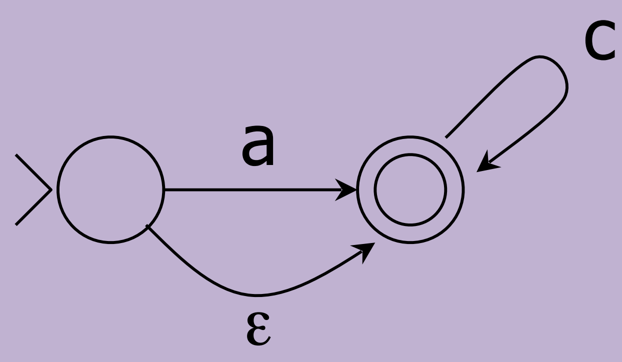
Finite-State Methods

Finite state acceptors (FSAs)



Defines the language a? c*
= {a, ac, acc, accc, …,
ε, c, cc, ccc, …}

- Things you may know about FSAs:
  - Equivalence to regexps
  - Union, Kleene *, concat, intersect, complement, reversal
  - Determinization, minimization
  - Pumping, Myhill-Nerode

n-gram models not good enough

- Want to model grammaticality
- A "training" sentence known to be grammatical:

BOS **mouse** **traps** **catch** **mouse** **traps** EOS

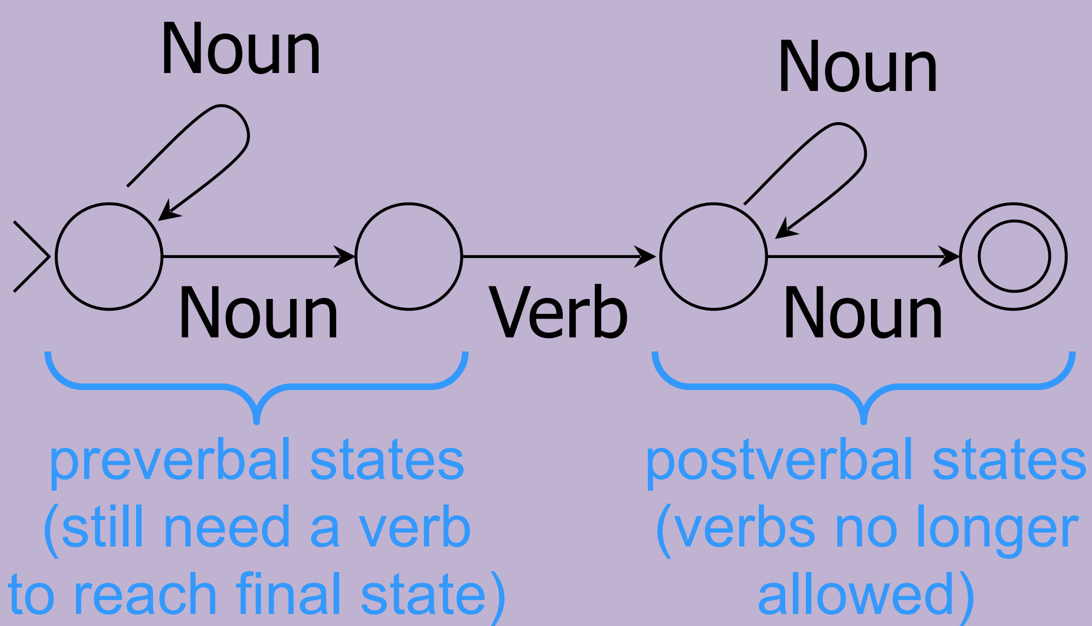
trigram model must allow these trigrams

- Resulting trigram model has to overgeneralize:
  - allows sentences with 0 verbs
    BOS **mouse traps** EOS
  - allows sentences with 2 or more verbs
    BOS **mouse traps catch mouse traps**
    catch mouse traps catch mouse traps EOS
- Can't remember whether it's in subject or object (i.e., whether it's gotten to the verb yet)

Finite-state models can "get it"

- Want to model grammaticality
  BOS **mouse traps catch mouse traps** EOS
- Finite-state can capture the generalization here:

Noun+ Verb Noun+



Allows arbitrarily long NPs (just keep looping around for another Noun modifier).

Still, never forgets whether it's preverbal or postverbal!

(Unlike 50-gram model)

How powerful are regexps / FSAs?

- More powerful than n-gram models
  - The hidden state may "remember" arbitrary past context
  - With k states, can remember which of k "types" of context it's in
- Equivalent to HMMs
  - In both cases, you observe a sequence and it is "explained" by a hidden path of states. The FSA states are like HMM tags.
- Appropriate for phonology and morphology

Word = Syllable+
= (Onset Nucleus Coda?)+
= (C+ V+ C*)+
= ((b|d|f|…)+ (a|e|i|o|u)+ (b|d|f|…)*)+

How powerful are regexps / FSAs?

- But less powerful than CFGs / pushdown automata
  - Can't do recursive center-embedding
  - Hmm, humans have trouble processing those constructions too …

- This is the rat that ate the malt.
- This is the malt that the rat ate.

- This is the cat that bit the rat that ate the malt.
- This is the malt that the rat that the cat bit ate.

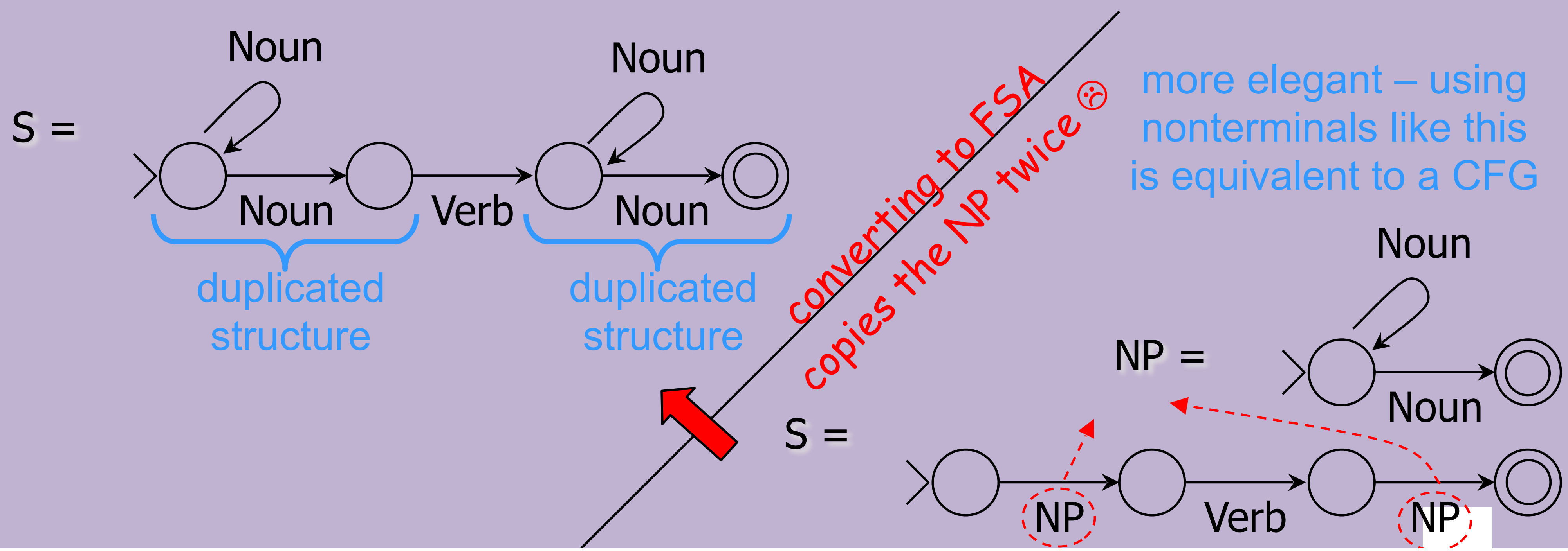
finite-state can handle this pattern (can you write the regexp?)

- This is the dog that chased the cat that bit the rat that ate the malt.
- This is the malt that [the rat that [the cat that [the dog chased] bit] ate].

but not this pattern, which requires a CFG

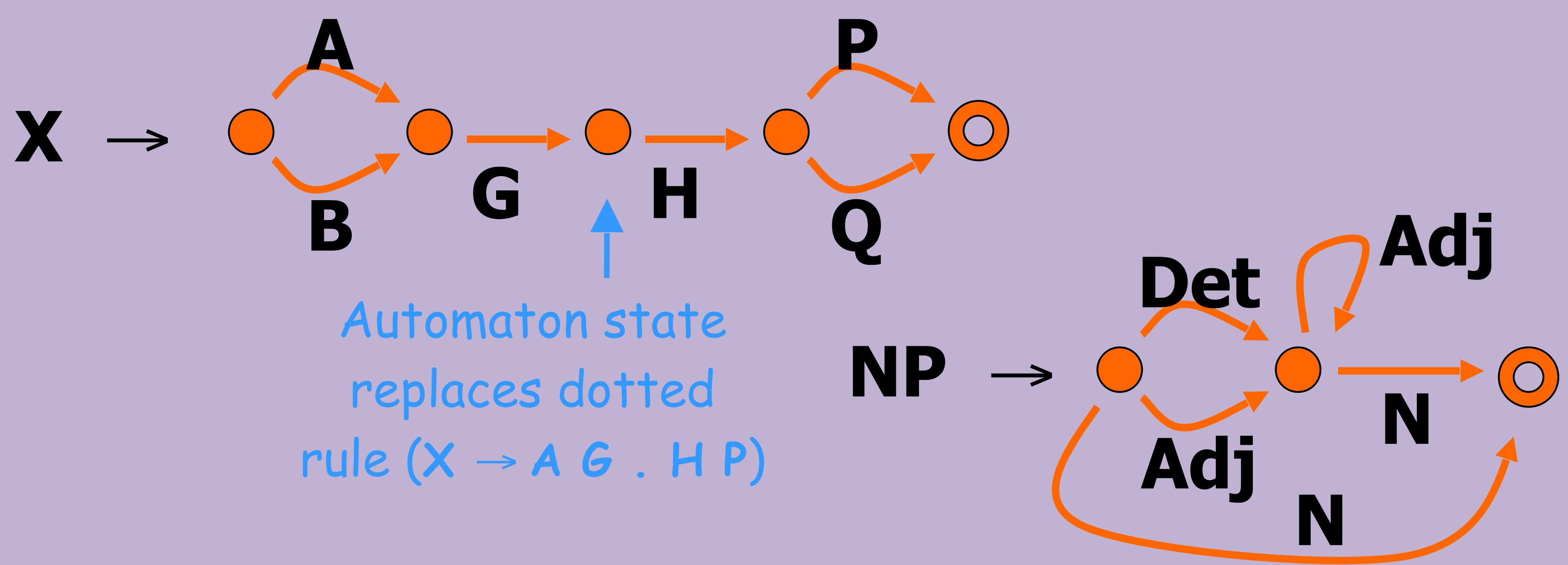
How powerful are regexps / FSAs?

