period=0&
#                      names=['ROW_ID',
#                             'DAT',
#                             'PRICE_ZONE_CODE',
#                             'CONSUMER_VOLUME',
#                             'CONSUMER_PRICE',
#                             'CONSUMER_RD_VOLUME',
#                             'CONSUMER_SPOT_VOLUME',
#                             'CONSUMER_PROVIDE_RD',
#                             'CONSUMER_MAX_PRICE',
#                             'CONSUMER_MIN_PRICE',
#                             'SUPPLIER_VOLUME',
#                             'SUPPLIER_PRICE',
#                             'SUPPLIER_RD_VOLUME',
#                             'SUPPLIER_SPOT_VOLUME',
#                             'SUPPLIER_PROVIDE_RD',
#                             'SUPPLIER_MAX_PRICE',
#                             'SUPPLIER_MIN_PRICE',
#                             'HOUR'],
#                      xpath='//row',
#                      parse_dates=['DAT'])
# # Make datetime
# price_data = price_data.set_index(pd.to_datetime(price_data['DAT'].astype(str)
# price_data.index.name ='Datetime'
# # We can now drop all unnecessary columns to reduce the dataframe
# price_data = price_data.drop(columns=['ROW_ID',
#                           'DAT',
```

```
#                             'CONSUMER_VOLUME',
#                             'CONSUMER_RD_VOLUME',
#                             'CONSUMER_SPOT_VOLUME',
#                             'CONSUMER_PROVIDE_RD',
#                             'CONSUMER_MAX_PRICE',
#                             'CONSUMER_MIN_PRICE',
#                             'SUPPLIER_VOLUME',
#                             'SUPPLIER_PRICE',
#                             'SUPPLIER_RD_VOLUME',
#                             'SUPPLIER_SPOT_VOLUME',
#                             'SUPPLIER_PROVIDE_RD',
#                             'SUPPLIER_MAX_PRICE',
#                             'SUPPLIER_MIN_PRICE']).dropna()
# price_data = price_data.loc[~price_data.index.isna()].sort_index()
# price_data['Weekday'] = price_data.index.day_of_week
# price_data.to_csv('./data/price_data_20140623_20240430.csv')
```

In [ ]:
```
price_data = pd.read_csv('./data/price_data_20140623_20240430.zip', index_col='D
```

In [ ]:
```
price_data.shape
```

Out[ ]:
```
(172800, 4)
```

In [ ]:
```
price_data.head(4)
```

Out[ ]:

Datetime	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday
2014-06-23 00:00:00	1	1147.08	0	0
2014-06-23 00:00:00	2	685.81	0	0
2014-06-23 01:00:00	2	684.27	1	0
2014-06-23 01:00:00	1	1037.39	1	0

In [ ]:
```
price_data.tail(4)
```

Out[ ]:

Datetime	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday
2024-04-30 22:00:00	2	1465.38	22	1
2024-04-30 22:00:00	1	1442.53	22	1
2024-04-30 23:00:00	2	1439.88	23	1
2024-04-30 23:00:00	1	1327.94	23	1

In [ ]:
```
price_data.describe()
```

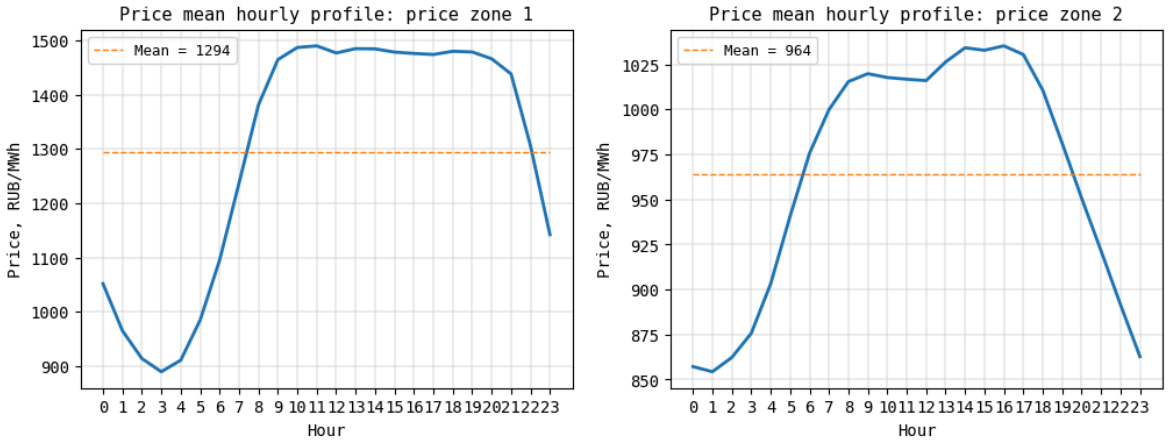
Out[ ]:

	PRICE_ZONE_CODE	CONSUMER_PRICE	HOUR	Weekday
count	172800.000000	172800.000000	172800.000000	172800.000000
mean	1.500000	1128.835519	11.500000	2.998611
std	0.500001	311.825787	6.922207	2.000352
min	1.000000	0.000000	0.000000	0.000000
25%	1.000000	901.610000	5.750000	1.000000
50%	1.500000	1061.570000	11.500000	3.000000
75%	2.000000	1355.850000	17.250000	5.000000
max	2.000000	2504.960000	23.000000	6.000000

In [ ]:
```
# Price zone 1 (European)
price_data_eur = price_data[price_data['PRICE_ZONE_CODE'] == 1]
# Price zone 2 (Siberian)
price_data_sib = price_data[price_data['PRICE_ZONE_CODE'] == 2]
```

In [ ]:
```
# Hour profiles
price_data_daily_agg_eur = price_data_eur.groupby('HOUR')['CONSUMER_PRICE'].mean
price_data_daily_agg_sib = price_data_sib.groupby('HOUR')['CONSUMER_PRICE'].mean
```

In [ ]:
```
plt.figure(figsize=(12, 4))
for i, hour_profile in zip(range(1, 3), [price_data_daily_agg_eur, price_data_da
    plt.subplot(1, 2, i)
    plt.plot(hour_profile.index, hour_profile, color='C0', lw=2)
    plt.hlines(hour_profile.mean(), xmin=hour_profile.index.min(), xmax=hour_pro
    plt.xlabel('Hour', size=10, family='monospace')
    plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
    plt.xticks(hour_profile.index, size=10, family='monospace')
    plt.yticks(size=10, family='monospace')
    plt.title(f'Price mean hourly profile: price zone {i}', size=11, family='mon
    plt.legend(prop={'size': 9, 'family': 'monospace'})
    plt.grid(lw=0.25, color='xkcd:cement');
```



In [ ]:
```
# Peak and off-peak hours
hour_max_eur = price_data_daily_agg_eur.idxmax()
hour_min_eur = price_data_daily_agg_eur.idxmin()
hour_max_sib = price_data_daily_agg_sib.idxmax()
```

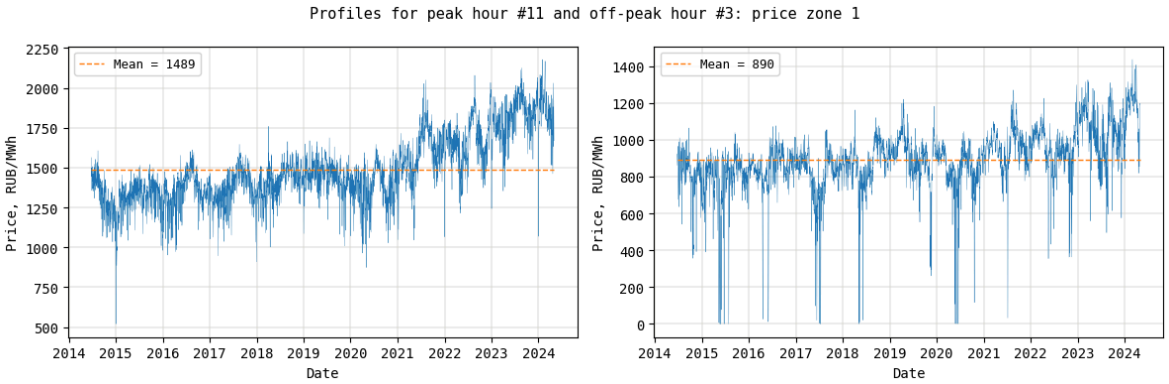
```
hour_min_sib = price_data_daily_agg_sib.idxmin()
(hour_max_eur, hour_min_eur), (hour_max_sib, hour_min_sib)
```

Out[ ]: ((11, 3), (16, 1))

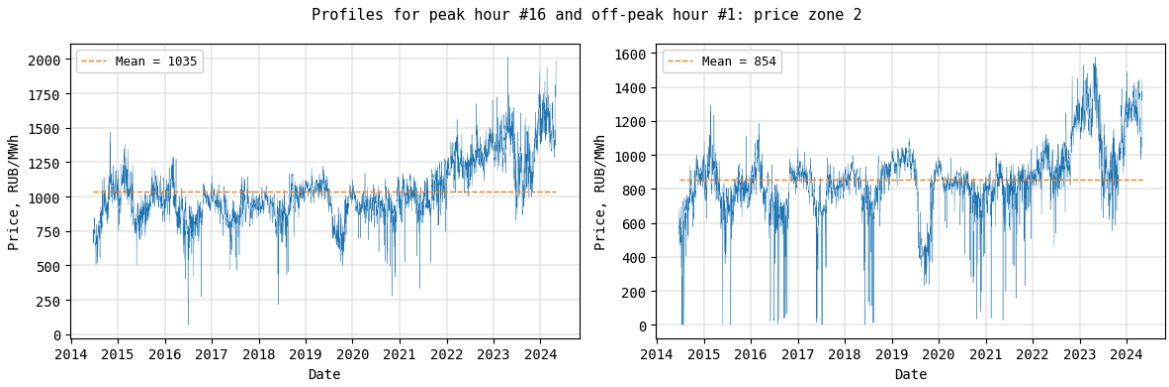
All cross-validation subsets for SV Baseline model.

In [ ]:
```
price_data_peak_eur = price_data_eur[price_data_eur['HOUR'] == hour_max_eur]
price_data_offpeak_eur = price_data_eur[price_data_eur['HOUR'] == hour_min_eur]
price_data_peak_sib = price_data_sib[price_data_sib['HOUR'] == hour_max_sib]
price_data_offpeak_sib = price_data_sib[price_data_sib['HOUR'] == hour_min_sib]
```

In [ ]:
```
plt.figure(figsize=(12, 4))
for i, hour_profile in zip(range(1, 3), [price_data_peak_eur, price_data_offpeak
    plt.subplot(1, 2, i)
    plt.plot(hour_profile.index, hour_profile['CONSUMER_PRICE'], color='C0', lw=
    plt.hlines(hour_profile['CONSUMER_PRICE'].mean(), xmin=hour_profile.index.mi
    plt.xlabel('Date', size=10, family='monospace')
    plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
    plt.xticks(size=10, family='monospace')
    plt.yticks(size=10, family='monospace')
    plt.legend(prop={'size': 9, 'family': 'monospace'})
    plt.grid(lw=0.25, color='xkcd:cement');
    plt.gca().set_axisbelow(True)
plt.suptitle(f'Profiles for peak hour #{hour_max_eur} and off-peak hour #{hour_m
plt.tight_layout();
```



In [ ]:
```
plt.figure(figsize=(12, 4))
for i, hour_profile in zip(range(1, 3), [price_data_peak_sib, price_data_offpeak
    plt.subplot(1, 2, i)
    plt.plot(hour_profile.index, hour_profile['CONSUMER_PRICE'], color='C0', lw=
    plt.hlines(hour_profile['CONSUMER_PRICE'].mean(), xmin=hour_profile.index.mi
    plt.xlabel('Date', size=10, family='monospace')
    plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
    plt.xticks(size=10, family='monospace')
    plt.yticks(size=10, family='monospace')
    plt.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'})
    plt.grid(lw=0.25, color='xkcd:cement');
    plt.gca().set_axisbelow(True)
plt.suptitle(f'Profiles for peak hour #{hour_max_sib} and off-peak hour #{hour_m
plt.tight_layout();
```



All SV Baseline models to cross-validate.

In [ ]:
```
with open('./models/sv_base_fit.stan', 'r') as fh:
  sv_base_code_fit = fh.read()
with open('./models/sv_base_predict.stan', 'r') as fh:
  sv_base_code_predict = fh.read()
num_samples = 1_000
```

In [ ]:
```
# 1) SV Baseline + Peak hour + Price zone 1
model_sv_base_peak_eur = StanModel(kind='sv_base',
                                   name='SV Baseline + Peak hour + Price zone 1',
                                   stan_code_fit=sv_base_code_fit,
                                   stan_code_predict=sv_base_code_predict,
                                   num_samples=num_samples)
```

In [ ]:
```
# 2) SV Baseline + Peak hour + Price zone 2
model_sv_base_peak_sib = StanModel(kind='sv_base',
                                   name='SV Baseline + Peak hour + Price zone 2',
                                   stan_code_fit=sv_base_code_fit,
                                   stan_code_predict=sv_base_code_predict,
                                   num_samples=num_samples)
```

In [ ]:
```
# 3) SV Baseline + Off-peak hour + Price zone 1
model_sv_base_offpeak_eur = StanModel(kind='sv_base',
                                   name='SV Baseline + Off-peak hour + Price zone
                                   stan_code_fit=sv_base_code_fit,
                                   stan_code_predict=sv_base_code_predict,
                                   num_samples=num_samples)
```

In [ ]:
```
# 4) SV Baseline + Off-peak hour + Price zone 2
model_sv_base_offpeak_sib = StanModel(kind='sv_base',
                                   name='SV Baseline + Off-peak hour + Price zone
                                   stan_code_fit=sv_base_code_fit,
                                   stan_code_predict=sv_base_code_predict,
                                   num_samples=num_samples)
```

Cross-validation strategy:

1. Choose some starting date;
2. Train for 360 days (approx. 1 year) from starting date;
3. Predict 90 days (approx. 3 months = 1 quarter) ahead using the most recent log volatility obtained during training (90 points);
4. Move the date sliding window and repeat from 2 until all folds are done.

The number of date sliding windows $N_w = \frac{3600 - 30 \times 12}{30 \times 3} = 36$.

Let's use scikit-learn's Time Series cross-validator to correctly split our data into train and test subsets.

In [ ]:
```
n_windows = 36 # Number of sliding windows
ts_cv_sv_base = TimeSeriesSplit(n_splits=n_windows, max_train_size=30*12, test_s
```

In [ ]:
```
print('Split # | Train fold                      | Test fold')
print('--------|---------------------------------|-----------------------------')
# All 4 subsets have equal size, so get split indices from any of the four
for i, split in enumerate(ts_cv_sv_base.split(price_data_peak_eur)):
    train_start = price_data_peak_eur.iloc[split[0]].index.min().date()
    train_end = price_data_peak_eur.iloc[split[0]].index.max().date()
    test_start = price_data_peak_eur.iloc[split[1]].index.min().date()
    test_end = price_data_peak_eur.iloc[split[1]].index.max().date()
    print(f'{i + 1:>7} | {train_start} -- {train_end} ({(train_end - train_start
```

