

INTRODUCTION

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STRING MATCHING WITH FINITE AUTOMATA:

- String matching algorithm builds a finite automaton
- A simple machine for processing information that scans the text string T for all occurrences of the pattern P

FINITE AUTOMATA...

FINITE AUTOMATA

- A *finite automaton* M , is a 5 tuple $(Q, q_0, A, \Sigma, \delta)$, where
 - Q is finite set of *states*,
 - $q_0 \in Q$ is the start *state*,
 - $A \subseteq Q$
 - is a distinguished set of *accepting states*,
 - Σ is a finite *input alphabet*,
 - δ is a function from $Q \times \Sigma$ into Q , called the *transition function* of M .

STRING MATCHING WITH FINITE AUTOMATA...

MATCHING AUTOMATA (SUFFIX Function)

The String Matching automata corresponding to a given pattern $P[1..m]$

- An auxiliary function σ , called the suffix function corresponding to P
- The function σ maps Σ^* to $\{0, 1, \dots, m\}$ such that $\sigma(x)$ is the length of the longest prefix of P that is also a suffix of x :
- $\sigma(x) = \max \{k: P_k \sqsubseteq x\}$

ALGORITHMS...

COMPUTE TRANSITION FUNCTION:

COMPUTE-TRANSITION-FUNCTION(P, Σ)

1. $m \leftarrow P.length$
2. for $q=0$ to m
3. for each character $a \in \Sigma$
4. $k \leftarrow \min(m+1, q+2)$
5. repeat
6. $K \leftarrow k-1$
7. until $P[k] \neq P[q+1] + a$
8. $\delta(q, a) \leftarrow k$
9. return δ

DRY RUNNING...

COMPUTE TRANSITION FUNCTION:

- TEXT=abababacaba
- PATTERN=ababaca
- $\Sigma=\{a,b,c\}$

Compute transition

function(P, Σ)

1. $m \leftarrow \text{length}[P]$
 $m \leftarrow 7$
2. for $q \leftarrow 0$ to m
for $q \leftarrow 0$ to 7
3. do for each character $a \in \Sigma$
// $\Sigma=\{a,b,c\}$

1st ITERATION:

$q=0, a=a$

3. for each character $a \in \Sigma$

4. $k \leftarrow \min(m+1, q+2)$

$= \min(7+1, 0+2) = 2$

$K \leftarrow 2$

7. $P_k \sqsupset P_q a = P_2 \sqsupset P_0 a$

$(ab \sqsupset a)$ FALSE

$K \leftarrow 1$

7. $P_k \sqsupset P_q a = P_1 \sqsupset P_0 a$

$(a \sqsupset a)$ TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(0, a) \leftarrow 1$

state	a	b	c	P
0	1			a
1				b
2				a
3				b
4				a
5				c
6				a
7				

CONT...

$q=0$

3. for each character $a \in b$

4. $K=2$

7. $P_k \sqsupset P_{qa}=P_2 \sqsupset P_0a$

$(ab \sqsupset b)$ FALSE

$K=1$

7. $P_k \sqsupset P_{qa}=P_1 \sqsupset P_0a$

$(a \sqsupset b)$ FALSE

$K=0$

$P_k \sqsupset P_{qa}=P_0 \sqsupset P_0b$

$(\epsilon \sqsupset b)$ TRUE

8. $\delta(q,a) \leftarrow k$

$\delta(0,b) \leftarrow 0$

state	a	b	c	P
0	1	0		a
1				b
2				a
3				b
4				a
5				c
6				a
7				

CONT...

$q=0$

3. for each character $a \in \Sigma$

4. $K=2$

7. $P_k \sqsupset P_{qa} = P_2 \sqsupset P_0a$

$(ab \sqsupset c)$ FALSE

$K=1$

7. $P_k \sqsupset P_{qa} = P_1 \sqsupset P_0a$

$(a \sqsupset c)$ FALSE

$K=0$

$P_k \sqsupset P_{qa} = P_0 \sqsupset P_0s$

$(\epsilon \sqsupset c)$ TRUE

8. $\delta(q,a) \leftarrow k$

$\delta(0,c) \leftarrow 0$

state	a	b	c	P
0	1	0	0	a
1				b
2				a
3				b
4				a
5				c
6				a
7				

2nd ITERATION:

2. for $q \leftarrow 1$ to 7

3. for each character $a \in \{a, b, c\}$

$q=1, a=a$

4. $k \leftarrow \min(m+1, q+2) = \min(7+1, 1+2) = 3$

$K \leftarrow 3$

$P_k \sqsupset P_q a = P_3 \sqsupset P_1 a = (aba \sqsupset aa)$

FALSE

$k \leftarrow 2$

7. $P_k \sqsupset P_q a = P_2 \sqsupset P_1 a$

$(ab \sqsupset aa)$ FALSE

$K=1$

7. $P_k \sqsupset P_q a = P_1 \sqsupset P_1 a$

$(a \sqsupset aa)$ TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(1, a) \leftarrow 1$

State	a	b	c	P
0	1	0	0	a
1	1			b
2				a
3				b
4				a
5				c
6				a
7				

CONT...

$q=1$

for each character $a \in b$

4. $K \leftarrow 3$

$P_k \sqcup P_{qa} = P_3 \sqcup P_1 a = (aba \sqcup ab)$

FALSE

$k \leftarrow 2$

7. $P_k \sqcup P_{qa} = P_2 \sqcup P_1 a$

$(ab \sqcup ab)$ TRUE

8. $\delta(1, b) \leftarrow 2$

State	a	b	c	P
0	1	0	0	a
1	1	2		b
2				a
3				b
4				a
5				c
6				a
7				

CONT...

$q=1$

for each character $a \in c$

4. $K \leftarrow 3$

$P_k \sqcup Pqa = P3 \sqcup P1a = (aba \sqcup ac)$

FALSE

$k \leftarrow 2$

7. $P_k \sqcup Pqa = P2 \sqcup P1a$

$(ab \sqcup ac)$ FALSE

$k \leftarrow 1$

$P_k \sqcup Pqa = P1 \sqcup P1a$

$(a \sqcup ac)$ FALSE

$k \leftarrow 0$

$P_k \sqcup Pqa = P0 \sqcup P1a$

$(\epsilon \sqcup ac)$ TRUE

8. $\delta(1, c) \leftarrow 0$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2				a
3				b
4				a
5				c
6				a
7				

3rd ITERATION:

2. for $q \leftarrow 2$ to 7

3. for each character $a \in \{a, b, c\}$

$q=2, a=a$

4. $k \leftarrow \min(m+1, q+2) = \min(8, 4) = 4$

$K \leftarrow 4$

$P_k \sqsupset P_{qa} = P_4 \sqsupset P_2 a = (abab \sqsupset aba)$

FALSE

$P_k \sqsupset P_{qa} = P_3 \sqsupset P_2 a = (aba \sqsupset aba)$

TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(2, a) \leftarrow 3$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3			a
3				b
4				a
5				c
6				a
7				

CONT...

