Master of Data Science Online Programme
Course: Discrete Mathematics
SGA #1: Midterm Quiz

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Problem 1

Can it be that $|A| = 5$, $|B| = 3$ and $|A \cup B| = 6$? If it is possible, provide an example, otherwise provide a proof that this is impossible.

Solution

Following the Generalized Rule of Sum [2, pp. 72–85]: number of elements in the union of two finite sets is:

$$|A \cup B| = |A| + |B| - |A \cap B|. \tag{1}$$

Since the size of both A and B is known, i.e. the sets are finite, we can apply (1) and re-write it as follows:

$$|A \cap B| = |A| + |B| - |A \cup B|.$$

Size of set is a non-negative number, i.e. $|A \cap B| \geq 0$ and thus $|A| + |B| - |A \cup B| \geq 0$. So, $5 + 3 - 6 = 2 \geq 0$, which is true. Sets A and B intersect and have two common elements.

Example

Let $A = \{ 1, 2, 3, 4, 5 \}$ and $|A| = 5$. Let $B = \{ 4, 5, 6 \}$ and $|B| = 3$. Then $A \cup B = \{ 1, 2, 3, 4, 5, 6 \}$ and $|A \cup B| = 6$. And $A \cap B = \{ 4, 5 \}$ and $|A \cap B| = 2$.

Answer

It is possible.

Problem 2

Can it be that $|A| = 5, |B| = 3, |A \cup B| = 6$ and $|A \cap B| = 1$? If it is possible, provide an example, otherwise provide a proof that this is impossible.

Solution

Following the Generalized Rule of Sum [2, pp. 72–85]: number of elements in the union of two finite sets is:

$$|A \cup B| = |A| + |B| - |A \cap B|. \tag{2}$$

Since the size of both A and B is known, i.e. the sets are finite, we can apply (2): $|A \cup B| = 5 + 3 - 1 = 7$. But we are given $|A \cup B| = 6$ which is a contradiction. In other words, union of sets A and B must have seven elements to make (2) true, but the union only have six elements.

Answer

It is impossible.

Problem 3

Check whether the following equivalence on sets holds by considering elements of each side and checking whether they are contained in the other side:

$$(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C). \tag{3}$$

Solution

To show the equivalence of two sets $F = G$, it is enough to show that $F \subseteq G$ and $G \subseteq F$. The idea is to show these two inclusions separately [1, p. 2]. To show that $F \subseteq G$ we can consider $x \in F$, decompose what it means by definitions and show that $x \in G$ also. To check that $G \subseteq F$, we can do a symmetric argument.

Denote the left-hand side of (3) as F and the the right-hand side as G: $F = (A \cup B) \setminus C$. $G = (A \setminus C) \cup (B \setminus C)$.

For F, suppose $x \in (A \cup B) \setminus C$. It means that $x \in A \cup B$ *and* $x \notin C$ by definition of set difference. And further $(x \in A$ *or* $x \in B)$ *and* $x \notin C$ by definition of set union. So x is at least in A or B and not in C at the same time. This can be re-written as $(x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C)$ which by definition of set difference and set union gives $x \in (A \setminus C) \cup (B \setminus C)$, so $x \in G$ also and thus $F \subseteq G$.

For G, suppose $x \in (A \setminus C) \cup (B \setminus C)$. It means that $(x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C)$ by definition of set difference and set union. So x is at least in A or B and not in C at the same time. This can be re-written as $x \in (A \cup B) \setminus C$, so $x \in F$ also and thus $G \subseteq F$.

Since we showed both $F \subseteq G$ and $G \subseteq F$, the equivalence holds.

Answer

The equivalence holds.

Problem 4

Suppose mobile company has 6 mobile plans. They make a survey among their clients asking for the client's favorite mobile plan, second favorite plan (that should be different from the first one) and the most overpriced mobile plan in client's opinion (that can be the same as his two favorite plans, or can be some other plan). What is the number of possible different outcomes of the survey? Provide a detailed explanation of your calculation. If you are using some of the rules (including the rule of sum and the rule of product), please explain why are you using these rules.

Solution

Since it is stated in the Problem that the mobile company is interested in different outcomes of the survey, i.e. distinct survey outcomes with one survey per each client, we will assume that if one client chose plans #1 (his/her first favourite), #2 (his/her second favourite), #3 (his/her overpriced), and the other client chose plans #2 (his/her first favourite), #1 (his/her second favourite), #3 (his/her overpriced), we will consider these two survey results as different. In this case we need to consider *ordered* sequences of chosen mobile plans.

