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CS 613 - Assignment 1 report

PART-1

Answer-1

We like to use quadratic error functions since they allow us to give satisfactory results when it is desired to reach a single minima/maxima in expectation minimization/maximization problems. In a highly dimensional space, for instance if use a 4th degree polynomial function it would provide us with 2 minima/maxima depending on the nature of the curve. But if our goal is to just reach a single minima/maxima instead of a globally optimal one then we would want use a quadratic error function instead of a higher degree polynomial function which would allow us to save computational effort and time and be a good fit to the model of data.

Answer-2 A

Data, X =

$$\begin{bmatrix} -2 & 1 \\ -5 & -4 \\ -3 & 1 \\ 0 & 3 \\ -8 & 11 \\ -2 & 5 \\ 1 & 0 \\ 5 & -1 \\ -1 & 3 \\ 6 & 1 \end{bmatrix}$$

$$\mu_1 = (-2-5-3+0-8-2+1+5-1+6)/10 = -0.9 \tag{1}$$

$$\mu_2 = (1-4+1+3+11+5+0-1-3+1)/10 = 1.4 \tag{2}$$

$$\sigma_1 = \sqrt{\Sigma(x_{i1}-\mu_1)} = 4.011 \tag{3}$$

$$\sigma_2 = \sqrt{\Sigma(x_{i2}-\mu_2)} = 4.0456 \tag{4}$$

$$X_{standardized} = \begin{bmatrix} -0.2601 & -0.0935 \\ -0.9696 & -1.2634 \\ -0.4966 & -0.0935 \\ 0.2128 & 0.3743 \\ -1.6791 & 2.2461 \\ -0.26015 & 0.8423 \\ 0.4493 & -0.3275 \\ 1.3953 & -0.5615 \\ -0.0236 & -1.0294 \\ 1.6318 & -0.0935 \end{bmatrix} \tag{5}$$

$$\Sigma(X) = \frac{X^T X}{N-1} \tag{6}$$

$$= \begin{bmatrix} 1 & -0.4083 \\ -0.4083 & 1 \end{bmatrix}$$

$$Now, \lambda^2 - \lambda(1+1) + (1 - 0.167) = \lambda^2 - 2\lambda + 0.833 = 0 \tag{7}$$

$$Solving for \lambda, \lambda = 1.4083, 0.5913$$

$$(\Sigma(X) - \lambda I)x = 0 \tag{8}$$

$$(\begin{bmatrix} 1 & -0.4083 \\ -0.4083 & 1 \end{bmatrix} - \begin{bmatrix} 1.4087 & 0 \\ 0 & 1.4087 \end{bmatrix}) \begin{bmatrix} x \\ y \end{bmatrix} = 0, for \lambda = 1.4083 \tag{9}$$

when x =1, y =-1

$$e1 = \begin{bmatrix} 0.7071 & -0.7071 \end{bmatrix}^T, normalized$$

$$(\begin{bmatrix} 1 & -0.4083 \\ -0.4083 & 1 \end{bmatrix} - \begin{bmatrix} 0.5913 & 0 \\ 0 & 0.5913 \end{bmatrix}) \begin{bmatrix} x \\ y \end{bmatrix} = 0, for \lambda = 0.5913 \tag{10}$$

when x = 1, y = 1

$$e2 = \begin{bmatrix} 0.7071 & 0.7071 \end{bmatrix}^T, normalized$$

<u>Answer-2 B</u>

Projecting Data Points onto e1, eigen vector with largest magnitude:
Z = XW

Therefore,

$$\begin{bmatrix} -0.2601 & -0.0935 \\ -0.9696 & -1.2634 \\ -0.4966 & -0.0935 \\ 0.2128 & 0.3743 \\ -1.6791 & 2.2461 \\ -0.26015 & 0.8423 \\ 0.4493 & -0.3275 \\ 1.3953 & -0.5615 \\ -0.0236 & -1.0294 \\ 1.6318 & -0.0935 \end{bmatrix} \times \begin{bmatrix} 0.7071 \\ -0.7071 \end{bmatrix} = \begin{bmatrix} 2.1213 \\ 0.7071 \\ 2.8284 \\ 2.1213 \\ 13.4350 \\ 4.9497 \\ -0.7071 \\ -4.2426 \\ -1.4142 \\ -3.5355 \end{bmatrix}$$

