Relative Lempel-Ziv Compression of Suffix Arrays

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SPIRE 2020

Problem (Pattern Matching)

Find all occurrences of a pattern P in a text T

Popular solution: find the interval of the suffix array (SA) that contains them

- Binary search using SA and text, or
- Backward search on the Burrows-Wheeler Transform of T (FM-index)
- Lots of compressed versions of the SA
  - Problem then becomes: how do we decompress the interval's contents?

Previous work decoding intervals

- r-index (Gagie et al., SODA 2018)
  - recent, very fast, very small — a huge leap forward in compressed indexing
- CDAWG: succinct acyclic word graph (Belazzougui et al., CPM 2015)
  - faster than r-index on some data
  - current implementations only work for DNA

Our contribution

A compressed SA that is bigger than the r-index, smaller than the CDAWG, and much much faster in practice than both

Our interest in the problem

Our recent algorithms for the variable-length gap pattern matching problem (SOFSEM 2020) make scans of intervals of uncompressed SA

How to compress a SA so that decompression of random intervals would be fast?

Core idea

SA					30	25	20	15	10	5											29	24	19	14	9							

repetitions that are off by 1 (Mäkinen, CPM 2000)

differences will turn into actual repetitions (González, Navarro, CPM 2007)

Overview

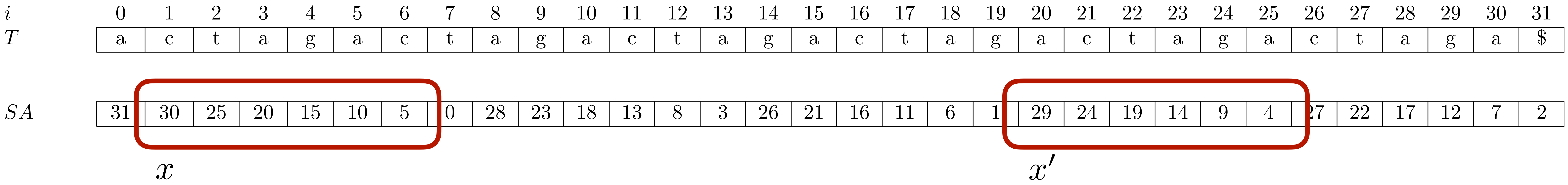
Compression:

1. form differentially encoded SA^{diff} from SA
2. form reference R by selecting substrings from SA^{diff}
3. use Relative Lempel-Ziv (RLZ) to parse SA^{diff} relative to R
4. output reference R plus set of phrases (pointers into R)

Decompression requires:

1. predecessor data structure containing phrase starting positions (in order to find the phrase covering the start of an interval)
2. absolute SA value for a starting position of the phrase

Example



i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2

$$SA[x,y] \text{ preceded by symbol } c \Rightarrow$$
$$\exists SA[x', x' + (y-x)] :$$
$$\forall i \in [0, y-x] \quad SA[x+i] = SA[x'+i] + 1$$

(González and Navarro, CPM 2007)

Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
									x'												x											

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i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27

$i \in [1, n-1]$ $\qquad SA^{diff}[i] = SA[i] - SA[i-1] + n$

Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				

Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases																																
	P								S																							

Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases																																
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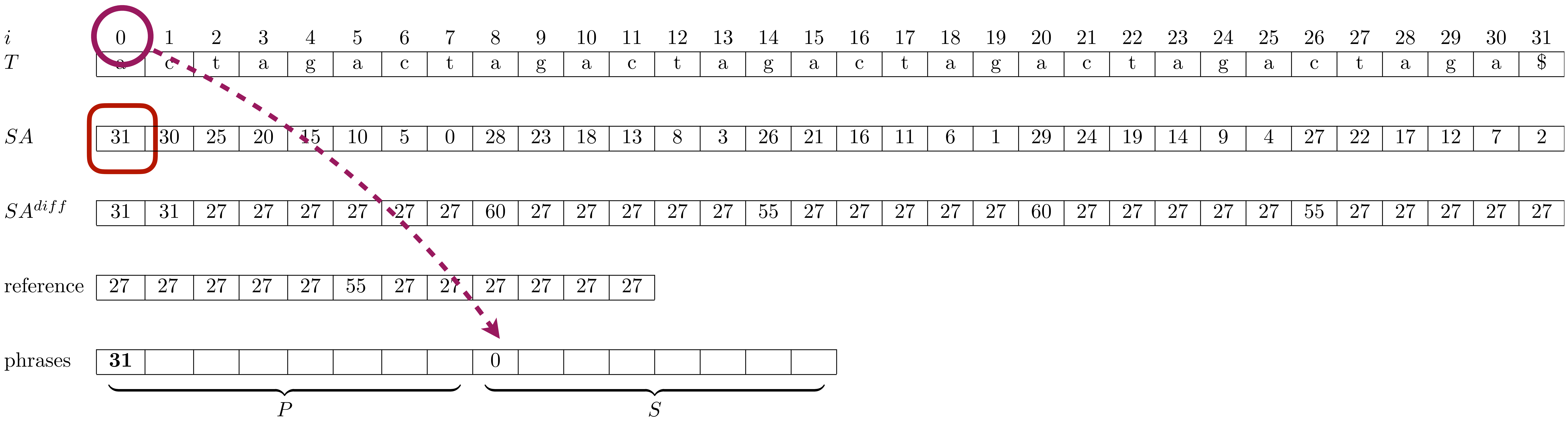
Example



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T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**								0																							

Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**								0																							

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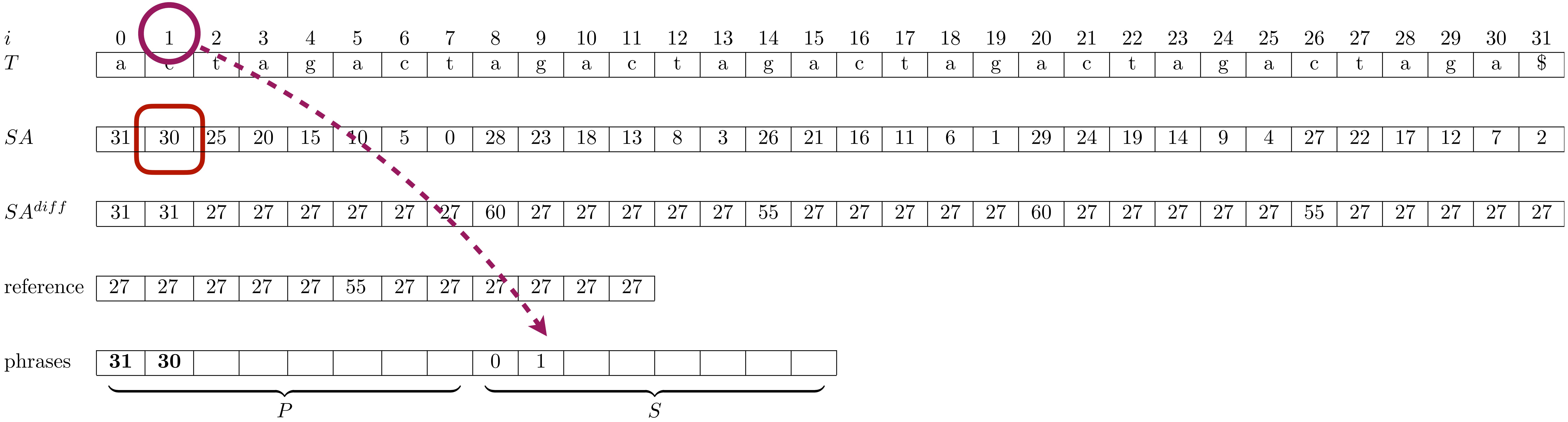
Example



i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
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reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**							0	1																						

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Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	\$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
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phrases	**31**	**30**							0	1																						
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Example