Let A be a set of all mobile plans, i.e. $A = \{1, 2, 3, 4, 5, 6\}$ and $|A| = 6$. Let B be a set of sequences (pairs) of the first and second favorite mobile plans, where the first mobile plan and the second mobile plan are not the same: $B = \{(a_i, a_j) \mid a_i, a_j \in A \wedge 1 \leq i, j \leq |A| \wedge i \neq j\}$, i.e. $B = \{(1, 2), (2, 1), \ldots, (5, 6), (6, 5)\}$. Let's denote the number of possible third (overpriced) mobile plans as M_{third_plan}.

Following the Rule of Product [2, pp. 105–122]: if there are k objects of the first type and there are n objects of the second type, then there are $n \times k$ pairs of objects. In our scenario we have $|B|$ objects of the first type: 2-element pairs for the first and second mobile plans and M_{third_plan} objects of the second type: the third mobile plan can be equal to the first mobile plan or equal to the second mobile plan or be any other mobile plan, it means that the third plan does *not* depend on choosing a pair of mobile plans from B. So, the total number of survey outcomes is

$$N_{outcomes} = |B| \times M_{third\_plan}. \tag{4}$$

Let's find $|B|$. Since we have ordered sequences without repetition, the number of all possible sequences is the number of k-permutations (ordered sequences without repetitions) [4, pp. 74–97]:

$$|B| = {}^{n}P_k = \frac{n!}{(n-k)!},$$

where n is the total number of possible options (mobile plans) which is $|A| = 6$, and k is the size of each sequence in B, i.e $k = 2$, since we permute two mobile plans: the first and the second one. Thus $|B| = \frac{6!}{(6-2)!} = \frac{6!}{4!} = 30$. In other words, we have 30 possible ordered pairs of the first and second mobile plans.

Now let's find M_{third_plan}. We need to choose only one mobile plan (the third plan) out of six plans. In this case it does not matter if it is an ordered or unordered sequence, and repetitions are not possible, since we choose only one element. So, the number of ways to choose $k = 1$ element out of $n = 6$ elements is simply $|A| = 6$.

Finally, as per (4), the total number of possible different outcomes of the survey is

$$N_{outcomes} = 30 \times 6 = 180.$$

Answer

Number of possible different outcomes of the survey is 180.

Problem 5

Suppose mortgage company has 10 houses. They want to calculate distance between each two (distinct) houses among them (just usual distance on the map). We want to count how many distances we will calculate. What is wrong with the following solution attempt? Explain the mistake and provide a correct answer to the problem.

Solution attempt: We need to count the number of pairs of houses. The first house can be picked in 10 ways. The second house can be picked in 9 possible ways, since one of the houses was already picked. By the rule of product the pairs of houses can be picked in $10 \times 9 = 90$ possible ways. So we will need to make 90 calculations.

Solution

Let H be a set of all houses, i.e. $H = \{ h_1, h_2, \dots, h_{10} \}$. It is clear that the distance between each two (distinct) houses does *not* depend on the order of choosing houses, in terms of usual distance on the map, i.e. distance between h_i and h_j ($i \neq j$, since we cannot measure distance between the same one house) is the same as distance between h_j and h_i. Thus we can count the number of all possible distances as the number of k-combinations (unordered sequences without repetitions) [3, pp. 25–60]:

$$N_{ways} = {}^nC_k = \binom{n}{k} = \frac{n!}{k! \cdot (n-k)!},$$

where n is the total number of possible options (number of houses) which is $|H| = 10$, and k is the size of each sequence of houses, which is two, since we calculate the distance between two houses, i.e $k = 2$. Thus $N_{ways} = \frac{10!}{2! \cdot (10-2)!} = \frac{10!}{2! \cdot 8!} = 45$. In other words, we have 45 distances between 10 houses.

Further Considerations

The proposed solution attempt is correct for counting *ordered* sequences of elements and this solution would be applicable if the distance between two house *depended* on the order of choosing houses. One possible example where the order of choosing two geographical objects is important is flight time between two cities: flight time from City A to City B is not necessarily the same as the flight time from City B to City A. In this case we would need to consider ordered sequences of flight times.

Answer

Number of distances between 10 houses is 45.