- But less powerful than CFGs / pushdown automata
- More important: Less explanatory than CFGs
  - An CFG without recursive center-embedding can be converted into an equivalent FSA – but the FSA will usually be far larger
  - Because FSAs can't reuse the same phrase type in different places



We've already used FSAs this way …

- CFG with regular expression on the right-hand side:
  X → (A | B) G H (P | Q)
  NP → (Det | ε) Adj* N
- So each nonterminal has a finite-state automaton, giving a "recursive transition network (RTN)"



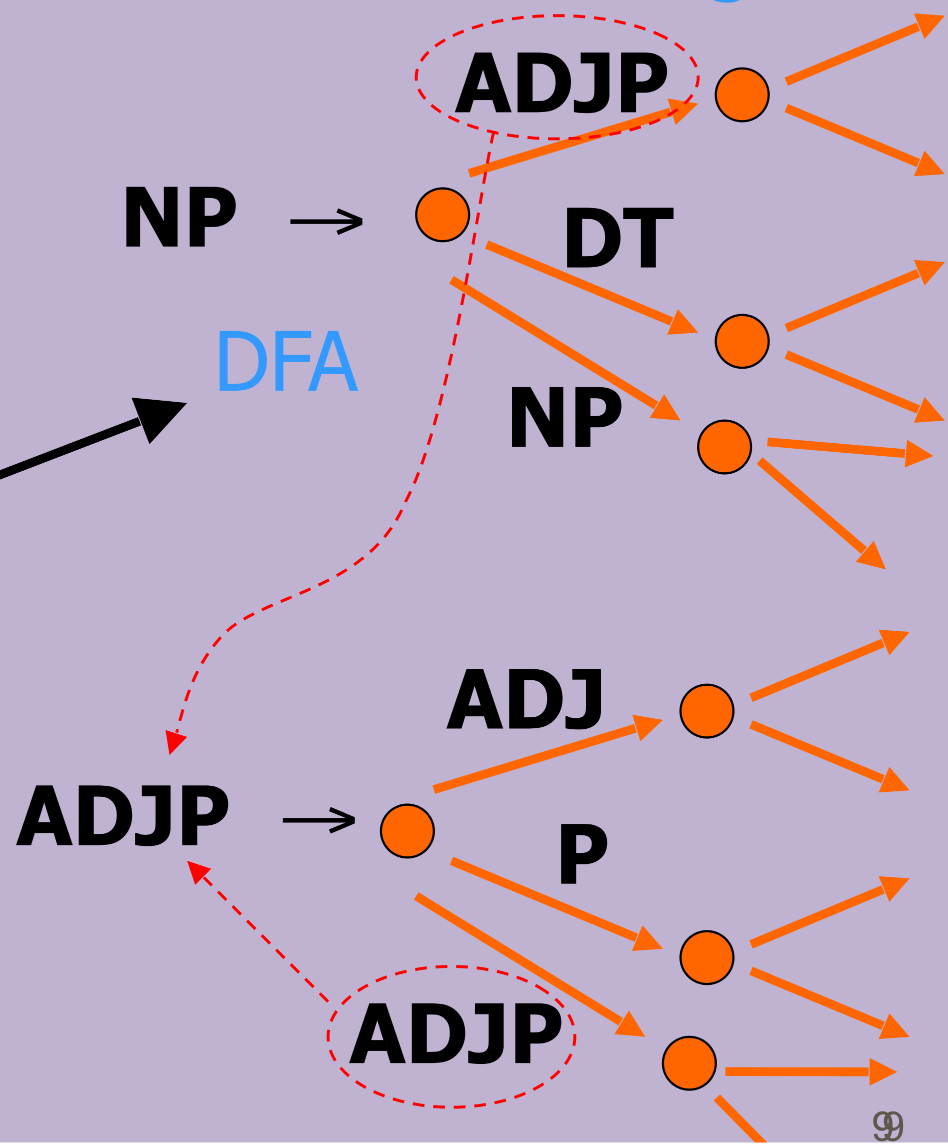
We've already used FSAs once ..

NP → rules from the WSJ grammar become a single DFA

NP → ADJP ADJP JJ JJ NN NNS
| ADJP DT NN
| ADJP JJ NN
| ADJP JJ NN NNS
| ADJP JJ NNS
| ADJP NN
| ADJP NN NN
| ADJP NN NNS
| ADJP NNS
| ADJP NPR
| ADJP NPRS
| DT
| DT ADJP
| DT ADJP , JJ NN
| DT ADJP ADJP NN
| DT ADJP JJ JJ NN
| DT ADJP JJ NN
| DT ADJP JJ NN NN

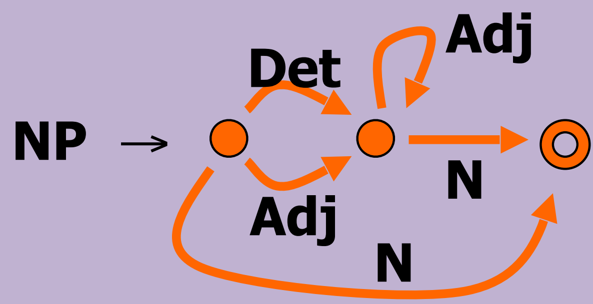
etc.

regular expression → DFA

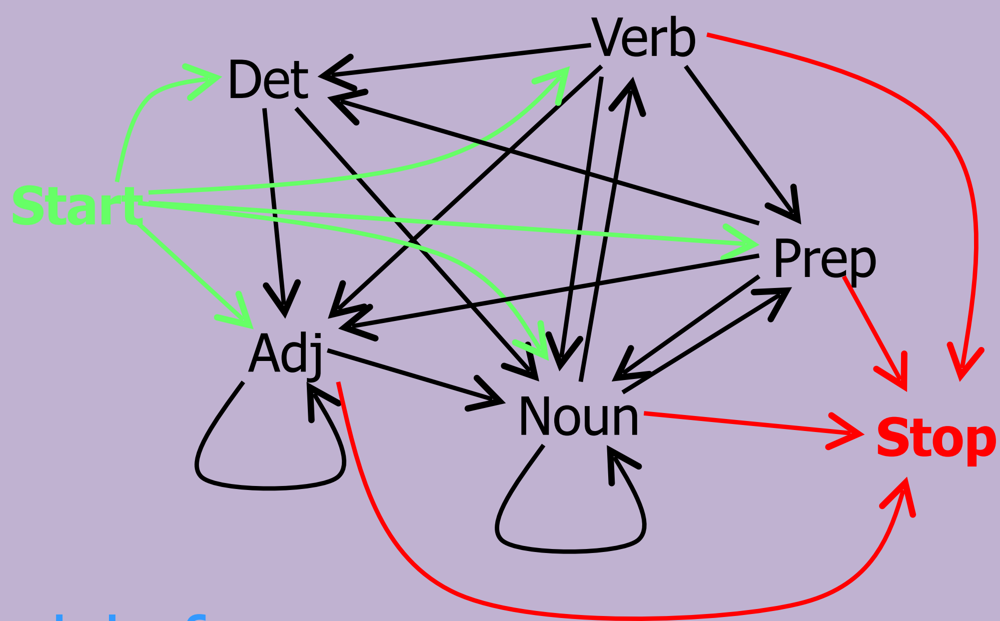


But where can we put our weights?

- CFG / RTN



- bigram model of words or tags (first-order Markov Model)



- Hidden Markov Model of words and tags together??

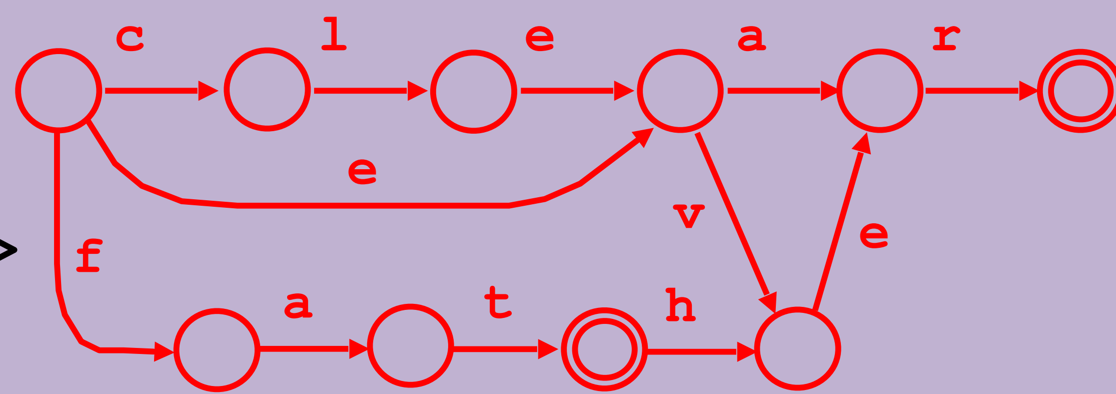
Another useful FSA ...