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SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
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T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
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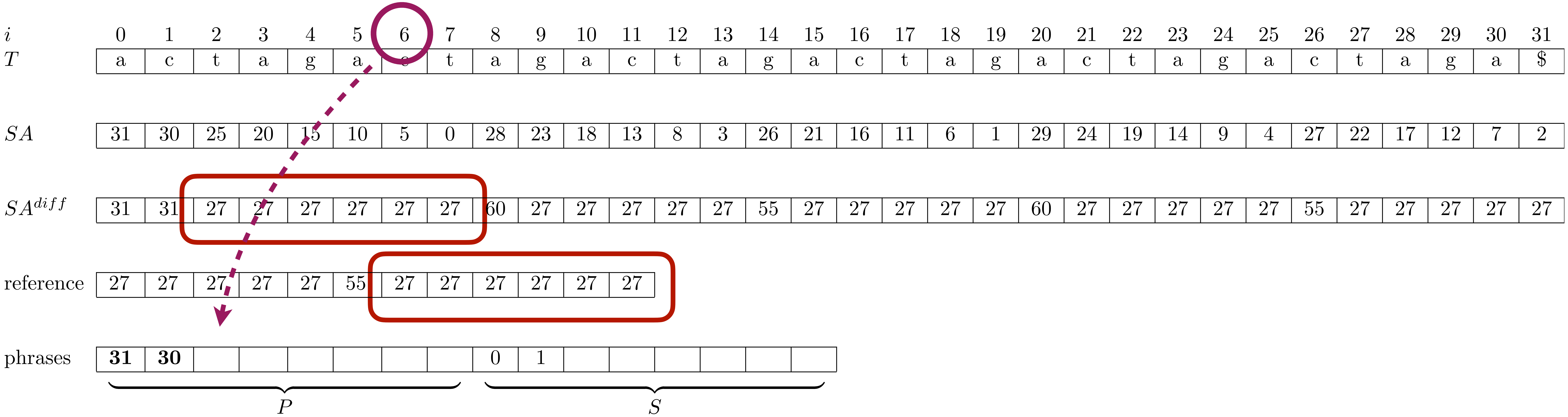
phrases	**31**	**30**							0	1																						
	P								S																							

Example



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T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**							0	1																						

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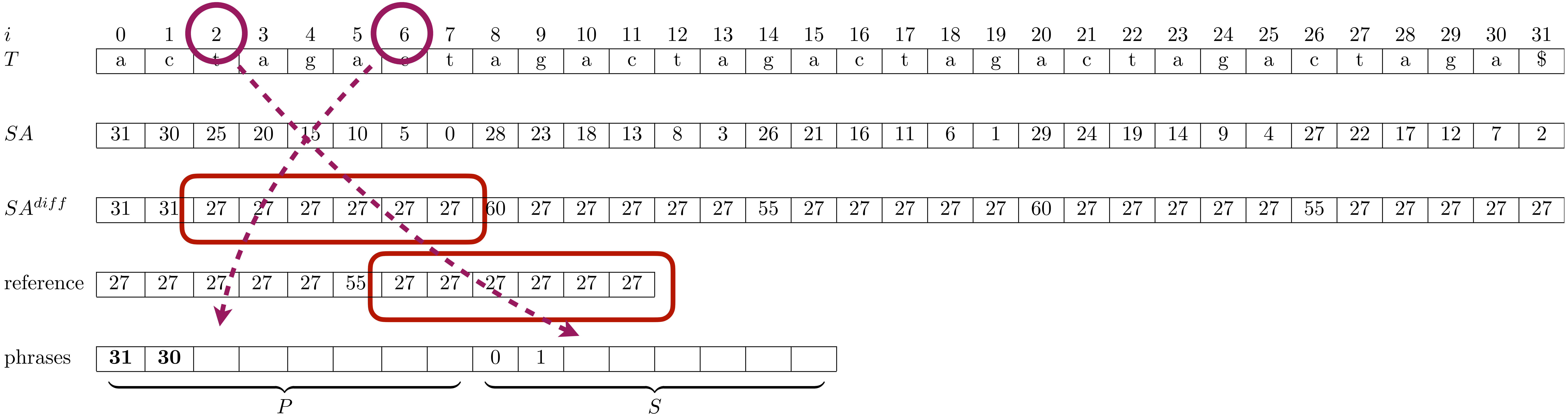
Example