Answer-3 A

$$Class1 = \begin{bmatrix} -2 & 1 \\ -5 & -4 \\ -3 & 1 \\ 0 & 3 \\ -8 & 11 \end{bmatrix} \tag{11}$$

$$Class2 = \begin{bmatrix} -2 & 5 \\ 1 & 0 \\ 5 & -1 \\ -1 & -3 \\ 6 & 1 \end{bmatrix} \tag{12}$$

feature1

$$p_0 = 1, p_{-2} = 1, p_{-3} = 1, p_{-5} = 1, p_{-8} = 1,$$

$$n_{-2} = 1, n_{-1} = 1, n_1 = 1, n_5 = 1, n_6 = 1,$$

Accounting for -2, rest 0

$$Entropy = \frac{2}{10} * (-\frac{1}{2} * \log_2(\frac{1}{2}) - \frac{1}{2}\log_2(\frac{1}{2}))$$

Entropy = 0.2

$$IG(1) = -\frac{5}{10} * \log_2(\frac{5}{10}) - \frac{5}{10} * \log_2(\frac{5}{10}) - 0.2 = 1 - 0.2 = 0.8 \tag{13}$$

feature2

$$p_1 = 2, p_{-4} = 1, p_3 = 1, p_{11} = 1,$$

$$n_1 = 1, n_5 = 1, n_0 = 1, n_{-1} = 1, n_{-3} = 1,$$

Accounting for 1, rest 0

$$Entropy = \frac{3}{10} * (-\frac{2}{3} * \log_2(\frac{2}{3}) - \frac{1}{3}\log_2(\frac{1}{3}))$$

Entropy = 0.3 * (0.924) = 0.2772

$$IG(1) = -\frac{5}{10} * \log_2(\frac{5}{10}) - \frac{5}{10} * \log_2(\frac{5}{10}) = 1 - 0.2772 = 0.7228 \tag{14}$$

Answer-3 B

Since Information Gain for Feature 1 is more than Information Gain for feature 2, therefore feature 1 seems to be more discriminating.

Answer-3 C

$$Class1 = \begin{bmatrix} -2 & 1 \\ -5 & -4 \\ -3 & 1 \\ 0 & 3 \\ -8 & 11 \end{bmatrix} \tag{15}$$

$$Class2 = \begin{bmatrix} -2 & 5 \\ 1 & 0 \\ 5 & -1 \\ -1 & -3 \\ 6 & 1 \end{bmatrix} \tag{16}$$

$$\mu_1 = \begin{bmatrix} (-2-5-3+0-8)/5 & (1-4+1+3+11)/5 \end{bmatrix} = \begin{bmatrix} -3.6 & 2.4 \end{bmatrix} \tag{17}$$

$$\mu_1 = \begin{bmatrix} (-2+1+5-1+6)/5 & (5+0-1-3+1)/5 \end{bmatrix} = \begin{bmatrix} 1.8 & 0.4 \end{bmatrix} \tag{18}$$

$$\Sigma(C1) = \frac{C1^T C1}{N-1} \tag{19}$$

$$= \begin{bmatrix} 9.30 & -7.45 \\ -7.45 & 29.80 \end{bmatrix}$$

$$\Sigma(C2) = \frac{C2^T C2}{N-1} \tag{20}$$

$$= \begin{bmatrix} 12.70 & -2.40 \\ -2.40 & 8.80 \end{bmatrix}$$

$$\sigma_1^2 = (5-1) * \Sigma(C1) = 4 * \begin{bmatrix} 9.30 & -7.45 \\ -7.45 & 29.80 \end{bmatrix} = \begin{bmatrix} 37.20 & -29.80 \\ -29.80 & 119.20 \end{bmatrix} \tag{21}$$

$$\sigma_2^2 = (5-1) * \Sigma(C2) = 4 * \begin{bmatrix} 12.70 & -2.40 \\ -2.40 & 8.80 \end{bmatrix} = \begin{bmatrix} 50.80 & -9.60 \\ -9.60 & 35.20 \end{bmatrix} \tag{22}$$