Problem 6

How many ways are there to write down numbers from 0 to 9 in a sequence in such a way that even numbers are positioned in the sequence in the increasing order and odd numbers are positioned in the decreasing order? Provide a detailed explanation for your solution.

Solution

Let A be a set of all numbers, i.e. $A = \{0,1,2,3,4,5,6,7,8,9\}$ and $|A| = 10$. Let $Evens = \{x \in A \mid x\ is\ even\}$, i.e. $Evens = \{0,2,4,6,8\}$ and $|Evens| = 5$. Let $Odds = \{x \in A \mid x\ is\ odd\}$, i.e. $Odds = \{1,3,5,7,9\}$ and $|Odds| = 5$. Number of all possible sequences is the number of k-permutations (ordered sequences without repetitions) out of $n = |A| = 10$ numbers (digits) with length $k = |A| = 10$ is ${}^{10}P_{10}$, or simply $|A|! = 10! = 3,628,800$. However, we need only sequences with fixed order, as per conditions:

a) even numbers increase,

b) odd numbers decrease.

In this case, since the order is fixed, i.e. only one order is possible, we should use k-combinations, not k-permutations. Example: $(0, 2, ...)$ is an allowed sequence, while $(2, 0, ...)$ is not allowed, since even numbers must increase. For odd number it is the same restriction: $(9, 7, ...)$ is an allowed sequence, while $(7, 9, ...)$ is not allowed, since odd numbers must decrease.

For our sequences we need to choose only even (or odd) numbers out of all numbers, because each combination of increasing even numbers (or decreasing odd numbers) is a unique solution. For example, $(even_1, even_2, odd_1, ...) \mid even_1 < even_2$ is one possible solution; $(even_1, odd_1, even_2, ...) \mid even_1 < even_2$ is another possible solution. The count of even (or odd) numbers in A is $|Evens| = 5$ (or $|Odds| = 5$), so $k = 5$. Thus the number of solutions satisfying both conditions a) and b) above is ${}^{n}C_k = \binom{n}{k} = \frac{n!}{k!\cdot(n-k)!} = \frac{10!}{5!\cdot(10-5)!} = \frac{10!}{5!} = 252$.

Examples

Let's show some shorter sequences, i.e. subsets of A.

In Figure 1 we have two combinations for $n = 2$ and $k = 1$. In Figure 2 we have three combinations for $n = 3$ and $k = 2$ (if combining even numbers), or same result for $k = 1$ (if combining odd numbers). In Figure 3 we have six combinations for $n = 4$ and $k = 2$.

Finally, in Figure 4 we have 252 combinations for $n = 10$ and $k = 5$ – shown only schematically, since this will be a large picture.

Some notes:

- sequence must start with either the smallest number in *Evens* (because even numbers must increase) or the largest number in *Odds* (because odd numbers must decrease),
- sequence must end with either the largest number in *Evens* (again, because even numbers must increase) or the smallest number in *Odds* (again, because odd numbers must decrease),