Wordlist

```
clear
clever
ear
ever
fat
father
```

compile

Network



`/usr/dict/words`

25K words
206K chars

0.6 sec

FSM

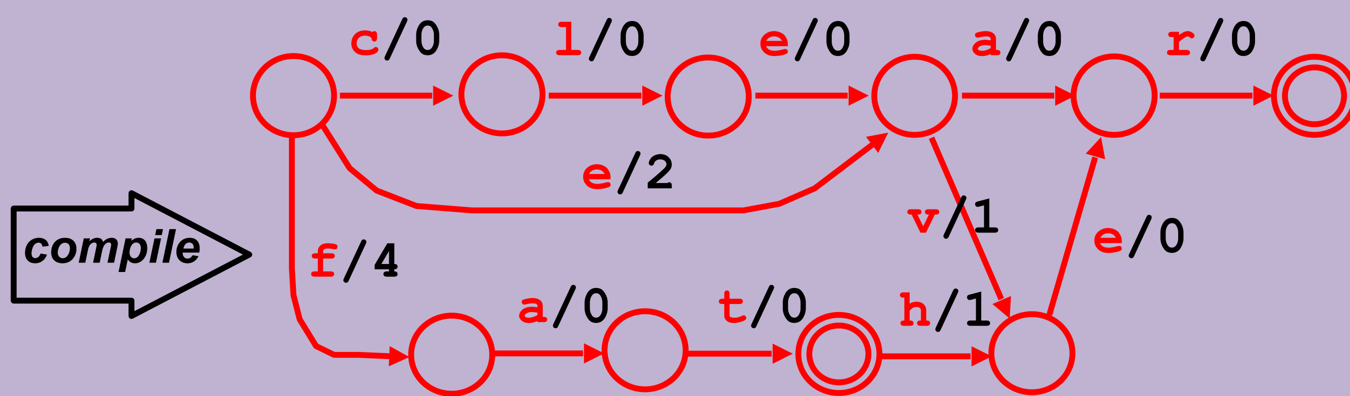
17728 states,
37100 arcs

Weights are useful here too!

Wordlist

clear	0
clever	1
ear	2
ever	3
fat	4
father	5

Network



Computes a perfect hash!
Sum the weights along a word's accepting path.

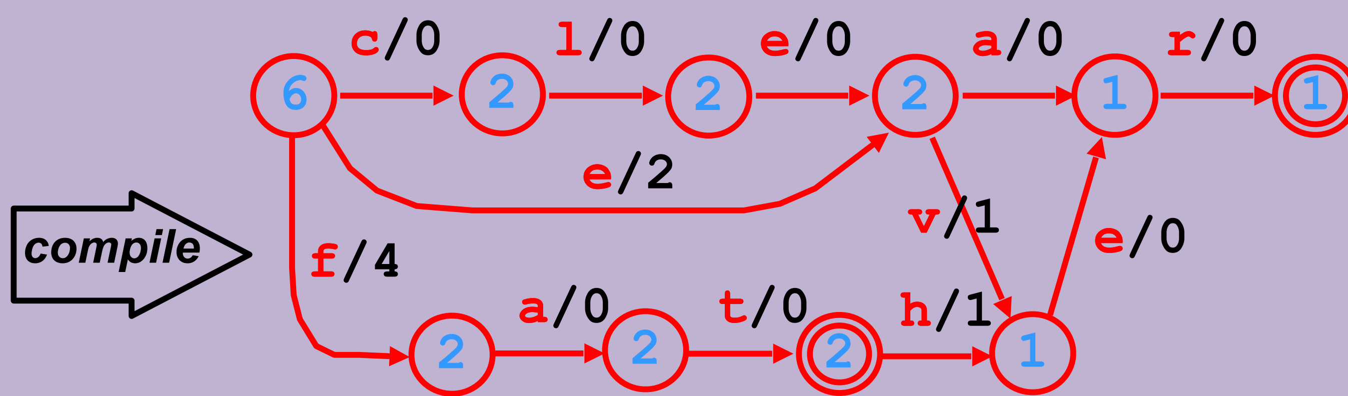
slide courtesy of L. Karttunen (modified)

Example: Weighted acceptor

Wordlist

clear	0
clever	1
ear	2
ever	3
fat	4
father	5

Network



- Compute number of paths from each state (**Q**: how?)
  A: recursively, like DFS
- Successor states partition the path set
- Use offsets of successor states as arc weights
- **Q**: Would this work for an arbitrary numbering of the words?

Example: Unweighted transducer

VP [head=vouloir,...]

V[head=vouloir,
tense=Present,
num=SG, person=P3]

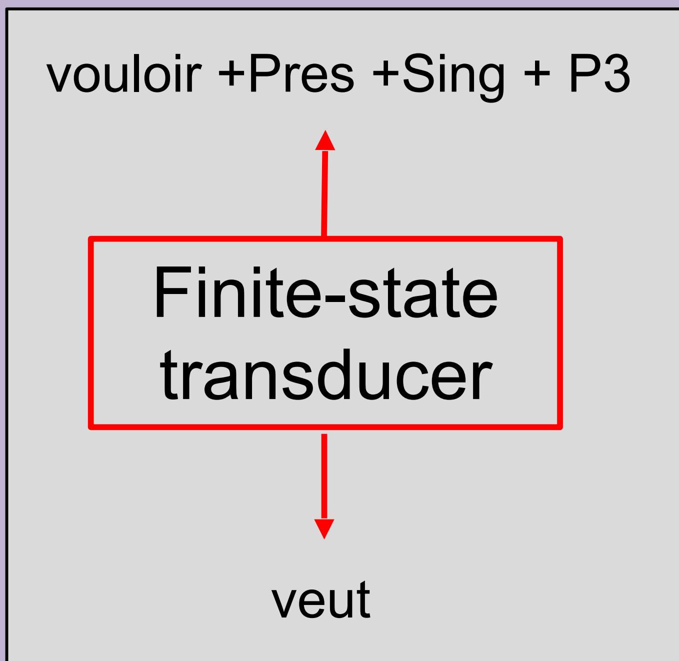
...

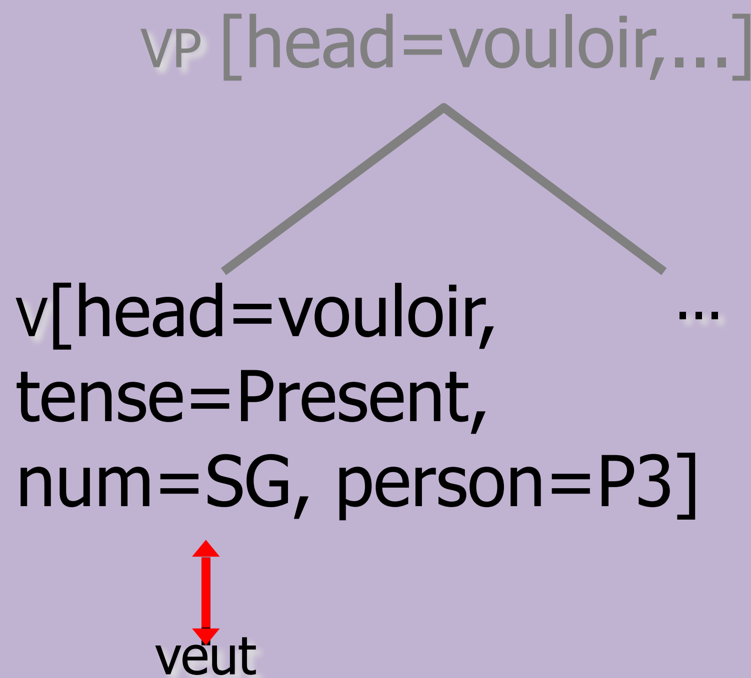
veut

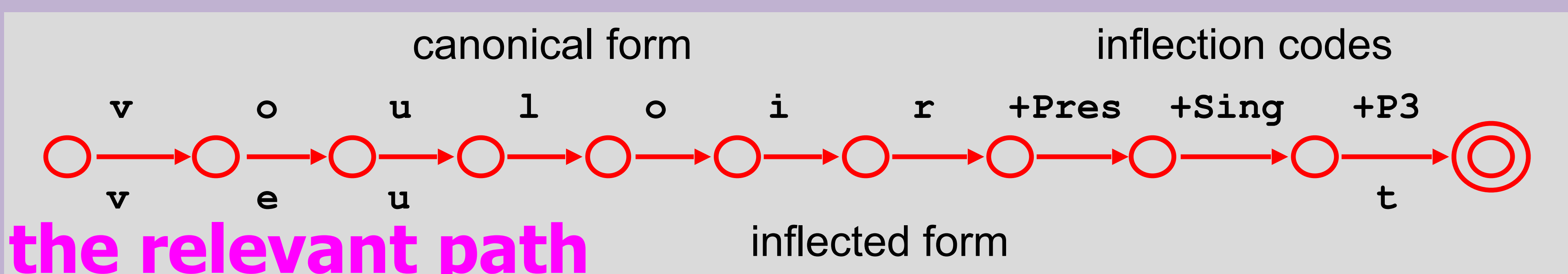
the problem
of **morphology**
("word shape") -
an area of linguistics

slide courtesy of L. Karttunen (modified)