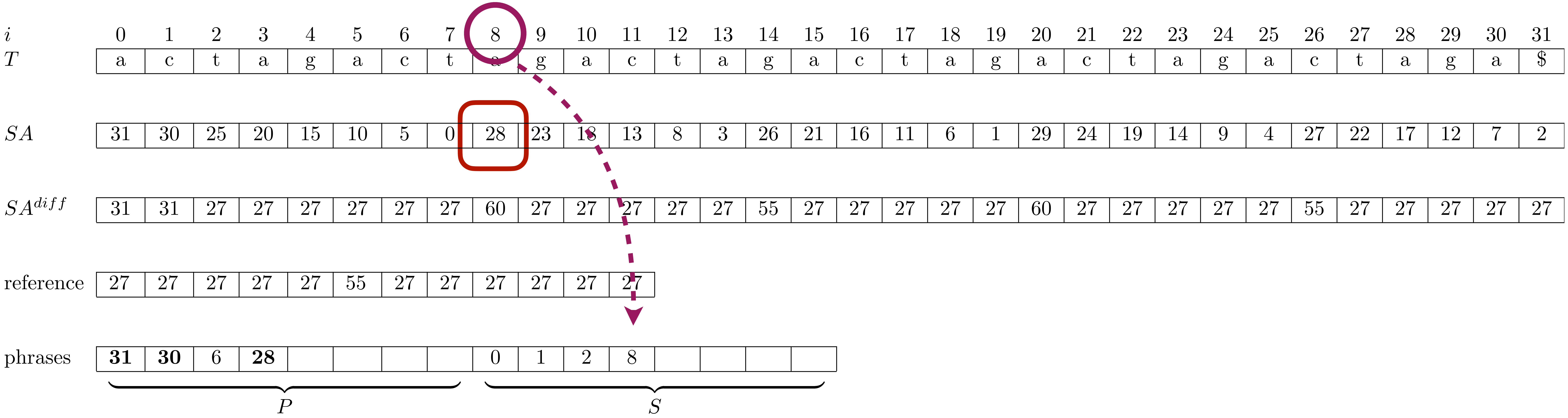


i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**							0	1																						

Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**	6	**28**					0	1	2	8																				
	P								S																							

Example



i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**	6	**28**					0	1	2	8																				

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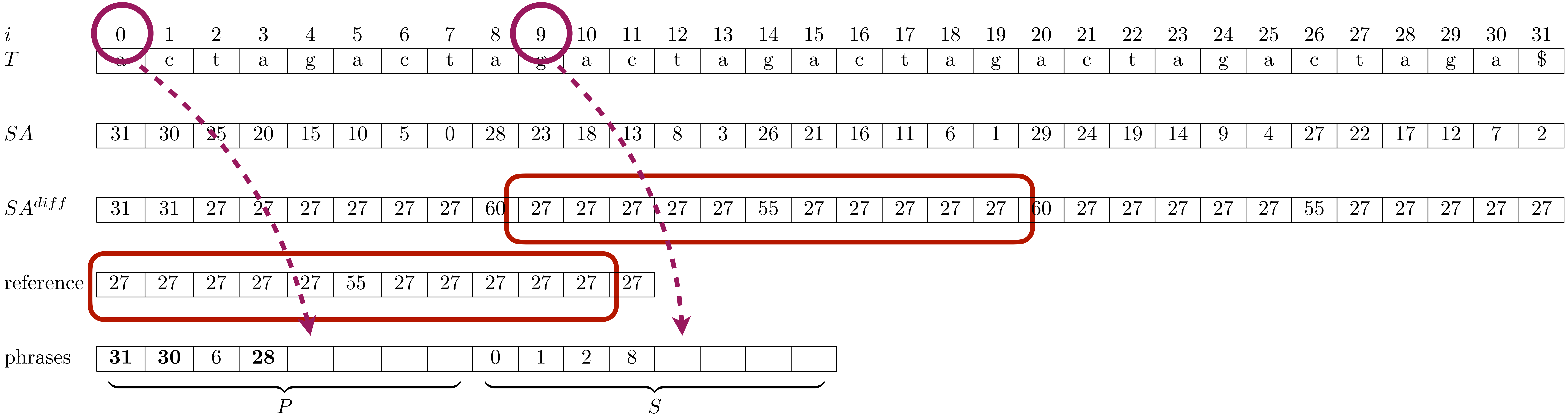
Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**	6	**28**					0	1	2	8																				
	P								S																							