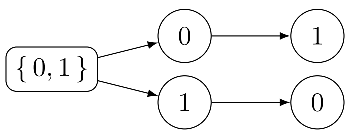

Figure 1: 2-number sequences: $\binom{2}{1} = 2$ solutions

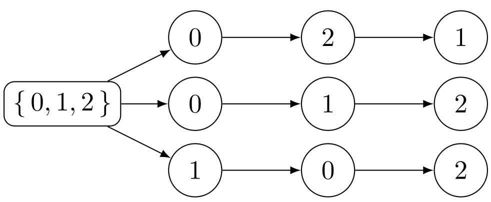

Figure 2: 3-number sequences: $\binom{3}{2} = 3$ solutions

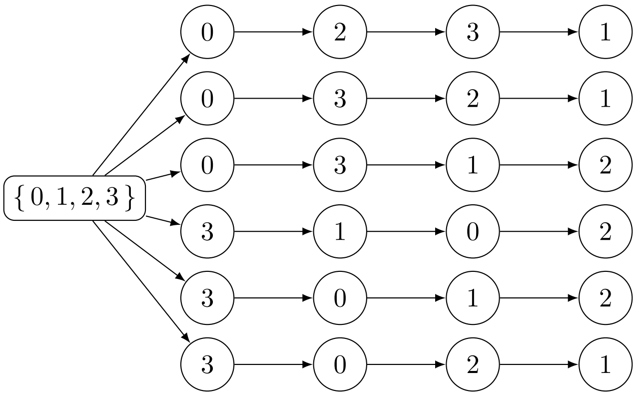

Figure 3: 4-number sequences: $\binom{4}{2} = 6$ solutions

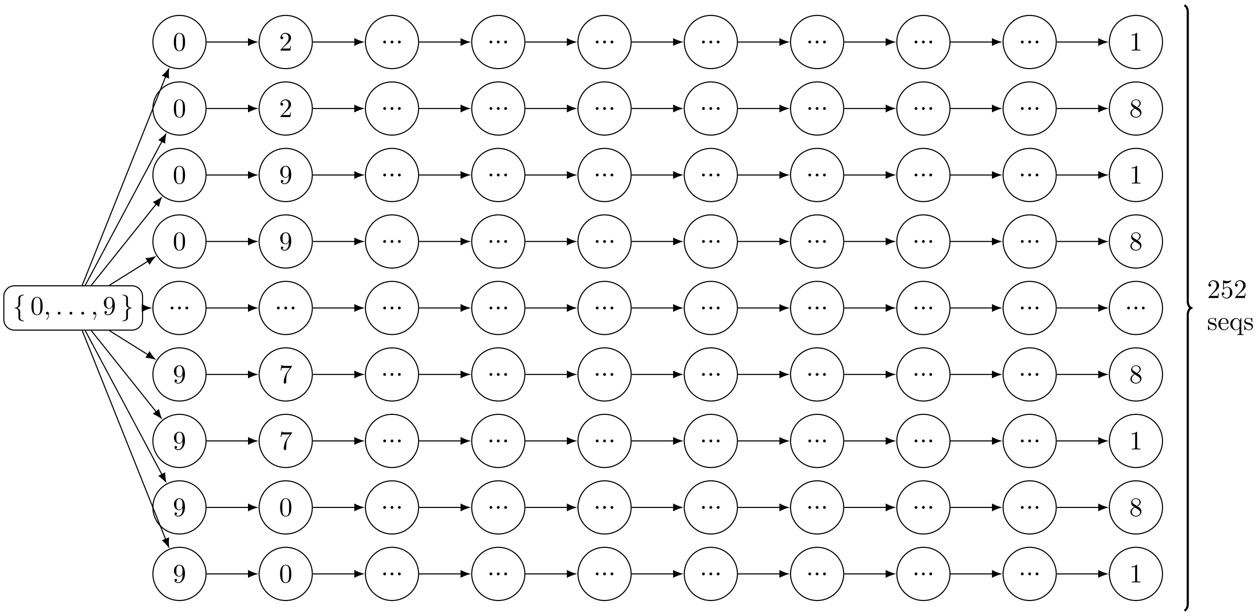

Figure 4: 10-number sequences: $\binom{10}{5} = 252$ solutions

Answer

There are 252 ways to place numbers from 0 to 9 in a sequence such that even numbers increase and odd numbers decrease.

So there are only 252 sequences out of 3,628,800 possible sequences which is only around 70 per one million!

Problem 7

Let A and B be events associated with some random experiment. Is it possible that these events never occur simultaneously and $P(A) = 1/2, P(B) = 1/3$? If yes, give an example of random experiment and events that satisfy these conditions. If no, give a proof.

Solution

Consider an urn with six identical symmetrical balls of three colors: three red balls, two blue balls, and one green ball. The researcher cannot see the colors of the balls.

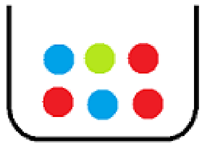

Figure 5: Urn ready for random experiment

Let's do a random experiment:

- take any one ball out of the urn at a time without replacement,
- taking any one ball does *not* depend on taking any other ball in the urn,
- the order of choosing the balls is not important.