Example: Unweighted transducer







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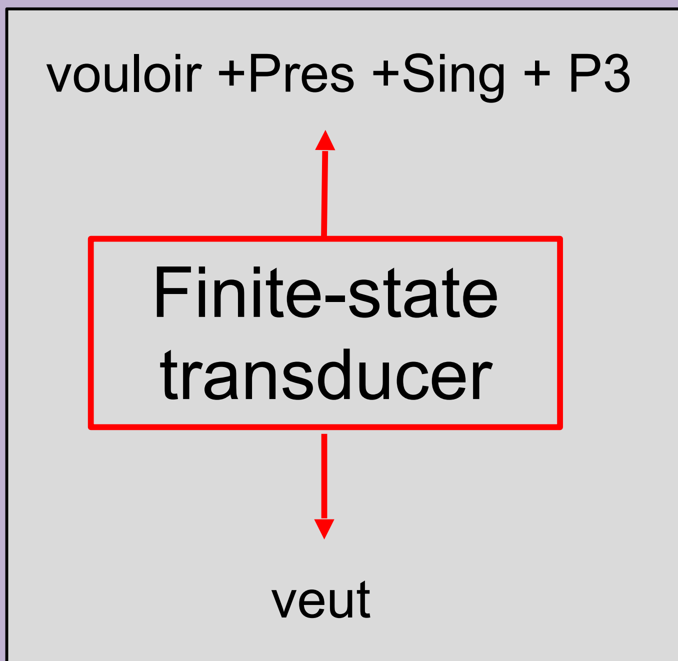
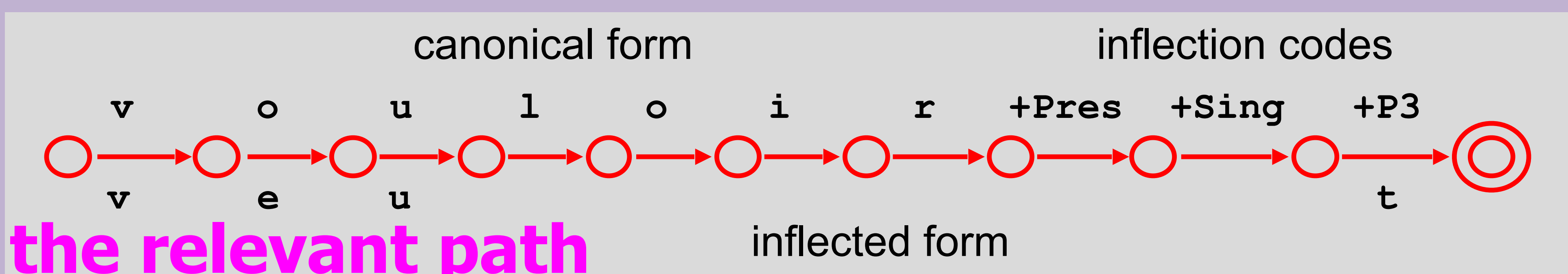
Example: Unweighted transducer



- Bidirectional: generation or analysis
- Compact and fast
- Xerox sells for about 20 languages including English, German, Dutch, French, Italian, Spanish, Portuguese, Finnish, Russian, Turkish, Japanese, ...
- Research systems for many other languages, including Arabic, Malay



What is a function?

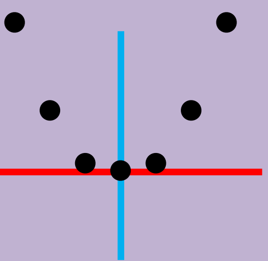
- Formally, a set of <input, output> pairs where each input $\in$ domain, output $\in$ co-domain.
- Moreover, $\forall$x $\in$ domain, $\exists$ <u>exactly one</u> y such that <x,y> $\in$ the function.

domain co-domain

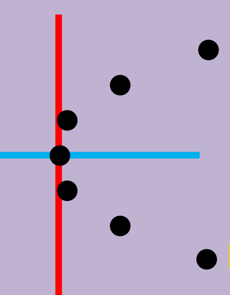
- **square: int** $\rightarrow$ **int**

  = { <0,0>, <1,1>, <-1,1>, <2,4>, <-2,4>, <3,9>, <-3,9>, ... }
- domain(**square**) = {0, 1, -1, 2, -2, 3, -3, ...}
- range(**square**) = {0, 1, 4, 9, ...}

What is a relation?

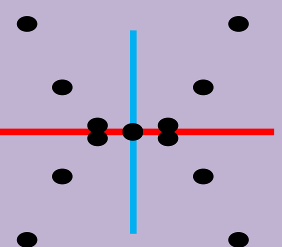


- **square: int** → **int**
  = { <0,0>, <1,1>, <-1,1>, <2,4>, <-2,4>, <3,9>, <-3,9>, ... }



- inverse(**square**): **int** → **int**
  = { <0,0>, <1,1>, <1,-1>, <4,2>, <4,-2>, <9,3>, <9,-3>, ... }

- Is inverse(**square**) a function?
  - 0 ↦ {0} 9 ↦ {3,-3} 7 ↦ {} -1 ↦ {}
- No - we need a more general notion!
  - A **relation** is <u>any</u> set of <input, output> pairs

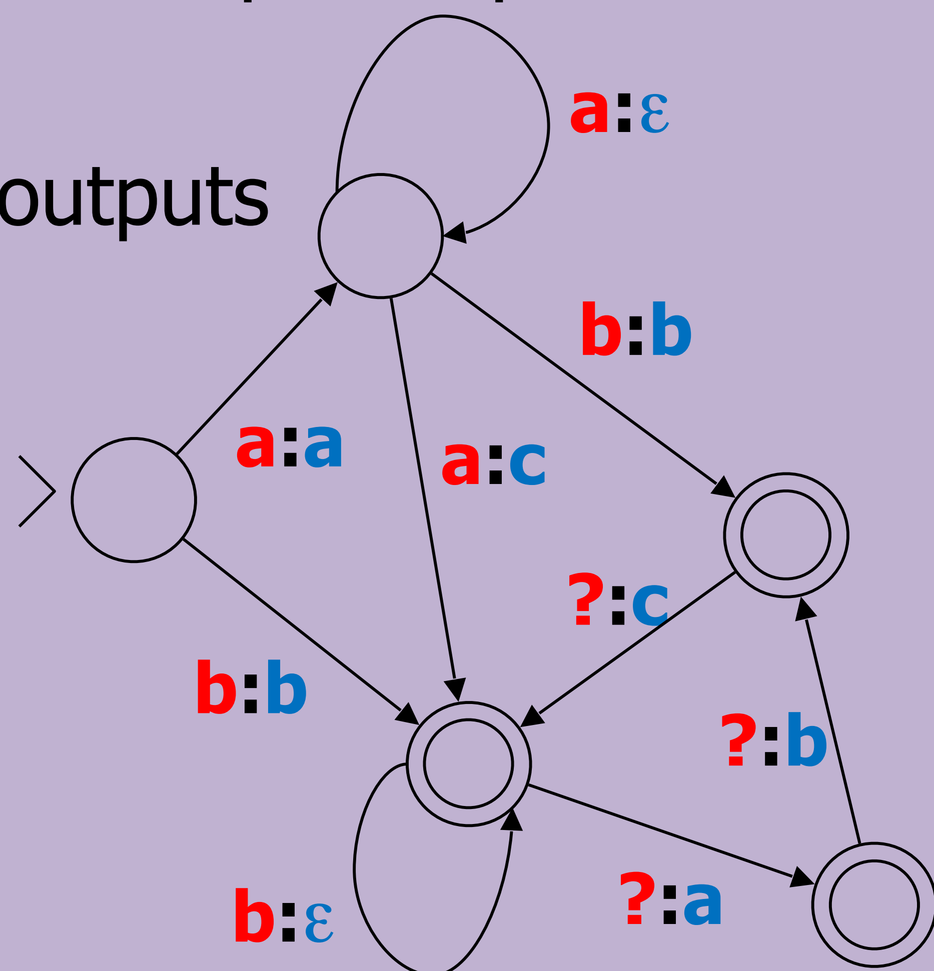


Regular Relation (of strings)

- Relation: like a function, but multiple outputs ok
- Regular: finite-state
- Transducer: automaton w/ outputs

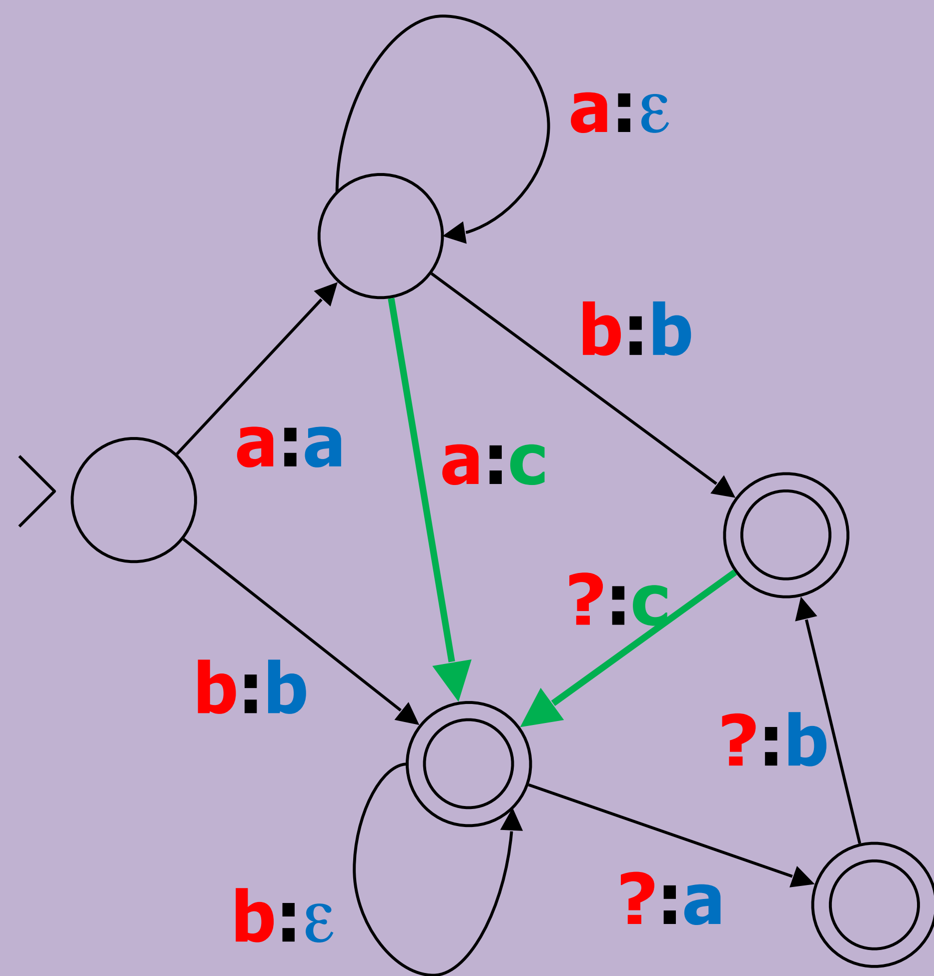
- b → {b} a → {}
- aaaaa → {ac, aca, acab, acabc}

- Invertible?
- Closed under composition?