Example



i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**	6	**28**					0	1	2	8																				

P

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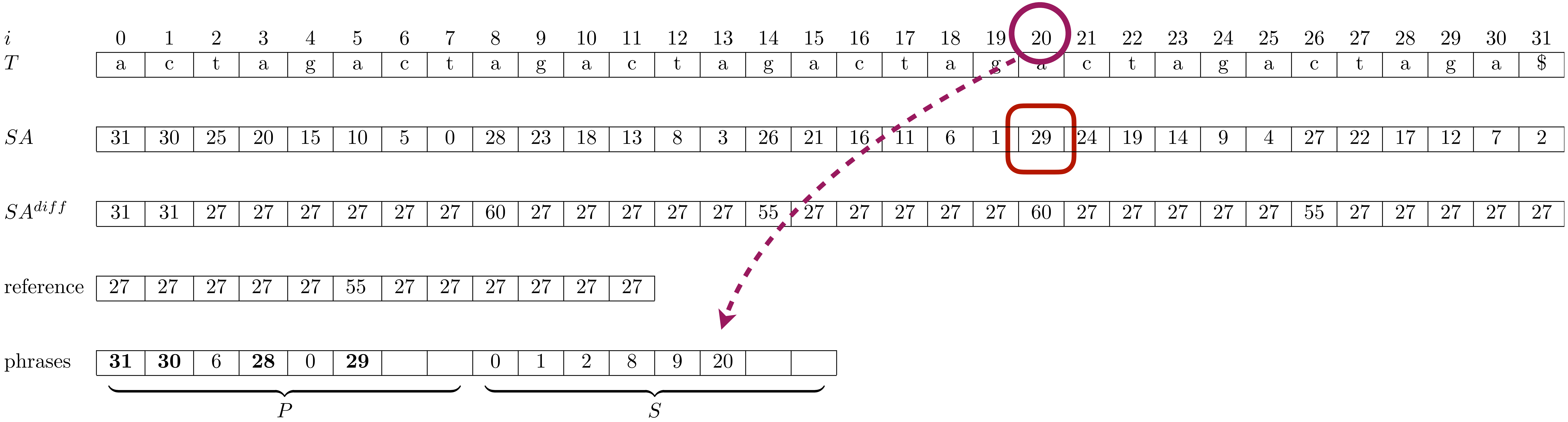
Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**	6	**28**	0	**29**			0	1	2	8	9	20																		
	P								S																							

Example



i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**	6	**28**	0	**29**			0	1	2	8	9	20																		

P S

Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**	6	**28**	0	**29**			0	1	2	8	9	20																		
	P								S																							

Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**	6	**28**	0	**29**			0	1	2	8	9	20																		

P

S

Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**	6	**28**	0	**29**			0	1	2	8	9	20																		
	P								S																							

Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27
reference	27	27	27	27	27	55	27	27	27	27	27	27																				
phrases	**31**	**30**	6	**28**	0	**29**	0	-	0	1	2	8	9	20	21	32																
	P								S																							