Consider the following two events:

1. Event A = 'We took a red ball.'
2. Event B = 'We tool a blue ball.'

Since we have identical and symmetrical balls and cannot see the color of the balls, taking any one ball has no advantage over taking any other ball, i.e. taking is fair, so we can safely assume that each elementary outcome is equally probable, and we can find classical probabilities of events A and B. As per [5], the classical probability of event A is

$$P(A) = \frac{|A|}{|\Omega|}, \tag{5}$$

where Ω is the sample space of the random experiment – it is a set of all possible elementary outcomes and each outcome is equally probable, $|\Omega|$ is the size of the sample space Ω, and $|A|$ is the size of the event set which is a subset of Ω. Our sample space has unordered sequences of elementary outcomes, where each outcome is one ball chosen out of six possible options (balls in the urn), i.e. the size of the sample space is the number of k-combinations with $n = 6$ and $k = 1$: $|\Omega| = {}^nC_k = \binom{n}{k} = \frac{n!}{k!\cdot(n-k)!} = \frac{6!}{1!\cdot(6-1)!} = \frac{6!}{5!} = 6$. So, all elementary outcomes of the sample space $\Omega = \{\, blue, blue, green, red, red, red\,\}$. The same apples to the size of event sets A and B:

1. For Event A we can choose one red ball out of three red balls in the urn: $|A| = {}^3C_1 = \binom{3}{1} = \frac{3!}{1!\cdot(3-1)!} = \frac{3!}{2!} = 3$. So, all outcomes of the Event $A = \{\, red, red, red\,\}$.
2. For Event B we can choose one blue ball out of two blue balls in the urn: $|B| = {}^2C_1 = \binom{2}{1} = \frac{2!}{1!\cdot(2-1)!} = \frac{2!}{1!} = 2$. So, all outcomes of the Event $B = \{\, blue, blue\,\}$.

Finally, applying (5) gives us the probabilities of events A and B: $P(A) = \frac{|A|}{|\Omega|} = \frac{3}{6} = \frac{1}{2}$, $P(B) = \frac{|B|}{|\Omega|} = \frac{2}{6} = \frac{1}{3}$.

It is clear that, by the definition of the random experiment, events A and B cannot occur simultaneously also, i.e. the probability of intersection of events A and B is $P|A \cap B| = 0$.

Further Considerations

We have one green ball in the urn. Let Event C = 'We took the green ball'. Let's find the probability of event C: $P(C) = \frac{|C|}{|\Omega|} = \frac{1}{6}$. Can we find the probability of Event C without calculating the probability by definition?

The probability of the sample space (all elementary outcomes in the random experiment which can happen) is $P(\Omega) = 1$. Since, by the definition of the random experiment, events cannot occur simultaneously, all probabilities of intersections of events is $P|A \cap B| = P|A \cap C| = P|B \cap C| = 0$ and thus $P(\Omega) = P(A) + P(B) + P(C)$, i.e. the sum of probabilities of taking either a red ball, or a blue ball, or the green ball. The probability of Event C is $P(C) = P(\Omega) - P(A) - P(B) = 1 - \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ which is exactly the same result if calculating the probability by definition.

Answer

It is possible.

Problem 8

There is a small village with only ten adult people living in it. Instead of elections, they form their "government" using random choice. Every adult villager has equal probability to be chosen. Consider two random experiments:

1. The villagers want to select a Village Council that consists of three persons. The researcher is interested in the list of members of one Council.

2. Every four months villagers select a President. The same person can be President arbitrary number of times. The same person can be a President and a member of Village Council at the same time. The researcher is interested in the ordered list of Presidents during one year.

Describe the sample spaces (sets of all elementary outcomes) for both experiments. Use the corresponding notions from combinatorics. Find the number of elements in both sample spaces. Provide all calculations and detailed explanations. What is the difference between these experiments?

Solution

Let V be a set of all villagers, i.e. $V = \{ v_1, v_2, \ldots, v_{10} \}$. Consider each of the random experiments:

1. Each Village Council is an unordered 3-villager sequence. Thus we can count the number of all possible Councils as the number of k-combinations (unordered sequences without repetitions) [3, pp. 25–60]:

$$N_{Councils} = {}^{n}C_k = \binom{n}{k} = \frac{n!}{k! \cdot (n-k)!},$$

   where n is the total number of possible options (number of villagers) which is $|V| = 10$, and k is the size of each sequence of villagers, which is three, since each Council consists of three persons, i.e $k = 3$. Thus $N_{Councils} = \frac{10!}{3! \cdot (10-3)!} = \frac{10!}{3! \cdot 7!} = 120$. In other words, we have 120 Councils in our sample space, i.e. $\Omega = \{ \omega_1, \omega_2, \ldots, \omega_{N_{Councils}} \}$, where $\omega_i = (v_a, v_b, v_c); a \neq b \neq c$, since one person *cannot* be present more than once in the same Council. So, Ω can be defined as follows: $\Omega = \{ (v_i, v_j, v_k) \mid v_i, v_j, v_k \in V \wedge 1 \leq i, j, k \leq |\, V| \wedge i \neq j \neq k \wedge i < j < k \}$. And $|\Omega| = 120$. Let's list some elementary outcomes ω_i, as an example:

   - $\omega_1 = (v_1, v_2, v_3)$: Council consists of villagers #1, #2, and #3;
   - $\omega_{60} = (v_2, v_7, v_9)$: Council consists of villagers #2, #7, and #9;
   - $\omega_{120} = (v_8, v_9, v_{10})$: Council consists of villagers #8, #9, and #10.