```
Split # | Train fold                      | Test fold
--------|---------------------------------|-----------------------------
      1 | 2014-06-23 -- 2015-06-17 (360) | 2015-06-18 -- 2015-09-15 (90)
      2 | 2014-09-21 -- 2015-09-15 (360) | 2015-09-16 -- 2015-12-14 (90)
      3 | 2014-12-20 -- 2015-12-14 (360) | 2015-12-15 -- 2016-03-13 (90)
      4 | 2015-03-20 -- 2016-03-13 (360) | 2016-03-14 -- 2016-06-11 (90)
      5 | 2015-06-18 -- 2016-06-11 (360) | 2016-06-12 -- 2016-09-09 (90)
      6 | 2015-09-16 -- 2016-09-09 (360) | 2016-09-10 -- 2016-12-08 (90)
      7 | 2015-12-15 -- 2016-12-08 (360) | 2016-12-09 -- 2017-03-08 (90)
      8 | 2016-03-14 -- 2017-03-08 (360) | 2017-03-09 -- 2017-06-06 (90)
      9 | 2016-06-12 -- 2017-06-06 (360) | 2017-06-07 -- 2017-09-04 (90)
     10 | 2016-09-10 -- 2017-09-04 (360) | 2017-09-05 -- 2017-12-03 (90)
     11 | 2016-12-09 -- 2017-12-03 (360) | 2017-12-04 -- 2018-03-03 (90)
     12 | 2017-03-09 -- 2018-03-03 (360) | 2018-03-04 -- 2018-06-01 (90)
     13 | 2017-06-07 -- 2018-06-01 (360) | 2018-06-02 -- 2018-08-30 (90)
     14 | 2017-09-05 -- 2018-08-30 (360) | 2018-08-31 -- 2018-11-28 (90)
     15 | 2017-12-04 -- 2018-11-28 (360) | 2018-11-29 -- 2019-02-26 (90)
     16 | 2018-03-04 -- 2019-02-26 (360) | 2019-02-27 -- 2019-05-27 (90)
     17 | 2018-06-02 -- 2019-05-27 (360) | 2019-05-28 -- 2019-08-25 (90)
     18 | 2018-08-31 -- 2019-08-25 (360) | 2019-08-26 -- 2019-11-23 (90)
     19 | 2018-11-29 -- 2019-11-23 (360) | 2019-11-24 -- 2020-02-21 (90)
     20 | 2019-02-27 -- 2020-02-21 (360) | 2020-02-22 -- 2020-05-21 (90)
     21 | 2019-05-28 -- 2020-05-21 (360) | 2020-05-22 -- 2020-08-19 (90)
     22 | 2019-08-26 -- 2020-08-19 (360) | 2020-08-20 -- 2020-11-17 (90)
     23 | 2019-11-24 -- 2020-11-17 (360) | 2020-11-18 -- 2021-02-15 (90)
     24 | 2020-02-22 -- 2021-02-15 (360) | 2021-02-16 -- 2021-05-16 (90)
     25 | 2020-05-22 -- 2021-05-16 (360) | 2021-05-17 -- 2021-08-14 (90)
     26 | 2020-08-20 -- 2021-08-14 (360) | 2021-08-15 -- 2021-11-12 (90)
     27 | 2020-11-18 -- 2021-11-12 (360) | 2021-11-13 -- 2022-02-10 (90)
     28 | 2021-02-16 -- 2022-02-10 (360) | 2022-02-11 -- 2022-05-11 (90)
     29 | 2021-05-17 -- 2022-05-11 (360) | 2022-05-12 -- 2022-08-09 (90)
     30 | 2021-08-15 -- 2022-08-09 (360) | 2022-08-10 -- 2022-11-07 (90)
     31 | 2021-11-13 -- 2022-11-07 (360) | 2022-11-08 -- 2023-02-05 (90)
     32 | 2022-02-11 -- 2023-02-05 (360) | 2023-02-06 -- 2023-05-06 (90)
     33 | 2022-05-12 -- 2023-05-06 (360) | 2023-05-07 -- 2023-08-04 (90)
     34 | 2022-08-10 -- 2023-08-04 (360) | 2023-08-05 -- 2023-11-02 (90)
     35 | 2022-11-08 -- 2023-11-02 (360) | 2023-11-03 -- 2024-01-31 (90)
     36 | 2023-02-06 -- 2024-01-31 (360) | 2024-02-01 -- 2024-04-30 (90)
```

In [ ]:
```
%%capture
models = [model_sv_base_peak_eur, model_sv_base_peak_sib, model_sv_base_offpeak_
subsets = [price_data_peak_eur, price_data_peak_sib, price_data_offpeak_eur, pri
cv_results_sv_base_df = pd.DataFrame(columns=['model_kind', 'model_name', 'train
print('Running cross-validation...')
for model, subset in tqdm(zip(models, subsets)):
    print(f'Model: {model.name}')
    cv_result = cross_validate(model,
                               X=subset['CONSUMER_PRICE'],
                               y=subset['CONSUMER_PRICE'],
                               cv=ts_cv_sv_base,
                               scoring=['neg_mean_absolute_error', 'neg_root_me
                               return_indices=True,
                               n_jobs=4)
    cv_result_df = pd.concat([pd.Series(cv_result['indices']['train'], name='tra
                              pd.Series(cv_result['indices']['test'], name='test
                              pd.Series(cv_result['fit_time'], name='fit_time'),
                              pd.Series(cv_result['score_time'], name='score_tim
                              pd.Series(-cv_result['test_neg_mean_absolute_error
                              pd.Series(-cv_result['test_neg_root_mean_squared_e
    cv_results_sv_base_df = pd.concat([cv_results_sv_base_df,
                                       pd.concat([pd.Series([model.kind] * n_win
                                       pd.Series([model.name] * n_windows, name=
                                       cv_result_df], axis=1)])
cv_results_sv_base_df = cv_results_sv_base_df.reset_index(drop=True)
print('Done!')
```

In [ ]:
```
cv_results_sv_base_df
```

Out[ ]:

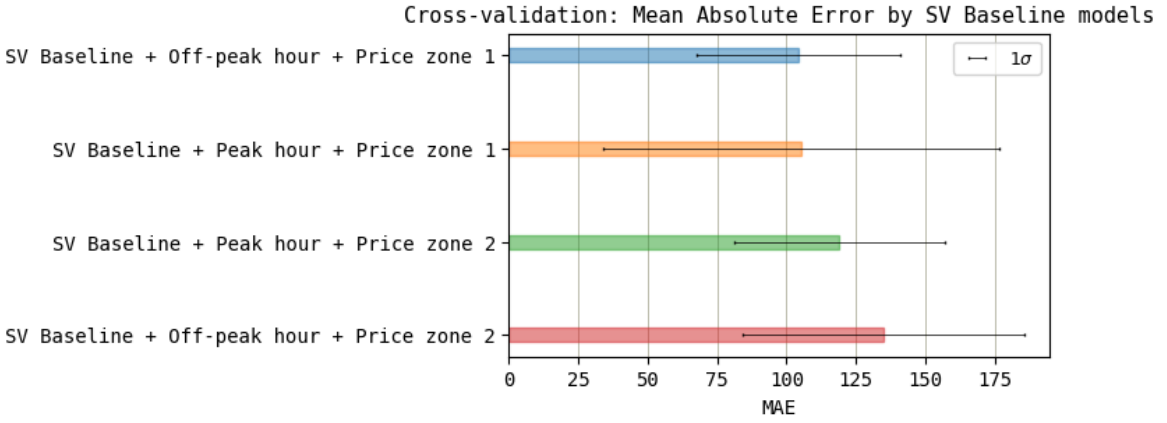
	model_kind	model_name	train_indices	test_indices	fit_time	score_time	test_
0	sv_base	SV Baseline + Peak hour + Price zone 1	[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,...	[360, 361, 362, 363, 364, 365, 366, 367, 368, ...	6.997288	0.245346	66.93
1	sv_base	SV Baseline + Peak hour + Price zone 1	[90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, ...	[450, 451, 452, 453, 454, 455, 456, 457, 458, ...	6.366097	0.221779	91.49
2	sv_base	SV Baseline + Peak hour + Price zone 1	[180, 181, 182, 183, 184, 185, 186, 187, 188, ...	[540, 541, 542, 543, 544, 545, 546, 547, 548, ...	10.321057	0.243930	98.05
3	sv_base	SV Baseline + Peak hour + Price zone 1	[270, 271, 272, 273, 274, 275, 276, 277, 278, ...	[630, 631, 632, 633, 634, 635, 636, 637, 638, ...	6.422443	0.218006	90.65
4	sv_base	SV Baseline + Peak hour + Price zone 1	[360, 361, 362, 363, 364, 365, 366, 367, 368, ...	[720, 721, 722, 723, 724, 725, 726, 727, 728, ...	5.611288	0.209697	144.87
...	...	...	...	...	...	...	
139	sv_base	SV Baseline + Off-peak hour + Price zone 2	[2790, 2791, 2792, 2793, 2794, 2795, 2796, 279...	[3150, 3151, 3152, 3153, 3154, 3155, 3156, 315...	11.440015	0.219902	303.76
140	sv_base	SV Baseline + Off-peak hour + Price zone 2	[2880, 2881, 2882, 2883, 2884, 2885, 2886, 288...	[3240, 3241, 3242, 3243, 3244, 3245, 3246, 324...	10.264223	0.221994	175.06
141	sv_base	SV Baseline + Off-peak hour + Price zone 2	[2970, 2971, 2972, 2973, 2974, 2975, 2976, 297...	[3330, 3331, 3332, 3333, 3334, 3335, 3336, 333...	10.199910	0.230959	261.34
142	sv_base	SV Baseline + Off-peak hour + Price zone 2	[3060, 3061, 3062, 3063, 3064, 3065, 3066, 306...	[3420, 3421, 3422, 3423, 3424, 3425, 3426, 342...	13.606772	0.241055	121.89
143	sv_base	SV Baseline + Off-peak hour + Price zone 2	[3150, 3151, 3152, 3153, 3154, 3155, 3156, 315...	[3510, 3511, 3512, 3513, 3514, 3515, 3516, 351...	8.310721	0.232118	159.86

144 rows × 8 columns

In [ ]:
```
# Mean MAE by model
mae_sv_base_cross = cv_results_sv_base_df.groupby('model_name')['test_mae']
mae_sv_base_cross.mean().sort_values()
```

Out[ ]:
```
model_name
SV Baseline + Off-peak hour + Price zone 1    104.422811
SV Baseline + Peak hour + Price zone 1        105.385055
SV Baseline + Peak hour + Price zone 2        119.140971
SV Baseline + Off-peak hour + Price zone 2    134.863451
Name: test_mae, dtype: float64
```

In [ ]:
```
plt.figure(figsize=(5, 3))
bars = plt.barh(range(mae_sv_base_cross.mean().shape[0]), mae_sv_base_cross.mean
plt.gca().invert_yaxis()
plt.xlabel('MAE', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(range(mae_sv_base_cross.mean().shape[0]), mae_sv_base_cross.mean().sc
plt.grid(axis='x', lw=0.5, color='xkcd:cement')
for i in range(len(bars)):
    bars[i].set_edgecolor(f'C{i}')
    bars[i].set_color(f'C{i}')
plt.gca().set_axisbelow(True)
plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
plt.title('Cross-validation: Mean Absolute Error by SV Baseline models', size=11
```

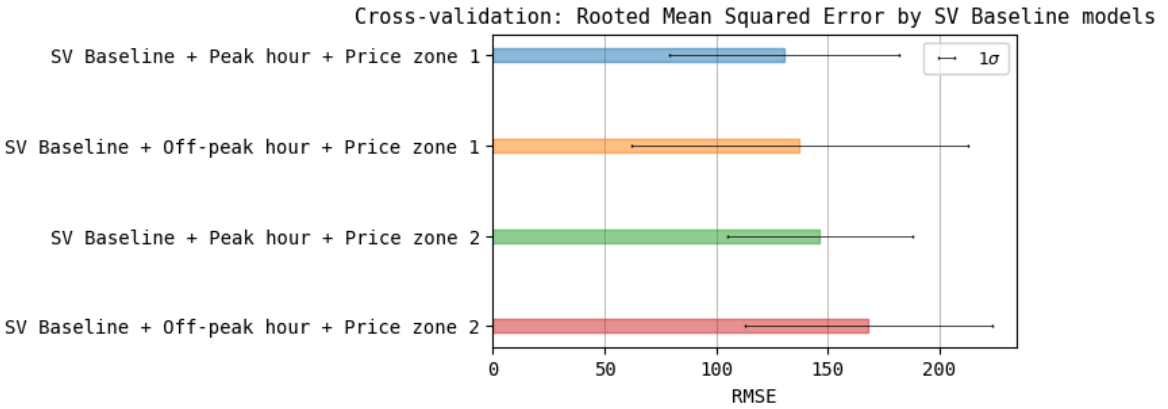


In [ ]:
```
# Mean RMSE by model
rmse_sv_base_cross = cv_results_sv_base_df.groupby('model_name')['test_rmse']
rmse_sv_base_cross.mean().sort_values()
```

Out[ ]:
```
model_name
SV Baseline + Peak hour + Price zone 1        130.526412
SV Baseline + Off-peak hour + Price zone 1    137.501447
SV Baseline + Peak hour + Price zone 2        146.445320
SV Baseline + Off-peak hour + Price zone 2    168.305299
Name: test_rmse, dtype: float64
```

In [ ]:
```
plt.figure(figsize=(5, 3))
bars = plt.barh(range(rmse_sv_base_cross.mean().shape[0]), rmse_sv_base_cross.me
plt.gca().invert_yaxis()
plt.xlabel('RMSE', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(range(rmse_sv_base_cross.mean().shape[0]), rmse_sv_base_cross.mean().
plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
plt.grid(axis='x', lw=0.5, color='xkcd:cement')
for i in range(len(bars)):
```