$q=2$

for each character $a \in b$

$K \leftarrow 4$

$P_k \sqsupset P_{qa} = P_4 \sqsupset P_{2a} = (abab \sqsupset abb)$

FALSE

$K=3$

$P_k \sqsupset P_{qa} = P_3 \sqsupset P_{2a} = (aba \sqsupset abb)$

FALSE

$k \leftarrow 2$

7. $P_k \sqsupset P_{qa} = P_2 \sqsupset P_{2a} = (ab \sqsupset abb)$

FALSE

$K=1$

7. $P_k \sqsupset P_{qa} = P_1 \sqsupset P_{2a}$

$(a \sqsupset abb)$ FALSE

$K=0$

7. $P_k \sqsupset P_{qa} = P_0 \sqsupset P_{2a}$

$(\epsilon \sqsupset abb)$ TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(2, b) \leftarrow 0$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0		a
3				b
4				a
5				c
6				a
7				

CONT...

$q=2$

for each character $a \in c$

$K \leftarrow 4$

$P_k \sqsupset P_{qa} = P_4 \sqsupset P_{2a} = (abab \sqsupset abc)$

FALSE

$K=3$

$P_k \sqsupset P_{qa} = P_3 \sqsupset P_{2a} = (aba \sqsupset abc)$

FALSE

$k \leftarrow 2$

7. $P_k \sqsupset P_{qa} = P_2 \sqsupset P_{2a} = (ab \sqsupset abc)$

FALSE

$K=1$

7. $P_k \sqsupset P_{qa} = P_1 \sqsupset P_{2a}$

$(a \sqsupset abc)$ FALSE

$K=0$

7. $P_k \sqsupset P_{qa} = P_0 \sqsupset P_{2a}$

$(\epsilon \sqsupset abc)$ TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(2, c) \leftarrow 0$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3				b
4				a
5				c
6				a
7				

4TH ITERATION:

2. for $q \leftarrow 3$ to 7

3. for each character $a \in \{a, b, c\}$

$q=3, a=a$

4. $k \leftarrow \min(m+1, q+2) = \min(8, 4) = 4$

$K \leftarrow 5$

$P_k \sqsupset P_q a = P_5 \sqsupset P_3 a = (ababa \sqsupset abaa)$

FALSE

$k \leftarrow 4$

$P_k \sqsupset P_q a = P_4 \sqsupset P_3 a = (abab \sqsupset abaa)$

FALSE

$K \leftarrow 3$

$P_k \sqsupset P_q a = P_3 \sqsupset P_3 a = (aba \sqsupset abaa)$

FALSE

$k \leftarrow 2$

$P_k \sqsupset P_q a = P_2 \sqsupset P_3 a = (ab \sqsupset abaa)$

FALSE

$K \leftarrow 1$

$P_k \sqsupset P_q a = P_1 \sqsupset P_3 a = (a \sqsupset abaa)$

TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(3, a) \leftarrow 1$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1			b
4				a
5				c
6				a
7				

CONT...

2. for $q \leftarrow 3$

3. for each character $a \in b$

$K \leftarrow 5$

P_k

$\neg P_{qa} = P_5 \neg P_3 a = (ababa \neg abab)$

FALSE

$k \leftarrow 4$

$P_k \neg P_{qa} = P_4 \neg P_3 a = (abab \neg abab)$

TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(3, b) \leftarrow 4$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4		b
4				a
5				c
6				a
7				

CONT...

2. $q=3$

3. for each character $a \in \Sigma$

$K \leftarrow 5$

$P_k \sqcap P_{qa} = P_5 \sqcap P_3 a = (ababa \sqcap abac)$

FALSE

$k \leftarrow 4$

$P_k \sqcap P_{qa} = P_4 \sqcap P_3 a = (abab \sqcap abac)$

FALSE

$K \leftarrow 3$

$P_k \sqcap P_{qa} = P_3 \sqcap P_3 a = (aba \sqcap abac)$

FALSE

$k \leftarrow 2$

$P_k \sqcap P_{qa} = P_2 \sqcap P_3 a = (ab \sqcap abac)$

FALSE

$K \leftarrow 1$

$P_k \sqcap P_{qa} = P_1 \sqcap P_3 a = (a \sqcap abac)$

FALSE

$K \leftarrow 0$

$P_k \sqcap P_{qa} = P_0 \sqcap P_3 a = (\epsilon \sqcap abac)$

TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(3, c) \leftarrow 0$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4				a
5				c
6				a
7				

5th ITERATION:

2. for $q \leftarrow 4$ to 7

3. for each character $a \in (a, b, c)$

$q = 4, a = a$

4. $k \leftarrow \min(m+1, q+2) = \min(8, 6) = 6$

$K \leftarrow 6$

$P_k \sqsupset P_q a = P_6 \sqsupset P_4 a = (ababac \sqsupset ababa)$

FALSE

$K \leftarrow 5$

$P_k \sqsupset P_q a = P_5 \sqsupset P_4 a = (ababa \sqsupset abaBA)$

TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(4, a) \leftarrow 5$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5			a
5				c
6				a
7				

CONT...

2. $q=4$

3. for each character $a \in b$

$K \leftarrow 6$

$P_k \sqcup P_{qa} = P_6 \sqcup P_4 a = (ababac \sqcup ababb)$

FALSE

$K \leftarrow 5$

$P_k \sqcup P_{qa} = P_5 \sqcup P_4 a = (ababa \sqcup ababb)$

FALSE

$k \leftarrow 4$

$P_k \sqcup P_{qa} = P_4 \sqcup P_4 a = (abab \sqcup ababb)$

FALSE

$K \leftarrow 3$

$P_k \sqcup P_{qa} = P_3 \sqcup P_4 a = (aba \sqcup ababb)$

FALSE

$k \leftarrow 2$

$P_k \sqcup P_{qa} = P_2 \sqcup P_4 a = (ab \sqcup ababb)$

FALSE

$k \leftarrow 1$

$P_k \sqcup P_{qa} = P_2 \sqcup P_4 a = (a \sqcup ababb)$

FALSE

$k \leftarrow 0$

$P_k \sqcup P_{qa} = P_2 \sqcup P_4 a = (\epsilon \sqcup ababb)$

TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(4, b) \leftarrow 0$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0		a
5				c
6				a
7				

CONT...

2. $q=4$

3. for each character $a \in c$

$K \leftarrow 6$

$P_k \sqcup P_{qa} = P_6 \sqcup P_4 a = (ababac \sqcup ababc)$

FALSE

$K \leftarrow 5$

$P_k \sqcup P_{qa} = P_5 \sqcup P_4 a = (ababa \sqcup ababc)$

FALSE

$k \leftarrow 4$

$P_k \sqcup P_{qa} = P_4 \sqcup P_4 a = (abab \sqcup ababc)$

FALSE

$K \leftarrow 3$

$P_k \sqcup P_{qa} = P_3 \sqcup P_4 a = (aba \sqcup ababc)$

FALSE

$k \leftarrow 2$

$P_k \sqcup P_{qa} = P_2 \sqcup P_4 a = (ab \sqcup ababc)$

FALSE

$k \leftarrow 1$

$P_k \sqcup P_{qa} = P_2 \sqcup P_4 a = (a \sqcup ababbc)$

FALSE

$k \leftarrow 0$

$P_k \sqcup P_{qa} = P_2 \sqcup P_4 a = (\epsilon \sqcup ababc)$

TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(4, c) \leftarrow 0$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5				c
6				a
7				

6th ITERATION:

2. for $q \leftarrow 5$ to 7

3. for each character $a \in \{a, b, c\}$

$q = 5, a = a$

4. $k \leftarrow \min(m+1, q+2) = \min(8, 7) = 7$

$K \leftarrow 7$

$P_k \sqcup P_q a = P_7 \sqcup P_5 a = (ababaca \sqcup ababaa)$

FALSE

$K \leftarrow 6$

$P_k \sqcup P_q a = P_6 \sqcup P_5 a = (ababac \sqcup ababaa)$

FALSE

$K \leftarrow 5$

$P_k \sqcup P_q a = P_5 \sqcup P_5 a = (ababa \sqcup ababaa)$ FALSE

$k \leftarrow 4$

$P_k \sqcup P_q a = P_4 \sqcup P_5 a = (abab \sqcup ababaa)$

FALSE

$K \leftarrow 3$

$P_k \sqcup P_q a = P_3 \sqcup P_5 a = (aba \sqcup ababaa)$ FALSE

$k \leftarrow 2$

$P_k \sqcup P_q a = P_2 \sqcup P_5 a = (ab \sqcup ababaa)$

FALSE

$k \leftarrow 1$

$P_k \sqcup P_q a = P_2 \sqcup P_4 a = (a \sqcup ababbaa)$

TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(5, a) \leftarrow 1$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1			c
6				a
7				

CONT...