2. Since we are interested in the results of the random experiment over one year, i.e. 12 months, and the President is selected every four months, by the end of the year (end of experiment) we will have $12 \div 4 = 3$ sub-experiments and thus the final result is a set of 3-President sequences.

   Since the same person *can* be President several times, i.e. repetitions are allowed, and we are interested in the ordered list of Presidents, we can count the number of all possible sequences of Presidents as the number of tuples (ordered sequences with repetitions) [4, pp. 2–23]:

$$N_{seq\_Presidents} = n^k,$$

   where n is the total number of possible options (number of villagers) which is $|V| = 10$, and k is the size of each sequence of Presidents, which is three, since President is selected three times during the year, i.e $k = 3$. Thus $N_{seq\_Presidents} = 10^3 = 1000$. In other words, we have 1000 sequences of Presidents in our sample space, i.e. $\Omega = \{ \omega_1, \omega_2, \ldots, \omega_{N_{seq\_Presidents}} \}$, where $\omega_i = (v_a, v_b, v_c)$; a, b, c are allowed to be equal, since one person *can* be selected as President more than once during the year. So, Ω can be defined as follows: $\Omega = \{ (v_i, v_j, v_k) \mid v_i, v_j, v_k \in V \wedge 1 \leq i, j, k \leq |\, V| \}$. And $|\Omega| = 1000$.

   As for the condition 'The same person can be a President and a member of Village Council at the same time', it does not affect the calculation since being President does *not* depend on being in the Village Council – it is 'can' but not 'must'.

   Let's list some elementary outcomes ω_i, as an example:

- $\omega_1 = (v_1, v_1, v_1)$: villager #1 was selected President three times in a row during the year;
- $\omega_{443} = (v_5, v_5, v_3)$: villager #5 was selected two times in a row, but then he/she was replaced by villager #3;
- $\omega_{637} = (v_7, v_4, v_7)$: villager #7 was selected first, but then he/she was replaced by the villager #4, however villager #7 regained trust and was again selected President.
- $\omega_{790} = (v_8, v_9, v_{10})$: villagers #8, #9, and #10 were selected President one after another;
- $\omega_{1000} = (v_{10}, v_{10}, v_{10})$: villager #10 was selected President three times in a row during the year;

Answer

- Experiment #1: $\Omega = \{ (v_i, v_j, v_k) \mid v_i, v_j, v_k \in V \wedge 1 \leq i, j, k \leq |V| \wedge i \neq j \neq k \wedge i < j < k \}$, $V = \{ v_1, v_2, \ldots, v_{10} \}$. The number of elements in the sample space $|\Omega| = 120$.
- Experiment #2: $\Omega = \{ (v_i, v_j, v_k) \mid v_i, v_j, v_k \in V \wedge 1 \leq i, j, k \leq |V| \}$, $V = \{ v_1, v_2, \ldots, v_{10} \}$. The number of elements in the sample space $|\Omega| = 1000$.
- The difference between these two experiments is that in the first experiment repetitions of villagers were not possible and the order of persons was not important, while the second experiment allowed repetitions of villagers and the order of persons was important.

Problem 9

Consider the following random experiment. We roll two identical indistinguishable symmetric dices once and record the result of this tossing (i.e. 'on one dice we obtained 1 and on another we obtained 2').

Describe sample space (set of all outcomes) of this experiment. What is the probability of event 'on one dice we obtained 1 and on another we obtained 2'? What is the probability of event 'we obtained two 1's'? What kind of sample space you consider in this problem: sample space with equal probabilities of outcomes or sample space with non-equal probabilities of outcomes? Explain your answer.