$$S_b = (\mu_1 - \mu 2)^T * (\mu 1_- \mu_2) = (\begin{bmatrix} 3.6 & 2.4 \end{bmatrix} - \begin{bmatrix} 1.8 & 0.4 \end{bmatrix})^T \times (\begin{bmatrix} -3.6 & 2.4 \end{bmatrix} - \begin{bmatrix} 1.8 & 0.4 \end{bmatrix}) = \begin{bmatrix} -5.4 \\ 2.0 \end{bmatrix} \times \begin{bmatrix} -5.4 & 2.0 \end{bmatrix} \tag{23}$$

$$= \begin{bmatrix} 29.16 & -10.80 \\ -10.80 & 4.0 \end{bmatrix}$$

$$S_w = \Sigma^2 + \Sigma^2 = \begin{bmatrix} 37.20 & -29.80 \\ -29.80 & 119.20 \end{bmatrix} + \begin{bmatrix} 50.80 & -9.60 \\ -9.60 & 35.20 \end{bmatrix} = \begin{bmatrix} 88 & -39.40 \\ -39.40 & 154.40 \end{bmatrix} \tag{24}$$

$$S_w = \Sigma^2 + \Sigma^2 = \begin{bmatrix} 37.20 & -29.80 \\ -29.80 & 119.20 \end{bmatrix} + \begin{bmatrix} 50.80 & -9.60 \\ -9.60 & 35.20 \end{bmatrix} = \begin{bmatrix} 88 & -39.40 \\ -39.40 & 154.40 \end{bmatrix} \tag{25}$$

$$S_w^{-1} = \frac{1}{12035} * \begin{bmatrix} 154.40 & 39.40 \\ 39.40 & 88 \end{bmatrix} = \begin{bmatrix} 0.0128 & 0.0033 \\ 0.0033 & 0.0073 \end{bmatrix} \tag{26}$$

$$X = S_w^{-1} * S_b = \begin{bmatrix} 0.0128 & 0.0033 \\ 0.0033 & 0.0073 \end{bmatrix} \times \begin{bmatrix} 29.16 & -10.80 \\ -10.80 & 4.0 \end{bmatrix} = \begin{bmatrix} 0.3387 & -0.1254 \\ 0.0165 & -0.0061 \end{bmatrix} \tag{27}$$

$$Now, \lambda^2 - \lambda(0.3326) + (-0.002 - 0.002) = \lambda^2 - 0.3326\lambda = 0 \tag{28}$$

$$Solving for \lambda, \lambda = 0.3326, 0$$

$$(\Sigma(X) - \lambda I)x = 0 \tag{29}$$

$$(\begin{bmatrix} 0.3387 & -0.1254 \\ 0.0165 & -0.0061 \end{bmatrix} - \begin{bmatrix} 0.3326 & 0 \\ 0 & 0.3326 \end{bmatrix}) \begin{bmatrix} x \\ y \end{bmatrix} = 0, for \lambda = 0.3326 \tag{30}$$

when x =1, y =0.04

$$e2 = \begin{bmatrix} 0.9988 & 0.0486 \end{bmatrix}^T, normalized$$

$$(\begin{bmatrix} 0.3387 & -0.1254 \\ 0.0165 & -0.0061 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}) \begin{bmatrix} x \\ y \end{bmatrix} = 0, for \lambda = 0 \tag{31}$$

e2 not to be considered (zero magnitude)

<u>Answer-3 D</u>

Projecting Data Points onto e1, eigen vector with largest magnitude (unit length):
Z1 = C1*W

Therefore,

$$\begin{bmatrix} -2 & 1 \\ -5 & -4 \\ -3 & 1 \\ 0 & 3 \\ -8 & 11 \end{bmatrix} \times \begin{bmatrix} 0.9988 \\ 0.0486 \end{bmatrix} = \begin{bmatrix} -0.66527 \\ -1.66319 \\ -0.9979 \\ 0 \\ -2.6611 \end{bmatrix}$$