```
    bars[i].set_edgecolor(f'C{i}')
    bars[i].set_color(f'C{i}')
plt.gca().set_axisbelow(True)
plt.title('Cross-validation: Rooted Mean Squared Error by SV Baseline models', s
```



Both metrics show that on average:

- SV Baseline models for Price zone 1 (European) have higher quality than for Price zone 2 (Siberian);
- the best model is for Price zone 1 for Peak hour;
- the worst model is for Price zone 2 for Off-peak hour.

4.2. SV X

Load air temperature data for the whole time frame and both price zones.

In [ ]:
```
# Price zone 1: Moscow
temp_data_eur = pd.read_csv(f'./data/UUEE.23.06.2014.30.04.2024.1.0.0.ru.utf8.00
                            sep=';',
                            encoding='utf-8',
                            skiprows=7,
                            index_col=0,
                            dayfirst=True,
                            usecols=[0, 1],
                            names=['Datetime', 'Temperature'],
                            parse_dates=True).dropna().sort_index()
```

In [ ]:
```
temp_data_eur
```

Out[ ]:

Datetime	Temperature
2014-06-23 00:00:00	8.0
2014-06-23 00:30:00	9.0
2014-06-23 01:00:00	8.0
2014-06-23 01:30:00	10.0
2014-06-23 02:00:00	12.0
...	...
2024-04-30 21:30:00	18.0
2024-04-30 22:00:00	19.0
2024-04-30 22:30:00	19.0
2024-04-30 23:00:00	18.0
2024-04-30 23:30:00	17.0

164068 rows × 1 columns

In [ ]:
```
# Price zone 2: Novosibirsk
temp_data_sib = pd.read_csv(f'./data/UNNT.23.06.2014.30.04.2024.1.0.0.ru.utf8.00
                            sep=';',
                            encoding='utf-8',
                            skiprows=7,
                            index_col=0,
                            dayfirst=True,
                            usecols=[0, 1],
                            names=['Datetime', 'Temperature'],
                            parse_dates=True).dropna().sort_index()
# Don't forget to convert local Novosibirsk time to Moscow time, since all price
temp_data_sib.index = temp_data_sib.index - pd.Timedelta(hours=4)
```

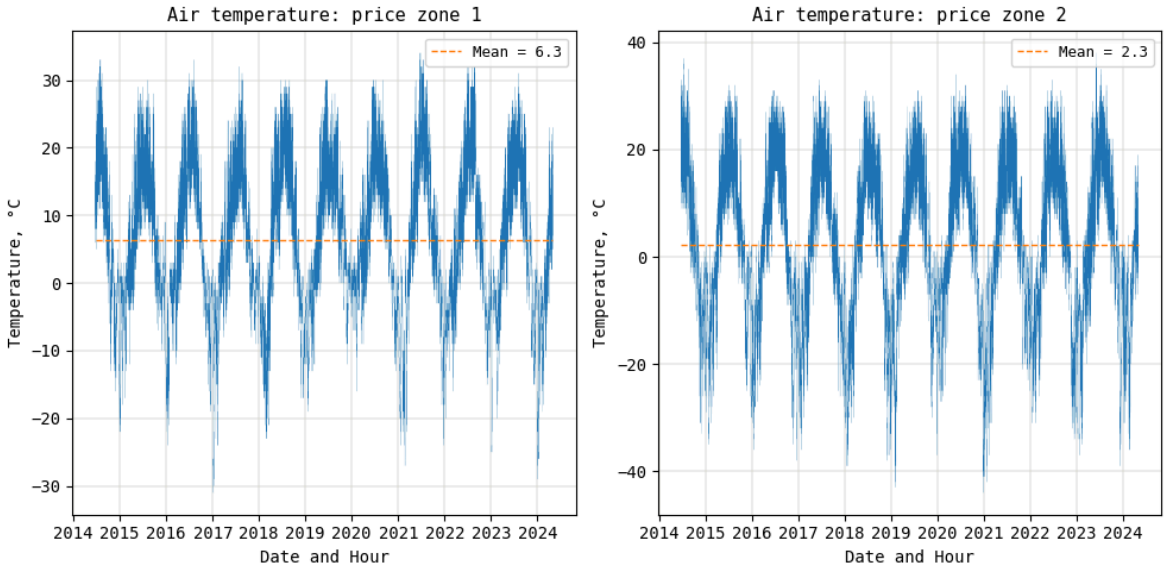
In [ ]:
```
temp_data_sib
```

Out[ ]:

Datetime	Temperature
2014-06-22 20:00:00	21.0
2014-06-22 20:30:00	20.0
2014-06-22 21:00:00	19.0
2014-06-22 21:30:00	18.0
2014-06-22 22:00:00	17.0
...	...
2024-04-30 17:30:00	3.0
2024-04-30 18:00:00	2.0
2024-04-30 18:30:00	1.0
2024-04-30 19:00:00	1.0
2024-04-30 19:30:00	-1.0

163741 rows × 1 columns

In [ ]:
```
plt.figure(figsize=(10, 5))
for i, temp_history in zip(range(1, 3), [temp_data_eur, temp_data_sib]):
    plt.subplot(1, 2, i)
    plt.plot(temp_history.index, temp_history['Temperature'], color='C0', lw=0.1
    plt.hlines(temp_history['Temperature'].mean(), xmin=temp_history.index.min()
    plt.xlabel('Date and Hour', size=10, family='monospace')
    plt.ylabel('Temperature, °C', size=10, family='monospace')
    plt.xticks(size=10, family='monospace')
    plt.yticks(size=10, family='monospace')
    plt.title(f'Air temperature: price zone {i}', size=11, family='monospace')
    plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
    plt.grid(lw=0.25, color='xkcd:cement')
plt.tight_layout();
```



In [ ]:
```
# Join consumer price and air temperature data
price_temp_data_peak_eur = pd.merge(price_data_peak_eur, temp_data_eur, on='Date
price_temp_data_offpeak_eur = pd.merge(price_data_offpeak_eur, temp_data_eur, on
price_temp_data_peak_sib = pd.merge(price_data_peak_sib, temp_data_sib, on='Date
price_temp_data_offpeak_sib = pd.merge(price_data_offpeak_sib, temp_data_sib, on
```

All SV X models to cross-validate.

In [ ]:
```
with open('./models/sv_x_fit.stan', 'r') as fh:
  sv_x_code_fit = fh.read()
with open('./models/sv_x_predict.stan', 'r') as fh:
  sv_x_code_predict = fh.read()
num_samples = 1_000
```

In [ ]:
```
# 5) SV X + Peak hour + Price zone 1
model_sv_x_peak_eur = StanModel(kind='sv_x',
                                name='SV X + Peak hour + Price zone 1',
                                stan_code_fit=sv_x_code_fit,
                                stan_code_predict=sv_x_code_predict,
                                num_samples=num_samples)
```

In [ ]:
```
# 6) SV X + Peak hour + Price zone 2
model_sv_x_peak_sib = StanModel(kind='sv_x',
                                name='SV X + Peak hour + Price zone 2',
                                stan_code_fit=sv_x_code_fit,
                                stan_code_predict=sv_x_code_predict,
                                num_samples=num_samples)
```

In [ ]:
```
# 7) SV X + Off-peak hour + Price zone 1
model_sv_x_offpeak_eur = StanModel(kind='sv_x',
                                name='SV X + Off-peak hour + Price zone 1',
                                stan_code_fit=sv_x_code_fit,
                                stan_code_predict=sv_x_code_predict,
                                num_samples=num_samples)
```

In [ ]:
```
# 8) SV X + Off-peak hour + Price zone 2
model_sv_x_offpeak_sib = StanModel(kind='sv_x',
                                name='SV X + Off-peak hour + Price zone 2',
                                stan_code_fit=sv_x_code_fit,
                                stan_code_predict=sv_x_code_predict,
                                num_samples=num_samples)
```

In [ ]:
```
n_windows = 36 # Number of sliding windows
ts_cv_sv_x = TimeSeriesSplit(n_splits=n_windows, max_train_size=30*12, test_size
```

In [ ]:
```
print('Split # | Train fold                    | Test fold')
print('--------|-------------------------------|-------------------------------'
# Due to air temperature having missing data, the 4 subsets are not strictly equ
# So get split indices from any of the four
for i, split in enumerate(ts_cv_sv_x.split(price_temp_data_peak_eur)):
    train_start = price_temp_data_peak_eur.iloc[split[0]].index.min().date()
    train_end = price_temp_data_peak_eur.iloc[split[0]].index.max().date()
    test_start = price_temp_data_peak_eur.iloc[split[1]].index.min().date()
    test_end = price_temp_data_peak_eur.iloc[split[1]].index.max().date()
    print(f'{i + 1:>7} | {train_start} -- {train_end} ({(train_end - train_start
```

