

Sparse Array Toolbox

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1. Sparse Array Toolbox

For sparse data, sparse formats can significantly increase speed and reduce memory requirements. However, MATLAB's built-in sparse format represents only vectors and matrices. This toolbox contains a new *sparse array structure* and associated operations – outer, entrywise, and inner products, addition, summation, convolution, permutation, (circular) shifts, and distance measures – that can be applied to sparse representations of N -dimensional arrays. All functions have been carefully designed to optimize speed.

A sparse array structure has the following fields: **Ind**, which is a column vector of linear indices of nonzero values in the full array that they represent; **Val**, which is a column vector of the values at those indices; **Size**, which is a row vector of the sizes of each dimension of the full array – for example, a column vector with N entries has a size of N ; a row vector with N entries has a size of $(1, N)$; a matrix with N entries has a size (J, K) , where $JK = N$; a three-way array with N entries has a size (J, K, L) , where $JKL = N$.

1.1. *array2spArray*

`spA = array2spArray(A)`: Convert a full array into a sparse array structure.

1.2. *spArray2Array*

`A = spArray2Array(spA)`: Convert a sparse array structure into a full array.

1.3. *spInd2spSub*

`subsA = spInd2spSub(spA)`: Convert a sparse array's linear index into a matrix of subscripts.

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1.4. *spSub2spInd*

`indC = spSub2spInd(siz,subsA)`: Convert a matrix of subscripts into a vector of the linear indices for an array of size `siz`.

1.5. *spOuter*

`spC = spOuter(varargin)`: The outer (tensor) product of full arrays, each represented as a sparse array structure (`spA`). The sparse array structures can entered as a comma separated list or as a members of a cell. The output is a sparse array structure. Calculations are performed from left to right in the list; that is, `spOuter(spA, spB, spC, spD)` corresponds to $((A \otimes B) \otimes C) \otimes D$. All singleton dimensions are collapsed: if A is a row vector of size $(1, M)$ and B is a matrix of size (N, P) , the resulting tensor has size (M, N, P) , not size $(1, M, N, P)$. The output is a sparse array structure.

1.6. *spTimes*

`spC = spTimes(varargin)`: Entrywise (Hadamard) product of full arrays and/or scalars, the former represented as sparse array structures. The arrays and scalars can be entered as a comma separated list or as a members of a cell. The output is a sparse array structure.

1.7. *spInner*

`c = spInner(varargin)`: Inner (scalar) product of two full arrays represented by sparse array structures. There are alternative definitions of ‘inner product’ for tensors/arrays. Here, it is their scalar product – the sum of entries resulting from their entrywise (Hadamard) product. The sparse arrays can entered as a comma separated list or as a members of a cell.

1.8. *spPlus*

`spC = spPlus(varargin)`: The sum of identically sized full arrays, each represented as a sparse array structure. The sparse arrays can entered as a comma separated list or as a members of a cell. The output is a sparse array structure.

1.9. *spSum*

`spC = spSum(spA,dim)`: Sum a full array, represented as a sparse array structure, over the dimension specified as a scalar in ‘dim’. The output is a sparse array structure.

1.10. *spConv*

`spC = spConv(spA,spB,shape)`: The N -dimensional convolution of two N -dimensional arrays represented as sparse array structures. The output is a sparse array structure.

`shape == ‘full’`: full convolution (default). Its size is the sum of the sizes of its arguments.

`shape == ‘same’`: central part of the convolution, same size as `spA`.

`shape == ‘circ’`: circular convolution over the size of `spA`.

1.11. *spPerm*

`spC = spPerm(spA,order)`: Permute the dimensions of the full array represented as a sparse array structure. The second argument is the vector of permutations. The output is a sparse array structure.

1.12. *spShift*

`spC = spShift(spA,shifts,isPer,isProg,collapse)`: Shift each dimension of the full array represented as a sparse array structure by the amounts specified in the integer vector `shifts`. The output is a sparse array structure.

`isPer == 1`: the shifts are circular .

`isProg == 1`: the shift for all dimensions except the last are multiplied by the index of the last dimension. For example, if the shift value for the first dimension is m , the shift of that dimension when the final dimension's index is n is mn .

`collapse == 1`: the array is summed over this last dimension.

Setting `isProg=1` and `collapse=1` is useful for converting absolute r -ad expectation tensors into relative r -ad expectation tensors.

1.13. *spCosSim*

`s = spCosSim(spA,spB)`: Cosine similarity of two vectorized full arrays, represented as sparse array structures, with same numbers of entries (`spInner`).

1.14. *spPDist*

`d = spPDist(spA,spB,p)`: The p -norm distance between two vectorized full arrays, represented as sparse array structures, with same numbers of entries.

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