4. $q \leftarrow 5$

3. for each character $a \in b$

$K \leftarrow 7$

$P_k \sqcup P_q a = P_7 \sqcup P_5 a = (\text{ababaca} \sqcup \text{ababab})$

FALSE

$K \leftarrow 6$

$P_k \sqcup P_q a = P_6 \sqcup P_5 a = (\text{ababac} \sqcup \text{ababab})$

FALSE

$K \leftarrow 5$

$P_k \sqcup P_q a = P_5 \sqcup P_5 a = (\text{ababa} \sqcup \text{ababab})$

FALSE

$k \leftarrow 4$

$P_k \sqcup P_q a = P_4 \sqcup P_5 a = (\text{abab} \sqcup \text{ababab})$

TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(5, b) \leftarrow 4$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4		c
6				a
7				

CONT...

4. $q \leftarrow 5$

3. for each character $a \in c$

$K \leftarrow 7$

$P_k \sqcup P_{qa} = P_7 \sqcup P_5 a = (ababaca \sqcup ababac)$

FALSE

$K \leftarrow 6$

$P_k \sqcup P_{qa} = P_6 \sqcup P_5 a = (ababac \sqcup ababac)$

TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(5, c) \leftarrow 6$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6				a
7				

7TH ITERATION:

2. for $q \leftarrow 6$ to 7

3. for each character $a \in \{a, b, c\}$

$q = 6, a = a$

4. $k \leftarrow \min(m+1, q+2) = \min(8, 8) = 8$

$K \leftarrow 8$

$P_k \sqsupset P_q a = P_8 \sqsupset P_6 a = (ababacaa \sqsupset ababaca)$

FALSE

$K \leftarrow 7$

$P_k \sqsupset P_q a = P_7 \sqsupset P_6 a = (ababaca \sqsupset ababaca)$

TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(6, a) \leftarrow 7$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7			a
7				

CONT...

2. $q \leftarrow 6$

3. for each character $a \in b$

$K \leftarrow 8$

$P_k \sqcup P_{qa} = P_8 \sqcup P_6a = (ababacaa \sqcup ababacb)$

FALSE

$K \leftarrow 7$

$P_k \sqcup P_{qa} = P_7 \sqcup P_6a = (ababaca \sqcup ababacb)$

FALSE

$K \leftarrow 6$

$P_k \sqcup P_{qa} = P_6 \sqcup P_6a = (ababac \sqcup ababacb)$ FALSE

$K \leftarrow 5$

$P_k \sqcup P_{qa} = P_5 \sqcup P_6a = (ababa \sqcup ababacb)$ FALSE

$K \leftarrow 4$

$P_k \sqcup P_{qa} = P_4 \sqcup P_6a = (abab \sqcup ababacb)$ FALSE

$K \leftarrow 3$

$P_k \sqcup P_{qa} = P_3 \sqcup P_6a = (aba \sqcup ababacb)$ FALSE

$K \leftarrow 2$

$P_k \sqcup P_{qa} = P_2 \sqcup P_6a = (ab \sqcup ababacb)$ FALSE

$K \leftarrow 1$

$P_k \sqcup P_{qa} = P_1 \sqcup P_6a = (a \sqcup ababacb)$ FALSE

$K \leftarrow 0$

$P_k \sqcup P_{qa} = P_0 \sqcup P_6a = (\epsilon \sqcup ababacb)$ TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(6, b) \leftarrow 0$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0		a
7				

CONT...

2. $q \leftarrow 6$

3. for each character $a \in c$

$K \leftarrow 8$

$P_k \sqcup P_{qa} = P_8 \sqcup P_6a = (ababacaa \sqcup ababac)$
FALSE

$K \leftarrow 7$

$P_k \sqcup P_{qa} = P_7 \sqcup P_6a = (ababaca \sqcup ababac)$ FALSE

$K \leftarrow 6$

$P_k \sqcup P_{qa} = P_6 \sqcup P_6a = (ababac \sqcup ababac)$ FALSE

$K \leftarrow 5$

$P_k \sqcup P_{qa} = P_5 \sqcup P_6a = (ababa \sqcup ababac)$ FALSE

$K \leftarrow 4$

$P_k \sqcup P_{qa} = P_4 \sqcup P_6a = (abab \sqcup ababac)$ FALSE

$K \leftarrow 3$

$P_k \sqcup P_{qa} = P_3 \sqcup P_6a = (aba \sqcup ababac)$ FALSE

$K \leftarrow 2$

$P_k \sqcup P_{qa} = P_2 \sqcup P_6a = (ab \sqcup ababac)$ FALSE

$K \leftarrow 1$

$P_k \sqcup P_{qa} = P_1 \sqcup P_6a = (a \sqcup ababac)$ FALSE

$K \leftarrow 0$

$P_k \sqcup P_{qa} = P_0 \sqcup P_6a = (\epsilon \sqcup ababac)$ TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(6, c) \leftarrow 0$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7				

8TH ITERATION:

2. $q \leftarrow 7$ to 7

3. for each character $a \in \{a, b, c\}$

$q = 7, a = a$

4. $k \leftarrow \min(m+1, q+2) = \min(8, 9) = 8$

$K \leftarrow 8$

$P_k \sqsupset P_q a = P_8 \sqsupset P_7 a = (ababacaa \sqsupset ababacaa)$

FALSE

$K \leftarrow 7$

$P_k \sqsupset P_q a = P_7 \sqsupset P_7 a = (ababaca \sqsupset ababacaa)$ FALSE

$K \leftarrow 6$

$P_k \sqsupset P_q a = P_6 \sqsupset P_7 a = (ababac \sqsupset ababacaa)$ FALSE

$K \leftarrow 5$

$P_k \sqsupset P_q a = P_5 \sqsupset P_7 a = (ababa \sqsupset ababacaa)$ FALSE

$K \leftarrow 4$

$P_k \sqsupset P_q a = P_4 \sqsupset P_7 a = (abab \sqsupset ababacaa)$ FALSE

$K \leftarrow 3$

$P_k \sqsupset P_q a = P_3 \sqsupset P_7 a = (aba \sqsupset ababacaa)$ FALSE

$K \leftarrow 2$

$P_k \sqsupset P_q a = P_2 \sqsupset P_7 a = (ab \sqsupset ababacaa)$ FALSE

$K \leftarrow 1$

$P_k \sqsupset P_q a = P_1 \sqsupset P_7 a = (a \sqsupset ababacaa)$ TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(7, a) \leftarrow 1$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1			

CONT...