```
Split # | Train fold                      | Test fold
--------|--------------------------------|-----------------------------
      1 | 2014-06-23 -- 2015-06-09 (352) | 2015-06-10 -- 2015-09-10 (93)
      2 | 2014-09-13 -- 2015-09-10 (363) | 2015-09-11 -- 2015-12-10 (91)
      3 | 2014-12-12 -- 2015-12-10 (364) | 2015-12-11 -- 2016-03-09 (90)
      4 | 2015-03-12 -- 2016-03-09 (364) | 2016-03-10 -- 2016-06-08 (91)
      5 | 2015-06-10 -- 2016-06-08 (365) | 2016-06-09 -- 2016-09-07 (91)
      6 | 2015-09-11 -- 2016-09-07 (363) | 2016-09-08 -- 2016-12-06 (90)
      7 | 2015-12-11 -- 2016-12-06 (362) | 2016-12-07 -- 2017-03-06 (90)
      8 | 2016-03-10 -- 2017-03-06 (362) | 2017-03-07 -- 2017-06-04 (90)
      9 | 2016-06-09 -- 2017-06-04 (361) | 2017-06-06 -- 2017-09-03 (90)
     10 | 2016-09-08 -- 2017-09-03 (361) | 2017-09-04 -- 2017-12-02 (90)
     11 | 2016-12-07 -- 2017-12-02 (361) | 2017-12-03 -- 2018-03-02 (90)
     12 | 2017-03-07 -- 2018-03-02 (361) | 2018-03-03 -- 2018-05-31 (90)
     13 | 2017-06-06 -- 2018-05-31 (360) | 2018-06-01 -- 2018-08-29 (90)
     14 | 2017-09-04 -- 2018-08-29 (360) | 2018-08-30 -- 2018-11-27 (90)
     15 | 2017-12-03 -- 2018-11-27 (360) | 2018-11-28 -- 2019-02-25 (90)
     16 | 2018-03-03 -- 2019-02-25 (360) | 2019-02-26 -- 2019-05-26 (90)
     17 | 2018-06-01 -- 2019-05-26 (360) | 2019-05-27 -- 2019-08-24 (90)
     18 | 2018-08-30 -- 2019-08-24 (360) | 2019-08-25 -- 2019-11-22 (90)
     19 | 2018-11-28 -- 2019-11-22 (360) | 2019-11-23 -- 2020-02-20 (90)
     20 | 2019-02-26 -- 2020-02-20 (360) | 2020-02-21 -- 2020-05-20 (90)
     21 | 2019-05-27 -- 2020-05-20 (360) | 2020-05-21 -- 2020-08-18 (90)
     22 | 2019-08-25 -- 2020-08-18 (360) | 2020-08-19 -- 2020-11-16 (90)
     23 | 2019-11-23 -- 2020-11-16 (360) | 2020-11-17 -- 2021-02-14 (90)
     24 | 2020-02-21 -- 2021-02-14 (360) | 2021-02-15 -- 2021-05-15 (90)
     25 | 2020-05-21 -- 2021-05-15 (360) | 2021-05-16 -- 2021-08-13 (90)
     26 | 2020-08-19 -- 2021-08-13 (360) | 2021-08-14 -- 2021-11-11 (90)
     27 | 2020-11-17 -- 2021-11-11 (360) | 2021-11-12 -- 2022-02-09 (90)
     28 | 2021-02-15 -- 2022-02-09 (360) | 2022-02-10 -- 2022-05-10 (90)
     29 | 2021-05-16 -- 2022-05-10 (360) | 2022-05-11 -- 2022-08-08 (90)
     30 | 2021-08-14 -- 2022-08-08 (360) | 2022-08-09 -- 2022-11-06 (90)
     31 | 2021-11-12 -- 2022-11-06 (360) | 2022-11-07 -- 2023-02-04 (90)
     32 | 2022-02-10 -- 2023-02-04 (360) | 2023-02-05 -- 2023-05-05 (90)
     33 | 2022-05-11 -- 2023-05-05 (360) | 2023-05-06 -- 2023-08-04 (91)
     34 | 2022-08-09 -- 2023-08-04 (361) | 2023-08-05 -- 2023-11-02 (90)
     35 | 2022-11-07 -- 2023-11-02 (361) | 2023-11-03 -- 2024-01-31 (90)
     36 | 2023-02-05 -- 2024-01-31 (361) | 2024-02-01 -- 2024-04-30 (90)
```

In [ ]:

```
%%capture
models = [model_sv_x_peak_eur, model_sv_x_peak_sib, model_sv_x_offpeak_eur, mode
subsets = [price_temp_data_peak_eur, price_temp_data_peak_sib, price_temp_data_o
cv_results_sv_x_df = pd.DataFrame(columns=['model_kind', 'model_name', 'train_in
print('Running cross-validation...')
for model, subset in tqdm(zip(models, subsets)):
    print(f'Model: {model.name}')
    cv_result = cross_validate(model,
                               X=subset[['Temperature', 'Weekday']],
                               y=subset['CONSUMER_PRICE'],
                               cv=ts_cv_sv_x,
                               scoring=['neg_mean_absolute_error', 'neg_root_me
                               return_indices=True,
                               n_jobs=4)
    cv_result_df = pd.concat([pd.Series(cv_result['indices']['train'], name='tra
                              pd.Series(cv_result['indices']['test'], name='test
                              pd.Series(cv_result['fit_time'], name='fit_time'),
                              pd.Series(cv_result['score_time'], name='score_tim
                              pd.Series(-cv_result['test_neg_mean_absolute_error
                              pd.Series(-cv_result['test_neg_root_mean_squared_e
    cv_results_sv_x_df = pd.concat([cv_results_sv_x_df,
```