Regular Relation (of strings)

- Can weight the arcs: → vs. →
- b → {b}   a → {}
- aaaaa → {ac, aca, acab,
  acabc}



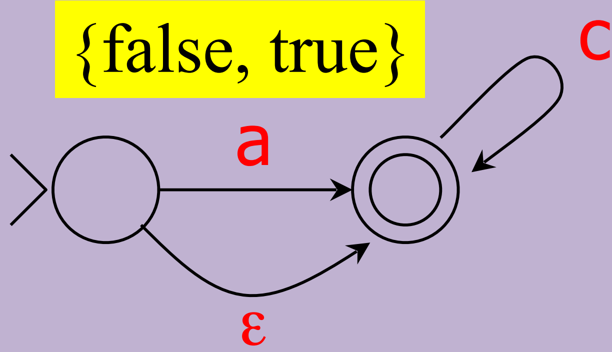
- How to find best outputs?
  - For aaaaa?
  - For all inputs at once?

Function from strings to ...

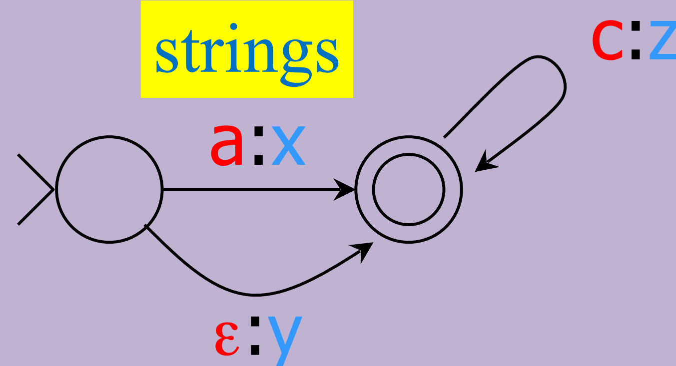
Acceptors (FSAs) | **Transducers (FSTs)**