Z2 = C2*W

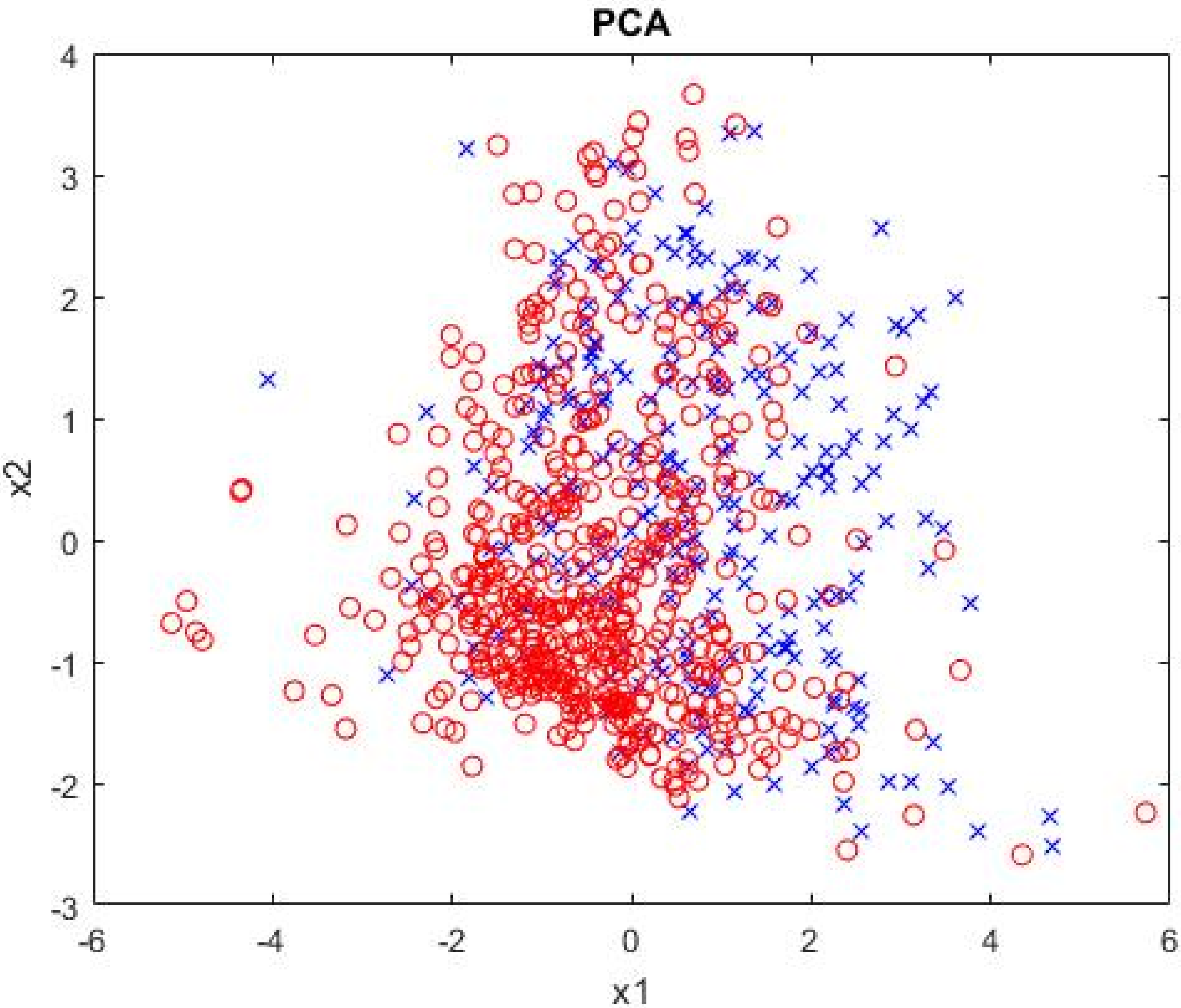
Therefore,

$$\begin{bmatrix} -2 & 5 \\ 1 & 0 \\ 5 & -1 \\ -1 & -3 \\ 6 & 1 \end{bmatrix} \times \begin{bmatrix} 0.9988 \\ 0.0486 \end{bmatrix} = \begin{bmatrix} -0.66527 \\ 0.3326 \\ 1.6631 \\ -0.3326 \\ 1.9958 \end{bmatrix}$$