```
                                        pd.concat([pd.Series([model.kind] * n_window
                                                   pd.Series([model.name] * n_window
                                                   cv_result_df], axis=1)])
cv_results_sv_x_df = cv_results_sv_x_df.reset_index(drop=True)
print('Done!')
```

In [ ]:
```
cv_results_sv_x_df
```

Out[ ]:

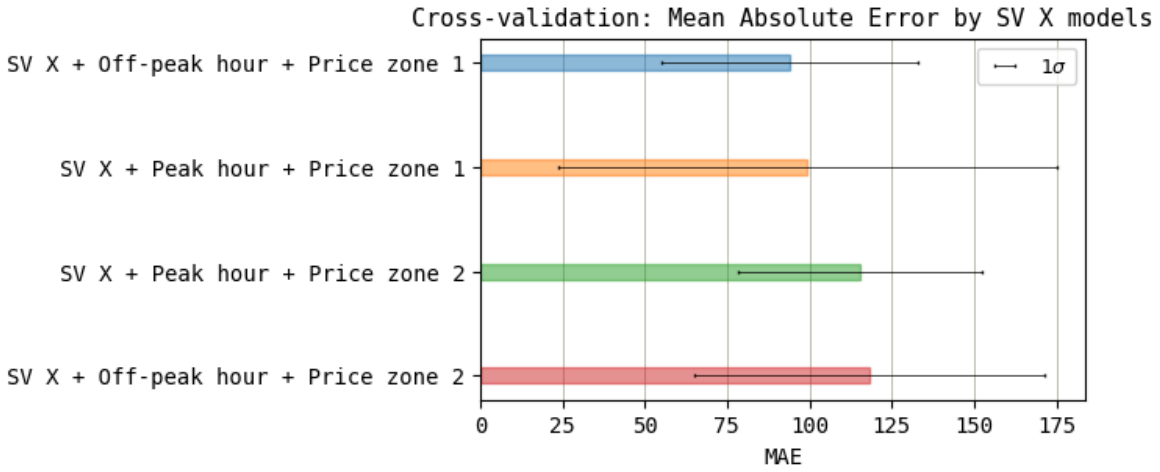
	model_kind	model_name	train_indices	test_indices	fit_time	score_time	test_
0	sv_x	SV X + Peak hour + Price zone 1	[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,...	[352, 353, 354, 355, 356, 357, 358, 359, 360, ...	62.712077	0.207005	59.32
1	sv_x	SV X + Peak hour + Price zone 1	[82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 9...	[442, 443, 444, 445, 446, 447, 448, 449, 450, ...	65.499112	0.212741	78.68
2	sv_x	SV X + Peak hour + Price zone 1	[172, 173, 174, 175, 176, 177, 178, 179, 180, ...	[532, 533, 534, 535, 536, 537, 538, 539, 540, ...	70.215640	0.211465	103.48
3	sv_x	SV X + Peak hour + Price zone 1	[262, 263, 264, 265, 266, 267, 268, 269, 270, ...	[622, 623, 624, 625, 626, 627, 628, 629, 630, ...	59.226787	0.230045	77.41
4	sv_x	SV X + Peak hour + Price zone 1	[352, 353, 354, 355, 356, 357, 358, 359, 360, ...	[712, 713, 714, 715, 716, 717, 718, 719, 720, ...	62.684058	0.202945	107.45
...	...	...	...	...	...	...	
139	sv_x	SV X + Off-peak hour + Price zone 2	[2760, 2761, 2762, 2763, 2764, 2765, 2766, 276...	[3120, 3121, 3122, 3123, 3124, 3125, 3126, 312...	48.887254	0.222090	247.34
140	sv_x	SV X + Off-peak hour + Price zone 2	[2850, 2851, 2852, 2853, 2854, 2855, 2856, 285...	[3210, 3211, 3212, 3213, 3214, 3215, 3216, 321...	65.654209	0.222598	171.59
141	sv_x	SV X + Off-peak hour + Price zone 2	[2940, 2941, 2942, 2943, 2944, 2945, 2946, 294...	[3300, 3301, 3302, 3303, 3304, 3305, 3306, 330...	52.050411	0.197723	182.41
142	sv_x	SV X + Off-peak hour + Price zone 2	[3030, 3031, 3032, 3033, 3034, 3035, 3036, 303...	[3390, 3391, 3392, 3393, 3394, 3395, 3396, 339...	52.940592	0.211724	97.46
143	sv_x	SV X + Off-peak hour + Price zone 2	[3120, 3121, 3122, 3123, 3124, 3125, 3126, 312...	[3480, 3481, 3482, 3483, 3484, 3485, 3486, 348...	46.458723	0.201555	101.64

144 rows × 8 columns

In [ ]:
```
# Mean MAE by model
mae_sv_x_cross = cv_results_sv_x_df.groupby('model_name')['test_mae']
mae_sv_x_cross.mean().sort_values()
```

Out[ ]:
```
model_name
SV X + Off-peak hour + Price zone 1     93.920306
SV X + Peak hour + Price zone 1         99.451586
SV X + Peak hour + Price zone 2        115.563463
SV X + Off-peak hour + Price zone 2    118.180067
Name: test_mae, dtype: float64
```

In [ ]:
```
plt.figure(figsize=(5, 3))
bars = plt.barh(range(mae_sv_x_cross.mean().shape[0]), mae_sv_x_cross.mean().sor
plt.gca().invert_yaxis()
plt.xlabel('MAE', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(range(mae_sv_x_cross.mean().shape[0]), mae_sv_x_cross.mean().sort_val
plt.grid(axis='x', lw=0.5, color='xkcd:cement')
for i in range(len(bars)):
    bars[i].set_edgecolor(f'C{i}')
    bars[i].set_color(f'C{i}')
plt.gca().set_axisbelow(True)
plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
plt.title('Cross-validation: Mean Absolute Error by SV X models', size=11, famil
```

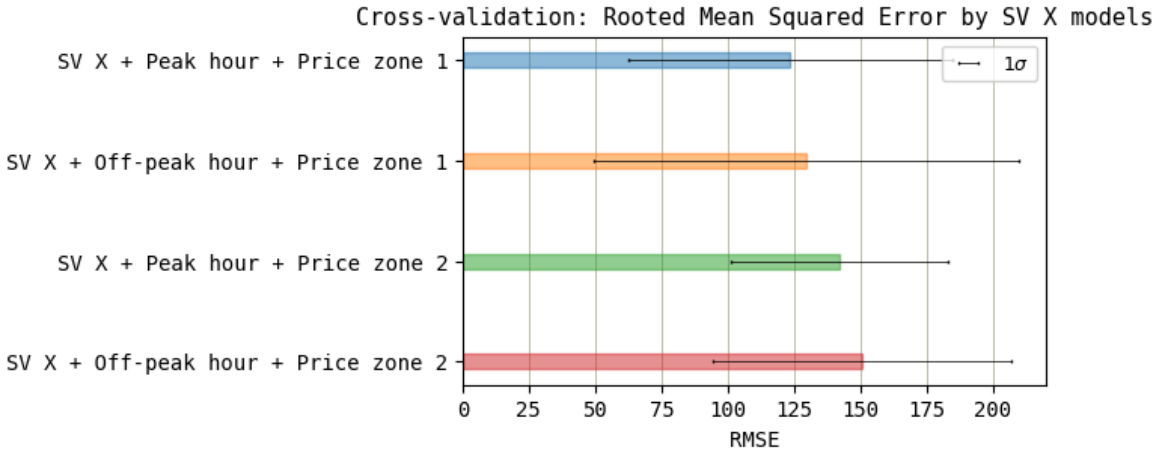


In [ ]:
```
# Mean RMSE by model
rmse_sv_x_cross = cv_results_sv_x_df.groupby('model_name')['test_rmse']
rmse_sv_x_cross.mean().sort_values()
```

Out[ ]:
```
model_name
SV X + Peak hour + Price zone 1        123.588498
SV X + Off-peak hour + Price zone 1    129.588961
SV X + Peak hour + Price zone 2        142.098038
SV X + Off-peak hour + Price zone 2    150.729970
Name: test_rmse, dtype: float64
```

In [ ]:
```
plt.figure(figsize=(5, 3))
bars = plt.barh(range(rmse_sv_x_cross.mean().shape[0]), rmse_sv_x_cross.mean().s
plt.gca().invert_yaxis()
plt.xlabel('RMSE', size=10, family='monospace')
plt.xticks(size=10, family='monospace')
plt.yticks(range(rmse_sv_x_cross.mean().shape[0]), rmse_sv_x_cross.mean().sort_v
plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
plt.grid(axis='x', lw=0.5, color='xkcd:cement')
```