Unweighted

{false, true}

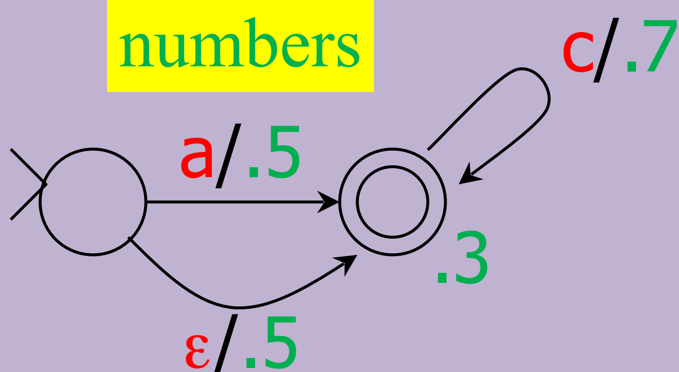


strings

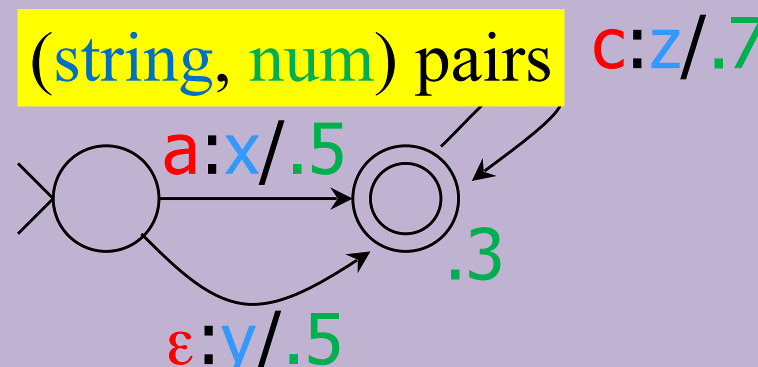


Weighted

numbers



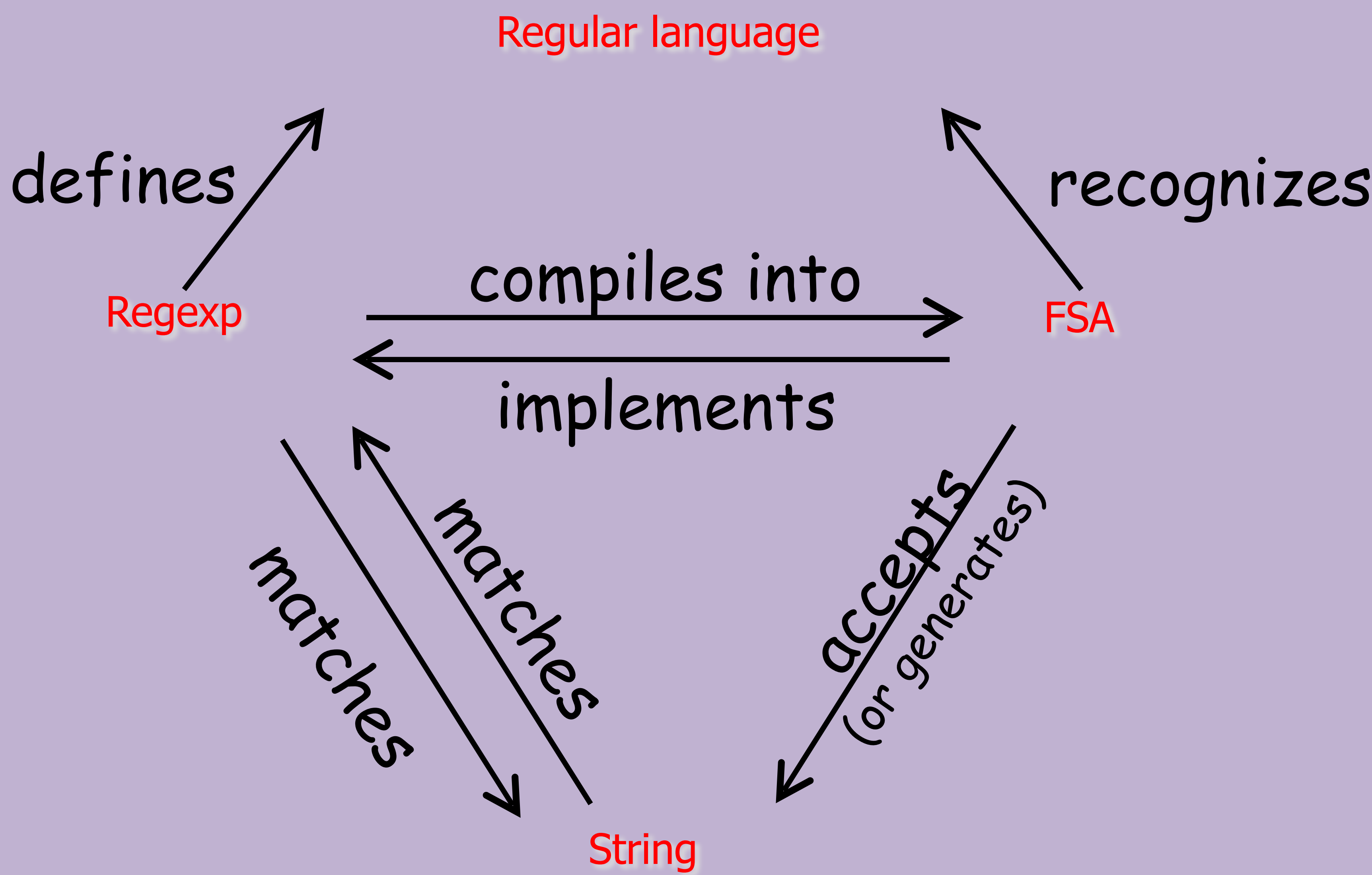
(string, num) pairs



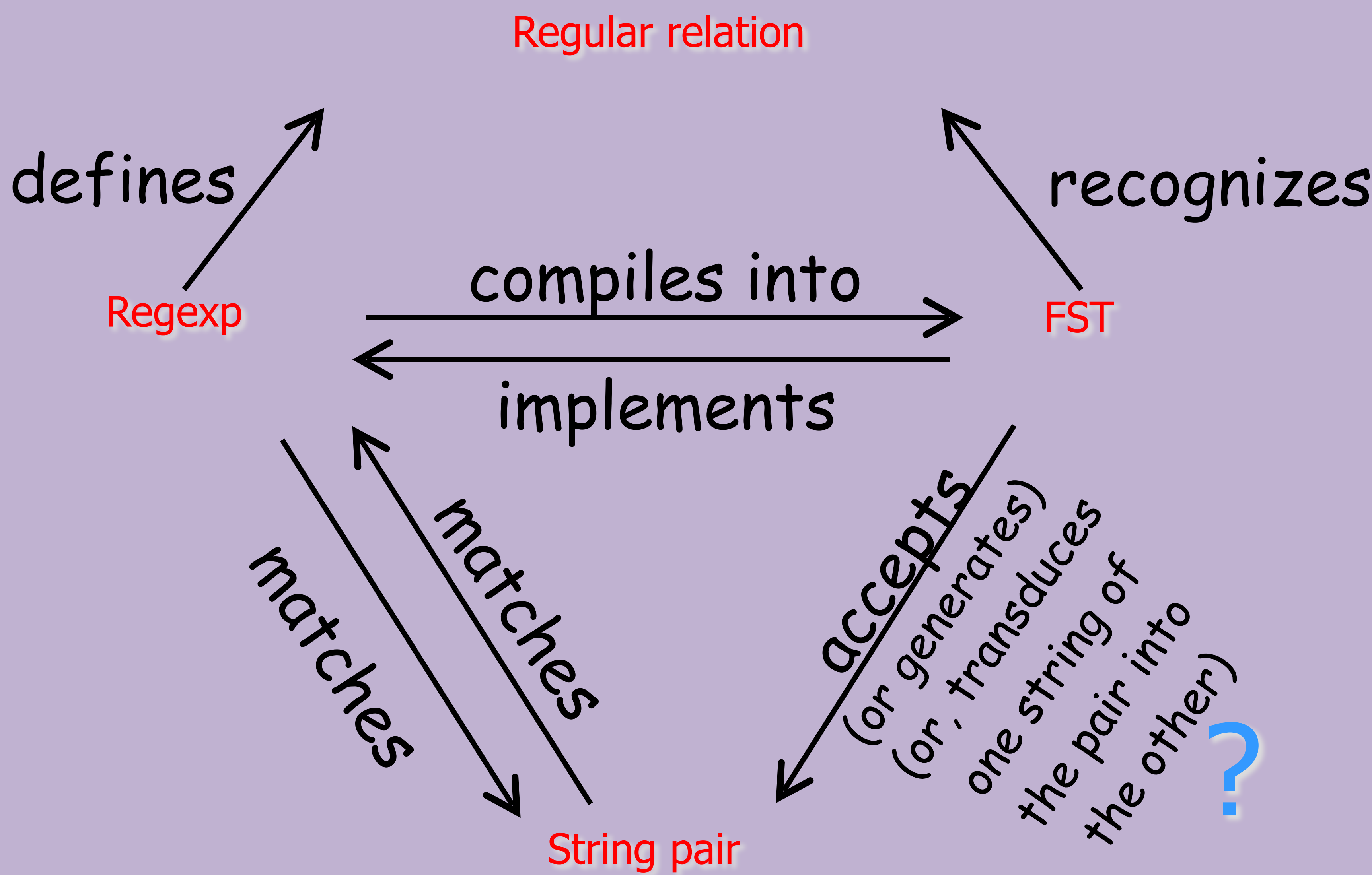
Sample functions

	Acceptors (FSAs)	Transducers (FSTs)
Unweighted	{false, true} Grammatical?	strings Markup Correction Translation
Weighted	numbers How grammatical? Better, how probable?	(string, num) pairs Good markups Good corrections Good translations

Terminology (acceptors)



Terminology (transducers)



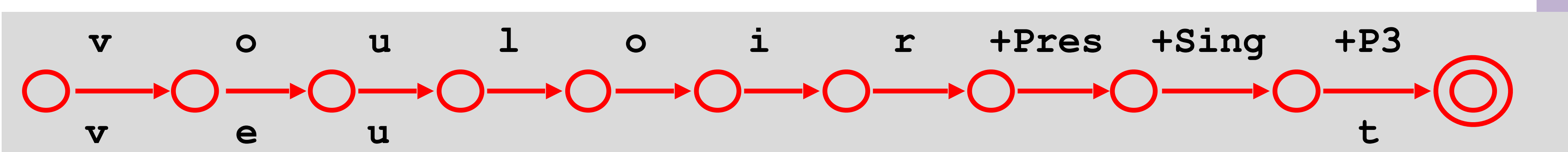
Perspectives on a Transducer

- Remember these CFG perspectives:

3 views of a context-free rule

- generation (production): $S \rightarrow NP\ VP$ (randsent)
- parsing (comprehension): $S \leftarrow NP\ VP$ (parse)
- verification (checking): $S = NP\ VP$

- Similarly, 3 views of a transducer:
  - Given 0 strings, generate a new string pair (by picking a path)
  - Given one string (upper or lower), transduce it to the other kind
  - Given two strings (upper & lower), decide whether to accept the pair



FST just defines the regular relation (mathematical object: set of pairs).
What's "input" and "output" depends on what one <u>asks</u> about the relation.
The 0, 1, or 2 given string(s) constrain which paths you can use.

Functions

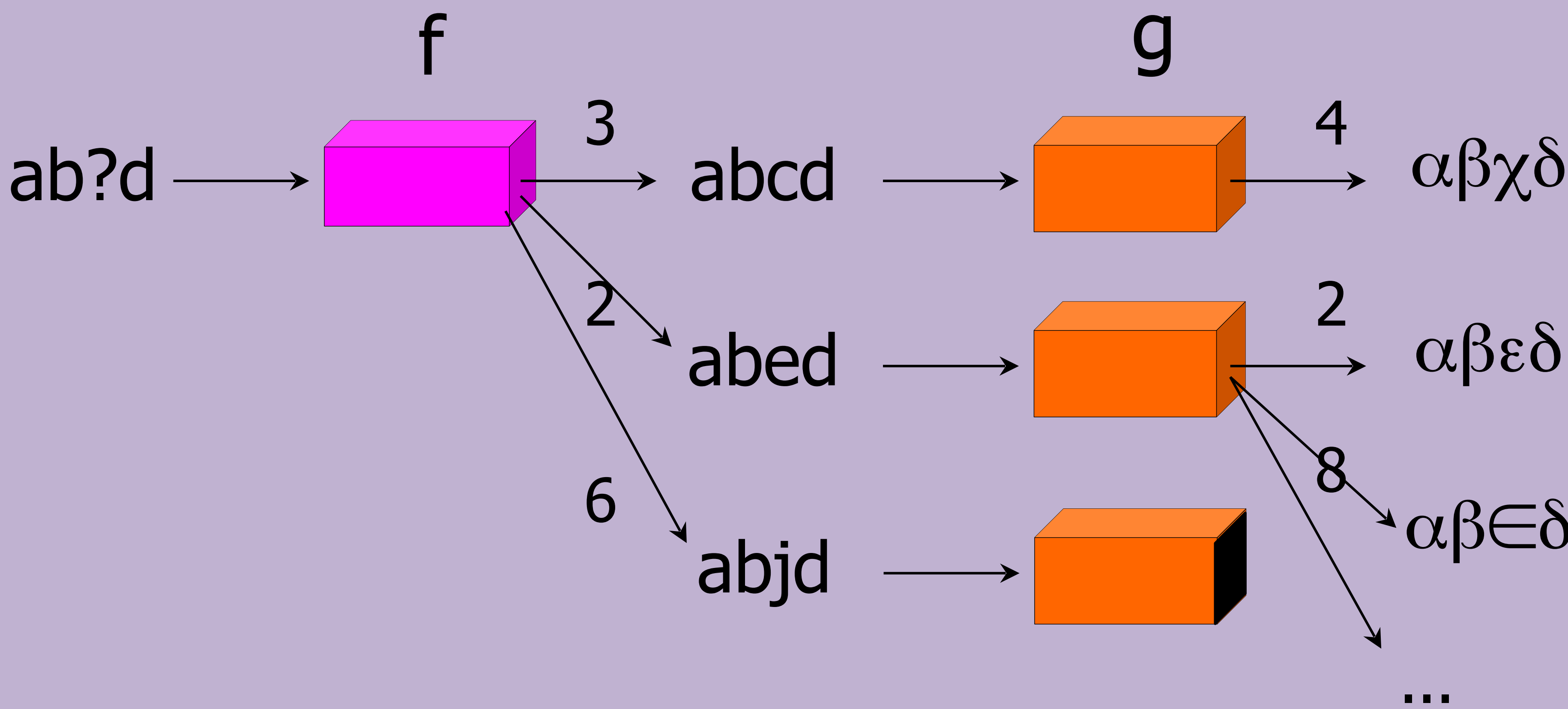
Functions

Function composition: $f \circ g$

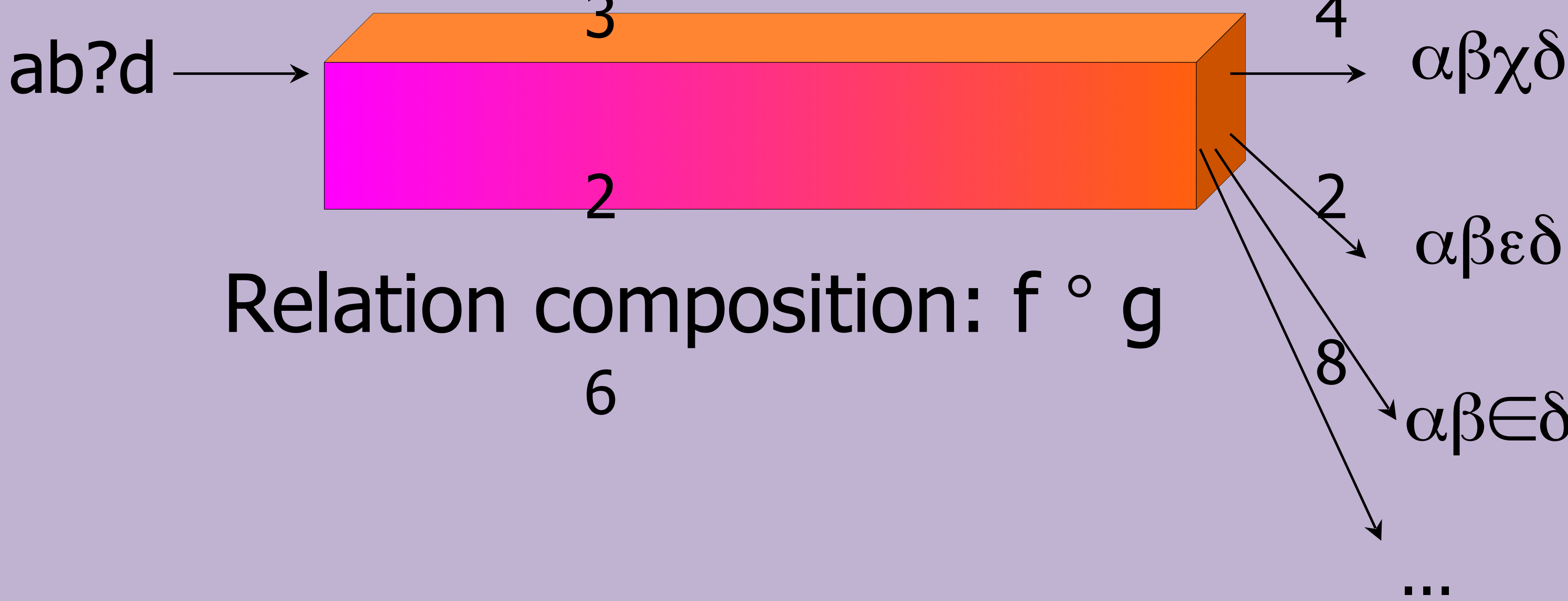
[first f, then g – intuitive notation, but opposite of the traditional math notation]

Like the Unix pipe: `cat x | f | g > y`
Example: Pass the input through a sequence of ciphers