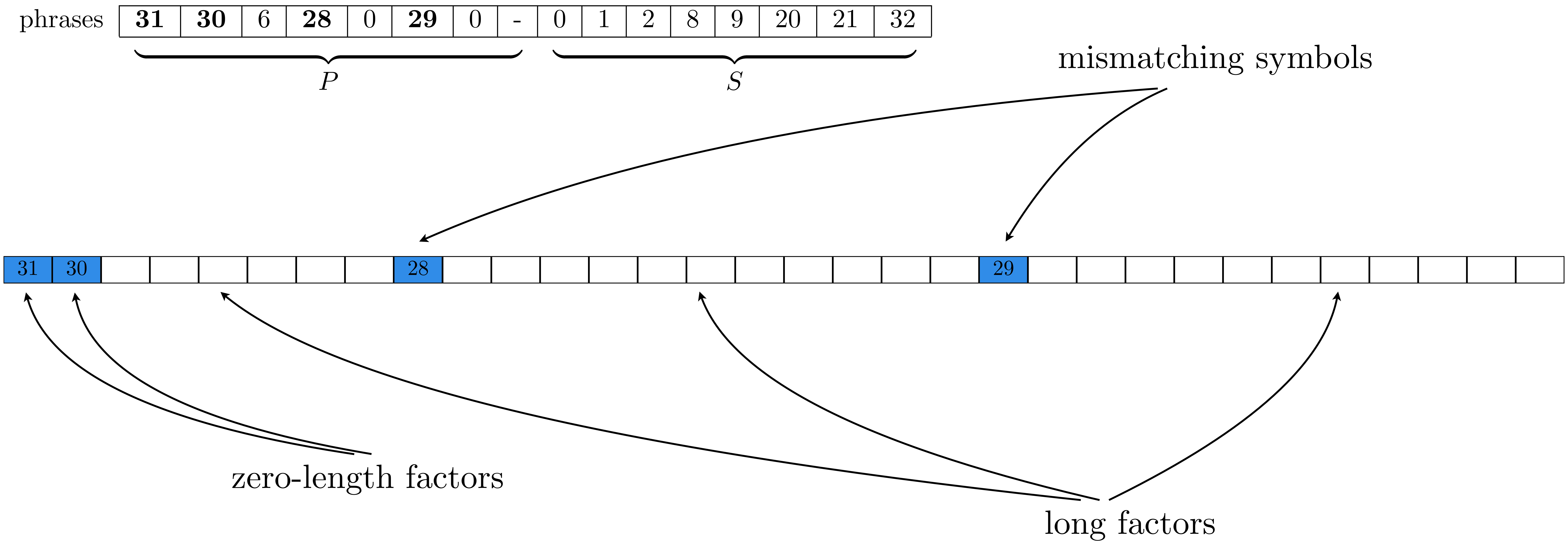
Example

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	c	t	a	g	a	$
SA	31	30	25	20	15	10	5	0	28	23	18	13	8	3	26	21	16	11	6	1	29	24	19	14	9	4	27	22	17	12	7	2
SA^{diff}	31	31	27	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27	60	27	27	27	27	27	55	27	27	27	27	27

reference	27	27	27	27	27	55	27	27	27	27	27	27

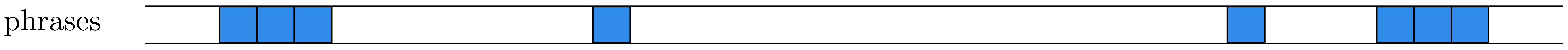
phrases	**31**	**30**	6	**28**	0	**29**	0	-	0	1	2	8	9	20	21	32
	P								S							

Example



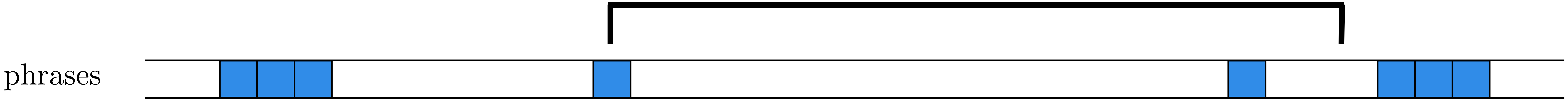
phrases	**31**	**30**	6	**28**	0	**29**	0	-	0	1	2	8	9	20	21	32