```
for i in range(len(bars)):
    bars[i].set_edgecolor(f'C{i}')
    bars[i].set_color(f'C{i}')
plt.gca().set_axisbelow(True)
plt.title('Cross-validation: Rooted Mean Squared Error by SV X models', size=11,
```



As with SV Baseline model, both metrics for SV X model show that on average:

- models for Price zone 1 (European) have higher quality than for Price zone 2 (Siberian);
- the best model is for Price zone 1 for Peak hour;
- the worst model is for Price zone 2 for Off-peak hour.

4.3. Model Comparison

Let's compare the distributions of MAE and RMSE metrics obtained during cross-validation.

In [ ]:
```
cv_results_sv_all_df = pd.concat([cv_results_sv_base_df, cv_results_sv_x_df], ax
cv_results_sv_all_df
```

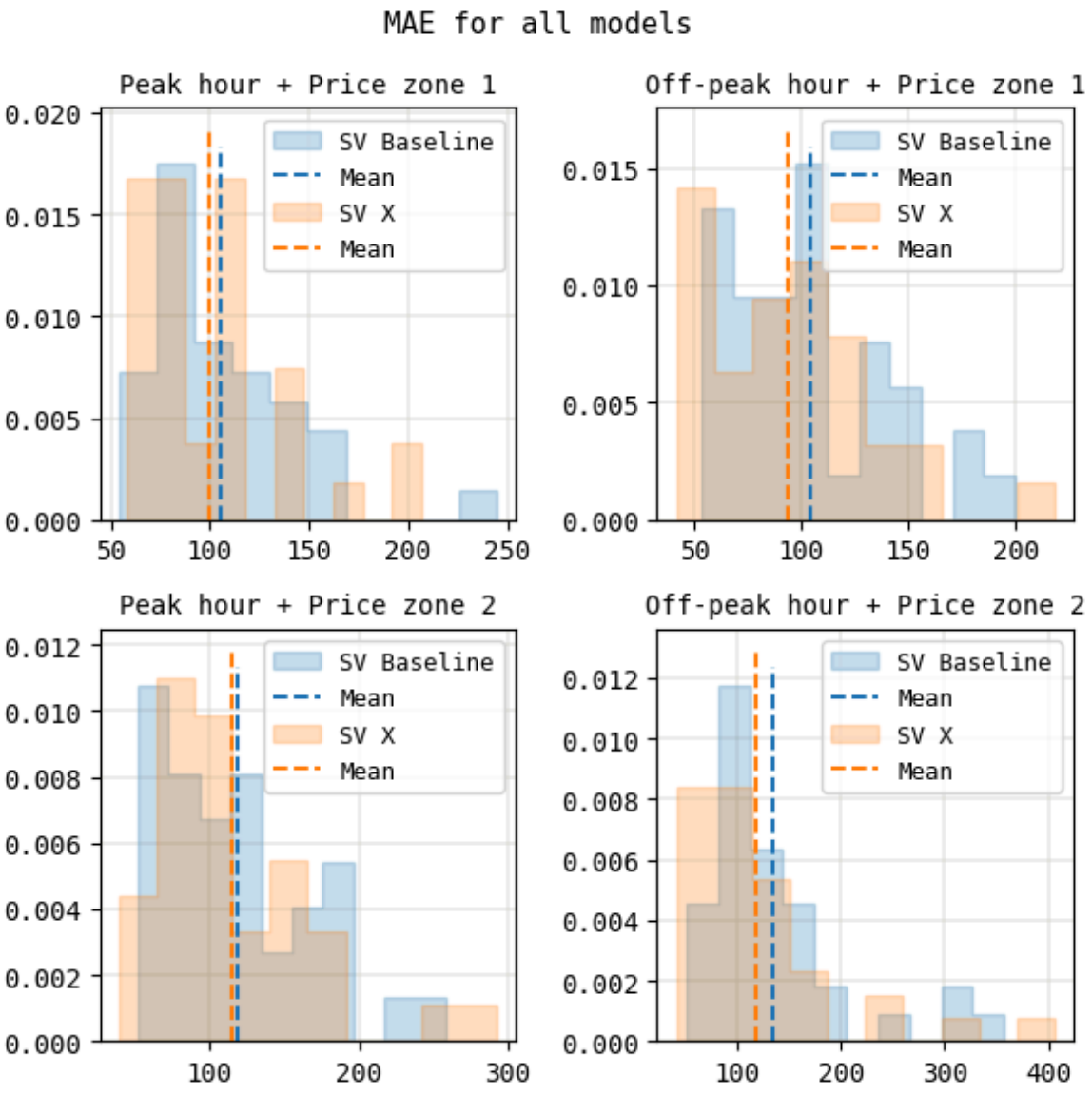
Out[ ]:

	model_kind	model_name	train_indices	test_indices	fit_time	score_time	test_
0	sv_base	SV Baseline + Peak hour + Price zone 1	[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,...	[360, 361, 362, 363, 364, 365, 366, 367, 368, ...	6.997288	0.245346	66.93
1	sv_base	SV Baseline + Peak hour + Price zone 1	[90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, ...	[450, 451, 452, 453, 454, 455, 456, 457, 458, ...	6.366097	0.221779	91.49
2	sv_base	SV Baseline + Peak hour + Price zone 1	[180, 181, 182, 183, 184, 185, 186, 187, 188, ...	[540, 541, 542, 543, 544, 545, 546, 547, 548, ...	10.321057	0.243930	98.05
3	sv_base	SV Baseline + Peak hour + Price zone 1	[270, 271, 272, 273, 274, 275, 276, 277, 278, ...	[630, 631, 632, 633, 634, 635, 636, 637, 638, ...	6.422443	0.218006	90.65
4	sv_base	SV Baseline + Peak hour + Price zone 1	[360, 361, 362, 363, 364, 365, 366, 367, 368, ...	[720, 721, 722, 723, 724, 725, 726, 727, 728, ...	5.611288	0.209697	144.87
...	...	...	...	...	...	...	
139	sv_x	SV X + Off-peak hour + Price zone 2	[2760, 2761, 2762, 2763, 2764, 2765, 2766, 276...	[3120, 3121, 3122, 3123, 3124, 3125, 3126, 312...	48.887254	0.222090	247.34
140	sv_x	SV X + Off-peak hour + Price zone 2	[2850, 2851, 2852, 2853, 2854, 2855, 2856, 285...	[3210, 3211, 3212, 3213, 3214, 3215, 3216, 321...	65.654209	0.222598	171.59
141	sv_x	SV X + Off-peak hour + Price zone 2	[2940, 2941, 2942, 2943, 2944, 2945, 2946, 294...	[3300, 3301, 3302, 3303, 3304, 3305, 3306, 330...	52.050411	0.197723	182.41
142	sv_x	SV X + Off-peak hour + Price zone 2	[3030, 3031, 3032, 3033, 3034, 3035, 3036, 303...	[3390, 3391, 3392, 3393, 3394, 3395, 3396, 339...	52.940592	0.211724	97.46
143	sv_x	SV X + Off-peak hour + Price zone 2	[3120, 3121, 3122, 3123, 3124, 3125, 3126, 312...	[3480, 3481, 3482, 3483, 3484, 3485, 3486, 348...	46.458723	0.201555	101.64

288 rows × 8 columns

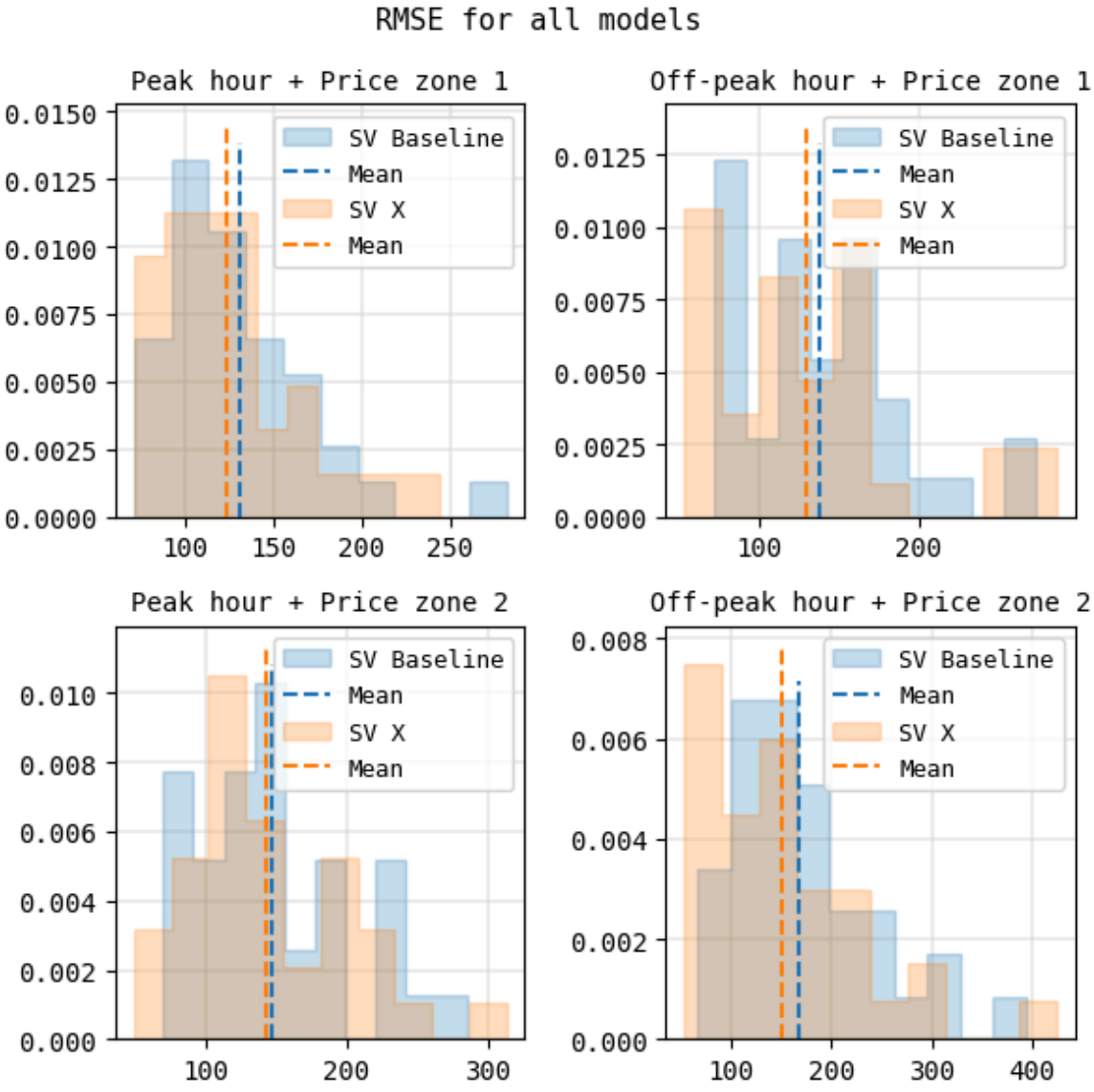
In [ ]:
```
cv_results_peak_eur = cv_results_sv_all_df[cv_results_sv_all_df['model_name'].st
cv_results_offpeak_eur = cv_results_sv_all_df[cv_results_sv_all_df['model_name']
cv_results_peak_sib = cv_results_sv_all_df[cv_results_sv_all_df['model_name'].st
cv_results_offpeak_sib = cv_results_sv_all_df[cv_results_sv_all_df['model_name']
```

In [ ]:
```
plt.figure(figsize=(6, 6))
for i, subset in zip(range(1, 5), [cv_results_peak_eur, cv_results_offpeak_eur,
    plt.subplot(2, 2, i)
    plt.hist(subset[subset['model_kind'] == 'sv_base']['test_mae'], bins=10, den
    plt.vlines(subset[subset['model_kind'] == 'sv_base']['test_mae'].mean(), ymi
    plt.hist(subset[subset['model_kind'] == 'sv_x']['test_mae'], bins=10, densit
    plt.vlines(subset[subset['model_kind'] == 'sv_x']['test_mae'].mean(), ymin=0
    plt.xticks(size=10, family='monospace')
    plt.yticks(size=10, family='monospace')
    plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
    plt.title(f"{' + '.join(subset['model_name'].unique()[0].split(' + ')[1:])}"
    plt.grid(lw=0.25, color='xkcd:cement')
    plt.gca().set_axisbelow(True)
plt.suptitle(f'MAE for all models', size=11, family='monospace')
plt.tight_layout();
```



In [ ]:
```
plt.figure(figsize=(6, 6))
for i, subset in zip(range(1, 5), [cv_results_peak_eur, cv_results_offpeak_eur,
```

```
    plt.subplot(2, 2, i)
    plt.hist(subset[subset['model_kind'] == 'sv_base']['test_rmse'], bins=10, de
    plt.vlines(subset[subset['model_kind'] == 'sv_base']['test_rmse'].mean(), ym
    plt.hist(subset[subset['model_kind'] == 'sv_x']['test_rmse'], bins=10, densi
    plt.vlines(subset[subset['model_kind'] == 'sv_x']['test_rmse'].mean(), ymin=
    plt.xticks(size=10, family='monospace')
    plt.yticks(size=10, family='monospace')
    plt.legend(loc='upper right', prop={'size': 9, 'family': 'monospace'})
    plt.title(f"{' + '.join(subset['model_name'].unique()[0].split(' + ')[1:])}"
    plt.grid(lw=0.25, color='xkcd:cement')
    plt.gca().set_axisbelow(True)
plt.suptitle(f'RMSE for all models', size=11, family='monospace')
plt.tight_layout();
```



We should also check the statistical significance of the difference in the metrics. We will use a non-parametric one-tailed Mann-Whitney U test for two independent samples to see if the distributions of metrics for SV Baseline and SV X models are identical or have a shift between each other.

$\mathcal{H}_0 : F_{X_1}(x) = F_{X_2}(x)$ (distributed identically)

$\mathcal{H}_1 : F_{X_1}(x) = F_{X_2}(x + \Delta x)$ (distributed with some shift Δx), where

$F_{X_i}(x)$ is the distribution of a given metric for a given model.

Statistic: Mann-Whitney's U

Null distribution: no assumption; generated empirically by permutation test; for large samples it can be approximated by $\mathcal{N}(\mu = \frac{n_1 n_2}{2}, \sigma^2 = \frac{n_1 n_2 (n_1+n_2+1)}{12})$.

In [ ]:
```
# Two independent samples one-tailed Mann-Whitney U test for MAE
alpha = 0.05
mwu_stats, mwu_pval = mannwhitneyu(cv_results_sv_base_df['test_mae'], cv_results
print(f'{mwu_stats = :.0f}, {mwu_pval = :.4f}', end=', ')
print("SV Baseline's MAE is greater than SV X's MAE (shifted to the right = wors
```

```
mwu_stats = 11785, mwu_pval = 0.0225, SV Baseline's MAE is greater than SV X's MA
E (shifted to the right = worse)
```

In [ ]:
```
# Two independent samples one-tailed Mann-Whitney U test for RMSE
alpha = 0.05
mwu_stats, mwu_pval = mannwhitneyu(cv_results_sv_base_df['test_rmse'], cv_result
print(f'{mwu_stats = :.0f}, {mwu_pval = :.4f}', end=', ')
print("SV Baseline's RMSE is greater than SV X's RMSE (shifted to the right = wo
```

```
mwu_stats = 11575, mwu_pval = 0.0439, SV Baseline's RMSE is greater than SV X's R
MSE (shifted to the right = worse)
```

The statistical significance of difference in both metrics, obtained during cross-validation, between SV Baseline and SV X models *can* be confirmed by hypothesis testing with one-tailed Mann-Whitney U test fro two independent samples.

Metrics for both models have heavy right tails, which signals that the models were not good for all splits. This might be due to market regime change or other factors.

In [ ]:
```
print(' Train / test fold with best RMSE:', price_temp_data_peak_eur.iloc[cv_res
'--', price_temp_data_peak_eur.iloc[cv_results_sv_base_df[cv_results_sv_base_df[

print(price_temp_data_peak_eur.iloc[cv_results_sv_base_df[cv_results_sv_base_df[
'--', price_temp_data_peak_eur.iloc[cv_results_sv_base_df[cv_results_sv_base_df[

print('Train / test fold with worst RMSE:', price_temp_data_peak_eur.iloc[cv_res
'--', price_temp_data_peak_eur.iloc[cv_results_sv_base_df[cv_results_sv_base_df[

print(price_temp_data_peak_eur.iloc[cv_results_sv_base_df[cv_results_sv_base_df[
'--', price_temp_data_peak_eur.iloc[cv_results_sv_base_df[cv_results_sv_base_df[
```

```
 Train / test fold with best RMSE: 2016-09-16 -- 2017-09-11 / 2017-09-12 -- 2017-
12-10
Train / test fold with worst RMSE: 2018-09-07 -- 2019-09-01 / 2019-09-02 -- 2019-
11-30
```

In [ ]:
```
cv_results_peak_eur.groupby('model_kind')[['test_mae', 'test_rmse']].mean().T.st
```

Out[ ]:

Peak hour + Price zone 1

model_kind	sv_base	sv_x
test_mae	105.385055	99.451586
test_rmse	130.526412	123.588498

In [ ]:
```
cv_results_peak_sib.groupby('model_kind')[['test_mae', 'test_rmse']].mean().T.st
```

Out[ ]: Peak hour + Price zone 2

model_kind	sv_base	sv_x
test_mae	119.140971	115.563463
test_rmse	146.445320	142.098038

In [ ]:
```
cv_results_offpeak_eur.groupby('model_kind')[['test_mae', 'test_rmse']].mean().T
```

Out[ ]: Off-peak hour + Price zone 1

model_kind	sv_base	sv_x
test_mae	104.422811	93.920306
test_rmse	137.501447	129.588961

In [ ]:
```
cv_results_offpeak_sib.groupby('model_kind')[['test_mae', 'test_rmse']].mean().T
```

Out[ ]: Off-peak hour + Price zone 2

model_kind	sv_base	sv_x
test_mae	134.863451	118.180067
test_rmse	168.305299	150.729970

In [ ]:
```
# MAE % improvement
print(f"{cv_results_sv_base_df['test_mae'].mean() / cv_results_sv_x_df['test_mae
```