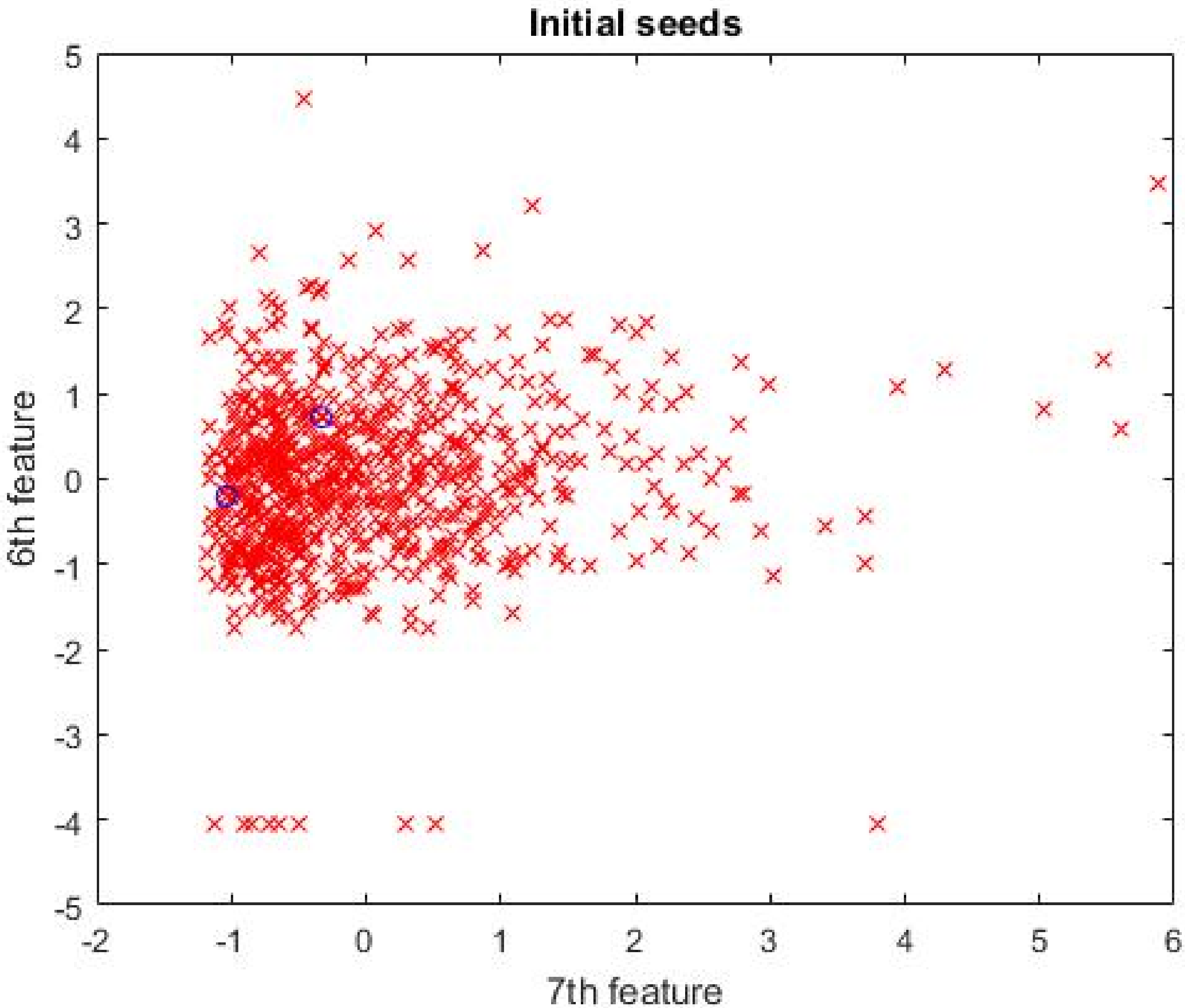
Answer-3 E

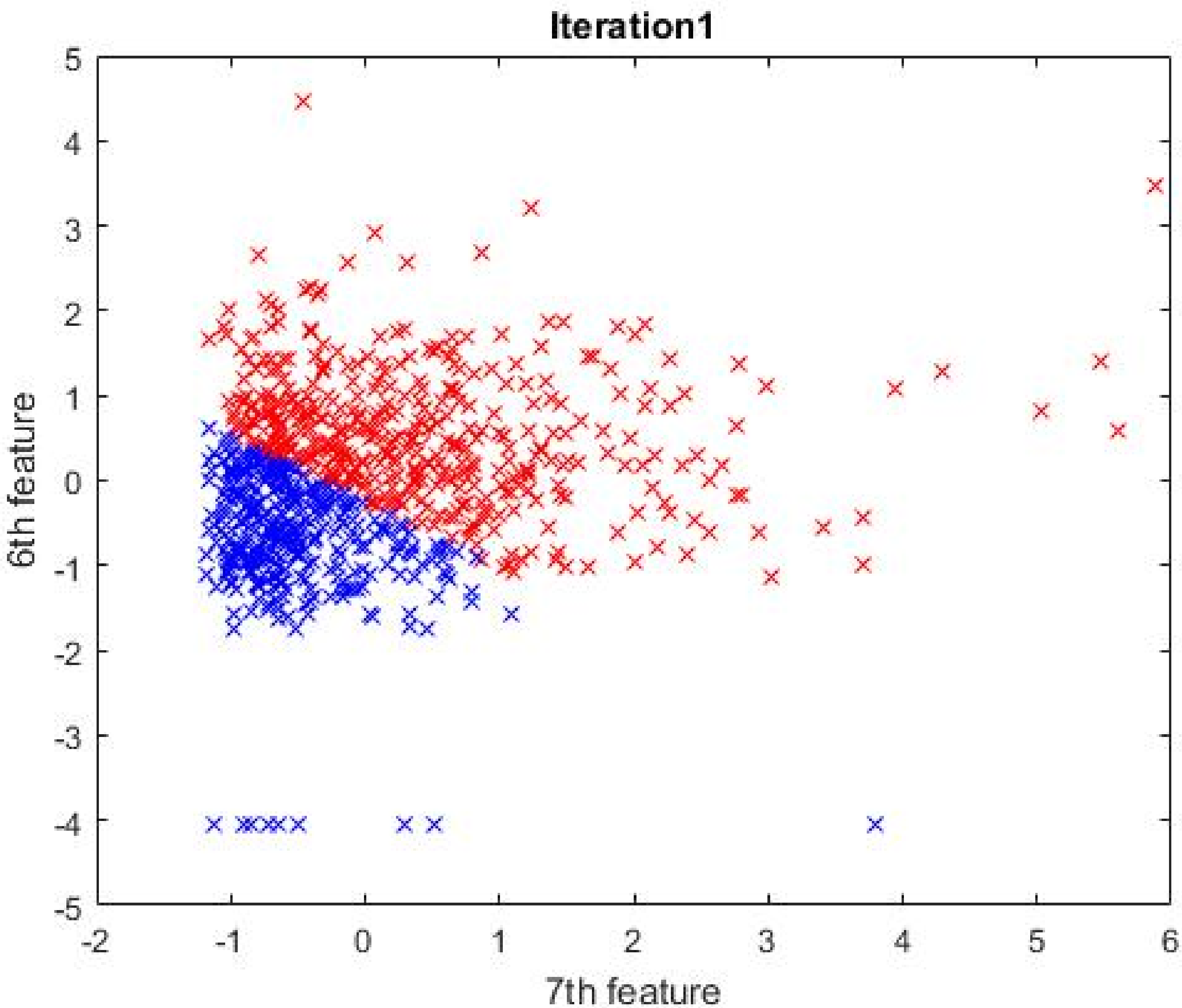
If we consider that we are projecting the points from 2 dimensions to 1 dimension then we can say that the projection from the previous part seems to provide some decent class separation. In Class 1, 3 out of the 5 points are on the negative scale of the axis and in Class 2, 3 out of the 5 points are on the positive scale of the axis. However, one of the points (-0.66527) is directly coinciding and 1 point (-0.3326) in Class 2 crosses and seems to be as a part of class 1. Therefore, I would say that the nature of the data cannot be clearly separable in a 1D space and we probably could do better when we consider a higher dimensional space.