From Functions to Relations

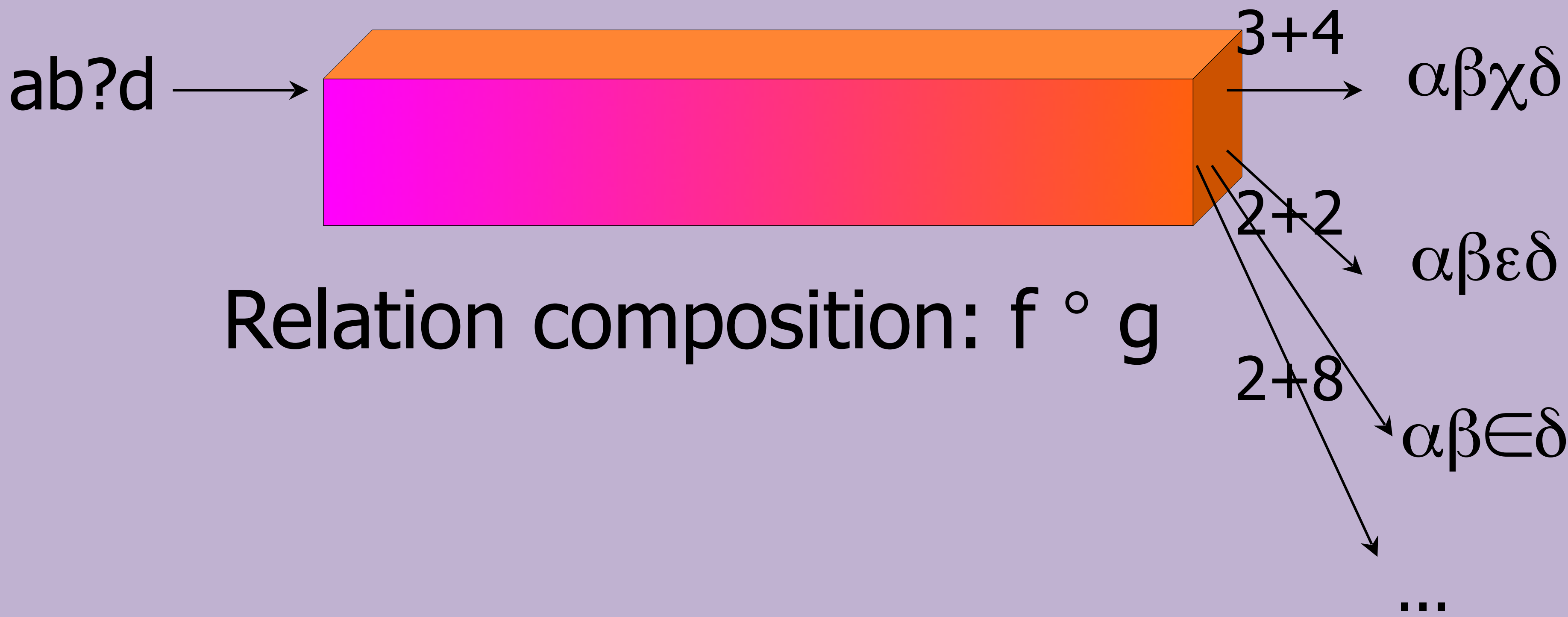


From Functions to Relations



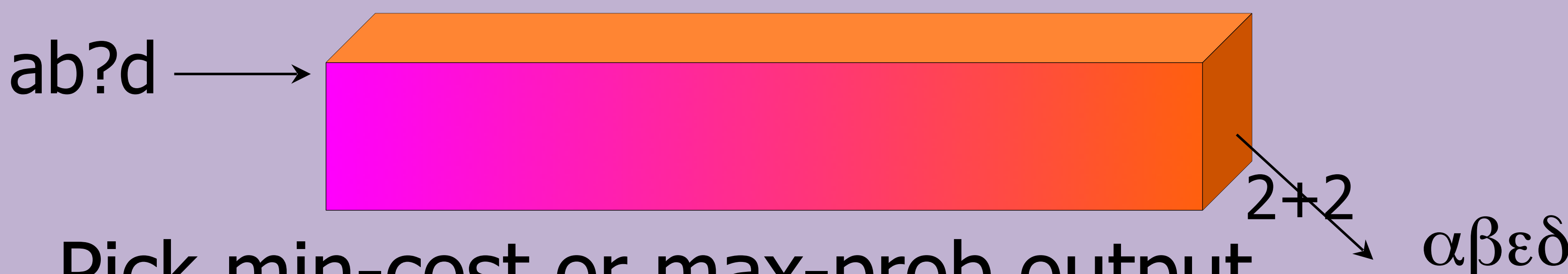
Relation composition: f ° g

From Functions to Relations



Relation composition: f ° g

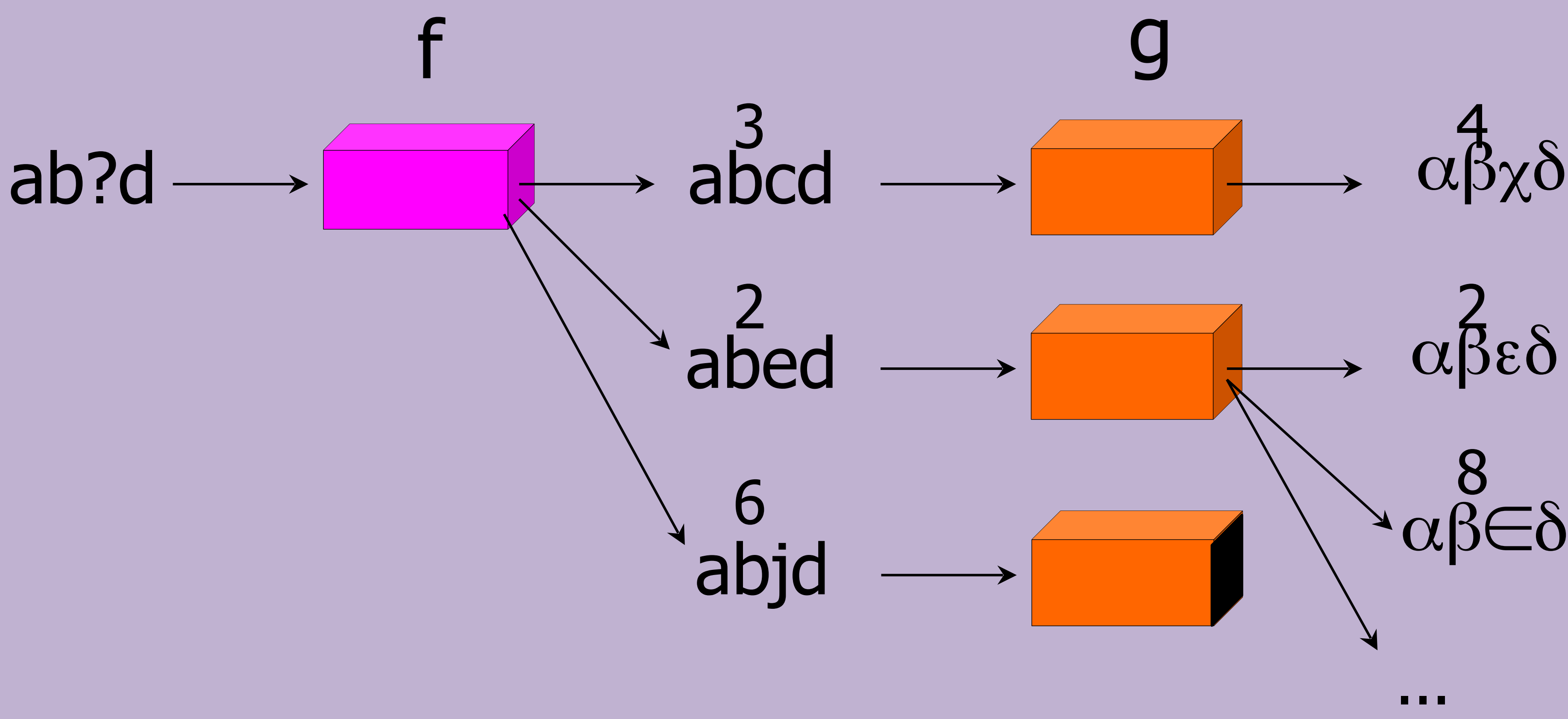
From Functions to Relations



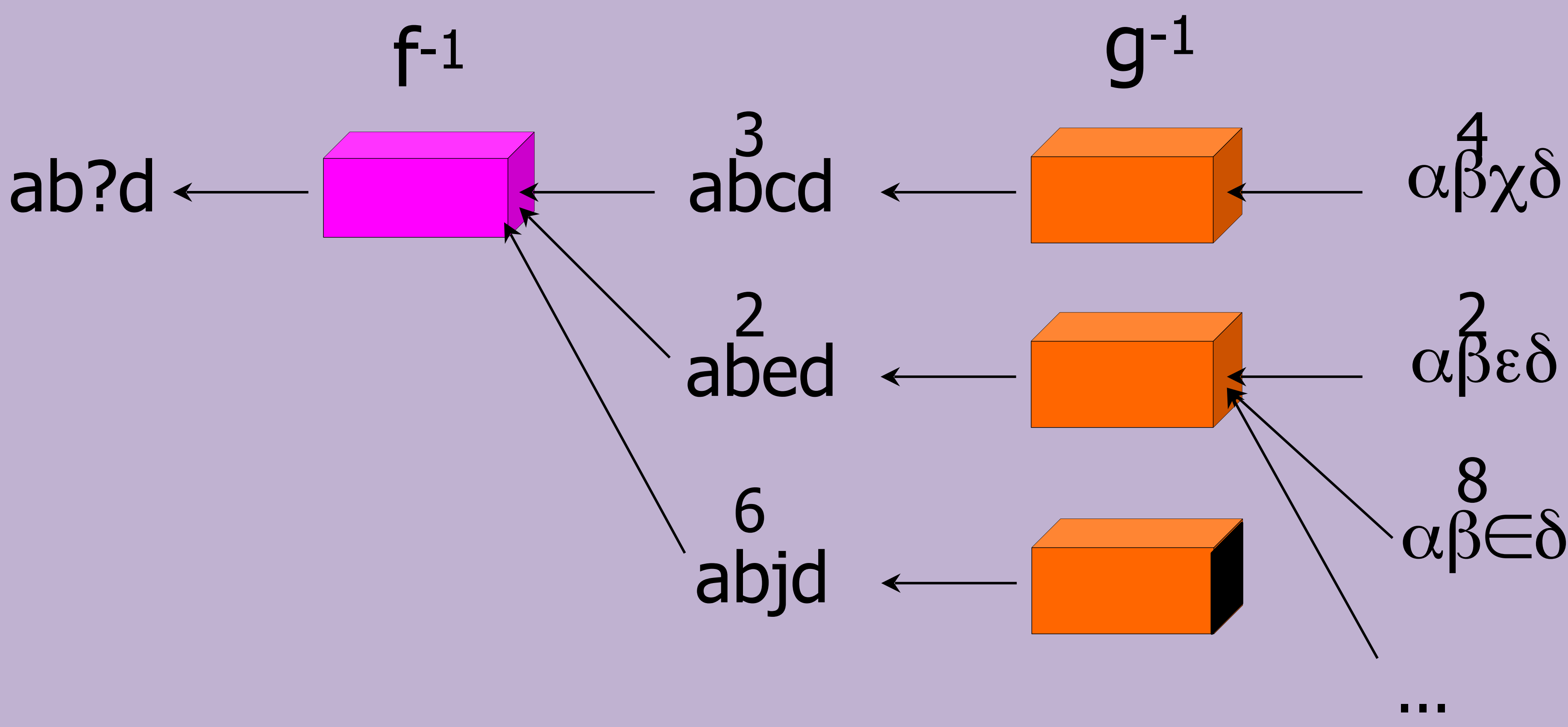
Pick min-cost or max-prob output

Often in NLP, all of the functions or relations involved can be described as finite-state machines, and manipulated using standard algorithms.

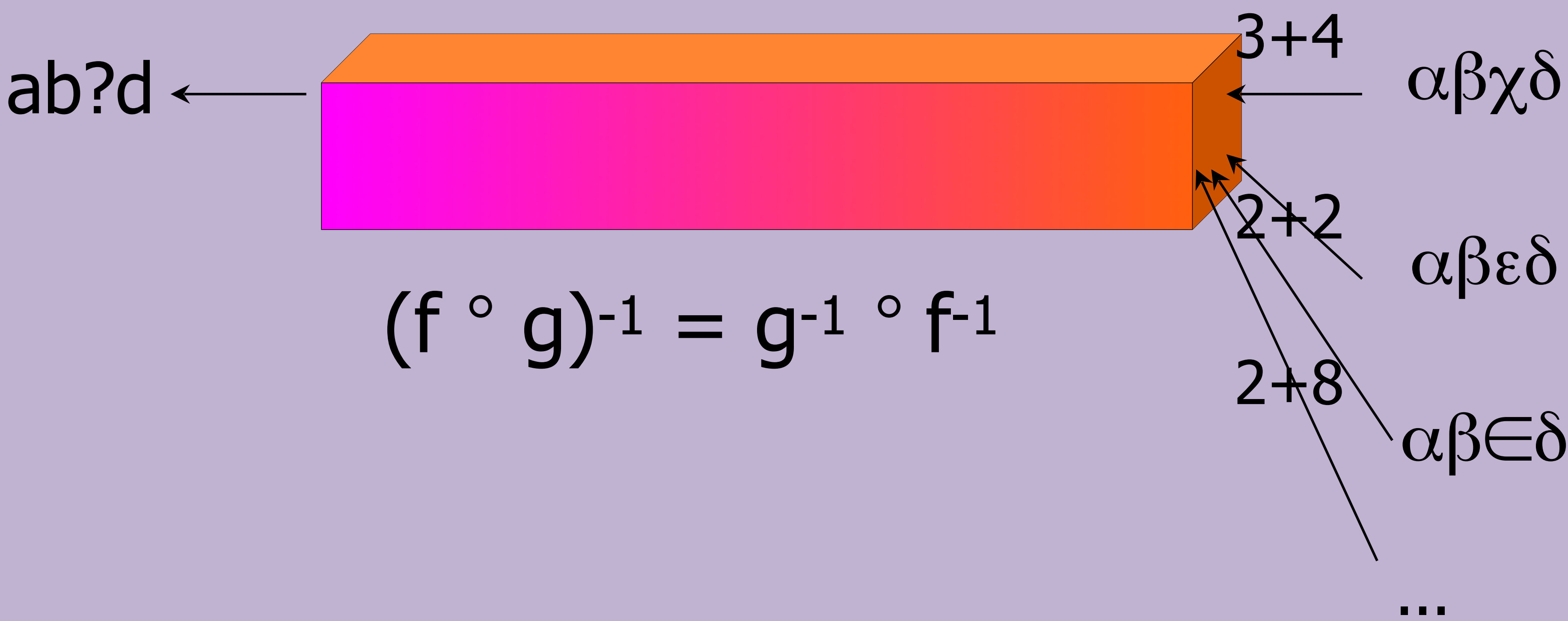
Inverting Relations



Inverting Relations

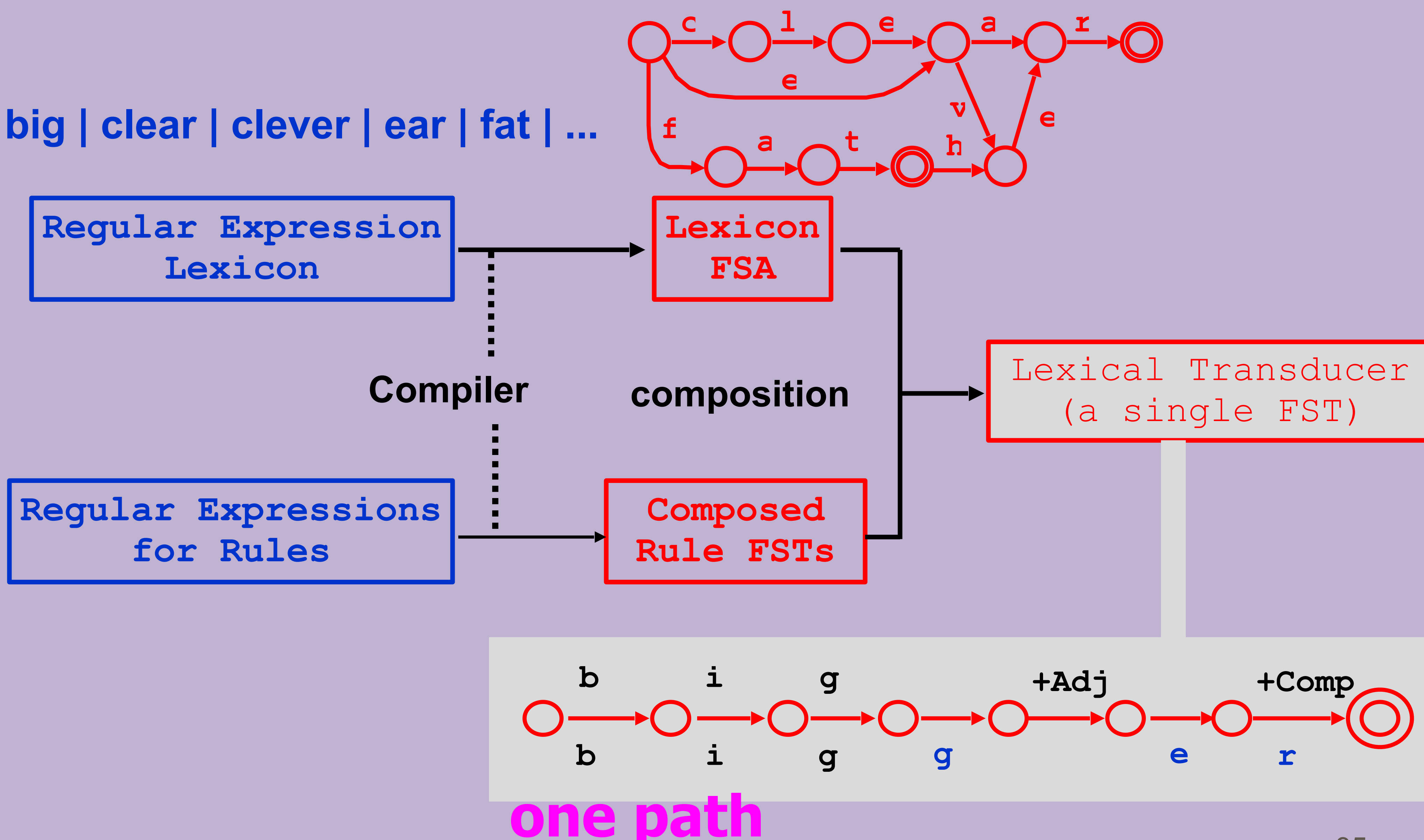