```
8.59%
```

In [ ]:
```
# RMSE % improvement
print(f"{cv_results_sv_base_df['test_rmse'].mean() / cv_results_sv_x_df['test_rm
```

```
6.73%
```

On average, all four SV X models have better MAE (-8.59%) and RMSE (-6.73%) metrics as compared with SV Baseline models obtained during cross-validation. The statistical significance of this difference *can* be confirmed by hypothesis testing with one-tailed Mann-Whitney U test for two independent samples: $\mathcal{H}_0$ can be rejected at $\alpha = 5\%$.

Back

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5. Forecasting

Finally, we would like to generate forecasts for the nearest day(s) ahead with our models. Just like with cross-validation, we will generate 8 forecasts:

1. SV Baseline + Peak hour + Price zone 1 (European);
2. SV Baseline + Peak hour + Price zone 2 (Siberian);

3. SV Baseline + Off-peak hour + Price zone 1;
4. SV Baseline + Off-peak hour + Price zone 2;
5. SV X + Peak hour + Price zone 1;
6. SV X + Peak hour + Price zone 2;
7. SV X + Off-peak hour + Price zone 1;
8. SV X + Off-peak hour + Price zone 2;

Forecast strategy:

1. Fit the model on a time frame right before the first forecasted day;
2. Generate one day-ahead prediction (forecast);
3. Move the time frame forward by one day;
4. Repeat from 1 for as many days as required.

First train time frame: 01.05.2023 -- 30.04.2024 (same as used for modeling in [2] and [3]).

Forecast time frame: 01.05.2024 -- 07.05.2024 (1 week).

In [ ]:
```
price_data = pd.read_csv('./data/price_data_20230501_20240507.zip', index_col='D
```

In [ ]:
```
# Price zone 1: Moscow
temp_data_eur = pd.read_csv(f'./data/UUEE.01.05.2023.07.05.2024.1.0.0.ru.utf8.00
                            sep=';',
                            encoding='utf-8',
                            skiprows=7,
                            index_col=0,
                            dayfirst=True,
                            usecols=[0, 1],
                            names=['Datetime', 'Temperature'],
                            parse_dates=True).dropna().sort_index()
```

In [ ]:
```
# Price zone 2: Novosibirsk
temp_data_sib = pd.read_csv(f'./data/UNNT.01.05.2023.07.05.2024.1.0.0.ru.utf8.00
                            sep=';',
                            encoding='utf-8',
                            skiprows=7,
                            index_col=0,
                            dayfirst=True,
                            usecols=[0, 1],
                            names=['Datetime', 'Temperature'],
                            parse_dates=True).dropna().sort_index()
# Don't forget to convert local Novosibirsk time to Moscow time, since all price
temp_data_sib.index = temp_data_sib.index - pd.Timedelta(hours=4)
```

In [ ]:
```
price_data_eur = price_data[price_data['PRICE_ZONE_CODE'] == 1]
price_data_sib = price_data[price_data['PRICE_ZONE_CODE'] == 2]
```

In [ ]:
```
price_data_daily_agg_eur = price_data_eur.groupby('HOUR')['CONSUMER_PRICE'].mean
price_data_daily_agg_sib = price_data_sib.groupby('HOUR')['CONSUMER_PRICE'].mean
hour_max_eur = price_data_daily_agg_eur.idxmax()
hour_min_eur = price_data_daily_agg_eur.idxmin()
hour_max_sib = price_data_daily_agg_sib.idxmax()
hour_min_sib = price_data_daily_agg_sib.idxmin()
(hour_max_eur, hour_min_eur), (hour_max_sib, hour_min_sib)
```

Out[ ]:
```
((14, 3), (17, 1))
```

In [ ]:
```
price_data_peak_eur = price_data_eur[price_data_eur['HOUR'] == hour_max_eur]
price_data_offpeak_eur = price_data_eur[price_data_eur['HOUR'] == hour_min_eur]
price_data_peak_sib = price_data_sib[price_data_sib['HOUR'] == hour_max_sib]
price_data_offpeak_sib = price_data_sib[price_data_sib['HOUR'] == hour_min_sib]
```

In [ ]:
```
price_temp_data_peak_eur = pd.merge(price_data_peak_eur, temp_data_eur, on='Date
price_temp_data_offpeak_eur = pd.merge(price_data_offpeak_eur, temp_data_eur, on
price_temp_data_peak_sib = pd.merge(price_data_peak_sib, temp_data_sib, on='Date
price_temp_data_offpeak_sib = pd.merge(price_data_offpeak_sib, temp_data_sib, on
```

In [ ]:
```
%%capture
# Peak hour + Price zone 1
forecast_sv_base_peak_eur_many = model_sv_base_peak_eur.forecast_many(X=price_te
```

In [ ]:
```
%%capture
forecast_sv_x_peak_eur_many = model_sv_x_peak_eur.forecast_many(X=price_temp_dat
```

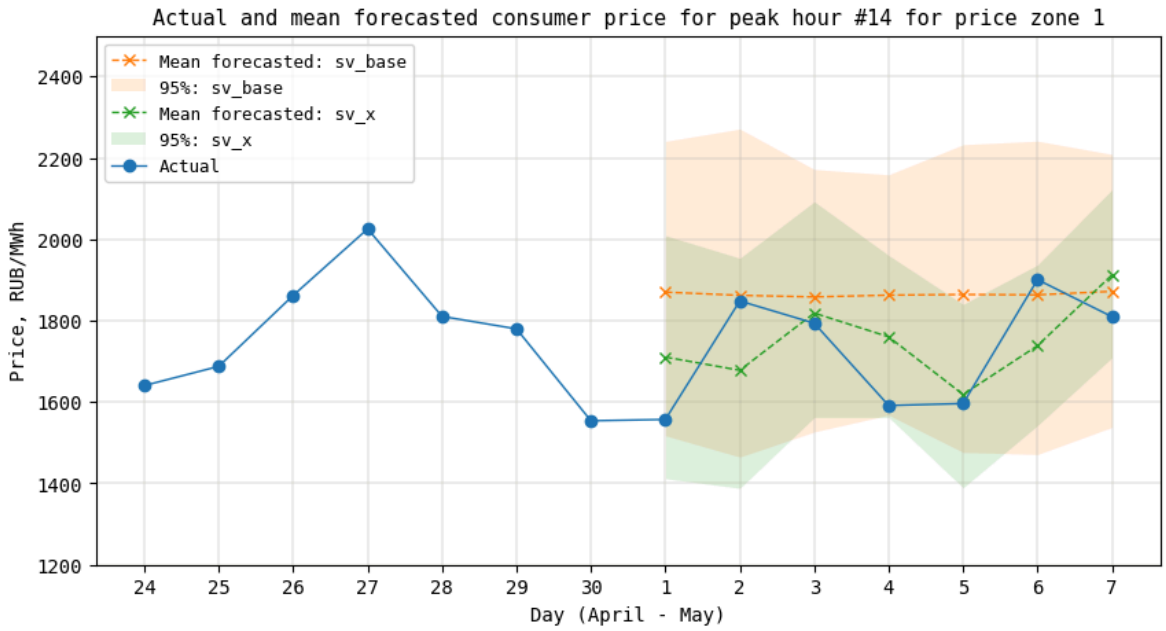
In [ ]:
```
pd.DataFrame({'sv_base': [
    mean_absolute_error(price_temp_data_peak_eur['CONSUMER_PRICE']['2024-05-01':
    root_mean_squared_error(price_temp_data_peak_eur['CONSUMER_PRICE']['2024-05-
    'sv_x': [mean_absolute_error(price_temp_data_peak_eur['CONSUMER_PRICE']['202
    root_mean_squared_error(price_temp_data_peak_eur['CONSUMER_PRICE']['2024-05-
    index=['mae', 'rmse']).style.highlight_min(color='palegreen', axis=1).set_ca
```

Out[ ]:

Peak hour + Price zone 1

	sv_base	sv_x
mae	147.187481	114.891628
rmse	190.090666	130.354028

In [ ]:
```
plt.figure(figsize=(10, 5))
for i, forecast, model in zip([1, 2], [forecast_sv_base_peak_eur_many, forecast_
    plt.plot(price_temp_data_peak_eur.loc['2024-05-01':]['CONSUMER_PRICE'].index
    plt.fill_between(price_temp_data_peak_eur.loc['2024-05-01':]['CONSUMER_PRICE
plt.plot(price_temp_data_peak_eur.loc['2024-04-24':]['CONSUMER_PRICE'].index, pr
plt.xlabel('Day (April - May)', size=10, family='monospace')
plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
plt.xticks(price_temp_data_peak_eur.loc['2024-04-24':]['CONSUMER_PRICE'].index,
plt.yticks(size=10, family='monospace')
plt.ylim([1200, 2500])
plt.title(f'Actual and mean forecasted consumer price for peak hour #{hour_max_e
plt.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'});