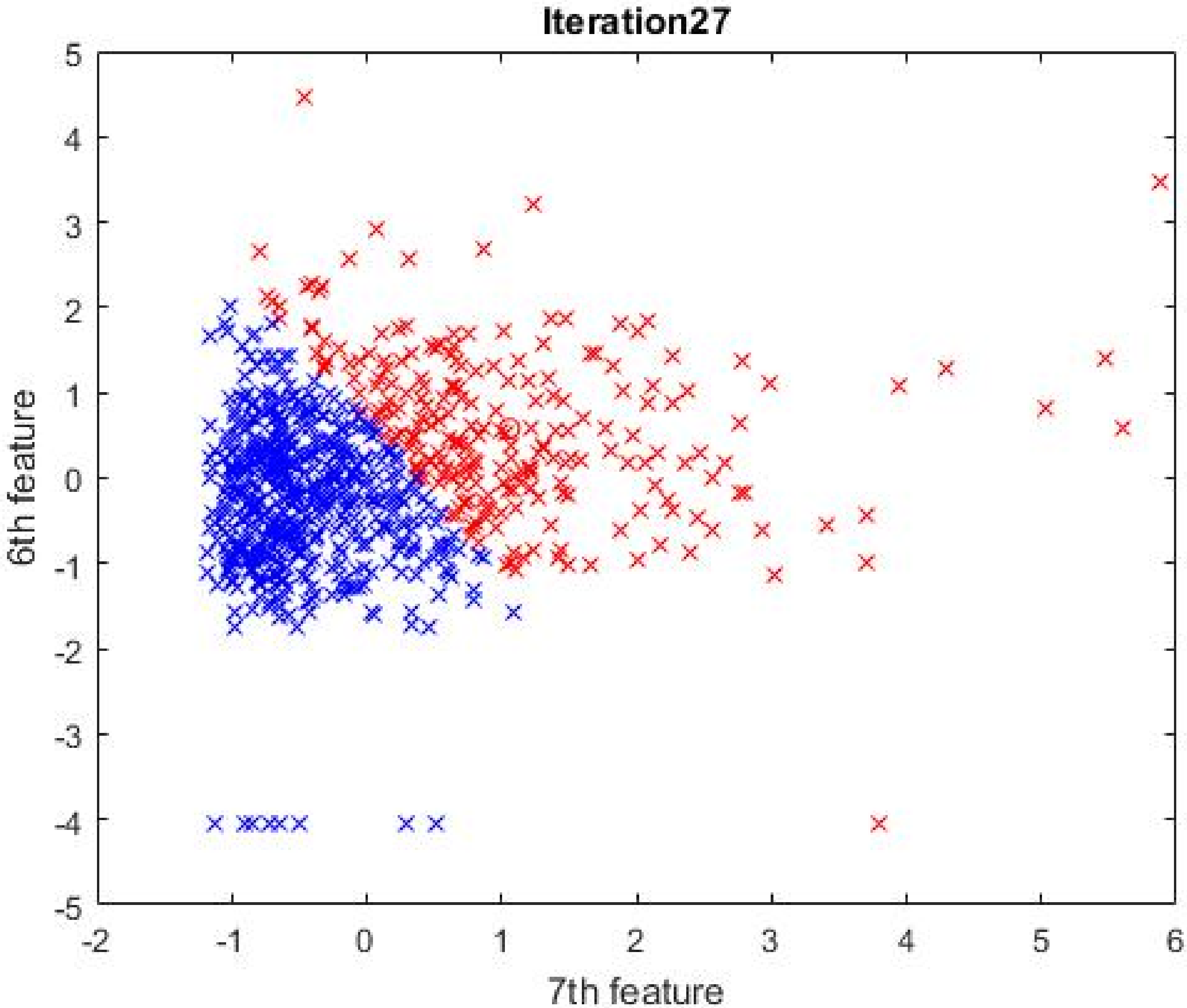
PART-2



PART-3 - It took 27 iterations for the algorithm to terminate




Iteration1
6th feature
7th feature
5
4
3
2
1
0
-1
-2
-3
-4
-5
-2
-1
0
1
2
3
4
5
6


Iteration27
6th feature
7th feature
5
4
3
2
1
0
-1
-2
-3
-4
-5
-2
-1
0
1
2
3
4
5
6