Example

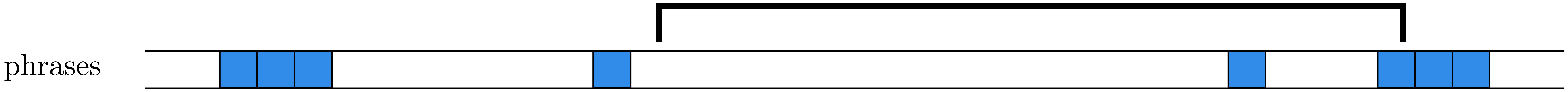


phrases

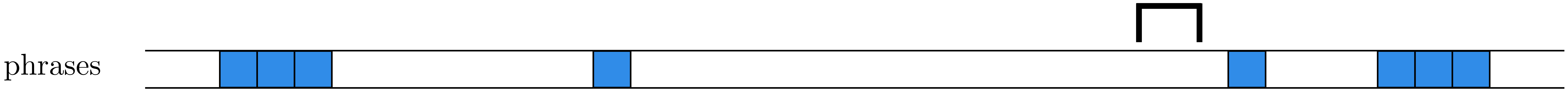
Example



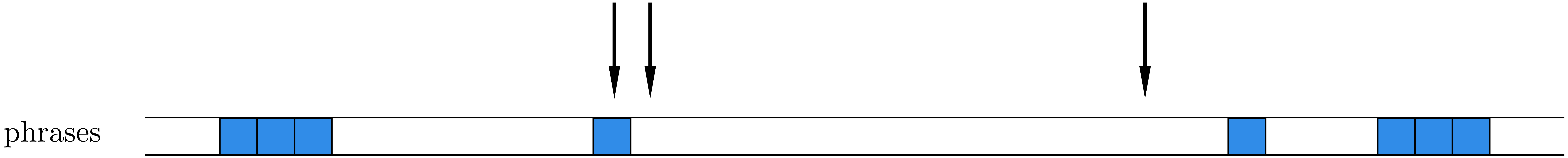
Example



Example

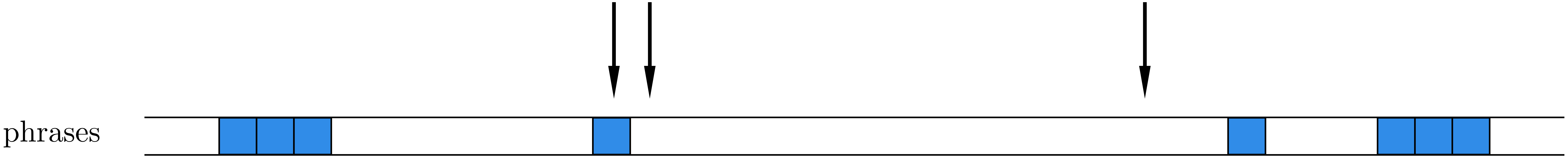


Example



- predecessor structure to find the phrase that contains the start of the SA interval

Example



- predecessor structure to find the phrase that contains the start of the SA interval

- $value[i] = \begin{cases} phrases[i], & \text{if } phraseLength[i] = 1 \\ reference[phrases[i]] + prevSaValue - n, & \text{otherwise} \end{cases}$

Experiment

We compared our prototype (*rlzsa*) to other compressed indexes, replicating the experimental design used in the *r-index* paper (Gagie, Navarro, and Prezza, SODA 2018)

Datasets:

- boost — concatenated versions of GitHub's boost library — 600Mbyte
- DNA — concatenated copies of a DNA sequence of length 1000 with mutations — 600 Mbyte
- einstein — concatenated versions of Wikipedia's Einstein page — 600 Mbyte
- world — pdf files of CIA World Leaders from Jan 2003 to Dec 2009 — 45Mbyte

Search queries:

- 1000 patterns
- length $= 8$

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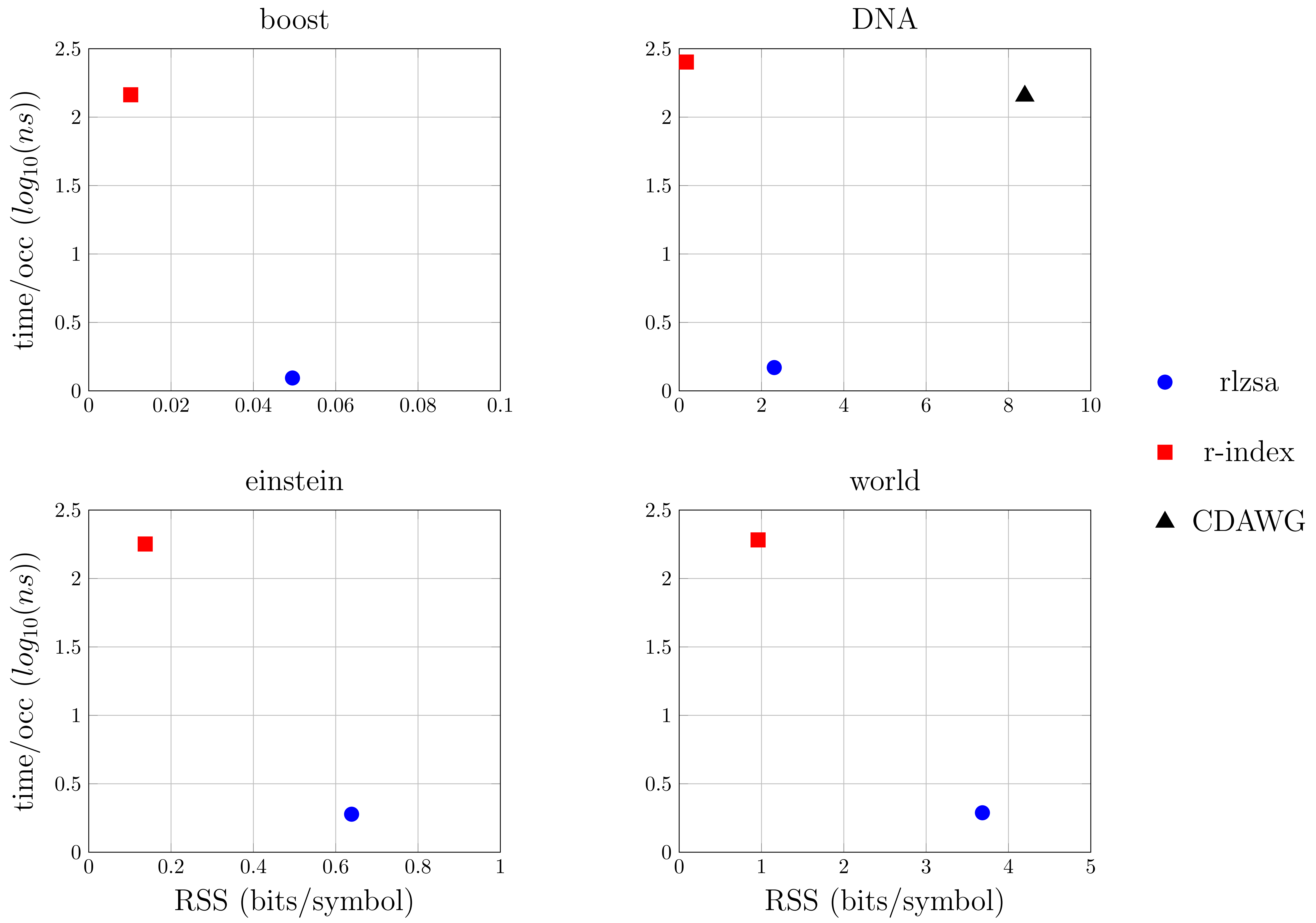
Search queries:

- 1000 patterns
- length = 8

References:

- boost — 21 samples * 4096
- DNA — 11377 samples * 2048
- einstein — 2089 samples * 3072
- world — 498 samples * 4096

Experimental results



Comparison to r-index

	more space	much faster
boost	4.88	115.32
DNA	13.19	149.10
einstein	4.65	96.27
world	3.84	89.08

Future work

- Reducing space:
  - Our prototype uses word-aligned units everywhere (16-, 32-, 64-bit ints)
  - We can save space by using succinct representations instead (Elias-Fano for the predecessor structure, packed int vectors for phrases, etc.) (progress here)
  - Improved reference construction (we have some progress here already as well)
- Apply it to document (D) array
- Best of both worlds?
  - Is there a way to derive a hybrid of the **r-index** and **rlzsa**?

Thank you!

r-index time $= O(occ * \log\log n)$

CDAWG time $= O(m(\log\log n + \log z) + pocc * \log^{\varepsilon} z + socc * \log\log n)$

rlzsa time $= O(\log\log n + occ + l_{max})$