plt.grid(lw=0.25, color='xkcd:cement');
```



In [ ]:
```
%%capture
# Peak hour + Price zone 2
forecast_sv_base_peak_sib_many = model_sv_base_peak_sib.forecast_many(X=price_te
```

In [ ]:
```
%%capture
forecast_sv_x_peak_sib_many = model_sv_x_peak_sib.forecast_many(X=price_temp_dat
```

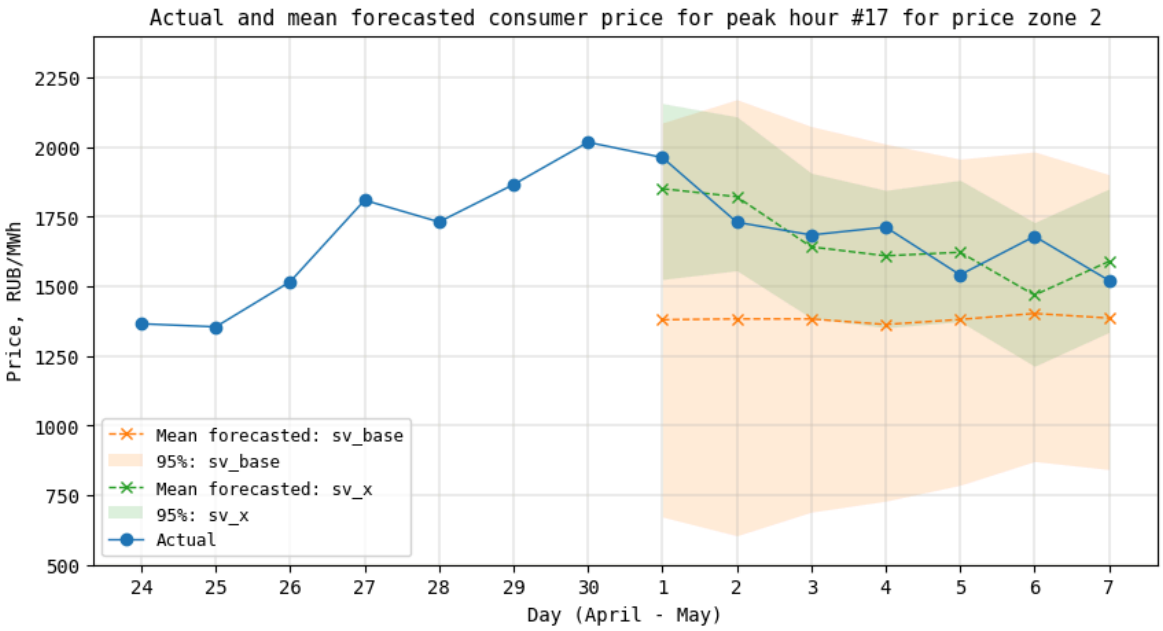
In [ ]:
```
pd.DataFrame({'sv_base': [
    mean_absolute_error(price_temp_data_peak_sib['CONSUMER_PRICE']['2024-05-01':
    root_mean_squared_error(price_temp_data_peak_sib['CONSUMER_PRICE']['2024-05-
    'sv_x': [mean_absolute_error(price_temp_data_peak_sib['CONSUMER_PRICE']['202
    root_mean_squared_error(price_temp_data_peak_sib['CONSUMER_PRICE']['2024-05-
    index=['mae', 'rmse']).style.highlight_min(color='palegreen', axis=1).set_ca
```

Out[ ]:

Peak hour + Price zone 2

	sv_base	sv_x
mae	307.673757	101.415521
rmse	336.820693	112.829166

In [ ]:
```
plt.figure(figsize=(10, 5))
for i, forecast, model in zip([1, 2], [forecast_sv_base_peak_sib_many, forecast_
    plt.plot(price_temp_data_peak_sib.loc['2024-05-01':]['CONSUMER_PRICE'].index
    plt.fill_between(price_temp_data_peak_sib.loc['2024-05-01':]['CONSUMER_PRICE
plt.plot(price_temp_data_peak_sib.loc['2024-04-24':]['CONSUMER_PRICE'].index, pr
plt.xlabel('Day (April - May)', size=10, family='monospace')
plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
plt.xticks(price_temp_data_peak_sib.loc['2024-04-24':]['CONSUMER_PRICE'].index,
plt.yticks(size=10, family='monospace')
plt.ylim([500, 2400])
plt.title(f'Actual and mean forecasted consumer price for peak hour #{hour_max_s
plt.legend(loc='lower left', prop={'size': 9, 'family': 'monospace'});
plt.grid(lw=0.25, color='xkcd:cement');
```



In [ ]:
```
%%capture
# Off-peak hour + Price zone 1
forecast_sv_base_offpeak_eur_many = model_sv_base_offpeak_eur.forecast_many(X=pr
```

In [ ]:
```
%%capture
forecast_sv_x_offpeak_eur_many = model_sv_x_offpeak_eur.forecast_many(X=price_te
```

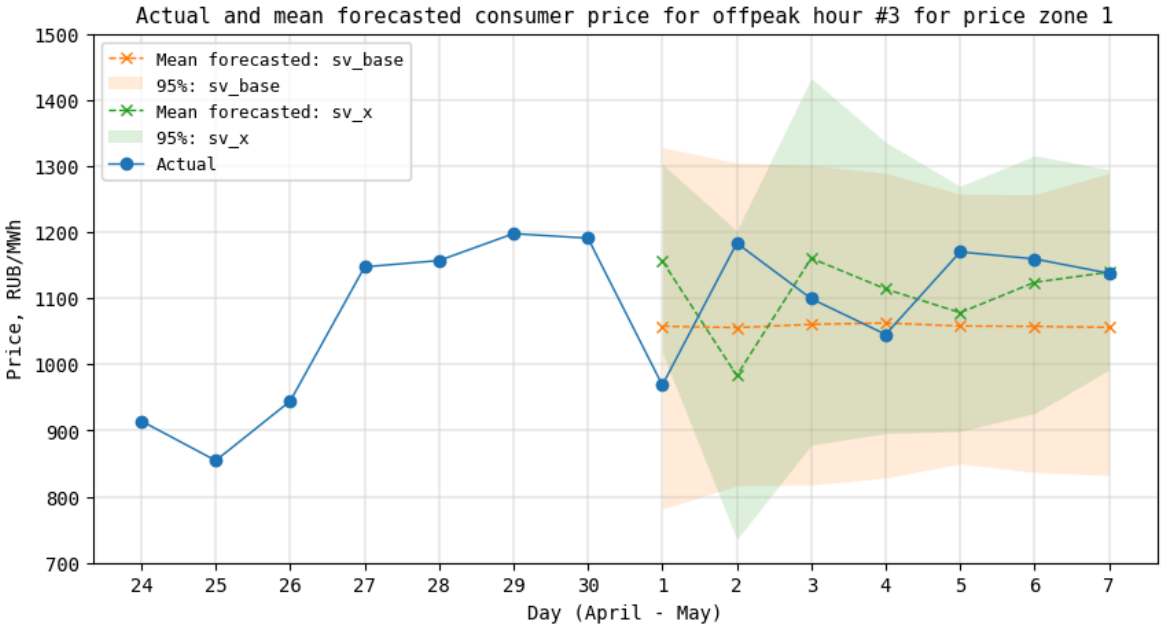
In [ ]:
```
pd.DataFrame({'sv_base': [
    mean_absolute_error(price_temp_data_offpeak_eur['CONSUMER_PRICE']['2024-05-0
    root_mean_squared_error(price_temp_data_offpeak_eur['CONSUMER_PRICE']['2024-
    'sv_x': [mean_absolute_error(price_temp_data_offpeak_eur['CONSUMER_PRICE']['
    root_mean_squared_error(price_temp_data_offpeak_eur['CONSUMER_PRICE']['2024-
    index=['mae', 'rmse']).style.highlight_min(color='palegreen', axis=1).set_ca
```

Out[ ]:

Off-peak hour + Price zone 1

	sv_base	sv_x
mae	81.280877	92.423632
rmse	89.244318	115.602406

In [ ]:
```
plt.figure(figsize=(10, 5))
for i, forecast, model in zip([1, 2], [forecast_sv_base_offpeak_eur_many, foreca
    plt.plot(price_temp_data_offpeak_eur.loc['2024-05-01':]['CONSUMER_PRICE'].in
    plt.fill_between(price_temp_data_offpeak_eur.loc['2024-05-01':]['CONSUMER_PR
plt.plot(price_temp_data_offpeak_eur.loc['2024-04-24':]['CONSUMER_PRICE'].index,
plt.xlabel('Day (April - May)', size=10, family='monospace')
plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
plt.xticks(price_temp_data_offpeak_eur.loc['2024-04-24':]['CONSUMER_PRICE'].inde
plt.yticks(size=10, family='monospace')
plt.ylim([700, 1500])
plt.title(f'Actual and mean forecasted consumer price for offpeak hour #{hour_mi
plt.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'});
plt.grid(lw=0.25, color='xkcd:cement');
```



In [ ]:
```
%%capture
# Off-peak hour + Price zone 2
forecast_sv_base_offpeak_sib_many = model_sv_base_offpeak_sib.forecast_many(X=pr
```

In [ ]:
```
%%capture
forecast_sv_x_offpeak_sib_many = model_sv_x_offpeak_sib.forecast_many(X=price_te
```

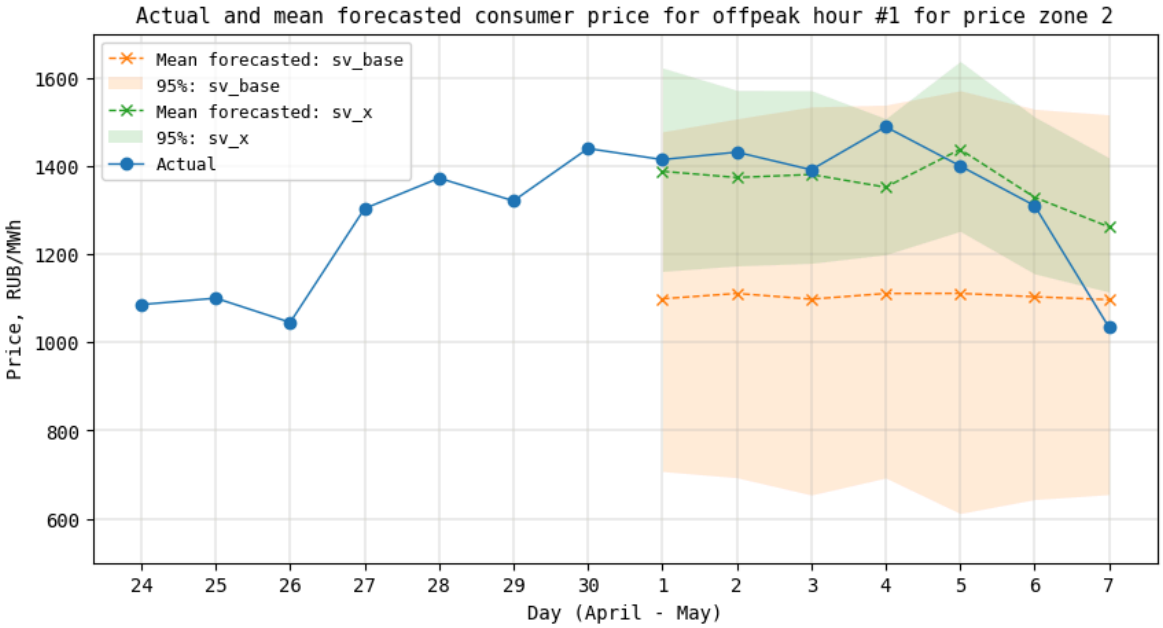
In [ ]:
```
pd.DataFrame({'sv_base': [
    mean_absolute_error(price_temp_data_offpeak_sib['CONSUMER_PRICE']['2024-05-0
    root_mean_squared_error(price_temp_data_offpeak_sib['CONSUMER_PRICE']['2024-
    'sv_x': [mean_absolute_error(price_temp_data_offpeak_sib['CONSUMER_PRICE']['
    root_mean_squared_error(price_temp_data_offpeak_sib['CONSUMER_PRICE']['2024-
    index=['mae', 'rmse']).style.highlight_min(color='palegreen', axis=1).set_ca
```

Out[ ]:

Off-peak hour + Price zone 2

	sv_base	sv_x
mae	266.628682	73.707465
rmse	283.308756	104.546248

In [ ]:
```
plt.figure(figsize=(10, 5))
for i, forecast, model in zip([1, 2], [forecast_sv_base_offpeak_sib_many, foreca
    plt.plot(price_temp_data_offpeak_sib.loc['2024-05-01':]['CONSUMER_PRICE'].in
    plt.fill_between(price_temp_data_offpeak_sib.loc['2024-05-01':]['CONSUMER_PR
plt.plot(price_temp_data_offpeak_sib.loc['2024-04-24':]['CONSUMER_PRICE'].index,
plt.xlabel('Day (April - May)', size=10, family='monospace')
plt.ylabel('Price, RUB/MWh', size=10, family='monospace')
plt.xticks(price_temp_data_offpeak_sib.loc['2024-04-24':]['CONSUMER_PRICE'].inde
plt.yticks(size=10, family='monospace')
plt.ylim([500, 1700])
plt.title(f'Actual and mean forecasted consumer price for offpeak hour #{hour_mi
plt.legend(loc='upper left', prop={'size': 9, 'family': 'monospace'});
plt.grid(lw=0.25, color='xkcd:cement');
```



Overall, both models' 95% CI of all sampled forecasts cover the actual price. SV X model provides a narrower forecast CI as compared to SV Baseline model.

Back

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6. Sources

- АО "АТС" (Администратор Торговой Системы)
- Stan. Time-Series Models
- Michael Clark. Bayesian Stochastic Volatility Model
- Kim Sangjoon, Neil Shephard, and Siddhartha Chib. Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models. Review of Economic Studies (65), 1998
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Back