Solution

Let A be a set of all possible points for a dice, i.e. $D = \{1, 2, 3, 4, 5, 6\}$ and $|D| = 6$. Since we have two identical indistinguishable dices, D is applicable to each of the dices.

By definition of the random experiment, two dices are rolled at the same time, so we can count the number of all possible sequences of points on two dices as the number of tuples (ordered sequences with repetitions) [4, pp. 2–23]:

$$N_{seq\_points} = n^k,$$

where n is the total number of possible options (number of points on each dice, which is the same for both dices) which is $|D| = 6$, and k is the size of each sequence of points, which is two, since we roll two dices simultaneously, i.e $k = 2$. Thus $N_{seq\_points} = 6^2 = 36$. In other words, we have 36 sequences of 2-point elementary outcomes in our sample space, i.e. $\Omega = \{\omega_1, \omega_2, \ldots, \omega_{N_{seq\_points}}\}$, where $\omega_i = (d_i, d_j)$; i, j are allowed to be equal, since the same point *can* be obtained on both dices after one roll. So, Ω can be defined as follows: $\Omega = \{(d_i, d_j) \mid d_i, d_j \in D \wedge 1 \leq i, j \leq |D|\}$. And $|\Omega| = 36$. Since Ω's size is not very large, let's list all elementary outcomes ω_i in Ω:

Elementary outcomes (all possible sequences of points on two dices)					
$\omega_1 = (1,1)$	$\omega_7 = (2,1)$	$\omega_{13} = (3,1)$	$\omega_{19} = (4,1)$	$\omega_{25} = (5,1)$	$\omega_{31} = (6,1)$
$\omega_2 = (1,2)$	$\omega_8 = (2,2)$	$\omega_{14} = (3,2)$	$\omega_{20} = (4,2)$	$\omega_{26} = (5,2)$	$\omega_{32} = (6,2)$
$\omega_3 = (1,3)$	$\omega_9 = (2,3)$	$\omega_{15} = (3,3)$	$\omega_{21} = (4,3)$	$\omega_{27} = (5,3)$	$\omega_{33} = (6,3)$
$\omega_4 = (1,4)$	$\omega_{10} = (2,4)$	$\omega_{16} = (3,4)$	$\omega_{22} = (4,4)$	$\omega_{28} = (5,4)$	$\omega_{34} = (6,4)$
$\omega_5 = (1,5)$	$\omega_{11} = (2,5)$	$\omega_{17} = (3,5)$	$\omega_{23} = (4,5)$	$\omega_{29} = (5,5)$	$\omega_{35} = (6,5)$
$\omega_6 = (1,6)$	$\omega_{12} = (2,6)$	$\omega_{18} = (3,6)$	$\omega_{24} = (4,6)$	$\omega_{30} = (5,6)$	$\omega_{36} = (6,6)$

Table 1: Full list of all elementary outcomes ω_i in Ω

Let Event A = 'on one dice we obtained 1 and on another we obtained 2'. Since A's description is simple, constructing the set for it is straightforward: $A = \{(1,2), (2,1)\}$ and $|A| = 2$. Since we roll two identical indistinguishable symmetric dices, i.e. so called fair dices, the probability of Event A may be found by applying (5): $P(A) = {}^2\!/_{36} = {}^1\!/_{18} = 0.0(5)$.

Let Event B = 'we obtained two 1's'. Again, since B's description is simple, constructing the set for it is straightforward: $B = \{(1,1)\}$ and $|A| = 1$. And again, since we roll two identical indistinguishable symmetric dices, i.e. so called fair dices, the probability of Event B may be found by applying (5): $P(B) = {}^1\!/_{36} = 0.02(7)$.

Answer

- Sample space $\Omega = \{(d_i, d_j) \mid d_i, d_j \in D \wedge 1 \leq i, j \leq |D|\}, D = \{1, 2, \ldots, 6\}$. The number of elements in the sample space $|\Omega| = 36$.
- The probability of event 'on one dice we obtained 1 and on another we obtained 2' is ${}^1\!/_{18} = 0.0(5)$.
- The probability of event 'we obtained two 1's' is ${}^1\!/_{36} = 0.02(7)$.
- In this Problem we consider sample space with equal probabilities of outcomes because it is clearly stated that 'We roll two identical indistinguishable symmetric dices', i.e. so called fair dices for which obtaining any elementary outcome is equally probable. It means that each elementary outcome ω_i has probability $p_i = const = {}^1\!/_{|\Omega|}$.

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