Inverting Relations

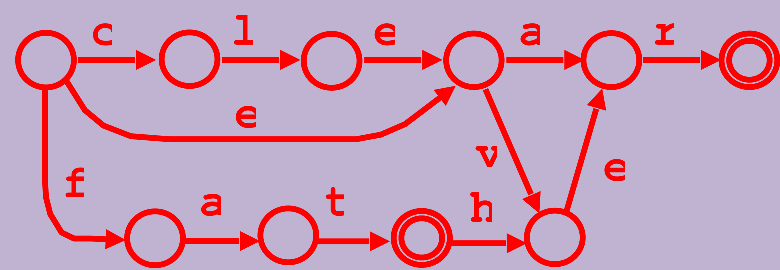


$$(f \circ g)^{-1} = g^{-1} \circ f^{-1}$$

Building a lexical transducer



Building a lexical transducer



big | clear | clever | ear | fat | ...

Regular Expression Lexicon → **Lexicon FSA**

- Actually, the lexicon must contain elements like
  big +Adj +Comp
- So write it as a more complicated expression:
  (big | clear | clever | fat | ...) +Adj (ε | +Comp | +Sup) ← ***adjectives***
  | (ear | father | ...) +Noun (+Sing | +Pl) ← ***nouns***
  | ... ← ***...***
- Q: Why do we need a lexicon at all?

Weighted version of transducer: Assigns a weight to each string pair