2. $q \leftarrow 7$

3. for each character $a \in b$

$K \leftarrow 8$

$P_k \sqcup P_{qa} = P_8 \sqcup P_7a = (\text{ababacaa} \sqcup \text{ababacab})$

FALSE

$K \leftarrow 7$

$P_k \sqcup P_{qa} = P_7 \sqcup P_7a = (\text{ababaca} \sqcup \text{ababacab})$ FALSE

$K \leftarrow 6$

$P_k \sqcup P_{qa} = P_6 \sqcup P_7a = (\text{ababac} \sqcup \text{ababacab})$ FALSE

$K \leftarrow 5$

$P_k \sqcup P_{qa} = P_5 \sqcup P_7a = (\text{ababa} \sqcup \text{ababacab})$ FALSE

$K \leftarrow 4$

$P_k \sqcup P_{qa} = P_4 \sqcup P_7a = (\text{abab} \sqcup \text{ababacab})$ FALSE

$K \leftarrow 3$

$P_k \sqcup P_{qa} = P_3 \sqcup P_7a = (\text{aba} \sqcup \text{ababacab})$ FALSE

$K \leftarrow 2$

$P_k \sqcup P_{qa} = P_2 \sqcup P_7a = (\text{ab} \sqcup \text{ababacab})$ TRUE

8. $\delta(q, a) \leftarrow k$

$\delta(7, b) \leftarrow 2$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2		

CONT...

2. $q \leftarrow 7$

3. for each character $a \in c$

$K \leftarrow 8$

$P_k \sqcup P_{qa} = P_8 \sqcup P_7a = (ababacaa \sqcup ababacac)$

FALSE

$K \leftarrow 7$

$P_k \sqcup P_{qa} = P_7 \sqcup P_7a = (ababaca \sqcup ababacac)$

FALSE

$K \leftarrow 6$

$P_k \sqcup P_{qa} = P_6 \sqcup P_7a = (ababac \sqcup ababacac)$ FALSE

$K \leftarrow 5$

$P_k \sqcup P_{qa} = P_5 \sqcup P_7a = (ababa \sqcup ababacac)$ FALSE

$K \leftarrow 4$

$P_k \sqcup P_{qa} = P_4 \sqcup P_7a = (abab \sqcup ababacac)$ FALSE

$K \leftarrow 3$

$P_k \sqcup P_{qa} = P_3 \sqcup P_7a = (aba \sqcup ababacac)$ FALSE

$K \leftarrow 2$

$P_k \sqcup P_{qa} = P_2 \sqcup P_7a = (ab \sqcup ababacac)$ FALSE

$K \leftarrow 1$

$P_k \sqcup P_{qa} = P_1 \sqcup P_7a = (a \sqcup ababacac)$

FALSE

$K \leftarrow 0$

$P_k \sqcup P_{qa} = P_0 \sqcup P_7a = (\epsilon \sqcup ababacac)$

TRUE

8. $\delta(q, a) \leftarrow k$

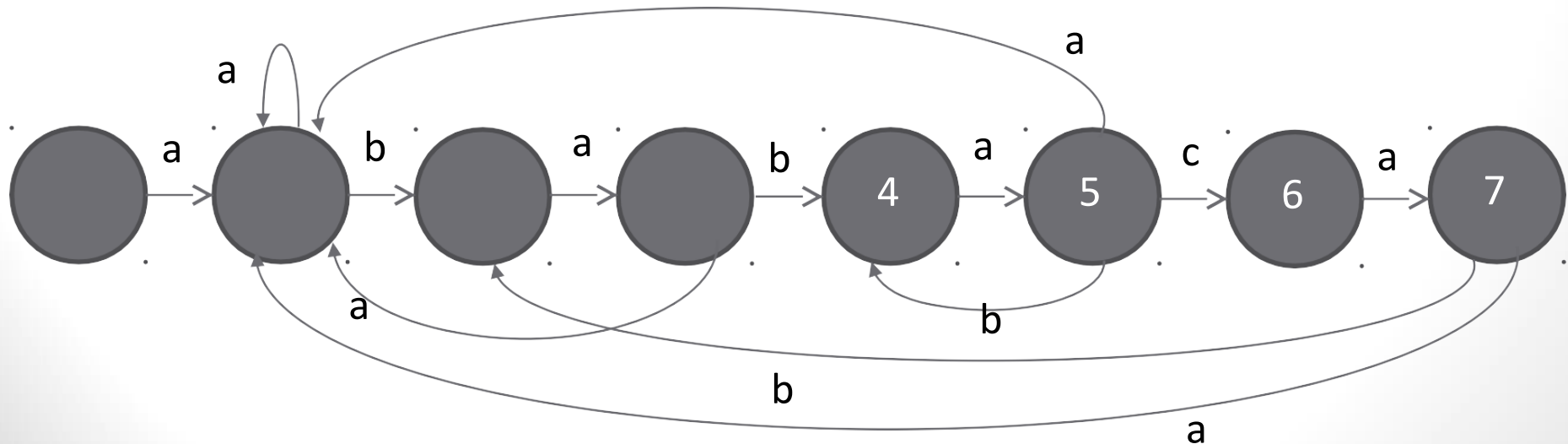
$\delta(7, c) \leftarrow 0$

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

REQUIRED
RESULT:

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

THE FINITE STATE
AUTOMATA IS:



ALGORITHM...

FINITE AUTOMATON MATCHER

FINITE-AUTOMATON-MATCHER(T, δ, m)

1. $n = T.length$
2. $q = 0$
3. **for** $i = 1$ **to** n
4. $q = \delta(q, T[i])$
5. **If** $q == m$
6. Print “Pattern occurs with shift” $i - m$

DRY RUNNING...

FINITE-AUTOMATON MATCHER

$i = 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11$

$T = a\ b\ a\ b\ a\ b\ a\ c\ a\ b\ a$

1. $n \leftarrow \text{length}[T]$

$n \leftarrow 11$

2. $q \leftarrow 0$

3. for $i \leftarrow 1$ to n

for $i \leftarrow 1$ to 11

1st ITERATION:

3. for $i \leftarrow 1$ to 11

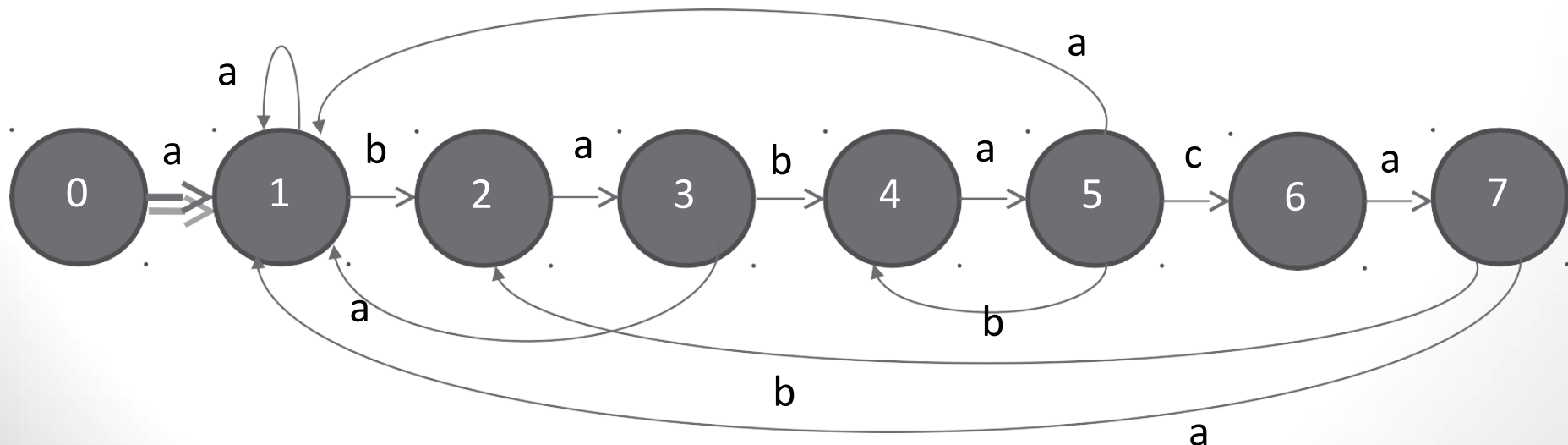
4. do $q \leftarrow \delta(q, T[i])$

$q \leftarrow \delta(0, T[1]) = \delta(0, a)$

$q \leftarrow (0, a) = 1$

5. if $q = m$

if $1 = 7$ FALSE



2ND ITERATION:

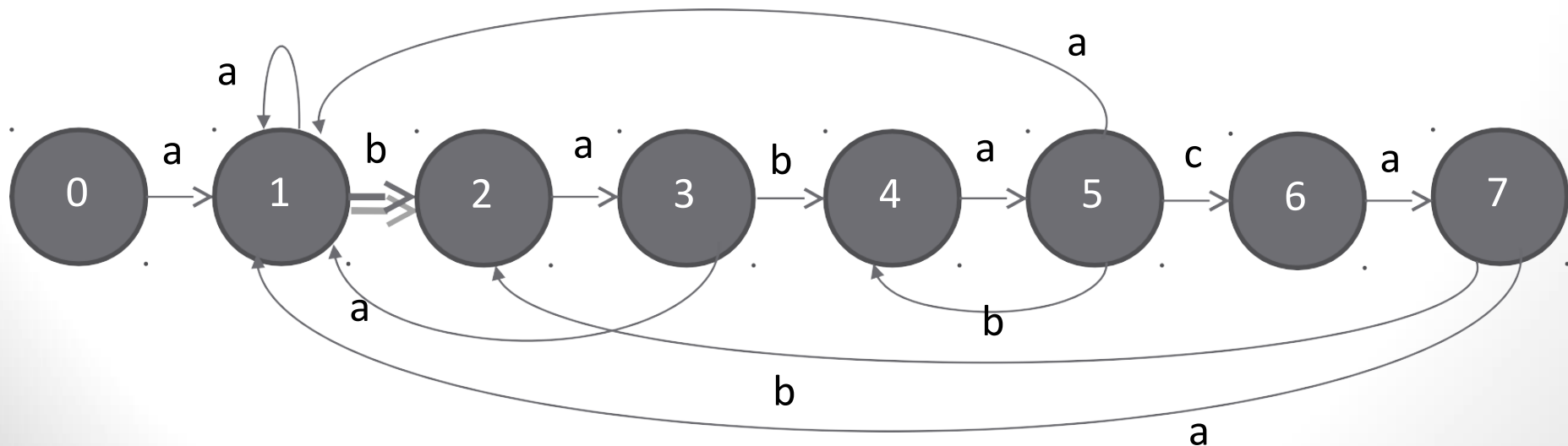
$i=2, q=2$

4. do $q \leftarrow \delta(1, T[2])$

$=\delta(1, b)$

$q \leftarrow 2$

5 if $2=7$ FALSE



3RD ITERATION:

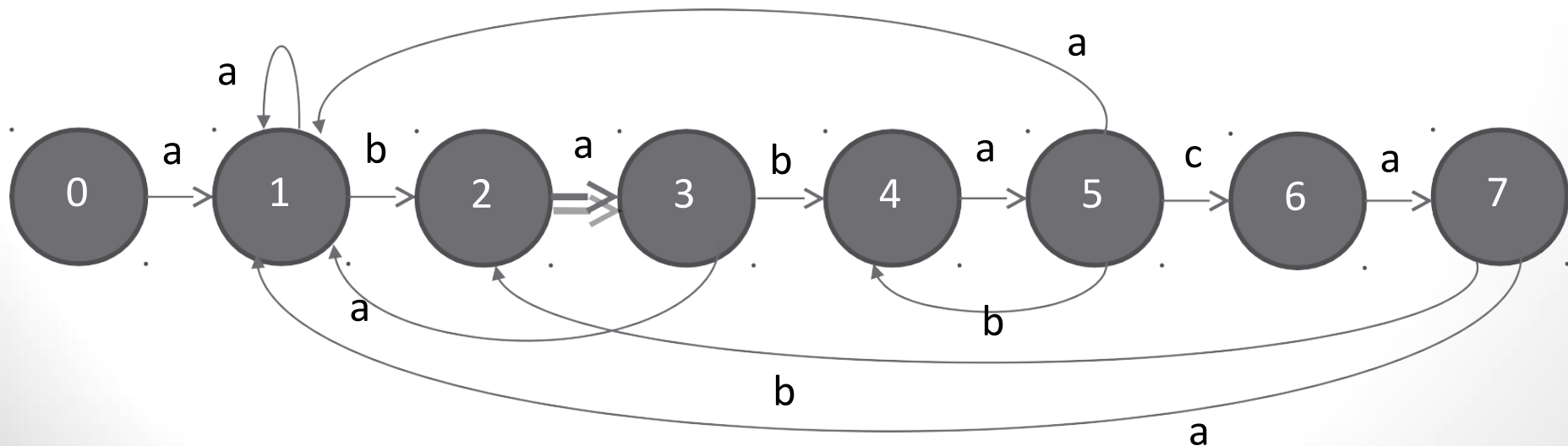
$i=3, q=2$

4. do $q \leftarrow \delta(2, T[3])$

$=\delta(2, a)$

$q \leftarrow 3$

5 if $3=7$ FALSE



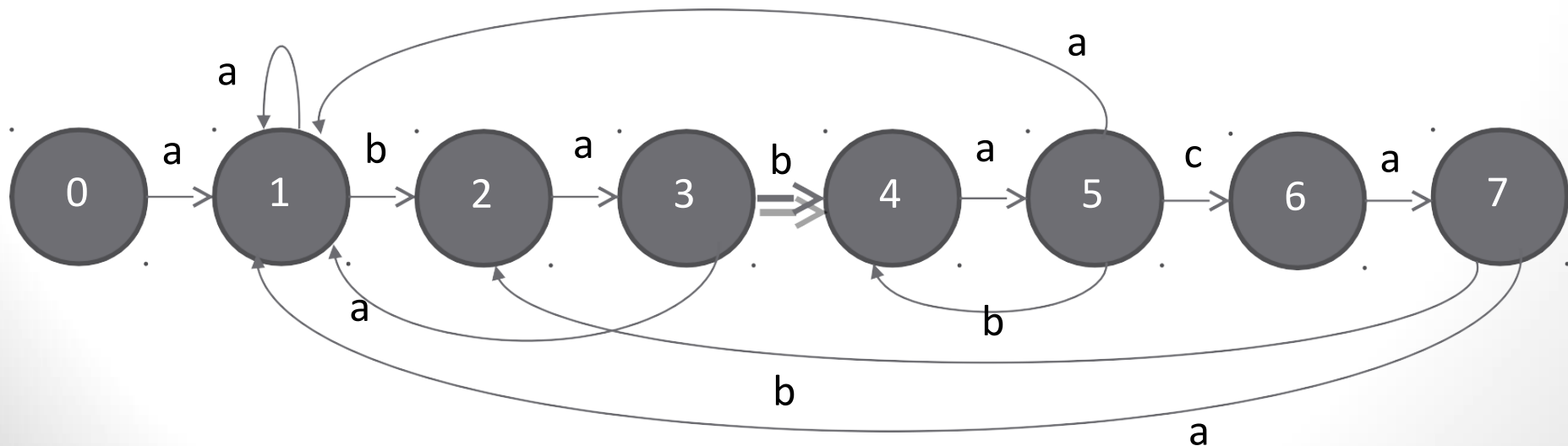
4th ITERATION:

$i=4, q=3$

4. do $q \leftarrow \delta(3, T[4]) = \delta(3, b)$

$q \leftarrow 4$

5 if $4=7$ FALSE



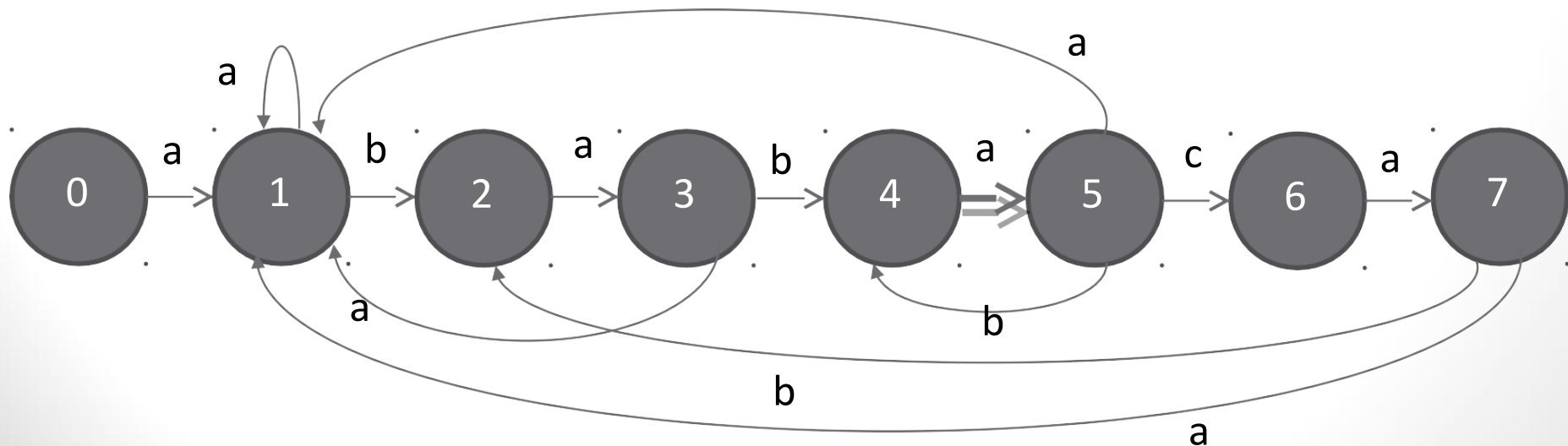
5th ITERATION:

$i=5, q=4$

4. do $q \leftarrow \delta(4, T[5])$
 $= \delta(4, a) = 5$

$q \leftarrow 5$

5 if $5=7$ FALSE



6TH ITERATION:

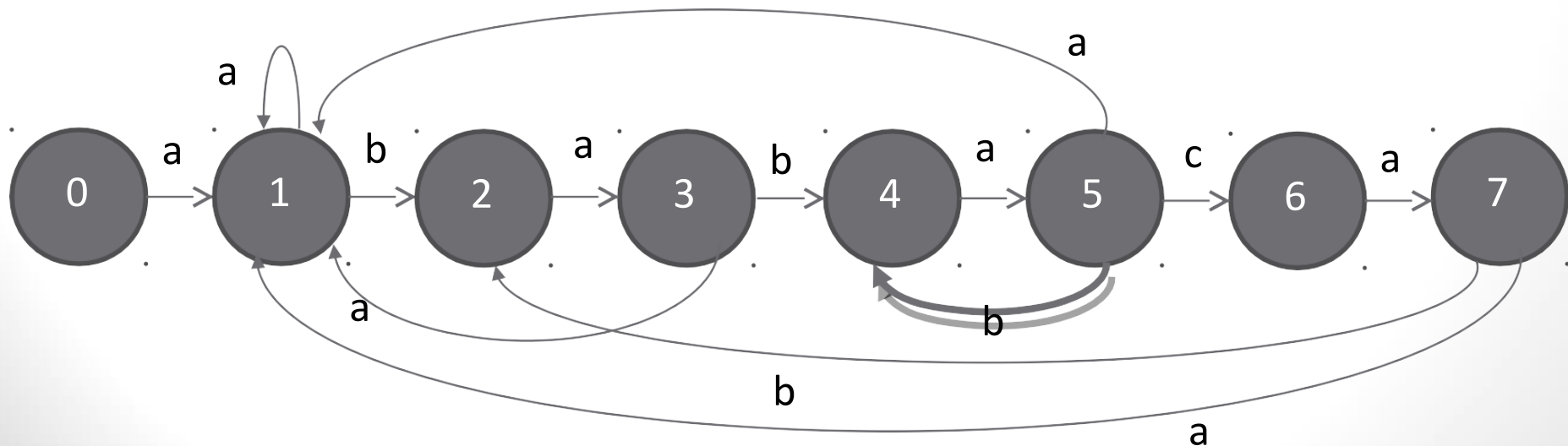
$i=6, q=5$

4. do $q \leftarrow \delta(5, T[6])$

$= \delta(5, b) = 4$

$q \leftarrow 5$

5 if $4=7$ FALSE



7TH ITERATION:

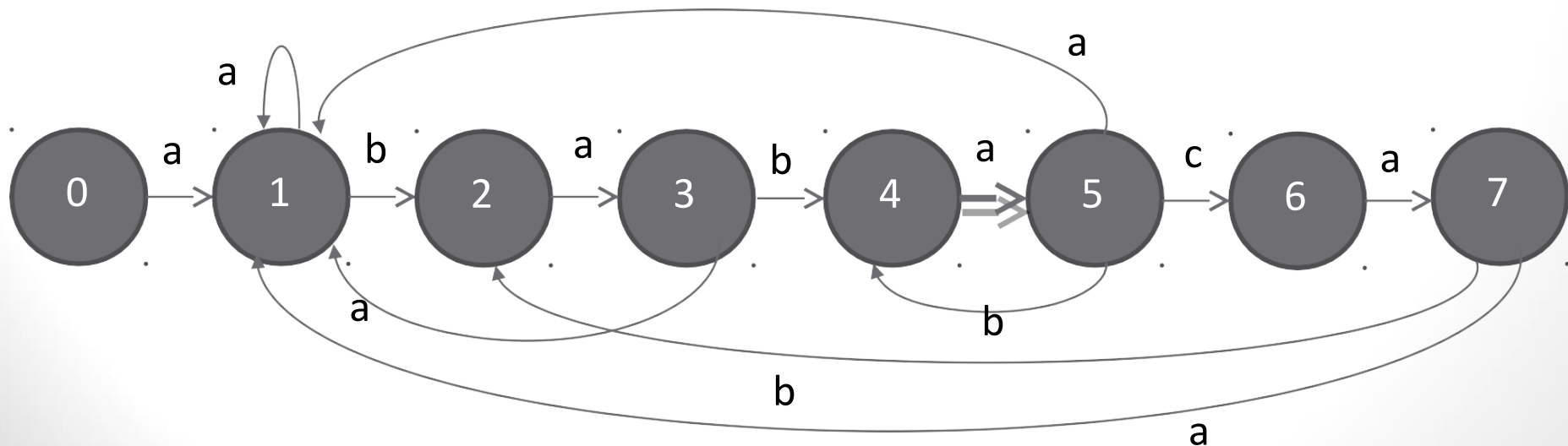
$i=7, q=4$

4. do $q \leftarrow \delta(4, T[7])$

$= \delta(4, a) = 5$

$q \leftarrow 5$

5 if $5=7$ FALSE



8TH ITERATION:

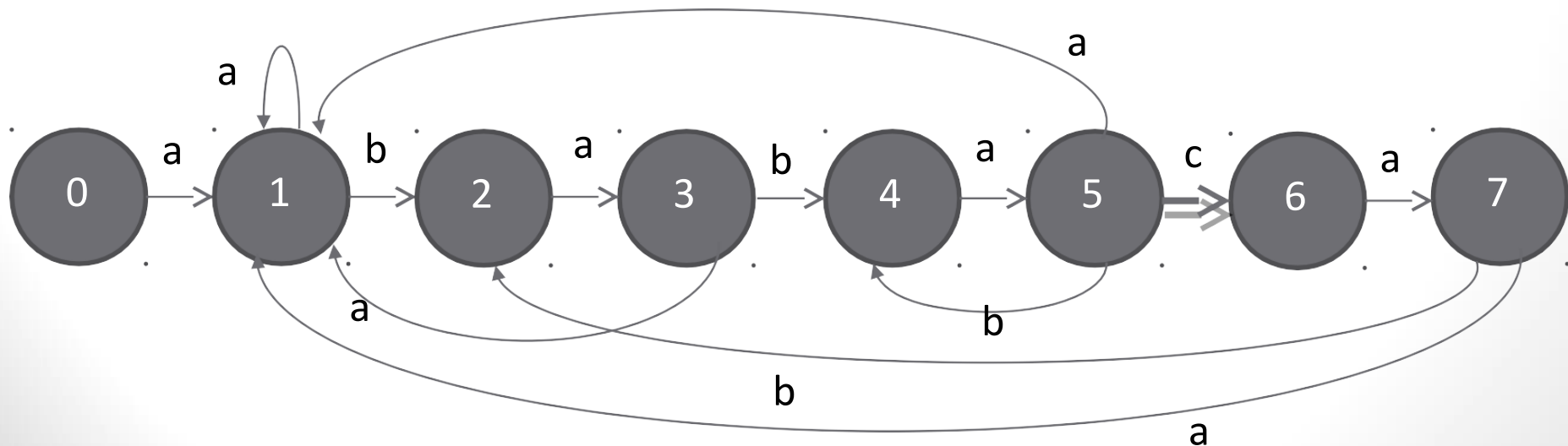
$i=8, q=5$

4. do $q \leftarrow \delta(5, T[8])$

$= \delta(5, c) = 6$

$q \leftarrow 6$

5 if $6=7$ FALSE



9TH ITERATION:

$i=9, q=6$

4. do $q \leftarrow \delta(6, T[9])$

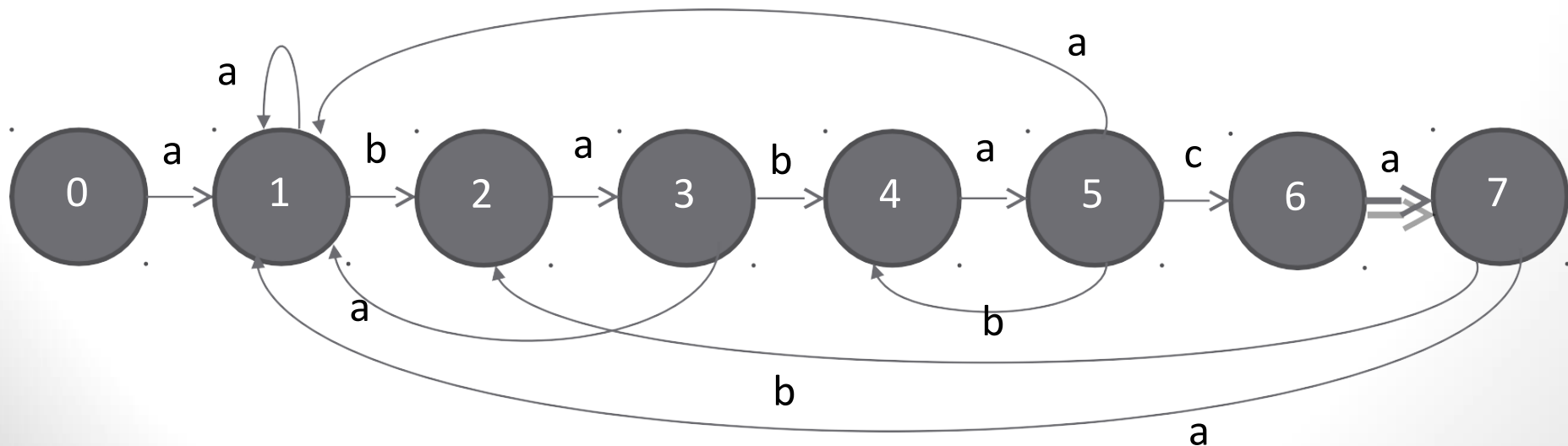
$= \delta(6, a) = 7$

$q \leftarrow 7$

5 if $7=7$ TRUE

Then print “pattern occurs with
shift” $i-m$

Print” pattern occurs with
shift” $9-7=2$



10th ITEARTION:

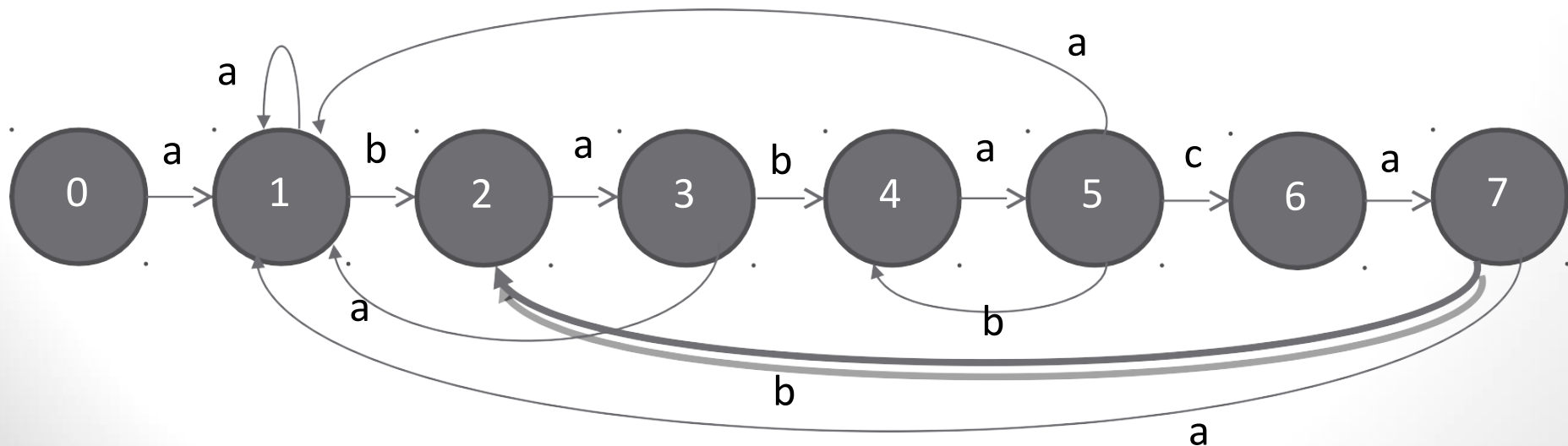
$i=10, q=7$

4. do $q \leftarrow \delta(7, T[10])$

$=\delta(7, b)=2$

$q \leftarrow 2$

5 if $2=7$ FALSE



11TH ITEARTION:

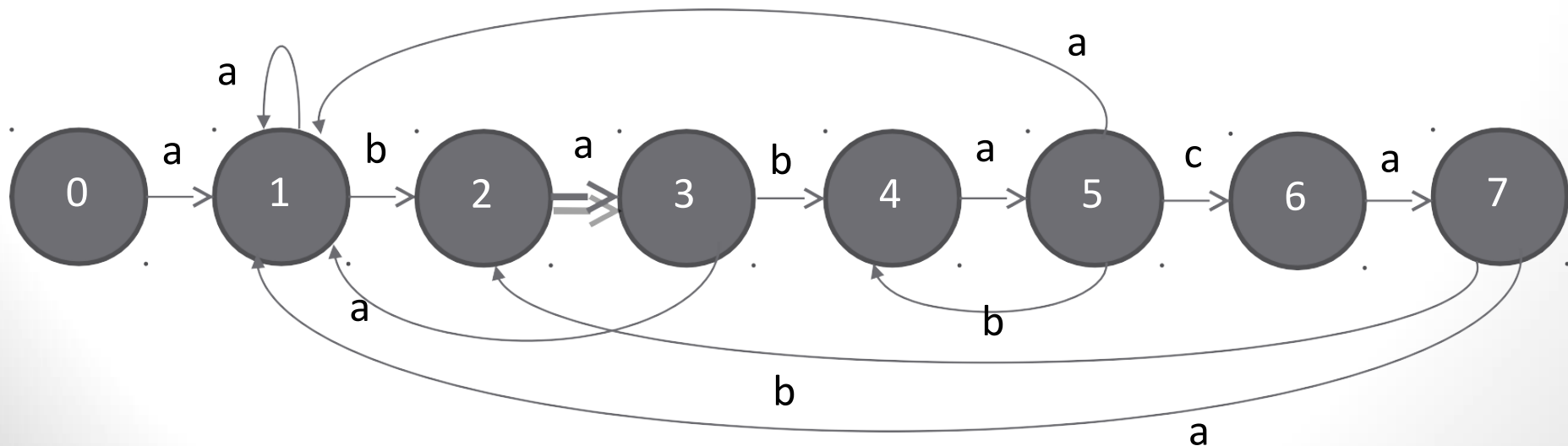
$i=11, q=2$

4. do

$q \leftarrow \delta(2, T[11]) = \delta(2, a) = 3$

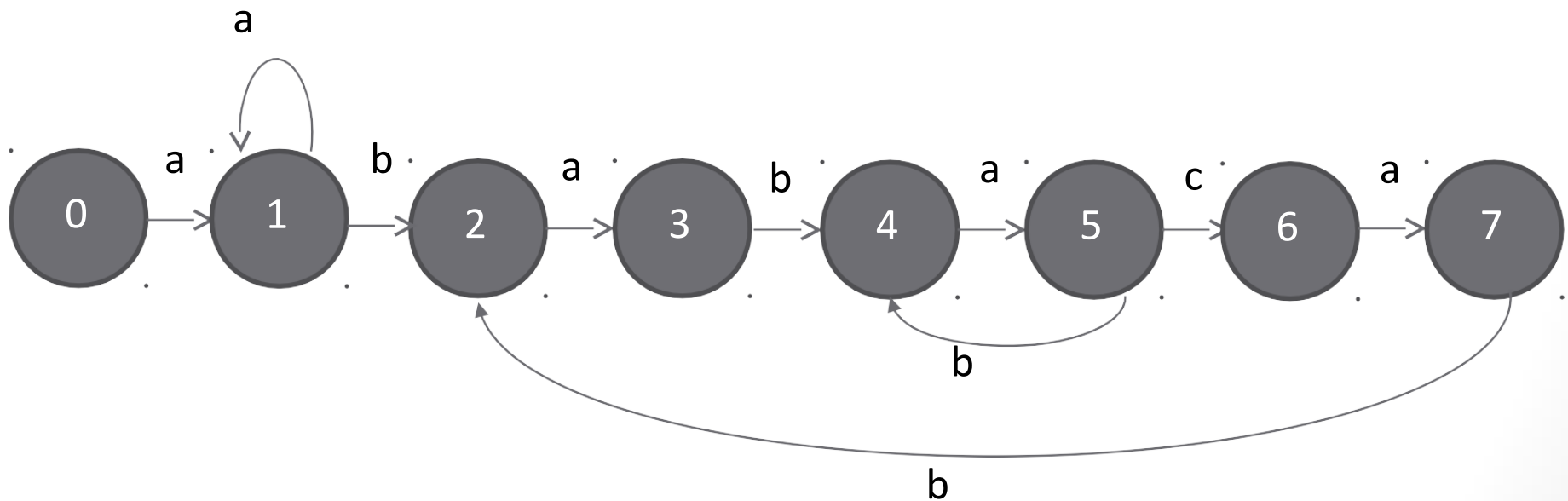
$q \leftarrow 3$

5 if $3=7$ FALSE



REQUIRED OUTPUT:

After removing useless edges:



CONT..

i= - 1 2 3 4 5 6 7 8 9 10
T[i] - a b a b a b a c a b a
state $\Phi T[i]$ 0 1 2 3 4 5 4 5 6 7 2 3

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

COMPLEXITY...

COMPLEXITY ANALYSIS:

- The simple loop time of FINITE-AUTOMATON-MATCHER implies that its matching time on the text string of length n is $\Theta(n)$. This matching time does not include the preprocessing time required to compute the transition function δ
- The running time of COMPUTE-TRANSITION-FUNCTION is $O(m^3 |\Sigma|)$, because the outer loops contribute a factor of $m |\Sigma|$, the inner repeat loop can run at most $m+1$ times, and the test $P_k \sqsubseteq P_{k+a}$ on line 7 can require computing up to m characters.
- Much faster procedures exist; the time required to compute δ from P can be improved to $O(m |\Sigma|)$ by utilizing some cleverly computed information about the pattern P . With this improved procedure for computing δ from P to $O(m |\Sigma|)$, we can find all occurrences of a length- m pattern in a length- n text over an alphabet Σ with $O(m |\Sigma|)$ preprocessing time and $\Theta(n)$ matching time.

ADVANTAGES & DISADVANTAGE...

ADVANTAGES & DISADVANTAGE:

- String matching automaton are very efficient
- Examine each text character exactly once taking constant time per text character.
- The time to build the automaton can be large if Σ is large